Are There Common Values in BC Timber Sales? A Tail-Index Nonparametric Test*

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Abstract

We develop a new nonparametric test of common values in first-price auctions and apply it to British Columbia (BC) Timber Sales. The test is based on the behavior of the CDF of bids near the reserve price. We show that the curvature of the CDF is drastically different under private values (PV) and common values (CV). We then show that the problem of discriminating between PV and CV is equivalent to estimating the lower tail index of the bid distribution. Our approach allows for unobserved auction heterogeneity of an arbitrary form, and in particular doesn't require the number of potential bidders to be observable. Drawing on the existing and recent literature on tail index estimation, we characterize the B. Hill (1975) tail index estimator for panels with stochastic dimension and a new semi-parametric estimator of the asymptotic variance for robust inference. For BC Timber Sales, we find overwhelming support for CV.

Keywords: first-price auctions, common values, private values, tail index, timber auctions

1 Introduction

The province of British Columbia (BC) in Canada is in legal possession of a massive forested area, and the sale of timber is one of its major sources of revenue. In 2003, partly in response to pressure from the US to create a more competitive market for timber in Canada or face a high import duty, BC initiated a major reform of its timber industry. In particular, the US proposal had called for the establishment of an auction mechanism as the main instrument of timber pricing.[1]

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[1]The proposal draft is available at `http://www.for.gov.bc.ca/het/softwood/softwood_lumber_framework.pdf`, accessed on 28 April, 2009.

Prior to 2003, the Ministry of Forests[2] sold some timber rights through auctions under the Small Business Forest Enterprise Program (SBFEP) first studied in Paarsch (1997). In 2003, this program was transformed into BC Timber Sales (BCTS), an independent organization that aims to generate the best possible financial return from publicly-owned timber, provide timber harvesting opportunities, and set a credible reference point for the price of harvested timber. An additional 10 percent of the annual cut was being gradually transferred from long-term lease holders, large forestry firms that still harvest about 67 percent of the cut, to be auctioned through BCTS (Niquidet (2008)). The auction prices now serve as the basis for a market pricing system (MPS) that determines stumpage rates for long-term lease holders.

The BCTS auctions are sealed-bid, first-price. But a very important practical question is whether this is the best auction format to use. Milgrom and Weber (1982) have shown that with common values, an open auction is revenue superior to the first-price, sealed-bid auction.[3] Other policy recommendations, e.g. how the reserve prices should be set, how many bidders to invite, what entry fee to impose etc. differ depending on whether the values are private or common. In addition, as Laffont and Vuong (1996) have shown, models with common values are often nonparametrically non-identified , while private value models are often identified.

Timber auctions have attracted significant amount of attention in the economics literature, most of which focusses on the US. But the question of which model, private values (PV) or common values (CV), is more appropriate, hasn't been fully resolved.[4] Baldwin (1995) and Athey and Levin (2001) argue for the presence of common values, while Baldwin, Marshall, and Richard (1997), Haile (2001), and Haile and Tamer (2003) adopted a PV paradigm.

In BCTS, a concurrent event, the Mountain Pine Beetle epidemic, may have greatly increased uncertainty about the quality of the timber, so common values might have been present at least to some extent over the period of our study. We develop a "reduced-form" nonparametric test of common versus private values that utilizes the main feature of BCTS: binding reserve prices. In BCTS, bidders submit bonus bids, equal to the amounts over and above the reserve price. The highest bidder pays the bonus bid plus the reserve price. No negative bonus bids are allowed, which makes the reserve price strictly binding. Other relevant features of BCTS, discussed in more length in the empirical section of the paper, include likely bidder symmetry: the bidders primarily are small logging firms, and competition is highly localized.

Our approach is structural in the sense that it is based on auction theory, but at the same time is "reduced-form" in the sense that it tests the prediction of the theory directly. We show that under PV, there must be excessive clustering of bids around the reserve price, relative to the degree of clustering under CV. A very rough intuition for this difference is as follows. Let r be the reserve price. Suppose bidders receive signals S_i that are unbiased estimates of the value of the object. First, let's look at what happens in a second-price auction. Under PV, it is well known that only the bidders with signals $S_i \geq r$ will

[2] Now known as the Ministry of Forests and Range.

[3] There is also empirical evidence to the importance of this effect, e.g. in Shneyerov (2006).

[4] When we say CV, we mean a general model with interdependent values, not necessarily pure common values.

participate, and they will bid their signals, $B_i = S_i$. Under CV, as Milgrom and Weber (1982) have shown, only bidders with signals above a certain cutoff s^*, where $s^* > r$, will participate The reason is the potential winner's curse. The bidders with signals just above the reserve can only win if their rivals do not participate, which is bad news. A more subtle fact is that these marginal bidders will bid strictly more than their values conditional on winning. In a second-price auction, the bid only affects the value to the extent it affects the price. The price however is only affected on the margin, when the rival highest bid Y_i "crosses" bid B_i. Because higher Y_i indicates higher ex-post object value, this event of "bid crossing", or winning on the margin, is not as bad news as simply winning the auction. In the latter event, the highest rival bid $Y_i \leq B_i$, while in the former, $Y_i = B_i$. This means that under CV, there will be a *gap* between the reserve price and the lowest bid.[5]

Things are a bit different in a first-price auction (our case), where bidders shade their bids relative to the "counterfactual" bids they would submit in the second-price auction. Under PV, bidders with signals slightly above the reserve price cannot shade too much - the reserve price anchors their bids. This leads to excessive bid clustering around the reserve. Under CV, there is much less clustering because the "counterfactual" bids of the marginal bidders are well above the reserve price.

Our innovation is to use the tail index κ of a distribution of bids as a measure of clustering around the reserve price. The tail index measures the curvature of the CDF in the tail, and is a popular approach to modelling extreme quantiles in economics and finance (see e.g. Chernozhukov and Du (2007) for a formal definition of the tail index and a review of applications). To our knowledge, our paper is the first to apply tail index methodology to a fundamental problem in industrial organization. We show that κ must be 1 under CV, and 1/2 under PV.[6] The test is based on nonparametric tail index estimation and inference (Hill (1975), Hsing (1993) and more recently Hill (2009b)). A version of Hill's (1975) estimator $\hat{\kappa}$ is used to estimate the tail index and perform tests. The estimator is remarkably easy to implement, and the asymptotically most powerful tail index-based test of CV versus PV is simply a one-sided t-test . Our testing approach works even when there is unobserved auction heterogeneity, and the number of potential bidders is unobservable, since the tail index is preserved when the distribution is aggregated along any dimension.[7]

The approach in this paper is inspired by Hendricks, Pinkse, and Porter (2003) (a more detailed discussion appears in Hendricks and Porter (2007)) who also noted that the behavior of bids around the reserve price is different under PV and CV. Specifically, the lower bound of the support of pseudo-values (see Guerre, Perrigne, and Vuong (2000), GPV hereafter, for the definition, and also see Athey and Haile (2002)) is equal to r under PV but is strictly greater than r under CV. Hendricks, Pinkse, and Porter (2003) did not propose a statistical test based on this restriction. Our contribution is to propose such a test. Moreover, our testing approach does not require nonparametric estimation of pseudo-valuations and results in simple null and alternative hypotheses. The power of the test is such that it allows us to give a definite answer for BCTS.

[5]This is also observed in Milgrom and Weber (1982) in footnote 26 on page 1111.

[6]Thus the distribution of bids has a *Paretian tail* under PV (see Chernozhukov and Du (2007) for a definition). This evidently marks a rare case of naturally occurring Paretian tails (cf. **?**).

[7]Krasnokutskaya (2003) argues for the importance of accounting for the unobserved heterogeneity in the estimation of auction markups. See also the discussion in Paarsch, Hong, and Haley (2006).

An early approach to testing for common values was to check if bids increases monotonically with the number of potential bidders; a non-monotonic pattern was believed to provide evidence for common values. This approach was initiated by Gilley and Karels (1981), and applied to second-price sealed-bid and English auctions by Paarsch (1991) and Bajari and Hortacsu (2004). However, Pinkse and Tan (2005) have shown that in first-price auctions, this pattern can also arise if values are private and affiliated.

The first paper that adopted a structural approach to this problem is Paarsch (1992), where a parametric testing method is developed and applied to the auctions of tree planting contracts in BC. The recent literature has focussed on nonparametric approaches. Haile, Hong, and Shum (2003) propose a nonparametric test of PV versus CV. Their approach is entirely different from ours and is based on the variation in the number of bidders across auctions. They implement their test on a sample of US Forest Service (USFS) timber auctions and obtain mixed results. Haile, Hong and Shum's approach does not require a binding reserve price, but requires the number of potential bidders to be observable.[8] This is not the case in our application, precisely because the reserve price is binding.

Recently, Hortacsu and Kastl (2008) proposed a test of common values when some bidders have information about rivals' bids, and applied it to Canadian Treasury Bill auctions.[9] Their approach is tailored to the environment of Canadian Treasury Bill auctions and is also entirely different from ours.

As well as applying the tail index methodology to an important problem in empirical auctions, our paper also makes a number of econometric contributions, by extending the Hill (1975) tail index estimator to imbalanced panels where bids are nonlinearly dependent within auctions of random size. Thus, the sample size is itself a random variable correlated with the bids in an unknown way. The literature is silent concerning extremal statistics with stochastic sample size, and there are only a few applications of tail index estimation for panel data (e.g. Mikosch and C. de Vreis (2006); Jongen, Verschoor, Wolff, and Zwinkels (2006)). By exploiting theory developed in Hsing (1991), Hill (2009a), and Hill (2009b), the celebrated Hill-estimator is shown to be asymptotically normal where the stochastic nature of bid counts is irrelevant.

Finally, a test of PV against CV requires robust estimators of the asymptotic variance of the tail index estimator. We propose a new consistent semi-parametric estimator of the asymptotic variance designed for auction data, and compare it to Hill (2009b)'s nonparametric estimator. In Monte-Carlo simulations, the nonparametric variance estimator strongly dominates the semi-parametric estimator under a null of PV, and both lead to sharp inference under the alternative of CV.

We implement our test on a BCTS dataset that contains all auctions conducted from January 14, 2004 to December 14, 2006. This period corresponds to an outbreak of the Mountain Pine Beetle, a factor that affected the quality of timber in the province. This epidemics was unexpected, and logging firms in BCTS likely faced elevated uncertainty about the quality of the timber over the period covered in our dataset. Common value factors may have played an increasingly important role over that period. Our test strongly

[8]In USFS auctions, the reserve price is typically non-binding and the number of potential bidders is observable. See Baldwin, Marshall, and Richard (1997), Haile (2001) and Haile and Tamer (2003).

[9]In Canadian Treasury bill auctions, bidders naturally fall into two groups - dealers and customers, and the former have an informational advantage over the latter.

rejects private values in favor of a model with a common value component.

2 The Model and Testable Restrictions

The model is a slight specialization of the canonical symmetric model of Milgrom and Weber (1982). There are $N \geq 2$ potential loggers that consider bidding in a sealed-bid, first-price auction for a tract of timber. The tract is assumed to have value U_i to logger i, and this value may not be fully known at the time of the auction. Prior to the auction, loggers cruise the tract area and collect information about the timber, such as its quality, the composition of the species, etc. Also, they may have some information about the market value of the logs, as well as about their own harvesting costs. All this information is summarized by a scalar signal S_i. Loggers may have common as well as private components in their valuations. The common component is V. The valuation of the bidder is $U_i = u(V, S_i)$, where u is a nonnegative, continuous and nondecreasing function.[10] As in Milgrom and Weber (1982), we assume that the vector $(V, S_1, ..., S_N)$ is drawn from some joint distribution F with density f that satisfies the affiliation property.[11] We assume that the support of F is $[\underline{v}, \bar{v}] \times [\underline{s}, \bar{s}]^N$, where $\underline{v} < \bar{v}$, $\underline{s} < \bar{s}$, that F has continuous partial derivatives of all orders on the interior of its support, i.e. $F \in C^\infty([\underline{v}, \bar{v}] \times [\underline{s}, \bar{s}]^N)$, and that the density f is positive everywhere on the support. The model is symmetric: the function u is the same for all bidders, and the distribution F is symmetric in bidders' signals.

If $u(V, S_i)$ does not depend on V, we have a PV model (an affiliated private values model, or APV, if the signals are strictly affiliated). Otherwise we have a CV model. Let $Y_i = \max_{j \neq i} S_j$. Milgrom and Weber (1982) have shown that in a symmetric equilibrium, the bidding strategy $B(s)$ satisfies the differential equation

$$(v(s) - B(s)) f_{Y_1|S_1}(s|s) - B'(s) F_{Y_1|S_1}(s|s) = 0, \tag{1}$$

where

$$v(s) \equiv E\{U_i | S_i = s, Y_i = s\},$$

the value of the object conditional on "just" winning the auction with bid $B(s)$. We assume that $v(s)$ is a differentiable function, and that its derivative is positive, $v'(s) > 0$ for all $s \in [\underline{s}, \bar{s}]$.

This differential equation has a unique solution subject to the boundary condition $B(s^*) = r$. Only the bidders with signals $S_i \geq s^*$ can win the auction, and we assume that only they bid. In general the *screening level* $s^* \in [\underline{s}, \bar{s}]$, but we assume that the reserve price is *binding*: $s^* \in (\underline{s}, \bar{s})$. In a symmetric equilibrium, a bidder with signal $S_i = s^*$ can only win if his potential rivals all draw signals below the screening level, $S_j \leq s^*$. His value conditional on winning is $w(s^*)$, where

$$w(s) \equiv E\{U_i | S_i = s, Y_i < s\}.$$

[10] Milgrom and Weber (1982) allow more generally $U_i = u(V, S_i, \{S_j\}_{j \neq i})$, i.e. allow the valuation of a given bidder to depend on rivals" signals directly, not only through the common component V. Nothing would change if we adopted this more general specification. We decided to stick to a simpler specification because it is easy to interpret empirically in our application.

[11] I.e., $f(\min\{x, y\}) f(\max\{x, y\}) \geq f(x) f(y)$ .See Milgrom and Weber (1982).

By definition, this bidder is a marginal bidder, i.e. is indifferent between entering or not, and makes zero expected profit. The marginal bidder bids r, and the zero expected profit condition together with the fact that the reserve price is binding implies that s^* is determined implicitly from the equation

$$w\left(s^*\right) = r.$$

As was first noted in Hendricks, Pinkse and Porter (2003) in their study of wildcat sales in Outer Continental Shelf (OCS) auctions, there is a difference in bidders' behavior around the reserve price under private and common values. Under PV, there is no scope for the winner's curse, and therefore

$$r = w\left(s^*\right) = v\left(s^*\right),$$

which implies $B'\left(s^*\right) = 0$. Under CV, on the other hand,

$$r = w\left(s^*\right) < v\left(s^*\right),$$

and the slope is positive, $B'\left(s^*\right) > 0$.

The difference in the slope $B'\left(s^*\right)$ under PV and CV leads to a difference in the curvature of the distribution of bids around $b = r$. Our testing approach is based on a measure of this curvature, formally captured in the notion of the tail index. It is convenient to normalize the bids

$$B^* \equiv \frac{B}{r} - 1.$$

Let $G^*\left(b\right)$ be the equilibrium distribution of the normalized bids,

$$G^*\left(b\right) \equiv P\{B^*\left(S_i\right) \leq b | S_i \geq s^*\}.$$

The lower bound of the support of G^* is 0. The (left) *tail index* of this distribution is defined as $\kappa > 0$ such that

$$G^*\left(b\right) = c \cdot b^\kappa \cdot \left(1 + o\left(1\right)\right) \quad \text{as } b \downarrow 0,$$

where $c > 0$.[12]

In the proposition below, we derive the tail indexes of $G^*(b)$ under CV and PV, and even stronger, characterize the decay scale of $G^*\left(b\right)$ as $b \downarrow 0$.

Proposition 1 *Under CV, $G^*\left(b\right) = c \cdot b^\kappa \cdot \left(1 + O\left(b^\kappa\right)\right)$ as $b \downarrow 0$ with $c > 0$ and $\kappa = 1$. Under PV, $G^*\left(b\right) = c \cdot b^\kappa \cdot \left(1 + O\left(b^\kappa\right)\right)$ as $b \downarrow 0$ with $c > 0$ and $\kappa = 1/2$.*

Proof. In this proof we smoothly extend the bidding strategy B^* and the distribution function $F_{S_i|S_i \geq s^*}$ from the domain $[s^*, \bar{s}]$ to an open domain D_ε that includes s^* as an interior point, $D_\varepsilon = (s^* - \varepsilon, s^* + \varepsilon)$ where $\varepsilon > 0$ is sufficiently small. Under CV, since $B^{*\prime}\left(s^*\right) > 0$, the Inverse Function Theorem implies that for a small enough $\varepsilon > 0$, B^* is a diffeomorphism, so that the inverse bidding strategy B^{*-1} is also smooth (on $B^*\left(D_\varepsilon\right)$).[13]

[12] See Chernozhukov and Du (2007) for a discussion of tail indexes and their applications in economics and finance.

[13] A smooth map $f : D \to Y \subset \mathbb{R}$ defined on some open domain $D \subset \mathbb{R}^m$ is called a *diffeomorphism* if it is one to one and onto, and the inverse map f^{-1} is smooth (Guillemin and Pollack (1974), page 3).

Then $F_{S_i|S_i \geq s^*} \circ B^{*-1}$ is smooth on D_ε as a composition of two smooth functions, and in particular is twice continuously differentiable. Therefore

$$\begin{aligned} P\left\{B^*\left(S_i\right) \leq b | S_i \geq s^*\right\} &= F_{S_i|S_i \geq s^*}(B^{*-1}(b)) \\ &= cb\left(1 + O\left(b\right)\right) \end{aligned}$$

as $b \downarrow 0$, where $c > 0$. This proves the tail index representation under CV.

Under PV, B^* has a critical point as $s = s^*$, $B^{*\prime}\left(s^*\right) = 0$, and the Inverse Function Theorem doesn't apply. However, the critical point is non-degenerate. Re-writing the differentiating equation (1) in terms of B^* and differentiating with respect to s at $s = s^*$ gives

$$\left(v'\left(s^*\right) - rB^{*\prime}\left(s^*\right)\right) f_{Y_1|S_1}\left(s^*|s^*\right) - rB^{*\prime\prime}\left(s^*\right) F_{Y_1|S_1}\left(s^*|s^*\right) = 0,$$

which implies

$$B''\left(s_*\right) = \frac{f_{Y_1|S_1}\left(s^*|s^*\right) v'\left(s^*\right)}{rF_{Y_1|S_1}\left(s^*|s^*\right)} > 0.$$

The *Morse Lemma* (Guillemin and Pollack (1974), p. 42) states that if a smooth function $f : D \to R$, defined on an open subset $D \subset \mathbb{R}^m$, has a *non-degenerate* critical point $a \in D$, i.e. the Hessian matrix (h_{ij}) of f at a is non-singular, then there exists an open neighborhood D_0 of a ($D_0 \subset D$), and diffeomorphisms $y_i : D_0 \to R$ such that $\forall x \in D_0$, $f\left(x\right) = f\left(a\right) + \sum_{i=1}^m \sum_{j=1}^m h_{ij} y_i\left(x\right) y_j\left(x\right)$. Our case is single-dimensional, and this lemma specializes to $f\left(x\right) = f\left(a\right) + f''\left(a\right) y\left(x\right)^2$ for some diffeomorphism $y : D_0 \to R$. If $f''\left(a\right) > 0$, y can be chosen as $+(f\left(x\right) - f\left(a\right))^{\frac{1}{2}}$ if $x \geq a$ and $-(f\left(x\right) - f\left(a\right))^{\frac{1}{2}}$ if $x < a$, and being a diffeomorphism, y has a smooth inverse.

Here the Morse Lemma implies that $\beta : D_\varepsilon \to R$ defined as

$$\beta\left(s\right) = \begin{cases} +B^*\left(s\right)^{\frac{1}{2}}, & s \geq s^* \\ -B^*\left(s\right)^{\frac{1}{2}}, & s < s^* \end{cases},$$

is a diffeomorphism on D_ε. Since

$$\begin{aligned} P\left\{B^*\left(S_i\right) \leq b | S_i \geq s^*\right\} &= P\{B^*\left(S_i\right)^{\frac{1}{2}} \leq b^{\frac{1}{2}} | S_i \geq s^*\} \\ &= F_{S_i|S_i \geq s^*}(\beta^{-1}(b^{\frac{1}{2}})), \end{aligned}$$

and $F_{S_i|S_i \geq s^*} \circ \beta^{-1}$ is smooth as a composition of two smooth functions, we conclude that under PV,

$$P\left\{B^*\left(S_i\right) \leq b | S_i \geq s^*\right\} = cb^{\frac{1}{2}}(1 + O(b^{\frac{1}{2}})),$$

as $b \downarrow 0$ for some $c > 0$. *Q.E.D.*

2.1 Discussion

Working directly with the first-order Bayesian-Nash equilibrium conditions of the bidding game as in GPV, Hendricks, Pinkse and Porter (2003) have shown that in a PV environment, the "markup" (the difference between the valuation and the bid) is 0 in the limit as $b \downarrow r$,

$$\lim_{b \downarrow r} \frac{G_{Y|B}(b|b)}{g_{Y|B}(b|b)} = 0, \tag{2}$$

while in a CV environment

$$\lim_{b \downarrow r} \frac{G_{Y|B}(b|b)}{g_{Y|B}(b|b)} > 0. \tag{3}$$

where $G_{Y|B}(y|b)$ is the distribution of the maximum rival's bid Y_i conditional on own bid $B_i = b$. They mention that these conditions are potentially testable. However, in OCS auctions, there are relatively few bids around the reserve price to implement such a test. The government often rejects high bids near the reserve price. In addition, in a model with a secret reserve price, (2) is no longer true.

But, if the reserve price is not secret and is strictly binding, as in our application, there may be quite a few bids around the reserve. As a matter of fact, *clustering* of bids around the reserve is observed in BC Timber Sales (see Figure 9 in Section 5) While our computer simulations suggest that such clustering can occur to some extent in both models (see Figure 1 in Section 4), there is much more pronounced, "excessive" clustering under PV. The reason for this is that under PV, the slope of the bidding strategy at s^* is 0, $B'(s^*) = 0$, which means that bidders with signals somewhat over the screening level will bid very close to the reserve price. Indeed, our Proposition 1 implies that $\lim_{b \downarrow r} g(b) = +\infty$. (However, unlike in Wilson's Drainage Tract Model (Wilson (1969)), there is no mass point at r.) Under CV, on the other hand, $B'(s^*) > 0$, so there is a positive and finite density of bids $g(b)$ at $b = r$.

In principle, the conditions (2) and (3) are testable, but there is a serious practical complication in that the density in the denominator in (2) becomes infinite as $b \downarrow r$. In addition, a practical implementation of such a test would require conditioning on various object characteristics, which is likely to lead to a curse of dimensionality given the sample sizes typically available in applications.

One could also attempt to test if the marginal density of bids is infinite at $b = r$ by using a transformation of bids proposed in GPV: $B^{\dagger}(s) = (B(s) - r)^{1/2}$, which leads to a finite density of $B^{\dagger}(S_i)$ at $b = 0$. The density of $B^{\dagger}(S_i)$ is positive at $b = 0$ under PV, and is zero under CV. One could then attempt to estimate this density nonparametrically. GPV do not consider testing of CV versus PV. Since the PV is often a natural null hypothesis, the power of the test would be weak since the null includes small positive values of the density that are practically indistinguishable from the alternative. Our approach, on the other hand, results in simple hypotheses for both the null ($\kappa = 1/2$) and the alternative ($\kappa = 1$).

Haile, Hong and Shum (2008) propose a test based on pseudo-valuations as in GPV, and on the exogenous variation in the number of bidders in the auction. (If the number of bidders is endogenous,they propose an instrumental variable approach). For a bidder who submitted bid b in the first-price auction, the pseudo-valuation

$$\xi(b; n) \equiv b + \frac{G_{Y|B}(b|b; n)}{g_{Y|B}(b|b; n)} \tag{4}$$

is equal to the corresponding counter-factual bid in the second-price auction, $\tilde{B}(b;n) = E\{U_i|B_i = b, \max_{j=1,...,n;j\neq i} B_j = b; n\}$.[14] Even though it is in general impossible to determine the direction of the effect of competition on bids in a first-price auction even under PV (Pinkse and Tan (2005)), it is possible to do so in a second-price auction under both PV and CV. For a fixed value of b, $\tilde{B}(b;n)$ is constant in n under PV but is decreasing in n under CV. Haile, Hong and Shum use this property as a basis of a nonparametric testing approach. But in order to correctly impute the pseudo-values when there is a binding reserve price, it is necessary to observe the number of potential bidders N. The maximum rival's bids Y is then set equal to either the actual bid (if the rival is active), or to 0 if the rival is not active. In our application, the reserve price is binding, and it is very difficult to obtain a precise measure of potential competition. For that reason, we have not attempted to implement the Haile, Hong and Shum testing approach on our data.

3 Estimation and Testing Framework

3.1 Data Generating Process (DGP)

We assume that a sample of L auctions is available, and index the auctions by $l = 1, .., L$. Each auction is characterized by a reserve price r_l, characteristics x_l and the number of potential bidders N_l. The data generating process is further specified as follows.

1. The vectors (r_l, x_l, N_l) are drawn independently across l from some distribution with support $[\underline{r}, \bar{r}] \times \mathcal{X} \times [\underline{N}, \bar{N}]$ ($\mathcal{X} \subset \mathbb{R}^d$ and compact, $2 \leq \underline{N} < \bar{N} < \infty$).

2. Conditional on (r_l, x_l, N_l) $(l = 1, ..., L)$, the signals $S_{i,l}$ of potential bidders $i = 1, ..., N_l$ and the common value components V_l are drawn independently across l from distributions with conditional densities $f(v, s_1, ..., s_N|r, x)$ symmetric in the s_i arguments, with support $[\underline{v}, \bar{v}] \times [\underline{s}, \bar{s}]^N$. The conditional density $f(v, s_1, ..., s_N|r, x)$ is smooth in $(v, s_1, ..., s_N)$ for all (r, x) on the support.

3. Only the bidders whose signals exceed the screening level, $S_{i,l} \geq s^*(r_l, x_l, N_l)$, submit bids. It is assumed that the reserve prices are always binding, i.e. $s^*(r_l, x_l, N_l) \in (\underline{s}, \bar{s})$. Let n_l be the number of active bidders in auction l. We index the active bidders as $i = 1, ..., n_l$. The total number of observations in the sample is $n = \sum_l n_l$. Our assumption $\bar{N} < \infty$ implies that $0 \leq n_l \leq \bar{n} \equiv \bar{N}$. The bids of active bidders are determined according to

$$b_{i,l} = B(S_{i,l}, r_l, x_l, N_l), \quad (i = 1, ..., n_l),$$

   where the bidding strategy B is found as the solution to (1).

The data available to the econometrician consists of an (independent and identically distributed across l) sample of observations

$$\left\{ \{b_{i,l}\}_{i=1,..,n_l}, r_l, x_l, n_l \right\}_{l=1,...,L}.$$

[14]This observation is due to Shneyerov (2006).

Consider the distribution of normalized bids

$$b_{i,l}^* = \frac{b_{i,l}}{r_l} - 1. \tag{5}$$

Working with this distribution has the advantage that the lower bound of the support of $b_{i,l}^*$ is 0 regardless of the covariates, and the tail behavior can be characterized around $b = 0$. We assume that the environment (PV or CV) is fixed, so that the conditional distribution of bids has the tail representation

$$G^* (b|r_l, x_l, N_l) = c (r_l, x_l, N_l) \cdot b^\kappa \cdot (1 + O (b^\kappa)) \quad \text{as } b \downarrow 0. \tag{6}$$

where $c (r_l, x_l, N_l) > 0$ on $[\underline{r}, \bar{r}] \times \mathcal{X} \times [\underline{N}, \bar{N}]$-*a.e.* and the tail index κ is independent of (r_l, x_l, N_l), and equal to 1 under PV and to 1/2 under CV by Proposition 1.

Remark 2 *We assume that N_l is unobservable, and in addition, certain components of x_l may also be unobservable. However, taking expected values in* (6) *conditional on any observable vector z_l, we still obtain the tail representation*

$$G^* (b|z_l) = E \{c (r_l, x_l, N_l) |z_l\} \cdot b^\kappa \cdot (1 + O (b^\kappa)) \quad \text{as } b \downarrow 0. \tag{7}$$

where $E \{c (r_l, x_l, N_l) |z_l\} > 0$. This means that under our assumptions, the tail index representation is preserved conditional on any observable vector z_l, and even unconditionally*.*

3.2 The Hill-Estimator

In view of Remark 2, in this paper we consider an unconditional version of the estimator. The Hill-estimator of the tail index is based on the following alternative representation:

$$\kappa^{-1} = E \left\{ \ln q - \ln b_{i,l}^* | \ln b_{i,l}^* \leq \ln q \right\} + O (q^\kappa) \text{ as } q \downarrow 0. \tag{8}$$

That is, κ^{-1} is the mean distance of the log-normalized bid $\ln b_{i,l}^*$ below some low threshold $\ln q$, as $q \downarrow 0$. Presentation (8) follows from properties of regularly varying functions:

$$\begin{aligned} E \left\{ \ln q - \ln b_{i,l}^* | \ln b_{i,l}^* \leq \ln q \right\} &= \ln q - \frac{\int_{-\infty}^{q} (\ln x) \, dG^* (x)}{G^* (q)} \\ &= \frac{\int_{-\infty}^{\ln q} G^* (e^x) \, dx}{G^* (q)} \\ &= \frac{\int_{-\infty}^{\ln q} c e^{\kappa x} dx \cdot (1 + O(q^\kappa))}{c q^\kappa \cdot (1 + O(q^\kappa))} \\ &= \kappa^{-1} + O(q^\kappa), \end{aligned}$$

where the second line follows from integration by parts, and the third line follows from Proposition 1, by substituting $G^*(e^x) = ce^{\kappa x}(1 + O(q^\kappa))$ for $G^*(q) = cq^\kappa \cdot (1 + O(q^\kappa))$.

Equation (8) suggests a natural way of estimating the (inverse of) the tail index κ^{-1} by a sample analogue, with an appropriately chosen sequence of $q \downarrow 0$ as the sample size goes to infinity.[15] Namely, let $\{b_t^*\}_{t=1}^n$ be the sample stacking all bids $b_{i,l}^*$, let $b_{(j)}^*$ be the j^{th} sample order statistic of b_t^*, $b_{(1)}^* \leq b_{(2)}^* \leq \cdots \leq b_{(n)}^*$, and let $\{m_n\}$ be an intermediate order sequence: $1 \leq m_n < n$, $m_n \to \infty$ and $m_n/n \to 0$ as $L \to \infty$ (e.g. Leadbetter, Lindgren, and Rootzen (1983)). Then the sequence of q is chosen as $b_{(m_n+1)}^*$, and the Hill (1975) estimator of κ^{-1} is simply the sample average[16]

$$\hat{\kappa}_{m_n}^{-1} = \frac{1}{m_n} \sum_{t=1}^{n} \left(\ln b_{(m_n+1)}^* - \ln b_t^* \right)_+ . \tag{9}$$

Since auction sizes $\{n_l\}_{l=1}^L$ are random variables, trivially n and therefore the number of bids near the reserve price m_n are also random.

In the i.i.d case with deterministic sample size n Hall (1982) shows $\hat{\kappa}_{m_n}^{-1}$ is consistent for κ^{-1} and asymptotically normal with the asymptotic variance equal to κ^{-2}. However, our setting is not i.i.d. because bids may be correlated within an auction. Even though the literature contains several result for the non i.i.d. case (e.g. Resnick and Starica (1998), Hsing (1991), Hill (2009b)), these results are for time-series data, and they do not transfer immediately to our setting of an unbalanced panel with stochastic auction sizes.

In Appendix A we establish asymptotic normality of $\hat{\kappa}_{m_n}^{-1}$ for bids governed by our auction data generating process. If the number of order statistics m_n used in the Hill-estimator grows to infinity with the sample size, but not too fast, $m_n \to \infty$ with probability one and $m_n = o_p(n^{2/3})$, then (Theorem 5)

$$\frac{m_n^{1/2}}{v_{m_n}} \left(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1} \right) \xrightarrow{d} N(0,1),$$

where $v_{m_n}^2$ is the mean-squared error of $\hat{\kappa}_{m_n}^{-1}$:

$$v_{m_n}^2 = E\left[m_n \left(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1} \right)^2 \right].$$

In view of the bid tail decay characterized in Proposition 1 and $m_n = o_p(n^{2/3})$, asymptotically the bias of $\hat{\kappa}_{m_n}^{-1}$ is negligible and the mean-squared error approximates the variance of $\hat{\kappa}_{m_n}^{-1}$.

Remark 3 *Even though the unconditional Hill estimator of κ is consistent, an estimator obtained by first conditioning on observable auction characteristics, and then averaging the conditional estimators, could be more efficient. However, the normalization of bids* (5) *is likely at least to some extent remove the effect of covariates on bids.*[17] *For example, if we assume a multiplicative structure $V_l = a(x_l)\tilde{V}_l$, $S_{i,l} = a(x_l) S_{i,l}$ and $r_l = a(x_l)\tilde{r}_l$, where $(\tilde{V}_l, S_{i,l}, \tilde{r}_l)$ and x_l are independent, then $b_{i,l} = a(x_l)\tilde{b}_{i,l}$, where $\tilde{b}_{i,l}$ and x_l are also independent.*[18] *This implies that $b_{i,l}^* = (\tilde{b}_{i,l}/r_l) - 1$ and x_l are likewise independent, and*

[15] In the following we write $(z)_+$ to denote $\max\{z, 0\}$.

[16] Other estimators exist but none have been shown to be as robust to unknown forms of dependence and heterogeneity. See the literature reviews in Hill (2009b) and Hill (2009a).

[17] See the recent paper by Roberts (2008) that proposes a general argument.

[18] A variant of this specification was also considered in Krasnokutskaya (2003).

conditioning on x_l is not necessary. However, even in this specification, the dependence between $b^*_{i,l}$ and n_l generally remains, so in principle one could benefit from conditioning on n_l. But our Monte-Carlo simulations have shown that in samples of typical size, even though the reduction in the variance of the estimator can be substantial, it is overwhelmed by the small sample bias.[19]

3.3 Variance Estimation

An important practical question is how to estimate $v^2_{m_n}$ in the presence of dependent bids in auctions of random size. We propose a new semi-parametric estimator $\hat{v}^2_{m_n}$, and compare it to Hill's (2009) nonparametric kernel estimator $\hat{\sigma}^2_{m_n}$.

Assume for the sake of exposition auction sizes n_l are deterministic.[20] The nonparametric estimator $\hat{\sigma}^2_{m_n}$ follows from a trivial expansion of the mean-squared-error $v^2_{m_n}$:

$$v^2_{m_n} = m_n \times E\left(\frac{1}{m_n}\sum_{t=1}^{n}\left\{\left(\ln\left(\frac{b^*_{(m_n+1)}}{b^*_t}\right)\right)_+ - \frac{m_n}{n}\kappa^{-1}\right\}\right)^2 = \frac{1}{m_n}\sum_{s,t=1}^{n} E\left[U_{m_n,s}U_{m_n,t}\right],$$

where we write $U_{m_n,t} := (\ln(b^*_{(m_n+1)}/b^*_t))_+ - (m_n/n)\kappa^{-1}$. Although a natural estimator of $v^2_{m_n}$ appears to be $1/m_n \sum_{s,t=1}^{n} \hat{U}_{m_n,s}\hat{U}_{m_n,t}$ with $\hat{U}_{m_n,t} := (\ln(b^*_{(m_n+1)}/b^*_t))_+ - (m_n/n)\hat{\kappa}^{-1}_{m_n}$, it is not guaranteed to be positive (Newey and West (1986)). Hill (2009b) exploits a now classic approach in the literature that trims the cross-products $\hat{U}_{m_n,s}\hat{U}_{m_n,t}$ (cf. Newey and West (1986), Andrews (1991), de Jong and Davidson (2000)):

$$\hat{\sigma}^2_{m_n} = \frac{1}{m_n}\sum_{s,t=1}^{n} k((s-t)/\gamma_n)\hat{U}_{m_n,s}\hat{U}_{m_n,t}, \tag{10}$$

where k denotes a standard kernel function with bandwidth $\gamma_n \to \infty$. The kernel $k((s-t)/\gamma_n)$ asymptotically negligibly trims cross-products $\hat{U}_{m_n,s}\hat{U}_{m_n,t}$ at large displacements $|s-t|$ so that $\hat{\sigma}^2_{m_n} > 0$ with probability one for all $n \geq 1$ while ostensibly retaining consistency (Newey and West (1986)). As a bonus, a fully nonparametric approach allows the analyst to have only a vague idea about cross auction dependence and heterogeneity.

Since the kernel $k((s-t)/\gamma_n)$ trims $\hat{U}_{m_n,s}\hat{U}_{m_n,t}$ and the bandwidth γ_n gauges the amount of trimming, some care for choosing γ_n must be taken. In general as long as $\gamma_n/n \to 0$ sufficiently fast then $\hat{\sigma}^2_{m_n} > 0$ with probability one and $\hat{\sigma}^2_{m_n}/v^2_{m_n} \xrightarrow{p} 1$ for a large class of kernels, including the popularly used Bartlett kernel $k(z) := (1-|z|)_+$ (Theorem 8).

The promise of $\hat{\sigma}^2_{m_n}$ lies in the fact that it directly approximates linear dependence in $\{U_{m_n,t}\}_{t=1}^{n}$ without any parametric information concerning this dependence. This is non-trivial since every component $U_{m_n,t} := (\ln(b^*_{(m_n+1)}/b^*_t))_+ - (m_n/n)\kappa^{-1}$ contains the order statistic $b^*_{(m_n+1)}$ which is dependent on other bids by the nature of our auction data. In fact, this implies we cannot exploit cross-auction independence to further simplify the above

[19]The simulations are available on request.

[20]We show in Theorem 5 of Appendix A that allowing stochastic auction sizes $\{n_l\}_{l=1}^{L}$ does not alter asymptotic arguments. We may without loss of generality simply treat $\{n_l\}_{l=1}^{L}$, and therefore the total number of bids n and bids near the reserve price m_n, as constants.

expansion of $v_{m_n}^2$. In order to see this, write the $v_{m_n}^2$-expansion in bid-auction notation:

$$v_{m_n}^2 = \frac{1}{m_n} \sum_{l_1,l_2=1}^{L} \sum_{i=1}^{n_{l_1}} \sum_{j=1}^{n_{l_2}} E\left[U_{m_n,i,l_1} U_{m_n,j,l_2}\right],$$

where $U_{m_n,i,l} := (\ln(b^*_{(m_n+1)}/b^*_{i,l}))_+ - (m_n/n)\kappa^{-1}$. Even though bids b^*_{i,l_1} and b^*_{j,l_2} are independent across auctions $l_1 \neq l_2$, the presence of $b^*_{(m_n+1)}$ makes each U_{m_n,i,l_1} nonlinearly dependent on *every* other U_{m_n,j,l_1}. Thus, we cannot say $E[U_{m_n,i,l_1} U_{m_n,j,l_2}] = 0$ for $l_1 \neq l_2$ without further information about bid dependence.

Nevertheless, we can exploit a useful asymptotic approximation of the Hill-estimator (Theorem 5) based on arguments in Hsing (1991) and Hill (2009a) to reduce $v_{m_n}^2$ under cross-auction independence. The resulting estimator of $v_{m_n}^2$ is (cf. Lemma 6 and Theorem 7)

$$\hat{v}_{m_n}^2 = \hat{\kappa}_{m_n}^{-2} + 2\frac{1}{m_n} \sum_{l=1}^{L} \sum_{i=1}^{n_l-1} (n_l - i) \times \hat{c}_{m_n}(i). \tag{11}$$

Since $\hat{\kappa}_{m_n}^{-2}$ is the asymptotic variance estimator for iid data (Hall (1982)), $\hat{v}_{m_n}^2$ includes a correction term due to within-auction bid dependence. In particular, $\hat{c}_{m_n}(i)$ estimates tail dependence between bids $b^*_{j,l}$ and $b^*_{j+i,l}$ for all possible displacements[21] $i \in \{1, ...\bar{n}_l - 1\}$:

$$\hat{c}_{m_n}(i) = \frac{1}{\tilde{L}_i} \sum_{l=1}^{L} \sum_{j=1}^{(n_l-i)_+} \left(\hat{U}_{m_n,j,l} - \hat{\kappa}_{m_n}^{-1} \hat{I}_{m_n,j,l}\right) \times \left(\hat{U}_{m_n,j+i,l} - \hat{\kappa}_{m_n}^{-1} \hat{I}_{m_n,j+i,l}\right). \tag{12}$$

Note we define $\hat{I}_{m_n,i,l} := I(b^*_t < b^*_{(m_n+1)}) - m_n/n$, and $\tilde{L}_i := \sum_{l=1}^{L} (n_l - i)_+$ denotes the total number of bid pairs $\{b^*_{j,l}, b^*_{j+i,l}\}$ that enter into $\hat{c}_{m_n}(i)$. By convention we set $\hat{c}_{m_n}(i) = 0$ if there is no more than one auction with displacement i.[22] See Appendix A for complete details on the derivation of (11), and for a proof of consistency $\hat{v}_{m_n}^2 / v_{m_n}^2 \xrightarrow{p} 1$ (Theorem 7).

A drawback to non-kernel estimators of the type $\hat{v}_{m_n}^2$ is $\hat{v}_{m_n}^2 < 0$ is possible with non-negligible probability for any finite sample, even though $\hat{v}_{m_n}^2 \xrightarrow{p} \lim v_{m_n}^2 > 0$. The kernel estimator by construction has $\hat{\sigma}_{m_n}^2 > 0$ with probability one for any sample.

3.4 Test of PV against CV

Since $m_n^{1/2}(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1})$ is asymptotically normal, an Asymptotically Most Powerful (AMP) test of PV $H_0 : \kappa = 1/2$ against CV $H_1 : \kappa = 1$ is equivalent to a one-sided test of PV against $\kappa > 1/2$. By an extension of the Neyman-Pearson Lemma[23] it is easy to show the

[21] Neighbor bids $b^*_{j,l}$ and $b^*_{j+1,l}$ have the smallest displacement of 1, and the greatest possible bid displacement is $\bar{n}_l - 1$ (first to last bid in the largest auction).

[22] For example, if there is only one auction l with the maximum observed number of bids $\bar{n}$, then there is only one bid pair in the sample $\{b^*_{1,l}, b^*_{\bar{n}-1,l}\}$ with bid displacement $\bar{n} - 1$. In this case we set $\hat{c}_{m_n}(\bar{n} - 1) = 0$.

[23] Cf. Wald (1941) and Karlin and Rubin (1956).

one-sided AMP test of H_0 at nominal significance level $\theta \in [0,1]$ reduces to

$$\lim_{L\to\infty} P\left(m_n^{1/2}\left(\hat{\kappa}_{m_n}^{-1}-2\right)/v_{m_n}^2 \le -Z_\theta/v_{m_n}\right)=\theta$$

where Z_θ is the upper θ^{th}-quantile of a standard normal distribution. Simply compute the t-ratio

$$t_{m_n}=m_n^{1/2}\left(\hat{\kappa}_{m_n}^{-1}-2\right)/\hat{v}$$

for any consistent estimator $\hat{v}^2$ of $v_{m_n}^2$. Under PV ($\kappa = 1/2$) we can easily use Theorem 5 in Appendix A and Cramér's theorem to deduce $t_{m_n} \xrightarrow{d} N(0,1)$. Conversely, under CV ($\kappa = 1$) it follows $|t_{m_n}| \to \infty$ with probability one.

4 A Monte-Carlo Study

4.1 Data Generating Processes

We consider a model where bidders may have a common as well as private value components in their valuations as in Wilson (1998).[24] Suppose the log of bidder i's true valuation u_i is a sum of a common value component v and an idiosyncratic component a_i: $u_i = v + a_i$, where v is normally distributed with mean μ_v and variance σ_v^2, while a_i is normally distributed with mean 0 and variance σ_a^2. Generally, the bidders do not observe their valuations, but observe signals s_i that are informative about the valuations: $s_i = u_i + \varepsilon_i$, where the "noise" term ε_i is also mean zero normally distributed, with variance σ_ε^2. This model nests naturally a private values environment within a common values one. If $\sigma_\varepsilon = 0$, then the environment is PV, and the private values are correlated to the extent that $\sigma_v > 0$. (If also $\sigma_v = 0$, then the environment is independent private values, IPV.) But if $\sigma_\varepsilon > 0$, then the true valuations are unobservable, and we have a model with a common value component.

Figure 1 shows numerically computed bidding strategies $B(s)$ and bid densities $g(b)$ for two examples of the above model.[25] In the first example (the CV example, on the left panel), we set $\sigma_v = \sigma_a = \sigma_\varepsilon = 0.3$. In the second example (the PV example, on the right panel) we set $\sigma_v = \sigma_a = 0.3$ and $\sigma_\varepsilon = 0$. In both examples, $N = 6$, the mean log valuation $\mu_v = \log 100$ and the reserve price is \$80. A barely noticeable, but important difference between the graphs in the top panel is that under PV the bidding strategy has zero slope at $s = s^*$ (the right graph), while it has a positive slope there under CV (the left graph). This behavior of the bidding strategy translates into a profoundly different behavior of the density of bids $g(b)$ around the reserve price, illustrating the power of our Proposition 1 (see the graphs on the lower panel). Under CV, $g(b)$ is continuous around r. But under PV, the density around the reserve price has a "spike". (The fact that the slope of $B(s)$ is zero at $s = s^*$ implies that the density is unbounded.)

The randomly generated PV and CV samples are $\{b_{i,l} : i = 1, ..., n_l\}_{l=1}^{L}$ for $L = 250,000$. The data set is broken into $R = 250$ samples of $L = 1000$ auctions. For each PV and CV

[24] See also Hong and Shum (2002) for an application to highway procurement auctions.

[25] A Mathematica notebook used to compute these examples is available at `http://artyom239.googlepages.com`

sample the Hill-estimator $\hat{\kappa}_{m_m}^{-1}$ and corresponding semi-parameter and nonparametric variance estimators $\hat{v}_{m_n}$ and $\hat{\sigma}_{m_n}^2$ are computed over the tail fractile window $m_n \in \{5, ..., 200\}$ with a Bartlett kernel and bandwidth[26] $\gamma_n = n^{.225}$. As a benchmark we also report the 2.5% and 97.5% quantiles of the simulation sample of $\hat{\kappa}_{m_n}^{-1}$.

4.2 Tail Inference

Let $\hat{v}^2$ denote either $\hat{v}_{m_n}^2$ or $\hat{\sigma}_{m_n}^2$. We plot the 95% asymptotic confidence band using each $\hat{v}^2$, and the 2.5% and 97.5% simulation quantiles. We also plot AMP test rejection frequencies under the one-sided null of PV and the alternative of CV, at the 5% level. See Figures 2-5.

The Hill-estimator uniformly hovers near $\hat{\kappa}_{m_n}^{-1} = 2$ when bids are generated under PV, and near $\hat{\kappa}_{m_n}^{-1} = 1$ under CV, both supporting Proposition 1. Using simulation sample quantiles as a benchmark, the most accurate asymptotic 95% band under CV is derived using the semi-parametric estimator $\hat{v}_{m_n}^2$.

The $\hat{v}_{m_n}$-based t-ratio, however, renders rejection frequencies of the PV null significantly above 5% for all $m_n \geq 25$. The $\hat{\sigma}_{m_n}$-based t-ratio's rejection frequencies, by comparison, hover near 5% for most fractiles m_n. Thus, when using asymptotic Gaussian critical values the kernel estimator results in the best approximation of the 5% nominal test size. The semi-parametric $\hat{v}_{m_n}^2$ leads to substantial over rejection of the null (Figure 3).

In the case of CV data rejection frequencies are above .95 in all cases for each t-test and all tail fractiles $m_n \geq 45$. Together, rejection frequencies under both PV and CV are the most uniformly accurate (near 5% under PV, near 100% under CV) when the kernel estimator $\hat{\sigma}_{m_n}^2$ is used.

Clearly $\hat{v}_{m_n}^2$ tends to be smaller than $\hat{\sigma}_{m_n}^2$ given the wider $\hat{\sigma}_{m_n}$-based band under PV and CV, and under PV the $\hat{v}_{m_n}$-based band is even tighter than the 2.5% and 97.5% simulation quantiles. But the simulation mean *and* variance of $\hat{v}_{m_n}^2$ are smaller than those for $\hat{\sigma}_{m_n}^2$ under both PV and CV: the simulation variance of $\hat{v}_{m_n}^2$ is roughly half that of $\hat{\sigma}_{m_n}^2$ for all m_n due to the relatively few auction pairs that enter $\hat{v}_{m_n}^2$. Further, $\hat{v}_{m_n}^2$ is slightly skewed left over most m_n while $\hat{\sigma}_{m_n}^2$ is slightly skewed right over all m_n. Thus, $\hat{v}_{m_n}^2$ tends to produce small outliers under PV and CV. Despite the $\hat{v}_{m_n}$-based 95% band being roughly identical to the simulation 2.5% and 97.5% quantiles under CV, the band is only an average over $R = 250$ samples, and the average band does not reveal the small but significant "outliers". The rejection frequency under the PV null, however, clearly captures these few outliers in the form of a disproportionately large rejection frequency.

But the latter discussion only partially explains why the two tests have different rejection frequencies: it fails to explain why the kernel-based ratio renders a frequency near the normal size when an asymptotic Gaussian critical value is used. Notice that although only $\hat{v}_{m_n}^2$ exploits cross-auction bid independence, it approximates the variance of a random variable that itself only *asymptotically approximates* $\hat{\kappa}_{m_n}^{-1}$ (see Theorem 5 and Lemma 6 in Appendix A). In fact, the approximation is required precisely so we can exploit bid

[26] The bandwidth must satisfy $\gamma_n = o(n^{1/2})$ to ensure consistency $|\hat{\sigma}_{m_n}^2 - v_{m_n}^2| \xrightarrow{p} 0$. See Theorem 5 in Appendix A. We repeatedly find $\gamma_n \sim n^{\delta}$ for $\delta \in [.2, .25)$ to be superior across data generating processes based on a massive array of simulations of a large variety data generating processes ranging from iid, to block-wise independent auction data, to dependent and heterogeneous time series.

independence across auctions. Conversely, $\hat{\sigma}^2_{m_n}$directly approximates the second moment of $\hat{\kappa}^{-1}_{m_n} - \kappa^{-1}$ for each n (Hill (2009b)) hence the t-ratio $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$ under PV is closer to the standard normal law than $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{v}_{m_n}$. Moreover, the average bid sample size is 3000 which is large enough to promote approximate normality for $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)$ under PV. Thus, $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$ is comparatively closer to a $(0,1)$-Gaussian law[27], and therefore a $(0,1)$-Gaussian critical region better describes $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$ under PV than $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{v}_{m_n}$. This is an important distinction: while the $\hat{v}_{m_n}$-based 95% bands better match the 2.5% and 97.5% simulation quantiles, other bands (99%, 90%, 85%, ...) based on $\hat{v}_{m_n}$ do not fair as well, in particular under PV since $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{v}_{m_n}$ does not closely follow the standard normal law.

5 So, Are There Common Values in BCTS?

The BCTS sells timber through first-price, sealed-bid auctions, and it has developed a complex system of reserve prices, also known in the industry as upset rates. The reserve prices are set in dollars per cubic meter of timber. Bids must be submitted as bonus payments, i.e. the dollar amounts per cubic meter over and above the reserve prices. Negative bonus bids are not allowed. This feature makes the reserve prices strictly binding, a necessary condition for our testing approach. When the timber is harvested and scaled, the successful bidders pay for the actual volume of timber.[28]

Besides maximizing the revenue to the Crown, BCTS also implements a market pricing system that uses auction high bids for setting stumpage rates for timber under long-term tenure contracts. (Approximately 70 percent of the annual cut is allocated to long-term tenure.) BCTS maintains an active database that contains historical information on timber sales in the province. Both stumpage rates and reserve prices are set using predicted high bids in comparable auctions.

The predicted high bids are computed using an estimated regression model, with adjustments for harvesting situations not well represented in the BCTS dataset (e.g. helicopter single standing stem selection etc.). The reserve price is set at 70 percent of the estimated high bid in the auction. A printout of a regression model used for coastal sales and the list of variables in exhibited in the Appendix.[29] The variables are selected as those thought by BCTS likely to explain a large portion in the variation of high bids, and it is reasonable to conjecture that these are the same main variables that affect bidders' valuations. They include average log selling prices in the region, net cruise volume, estimated species composition on the tract, and measures of distance to the closest major location (e.g. Vancouver, Nanaimo, Prince Rupert, etc.). The list of variables is not long, but probably detailed

[27]This is demonstrated by Kolmogorov-Smirnov tests of standard normality on simulated sequences of $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{v}_{m_n}$ and $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$. The KS p-value for $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$ is significantly larger than the p-value for $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/v_{m_n}$ for every $m_n \geq 60$, a fractilce range on which the empirical test size based on $m_n^{1/2}(\hat{\kappa}^{-1}_{m_n} - 2)/\hat{\sigma}_{m_n}$ is roughly 5%. The simulation results are available upon request.

[28]Some Further details of BCTS are described in Roise (2005) and Niquidet (2008).

[29]The printout extracted from the booklet that describes the market pricing system. The booklet was published by the BC Minstry of Forests and Range online, and is avaiable at `http://www.llbc.leg.bc.ca/public/PubDocs/bcdocs/370415/MPSCoast.pdf`.

enough to capture the main aspects of auction heterogeneity, so normalizing the bids by the reserve price is likely to homogenize the bids to a large extent. For this reason, we do not incorporate any covariates in the empirical implementation of our test beyond the normalization of bids by the reserve prices.

5.1 Why Should We Expect Common Values in BCTS?

Paarsch (1997) assumed a private values model in his study of SBFEP, the predecessor of BCTS. But things have changed in BC since then, both because of new features in the BCTS program, and because of certain concurrent events discussed below.

The value of a timber tract to a bidder depends the on the price of logs produced from the timber. The price depends on the composition of timber species, the quality of timber, and on market conditions. BCTS collects data on log prices for various timber species on a regular basis, and publishes them on its website in log price reports. A report for the coastal area covering the 12 months period ending December, 2006 is included in the Appendix. As is evident from this report, there were considerable price differences both across species and across quality grades. The average price over the period covered by the report was about $83/m^3. White Pine had a considerably lower price, about $53/m^3, while Cedar had a considerably higher price, about $123/m^3. Also, grade "D" Fir had a price of about $393/m^3, almost 10 times as much as grade "J".

BCTS provides estimates of the species composition at the time of sale, but these are just estimates, and the actual fractions of species on the tract can deviate considerably, as documented in Athey and Levin (2001) for the US Forest Service auctions. Given that logs from different species may have considerably different market prices, this "composition" effect may create uncertainty about the average price of logs from the auctioned timber. In SBFEP auctions studied in Paarsch (1997), even though bidders could only submit one bonus bid as in BCTS, the upset rates varied according to the species. In effect, the winner of the auction ended up paying different prices for different species. This may have allowed bidders to insure against the "composition" effect, by equalizing profit margins on different species. In BCTS, bidders end up paying one "total" price, and cannot insure away the "composition" effect, facing larger uncertainty in log prices.

Over the period covered in our dataset, an additional factor influencing the quality of timber in the province was in play: the epidemic of Mountain Pine Beetle.[30] It affected the predominant merchantable timber species, the Lodgepole Pine. According to an estimate by the BC Ministry of Forests and Range, since late 1990s beetles have killed over 620 million cubic meters of timber, covering over 130,000 square kilometers (see Figure 6). The BCTS conducts aerial surveys to determine the degree of beetle infestation of a forest area. A definite sign of beetle infestation is the change of the color of pine needles, from green to yellow to red. However, these measurements are imprecise because the needles don't start to turn colors until many months after the initial attack. The implication for our study is that not only the quality of the logs was lower over the period in the data, but also that there may have been substantial uncertainty about the quality at the time of bidding for a

[30] Some information about the Mountain Pine Beetle epidemic in BC can be found in the Mountain Pine Beetle Action Plan, available at the BC Ministry of Forests and Range website, at `http://www.for.gov.bc.ca/hfp/mountain_pine_beetle/MPB_ActionPlan_ProgressReport.pdf` (accessed on 28 April, 2009).

tract.

Our data were collected from the BCTS website.[31] The raw data is in the form of PDF files for each sale. We have all bonus bids, the reserve price, the identities of the bidders and also data on the characteristics of the sale (e.g. the location, the species of trees present, slope etc.). The reserve prices and bids are quoted per $1m^3$ of timber. Our dataset comprises of all auctions conducted from January 14, 2004 to December 14, 2006. The sample contains 611 auctions and a total of 1874 bids. For each auction, the reserve price as well as the bonus bids are available for all bidders. The summary statistics of the sample are shown in Table 1. It is evident that there is substantial heterogeneity in timber tract values, as reflected by the variances of both bids and reserve prices. The heterogeneity persists even when bids are normalized by the reserve prices.

5.2 Can Bidders Be Treated Symmetrically?

Our testing approach has been developed under the assumption that bidders are symmetric. Paarsch (1997) also treated bidders symmetrically in his study of SBFEP. But how well is symmetry supported in BCTS? Since one of the goals of BCTS is to obtain market value for the timber by promoting entry, the eligibility requirements in BCTS were somewhat relaxed. In particular, the eligibility is no longer restricted to small logging firms. In principle, any entity in BC that (i) is an individual 19 years of age or older or (ii) a corporation registered in British Columbia; and, has one year logging experience or own a timber processing facility, can participate for a two-year period. by paying a small registration fee (250 Canadian dollars). But many features of the SBFEP have been preserved in BCTS, e.g. the tracts are still quite small and, as important, no firm is allowed to hold more than five licenses at the same time. This alone is likely to limit the participation of large mills and timber processing companies.

In our data, we find no evidence of major players. Figure 7 contains the frequencies of participation by firms in the auctions over the period covered by our data. About half of the bidders participated in only one auction. Even the maximum of the participation rate is very small, 15 out of 611 auctions, or the rate of about 0.02.This also implies that information asymmetries, for example due to superior information about the beetle infestation, are unlikely in this market.

But even though the participating firms are primarily small loggers, relatively high transportation costs may create asymmetries if there is entry by distant firms. However, there is evidence that competition in BCTS is highly localized. Paarsch (1997) reports that in SBFEP, 90 percent of the sales in a particular district only involving firms from that district. Our data on firm locations allow us to provide even stronger evidence on a highly localized nature of this market.

We have information on the addresses of most firms (we were able to identify the locations of 580 out of 684 firms). Some information about site locations is also available, but unfortunately not sufficiently precise to pinpoint their precise geographic coordinates. Often, the raw data files contained a description of the location relative to some identifier such as a road, a lake etc. In principle, this information could be used to determine approximate coordinates of the site, but this is likely to be hard and wasn't attempted in this study.

[31] `http://www.for.gov.bc.ca/bcts/`, accessed in January 2007.

We can still measure the degree of firms' distance asymmetry by looking at within-auction differences in the coordinates of bidding firms. In each auction, we determined the "center" by averaging latitudes and longitudes of the firms that participated. A distance variable for each firm was constructed by computing its distance in miles to the auction's "center". The frequency plot of the distance variable is given in Figure 8. Over a third of the bids (657 out of 1874) were submitted by firms within 20 miles of the corresponding center, and only 207 bids (just slightly over 15%) by bidders farther than 100 miles. According to this measure, the competition in BCTS is indeed highly localized. This and our previous discussion imply that bidder asymmetries are not substantial.

5.3 Test Results

Figure 9 shows the histogram of normalized bonus bids. The histogram exhibits an overall declining pattern consistent with our findings in the numerical examples (Figure 1), for both PV and CV environments.[32] There is some evidence of bid clustering around the reserve price. This could be evidence of a PV environment, but could also be a statistical artifact, especially since our numerical examples have also identified some clustering under CV. The tail index test however provides robust and conclusive evidence. The results of the estimation of the tail index κ^{-1} are reported in Figure 10, where we plot the asymptotic 95% confidence bands $\hat{\kappa}_{m_n}^{-1} \pm 1.96\hat{v}/\sqrt{m_n}$ over the same fractile window $m_n \in \{5, ..., 200\}$ used in the simulation, where $\hat{v}^2$ denotes either semi-parametric or nonparametric estimator $\hat{v}_{m_n}^2$ or $\hat{\sigma}_{m_n}^2$. The CV value $\kappa^{-1} = 1$ lies within the confidence band for over 70% (55%) of the fractiles m_n when $\hat{\sigma}_{m_n}^2$ ($\hat{v}_{m_n}^2$) is used, while the PV value $\kappa^{-1} = 2$ never lies in either band.

Test p-values based on both asymptotic variance estimators are displayed in Figure 11. The fractile is again restricted to the window $m_n \in \{5, ..., 200\}$. There is no evidence at any fractile m_n for PV: p-values are no larger than .0045 for the test based on $\hat{v}_{m_n}^2$ or $\hat{\sigma}_{m_n}^2$. Since $\hat{\sigma}_{m_n}^2$ not only trumps $\hat{v}_{m_n}^2$ in simulation experiments but leads to accurate test sizes under the PV null and impressive power under CV, the evidence overwhelmingly points to CV.

6 Concluding Remarks

In this paper, we have developed a new tail-index nonparametric, asymptotically most powerful test of common values in first-price auctions and applied it to BC Timber Sales, an important institution both in the economy of British Columbia and in the larger context of the United States-Canada soft lumber dispute.

The test is based on auction theory, but is reduced-form in the sense that it is based on the properties of bids distribution, which is directly observable. The test exploits the difference in the clustering of bids near the reserve price. A measure of such clustering is provided by the tail index. We have shown that the tail index of the bid distribution is equal to one-half under private values, but is equal to one if there is a common-value component in bidders' valuations. The estimation of the tail index is a well-studied problem in econometrics, where the Hill-estimator is by far the most widely-used method. But the available

[32] Roise (2005) presents a similar pattern in his Figure 8, without normalizing bids by the reserve price.

asymptotic results do not cover our setting of imbalanced panels with stochastic dimension and conditional sub-sample heterogeneity. Our econometric contribution is to develop a nonparametric asymptotic inference framework for the Hill-estimator in this setting. These methods are potentially useful in other contexts.

Our nonparametric tail-index test has uncovered common values in BCTS beyond any reasonable doubt. Simply put, the bids do not cluster around the reserve price as much as they should if values were purely private. Why is this important? Recall that BCTS pursues a goal of obtaining the maximal revenue for the Crown, and currently uses first-price, sealed-bid auction mechanism. To the extent that common values are present, theory (Milgrom and Weber (1982)) suggests that BCTS would obtain a higher expected revenue if it adopted an ascending-bid auction. Shneyerov (2007) proposes nonparametric methods for estimation and bounding of such counter-factual revenues. In addition, Tang (2009) has recently developed nonparametric bounds for expected revenues under counter-factual reserve prices. An application of these methods to BC Timber Sales would be an interesting topic for future research.

Appendix A: Tail Index Asymptotics and Inference

In this appendix we characterize the limit distribution of the Hill-estimator for imbalanced panels with stochastic dimension. We derive the new semi-parametric asymptotic variance estimator $\hat{\upsilon}^2_{m_n}$ in (11) specifically tailored to auction data, and prove Hill (2009a)'s nonparametric estimator $\hat{\sigma}^2_{m_n}$ in (10) is trivially consistent for auction data.

It is convenient to denote

$$\digamma_L := \sigma\left(\{n_l\}_{l=1}^L\right),$$

the sigma-algebra we use for conditioning on the realizations of auction sizes n_l for a given sample of auctions of size L. In view of Proposition 1 and Remark 2, we have the following conditions satisfied for bids $b^*_{i,l}$

A1 (conditional tail) *The left-tail behavior of $b^*_{i,l}$ satisfies for every i and l* as $b \downarrow 0$

$$G^*\left(b|\digamma_L\right) := P\left(b^*_{i,l} < b|\digamma_L\right) = c_l \times b^\kappa\left(1 + O(b^\kappa)\right), \, \kappa > 0, \tag{13}$$

where the $\digamma_L$-measurable random variable $c_l > 0$ $\digamma_L$-a.e..

A2 (cross-auction independence) *$b^*_{i,l}$ is possibly dependent over i within auction l, and independent across auctions l.*

We require one more piece of information concerning tail dependence. Since our DGP implies $b_{i,l}$ is stationary in the tails, for a given auction size n_l we assume without loss of generality tail dependence depends only on bid displacement in the following sense.

A3 (tail stationarity) *$b^*_{i,l}$ is conditionally tail stationary: $P(b^*_{i,l} < be^{-u}, b^*_{j,l} < be^{-v}|\digamma_L)$ for any scale $u, v \geq 0$ depends only on auction size n_l and bid displacement $|i-j|$ as $b \searrow 0$. In particular, $P(b^*_{i,l} < q_{m_n}e^{-u}, b^*_{j,l} < q_{m_n}e^{-v}|\digamma_L) = P(b^*_{1,l} < q_{m_n}e^{-u}, b^*_{1+|i-j|,l} < q_{m_n}e^{-v}|\digamma_L) + o_p\left(m_n/n\right) \times e^{-u-v}$ for every bid-pair $i, j \in \{1, ..., n_l\}$ and every auction $l \in \{1, ..., L\}$.*

Now define the lower m_n/n^{th}-conditional quantile sequence $\{q_{m_n}\}$ according to[33]

$$\frac{n}{m_n}G^*\left(q_{m_n}|\digamma_L\right) \to 1 \text{ as } L \to \infty. \tag{14}$$

Since auction sizes n_l are random, so are the number of sample extreme bids m_n and therefore the extreme bid threshold q_{m_n}.

A.1 Asymptotic Normality

Since $\hat{\kappa}^{-1}_{m_n} = 1/m_n \sum_{t=1}^n (\ln b^*_{(m_n+1)}/b^*_t)_+$ is constructed from dependent bids b^*_t and therefore a dependent order statistic $b^*_{(m_n+1)}$, asymptotics are greatly simplified by a tail-array expansion that allows us to replace $b^*_{(m_n+1)}$ with the bid threshold q_{m_n}. Certain aspects of the following argument are inspired by Hsing (1991). See Hill (2009a) and Hill (2009b).

[33] See Leadbetter et al (1983: Theorem 1.7.13).

Since n, m_n and q_{m_n} are all random variables, throughout we operate conditionally by treating the auction sizes as known constants $\{n_l\}_{l=1}^{L}$. We then demonstrate that conditional and unconditional inference are the same asymptotically.

Denote by $(\ln q_{m_n}/b_t^*)_+|\mathcal{F}_L$ and $I(b_t^* < q_{m_n}e^{-u})|\mathcal{F}_L$ bid tail information for a given random draw $\{n_l\}_{l=1}^{L}$. Similarly, define a zero mean tail array conditioned on $\{n_l\}_{l=1}^{L}$,

$$\left\{T_{m_n,t}^{(L)}(u) : 1 \leq t \leq n\right\}_{n\geq 1} = \left\{T_{m_n,t}^{(L)}(u)\right\} := U_{m_n,t}^{(L)} - \kappa^{-1} I_{m_n,t}^{(L)}(u)$$

where

$$U_{m_n,t}^{(L)} := (\ln q_{m_n}/b_t^*)_+ |\mathcal{F}_L - E\left[(\ln q_{m_n}/b_t^*)_+ |\mathcal{F}_L\right]$$

$$I_{m_n,t}^{(L)}(u) := I\left(b_t^* < q_{m_n}e^{-u}\right)|\mathcal{F}_L - G^*\left(q_{m_n}e^{-u}|\mathcal{F}_L\right), \;\; u \geq 0,$$

and compactly write

$$I_{m_n,t}^{(L)} := I_{m_n,t}^{(L)}\left(u/m_n^{1/2}\right) \;\; \text{and} \;\; T_{m_n,t}^{(L)} := T_{m_n,t}^{(L)}\left(u/m_n^{1/2}\right).$$

We write interchangeably the stacked $\{U_{m_n,t}^{(L)}, I_{m_n,t}^{(L)}, T_{m_n,t}^{(L)}\}$ or the auction/bid array $\{U_{m_n,i,l}^{(L)}, I_{m_n,i,l}^{(L)}, T_{m_n,i,l}^{(L)}\}$.

Now define the conditional and population mean-squared-error:

$$v_{m_n|L}^2 := E\left[m_n\left(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1}\right)^2|\mathcal{F}_L\right] \;\; \text{and} \;\; v_{m_n}^2 := E\left[v_{m_n|L}^2\right].$$

Note $v_{m_n}^2$ is not in general the variance of $m_n^{1/2}(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1})$ because $\hat{\kappa}_{m_n}^{-1}$ is biased (e.g. Hall (1982), Segers (2002)). Nevertheless, a characterization of the asymptotic variance of $\hat{\kappa}_{m_n}^{-1}$ for a stochastic dimensional panel is expedited by the following result.

THEOREM 5 *Under A1 and A2, for any $\sigma(\cup_{L\geq 1}\mathcal{F}_L)$-measurable intermediate order sequence $\{m_n\}$, $m_n \to \infty$ a.s. and $m_n = o_p(n^{2/3})$, the Hill-estimator conditionally on $\{n_l\}_{l=1}^{L}$*

$$m_n^{1/2}\left(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1}\right)|\mathcal{F}_L = \frac{1}{m_n^{1/2}}\sum_{t=1}^{n} T_{m_n,t}^{(L)} + o_p(1). \tag{15}$$

Further, unconditionally

$$\frac{m_n^{1/2}}{v_{m_n}}\left(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1}\right) \stackrel{d}{\to} N(0,1) \;\; \text{where} \;\; v_{m_n}^2 = O(1) \tag{16}$$

where

$$\frac{v_{m_n}^2}{v_{m_n|L}^2} \stackrel{p}{\to} 1. \tag{17}$$

Remark 1: Limit (16) is identical to the ones established in Hill (2009a: Theorem

5; 2009b: Theorem 5.1), hence the stochastic nature of random auction sizes $\{n_l\}$ is non-influential in the limit. Indeed, (17) means asymptotic variance estimation can proceed as if $\{n_l\}_{l=1}^{L}$ were deterministic.

Remark 2: Limit (16) is arguably supported by a weak limit theorem for cadlag functionals with stochastic index (Theorem 14.4 of Billingsley (1999)). In that context it is assumed there exists a mapping $g : \mathbb{N} \rightarrow \mathbb{R}_+$ such that $n/g(L) \rightarrow \theta$ where θ is a constant, or a random variable under sharp regulatory conditions. In our case we do not require any information on the nature of n although $\text{plim}_{L\rightarrow\infty} n/L = E[n_l] \leq \bar{n}$ clearly exists since n_l is iid and bounded.

Remark 3: In the iid case $v_{m_n}^2 \rightarrow \kappa^{-2}$ (Hall (1982)), but other auction data special cases lead to the same result. See below.

A.2 Semi-Parametric Inference

Since bids $b_{i,l}^*$ are independent across auctions under A2, use asymptotic approximation (15) to deduce, conditional on auction sizes $\{n_l\}_{l=1}^{L}$,

$$v_{m_n|L}^2 \sim E\left(\frac{1}{m_n^{1/2}}\sum_{l=1}^{L}\sum_{i=1}^{n_l} T_{m_n,i,l}^{(L)}|\digamma_L\right)^2 = \frac{1}{m_n}\sum_{l=1}^{L}\sum_{i,j=1}^{n_l} E\left[T_{m_n,i,l}^{(L)} \times T_{m_n,j,l}^{(L)}|\digamma_L\right].$$

Cross-auction independence A2 and tail stationarity A3 imply for n sufficiently large $E[T_{m_n,i,l}^{(L)}T_{m_n,j,l}^{(L)}|\digamma_L]$ depends only on the auction size n_l and bid displacement $|i-j|$. In particular, by stationarity A3 and Lemma B.1 in Appendix B

$$v_{m_n|L}^2 \sim \frac{1}{m_n}\sum_{l=1}^{L}\sum_{i=1}^{n_l} E\left[\left(T_{m_n,i,l}^{(L)}\right)^2|\digamma_L\right] + 2\frac{1}{m_n}\sum_{l=1}^{L}\sum_{i=1}^{n_l-1}(n_l-i)\times E\left[T_{m_n,1,l}^{(L)} \times T_{m_n,i+1,l}^{(L)}|\digamma_L\right].$$

In the first term observe under A1 Lemma B.2 asserts $(n/m_n)E[(T_{m_n,1,l}^{(L)})^2|\digamma_L] \xrightarrow{p} \kappa^{-2}$. Along with $n = \sum_{l=1}^{L} n_l$ and (17) this proves the following claim.

LEMMA 6 *Under A1-A3 the unconditional variance satisfies*

$$\lim_{L\rightarrow\infty} v_{m_n}^2 = \kappa^{-2} + 2\lim_{L\rightarrow\infty}\frac{1}{m_n}\sum_{l=1}^{L}\sum_{i=1}^{n_l-1}(n_l-i)\times E\left[T_{m_n,1,l}^{(L)} \times T_{m_n,1+i,l}^{(L)}|\digamma_L\right].$$

Remark: If bids are everywhere independent then $E[T_{m_n,1,l}^{(L)} \times T_{m_n,1+i,l}^{(L)}|\digamma_L] = E[T_{m_n,i,l}^{(L)}|\digamma_L] \times E[T_{m_n,j,l}^{(L)}|\digamma_L] = 0$ hence the classic result $\lim_{L\rightarrow\infty} v_{m_n}^2 = \kappa^{-2}$. Synonymously and trivially, if all auctions have one bid $n_l = 1$ with probability one then $\lim_{L\rightarrow\infty} v_{m_n}^2 = \kappa^{-2}$. As long as asymptotically there are infinitely many auctions with more than one bid ($n_l > 1$) the iid asymptotic variance κ^{-2} is wrong, and without more information on the nature of bid dependence within auctions $E[T^{(L)}_{m_n,i,l} \times T^{(L)}_{m_n,j,l}|\digamma_L]$ cannot be simplified.

Lemma 6 provides the basis for the structure of the semi-parametric estimator $\hat{v}^2_{m_n}$ in (11), where $\hat{c}_{m_n}(i)$ defined in (12) estimates within-auction bid tail dependence $E[T^{(L)}_{m_n,1,l} \times T^{(L)}_{m_n,1+i,l}|\digamma_L]$. In order to ensure $\hat{c}_{m_n}(i)$ is consistent for $E[T^{(L)}_{m_n,1,l} \times T^{(L)}_{m_n,1+i,l}|\digamma_L]$ we must characterize how many auctions contain bid displacement i. Recall $\tilde{L}_i = \sum_{l=1}^{L}(n_l - i)_+$ denotes the number of bid pairs $\{b^*_{j,l}, b^*_{j+i,l}\}$ at displacement i that enter into $\hat{c}_{m_n}(i)$.

A4 (bid displacement) *Asymptotically every bid displacement occurs infinitely often:* $\tilde{L}_i \to \infty$ *as* $L \to \infty$ *for each* $i = 1...\bar{n} - 1$.

THEOREM 7 *Under A1-A4,* $\liminf_{n\geq 1} v^2_{m_n} > 0$ *and* $m_n/n^{1/2} \to \infty$ *a.s.,* $\hat{v}^2_{m_n}/v^2_{m_n} \xrightarrow{p} 1$.

Finally, Hill (2009a)'s kernel asymptotic variance estimator $\hat{\sigma}^2_{m_n}$ in (10) is easily consistent for blockwise independent bids in auctions of stochastic size. Although a large class of kernel functions $k(z)$ can be considered in the following arguments, including Parzen, Tukey-Hanning, and Quadratic-Spectral, by far the Bartlett kernel $k(z) = (1 - |z|)_+$ is the most popular in the economics literature. See Hill (2009b), cf. Andrews (1991) and de Jong and Davidson (2000). Since n is random and in practice the bandwidth γ_n is a function of n, we must treat γ_n as a random variable.

A5 (kernel) $k(z) = (1 - |z|)_+$, $\gamma_n \to \infty$ *a.s.,* $\gamma_n = o_p(m_n/n^{1/2})$, $m_n = o_p(n)$ *and* $m_n/n^{1/2} \to \infty$ ∞ *a.s.*

THEOREM 8 *Under A1-A3 and A5* $\hat{\sigma}^2_{m_n} > 0$ *with probability one. Additionally if* $\liminf_{n\geq 1} v^2_{m_n} > 0$ *then* $\hat{\sigma}^2_{m_n}/v^2_{m_n} \xrightarrow{p} 1$.

Remark 1: Notice $\gamma_n = o_p(m_n/n^{1/2}) = o_p(n)$ and $m_n = o_p(n)$ imply $\gamma_n = o_p(n^{1/2})$ must hold. In general $\gamma_n = o_p(n)$ ensures $\hat{\sigma}^2_{m_n} > 0$ *a.s.* (e.g. de Jong and Davidson (2000)), while specifically $\gamma_n = o_p(m_n/n^{1/2})$ promotes consistency by reducing the effects of persistence between distant events (Hill (2009b)). This is trivially satisfied under cross-auction independence since "distant events" are simply bids in different auctions of uniformly finite size $n_l \leq \bar{n} < \infty$.

Remark 2: We require sufficiently many tail observations $m_n/n^{1/2} \to \infty$ *a.s.* to ensure the estimators $b^*_{(m_n+1)}$ and $\hat{\kappa}^{-1}_{m_n}$ that occur in every $\hat{U}_{m_n,t}$ do not affect the limit of $\hat{\sigma}^2_{m_n}$.

A.3 Formal Proofs

Proof of Theorem 5.

Step 1: Consider approximation (15) and write $b^{*(L)}_t := b^*_t|\digamma_L$, an arbitrary bid given all auction sizes. By conditioning on $\{n_l\}_{l=1}^L$ it follows $\{m_n\}$ and $\{q_{m_n}\}$ are sequences of known constants. We therefore need only show the conditions of Lemma A.7 of Hill (2009a) are satisfied for the data generating process of conditional bids $\{b^{*(L)}_t\}$. It suffices if $b^{*(L)}_t$ has tail (13), and $\{I(b^*_t < q_{m_n}e^{-u})|\digamma_L\}$ is geometrically L_2-Near Epoch Dependent on some geometrically strong mixing base $\{\epsilon_t\}$. Condition A1 ensures the regularly varying tail (13), cf. Proposition 1.

Stationary blockwise independence A2 with blocks of finite size $n_l \leq \bar{n} < \infty$ ensures $\{I(b_t^* < q_{m_n}e^{-u})|F_L\}$ is no more than $\bar{n}$-dependent and therefore stationary geometrically strong mixing (Ibragimov and Linnik (1971)), hence $\{I(b_t^* < q_{m_n}e^{-u})|F_L\}$ is stationary geometrically L_2-NED on itself as a geometrically strong mixing base (e.g. Ibragimov and Linnik (1971); Gallant and White (1988), Hill (2009b))[34]. This proves (15).

Step 2: In order to prove the unconditional limit (16) we will first verify the conditional limit:

$$\frac{m_n^{1/2}}{v_{m_n|L}}\left(\hat{\kappa}_{m_n}^{-1}-\kappa^{-1}\right)|F_L \xrightarrow{d} N(0,1) \text{ where } v_{m_n|L}^2 = O_p(1). \tag{18}$$

The latter follows from approximation (15) since under Corollary 3.3 of Hill (2009a)

$$\frac{1}{m_n^{1/2}}\sum_{t=1}^{n} T_{m_n,t}^{(L)}/\sigma_{m_n|L} \xrightarrow{d} N(0,1), \tag{19}$$

where $\sigma_{m_n|L}^2 := E(1/m_n^{1/2}\sum_{t=1}^n T_{m_n,t}^{(L)}|F_L)^2$. The conditions of Hill (2009a)'s Corollary 3.3 are easily satisfied since $b_t^{*(L)}$ has tail (13) under A1, and $\{I(b_t^* < q_{m_n}e^{-u})|F_L\}$ is stationary L_2-NED with arbitrary size on a strong mixing base with arbitrary size by Step 1. Notice (15), (18) and (19) imply $v_{m_n|L}^2/\sigma_{m_n|L}^2 \xrightarrow{p} 1$.

Step 3: Now consider claims (16) and (17). Approximation (15) and limit (19) imply

$$\begin{aligned}
&\lim_{L\to\infty} P\left(\frac{m_n^{1/2}}{v_{m_n|L}}\left(\hat{\kappa}_{m_n}^{-1}-\kappa^{-1}\right) \leq z|F_L\right) \\
&\qquad = \lim_{L\to\infty} P\left(\frac{1}{m_n^{1/2}}\sum_{t=1}^{n} T_{m_n,t}^{(L)}/\sigma_{m_n|L} + o_p(1) \leq z\right) = P(Z \leq z),
\end{aligned}$$

where Z is an unconditional Gaussian law with zero mean and unit variance. Bounded convergence therefore implies

$$\begin{aligned}
\lim_{L\to\infty} P\left(\frac{m_n^{1/2}}{v_{m_n|L}}\left(\hat{\kappa}_{m_n}^{-1}-\kappa^{-1}\right) \leq z\right) &= \lim_{L\to\infty} E\left[P\left(\frac{m_n^{1/2}}{v_{m_n|L}}\left(\hat{\kappa}_{m_n}^{-1}-\kappa^{-1}\right) \leq z|F_L\right)\right] \\
&= E\left[\lim_{L\to\infty} P\left(\frac{m_n^{1/2}}{v_{m_n|L}}\left(\hat{\kappa}_{m_n}^{-1}-\kappa^{-1}\right) \leq z|F_L\right)\right] \\
&= P(Z \leq z).
\end{aligned}$$

[34]Specifically, if $\Im_t$ is the σ-field induced by the infinite past of ϵ_t, $\Im_t := \sigma(\epsilon_\tau : \tau \leq t)$, then some stochastic process $\{z_t\}$ is geometrically L_2-NED on $\{\Im_t\}$ if $\sup_{1\leq t\leq n} E(z_t - E[z_t|\Im_{t-l_n}^{t+l_n}])^2 = o(\rho^{l_n})$ for any sequence of positive integers $\{l_n\}$, $l_n \to \infty$, and $\rho \in (0,1)$. If $\Im_t$ is adapted to z_t then the L_2-NED property is trivial since $\sup_{1\leq t\leq n} E(z_t - E[z_t|\Im_{t-l_n}^{t+l_n}])^2 = 0$. Hill (2009a)'s result requires the L_2-NED base to be strong mixing. But block-wise independence implies bids $\{b_t^{*(L)}\}$ are geometrically strong mixing and therefore L_2-NED on themselves as a geometrically strong mixing base.

Since Z is a $N(0,1)$-law it follows instantly $v_{m_n}^2 / v_{m_n|L} \xrightarrow{p} 1$ by the definition $v_{m_n}^2 = E[m_n(\hat{\kappa}_{m_n}^{-1} - \kappa^{-1})^2]$. ■

Proof of Theorem 7. Assume auction sizes $\{n_l\}_{l=1}^L$, and therefore n and m_n, are deterministic since by Theorem 5 there is no improvement in generality by allowing stochastic n_l. Define

$$c_{m_n}(i) = \frac{1}{\tilde{L}_i} \sum_{l=1}^{L} \sum_{j=1}^{(n_l - i)_+} T_{m_n,j,l}^{(L)} \times T_{m_n,j+i,l}^{(L)}.$$

Use Lemma B.1 in in Appendix B to deduce tail stationarity A3 and the construction $\tilde{L}_i = \sum_{l=1}^L (n_l - i)_+$ imply

$$\begin{aligned} E\left[c_{m_n}(i)|\digamma_L\right] &= \frac{1}{\tilde{L}_i} \sum_{l=1}^{L} (n_l - i)_+ \times E\left[T_{m_n,1,l}^{(L)} \times T_{m_n,1+i,l}^{(L)} | \digamma_L\right] + o_p\left(m_n/n\right) \\ &= E\left[T_{m_n,1,l}^{(L)} \times T_{m_n,1+i,l}^{(L)} | \digamma_L\right] + o_p\left(m_n/n\right). \end{aligned}$$

Now use Lemma 6 to write

$$v_{m_n}^2 = \kappa^{-2} + 2\frac{1}{m_n} \sum_{l=1}^{L} \sum_{i=1}^{n_l - 1} (n_l - i) \times E\left[c_{m_n}(i)|\digamma_L\right] + o_p(1).$$

The Theorem 5 implication $\hat{\kappa}_{m_n}^{-2} = \kappa^{-2} + O_p(1/m_n^{1/2})$, the Lemma B.3 assertions $|\hat{c}_{m_n}(i) - c_{m_n}(i)| = o_p(1/m_n)$ and $|c_{m_n}(i) - E[c_{m_n}(i)|\digamma_L]| = o_p(1/m_n)$, auction size boundedness $n_l \le \bar{n} < \infty$, $\sum_{l=1}^L \sum_{i=1}^{n_l - 1} = n$ by construction, and $m_n/n^{1/2} \to \infty$ *a.s.* and $\liminf_{n \ge 1} v_{m_n}^2 > 0$ by supposition together imply $|\hat{v}_{m_n}^2 / v_{m_n}^2 - 1|$ is bounded by

$$\begin{aligned} K\left|\hat{v}_{m_n}^2 - v_{m_n}^2\right| &\le K\left|\hat{\kappa}_{m_n}^{-2} - \kappa^{-2}\right| + K\frac{1}{m_n} \sum_{l=1}^{L} \sum_{i=1}^{n_l - 1} (n_l - i)\left|\hat{c}_{m_n}(i) - c_{m_n}(i)\right| \\ &\quad + K\frac{1}{m_n} \sum_{l=1}^{L} \sum_{i=1}^{n_l - 1} (n_l - i)\left|c_{m_n}(i) - E\left[c_{m_n}(i)|\digamma_L\right]\right| + o_p(1) \\ &= O_p(1/m_n^{1/2}) + o_p(n/m_n^2) = o_p(1). \end{aligned}$$

■

Proof of Theorem 8. Under the stated assumptions and the line of proof of Theorem 5, all conditions of Hill (2009b)'s Theorem 6 hold for bids $\{b_t^{*(L)}\}$ conditioned on auction sizes $\{n_l\}_{l=1}^L$. Therefore $\text{plim}_{L\to\infty}\, \hat{\sigma}_{m_n}^2 / v_{m_n|L}^2 = 1$. Since $v_{m_n}^2 / v_{m_n|L}^2 \xrightarrow{p} 1$ by Theorem 5 the proof is complete. ■

Appendix B: Supporting Lemmata

The following results are straightforwardly verified under the maintained assumptions. Consult Hill and Shneyerov (2009) for proofs.

LEMMA B.1 *Under A1-A3 for n sufficiently large* $E[T^{(L)}_{m_n,i,l}T^{(L)}_{m_n,j,l}|\mathcal{F}_L]$ *depends only on auction size* n_l *and bid displacement* $|i-j|$. *In particular* $E[T^{(L)}_{m_n,i,l}T^{(L)}_{m_n,j,l}|\mathcal{F}_L] = E[T^{(L)}_{m_n,1,l}T^{(L)}_{m_n,1+|i-j|,l}|\mathcal{F}_L] + o_p(m_n/n)\ \forall i,j \in \{1,...,n_l\}$ *for all* $1 \leq l \leq L$.

LEMMA B.2 *Under A1* $(n/m_n)E[(T^{(L)}_{m_n,i,l})^2|\mathcal{F}_L] \xrightarrow{p} \kappa^{-2}$ $\mathcal{F}_L$*-a.e.*

LEMMA B.3 *Under A1, A2, A4 and* $m_n/n^{1/2} \to \infty$ *a.s.*, $\sup_{1\leq i\leq \bar{n}-1} |\hat{c}_{m_n}(i) - c_{m_n}(i)| = o_p(1/m_n)$ *and* $\sup_{1\leq i\leq \bar{n}-1} |c_{m_n}(i) - E[c_{m_n}(i)|\mathcal{F}_L]| = o_p(1/m_n)$.

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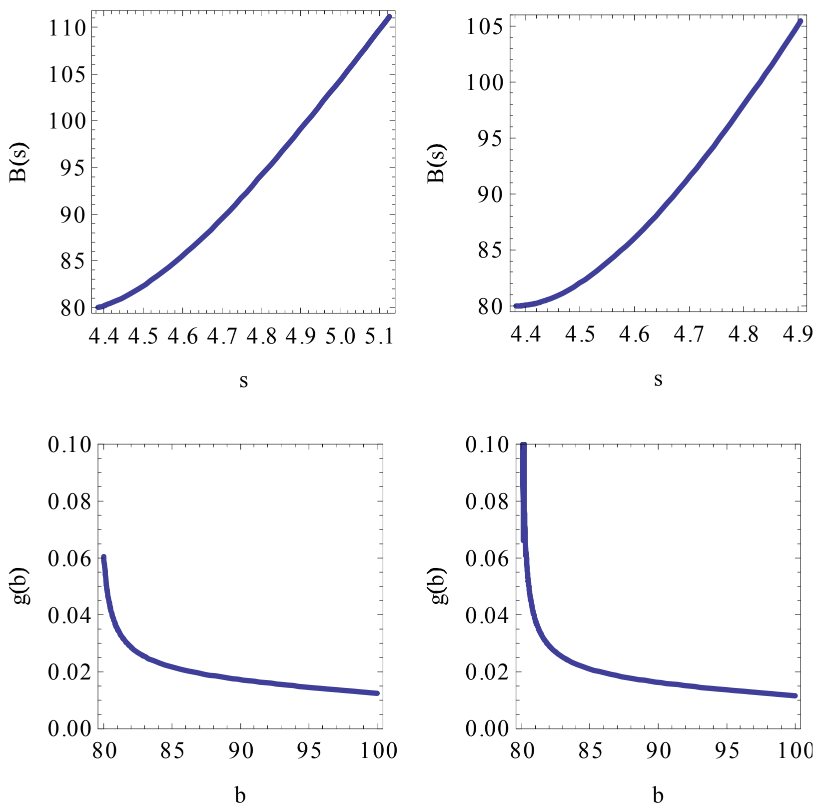

Figure 1: Bidding strategies and bid densities for CV (left panel) and PV (right panel) models.

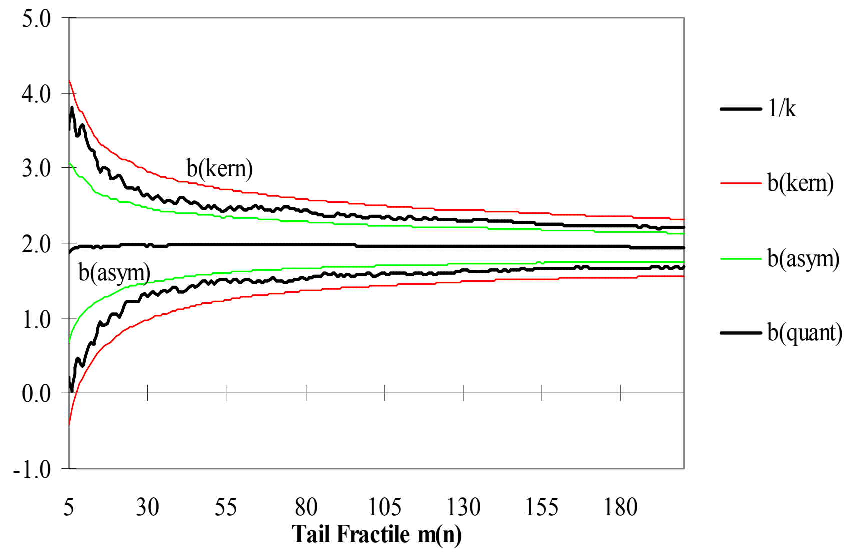

Figure 2: Simulated PV data : The tail index estimator $\hat{\kappa}_{m_n}^{-1}$. b(·) denotes the asymptotic 95% confidence band based on the variance estimator $\hat{\sigma}^2_{m_n}$ (kern) or $\hat{v}^2_{m_n}$ (asymp), or the sample quantiles (quant).

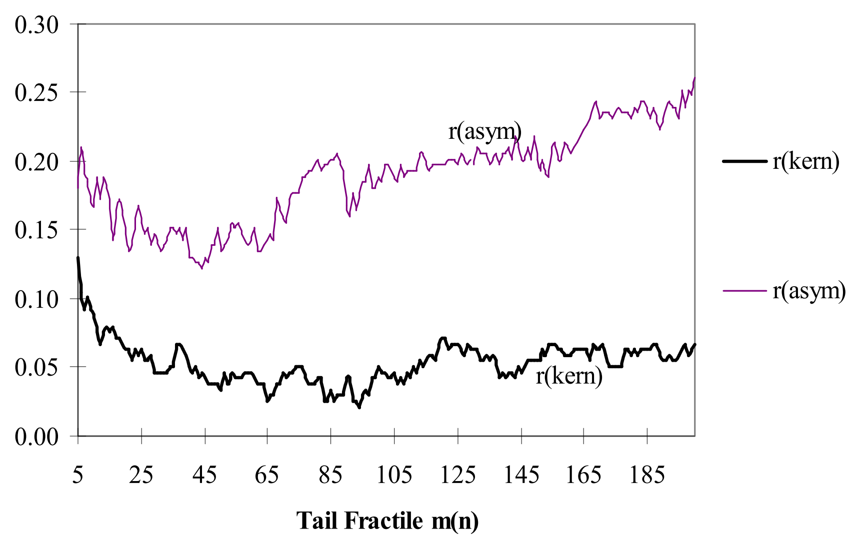

Figure 3: Simulated PV data test rejection frequencies: H_0 : PV $\kappa = 1/2$ against $\kappa > 1/2$. r(·) is the rejection frequency of the null of PV at the 5% level.

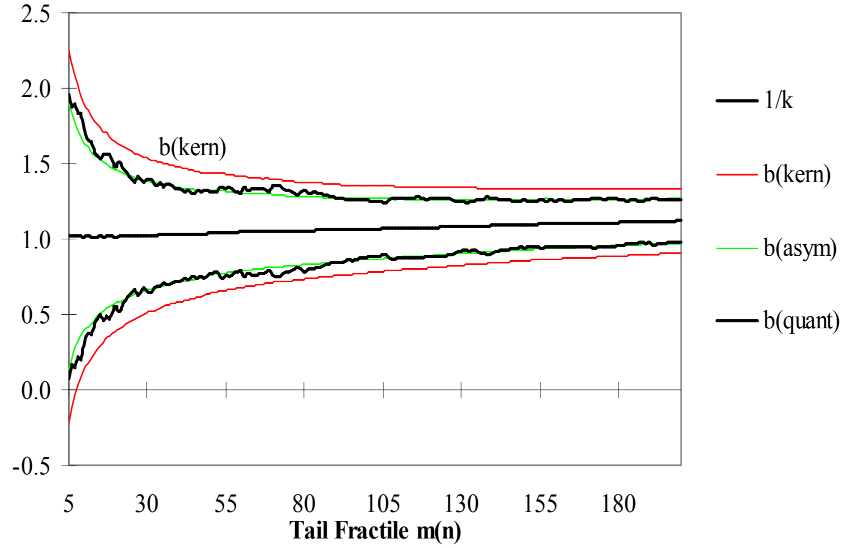

Figure 4: Simulated CV data : $\hat{\kappa}_{m_n}^{-1}$. b($\cdot$) denotes the asymptotic 95% confidence band based on the variance estimator $\hat{\sigma}_{m_n}^2$ (kern) or $\hat{v}_{m_n}^2$ (asymp), or the sample quantiles (quant).

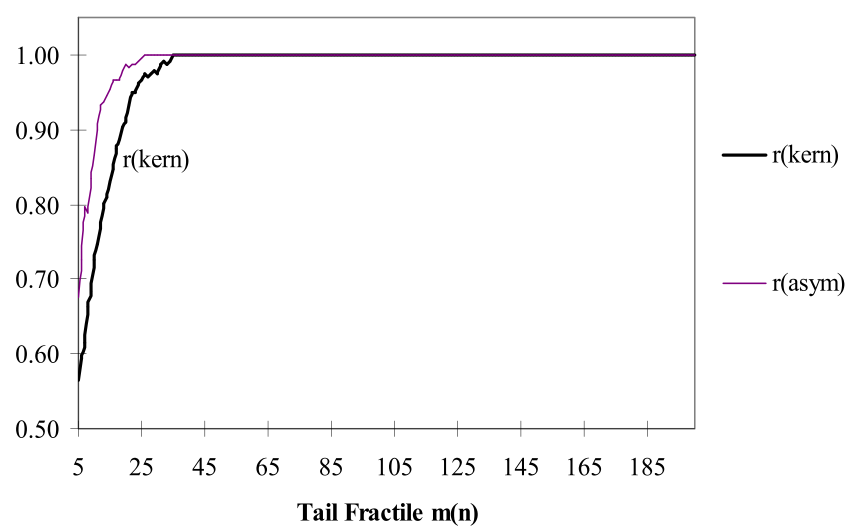

Figure 5: Simulated CV data test rejection frequencies for H_0 : PV $\kappa = 1/2$ against $\kappa > 1/2$. r($\cdot$) is the rejection frequency of the null of PV at the 5% level,

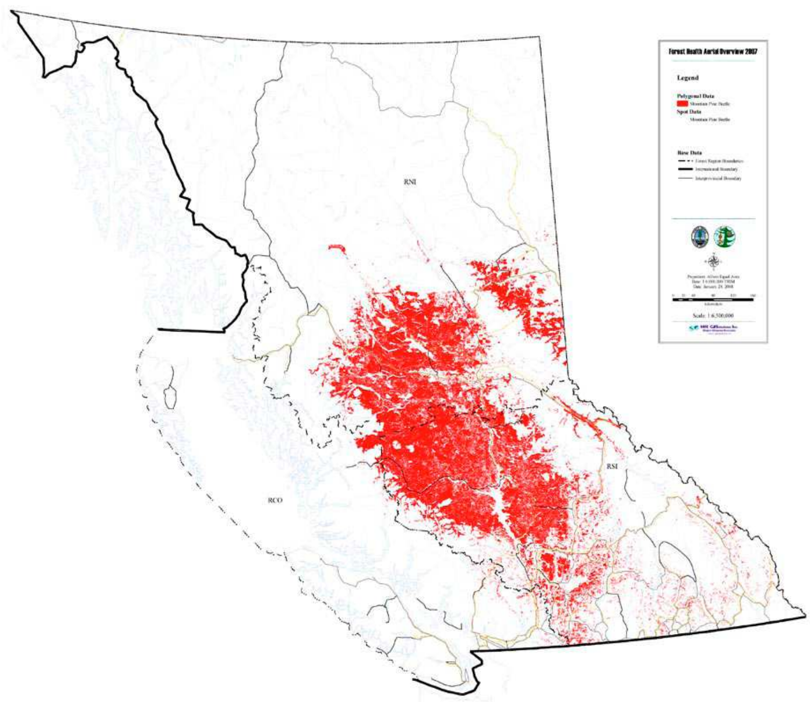

Figure 6: The outbreak of Mountain Pine Beetle in the province of British Columbia.

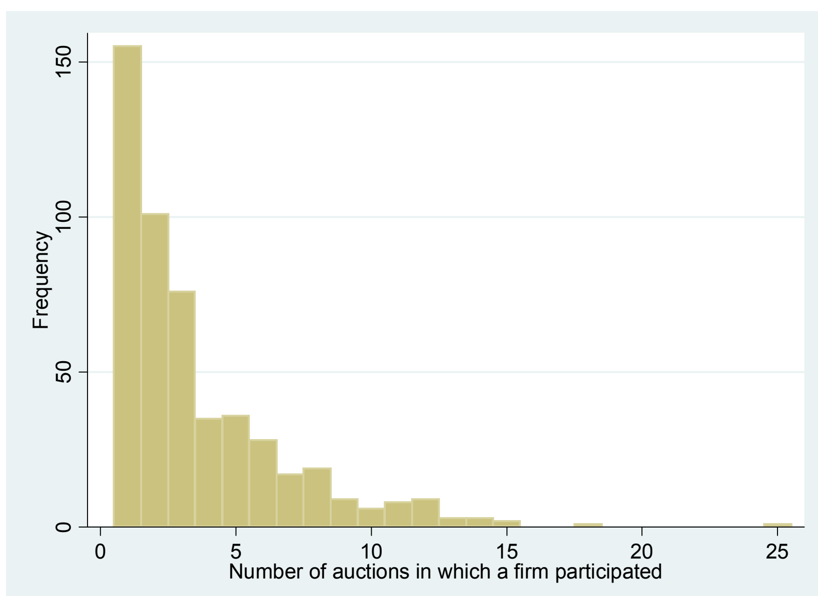

Figure 7: The histogram of firms' participation rates.

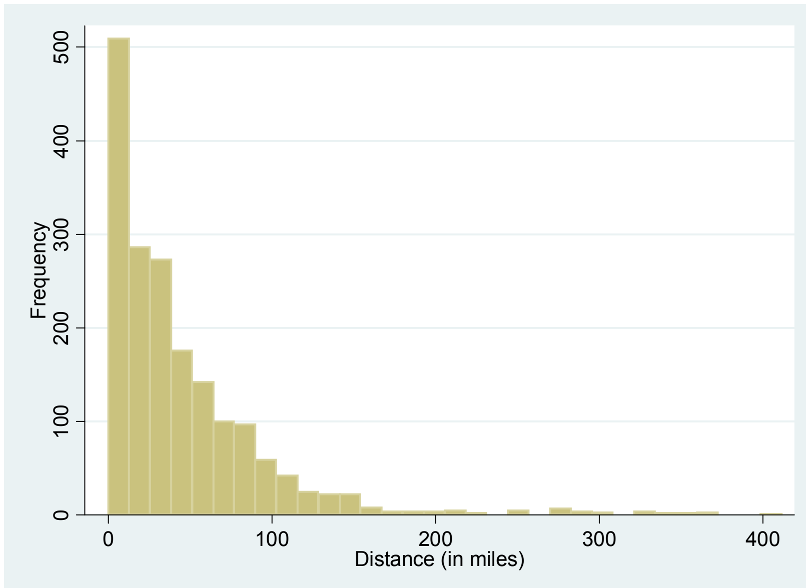

Figure 8: The histogram of firms' within-auction distances.

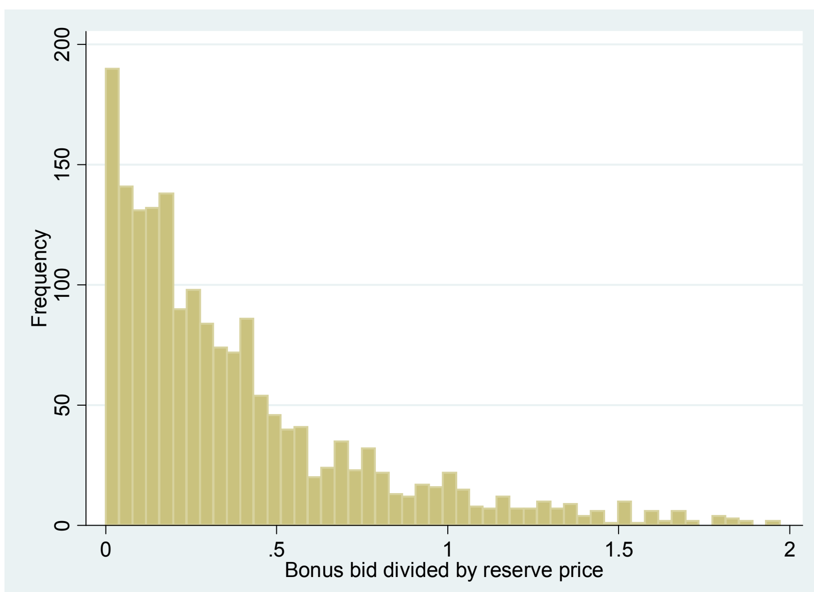

Figure 9: The histogram of normalized bonus bids.

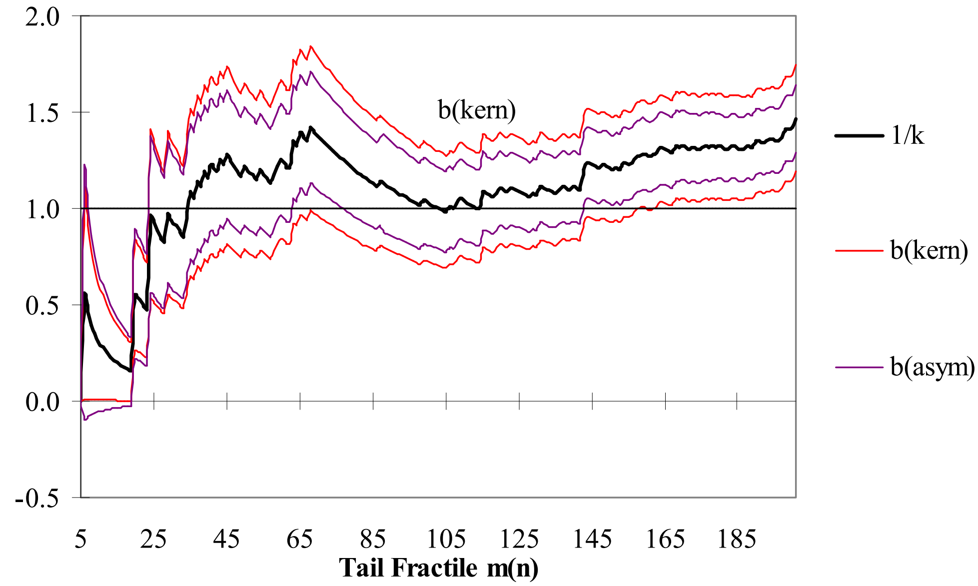

Figure 10: The tail index estimator $\hat{\kappa}_{m_n}^{-1}$ for BCTS data and 95% confidence bands computed from the variance estimator $\hat{\sigma}_{m_n}^2$ (kern) or $\hat{v}_{m_n}^2$ (asym).

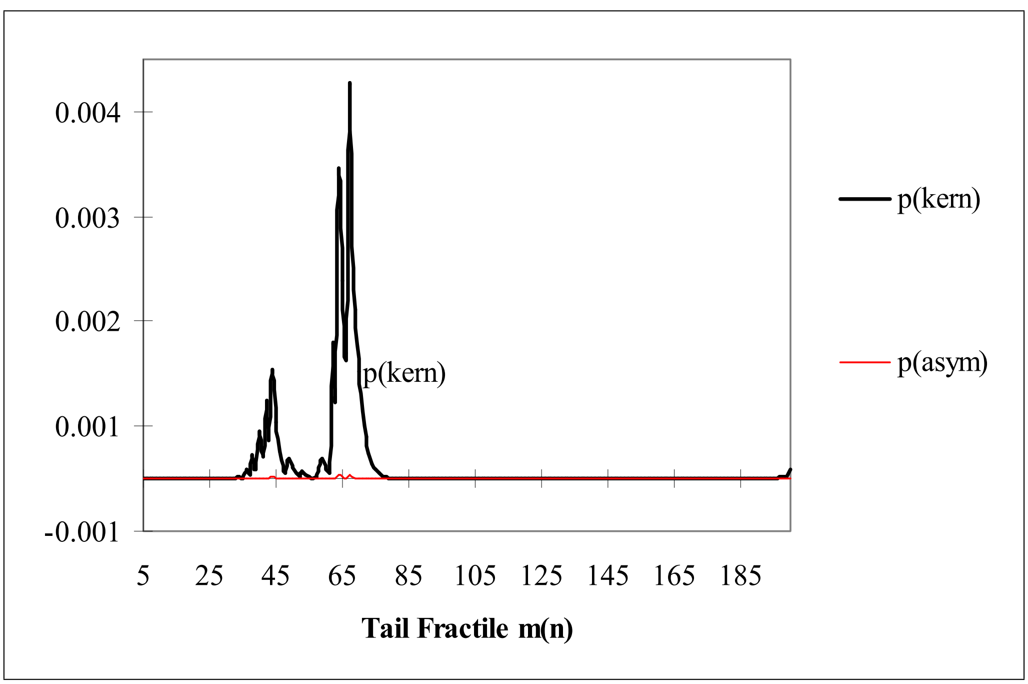

Figure 11: Test p-values for BCTS data based on the variance estimator $\hat{\sigma}_{m_n}^2$ (kern) or $\hat{v}_{m_n}^2$ (asym). The null hypothesis is PV.

Table 1: Summary statistics

Variable	Description	Mean	Std. Dev.	Min	Max
Auction date	Date on which the auction was held			14-Jan-05	14-Dec-06
Bonus bid	$ amount over the reserve price per m3	10.277	9.947	0.000	63.550
Price	Maximum bid	12.749	12.271	0.010	63.550
Reserve price	Upset rate per m3	28.655	10.858	0.250	50.220
Normalized bid	Bid divided by upset rate	0.457	0.599	0.000	5.278
Normalized price	Price divided by upset rate	0.581	0.740	0.000	5.278
Distance	Distance from bidder's to site location	44.898	52.278	0.000	411.364
Volume	Estimated volume of timber (m3)	26,372	20,944	200	122,113
Number of bidders	Number of bidders in the auction	4.560	2.230	1	10
Species composition:	Estimated fractions of various wood species in percentages				
Lodgepole		48.068	38.508	0.000	100.000
Spruce		12.696	15.609	0.000	93.000
Douglas-fir		9.382	20.618	0.000	99.000
Other		29.867	37.449	0.000	100.000
Number of observations		1835			
Number of auctions		611			

MARKET PRICING SYSTEM - COAST

APPENDIX 1

List of Variables

Predicted Bid	Estimated winning bid for the cutting authority ($\$/m^3$).
3-Month Average Log Selling Price	Average coniferous log selling price estimate ($\$/m^3$) based upon log grades and species for the cutting authority area, and schedules of log market values collected and published by Revenue Branch.
2^{nd} Growth Fir %	The fraction of the coniferous cruise volume that is second growth Douglas-fir.
2^{nd} Growth Hembal %	The fraction of the coniferous cruise volume that is second growth hemlock.
Old Growth Hembal %	Fraction of coniferous cruise volume that is old growth hemlock and balsam.
Slope	Average side slope percentage for the cutting authority area that is not harvested by helicopter.
Volume per Hectare	Volume per hectare in m^3/ha. Calculated by dividing the total net coniferous cruise volume (m^3) by the total merchantable area (ha).
Helicopter Logging %	Fraction of the total net cruise volume (which includes deciduous volume) that is required to be helicopter yarded plus the volume yarded by skyline (i.e., logs fully suspended) greater than 600 m straight line yarding distance measured from the centre of the closest possible landing into the cut block.
Haul Distance	Truck haul distance (km).
Number of Bidders	Estimated number of bidders that would compete for the cutting authority.
Volume	Total net cruise volume of coniferous timber (m^3).
Cable Yarding %	Fraction of the total net cruise volume (which includes deciduous volume) that needs to be cable yarded.
Cruise Grades	Cruise Grades = 1, where 50 percent or greater of the total net cruise volume has used the cruise compilation as the source of log grades for the appraisal, otherwise Cruise Grades = 0.
Location	The straight line distance (km) from the geographic centre of the cutting authority area to the nearest location listed below: Vancouver, Chilliwack, Merritt, Victoria, Nanaimo, Campbell River, Prince Rupert, Terrace, Houston.
Barge Distance	Barge distance is the barging distance (km) between the appraised point of origin and the point of appraisal for the cutting authority area.

MARKET PRICING SYSTEM - COAST

APPENDIX 2

MPS Coast Equation –Winning Bid

Dependent Variable: Real Winning Bid (for stands > 2,500 m3)
Method: Least Squares
Sample: January 1, 1999 to December 31, 2002
Included observations: 248
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-22.14037	5.944577	-3.724466	0.0002
Cruise Grades	3.460424	1.651467	2.095363	0.0372
2nd Growth HemBal_%	-19.00256	4.440790	-4.279094	0.0000
3-Month Average Log Selling Price	0.784393	0.061253	12.80572	0.0000
*LN (Old Growth HemBal_% + 0.01)	-2.879611	0.605312	-4.757236	0.0000
Slope	-0.166169	0.052742	-3.150589	0.0018
Helicopter Logging_%	-40.09100	3.506940	-11.43190	0.0000
*LN(Volume per Hectare/1000)	11.94704	1.827940	6.535793	0.0000
*LN(Number of Bidders)	10.06841	1.477136	6.816169	0.0000
Haul Distance	-0.034161	0.020904	-1.634167	0.1036
Barge Distance	-0.011281	0.002742	-4.114145	0.0001

R-squared	0.757806	Mean dependent var	44.39302
Adjusted R-squared	0.747587	S.D. dependent var	22.83775
S.E. of regression	11.47385	Akaike info criterion	7.761359
Sum squared resid	31200.86	Schwarz criterion	7.917196
Log likelihood	-951.4085	F-statistic	74.15553
Durbin-Watson stat	1.608942	Prob(F-statistic)	0.000000

*LN means natural logarithm.

REPORT ID : CSP-R11
RUN DATE 2007-01-24
RUN TIME : 07:34 AM

MINISTRY OF FORESTS
REVE. JE BRANCH

Log Monthly Report - Average Log Prices

FROM 2006-01-01 THROUGH 2006-12-31

Sale Type : O - Old Growth

DOMESTIC

Grade		Alder	Birch	Cedar	Cottonwood	Cypress	Fir	Hembal	Maple	Spruce	White Pine
B	AMV :	0.00	0.00	0.00	0.00	0.00	197.48	0.00	0.00	0.00	0.00
	% :	0.00	0.00	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.00
C	AMV :	0.00	0.00	0.00	0.00	0.00	140.95	0.00	0.00	0.00	0.00
	% :	0.00	0.00	0.00	0.00	0.00	8.52	0.00	0.00	0.00	0.00
D	AMV :	0.00	0.00	253.66	0.00	475.01	393.13	142.09	0.00	282.62	62.18
	% :	0.00	0.00	2.51	0.00	1.35	1.80	1.58	0.00	1.84	0.58
E	AMV :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	251.18	0.00
	% :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.38	0.00
F	AMV :	0.00	0.00	223.04	0.00	245.69	264.40	102.69	0.00	221.90	51.20
	% :	0.00	0.00	1.17	0.00	2.09	2.25	1.65	0.00	1.00	0.62
G	AMV :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	185.60	0.00
	% :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	0.00
H	AMV :	0.00	0.00	164.27	0.00	123.82	134.08	73.90	0.00	80.38	46.14
	% :	0.00	0.00	14.64	0.00	23.07	19.76	12.34	0.00	25.39	18.89
I	AMV :	0.00	0.00	114.57	0.00	77.95	102.26	55.13	0.00	61.20	33.74
	% :	0.00	0.00	7.79	0.00	8.12	13.28	12.54	0.00	18.00	10.36
J	AMV :	0.00	0.00	134.78	0.00	62.08	85.21	52.33	0.00	64.32	60.40
	% :	0.00	0.00	11.12	0.00	38.34	37.94	31.10	0.00	27.90	56.93
K	AMV :	0.00	0.00	152.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	% :	0.00	0.00	10.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L	AMV :	0.00	0.00	128.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	% :	0.00	0.00	21.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M	AMV :	0.00	0.00	90.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	% :	0.00	0.00	14.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
U	AMV :	0.00	0.00	64.98	0.00	39.89	55.19	38.79	0.00	37.46	32.25
	% :	0.00	0.00	12.31	0.00	15.15	10.49	17.01	0.00	10.81	7.41
X	AMV :	78.75	0.00	46.75	0.00	27.03	34.20	36.96	64.17	35.47	26.63
	% :	96.64	0.00	2.60	0.00	6.91	2.76	14.82	99.73	4.63	3.41

MINISTRY OF FORESTS
REVE JE BRANCH

Log Monthly Report - Average Log Prices

FROM 2006-01-01 THROUGH 2006-12-31

Sale Type : O - Old Growth
DOMESTIC

Grade		Alder	Birch	Cedar	Cottonwood	Cypress	Fir	Hembal	Maple	Spruce	White Pine
Y	AMV :	29.34	0.00	9.53	34.64	10.79	30.34	36.03	17.25	35.34	28.54
	% :	3.36	0.00	0.95	100.00	4.97	2.40	8.96	0.27	6.62	1.81
Z	AMV :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	% :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Volume (m3)		55,439.50	0.00	1,193,315.21	27,718.81	231,351.30	699,473.51	1,852,429.86	3,680.70	149,453.86	17,592.80
Species %		1.31	0.00	28.21	0.66	5.47	16.53	43.79	0.09	3.53	0.42
AMV :		77.09	0.00	123.45	34.64	78.70	106.47	51.55	64.04	72.80	51.08
Rounded AMV :		77.10	0.00	123.50	34.60	78.70	106.50	51.50	64.00	72.80	51.10

Total volume (m3)	4,230,455.55
Total value ($)	352,727,162.87
$/m3 :	83.38