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The Optimal Tax Rule in the Presence of Time Use*

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Abstract: Using Mexican data on household time use and consumption, we find significant substitution between goods and time in home production and different elasticities of substitution for different household commodities. Adding these findings to the Ramsey optimal tax problem, we show it is optimal to impose higher taxes on market goods used in the production of commodities with a lower elasticity of substitution between goods and time. The reason is that government wants to minimize the distortionary substitution from market purchases toward untaxed time use in home production. This is an analog of the classical Corlett and Hague (1953-1954) result, differing in that we allow for the possibility of substitution between goods and time in the production of commodities. Leaving aside distributional considerations, we conclude that higher taxes should be imposed on market goods used in the production of 'Eating' and lower taxes imposed on market goods used in the production of 'Recreation'.
Keywords: Optimal taxation; Time use; Elasticity of substitution.
JEL Classification: H21; J22; D13.

Resumen: Usando datos de México sobre uso del tiempo y consumo de los hogares, hay evidencia estadísticamente significativa de sustitución entre bienes de mercado y tiempo en la producción en el hogar, así como de diferentes elasticidades sustitución para diferentes bienes finales producidos en el hogar. Añadiendo estos resultados al problema del impuesto óptimo de Ramsey, demostramos que es óptimo imponer impuestos más altos a los bienes de mercado usados en la producción de bienes finales con menor elasticidad sustitución entre bienes de mercado y tiempo. La razón es que el gobierno busca minimizar el efecto distorsionante en la producción en el hogar de sustituir bienes de mercado por tiempo, ya que el uso de este último no genera impuestos. Esta derivación es análoga al resultado clásico de Corlett and Hague (1953-1954). La diferencia radica en que nosotros permitimos la posibilidad de sustitución entre bienes de mercado y tiempo en la producción de bienes finales. Dejando las consideraciones redistributivas a un lado, concluimos que los bienes de mercado utilizados en producir 'comer' deben tener impuestos más altos relativos a los bienes de mercado utilizados en producir 'recreación'.
Palabras Clave: Impuestos óptimos; Uso del tiempo; Elasticidad de sustitución.

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1 Introduction

Following Becker (1965), we assume that individuals combine market goods and time to produce commodities which ultimately yield utility. For example, consider a household that wants to change their car's engine oil. In order to get this commodity, the household needs to combine both market goods and time. Having only the engine oil does not give any utility, it has to be put in the car's engine, which requires time. Then we allow for the possibility of substitution between market goods and time to produce commodities. In the case of changing the car's engine oil, one way to get the commodity is to stop by Firestone and pay someone to do the job. This solution saves the household's time but requires payment, including taxes, for the service. Alternatively, members of the household can perform the maintenance themselves. This solution can save money and avoid taxation, but requires more time, assuming the professional working at Firestone has an absolute advantage in production, which is likely true in most cases. Take another example given by Burda et al. (2008). An American couple has to choose between goods-intensive and time-intensive summer vacations facing a limited budget constraint. The goods-intensive solution is to spend their time flying to the Côte d'Azur for a one-week holiday. On the other hand, the time-intensive solution is to take a two-week caravan trip to the Great Smoky Mountains National Park.

How do taxes on market goods affect the household's decision of how many market goods and how much time to use in home production? These two examples show that taxes on market goods can affect the choice of the household between goods-intensive and time-intensive solutions. Specially, an increase in taxes on market goods encourages households to substitute away from the market goods input in favor of the untaxed non-market time input. Therefore, if the government decides to change the tax rate on a specific market good, the government has to take the possibility of substitution into account.

In this paper, we first state theoretically how taxes on market good relate to the elasticities of substitution assuming that each commodity production function has a constant elasticity of substitution functional form, and solve the Ramsey optimal commodity tax problem for a benevolent government. Within the framework of a three-commodity economy proposed by Corlett and Hague (1953-1954) and the assumption of a Cobb-Douglas utility function, we find that the optimal tax rule is to impose a higher tax rate on market goods used in the production of commodities with a lower elasticity of substitution between goods and time.

Then we check how this optimal tax rule compares to what we see in reality. To this purpose, we need to calculate the elasticities of substitution between market goods and time for different commodities. We use the 2002 Mexican time use data for which we observe disaggregated market good expenditures and time uses for the same household and for various different commodities. We find that 'Eating' has the lowest elasticity of substitution and 'Recreation' has the highest elasticity of substitution. According to our theory, these results imply that 'Eating' should be taxed at a very high rate and 'Recreation' at a very low rate. The optimal value added tax system for Mexico would impose 7.0% tax rate on food and 5.5% on market goods used in the production of 'Lodging, Appearance, and Recreation'. This optimal tax structure is more regressive compared to the actual Mexican tax system in which the government gives more weight to equity considerations than to economic efficiency.

The paper proceeds as follows. The next section provides a theoretical model of the optimal taxation problem. Section 3 describes our data set and summarizes key variables. The econometric framework and estimation results are presented in section 4. Section 5 provides policy implications, and section 6 concludes.

2 Theoretical Model

2.1 Background

Since Becker's (1965) pioneering idea of household production as a combination of goods and time, a substantial amount of theoretical and empirical work on the household production has been done in a variety of areas in economics (Hamermesh (2007)). However, relatively little work has been conducted in public finance (see, e.g., Zhang et al. (2008)). The exception is the topic of optimal tax theory and the relevant literature includes Sandmo (1990), Gahvari and Yang (1993), Kleven (2000, 2004), and Boadway and Gahvari (2006).

Sandmo (1990) introduced the home production approach into the optimal taxation problem and found that the income tax creates distortions, giving an incentive to use too much time in home production. However, even though time spent preparing meals may be qualitatively different from time spent listening to music, Sandmo (1990) did not deal with the possibility that different household activities can give different utilities. Gahvari and Yang (1993) first related optimal commodity taxes to the Becker's (1965) idea of home production. They assumed that households consume a bundle of goods, each of which requires time in fixed, but different, proportions to yield utility. Then they found that optimal commodity tax rates depend on time spent consuming each good. Using the same formulation as Gahvari and Yang (1993), Kleven (2004) proposed that the optimal commodity taxation is governed by factor shares in household activities. That is, any market good which requires little time should carry a relatively low tax rate. Boadway and Gahvari (2006) studied the optimal commodity taxation problem under two assumptions: that consumption time is either a perfect substitute for labor or a perfect substitute for leisure and that time spent consuming any particular good is taken to be a fixed proportion of the quantity of the good. They showed that while labor substitutability affects the optimal tax structure, leisure substitutability leaves the classical optimal tax results intact.

Although these studies that have followed the original contribution of Gahvari and Yang (1993) give us useful insights into what the optimal commodity tax system looks like when households combine goods and time to produce commodities, they rule out the possibility of substitution between goods purchases and time use in the production of commodity. Both studies make use of a Leontief home production function, assuming that the amount of time devoted to the consumption of goods is fixed. Introducing a Leontief home production function has a great advantage, it simplifies the optimal commodity taxation problem by reducing it to the classical optimal commodity tax problem without home production. Allowing the possibility of substitution between market goods and time use complicates the problem.[1]

It is true that the assumption of a Leontief home production function does not completely rule out the possibility of substitution in household production. Kleven (2004) takes dish-washing as an example. Dish-washing may be carried out by the use of a brush or a machine and these two production processes involve fixed, but different ratios between market goods and time. So Kleven (2004) argues that washing up with a brush or a machine are two different commodities. Even if this is true, the problem is that the assumption of Leontief home production function requires too many commodities since there are numerous ways to wash dishes other than using a brush and a machine. For example,

[1]Under the Leontief production function ($Z_j = \min\left(\frac{X_j}{a_j}, T_j\right)$ where X_j and T_j represent market goods and time use, respectively), the optimal commodity taxation problem becomes the classical optimal tax problem without home production, that is, $U(Z_0, Z_1, ..., Z_n) = U(X_0/a_0, X_1/a_1, X_2/a_2, ..., X_n/a_n)$.

you can hire a maid. In contrast, if we explicitly allow the possibility of substitution between goods input and time input in home production, we can think of dish-washing as a composite commodity incorporating many different combinations of goods and time. So the aggregation of commodities can reduce the number of tax rates. This reduction is important in practical point of view, since it is impossible in real world to implement the Leontief-based optimal tax system; many different commodities should be taxed at different rates. As Belan et al. (2008) pointed out, the grouping of commodities should be done when there is a constraint on the number of tax rates.

Kleven (2000) provided a more general approach than Kleven (2004). Kleven (2000) showed that the optimal tax is related to factor shares and elasticities of substitution. However, the relationship is not clear without specific functional forms of home production, since household will optimally change factor shares in response to the change in tax rate. The relationship between the optimal tax and elasticities of substitution in household production varies depending on the functional forms of home production. While Kleven (2004) circumvents this problem by assuming Leontief production function,[2] we use a Constant Elasticity of Substitution (CES) function that has better advantages over Leontief function.

Our contribution is to allow explicitly the possibility of substitution between goods and time in home production by assuming a CES production function. In the theory section, we emphasize the importance of elasticity of substitution between goods and time uses in designing optimal tax system and derive the optimal tax rule under this possibility. The empirical analysis is based on Gronau and Hamermesh's (2006) commodity classifications and we estimate the elasticity of substitution between market goods and time for each commodity. From an optimal tax perspective, the magnitude of elasticity of substitution is important. So we test the hypothesis that these elasticities are equal and derive the corresponding policy implications. This new example shows that the restrictive Leontief assumption can be relaxed to allow for estimation of elasticities that are directly useful for policy.

[2]In case of Leontief production function factor shares do not change in respond to the change in tax rates. Factor shares are determined by parameters of the Leontief production function.

2.2 Household Maximization Problem

2.2.1 Utility Maximization

Households combine market goods and time to produce commodities that directly enter their utility function. Assume that $q_j = p_j + s_j$ where q_j is the consumer price of market good X_j, p_j is the producer price of X_j, and s_j is the tax on X_j. We also assume that w and T represent the wage rate and total time available, respectively, and M is non-labor income.[3] Then we can write the household utility maximization problem in the following way. If there are $n+1$ commodities and we take $q_1, q_2, ..., q_n, w, T$, and M as given, then the household's problem is:

$$\max_{\{X_j\}_{j=1}^n, \{T_j\}_{j=0}^n} U(Z_0, Z_1, ..., Z_n) \text{ such that } \sum_{j=1}^n q_j X_j = w\left(T - \sum_{j=0}^n T_j\right) + M$$

$$\text{where } Z_j = \begin{cases} T_0 & if \quad j = 0 \\ \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}} & if \quad j = 1...n, \text{ and } \theta_j < 1. \end{cases}$$

Z_0 is pure leisure that does not need market goods, but needs time. However, other commodities $Z_{j \neq 0}$ are produced with both goods X_j and time T_j and with specific technology having constant elasticity of substitution between X_j and T_j.[4] Let σ be the elasticity of substitution between market goods and time. So σ is equal to $\frac{1}{1-\theta}$. Since solving this problem directly is algebraically laborious, we solve this maximization problem in two steps. At the first stage, the household determines the optimal amount of goods and time input for each commodity by solving the cost minimization problem for given $\bar{Z}_j$. Then, in the second stage, the household makes a decision on the amount of consumption of each commodity.

First Step Note that the price of X_j is q_j $(= p_j + s_j)$ and the price of T_j is w. The household cost minimization problem is the following. Given $\bar{Z}_j$, q_j, and w,

$$\min_{X_j, T_j} q_j X_j + wT_j \qquad \text{such that} \quad \bar{Z}_j = \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}}.$$

Taking first order conditions yields:

$$\left(\frac{X_j}{T_j}\right) = \left(\frac{w}{q_j}\right)^{\frac{1}{1-\theta_j}}. \tag{1}$$

[3] Results do not change if we do not include non-labor income.

[4] CES functional form with $0 \leq \theta_j < 1$ guarantees that optimal X_j and T_j are strictly positive.

To measure how goods and time are combined to produce a commodity let the goods intensity of commodity j be X_j/T_j. Then equation (1) tells us how the goods intensity is related to w, q, and θ. Taking the derivative of X_j/T_j with respect to w and s_j we know that:

$$\frac{\partial}{\partial w}\left(\frac{X_j}{T_j}\right) > 0 \qquad \frac{\partial}{\partial s_j}\left(\frac{X_j}{T_j}\right) < 0 \qquad \frac{\partial^2}{\partial w \partial s_j}\left(\frac{X_j}{T_j}\right) < 0.$$

First, an increase in the wage, w, raises the goods intensity. This suggests that the goods intensity is increasing in household income,[5] which is consistent with empirical evidence.[6] Hamermesh (2007) calculates the goods intensity of eating at various percentiles of the income distribution for 1985 and 2003 and shows that the goods intensity increases when you move to the upper end of the income distribution.[7] Second, the increase in tax s_j reduces the goods intensity, but the magnitude of the effect depends on w. The effect becomes larger as wage decreases, which means that lower-income households are likely to be more sensitive to the tax change. Third, the goods intensity of commodity j depends on w, q_j, and θ_j, but does not depend on taxes on other goods $s_{j\neq k}$.

The solution to the cost minimization problem is:

$$X_j^* = \alpha_j \bar{Z}_j, \quad T_j^* = \beta_j \bar{Z}_j \tag{2}$$

$$\text{where } \alpha_j \equiv \left(1 + \left(\frac{q_j}{w}\right)^{-\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}} \text{ and } \beta_j \equiv \left(1 + \left(\frac{q_j}{w}\right)^{\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}.$$

This result looks like the assumption of Kleven (2004). However, the difference is that coefficients α_j and β_j depend on the tax rate s_j. Kleven (2004) assumes that these coefficients are fixed regardless of the tax rate s_j. Our result shows that when government increases the tax rate s_j on good X_j, households optimally respond by using less of the good and more time in the production of commodity $\bar{Z}_j$.

Second Step This step solves the utility maximization problem of the household. Given q_j for $j = 1, ..., n$, w, and the solution from the first step, the problem becomes:

$$\max_{Z_0, Z_1, \cdots, Z_n} U(Z_0, Z_1, \cdots, Z_n) \text{ such that } \sum_{j=1}^{n} q_j X_j = w\left(T - \sum_{j=0}^{n} T_j\right) + M.$$

[5] This is true as long as wage is a proxy for household income.

[6] It might be interesting to compare goods intensities across countries. We expect that the goods intensity will be higher in countries with a higher real wage (w/q).

[7] See Table 5 in Hamermesh (2007).

By using (2), we can rewrite the budget constraint as:

$$\sum_{j=0}^{n} \gamma_j Z_j = wT + M$$

$$\text{where } \gamma_j = \begin{cases} w & if \quad j = 0 \\ q_j \alpha_j + w\beta_j & if \quad j = 1, ..., n. \end{cases}$$

This relation tells us that the price of Z_j is γ_j which is the weighted sum of the price of good X_j, q_j, and the price of time, w. The price of Z_0 is only w since it does not require market goods for its production. From the first order conditions, we obtain $U_j = \lambda \gamma_j$ for $j = 0, 1, ..., n$.

2.3 Optimal Government Policy

Following the standard Ramsey taxation theory, the benevolent government's optimal tax problem is to choose s_1,...,s_n to maximize the indirect utility of the representative household, denoted by $V(\cdot)$, subject to the requirement that taxes yield an exogenous amount of revenue $\bar{R}$. If the government changes the tax rate on market goods, the household responds by changing both market purchases and time use. The social planner has to consider the effect of the tax change on both goods and time spent by the household. The government problem is:

$$\max_{s_1,...,s_n} V(q_1, .., q_n, w) \text{ such that } \sum_{j=1}^{n} s_j X_j = \bar{R}$$

$$\text{where } q_j = p_j + s_j \text{ for } j = 1, ..., n.$$

The first-order conditions are:

$$\frac{\partial V}{\partial q_k} + \mu \left(X_k + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial q_k} \right) = 0 \text{ for } k = 1, .., n.$$

By the envelope theorem, we can rewrite first order conditions as follows:

$$\frac{\lambda - \mu}{\mu} = \sum_{j=1}^{n} \frac{s_j}{X_k} \frac{\partial X_j}{\partial q_k}.$$

Then, using the Slutsky equation and Slutsky symmetry,[8] we can rewrite these conditions as:

$$\frac{\lambda - \mu}{\mu} + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial M} = \sum_{j=1}^{n} \frac{s_j}{q_j} \varepsilon_{kj}^c, \tag{3}$$

[8] For detailed derivations of these equations, please refer to the Appendix A.1.

where $\varepsilon^c_{kj} \equiv \frac{q_j}{X_k}\frac{\partial X^c_k}{\partial q_j}$ is the compensated elasticity of X_k with respect to the change in the price of X_j. Note that the left hand side of equation (3) does not depend on $k \neq j$. Therefore it is constant. Let $-\Phi \equiv \frac{\lambda-\mu}{\mu} + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial M}$. Then we can derive the Ramsey Rule as follows:

$$-\Phi = \sum_{j=1}^{n} \frac{s_j}{q_j}\varepsilon^c_{kj} \text{ for k =1,...,n.} \tag{4}$$

This Ramsey rule has the standard form of the optimal commodity tax expression which emphasizes the importance of compensated price responses. (Diamond and Mirrlees (1971), Sandmo (1987), Sandmo (1990)).[9]

Three-commodity Economy

Next, we examine a three-commodity economy first proposed by Corlett and Hague (1953-1954), and then used by Kleven (2004) and Boadway and Gahvari (2006). In this case, there are one untaxable commodity (Z_0) and two taxable commodities (Z_1, Z_2) with different elasticities of substitution between goods and time. The Ramsey rule in the elasticity form becomes:

$$\begin{aligned} -\Phi &= \frac{s_1}{q_1}\varepsilon^c_{11} + \frac{s_2}{q_2}\varepsilon^c_{12} \text{ and} \\ -\Phi &= \frac{s_1}{q_1}\varepsilon^c_{21} + \frac{s_2}{q_2}\varepsilon^c_{22}. \end{aligned}$$

If we use the homogeneity property of compensated demand functions,[10] we can solve for the tax rates as:

$$\begin{pmatrix} \frac{s_1}{q_1} \\ \frac{s_2}{q_2} \end{pmatrix} = -\frac{\Phi}{\Pi} \begin{pmatrix} \varepsilon^c_{11} + \varepsilon^c_{22} + 3\varepsilon^c_{10} \\ \varepsilon^c_{11} + \varepsilon^c_{22} + 3\varepsilon^c_{20} \end{pmatrix}$$

where $\Pi \equiv \varepsilon^c_{11}\varepsilon^c_{22} - \varepsilon^c_{21}\varepsilon^c_{12}$. This result suggests that if the compensated elasticity of X_1 with respect to the price of leisure is lower than the compensated elasticity of X_2 with respect to the price of leisure then a higher tax should be imposed on X_1. Symbolically, $\varepsilon^c_{10} < \varepsilon^c_{20} \rightarrow s_1/q_1 > s_2/q_2$.[11] This result is the analog of standard Corlett-Hague rule: the highest tax rate ought to be levied on the

[9] A detailed explanation of the Ramsey rule can be found in Diamond and Mirrlees (1971, p.262) and Sandmo (1990, p.92).

[10] For the detailed derivation of the property of compensated elasticity, please refer to the Appendix A.2.

[11] Diamond and Mirrlees (1971, p.262) prove that Φ is positive. Π is also positive, which can be proved using the determinant of the matrix of substitution effects (Sandmo (1987, p.93)).

commodity with the highest degree of complementarity with leisure. This result, however, differs from the standard Corlett-Hague rule, because of the last term on the right-hand side of each equation. In case of the standard Corlett-Hague rule, the last term on the right-hand side of the equation is ε_{10}^{c}, not $3\varepsilon_{10}^{c}$. This difference can be easily understood from the fact that the price of time is the same whether the time is used for the production of Z_0, Z_1, or Z_2.

Ramsey rule is hard to apply in practice because little is known about the magnitudes of the compensated elasticities (Kleven (2004)). However, the elasticities of substitution can be estimated easily if you have the necessary data. This is why we study the relationship between the compensated elasticity and the elasticity of substitution between goods and time. To do this, we assume a specific functional form for the utility function.[12] Specifically, we assume the following log utility function[13]:

$$u\left(Z_0, Z_1, Z_2\right) = \delta_0 \ln Z_0 + \delta_1 \ln Z_1 + \delta_2 \ln Z_2 \tag{5}$$

$$\text{where} \qquad Z_j = \begin{cases} T_0 & if \quad j = 0 \\ \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}} & if \quad j = 1, 2 \text{ and } \theta_1 < \theta_2 < 1, \end{cases}$$
$$\text{and } \delta_0 + \delta_1 + \delta_2 = 1.$$

Conventional wisdom contends that the price of a necessity is lower than the price of a luxury. If this is the case, we can show that the smaller the elasticity of substitution between goods and time, the smaller the compensated elasticity in a three-commodity economy with the logarithmic preferences stipulated by equation (5).[14] Symbolically, $\sigma_1 < \sigma_2 \rightarrow \varepsilon_{10}^{c} < \varepsilon_{20}^{c}$. Even in case that the price of a necessity is higher than the price of a luxury, if a necessity tends to have a lower elasticity of substitution than a luxury which is shown empirically in Section 4, the smaller the elasticity of substitution between goods and time, the smaller the compensated elasticity. This relationship has a quite important implication. The elasticity of substitution between goods and time is determined by the technology of home production, but the compensated elasticity represents the market. So

[12] Notice that previous results do not rely on any assumption about the functional form of the utility function.

[13] The merit of using a logarithmic utility function is that the income and substitution effects exactly balance. If we use another functional form, it would be hard to get meaningful results from the analysis without making further assumptions about the income and substitution effects. In future extensions of this paper we will test whether we can get the same result with less restrictive assumptions or other functional forms.

[14] For the detailed derivations, please refer to the technical Appendix A.2.

the relationship shows us how the home production technology is related to the market response. In response to the change in the wage rate, goods with higher elasticity of substitution between goods and time have larger compensated elasticity.

Proposition *In a three-commodity economy with logarithmic preferences, the optimal tax policy requires that a higher tax should be placed on goods with a lower elasticity of substitution between goods and time. Symbolically,* $\sigma_1 < \sigma_2 \rightarrow s_1/q_1 > s_2/q_2$.

3 Data

To demonstrate the applicability of these results we use the National Time Use Survey 2002[15] (ENUT) from Mexico. This is a nationally representative sample including urban and rural communities. It surveys all individuals[16] who were aged 12 years or older at the time of the survey. The total sample includes 4,783 households and 20,342 individuals. The objective of the survey is to measure the activities undertaken by men and women within the household.

One disadvantage of the ENUT data set is that the questionnaire is not based on time use diaries where individuals are asked to report the activities undertaken on a given day. Instead, individuals are only asked to report how many hours in the week were spent doing a finite number of activities listed in the questionnaire. Hence, the total time use for each individual does not add up to 168 hours, the total number of hours in a week. In fact, total time use averages 163.15 hours for our analysis sample. Although it is well known that diary time use questionnaires are more detailed and more reliable for research, the majority of time use surveys, including ENUT, instead use recall questionnaires for major activities due to the cost and complexity of the survey design.

This disadvantage is compensated by a very important advantage. The ENUT is a sub-sample of the National Household Survey of Income and Expenditure 2002[17] (ENIGH), the Mexican national income and expenditure data set. Therefore, we can match the time use data with the expenditure data

[15] Encuesta Nacional del Uso del Tiempo 2002, http://www.inegi.gob.mx.

[16] By all individuals we mean residents and non-residents. The latter group includes personnel who help with household activities and individuals staying there temporarily.

[17] Encuesta Nacional de Ingreso y Gasto de los Hogares 2002, http://www.inegi.gob.mx.

by household. To our knowledge, only Mexican data provides information (for the same household) on both time uses and goods expenditures for a large number of commodities, although statistical agencies in a number of countries are moving to generate combined time use and expenditure files.

3.1 Definitions of Commodities

A household engages in numerous activities every day, for example, having breakfast and dinner, or taking a shower and watching television. All these activities need both market goods and time as inputs. To simplify the analysis we implicitly allocate activities into ten mutually exclusive categories, which are called commodities. The commodities are 'Sleep', 'Eating', 'Lodging', 'Appearance', 'Recreation', 'Health', 'Child-care', 'Travel', 'Miscellaneous' and 'Work'. Classification of time uses and goods expenditures is not straightforward because any classification is somewhat arbitrary. In order to be consistent with previous literature and to avoid as much subjectivity on our part, we use Gronau and Hamermesh's (2006) definition of commodities. Tables 1 and 2 define the time use and goods expenditure categories, respectively. In both tables we exhaust all reported time uses and expenditures from the data.

The classifications are not exactly the same as in Gronau and Hamermesh (2006). There are three minor variations in the time use categories due to differences in the questionnaire structure between their data sets and ours. In our case, time use for 'Eating' includes not only eating at home and away, meal preparation, clean-up, and grocery shopping, but also raising corral animals, collecting fruits, hunting, fishing, and taking care of the orchard. Also, in our classification, 'Health' does not include medical care at hospitals. Given the available data, 'Health' only includes time spent recovering from an illness, taking care of a family member that is temporarily ill, and personal health care. Finally, the other difference is in the 'Travel' time use category. In our data set, this only includes time spent accompanying a member of the family to go somewhere and taking or picking up any member of the family to go somewhere, so it does not includes all non-working travel time. With respect to goods expenditures categories, there are essentially no differences between our classification and that in Gronau and Hamermesh (2006). The only minor discrepancy is that 'Lodging' includes materials and services to repair, maintain, or extend the dwelling besides housing, a fraction of appliances expenditures, and a fraction of communication expenditures. In both classifications, 'Sleep' and

Table 1: Time Use Categories[a]

Commodity	Category
Sleep	Night sleep and .5(rest or recovery from an illness).
Eating	Eating at home and away, meal preparation, clean-up, grocery shopping, raising corral animals, collecting fruits, hunting, fishing, and taking care of orchard.
Lodging	House cleaning, outdoor chores, home and car repairs, gardening and animal care, durable goods shopping, misc. household duties, and, .5(make furniture, ornament or traditional craft for the house).
Appearance	Laundry and clothes care, personal and beauty care, and personal hygiene.
Recreation	Sex, nonreligious organizations, entertainment, culture, visits, social events, sports, hobbies, crafts, games, reading, writing,TV and radio, conversing, thinking, .5(make furniture, ornament or traditional craft), and non-travel educational activities if no children and individual is aged $>$ 59.
Health	.5(Rest or recovery from illness), taking care of a family member that is temporarily ill, and personal health care.
Child-care	All infant and child-care non-travel activities if children.
Travel	Accompany any member of the family to somewhere, take or pick up any member of the family to somewhere and travel to education-related activities if no children.
Miscellaneous	Taking care of family documents, helping other households voluntarily, taking care of other members of the family with a physical or mental limitation, volunteering, religious activities, making payments, personal proceedings, taking food to another member of the family to school or work, attending funeral services, non-travel education-related activities if no children and individual is aged $<$60, and all infant and child-care non-travel activities if no children.
Work	Working at a paid job, job search time, and work commuting time.

[a] We exhaust all time uses reported in the ENUT 2002 into these ten mutually exclusive categories which we called commodities. Note that ‘Health’ does not include medical care at hospitals. Also, ‘Travel’ does not include all non-working travel time.

Table 2: Goods Expenditure Categories[a]

Commodity	Category
Sleep	None
Eating	Food+.5(beverages)+.33(appliances).
Lodging	Housing+.33(appliances)+.5(communications)+ materials and services to repair, maintain or extend the house.
Appearance	Apparel and services+.33(appliances)+personal care.
Recreation	Entertainment+tobacco+.5(beverages)+.5(communications)+ education expenses if no children and individual is aged > 60.
Health	If no children: Hospital care, doctor care, medicine expenses without prescription. If children: Health*(1-number of children/size of the family)
Child-care	boys' and girls' apparel+ education+ Health*(number of children/size of the family) if children.
Travel	Private and public transportation prorated by nonwork travel divided by total travel time.
Miscellaneous	Other expenditures and transfers+education expenses if no children and individual is aged < 60+boys' and girls' apparel if no children.
Work	None

[a] We exhaust all goods expenditures reported in the ENIGH 2002 into these ten mutually exclusive categories which we called commodities. We assume that 'Sleep' have no goods expenditures related to it. Any expenditures seemingly related to 'Sleep' were included either in 'Lodging' or 'Appearance' .

'Work' are assumed to have no expenditures related to them.

3.2 Households

The unit of analysis is the household, not individuals, because in the ENIGH only household expenditures are reported. In the sample we only include nuclear households (only one family within the dwelling) to keep the sample as homogeneous as possible, because different types of families have different time use patterns.[18] For instance, we expect married couples to be more efficient in home production than single individuals due to specialization by husband and wife in certain activities. In fact, single men spend on average 16 hours on the 'Eating', while husbands spend on average 12 hours per week on the same commodity. On the other hand, wives spend on average 34 hours per week on 'Eating', whereas single women spend only 22 hours. In the case of extended families (more than one family within the dwelling) it is easy to imagine that these families are different from nuclear families in terms of household expenditures and time uses. It could be the case that families within the extended household do not pool their incomes. Even in those cases, it is possible that such families share time uses. For example, a member of one of the families takes care of all the children within the dwelling, making all other members of the extended household more efficient in their allocation of time. Because of these differences we eliminated 1,286 households from the sample. In addition, 500 observations were dropped because only one spouse was present at the time of the survey. Finally, 57 households were removed because they had no income or were missing other variables. The total number of households in our sample is 2,940.

In Table 3, we summarize the demographic characteristics of husbands and wives as well as their time uses. In this table and throughout the paper, we define earnings as all labor earnings, specifically, salaries, wages, overtime payments, and self-employment income.

Based on the summary statistics in Table 3, we know that husbands are on average 4 years older than wives in the sample. In terms of years of schooling, both spouses are very similar, averaging about 7 years of education. It is also worth noting that wives' earnings are significantly lower than their husbands. This is directly related to the labor force participation decision of both husbands and

[18]Nuclear households represents 70% of the sample. The other 30% is composed of one-person households (7%) and extended households (23%).

Table 3: Demographic Characteristics and Time Uses of Husbands and Wives[b]

	Husbands		Wives	
Variable	Mean	Std. Dev.	Mean	Std. Dev.
Age	42.70	13.66	39.11	12.91
Years of Schooling	6.73	4.92	6.31	4.37
Labor Force Participation	.907	.290	.388	.487
Earnings[a]	928.14	1249.96	207.47	578.21
Firm size[c]	50.46	303.09	7.55	66.34
Unionized worker[d]	.074	.262	.028	.165
Time Uses[e] (hrs/week)				
Sleep	56.04	16.40	57.81	11.38
Eating	11.68	9.74	33.63	15.78
Lodging	4.15	6.01	16.41	10.18
Appearance	4.36	3.07	13.85	6.95
Health	3.34	5.69	2.77	5.09
Recreation	16.98	14.19	16.04	13.26
Child-care	1.59	6.21	6.32	18.00
Miscellaneous	4.64	9.47	12.90	22.78
Travel	.42	1.67	.94	2.30
Work	50.10	24.29	12.14	21.28

[a] In Mexican pesos as of 2002, per week. We define earnings as all labor earnings, specifically, salaries, wages, overtime payments, and self-employment income.

[b] Number of observations: 2,940.

[c] Firm size refers to the number of workers in the firm where the husband or the wife works.

[d] Unionized worker is a indicator variable equal to one if the firm is unionized and zero otherwise.

[e] The use of time for each individual does not add up to 168 hours, the total number of hours in a week, because the ENUT 2002 is based on recall questionnaires on major activities and not on time use diaries.

wives. A total of 91 percent of husbands participate in the labor force, whereas only 39 percent of wives do.

Husbands and wives have different time use patterns as a result of specialization. Husbands report 50 hours of work on average, while wives only work, on average, 12 hours a week in a paid job. However, wives dedicate 34 hours of the week, on average, to 'Eating' and 16 hours to 'Lodging', while men spend only 12 and 4 hours, respectively. Also wives dedicate more time to 'Appearance', 'Child-care' and 'Miscellaneous' commodities than husbands. With respect to 'Sleep' and 'Recreation', both husbands and wives devote similar amounts of time, around 56 and 16 hours a week, respectively.

3.3 Time Use and Goods Expenditure

3.3.1 Time Use

In Table 4, we summarize both expenditures and time use of the household.[19] We define household time use as the sum of the husband's and wife's time use. The household allocates 62 hours for 'Work' a week, on average. A total of 45 hours a week are devoted to 'Eating' and 21 hours are used on 'Lodging'. The household sleeps an average of 114 hours a week and 33 hours are used for 'Recreation'per week. Notice that average time spent on 'Travel' is about 2 hours per week. This reflects that the measure we have for 'Travel' time use is poor. The household allocates only 8 hours per week to 'Child-care', on average.[20]

In principle we could also add the time use of other members of the family to the household time use. However, most of the other members are children whose opportunity cost of time is not determined by the labor market. In fact, we could argue that there is no opportunity cost for their time. Nonetheless, in an attempt to capture any effect children could have on the allocation of goods or time in the household production of commodities, we control for the number of children in our estimation.

[19] For the time use variables the week of reference was the week from Monday to Sunday before the day of the survey. For the non-time variables the unit of time was daily, monthly, quarterly, or every six months depending on the type of expenditure. All variables were converted into a weekly basis.

[20] Around 40% of households do not have children.

Table 4: Summary Statistics of the Households[c]

	Expenditures[a]		Time Use[b]	
Variable	Mean	Std. Dev.	Mean	Std. Dev.
Sleep	–	–	113.85	22.96
Eating	389.77	321.10	45.31	20.86
Lodging	204.18	270.70	20.56	11.99
Appearance	156.89	187.47	18.21	7.95
Health	36.09	157.79	6.11	9.35
Recreation	104.22	201.23	33.02	23.70
Child-care	124.26	348.85	7.91	22.02
Miscellaneous	62.26	254.83	17.54	29.24
Travel	5.45	54.82	1.36	3.25
Work	–	–	62.24	33.17

[a] In Mexican pesos as of 2002, per week.

[b] The time use of the household is defined as the sum of the time use of the husband and the wife, per week.

[c] Number of observations: 2,940.

3.3.2 Market Goods Expenditure

Household expenditures are summarized in Table 4. 'Sleep' is assumed to have no expenditures related to it. Although almost negligible, any expenditures seemingly related to 'Sleep' were included either in 'Lodging' or 'Appearance'. On average, families in this sample spent 400 pesos per week on 'Eating', 200 pesos per week on 'Lodging', 150 pesos per week on 'Appearance' and 124 pesos per week on 'Child-care'. These four categories comprise the four largest components of the household total expenditures.

Households can hire workers such as maids, nannies, or drivers to produce household commodities. The employees carry out activities that are included in 'Eating', 'Lodging', 'Appearance', 'Travel' or 'Child-care' commodities. Therefore, we include the monetary payments the workers receive as household good expenditures because they represent market goods used to produce household commodities. However, we do not observe the salary these employees actually receive for their services, so we use the hourly minimum wage[21] to construct the market value of their hours of work. For example, if the employee dedicated 10 hours a week to the production of the 'eating' commodity and 25 hours to the 'Lodging' commodity then we include 10*minimum wage in the 'Eating' expenditure category and 25*minimum wage in the 'Lodging' expenditure category.

4 Estimation

In this section we report our estimates of the elasticities of substitution between time and market goods for four commodities. We are not aware of other research that attempts to estimate the particular elasticity of substitution between time and market goods. Nevertheless, there is a large number of econometric studies that estimate the elasticity of substitution between labor and capital. For a summary of such estimates see Berndt (1976) and Caddy (1976). Both of these authors mentioned that there is a substantial disagreement over the value of the elasticity of substitution due to the apparent sensitivity of the estimates to the database used, the choice of functional form, and the estimating technique.[22] Much closer to what we do in here are the estimates by Rupert et al. (1994) of the elasticity of substitution between market and home consumption goods. Using the Panel Study of

[21]The average minimum wage in Mexico for 2002 was 4.96 pesos per hour.

[22]For recent estimates of the elasticity of substitution between labor and capital see Antras (2004).

Income Dynamics, they estimated such elasticity for single males, single females and married couples. The results indicate that for single females and married couples there is high substitution elasticity between market and home consumption goods.

We estimate the elasticity of substitution for the 'Eating', 'Lodging', 'Appearance', and 'Recreation' commodities. 'Health' and 'Travel' are not included in the estimation because, as explained in Section 3.1, we have poor measures of time use for these categories. We also ignore 'Child-care'. Significant proportion of families do not have children, and for most families with children, child-care is most probably a secondary activity. That is, parents take care of their children under 13 while doing something else as the main activity.

4.1 Estimation Specification

Assuming the household production function for commodity j is CES, the relative demand function for the ratio of market goods expenditure Y_j, defined as $p_j X_j$, and time expenditure T_j is:

$$ln(Y_j/T_j) = constant + \sigma_j ln(\rho_j w_m + (1 - \rho_j) w_f) \tag{6}$$

where w_m and w_f are the wage rate of the husband and wife respectively, ρ_j is the weight on the husband's price of time, and σ_j is the elasticity of substitution between market goods and time.[23]

We use nonlinear least squares to estimate equation (6). The resulting parameter estimates for 'Eating', 'Lodging', 'Appearance' and 'Recreation' are reported in Table 5. The control variables included when estimating equation (6) are an urban dummy, state dummies, number of children less than 12 years old, number of daughters over 12 years old, and number of sons over 12 years old. Our main interest centers on the estimates of the elasticity of substitution, $\hat{\sigma}$.

Once we control for other characteristics of the household, we find that 'Eating' has the lowest elasticity of substitution between market goods and time. This is very intuitive given that food can not be substituted with anything else, not even time. Also, the most important activity in this commodity is actually eating which is very time intensive and, in contrast to other activities like meal preparation or dish washing, cannot be paid to be done by someone else.

'Lodging' has the second lowest elasticity of substitution. In the city, activities such as house-cleaning, outdoor chores, and home repairs are very easy to buy in the market by paying someone to

[23]The coefficient σ_j is defined as $1/(1 - \theta_j)$ where θ_j is the parameter of the CES function for commodity j.

Table 5: NLLS Equation by Equation[a, b]

	N	*constant*	$\hat{\sigma}$	$\hat{\rho}$
Eating	2727	-.273	.344	.327
		(.114)	(.015)	(.031)
Lodging	2738	-.620	.447	.283
		(.148)	(.019)	(.027)
Appearance	2733	-.852	.462	.289
		(.138)	(.018)	(.025)
Recreation	2367	-2.691	.573	.359
		(.222)	(.029)	(.036)

[a] Standard errors in parenthesis.

[b] $\hat{\rho}$ is the weight on the husband's price of time, and $\hat{\sigma}$ is the elasticity of substitution between market goods and time. N refers to the number of observations used in each estimation. Control variables are urban dummy, state dummies, number of children less than 12 years old, number of daughters over 12 years old and number of sons over 12 years old.

do such works for you. However, in rural areas this substitution between the household's time and the corresponding market goods is very rare, and these activities are in most cases performed by the members of the household. Once we consider this difference, 'Lodging' has a very low elasticity of substitution. In the Mexican case, the majority of these activities are responsibility of the wife and such activities absorb most of her time.

'Appearance' has the next to largest estimate of the elasticity of substitution between market goods and time. Although it is true that activities such as personal hygiene are very time-intensive, you can certainly spend a lot of money, relative to time, on such activities. Also, activities like laundry and clothes care could be done in various ways that range from the very time-intensive to the very goods-intensive.

Finally, 'Recreation' has the highest elasticity of substitution. It is not difficult to find examples of recreational activities in which the substitution between market goods and time is very easy. Moreover, this commodity includes very time-intensive activities such as reading, writing, conversing and thinking, as well as very market-good intensive activities such as social events, sports or some hobbies.

Given that ρ_j does not play any role in our analysis we can simplify our estimation by writing equation (6) as:

$$ln(Y_j/T_j) = constant + \sigma_j ln(wage_{HH}) \tag{7}$$

where $wage_{HH}$ is the sum of the husband's and wife's wage rates.

The benefit of this simplification is that equation (7) is now linear. In Table 6 we compare estimates of the elasticity of substitution for 'Eating', 'Lodging', 'Appearance', and 'Recreation' using equations (6) and (7). Comparing OLS and NLLS columns, we conclude there is no statistically significant difference in the estimates of σ regardless of whether we use equation (6) or (7).

By defining $wage_{HH}$ as the sum of the wages of the spouses, we are implicitly assuming that the wages of the husband and wife have the same weight. However, estimates of ρ_j using non-linear least squares are significantly different from 0.5. Thus to check whether implicitly assuming equal weights makes a difference in the estimates of σ_j we estimate the following equation:

$$ln(Y_j/T_j) = constant + \sigma_j ln(\hat{\rho}_j w_m + (1-\hat{\rho}_j)w_f) \tag{8}$$

where $\hat{\rho}_j$ comes from the estimates of ρ_j in Table 5. When comparing the estimates of the elasticities from this equation with the OLS estimates from equation (7), it turns out that the estimates of the

Table 6: OLS and NLLS Equation by Equation[a, b]

	N	OLS	NLLS
Eating	2727	.345	.344
		(.015)	(.015)
Lodging	2738	.449	.447
		(.019)	(.019)
Appearance	2733	.465	.462
		(.018)	(.018)
Recreation	2367	.576	.573
		(.029)	(.029)

[a] Standard errors in parenthesis.

[b] Estimates in this table refer to $\hat{\sigma}$, the elasticity of substitution between market goods and time. Control variables are urban dummy, state dummies, number of children less than 12 years old, number of daughters over 12 years old and number of sons over 12 years old.

elasticities under equation (8) are very similar to the estimates under equation (7).[24] Hence, assuming equal weights or using the optimal weights from equation (6) makes little difference in the estimates of the elasticities of substitution between market goods and time. Therefore, the remainder of the study will use the estimation based on equation (7).

To test whether the coefficients are the same across commodity equations we estimate the four commodity equations as a system.[25] We test and reject the hypothesis that all coefficients are equal using a Wald test. We also test the same hypothesis and reject the null for all different pairs of coefficients, except for the case when we compare 'Lodging' and 'Appearance' commodities.

4.2 Instrumental Variables Estimation

We suspect $wage_{HH}$ is endogenous in equation (7). There are unobservable characteristics, such as diligence or attitude toward planning, that are highly valued both in the labor market and in home production. Therefore, households which are efficient at home production are usually also efficient in the labor market, which translates into higher salaries. Without correcting the omitted variables problem the estimates of the elasticity of substitution will be inconsistent. To obtain consistent estimates of the elasticity we need instruments, variables correlated with family labor earnings but not directly with household production.

The set of instruments for the household labor earnings that we are using are: whether the firm in which the husband works is unionized and the size of the firm in which the husband and the wife are employed (measured by the number of workers). All our instruments are valid. The union dummy and size of the firm variables are clearly not related to the household decision of how much market goods and how much time to use in the production of a certain commodity, but certainly explain a lot of the wages of the husband and the wife, and therefore the household earnings. The prices that households pay for the market goods (implicit in the dependent variable) are clearly not correlated with our instrumental variables. Such prices are taken as given by the household and are not influenced by whether the spouse is a unionized worker or not, or whether he or his wife works in a big or a small company.

To test whether the coefficients are significantly different across the four commodities we estimate

[24] Estimates are available upon request.

[25] Estimates of the system of equations using SUR and the values of all Wald tests are available upon request.

a system of equations using GMM. We estimate system GMM using the set of instruments described above. For the first iteration, we used the estimates from GMM equation by equation. The system includes the household labor earnings equation as well as the four commodity equations. The regressors in the household labor earnings equation are years of education of both spouses, age and age squared of both spouses, firm size for both spouses, and a union dummy for the husband. Estimates of the elasticities of substitution are in Table 7. All coefficients in the table are significantly different from zero.

Table 7: System GMM with Four Commodities: Elasticity of Substitution[a]

Eating	Lodging	Appearance	Recreation
.343	.526	.576	.742
(.085)	(.099)	(.086)	(.117)

[a] Standard errors in parenthesis. Estimates in this table refer to $\hat{\sigma}$, the elasticity of substitution between market goods and time. Control variables are urban dummy, state dummies, number of children less than 12 years old, number of daughters over 12 years old and number of sons over 12 years old. N=2,354.

Similar to the previous estimates, it is the case that 'Eating' has the lowest elasticity of substitution and 'Recreation' has the highest elasticity of substitution. In between we have 'Lodging' and 'Appearance', in that order.

One important difference between the estimates in Table 6, without taking care of the endogeneity problem, and the estimates in Table 7, when the endogeneity problem is appropriately solved, is the value of the estimates. For all commodities except 'Eating', the elasticities of substitution between market goods and time are higher. This suggests that estimation without controlling for possible endogeneity problem is likely to underestimate the true effect of household earnings on the decision between market goods and time.

Using the results in Table 7 we test the hypothesis that the four elasticities of substitution are

Table 8: Wald Tests for System GMM Results

Hypothesis	P-Values
$\hat{\sigma}_{\text{Eating}} = \hat{\sigma}_{\text{Lodging}} = \hat{\sigma}_{\text{Appearance}} = \hat{\sigma}_{\text{Recreation}}$	0.016
$\hat{\sigma}_{\text{Lodging}} = \hat{\sigma}_{\text{Appearance}} = \hat{\sigma}_{\text{Recreation}}$	0.305
$\hat{\sigma}_{\text{Eating}} = \hat{\sigma}_{\text{Lodging}}$	0.091
$\hat{\sigma}_{\text{Eating}} = \hat{\sigma}_{\text{Appearance}}$	0.022
$\hat{\sigma}_{\text{Eating}} = \hat{\sigma}_{\text{Recreation}}$	0.002
$\hat{\sigma}_{\text{Lodging}} = \hat{\sigma}_{\text{Appearance}}$	0.639
$\hat{\sigma}_{\text{Lodging}} = \hat{\sigma}_{\text{Recreation}}$	0.131
$\hat{\sigma}_{\text{Appearance}} = \hat{\sigma}_{\text{Recreation}}$	0.204

equal. P-Values of the corresponding Wald tests are reported in Table 8. In the first row we test the hypothesis that all elasticities are the same and we reject it. However, according to the second row, we cannot reject the null that the elasticities for 'Lodging' , 'Appearance', and 'Recreation' are the same. This result is supported by the corresponding p-values in the last three rows where we test the hypothesis that each pair of these commodities' elasticities are the same.

For this reason, we calculated the elasticities of substitution using system GMM with instrumental variables for the commodities defined as 'Eating', and the composite commodity 'Lodging-Appearance-Recreation'. The results are in Table 9.

Based on Table 9, it is again the case that the 'Eating' elasticity of substitution is the smallest. These results are used to analyze the policy implications of our theoretical model. The elasticity of substitution for 'Eating' is 0.440 and 0.681 for 'Lodging-Appearance-Recreation'.

5 Policy Implications

The differences in the goods-time substitution of each commodity suggest the importance of setting differential goods taxes. This section calculates the optimal goods taxes in Mexico. Based on the results in Table 8, we denote Z_0, Z_1, and Z_2 as 'Sleeping', 'Eating', and 'Lodging-Appearance-Recreation'. Table 4 shows that Mexican households spend on average 389.77 pesos and 465.29 pesos on Z_1 and Z_2,

Table 9: System GMM with Two Commodities: Elasticity of Substitution[a]

Eating	Lodging + Appearance + Recreation
.440	.681
(.029)	(.028)

[a] Standard errors in parenthesis. Estimates in this table refer to $\hat{\sigma}$, the elasticity of substitution between market goods and time. Control variables are urban dummy, state dummies,number of children less than 12 years old, number of daughters over 12 years old and number of sons over 12 years old. N = 2,354.

respectively. They also spend 113.87 hours a week on T_0, 45.33 hours on T_1, and 71.80 hours on T_2, and they work 62.18 hours per week. In addition, the elasticities of substitution between goods and time for Z_1 and Z_2 are 0.440 and 0.681 in that order. We assume these observed goods expenditures and time use patterns are the outcome of the optimal choice made by Mexican consumers under the current tax system in Mexico. We simplify the actual Mexican tax system by setting tax rates on Z_1 equal to 0% and Z_2 equal to 15%.[26]

For policy analysis we use the same log-utility function in equation (5). We have to recover values for the underlying parameters from our data set. Note that we need values for the following 10 parameters: θ_1, θ_2, w, T, p_1, p_2, δ_0, δ_1, δ_2, and M. The system GMM estimation in Table 9 gives the values for θ_1 and θ_2. We set $w = T = 1$.[27] From the solution of the utility optimization problem we can solve for X_1^*, X_2^*, T_0^* T_1^*, and T_2^*. Then we have six equations[28] and six parameters. Solving the system, we get $p_1 = 0.24$, $p_2 = 0.44$, $\delta_0 = 0.19$, $\delta_1 = 0.31$, $\delta_2 = 0.49$, and $M = 0.97$.[29]

[26] In reality, appliances and eating outside are taxed, but the expenditures on these goods are small.

[27] Think of p_1 and p_2 as the prices of goods relative to the wage rate. T_j for j=0,1,2 is the ratio of hours to the total time spending, that is $T_0 = 38.8\%$, $T_1 = 17.8\%$, $T_2 = 18.4\%$, and $L = 25.0\%$.

[28] Five equations from the solution of utility optimization problem and one equation from the parameter restriction; $\delta_0 + \delta_1 + \delta_2 = 1$. For detailed solutions to this system of equations, please refer to Technical Appendix A.3

[29] We used the $fsolve$ function built in MATLAB to solve the six equations simultaneously. The initial vector is [p_1 p_2 δ_0 δ_1 δ_2 M] = [1 1 0.33 0.33 0.33 1].

Table 10: Optimal Tax Rate[a]

		Current(A)	Optimal(B)	$(B)-(A)$
Tax rate	Eating	0.0%	7.0%	
	Lodging + Appearance + Recreation	15.0%	5.5%	
Expenditure[a]	Eating	389.77	288.76	-101.01
	Lodging + Appearance + Recreation	465.29	599.39	134.10
Time spending[b]	Sleeping	113.87	113.87	
	Eating	45.33	52.22	6.89
	Lodging + Appearance + Recreation	71.80	53.80	-18.00
	Work	62.18	73.30	11.11

[a] Mexican pesos.

[b] Hours per week.

Now we have all the values we need to calculate the optimal tax rates. From the $10,201\left(=101^2\right)$ possible tax rate combinations (s_1, s_2),[30] we pick all combinations that give the government the same revenue as in the current tax system. For each of these combinations we calculate the corresponding indirect utility value $V(s_1, s_2)$. The pair (7.0%, 5.5%) gives the highest possible indirect utility, therefore this vector is the optimal tax combination.

Table 10 shows the household's behavior under the optimal tax system. Under the optimal tax rates, our model predicts Mexican household spends 288.76 pesos and 52.22 hours on Z_1 weekly on average. They also spend 599.39 pesos and use 53.80 hours on Z_2 a week on average. They work 73.30 hours a week. Compared with the current tax rates, the optimal tax system requires government to increase the tax rate on Z_1 by 7 percentage points (from 0% to 7.0%) and reduce the tax rate on Z_2 by 9.5 percentage points (from 15% to 5.5%).

6 Conclusions

We relax the usual assumption that individuals get utility directly from market goods. Instead, following Becker (1965), we assume that individuals combine market goods and time to produce commodities which ultimately yield utility. Previous research has incorporated Becker's idea that goods have to be combined with time to yield utility, but it simplifies the analysis by assuming a Leontief commodity production function. Thus, our contribution consists of allowing substitution

[30] For each $s_j \in \{0.000, 0.005, 0.010, ..., 0.490, 0.495, 0.500\}$ for j=1,2.

between market goods and time in the production of commodities by assuming a CES commodity production function. By incorporating these assumptions into the Ramsey optimal tax problem we show it is optimal to impose lower taxes on goods used in the production of commodities with a higher elasticity of substitution because these goods are easily substitutable for time. Likewise, goods used to produce a commodity in which it is difficult to substitute away from market goods toward time should be taxed at a higher rate. The goal is to minimize the distortionary effects of taxes over household utility maximization. This is an analog of the classical Corlett and Hague (1953-1954) result, differing in that we allow for substitution between time and goods expenditures.

Using the Mexican time use data set from 2002, we estimate the elasticity of substitution between goods expenditures and time in the production of four different commodities: 'Eating', 'Lodging', 'Appearance', and 'Recreation'. For these four commodities, we find that the elasticity is significantly different from zero and 'Eating' has a significantly different elasticity from 'Lodging', 'Appearance', and 'Recreation'. The elasticity of substitution for 'Recreation' is highest. However, we cannot reject the hypothesis that the elasticity of substitution for 'Lodging' is equal to the elasticity of substitution for 'Appearance' and 'Recreation'.

Combining these estimates of the elasticity of substitution with our theoretical results, we conclude that higher taxes should be imposed on the market goods, like food, used in the production of 'Eating'. Along the same lines, lower taxes should be imposed on the market goods used in the production of 'Lodging', 'Appearance', and 'Recreation'. The optimal tax structure is regressive, it goes against the common practice of exempting necessities such as food from sales tax bases. Comparing this optimal tax system to the actual one, we can argue that the Mexican government has traded off efficiency for equity. The actual system in Mexico has a zero tax rate on food and a 15 percent value added tax on all other goods except medicines. Households are very heterogeneous in their earning ability, so by exempting food the government may be attempting to make sales taxes less regressive. This regressivity suggests that future research needs to address the efficiency-equity trade-off of commodity taxation.

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Appendix

A. Derivations of Theoretical Model

A.1 Household Utility Maximization Problem

A.1.1 Step One

Given $\bar{Z}_j, q_j$, and w,

$$\min_{X_j, T_j} q_j X_j + w T_j \quad \text{such that } \bar{Z}_j = \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}}.$$

The lagrangian is:

$$L = q_j X_j + w T_j + \eta_j \left(\bar{Z}_j - \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}}\right).$$

Differentiate with respect to X_j, and T_j, we get first-order conditions:

$$q_j = \eta_j \left(\frac{1}{\theta_j}\left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}-1}\right)\theta_j X_j^{\theta_j - 1}, \quad w = \eta_j \left(\frac{1}{\theta_j}\left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}-1}\right)\theta_j T_j^{\theta_j - 1}.$$

Using the first-order conditions, we can get

$$\frac{X_j}{T_j} = \left(\frac{w}{q_j}\right)^{\frac{1}{1-\theta_j}}. \tag{9}$$

From the home production function $\left(\bar{Z}_j = \left(X_j^{\theta_j} + T_j^{\theta_j}\right)^{\frac{1}{\theta_j}}\right)$ and equation (9), we have:

$$X_j = \alpha_j \bar{Z}_j \tag{10}$$

$$T_j = \beta_j \bar{Z}_j \tag{11}$$

where $\alpha_j \equiv \left(1 + \left(\frac{p_j + s_j}{w}\right)^{-\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}, \quad \beta_j \equiv \left(1 + \left(\frac{p_j + s_j}{w}\right)^{\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}.$

A.1.2 Step Two

$$\max_{Z_0, Z_1, \cdots, Z_n} U\left(Z_0, Z_1, \cdots, Z_n\right)$$
$$\text{such that } q_1 X_1 + ... + q_n X_n = w\left(T - T_1 - ... - T_n - T_0\right) + M$$

We can rewrite the budget constraint by using (10) and (11).

$$q_1 X_1 + ... + q_n X_n = w\left(T - T_1 - ... - T_n - T_0\right) + M$$

$$\gamma_0 Z_0 + \gamma_1 Z_1 + ... + \gamma_n Z_n = wT + M$$

$$\text{where } \gamma_j = \begin{cases} w & j=0 \\ q_j\alpha_j + w\beta_j & j=1,..,n \end{cases}$$

So the maximization problem is:

$$\max_{Z_0,Z_1,\cdots,Z_n} U\left(Z_0, Z_1, \cdots, Z_n\right) \text{ such that } \gamma_0 Z_0 + \gamma_1 Z_1 + ... + \gamma_n Z_n = wT + M.$$

Then solutions are $U_j = \lambda\gamma_j \quad$ for $j = 0, 1, .., n$ where λ is the lagrangian multiplier.

A.2 Optimal Government Policy Problem

A.2.1 Optimal Government Policy Problem

The Government problem is

$$\max_{s_1,\ldots,s_n} V\left(q_0, q_1, \cdots, q_n, w\right) \text{ such that } s_1 X_1 + \cdots + s_n X_n = \bar{R}.$$

The lagrangian is:

$$L = V\left(q_0, q_1, \cdots, q_n, w\right) + \mu\left(s_1 X_1 + \cdots + s_n X_n - R\right)$$

where μ is the lagrangian multiplier. Differentiate the Lagrangian with respect to $s_1, ..., s_n$. we get:

$$\frac{dL}{ds_k} = \frac{\partial V}{\partial q_k}\frac{dq_k}{ds_k} + \mu\left(X_k + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial q_k}\frac{dq_k}{ds_k}\right) = 0 \text{ for } k = 1, ..., n.$$

Using $dq_k/ds_k = 1$, we get

$$\begin{aligned} \lambda\left(\frac{1}{\alpha_k}X_k\right)\frac{\partial\gamma_k}{\partial q_k} &= \mu\left(X_k + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial q_k}\right) \\ \frac{\lambda\left(\frac{1}{\alpha_k}\frac{\partial\gamma_k}{\partial q_k}\right) - \mu}{\mu} &= \sum_{j=1}^{n} \frac{s_j}{X_k}\frac{\partial X_j}{\partial q_k}. \end{aligned}$$

Then using $\partial\gamma_k/\partial q_k = \alpha_k$, we have

$$\frac{\lambda - \mu}{\mu} = \sum_{j=1}^{n} \frac{s_j}{X_k}\frac{\partial X_j}{\partial q_k}. \tag{12}$$

With the property of slutsky equation and slutsky symmetry, equation (12) becomes

$$\frac{\lambda - \mu}{\mu} + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial M} = \sum_{j=1}^{n} \frac{s_j}{X_k}\frac{\partial X_k^c}{\partial q_j}. \tag{13}$$

And the left hand side of equation (13) does not depend on k. So let $-\Theta \equiv \frac{\lambda-\mu}{\mu} + \sum_{j=1}^{n} s_j \frac{\partial X_j}{\partial M}$, then equation (13) is:

$$-\Phi = \sum_{j=1}^{n} \frac{s_j}{q_j}\varepsilon_{ki}^c \quad \text{where } \varepsilon_{ki}^c \equiv \frac{q_j}{X_k}\frac{\partial X_k^c}{\partial q_j} \tag{14}$$

A.2.2 Three-commodity Economy

To derive the property of compensated elasticity, we differentiate $\bar{U} = U(T_0, X_1, T_1, X_2, T_2)$ with respect to q_1. Then by using the envelope theorem and slutsky symmetry, we derive

$$\begin{aligned}
0 &= U_{T_0}\frac{\partial T_0^c}{\partial q_1} + U_{X_1}\frac{\partial X_1^c}{\partial q_1} + U_{T_1}\frac{\partial T_1^c}{\partial q_1} + U_{X_2}\frac{\partial X_2^c}{\partial q_1} + U_{T_2}\frac{\partial T_2^c}{\partial q_1} \\
&= \lambda w\frac{\partial T_0^c}{\partial q_1} + \lambda q_1\frac{\partial X_1^c}{\partial q_1} + \lambda w\frac{\partial T_1^c}{\partial q_1} + \lambda q_2\frac{\partial X_2^c}{\partial q_1} + \lambda w\frac{\partial T_2^c}{\partial q_1} \\
&= \frac{w}{X_1}\frac{\partial X_1^c}{\partial w} + \frac{q_1}{X_1}\frac{\partial X_1^c}{\partial q_1} + \frac{w}{X_1}\frac{\partial X_1^c}{\partial w} + \frac{q_2}{X_1}\frac{\partial X_1^c}{\partial q_2} + \frac{w}{X_1}\frac{\partial X_1^c}{\partial w} \\
&= \varepsilon_{11}^c + \varepsilon_{12}^c + 3\varepsilon_{10}^c.
\end{aligned}$$

Using $u(Z_0, Z_1, Z_2) = \delta_0 \ln Z_0 + \delta_1 \ln Z_1 + \delta_2 \ln Z_2$, let's calculate compensated demand.

$$\min \gamma_0 Z_0 + \gamma_1 Z_1 + \gamma_2 Z_2 \;\; s.t \;\; \bar{U} = \delta_0 \ln Z_0 + \delta_1 \ln Z_1 + \delta_2 \ln Z_2$$

Then we can obtain the following compensated demand function for X_1 and X_2:

$$X_1^c = \alpha_1 \bar{U}\left(\frac{\delta_1}{\delta_0}\frac{\gamma_1}{\gamma_0}\right)^{\delta_0}\left(\frac{\delta_1}{\delta_2}\frac{\gamma_2}{\gamma_1}\right)^{\delta_2}, \quad X_2^c = \alpha_2 \bar{U}\left(\frac{\delta_1}{\delta_0}\frac{\gamma_1}{\gamma_0}\right)^{\delta_0}\left(\frac{\delta_1}{\delta_2}\frac{\gamma_2}{\gamma_1}\right)^{\delta_2 - 1}$$

where $\alpha_j \equiv \left(1 + \left(\frac{q_j}{w}\right)^{-\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}$ for $j = 1, 2$. Then

$$\frac{w}{X_1^c}\frac{dX_1^c}{dw} = \frac{w}{\alpha_1}\frac{d\alpha_1}{dw} - \delta_0\frac{w}{\gamma_0}\frac{d\gamma_0}{dw} + (\delta_0 - \delta_2)\frac{w}{\gamma_1}\frac{d\gamma_1}{dw} + \delta_2\frac{w}{\gamma_2}\frac{d\gamma_2}{dw} \tag{15}$$

$$\begin{aligned}
\frac{w}{X_2^c}\frac{dX_2^c}{dw} = &\frac{w}{\alpha_2}\frac{d\alpha_2}{dw} + \delta_0\frac{w}{\gamma_1}\frac{d\gamma_1}{dw} - \delta_0\frac{w}{\gamma_0}\frac{d\gamma_0}{dw} + (\delta_2 - 1)\frac{w}{\gamma_2}\frac{d\gamma_2}{dw} \\
&- (\delta_2 - 1)\frac{w}{\gamma_1}\frac{d\gamma_1}{dw}
\end{aligned} \tag{16}$$

From equation (15) and (16),

$$\begin{aligned}
\frac{w}{X_2^c}\frac{dX_2^c}{dw} - \frac{w}{X_1^c}\frac{dX_1^c}{dw} &= \left(\frac{w}{\alpha_2}\frac{d\alpha_2}{dw} - \frac{w}{\gamma_2}\frac{d\gamma_2}{dw}\right) - \left(\frac{w}{\alpha_1}\frac{d\alpha_1}{dw} - \frac{w}{\gamma_1}\frac{d\gamma_1}{dw}\right) \\
&= \frac{\frac{\theta_2}{1-\theta_2}}{1 + \left(\frac{w}{q_2}\right)^{\frac{\theta_2}{1-\theta_2}}} - \frac{\frac{\theta_1}{1-\theta_1}}{1 + \left(\frac{w}{q_1}\right)^{\frac{\theta_1}{1-\theta_1}}}
\end{aligned}$$

This does not immediately translate into $\sigma_1 < \sigma_2 \rightarrow \varepsilon_{10}^c < \varepsilon_{20}^c$. However this result always holds if the price of the necessity (q_1) is lower than the price of the luxury (q_2). Even if the price of the necessity is higher than the price of the luxury, the result holds as long as the elasticity of substitution of Z_2 is sufficiently larger than that of Z_1. Conventional wisdom contends that a necessity tends to have a lower elasticity of substitution than a luxury. As shown empirically in Section 4, the elasticity of substitution for a necessity is significantly lower than that of a luxury.

A.3 Policy Implication

A.3.1 The Solution to The Household Maximization Problem

$X_j^* = \alpha_j \frac{\delta_j}{\gamma_j} (wT + M)$, $T_j^* = \beta_j \frac{\delta_j}{\gamma_j} (wT + M)$

where $\alpha_j \equiv \left(1 + \left(\frac{q_j}{w}\right)^{-\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}$, $\beta_j \equiv \left(1 + \left(\frac{q_j}{w}\right)^{\frac{\theta_j}{\theta_j - 1}}\right)^{-\frac{1}{\theta_j}}$

and $\gamma_j = \begin{cases} w & if \quad i = 0 \\ q_j \alpha_j + w \beta_j & if \quad i = 1, 2. \end{cases}$

A.3.2 Six Equations and Six Unknown Parameters

We solved 6 equations simultaneously to get values of 6 unknown parameters. The six unknown parameters are $p_1, p_2, \delta_0, \delta_1, \delta_2, M$, and the six equations are: $T_0^* = 0.389$, $T_1^* = 0.178$, $T_2^* = 0.184$, $\frac{p_1 X_1^*}{p_2 X_2^*} = 0.838 \left(= \frac{389.77}{465.29}\right)$, $\sum_{i=0}^{2} \delta_i = 1$, $\sum_{i=1}^{2} q_i X_i^* = w\left(T - \sum_{i=1}^{2} T_i^*\right) + M$. Solving the system, we get $p_1 = 0.2493$, $p_2 = 0.4489$, $\delta_0 = 0.1962$, $\delta_1 = 0.3103$, $\delta_2 = 0.4936$, and $M = 0.9797$.