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Tax Morality and Progressive Wage Tax

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Tax Morality and Progressive Wage Tax

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Abstract

We analyze the impact of tax morality on progressive income (wage) taxation. We assume that transfers (cash-back) and public expenditures are financed from linear wage taxes. We derive the reported wages from individual utility maximization, when individuals obtain partial satisfaction from reporting wages (depending on their tax morality), and cannot be excluded from the use of public services. The government maximizes a utilitarian social welfare function, also taking into account the utility of public services. The major conjecture is illustrated by numerical examples: the optimal degree of redistribution and the size of the public services are increasing functions of the individuals' tax morality.

Keywords: tax moral, reporting earnings, progressive income tax, welfare economics

JEL: H55, D91

Adómorál és adórendszer

Simonovits András

Összefoglaló

A tanulmányban az adómorál hatását elemezzük a társadalmilag optimális progresszív személyi jövedelemadó-rendszerre. Feltesszük, hogy a mindenkinek egyformán járó alapjáradékot és a közkiadást lineáris béradó fedezi. A bevallott kereseteket egyéni hasznosságmaximalizálásból származtatjuk, amikor a dolgozók – adómoráljuktól függően – némi kielégülést nyernek keresetük bevallásából, és nem zárhatók ki a közjavak fogyasztásából. A kormányzat utilitarista társadalmi jóléti függvényt maximalizál, amely figyelembe veszi a közkiadásból fakadó egyéni hasznosságokat is. Számpéldán igazoljuk, hogy a társadalmilag optimális újraelosztás foka és a közkiadások szintje az egyének adómoráljának egyaránt növekvő függvénye.

Tárgyszavak: adómorál, keresetbevallás, progresszív jövedelemadó, jóléti gazdaságtan

JEL: H55, D91

1. Introduction

In the last decades, tax and social contribution rates have increased in most of the developed countries, and as a result, the size of the hidden (or equivalently, shadow) economy has also risen (e.g. Frey and Weck-Hannemann, 1984; Schneider and Enste, 2000). The most popular expression of this connection is the so-called Laffer-curve, depicting the dependence of total tax revenues on marginal tax rates. In addition, Frey and Weck-Hannemann (1984) introduced a parameter "tax morality", which may have a quantitatively larger and statistically stronger influence on the tax revenue than the direct tax share has (cf. also Friedman et al., 2000). Even if one cannot measure tax morality, it is obvious that different countries with the same total tax share may have characteristically different sizes of the hidden economy. It is enough to look at Table 10 of Schneider (2002, p. 29) and discover that in 1996, in terms of the GDP, Greece and Austria had a total tax and social security burden about 70%, while the sizes of the shadow economies were 28.5% and 8.3%, respectively. Similar phenomena can be observed in transition (Schneider, 2002 and Lackó, 2007) and other countries.

To change the income distribution, one part of the tax revenue is paid back to the individuals. The other part, called the net tax is the difference between gross tax and cash-back, it finances public services. We speak of a *progressive* tax system if the ratio of net tax to reported wage is a decreasing function of the reported wage. In the present paper we construct a very simple model to study the interrelation between tax morality and progressive income (wage) taxation. The main message of the paper is as follows: for any level of tax morality, there exists a socially optimal wage-tax system, the size and progressivity of which grow with the tax morality.

Mirrlees (1971) was the first to study a market economy, where the labor supply is a decreasing function of the net wage and the government maximizes the social welfare under incentive constraints. To simplify the model, Sheshinski (1972) (see also Atkinson and Stiglitz, 1980) confined the welfare discussion to linear tax functions. He summarized his main result as follows: "...the optimal marginal tax rate is bounded by a fraction that decreases with the minimum elasticity of labor supply". Representing a progressive system as an inhomogeneous linear function with a negative incept (cash-back) can be a good approximation and becoming quite popular in transition economies by now. Nevertheless, the introduction of tax credits diminishes the burden of a negative income tax (cf. Akerlof, 1978). A third dimension of taxation was modeled by Varian (1980): social insurance against sickness or unemployment. Among the first modelers, Romer (1975) obtained interesting results concerning the majority voting on linear income tax—an alternative to welfare analysis.

Studying the effects of the Russian flat tax reform of 2001, Gorodnichenko et al.

(2009, p. 505), concluded that "(i) the tax evasion response to the tax change was large, but that (ii) the productivity response was relatively weak ...." Relying on this type of observations, the present paper models underreporting rather than restrained labor supply. As a technical simplification, we assume full employment. Following the methodology of neoclassical economics, we derive the reported (declared) wages from individual utility maximization, where the individual finds certain satisfaction in partially reporting wages, while free-rides on public goods and services. Note that here an individual only reports his wages to increase his resulting moral satisfaction. (In reality, the employer also influences this decision but we neglect this complication!) We call the reader's attention to a special feature of our formulation: though with rising marginal tax rate, the reported wage decreases, it does not disappear even at a 100 percent. As a result, the tax revenue may increase with the marginal tax rate in the entire interval! This result is in harmony with that of Malcomson (1986) with an entirely different approach.

While one part of the taxes serves income redistribution, another part finances public goods, to be used freely by everybody (or alternatively, assume a balanced tax system, where public goods are paid from individual incomes). Note that the health contributions can be considered as taxes because there is no strong relation between the individual tax and the resulting public health services. But the issue of unemployment or pension contributions and benefits needs a separate treatment (Baumann, 2009 and Simonovits, 2009, respectively), especially for countries with wage-related benefits. Of course, there are a lot of studies modeling the impact of redistribution on the labor supply in an overlapping generations model, incorporating a pension system (see the References in Simonovits, 2009).

In our model, the government has a utilitarian social welfare function, i.e. the expected value of the individual utility functions (with due respect to public goods and services). The parameter values of the socially optimal tax system are obtained from social welfare maximization. Following others, we also add another idealization to the model: there is only a wage tax but no value added tax (for the difference between income and value added taxes under tax evasion, see Richter and Boadway, 2005). We investigate the impact of the size and of the structure of the tax system on the reported wages of individuals with heterogeneous wages and tax moralities in a general equilibrium framework. We also assume that the individuals fully understand the mechanism of the system and the government determines its tax policy as a benevolent dictator.

The major conjecture of the present paper—an analog to Sheshinski's result cited above—is as follows: *The higher the tax morality, the higher are the socially optimal marginal tax rate and—above a certain critical morality value—the cash-back.* In the limiting case, when the individuals report their full wages (white economy), and the labor supply is still exogenously given, it is worthwhile to introduce a fully redistributive wage tax, equalizing individual consumption volumes (like in a kibbutz).

Having outlined the general model, we analyze the issue with a simple example, where the utility functions are logarithmic (though the utility function of underreporting also contains a linear term). Even then we were only able to derive the conditional individual decisions, without solving the general equilibrium problem in a closed form. The only exception is the white economy, where even the socially optimal tax parameter values can analytically be determined.

When we turn on the computer, and fill the model with illustrative numbers, we are able to consider the balance conditions, too. The most important illustrative results are contained in Table 6: as the common tax morality rises, both the size and the progressivity of the socially optimal tax system increase.

The technical problems of modeling required utmost simplifications. We hope that notwithstanding its abstractness, our toy model contributes to the better understanding of the interaction among reporting of wages, taxation and the public services. Anyway, we must consider the conclusions with utmost care.

Having presented the model, now we outline the related literature. The classical paper of tax evasion is Allingham and Sandmo (1972), which determined the optimal reported wage under given wage, marginal tax rate, audit probability and fine. Pestieau and Possen (1991) modeled the connection between tax evasion behavior and the choice of work place, anticipating a most serious problem of any transition economy. Feldstein (1999) and Chetty (2009) analyzed tax avoidance and the deadweight loss of the income tax, including underreported incomes into the generalized labor supply.

Turning from flexible labor supply to reporting wages, Valdés-Prieto (2009) analyzed a model, including a pension sector, where individuals choose the *density of contribution*, i.e. "the proportion of active life worked in fully covered jobs (density) rather than hours worked." Instead of diminishing the individual's utility as underreporting does in our model, he explicitly modeled the lower productivity achieved in the informal sector and compared the efficiency of various insurance systems, including pension credit.

In their survey, Andreoni et al. (1998) extended the narrow neoclassical model and introduced soft but relevant concepts like moral sentiments and the satisfaction of the taxpayer with public goods and services. From our point of view, they made three important claims: (i) the morally more sensitive citizens declare a higher share of their true wages; (ii) the more unfair the tax system is in the eyes of the citizens, the less wage they declare; (iii) the less satisfied the taxpayers are with the public goods and services, the less wage they declare. In a related study, Lackó (2007) demonstrated that the tax morality is closely related to the citizens' opinion of the utility of public goods and services.

Christie and Holzner (2006) considered both old and new EU members and obtained interesting estimates on the degree of tax evasion. For example, according to their Table 1.19, in Hungary, 2002, individuals reported 69.8 percent of their personal income tax base. In contrast, their Table 1.10 contains a pair 76.6 percents on the Czech Republic, 2003. Similarly, Janky (2007) considered the public attitudes toward tax evasion in Hungary and he cited remarkable data from CEORG Omnibus survey 1999 (Table 2). To the question "Is it improper for a tax-payer to provide a false income statement in order to avoid taxes?", 53% of the Hungarians and 77% of the Czechs answered with disapproval, while 42% of the Hungarians and 18% of the Czechs found it excusable. Bakos et al. (2008) discussed the tax evasion behavior of the Hungarian employees, using the reactions to the Hungarian tax reform of 2005.

OECD (2002) gave a wide survey on the 'non-observed' economy. For a comparative analysis of the hidden economies of transition countries, see also Lackó (2000). Tonin (2007) and Elek et al. (2009) discussed the related interaction of minimum wage and tax evasion.

We outline the structure of the present paper. Section 2 develops the model and then

Section 3 presents some analytical results. Sections 4 displays numerical illustrations and Section 5 concludes. An Appendix proves the corollaries.

2. The model

This section starts with the determination of that reported wage which maximizes the utility function of a given individual. Then we outline the macro framework and the social welfare function to be maximized by the government.

Utility maximization

In a given period of time, say in a year, an individual earns wage $w > 0$ and files in a report v on this wage, $v \in [0, w]$. He pays a linear wage tax θv with a marginal tax rate θ , $0 < \theta < 1$ and receives a lump-sum return, i.e. cash-back $\varepsilon \geq 0$. The individual's net tax is $a = \theta v - \varepsilon$. (This is the simplest formulation of progressive wage taxation.) The average net tax finances public goods and services including health care.

The individual's consumption is given by

$$c = w - \theta v + \varepsilon.$$

Anticipating the macro framework below, we define the net average tax by

$$\bar{a} = \theta \bar{v} - \varepsilon > 0,$$

where $\bar{v}$ denotes the average reported wage (see below). If one does not want to consider public goods, (s)he can get rid of it by adding a balance condition, namely $\bar{a} = 0$ or equivalently, $\varepsilon = \theta \bar{v}$.

In the present model, every individual has two characteristics, namely his *wage* w and his *tax morality* p. The *subjective* utility function consists of three parts: (i) the individual's utility $u(c)$, (ii) the utility of reporting wage v when one earns w and has a tax morality p: $z(w, p, v)$ and (iii) the utility of the public good $q(\bar{a})$ financed by the average net tax $\bar{a}$. (The second term corresponds to the moral sentiments in Andreoni et al. (1998) plus the utility loss due to the cost of underreporting in Feldstein (1999).) In sum,

$$\hat{U}(w, p, c, v) = u(c) + z(w, p, v) + q(\bar{a}).$$

(The hat distinguishes this function from the one to be used below.) As is usual, $u(\cdot)$, $z(w, p, \cdot)$ and $q(\cdot)$ are smooth, strictly increasing and strictly concave functions in the feasibility intervals. In addition, the usual properties are assumed:

$$u'(0) = \infty, \quad u'(\infty) = 0, \quad z'_v(w, p, 0) = \infty, \quad z'_v(w, p, w) = 0, \quad q'(0) = \infty, \quad q'(\infty) = 0.$$

As *a detour*, we mention that in the classical case of flexible labor supply, where l stands for the labor supply, consumption is $c = (1-\theta)wl + \varepsilon$ and the utility of reporting is replaced by labor disutility $z(l)$.

By definition, the individual determines his reported wage $v(w, p)$ so as to maximize the subjective utility $\hat{U}(w, p, c, v)$. By substituting the consumption equation into $\hat{U}(w, p, c, v)$ and replacing the hat by tilde, results in another form of the utility function:

$$\tilde{U}(w, p, v) = u(w - \theta v + \varepsilon) + z(w, p, v) + q(\bar{a}).$$

Neglecting the individually uncontrollable third term, partial derivation by v yields the individual optimum:

Theorem 1. *Under the above assumptions, the individually optimal reported wage $v(w, p)$ lies in the interval $(0, w)$ and satisfies*

$$-\theta u'(w - \theta v + \varepsilon) + z'_v(w, p, v) = 0.$$

Note two simple consequences: (i) If there is no tax, then people report their full wages: for $\theta = 0 = \varepsilon$, $v = w$ holds. (ii) Even for a 100 percent marginal tax rate, people report some wage: for $\theta = 1$, $v > 0$ holds.

To make the optimal reported wage increasing with the tax morality, wage and cash back but decreasing with marginal tax rate, we need additional technical assumptions on the utility to report function:

$$z''_{vw}(w, p, v) > 0, \qquad z''_{vp}(w, p, v) > 0, \qquad z''_{vv}(w, p, v) < 0.$$

Corollary 1. *Under our additional assumptions, the optimal reported wage is an increasing function of the tax morality, wage and cash back but is a decreasing function of the marginal tax rate:*

$$v'(p) > 0, \qquad v'(w) > 0, \qquad v'(\varepsilon) > 0, \qquad v'(\theta) < 0.$$

The proof is relegated to an appendix.

Macro framework

Until now we have only considered a single, though arbitrary individual. In reality, there exists a lot of types, with a 2-dimensional probability distribution function F: $F(W, P) = \mathbf{P}(w < W, p < P)$, where w and p are the random variables of wage and tax morality, respectively and $\mathbf{P}$ is the probability operator.

It is time to define the average reported wages:

$$\bar{v} = \mathbf{E}v,$$

where $\mathbf{E}$ is the expectation operator. The optimal v depends on the government tax parameters θ and ε and so does $\bar{v}$. Finally, let T be the per-capita total tax revenues: $T = \theta\bar{v} = \bar{a} + \varepsilon$. Note that the requirement of positive net average tax is equivalent to $T > \varepsilon$, which limits the redistribution.

As has been mentioned in the Introduction, since the advent of supply-side economics around 1980, the Laffer-curve, the per capita tax revenue–marginal tax rate schedule $T(\theta)$, has been enjoying a central role in public finance. Theorem 1 implies that the individual reported wage is a decreasing function of the marginal tax rate, hence the same is true for the average reported wage. It is clear that $T(0) = 0$ and $T'(0) \geq 0$. Since $T(\theta)$ is continuous in the interval $[0, 1]$, there is a marginal tax rate θ_L which maximizes the tax revenues. If that marginal tax rate is low, then the scope of redistribution and public expenditure is limited. Because of $\bar{v}(1) > 0$, it may occur (see Corollary 2 below), however, that $T(\theta)$ is increasing all over the interval $[0, 1]$, yielding a global maximum per capita tax at $\theta_L = 1$, i.e. $T(1) = \bar{v}(1)$.

At this point, we define the *minimal tax morality*, as the common value of individual tax moralities yielding zero net tax rate. In formula: p_m is the solution to the equation $\theta\bar{v}(p,\theta,\varepsilon) = \varepsilon$ for any given pair (θ,ε), with $\theta > \varepsilon$. Such a number exists because $\bar{v}(\cdot,\theta,\varepsilon)$ is an increasing function, $\bar{v}(0,\theta,\varepsilon) = 0$ and $\bar{v}(\infty,\theta,\varepsilon) = 1$, hence $\theta\bar{v}(p,\theta,\varepsilon) - \varepsilon$ satisfies the conditions of Bolzano's theorem.

From now on we always assume implicitly that the tax system is consistent with tax morality, i.e. the tax morality is higher than the minimal value: $p > p > p_m$. This notion can be extended to heterogeneous tax moralities but we skip it.

We conjecture that the higher is θ or ε, the higher is p_m.

Finally, we outline the government's welfare maximizing task. The utilitarian social welfare function is defined as the average of the objective utilities taken at the subjective optima (starred) $U(c^*, v^*, \bar{a}^*)$:

$$V(\theta,\varepsilon) = \mathbf{E}U(c^*, v^*, \bar{a}^*) = \mathbf{E}u(c^*) + \mathbf{E}z(v^*) + q(\bar{a}^*).$$

The government looks for a pair (θ,ε) such that maximizes the social welfare function.

Theorem 2. *The government's interior optimum satisfies the following first-order necessary conditions:*

$$0 = V'_\theta(\theta,\varepsilon) = \mathbf{E}[-u'(c^*)v^* + z'_v(w,p,v^*)v'_\theta + q'(\bar{a}^*)(\bar{v}^* + \theta\bar{v}'_\theta)]$$

and

$$0 = V'_\varepsilon(\theta,\varepsilon) = \mathbf{E}[u'(c^*) + z'_v(w,p,v^*)v'_\varepsilon + q'(\bar{a}^*)(\theta\bar{v}'_\varepsilon - 1)].$$

Proof. By elementary calculus,

$$0 = V'_\theta(\theta,\varepsilon) = \mathbf{E}[u'(c^*)c'_\theta + z'_v(w,p,v^*)v'_\theta] + q'(\bar{a}^*)\bar{a}'_\theta$$

and

$$0 = V'_\varepsilon(\theta,\varepsilon) = \mathbf{E}[u'(c^*)c'_\varepsilon + z'_v(w,p,v^*)v'_\varepsilon] + q'(\bar{a}^*)\bar{a}'_\varepsilon.$$

Using the facts that $c'_\theta = -v^*$ and $c'_\varepsilon = 1$, $\bar{a}'_\theta = \bar{v}^* + \theta\bar{v}'_\theta$ and $\bar{a}'_\varepsilon = \theta\bar{v}'_\varepsilon - 1$, these conditions simplify to Theorem 2. ∎

It is not enough to determine the socially optimal tax system, it is worth studying the welfare implications of deviating from it. A standard procedure is to define the social welfare $V(\omega,\theta,\varepsilon)$ as a function of tax system (θ,ε) and a uniform multiplying of wages by a common factor ω. Then the inefficiency of using a tax system (θ,ε) rather than the socially optimal $(\theta^{\mathrm{o}},\varepsilon^{\mathrm{o}})$ can be defined by

$$V(1,\theta,\varepsilon) = V(\omega,\theta^{\mathrm{o}},\varepsilon^{\mathrm{o}})$$

where $\omega \le 1$.

Note that Theorem 2 is quite cumbersome and at least at this stage of research, it seems to be suitable to simplify the discussion by invoking simple utility functions and other simplifying assumptions.

3. Analytical results

In this Section we consider simple cases which can be studied analytically: logarithmic utility functions and white economy.

Logarithmic utility functions

Following several studies, we use logarithmic specification, although it eliminates certain subtler relations arising with CRRA. In order to exclude the anomaly that any individual can increase his subjective utility by overreporting his true wage, a linear term had to be added to the logarithmic function $z(w, p, \cdot)$. Here are the parametric functions:

$$u(c) = \log c, \qquad z(w,p,v) = p(\log v - v/w), \qquad q(\bar{a}) = \kappa \log \bar{a},$$

where $p > 0$ is the parameter of *the tax morality*, a key parameter of the present model and $\kappa \geq 0$ is the parameter of the efficiency of public goods and services. If $\kappa = 0$ (or minimal tax morality), we obtain the economy without public services, where the socially optimal net average tax is zero: $\theta\bar{v} = \varepsilon$. Note that $u'(c) = 1/c$, $z'_v(w,p,v) = p(1/v - 1/w)$. In the interval $0 < v \leq w$,

$$z''_{vw}(w,p,v) = p/w^2 > 0, \quad z''_{vp}(w,p,v) = 1/v - 1/w > 0, \quad z''_{vv}(w,p,v) = p/v^2 < 0,$$

as the additional assumptions require. Also $q'(\bar{a}) = \kappa/\bar{a} \geq 0$.

Theorem 3. *In the logarithmic specification, the optimal report v^* is equal to*

$$v^* = \frac{B - \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = p\theta, \qquad B = \theta w + p(\theta w + w + \varepsilon), \qquad C = pw(w + \varepsilon).$$

Proof. The optimality condition of Theorem 1 reduces to

$$Z(v) = \frac{-\theta}{w - \theta v + \varepsilon} + p\left(\frac{1}{v} - \frac{1}{w}\right) = 0.$$

Since $Z(0) = \infty$, $Z(w) < 0$ and $Z(\infty) > 0$, there exists a unique root $v^* \in (0, w)$ to $Z = 0$. (The other root lies to the right of w.) Since $Z = 0$ can be transformed into a quadratic equation

$$Av^2 - Bv + C = 0$$

with coefficients above, we have a closed-form expression for v^*. ∎

We present two corollaries to Theorem 3.

Corollary 2. *For the proportional taxation ($\varepsilon = 0$), the Laffer-curve $T(\theta)$ is an increasing function of the marginal tax rate θ.*

Corollary 3. *For homogeneous wages ($w \equiv 1$), the minimal tax morality is equal to*

$$p_m = \frac{\theta\varepsilon}{\theta - \varepsilon}, \qquad \theta > \varepsilon.$$

The proofs are relegated to an appendix.

White economy

Before giving up analytical hopes, it is worthwhile considering the limiting case of *white* economy, where every individual reports his full wage. We discuss first in general: $v = w$, then for the logarithmic case: $p = \infty$.

In the general case, the formula for consumption simplifies to

$$c = (1 - \theta)w + \varepsilon.$$

The relation $\bar{v} = \bar{w} = 1$ makes the Laffer-curve and equation for the net average tax rather trivial: $T(\theta) = \theta$ and $\bar{a} = \theta - \varepsilon$ or $\varepsilon = \theta - \bar{a}$. The social welfare function is now

$$V(\theta, \bar{a}) = \mathbf{E}u((1 - \theta)w + \theta - \bar{a}) + q(\bar{a}).$$

Theorem 4. *In a white economy, the socially optimal tax system redistributes everything:*

$$\theta^{\mathrm{o}} = 1 \quad \text{and} \quad \varepsilon = 1 - \bar{a}^{\mathrm{o}} = c^{\mathrm{o}},$$

where the optimal net tax rate is determined by

$$u'(1 - \bar{a}^{\mathrm{o}}) = q'(\bar{a}^{\mathrm{o}}).$$

Proof. To determine the social optimum note that by Jensen's inequality,

$$\mathbf{E}u((1 - \theta)w + \theta - \bar{a}) \leq u(1 - \bar{a}).$$

The upper bound on the social welfare function—which is sharp for uniform wages or $\theta = 1$—simplifies to

$$V[\bar{a}] = u(1 - \bar{a}) + q(\bar{a}), \quad \text{i.e.} \quad V(\theta, \bar{a}) \leq V[\bar{a}].$$

Taking the partial derivative with respect to $\bar{a}$, results in

$$V'[\bar{a}] = -u'(1 - \bar{a}) + q'(\bar{a}) = 0.$$

By our assumptions, there exists a unique solution, $\bar{a}^{\mathrm{o}}$. If wages are uniform, then the social optimum is indeterminate, only the net tax rate is determinate. In the relevant case of heterogeneous wages, however, the optimal marginal tax rate θ^{o} is equal to 1, implying $c^* = \varepsilon = 1 - \bar{a}^{\mathrm{o}}$ for any type. ∎

The economic content is clear: the higher the economic efficiency κ of public services, the lower is the share of (private) consumption of the total output.

For the special utility functions, Theorem 4 yields

$$\bar{a}^{\mathrm{o}} = \frac{\kappa}{1 + \kappa}.$$

Grey economy

As is usual, we shall call economies *grey*, where only part of the wage is reported. The lower the ratio of average reported wage to average wage, i.e. $\bar{v}$, the grayer (the darker) the economy is. Corollary 1 implies that for fixed tax parameter values, $\bar{v}(p)$ is an increasing function, i.e. the economy gets less grey as the tax morality rises. We add

Conjecture 1. *Consider an economy with a uniform tax morality p and heterogeneous wages. There is a critical morality value p^* with the following property. With the rise of tax morality p, the socially optimal marginal tax rate $\theta(p)$ rises together with the average reported wage $\bar{v}^*(p)$ and the optimal cash-back $\varepsilon(p)$ is zero for $p \le p^*$ and increasing otherwise. In the limit, $\theta(\infty) = 1$ and $\varepsilon(\infty) = 1/(1+\kappa)$ together with $\bar{v}^*(\infty) = 1$.*

4. Numerical illustrations

We continue our investigation by numerical illustrations. First consider the white economy. After some experimentation, we have chosen $\kappa = 1/3$ for the utility coefficient of the public services. This choice renders the results for the white economy (Theorem 4) reasonable, the socially optimal consumption is $\varepsilon^{\rm o} = 3/4 = c^*$ with $\bar{a}^* = 1/4$.

Next we move to the grey economy with various shades, represented by p. To begin with, we are studying the quantitative impact of tax morality parameter p on individual behavior in a *proportional* tax system with $\varepsilon = 0$. We have already shown that in the white economy, the socially optimal marginal tax rate is $\theta^{\rm o} = \kappa/(1+\kappa) = 0.25$. What is surprising is that this relation approximately remains valid for the grey economies with various shades!

Table 1 illustrates the quantitative form of Corollary 1: as the value of the tax morality rises, the reported wage increases, the individual consumption decreases. To display the white economy, we present the solution for $p = \infty$, in the last row of Table 1.

Table 1. *Impact of tax morality with proportional taxes: $w = 1$*

Tax morality p	Reported wage $v^{(P)}$	Consumption $c^{(P)}$	Net tax $a^{(P)}$
0.5	0.628	0.843	0.157
1.0	0.764	0.809	0.191
1.5	0.826	0.793	0.207
2.0	0.863	0.784	0.216
3.0	0.903	0.774	0.226
5.0	0.939	0.765	0.235
∞	1	0.750	0.250

Remark: $\theta = 0.25$ and $\varepsilon = 0$

To study the impact of progressivity, we choose $\varepsilon = 0.15$ and raise the marginal tax rate to $\theta = 0.3$. Table 2 displays the results. Because of the lower net tax rate $\theta - \varepsilon$, consumption becomes higher and public expenditure becomes lower in the limit than in Table 1. We start now with $p = 0.3$, the minimal tax morality of Corollary 3.

Table 2. *Impact of tax morality with progressive taxes: $w = 1$*

Tax morality p	Reported wage v^*	Consumption c	Net tax a
0.3	0.500	1	0
0.5	0.617	0.965	0.035
1.0	0.755	0.924	0.076
1.5	0.819	0.904	0.096
2.0	0.856	0.893	0.107
3.0	0.898	0.881	0.119
5.0	0.935	0.869	0.131
∞	1	0.850	0.150

Remark: $\theta = 0.3$ and $\varepsilon = 0.15$

Next we present a generalization of Corollary 2 for a progressive tax system, already discussed in Table 2. We pick up three different morality values: $p = 0.5$, 2 and 5 and vary the marginal tax rate. Note that too low θs and ps result in $\bar{a} < 0$, then we write – in the corresponding entry of Table 3. Indeed, the second and third Laffer-curves are increasing in the interval $[0.2, 1]$, and the first one in $[0.3, 1]$.

Table 3. *Laffer-curves with progressive tax and various moralities*

Marginal tax rate θ	Tax revenue for $p = 0.5$ $T_{0.5}$	for $p = 2$ T_2	for $p = 5$ T_5
0.2	–	0.181	0.192
0.3	0.185	0.257	0.281
0.4	0.216	0.322	0.363
0.5	0.238	0.378	0.438
0.6	0.256	0.424	0.506
0.7	0.270	0.464	0.565
0.8	0.281	0.496	0.616
0.9	0.291	0.524	0.659
1.0	0.299	0.547	0.695

Remark: $\varepsilon = 0.15$

In Tables 1–3, we assumed perfect homogeneity. Next we introduce heterogeneous tax moralities, where we choose two values: $p_L = 0.5$ and $p_H = 2$ with relative frequencies f_L and $f_H = 1 - f_L$, respectively. As can be expected, as the share of low reporters decreases, the average reported wage and the net tax increase, while the consumption drops.

Table 4. *The impact of the frequency of low reporters*

Frequency of L f_L	Average reported wage $\bar{v}$	consumption $\bar{c}$	net tax $\bar{a}$
1.0	0.617	0.965	0.035
0.8	0.665	0.951	0.049
0.6	0.712	0.936	0.064
0.4	0.760	0.922	0.078
0.2	0.808	0.908	0.092
0.0	0.856	0.893	0.107

Remark: $\theta = 0.3$ and $\varepsilon = 0.15$, $p_L = 0.5$ and $p_H = 2$.

Until now we assumed identical wages. In the further calculations, we shall arbitrarily apply a very simple wage distribution: (w_L, w_H) with $f_H = 1/3$ and $f_L = 2/3$. To enhance its impact, we have chosen a rather extreme distribution with $w_L = 0.5$ and $w_H = 2$, yielding a unitary average wage. To simplify the exposition, we return to homogeneous tax morality, p.

Considering heterogeneous wages, we first display the numerical value of minimal tax morality, yielding close to zero net tax rate. We select a series of tax parameter values, which in turn are proportional to the absolute social optimum: $\theta^* = 1$ and

$\varepsilon^* = 3/4$, i.e. $\varepsilon = 3\theta/4$, $0 \le \theta \le 1$. We shall see that the closer the tax system to the absolute optimum, the higher is the minimal value of tax morality, p_m. (cf. Table 4). In words: higher taxes require higher tax morality. Furthermore, the stronger the taxation, the higher are the lower-paid's reported wage and consumption and the lower are the higher-paid's reported wage and consumption! If wages were the same, then $p_m = 3\theta$ would hold; but wages are different, therefore the actual p_m is higher.

Table 5. *Minimal tax morality as a function of the tax system*

Marginal tax rate θ	Cash-back ε	Minimal tax morality p_m	Wage w	Reported wage v	Consumption c
0.333	0.25	1.1			
			0.5	0.401	0.616
			2.0	1.487	1.754
0.667	0.50	2.2			
			0.5	0.414	0.724
			2.0	1.436	1.543
1.000	0.75	3.5			
			0.5	0.426	0.824
			2.0	1.404	1.346

Remark: $w_L = 0.5$ and $w_H = 2$.

Next we display the dependence of socially optimal tax parameters on the tax morality. Because this dependence is very weak, we rest satisfied with rounded-off values. We select four values of tax morality: $p = 0.5$, 2, 5 and ∞. With rising tax morality, the socially optimal marginal tax rate $\theta(p)$ rises until it reaches its maximum, 1. The cash-back $\varepsilon(p)$ also rises from $p^* \approx 0.5$, namely to 0.75, in harmony with our Conjecture 1. The reported wages and the public expenditures grow in parallel!

Table 6. *Socially optimal tax system—the tax morality*

Tax morality p	Optimal marginal tax rate θ	Optimal cash-back ε	Wage w	Reported wage v	Consumption c	Net tax $\bar{a}$
0.5	0.25	0.0				0.157
			0.5	0.314	0.422	
			2.0	1.255	1.686	
2	0.5	0.2				0.181
			0.5	0.400	0.500	
			2.0	1.489	1.456	
5	0.75	0.4				0.221
			0.5	0.442	0.569	
			2.0	1.600	1.200	
∞	1	0.75				0.25
			0.5	0.5	0.75	
			2.0	2.0	0.75	

Remark: $w_L = 0.5$ and $w_H = 2$.

To evaluate the efficiency loss of using a suboptimal tax system, we compare the taxes that are socially optimal in a dark/light grey economy in a light/dark grey economy, respectively. Here are the results: $\omega_{0.5|2} = 0.905$ and $\omega_{2|0.5} = 0.987$. That is, using a tax system which is optimal at $p = 2$ in a dark economy ($p = 0.5$) yields an efficiency loss of 9.5%, while using a tax system which is optimal at $p = 0.5$ in a light economy ($p = 2$) yields an efficiency loss of 1.3%. These numbers are probably too small and indicate a misspecification of our utility formulas.

Finally, we check the sensitivity of our results by reducing the inequality of the wages, with $w_L = 0.75$ rather than 0.5, i.e. $w_H = 1.5$ rather than 2. It is to be expected that in the lighter grey economies, the weaker the wage inequality, the lower are the socially optimal marginal tax rate and the cash-back but its extent is still surprising. Now the critical parameter p^* is around 2 rather than 0.5 and we had to raise the tax morality to 10 to obtain observable cash-back, arising already at $p = 2$ with a stronger wage inequality!

Table 7. *Socially optimal tax system—tax morality, weak wage inequality*

Tax morality p	Optimal marginal tax rate θ	Optimal cash-back ε	Wage w	Reported wage v	Consumption c	Net tax $\bar{a}$
0.5	0.25	0				0.157
			0.75	0.471	0.632	
			1.50	0.942	1.265	
10	0.45	0.2				0.225
			0.75	0.712	0.630	
			1.50	1.411	1.065	

Remark: $w_L = 0.75$ and $w_H = 1.5$.

5. Conclusions

We have constructed a model, in which—reflecting mainly the characteristics of transitional and developing countries—the size and the progressivity of the tax system influence the reported wages rather than the labor supply. We have just made the first theoretical and numerical computations. The results are promising but a lot of further analytical investigations and numerical trials are needed to corroborate the temporary conclusions: the introduction of a significant cash-back crowds out the public services, especially in economies with low tax morality.

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Appendix

The proof of Corollary 1. To prove the inequalities, we take the total derivative of both sides of

$$\theta u'(w - \theta v + \varepsilon) = z'_v(w, p, v)$$

with respect the foregoing variables or parameters.

(i) With respect to w:

$$\theta u''(c)[1 - \theta v'(w)] = z''_{vw} + z''_{vv} v'(w)$$

or expressing $v'(w)$:

$$\theta u''(c) - z''_{vw} = [\theta^2 u''(c) + z''_{vv}] v'(w).$$

Since $u''(c) < 0$, $z''_{vw} > 0$ and $z''_{vv} < 0$, $v'(w) > 0$.

(ii) With respect to p:

$$-\theta^2 u''(c)v'(p) = z''_{vp} + z''_{vv}v'(p)$$

or expressing $v'(p)$:

$$[-\theta^2 u''(c) - z''_{vv}]v'(p) = z''_{vp},$$

yielding $v'(p) > 0$.

(iii) With respect to ε:

$$\theta u''(c)[1 - \theta v'(\varepsilon)] = z''_{vv}v'(\varepsilon)$$

or expressing $v'(\varepsilon)$:

$$\theta u''(c) = [\theta u''(c) + z''_{vv}]v'(\varepsilon)$$

implying $v'(\varepsilon) > 0$.

(iv) With respect to θ:

$$u'(c) + \theta u''(c)[-v - \theta v'(\theta)] = z''_{vv}v'(\theta)$$

or expressing $v'(\theta)$:

$$u'(c) - \theta u''(c)v = [\theta^2 u''(c) + z''_{vv}]v'(\theta),$$

hence $v'(\theta) < 0$. ∎

The proof of Corollary 2. From Theorem 3,

$$T(\theta) = \frac{p + (1+p)\theta - \sqrt{(1+p)^2\theta^2 + 2p(1-p)\theta + p^2}}{2p}.$$

To demonstrate $T'(\theta) \geq 0$, we can drop the constant denominator, and show that the numerator's derivative is nonnegative:

$$1 + p - \frac{2(1+p)^2\theta + 2p(1-p)}{2\sqrt{(1+p)^2\theta^2 + 2p(1-p)\theta + p^2}} \geq 0.$$

Rearranging,

$$\begin{aligned}(1+p)^2[(1+p)^2\theta^2 + 2p(1-p)\theta + p^2] &\geq [(1+p)^2\theta + p(1-p)]^2 \\ &= (1+p)^4\theta^2 + 2(1+p)^2\theta p(1-p) + p^2(1-p)^2.\end{aligned}$$

With simple algebra, the last inequality is established. ∎

The proof of Corollary 3. For homogeneous wage and the minimal tax morality, $\bar{v} = v = \varepsilon/\theta$. Substituting this value and A, B, C into $Av^2 - Bv + C = 0$ yields

$$p_m\theta\frac{\varepsilon^2}{\theta^2} - [\theta + p(\theta + 1 + \varepsilon)]\frac{\varepsilon}{\theta} + p_m(1+\varepsilon) = 0.$$

Simple algebra yields p_m. ∎

References

Akerlof, G. A. (1978): "The Economics of "Tagging" as Applied to the Optimal Wage Tax, Welfare Programs and Manpower Planning", *American Economic Review, 68*, 8–19.

Allingham, M. and Sandmo, A. (1972): "Wage Tax Evasion: A Theoretical Analysis", *Journal of Public Economics, 1*, 323–338.

Andreoni, J.; Erard, B. and Feinstein, J. (1998): "Tax Compliance", *Journal of Economic Literature 36*, 818–860.

Atkinson, A. B. and Stiglitz, J. E. (1980): *Lectures on Public Economics*, Maidenhead, McGraw Hill.

Bakos, P., Benczúr, P. and Benedek, D. (2008): "Elasticity of Taxable Wage: Estimates and Flat Tax Predictions Using the Hungarian Tax Changes in 2005", Budapest, MNB WP 7.

Baumann, F.; Friehe, T. and Jansen, M. (2009): "On the Economics of Contribution Evasion", *FinanzArchiv/Public Finance Analysis* 65, 162–177.

Chetty, R. (2009): "Is Taxable Income Elasticity Sufficient to Calculate Deadweight Loss? The Implications of Evasion and Avoidance", *American Economic Journal: Economic Policy 1:1*, 31–52.

Christie, E. and Holzner, M. (2006): "What Explains Tax Evasion? An Empirical Assessment Based on European Data", Vienna, WIIW Working Paper 40.

Feldstein, M. (1999): "Tax Avoidance and the Deadweight Loss of the Income Tax", *Review of Economics and Statistics, 81* 674–680, full text: NBER WP 5055, 1996.

Frey, B. S. and Weck-Hannemann, H. (1984): "The Hidden Economy as an 'Unobserved' Variable", *European Economic Review 26*, 33–53.

Friedman, E; Johnson, S.; Kaufmann, S. and Zoido-Lobaton, P. (2000): "Dodging the Grabbing Hand: The Determinants of Unofficial Activity in 69 Countries", *Journal of Public Economics 76*, 459–493.

Gorodnichenko, Y.; Martinez-Vazquez, J. and Sabirianova Peter, K. (2009): "Myth and Reality of Flat Tax Reform: Micro Estimates of Tax Evasion Response and Welfare Effects in Russia", *Journal of Political Economy 117*, 504–554.

Hayoz, N. and Hug, S. eds. (2007): *Tax Evasion, Trust, and State Capacities*, Bern, P. Lang.

Janky, B. (2007): "Social Structure, Transition and Public Attitudes towards Tax Evasion in Hungary", *Hayoz, N. and Hug, S. eds.* 269–287.

Elek, P.; Köllő, J. and Szabó, P. A. (2009): The Implications of Ignoring 'Envelop' Wages in the Estimation of the Minimum Wage Effects, lecture at the Institute of Economics, HAS, Budapest.

Krekó, J. and Kiss P. G. (2007): "From Tax Optimization to Optimal Taxation – The First Step: Tax Compliance and the Hungarian Tax System", Budapest, MNB Studies 65.

Lackó, M. (2000): "Hidden Economy – An Unknown Quantity? Comparative Analysis of Hidden Economy in Transitional Countries", *Economics of Transition 8,* 117–149.

Lackó, M. (2007): "Interrelationships of the Hidden Economy and Some Visible Segments of the Labor-market", IE-HAS Discussion Paper, 7.

Malcomson, J. (1986): "Some Analytics of the Laffer Curve", *Journal of Public Economics 29*, 263–279.

Mirrlees, J. A. (1971): “An Exploration in the Theory of Optimum Income Taxation”, *Review of Economic Studies 38* 175–208.

OECD (2002): *Measuring the Non-Observed Economy – A Handbook*, Paris, OECD.

Pestieau, P. and Possen, U. (1991): “Tax Evasion and Occupation Choice”, *Journal of Public Economics 45*, 107–125.

Richter, W. F. and Boadway, R. (2005): “Trading off Tax Distortion and tax Evasion”, *Journal of Public Economic Theory*, 7, 361–381.

Romer, T. (1975): “Individual Welfare, Majority Voting and the Properties of a Linear Income Tax”, *Journal of Public Economics 4,* 163–185.

Schneider, F. (2002): “The Size and Development of the Shadow Economies of 22 Transition and 21 OECD Countries”, *IZA Discussion Paper 514*.

Schneider, F. and Enste, D. (2000): “Shadow Economies: Size, Causes and Consequences”, *Journal of Economic Literature 38* 77–114.

Sheshinski, E. (1972): “Optimal Linear Income Tax”, *Review of Economic Studies 39*, 297–302.

Simonovits, A. (2009): Underreported Earnings and Age-specific Income Redistribution in Post-socialist Economies, Budapest, IE-HAS Working Paper

Tonin, M. (2007): “Minimum Wage and Tax Evasion: Theory and Evidence”, IE-HAS, Discussion Paper 1.

Valdés-Prieto, S. (2009): “Time Inconsistency, Contribution Density and Contributory Social Insurance,” lecture at the 65th congress of the IIPF, Cape Town.

Varian, H. R. (1980): “Redistributive Taxation as Social Insurance”, *Journal of Public Economics 14*, 49–68.

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