JENA ECONOMIC
RESEARCH PAPERS

2010 – 014

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www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

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D-07743 Jena
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Kahlaische Str. 10
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Rules (of Bidding) to Generate Equal Stated Profits
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March 2010

Abstract

Although one may hope to achieve equality of stated profits without enforcing it, one may not trust in such voluntary equality seeking and rather try to impose rules (of bidding) guaranteeing it. Our axiomatic approach is based on envy-free net trades according to bids which, together with the equality requirement, characterize the first-prize auction and fair division game.

JEL classification: D44, D63, C72, D74

Keywords: Auctions, Fair Division, Procedural fairness, Envy-Freeness

1. Introduction

Equality of profits is often a major concern of cooperating parties like, for instance, of firms, other organizations, or private persons engaged in joint ventures. The problem we study is not how to share profits equally during the joint venture but rather to induce equal profits when terminating it. For this situation, we assume that an indivisible object, e.g. the joint venture firm but also any other unique indivisible commodity like a house or a painting, has to be allocated to one of several parties who state “profits” by bidding.

The rules of bidding thus have to determine for all possible bid vectors who “wins” the object and what the “winner” must pay. In case of an auction, the payment goes to the seller who is not a member of the bidder group for which we want to guarantee equality of stated profits. In fair division conflicts where the bidders collectively own the object what the “winner” pays are the monetary compensations for the non-winning bidders.

From imposing envy-freeness of net trades according to bids (Güth, 1986), it follows that the winner is the highest bidder and that the price must be in the interval of the second-highest and highest bid. In case of fair division conflicts, the “price” is the sum of equal monetary compensations. Additionally requiring equal stated profits of all bidders uniquely determines the price rule asking the winner to pay a price equal to his own bid. The two axioms together thus characterize the first-price auction and fair division game.

2. Axiomatic analysis

Let $N = \{1, ..., n\}$ with integer $n \geq 2$ denote the group of bidders $i \in N$ and assume some indivisible valuable object which

- in case of an auction is owned by an external seller offering to sell the object to one of the bidders in N

- in case of a fair division conflict, it is collectively owned by the group N of bidders who, however, want to sell the object to one of them, e.g. to terminate a joint venture.

As usual for the legal rules, e.g. those of public procurement auctions (see Gandenberger, 1961, for some historical account) or those for dissolving joint ventures and settling inheritance and divorce conflicts, we assume bidders $i \in N$ to submit monetary bids $b_i\left(\geq 0\right)$ declaring their idiosyncratic evaluation of the object. In the following we will refer to these bids b_i as the stated evaluations of the object by bidders i and speak of equality of stated profits when the net trades as evaluated by bids are equally profitable.

The rationale for guaranteeing equality of stated rather than true profits is that true evaluations are nearly always private information what renders guaranteeing equal true profits impossible. One might, of course, impose incentive compatibility to induce truthful bidding. But this requires rather special rules with serious drawbacks (see, for instance, Fehl and Güth, 1987, for auctions and, more generally, Wilson, 1987, asking for robust mechanisms whose rules do not depend on “volatile” parameters like private beliefs).

What the rules of bidding have to determine is for all possible bid vectors $b = \left(b_1, ..., b_n\right)$

- who “wins”, i.e., the winner $w\left(b\right) \in N$, and
- what $w\left(b\right)$ has to pay, i.e., the price $p\left(b\right)$ paid to the seller in case of an auction and the monetary compensations $t_i\left(b\right)$ which $w\left(b\right)$ pays to all non-winners $i \in N, i \neq w\left(b\right)$, in case of a fair division conflict.

We determine these rules axiomatically by imposing

Axiom 1: According to their bids, i.e., to their stated evaluations, no bidder $i \in N$ prefers the net trade of another bidder $j \in N, j \neq i$, to his own one.

Clearly, in case of a fair division conflict, axiom 1 requires

$$t_i(b) = t_j(b) =: t(b)$$

for all $i, j \in N$ with $i, j \neq w(b)$ and for all bid vectors b. To jointly characterize the rules of bidding for auctions and fair division conflicts, we also define a "price" for fair division conflicts via

$$p(p) := nt(b) \text{ for all bid vectors } b.$$

Thus, envy-freeness in the sense of axiom 1 requires

$b_{w(b)} - p(b) \geq 0$ for $w(b)$ and

$0 \geq b_i - p(b)$ for all $i \in N$ with $i \neq w(b)$

in case of an auction as well as

$b_{w(b)} - \frac{n-1}{n} p(b) \geq p(b)/n$ for $w(b)$ and

$p(b)/n \geq b_i - \frac{n-1}{n} p(b)$ for all $i \in N$ with $i \neq w(b)$

in case of a fair division conflict. Adding $p(b)$, resp. $\frac{n-1}{n} p(b)$ in case of an auction, resp. fair division conflict yields

$b_{w(b)} \geq p(b) \geq b_i$ for all $i \in N$ with $i \neq w(b)$.

According to axiom 1, the rules thus allocate the object to the highest bidder[1] who has to pay a "price" in the (closed) interval of the second-highest and highest bid. The ambiguity of the pricing rule $p(b)$, allowed by axiom 1, can be resolved by additionally imposing

[1] In case of more than one highest bidder, anonymity would require to determine each of them with equal probability as the winner $w(b) \in N$ what, in the following, will be neglected.

Axiom 2: According to their bids, i.e., to their stated evaluations, the stated profits of all bidders $i \in N$ are equal.

In case of an auction, all non-winners $i \in N$, $i \neq w(b)$, do not exchange anything, i.e., have net trades evaluated by 0. Requiring a net trade with 0-stated profit also for $w(b)$, thus requires

$b_{w(b)} - p(b) = 0$ or $b_{w(b)} = p(b)$.

Similarly, for fair division conflicts the requirement

$b_{w(b)} - \frac{n-1}{n} p(b) = p(b)/n$ or $b_{w(b)} = p(b)$

determines the first-price rule $b_{w(b)} = p(b)$ for all bid vectors b .

Proposition: For auctions as well as for fair division conflicts, axiom 1 and 2 together imply to allocate the object to the highest bidder at the price of his bid, i.e., according to the first-price rule.

General truth telling, i.e., bids expressing true evaluations, guarantees thus both, envy-freeness of net trades and equality of profits for all bidders, not only in view of stated but also in view of true evaluations. In case of auctions, our axiomatic approach could be more relevant for procurement by public authorities or agencies, so-called public tender auctions, than for private sales auctions where the seller may prefer revenue maximizing rules (e.g. Riley and Samuelson, 1981; Mascin and Riley, 1984) over procedurally fair ones. This may be different when private sellers employ auction houses or (Internet) platforms which may feel obliged to procedural fairness to attract many bidders. In case of fair division conflicts, procedural fairness seems to be of utmost importance and is often legally implemented to resolve conflicts when the parties cannot agree among themselves on the rules of bidding to be used.

3. Discussion

The two axioms, applied above, define fair rules (of bidding) without necessarily claiming that this implies fair allocation results as, for instance, judged by the bidders' true rather than stated evaluations where even that would be guaranteed if bids were truthful. Imposing incentive compatibility (in the sense of dominance solvability) instead of axiom 2 would have characterized the second-price auction (Vickrey, 1961) and implied an impossibility result for fair division conflicts (Güth, 1986).

The rules (of bidding) which we have derived above are not yet complete in the sense of yielding well-defined games. To complete the rule specification, one would have to include true evaluations and what bidders know about the true evaluations of their co-bidders (see Güth and van Damme, 1986, and for experimental studies Güth, 1986; Güth et al., 2002 and 2003; Becker and Brünner, 2009).

Axiom 1 is rather universally satisfied what renders it very convincing. One rare exception is the hiring of, e.g. academic teachers in the tradition of Continental Europe. Here a position is announced for a fixed honorarium $h(>0)$ without paying attention to competition on the supply side. If, for instance, two equally qualified candidates X and Y with outside option payoffs x and y, respectively, satisfying $h > x > y > 0$ would apply, each of them would envy the net trade of the other when the other is hired, earning the honorarium h. Envy-freeness would require to lower the honorarium so that $x \geq h \geq y$ applies, i.e., the honorarium is reduced to a range where only candidate Y prefers his being hired to his outside option y.

Equality is postulated by equity theory (Homans, 1961) in situations where all bidders $i \in N$ have contributed equally to the success, e.g. of a joint venture, and is seen as an important goal which, however, may be conflicting with own profit maximization according to the concept of inequity aversion (e.g. Bazerman, Löwenstein and Thomson, 1989; Bolton, 1991; Bolton and Ockenfels, 2000; Fehr and Schmidt, 1999). The (experimental) confirmation so far relies on small group interaction without stochastic uncertainty, e.g. in the sense of private information. With private information people may not be equality seeking, as judged by their true evaluations, but may rather try to guarantee only equality of stated profits.

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