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Abstract

This paper shows a detail-free idea of multi-object large double auction design in general trading environments, where the auctioneer randomly divides agents into two groups, and agents in each group trade at the market-clearing price vector in the other group. With private values, any dominant strategy profile mimics price-taking behavior, and the auctioneer achieves approximate efficiency. With interdependent values, any twice iteratively undominated strategy profile mimics fully revealing rational expectations equilibrium, and the auctioneer approximately achieves ex post efficiency. We need only a very weak common knowledge assumption on rationality.

Keywords: Random Grouping, Detail-Free Auction Design, Dominance, Perfect Competition, Rational Expectations Equilibrium, Asymptotic Efficiency.

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1. Introduction

The hypothesis of perfect competition assumes that traders, or agents, are non-strategic, and adopt price-taking behavior. In order to provide its strategic foundation, several works after Wilson (1977) investigated naïve models of large private value double auction with a single object, where many sellers and buyers announce their supply and demand functions, and trade at the market-clearing price. With the continuum of agents, every agent has a dominant strategy to behave as a price taker, because her demand or supply never influences the market-clearing price. As long as the number of agents is finite, however, each agent may be able to manipulate the market-clearing price on behalf of her benefit. In fact, each agent has no dominant strategy, and is involved in complicated strategic interaction. Hence, most of these works replaced dominant strategy with Bayesian Nash equilibrium. Rustichini, Satterthwaite, and Williams (1994) showed that in the independent private signal case with single-unit demands and supplies, any symmetric Bayesian Nash equilibrium, if exists, approximates price-taking behavior. Fudenberg, Mobius, and Szeidl (2003) investigated the correlated private signal case with single-unit demands and supplies, and showed the existence of approximately efficient symmetric Bayesian Nash equilibrium. Jackson and Swinkels (2004) investigated a variety of multi-unit double auctions with private values, and showed the existence of non-trivial mixed strategy Bayesian Nash equilibrium. The analyses of Bayesian Nash equilibrium typically assume that, not only the model specification such as payoff and information structures, but also agents' rational behavior, is common knowledge among all agents. This assumption is quite restrictive, especially in large economies. Hence, the strategic foundation without such common knowledge assumptions is very important to consider.

Based on the above observations, the present paper investigates alternative models of double auction instead of the naïve models, and shows the possibility that, in a very wide class of multi-object and multi-unit trading environments, each agent has a dominant strategy that mimics price-taking behavior, and the auctioneer can achieve efficiency in the limit as the number of agents grows. We introduce a new idea of auction design, whose intuition is as follows. The auctioneer randomly divides sellers and buyers into two groups. Each seller (buyer) announces a supply function (demand function, respectively). The auctioneer deals with each group as being separate, and calculates the price vector that equalizes the total demand and supply in every commodity markets that are announced by the agents who belong to this group. And then, agents in each group trade at the market-clearing price vector in the other group. Hence, each agent's announcement never influences the price vector at which she trades. By specifying a rationing rule appropriately, the auctioneer succeeds to induce agents to announce honest competitive demands and supplies in every commodity market as their dominant strategies. The law of large numbers guarantees that the market-clearing price vector in each group converges in probability to the market-clearing price vector in the whole commodity markets that combine both groups. Hence, the auctioneer achieves efficiency in the limit as the number of agents grows, and our double auctions can be regarded as stochastic approximation of perfect competition.

Of particular importance, our double auctions are detail-free in that the auctioneer needs no information about the model specification. This point is in contrast with the mechanism design literature, where the central planner possesses full knowledge on the model specification, and the designed mechanisms depend crucially on its fine detail. As Wilson (1987) has admonished, the restriction to detail-free mechanisms is very important to consider from the practical viewpoint.

Barberà and Jackson (1995) showed that in economic environments, any social choice function that is strategy-proof in terms of dominant strategy is inefficient, even in the limit as the number of agents grows. This impossibility relies on the restriction that excludes stochastic social choice functions that map preference profiles to lotteries over pure allocations. Gibbard (1977) and Benoit (2002) showed that in general social choice environments, no non-trivial stochastic social choice function is strategy-proof. In contrast to these works, this paper shows that stochastic decision does play a powerful role, particularly in economic environments.

This paper assumes that agents' preferences are quasi-linear and risk neutral. With this assumption, Vickery (1961), Clarke (1971), and Groves (1973) designed so-called VCG mechanisms, where truth-telling is a dominant strategy and achieves efficiency. The drawback of their works is that the VCG mechanisms do not satisfy budget-balancing. McAfee (1992) showed an alternative idea of double auction design, where the budget is not balanced, but the budgetary deficit never occurs. McAfee's analysis relies crucially on the assumption that each buyer (seller) has only single-unit demand (supply, respectively). In contrast to these works, our double auctions satisfy budget-balancing, and can be applied to the very general private value cases with multi-object and multi-unit demands and supplies, where we allow any mixture of complements and substitutes for each trader.

The basic idea of random grouping can be applied to the interdependent value case also, where each agent's payoff depends, not only on her private signal, but also on the other traders' private signals. The auctioneer randomly divides sellers and buyers into two groups. Each buyer (seller) announces a triplicate of messages, where the first message is a demand (supply) function, and the latter two messages are demand (supply, respectively) functions contingent on the other agents' first messages. The auctioneer calculates the market-clearing price vector in each group according to the second messages announced in this group. Agents in the other group, almost certainly, trade at this price vector, where the auctioneer uses their third messages as their demands and supplies. With small but positive probability, all agents trade at a randomly chosen price vector, where the auctioneer uses their first messages as their demands and supplies. This will provide agents with the incentive to announce competitive demand and supply functions honestly as their first messages. Hence, with a minor informational condition, agents' first messages will fully reveal their private signals. Based on this intuition, the present paper designs detail-free double auctions in the Bayesian framework, where any iteratively undominated strategy profile describes price-taking behavior, is fully revealing, and achieves ex post efficiency in the limit as the number of traders grows.

This possibility result is closely related to rational expectations equilibrium in competitive economies. Since the seminal work by Lucas (1972), the notion of rational expectations equilibrium has been pervasive in many fields of economics. The rational expectations equilibrium hypothesis assumes that agents act rationally with respect to information, while they adopt price-taking behavior in a non-strategic way. In order to provide its strategic foundation, Reny and Perry (2003) investigated naïve models of single-object and single-unit double auction with interdependent values and with finite agents. Reny and Perry showed that when agents' private signals are strictly affiliated and the number of agents is sufficiently large, there exists a fully-revealing and approximately efficient pure strategy Bayesian Nash equilibrium that mimics price-taking behavior.

The rational expectations hypothesis presumes that all agents' rational behavior is common knowledge among them. In fact, even with the continuum of agents, price-taking behavior is never described by dominant strategy. In contrast to rational expectations equilibrium, this paper needs to assume only a very weak common knowledge assumption on rationality, i.e., assume only that it is common knowledge among all agents that any agent never plays dominated strategies. Hence, all we need to do for derivation of iteratively undominated strategies is to check only two rounds of iterative removal of dominated strategies. Moreover, the set of iteratively undominated strategy profiles satisfies interchangeability, i.e., any combination of strategies that survive after two rounds of iterative removal is a Bayesian Nash equilibrium.

This paper covers a very wide class of trading environments even with interdependent values. We do not require the private signals to be affiliated. We allow multiple objects to be traded. We allow any mixture of complements and substitutes for every agent.

The organization of this paper is as follows. Section 2 considers the private value case. Subsection 2.1 shows the model. Subsection 2.2 designs double auction mechanisms. Subsections 2.3 and 2.4 show that price-taking behavior is described by dominant strategies and the auctioneer can achieve efficiency in the limit as the number of agents grows.

Section 3 considers the interdependent value case. Subsection 3.1 shows the model. Subsection 3.2 designs double auction mechanisms. Subsections 3.3 and 3.4 show that price-taking behavior is described by twice iteratively undominated strategy profiles and the auctioneer can achieve ex post efficiency in the limit as the number of agents grows, where it is assumed that only buyers have interdependent values. Subsection 3.5 extends our analysis to the general interdependent case, where the auctioneer randomly divides agents into three or more groups.

2. Private Values

This section assumes *private values* in that each agent receives no information about the other agents' payoffs that they do not know.

2.1. The Model

There exist $4n$ agents, where the first $2n$ agents are called *sellers*, and the latter $2n$ agents are called *buyers*. There are k different commodities to be traded. Seller $i \in \{1,...,2n\}$ can supply each commodity up to l units. Buyer $i \in \{2n+1,...,4n\}$ will demand each commodity up to l units. Fix a positive integer T arbitrarily, which may be sufficiently large. Let $P = \{0, \frac{1}{T}, \frac{2}{T}, ..., 1\}^k$ denote the finite set of price vectors. An *allocation* is defined as a combination $a = (x, q)$ where

$$x = (x_1, ..., x_{4n}),\ x_i = (x_i(1), ..., x_i(k)) \in \{0, ..., l\}^k \text{ for all } i \in \{1, ..., 4n\},$$

$$q = (q_1, ..., q_{4n}),\ q_i = (q_i(1), ..., q_i(k)) \in P \text{ for all } i \in \{1, ..., 4n\},$$

(1) $$\sum_{i=1}^{2n} x_i(h) = \sum_{i=2n+1}^{4n} x_i(h) \text{ for all } h \in \{1, ..., k\},$$

and

(2) $$\sum_{i=1}^{2n} \sum_{h=1}^{k} x_i(h) q_i(h) = \sum_{i=2n+1}^{4n} \sum_{h=1}^{k} x_i(h) q_i(h).$$

Seller $i \in \{1,...,2n\}$ sells the amount $x_i(h) \in \{0,...,l\}$ of commodity $h \in \{1,...,k\}$ at the unit price $q_i(h) \in P$. Buyer $i \in \{2n+1,...,4n\}$ buys the amount $x_i(h)$ of commodity h at the unit price $q_i(h)$. Equalities (1) imply that the total buying and selling amounts are balanced. Equality (2) implies *budget balancing* in that buyers' payments and sellers' revenues are balanced. Let A denote the set of allocations. Let Δ denote the set of simple lotteries over allocations. Agent $i's$ payoff function with expected utility hypothesis is given by $u_i : A \to R$. For every $\alpha \in \Delta$, let $u_i(\alpha) = \sum_{a \in \Gamma} u_i(a)\alpha(a)$, where Γ is the support of α.

A *mechanism* is defined by $G = (M, g)$, where M_i is the set of messages for agent $i \in \{1,...,4n\}$, $M = \prod_{i=1}^{4n} M_i$, and $g : M \to \Delta$. When agents announce a message profile $m = (m_i)_{i=1}^{4n} \in M$, the mechanism G chooses any allocation $(x, q) \in \Gamma$ with probability $g(m)(x, q) \in (0,1]$, where Γ is the support of the lottery $g(m)$, and $\sum_{a \in \Gamma} g(m)(a) = 1$. A combination $(G, (u_i)_{i=1}^{4n})$ defines a *game*. A message $m_i \in M_i$ for agent $i \in \{1,...,4n\}$ is said to be *dominant* in the game $(G, (u_i)_{i=1}^{4n})$ if

$$u_i(g(m_i, m'_{-i})) \geq u_i(g(m')) \text{ for all } m' \in M.$$

A message profile $m \in M$ is said to be *ex post individually rational* in $(G,(u_i)_{i=1}^{4n})$ if for every $a \in A$,

$$u_i(a) \geq 0 \text{ for all } i \in \{1,...,4n\} \text{ whenever } q(m)(a) > 0 .$$

2.2. Auction Design

We consider the following mechanism denoted by G^*, where sellers and buyers are randomly divided into groups 1 and 2. Each seller announces a supply function, whereas each buyer announces a demand function. Almost certainly, sellers in group 1 (group 2) trade with buyers in group 1 (group 2) at the price vector that approximates the market-clearing price vector in group 2 (group 1, respectively).

We construct $G^* = (M, g)$ as follows. Let D denote the set of functions $d : P \to \{0,...,l\}^k$. Let

$$M_i = D \text{ for all } i \in \{1,...,4n\},$$

where we denote $m_i : P \to \{0,...,l\}^k$ and $m_i(p) = (m_i(p)(h))_{h=1}^k$. Let Φ denote the set of one-to-one mappings $\phi : \{1,...,4n\} \to \{1,...,4n\}$ where

$$\phi(i) \in \{1,...,2n\} \text{ for all } i \in \{1,...,2n\},$$

and

$$\phi(i) \in \{2n+1,...,4n\} \text{ for all } i \in \{2n+1,...,4n\}.$$

A function $\phi \in \Phi$ implies permutations on the set of sellers $\{1,...,2n\}$ and on the set of buyers $\{2n+1,...,4n\}$. Sellers $\phi(1),...,\phi(n)$ and buyers $\phi(2n+1),...,\phi(3n)$ belong to group 1. Sellers $\phi(n+1),...,\phi(2n)$ and buyers $\phi(3n+1),...,\phi(4n)$ belong to group 2. For every $(\phi, m) \in \Phi \times M$, we define

$$\hat{p}^*(\phi, m) \in \arg\min_{p \in P}[\max_{h \in \{1,...,k\}} \left| \sum_{i=1}^{n} \{m_{\phi(i)}(p)(h) - m_{\phi(2n+i)}(p)(h)\} \right|],$$

and

$$\tilde{p}^*(\phi, m) \in \arg\min_{p \in P}[\max_{h \in \{1,...,k\}} \left| \sum_{i=n+1}^{2n} \{m_{\phi(i)}(p)(h) - m_{\phi(2n+i)}(p)(h)\} \right|],$$

where we assume that $\hat{p}^*(\phi, m)$ ($\tilde{p}^*(\phi, m)$) does not depend on the messages announced in group 2, i.e., $\tilde{m}^\phi = (m_{\phi(i)}, m_{\phi(2n+i)})_{i=n}^{2n}$ (the messages announced in group 1, i.e., $\hat{m}^\phi = (m_{\phi(i)}, m_{\phi(2n+i)})_{i=1}^{n}$, respectively). Hence, $\hat{p}^*(\phi, m)$ and $\tilde{p}^*(\phi, m)$ approximate the market-clearing price vectors in group 1 and group 2, respectively.

Fix $(\phi, m) \in \Phi \times M$ arbitrarily. With probability $\frac{1}{(2n!)^2}$, the mechanism G^* chooses the allocation $a^*(\phi, m) = (x, q)$, which is defined as follows. Let

$$q_{\phi(i)} = q_{\phi(2n+i)} = \tilde{p}^*(\phi, m) \text{ for all } i \in \{1,...,n\},$$

and

$$q_{\phi(i)} = q_{\phi(2n+i)} = \hat{p}^*(\phi, m) \text{ for all } i \in \{n+1,...,2n\}.$$

Hence, *in each group, sellers and buyers trade at the approximate market-clearing price vector in the other group.*

We specify the selling and buying amounts $(x_{\phi(i)}, x_{\phi(2n+i)})_{i=n+1}^{2n}$ in group 1 in the following way. Fix any commodity $h \in \{1,...,k\}$ arbitrarily. If there exists excessive supply in group 1, i.e.,

(3) $$\sum_{i=1}^{n} \{m_{\phi(i)}(\tilde{p}^*(\phi,m))(h) - m_{\phi(2n+i)}(\tilde{p}^*(\phi,m))(h)\} \geq 0,$$

then

$$x_{\phi(2n+i)} = m_{\phi(2n+i)}(\tilde{p}^*(\phi,m))(h) \text{ for all } i \in \{1,...,n\},$$

and there exists $\hat{i} \in \{1,...,n\}$ such that

$$x_{\phi(\hat{i})} = \sum_{i=1}^{n} m_{\phi(2n+i)}(\tilde{p}^*(\phi,m))(h) - \sum_{i=1}^{\hat{i}-1} m_{\phi(i)}(\tilde{p}^*(\phi,m))(h)$$
$$\leq m_{\phi(\hat{i})}(\tilde{p}^*(\phi,m))(h),$$
$$x_{\phi(i)} = m_{\phi(i)}(\tilde{p}^*(\phi,m))(h) \text{ for all } i \in \{1,...,\hat{i}-1\},$$

and

$$x_{\phi(i)} = 0 \text{ for all } i \in \{\hat{i}+1,...,n\}.$$

Each buyer buys the same amount of commodity h as what she intends to demand. Each seller before $\phi(\hat{i})$ sells the same amount of commodity h as what she intends to supply, whereas each seller after $\phi(\hat{i})$ cannot sell commodity h. If there exists excessive demand in group 1, i.e., inequality (3) does not hold, then

$$x_{\phi(i)} = m_{\phi(i)}(\tilde{p}^*(\phi,m))(h) \text{ for all } i \in \{1,...,n\},$$

and there exists $\hat{i} \in \{1,...,n\}$ such that

$$x_{\phi(2n+\hat{i})} = \sum_{i=1}^{n} m_{\phi(i)}(\tilde{p}^*(\phi,m))(h) - \sum_{i=1}^{\hat{i}-1} m_{\phi(2n+i)}(\tilde{p}^*(\phi,m))(h)$$
$$\leq m_{\phi(2n+\hat{i})}(\tilde{p}^*(\phi,m))(h),$$
$$x_{\phi(2n+i)} = m_{\phi(2n+i)}(\tilde{p}^*(\phi,m))(h) \text{ for all } i \in \{1,...,\hat{i}-1\},$$

and

$$x_{\phi(2n+i)} = 0 \text{ for all } i \in \{\hat{i}+1,...,n\}.$$

Each seller sells the same amount of commodity h as what she intends to supply. Each buyer before $\phi(2n+\hat{i})$ buys the same amount of commodity h as what she intends to demand, whereas each seller after $\phi(2n+\hat{i})$ cannot buy commodity h. We specify the selling and buying amounts $(x_{\phi(i)}, x_{\phi(2n+i)})_{i=n+1}^{2n}$ in group 2 in the same way. Note that $a^*(\phi, m) = (x, q)$ satisfies equalities (1) and (2).

2.3. Dominant Strategies

We specify agents' payoff functions $(u_i)_{i=1}^{4n}$ as follows. Each seller $i's$ production technology is described by a cost function $c_i : \{0,....,l\}^k \to [0,\infty)$, where $c_i(0,...,0) = 0$. Each seller $i's$ payoff for allocation (x,q) is given by a quasi-linear form

$$u_i(x,q) = \sum_{h=1}^{k} q_i(h)x_i(h) - c_i(x_i).$$

Each buyer $i's$ valuation is described by $v_i : \{0,...,l\}^k \to [0,\infty)$, where $v_i(0,...,0) = 0$. Buyer $i's$ payoff is given by a quasi-linear form

$$u_i(x,q) = v_i(x_i) - \sum_{h=1}^{k} q_i(h)x_i(h).$$

For every seller $i \in \{1,...,2n\}$, let $D_i \subset D$ denote the set of functions d such that for every $p \in P$,

$$d(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i)],$$

which implies seller $i's$ payoff-maximizing supply when she is a *price taker*. For every buyer $i \in \{2n+1,...,4n\}$, let $D_i \subset D$ denote the set of functions d such that for every $p \in P$,

$$d(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [v_i(x_i) - \sum_{h=1}^{k} p(h)x_i(h)],$$

which implies buyer $i's$ payoff-maximizing demand when she is a *price taker*.

Condition 1: For every $i \in \{1,...,2n\}$, there exists $c_i : \{1,...,k\} \times \{1,....,l\} \to [0,\infty)$ such that

$$c_i(x_i) = \sum_{h=1}^{k} \sum_{t=1}^{x_i(h)} c_i(h,t) \text{ for all } x_i \in \{0,...,l\}^k,$$

and

$$c_i(h,1) \le \cdots \le c_i(h,l) \text{ for all } h \in \{1,...,k\}.$$

For every $i \in \{2n+1,...,4n\}$, there exists $v_i : \{1,...,k\} \times \{1,....,l\} \to [0,\infty)$ such that

$$v_i(x_i) = \sum_{h=1}^{k} \sum_{t=1}^{x_i(h)} v_i(h,t) \text{ for all } x_i \in \{0,...,l\}^k,$$

and

$$v_i(h,1) \ge \cdots \ge v_i(h,l) \text{ for all } h \in \{1,...,k\}.$$

The former part of Condition 1 implies that each seller's production technology has *no* externality among different commodities, and the unit cost for each commodity is *non-*

decreasing. The latter part of Condition 1 implies that we do *not* allow any mixture of complements and substitutes among different commodities, and the unit valuation for each commodity is *non-increasing*.

Theorem 1: *Suppose that Condition 1 holds. Then, for every* $i \in \{1,...,4n\}$ *, a message* $m_i \in M_i$ *is dominant in* $(G^*, (u_i)_{i=1}^{4n})$ *if and only if*

$$m_i \in D_i .$$

Any dominant message profile is ex post individually rational in $(G^*, (u_i)_{i=1}^{4n})$.

Proof: For every $(\phi, m) \in \Phi \times M$, each seller $i \in \{1,...,2n\}$ receives the payoff given by

(4) $$\sum_{h=1}^{k} x_i(h) q_i(h) - c_i(x_i) ,$$

where $(x, q) = a^*(\phi, m)$,

$$q_i = \widetilde{p}^*(\phi, m) \text{ if seller } i \text{ belongs to group 1,}$$

and

$$q_i = \widetilde{p}^*(\phi, m) \text{ if seller } i \text{ belongs to group 2,}$$

which implies that q_i does not depend on m_i , and therefore, seller $i's$ message never influences the price vector at which she trades. Hence, it follows from the specification of $a^*(\phi, m)$ and the former part of Condition 1 that m_i always maximizes the value (4) if and only if $m_i \in D_i$. (Under the former part of Condition 1, seller i always prefers any selling amount of each commodity $h \in \{1,...,k\}$ closer to $m_i(q_i)(h)$ where $m_i \in D_i$. This implies that she is willing to announce any message in D_i even if she may sell only less than what she intends to supply.) It is clear that any message in D_i provides seller i with a non-negative payoff.

For every $(\phi, m) \in \Phi \times M$, each buyer $i \in \{2n+1,...,4n\}$ receives the payoff given by

(5) $$v_i(x_i) - \sum_{h=1}^{k} q_i(h) x_i(h) ,$$

where $(x, q) = a^*(\phi, m)$,

$$q_i = \widetilde{p}^*(\phi, m) \text{ if buyer } i \text{ belongs to group 1,}$$

and

$$q_i = \widetilde{p}^*(\phi, m) \text{ if buyer } i \text{ belongs to group 2,}$$

which implies that q_i does not depend on m_i , and therefore, buyer $i's$ message never influences the price vector at which she trades. Hence, it follows from the specification of $a^*(\phi, m)$ and the latter part of Condition 1 that m_i always maximizes the value (5) if and only if $m_i \in D_i$. (Under the latter part of Condition 1, buyer i always prefers any buying amount of each commodity $h \in \{1,...,k\}$ closer to $m_i(q_i)(h)$, where $m_i \in D_i$. This implies that she is willing to announce any message in D_i even if she can buy only less than what she intends to demand.) It is clear that any message in D_i provides buyer i with a non-negative payoff. Hence, we have proved this theorem.

Q.E.D.

2.4. Asymptotic Efficiency and Generalization

For every $\varepsilon > 0$ that is close to zero, a message profile $m \in M$ is said to be $\varepsilon -$ *efficient* in $(G,(u_i)_{i=1}^{4n})$ if

$$\frac{\left| \max_{a\in A} \sum_{i=1}^{4n} u_i(a) - \sum_{i=1}^{4n} u_i(g(m)) \right|}{4n} \le \varepsilon .$$

This implies that the average payoff $\frac{\sum_{i=1}^{4n} u_i(g(m))}{4n}$ induced by message profile m approximates the maximal total surplus per capita $\frac{\max_{a\in A} \sum_{i=1}^{4n} u_i(a)}{4n}$. For every $\varepsilon > 0$ that is close to zero, a message profile $m \in M$ is said to be $\varepsilon -$ *uniform pricing* in $(G,(u_i)_{i=1}^{4n})$ if the probability of the mechanism G choosing any allocation (x,q) such that

$$\left| q_i(h) - q_j(h) \right| < \varepsilon \text{ for all } (i,h) \in \{1,...,4n\} \times \{1,...,k\} \text{ and all } j \in \{1,...,4n\}$$

is more than $1-\varepsilon$. This implies that it is almost certain that all agents trade at almost the same price vector.

We will show that when the number of agents is sufficiently large, the mechanism G^* satisfies approximate efficiency and uniform pricing as follows. Denote $c = c^{(n)}$, $v = v^{(n)}$, $T = T^{(n)}$, and so on. Fix a non-decreasing and continuous function $\rho^* : [0,1]^k \to R$ and $p^* \in [0,1]^k$ arbitrarily, where

$$\rho^*(p^*) = (0,...,0) .$$

Fix an infinite sequence $(c^{(n)}, v^{(n)}, T^{(n)})_{n=1}^{\infty}$ arbitrarily, where

$$\lim_{n\to\infty} T^{(n)} = \infty .$$

Assume that for every infinite sequence of price vectors $(p^{(n)})_{n=1}^{\infty}$, every $p \in [0,1]^k$, and every infinite sequence of message profiles $(m^{(n)})_{n=1}^{\infty}$, whenever

(6) $$m^{(n)} \in \prod_{i=1}^{4n} D_i^{(n)} \text{ for every sufficiently large } n ,$$

and

$$\lim_{n\to\infty} p^{(n)} = p ,$$

then

$$\lim_{n\to\infty}\frac{\sum_{i=1}^{2n}\{m_i^{(n)}(p^{(n)})-m_{2n+i}^{(n)}(p^{(n)})\}}{2n}=\rho^*(p).$$

Hence, $\rho^*(p)$ approximates the excessive supply when n is sufficiently large and all agents play price-taking behavior. For every $n\geq 1$ and every $m=m^{(n)}\in M^{(n)}$, we define

$$p^*(m)=p^{*(n)}(m^{(n)})\in\arg\min_{p^{(n)}\in P^{(n)}}[\max_{h\in\{1,\ldots,k\}}\left|\sum_{i=1}^{2n}\{m_i^{(n)}(p^{(n)})(h)-m_{2n+i}^{(n)}(p^{(n)})(h)\}\right|],$$

which approximates the market-clearing price vector associated with all trades. Assume that for every infinite sequence of message profiles $(m^{(n)})_{n=1}^{\infty}$, whenever property (6) holds, then

$$\lim_{n\to\infty}p^{*(n)}(m^{(n)})=p^*.$$

Hence, p^* approximates the market-clearing price vector associated with all trades when n is sufficiently large and all agents play price-taking behavior. For every sufficiently large n, the maximal total surplus per capita $\dfrac{\max_{a^{(n)}\in A^{(n)}}\sum_{i=1}^{2n}u_i^{(n)}(a^{(n)})}{4n}$ is approximated by the average playoff when all agents trade at the uniform price vector p^*, i.e.,

$$\frac{\sum_{i=1}^{2n}\{v_{2n+i}^{(n)}(m_{2n+i}^{(n)}(p^*))-c_i^{(n)}(m_i^{(n)}(p^*))\}}{4n}.$$

Since whether each agent belongs to group 1 or group 2 is determined according to the uniform distribution on Φ, it follows that for every sufficiently large n and every message profile $m^{(n)}\in M^{(n)}$, it is almost certain that both $\hat{p}^{*(n)}(\phi,m^{(n)})$ and $\tilde{p}^{*(n)}(\phi,m^{(n)})$ are approximated by $p^{*(n)}(m^{(n)})$. Hence, whenever property (6) holds, then it is almost certain that both $\hat{p}^{*(n)}(\phi,m^{(n)})$ and $\tilde{p}^{*(n)}(\phi,m^{(n)})$ are approximated by p^*. From these observations, we have proved the following theorem.

Theorem 2: *For every $\varepsilon>0$, there exists a positive integer n^* such that for every $n\geq n^*$, any message profile $m^{(n)}\in\prod_{i=1}^{4n}D_i^{(n)}$ is $\varepsilon-$ efficient and $\varepsilon-$uniform pricing in $(G^{*(n)},(u_i^{(n)})_{i=1}^{4n})$.*

We will show that whenever n is sufficiently large, then the result of Theorem 1 holds even *without* Condition 1. Assume that for every agent $i\in\{1,\ldots,4n\}$, $D_i^{(n)}$ is a *singleton*, where we denote

$$D_i^{(n)}=\{m_i^{*(n)}\}.$$

Since the number of agents is sufficiently large and agents are randomly divided into groups 1 and 2 according to the uniform distribution on Φ, it follows that for each seller $i \in \{1,...,2n\}$ (each buyer $i \in \{2n+1,...,4n\}$), it is almost certain that she can sell (buy) the same amount of each commodity as what she intends to sell (buy, respectively), i.e., $x_i = m_i(q_i)$ almost certainly holds for all agents $i \in \{1,...,4n\}$. This implies that $m^{*(n)} = (m_i^{*(n)})_{i=1}^{4n}$ is the *unique* dominant message profile in G^*.

Unfortunately, without Condition 1, $m^{*(n)}$ does not satisfy ex post individual rationality in $G^{*(n)}$. Note, however, that whenever n is sufficiently large, then, for every agent $i \in \{1,...,4n\}$, the probability of $x_i \neq m_i^{*(n)}(q_i)$ occurring is very small. This implies that whenever n is sufficiently large, then $m^{*(n)}$ satisfies *participation constraint* in $(G^{*(n)}, (u_i^{(n)})_{i=1}^{4n})$ in the sense that

$$u_i^{(n)}(g(m^{*(n)})) \geq 0 \text{ for all } i \in \{1,...,4n\}.$$

Hence, we have proved the following theorem.

Theorem 3: *There exists a positive integer n^* such that for every $n \geq n^*$, $m^{*(n)}$ is the unique dominant message profile in $(G^{*(n)}, (u_i^{(n)})_{i=1}^{4n})$, and it satisfies participation constraint.*

3. Interdependent Values

This section assumes *interdependent values* in that each agent's payoff depends on the other agents' private signals.

3.1. The Model

We modify the model by assuming that each agent receives a *private signal* ω_i. Let Ω_i denote the finite set of private signals for agent $i \in \{1,...,4n\}$. The probability of private signal profile $\omega = (\omega_i)_{i=1}^{4n} \in \Omega = \prod_{i=1}^{4n} \Omega_i$ occurring is given by $f(\omega) \in (0,1]$. Each agent $i's$ payoff function with the expected utility hypothesis is redefined as $u_i : A \times \Omega \to R$. For every lottery $\alpha \in \Delta$, let $u_i(\alpha, \omega) = \sum_{a \in \Gamma} u_i(a, \omega)\alpha(a)$, where Γ is the support of α. A *strategy* for agent i is defined as a function $s_i : \Omega_i \to M_i$, where agent i with private signal $\omega_i \in \Omega_i$ announces $s_i(\omega_i) \in M_i$. Let S_i denote the set of strategies for agent i.

A combination $(G, (u_i)_{i=1}^{4n}, f)$ defines a *Bayesian game*. The expected payoff for agent i with private signal ω_i when agents play strategy profile $s = (s_i)_{i=1}^{4n} \in S = \prod_{i=1}^{4n} S_i$ in the Bayesian game $(G, (u_i)_{i=1}^{4n}, f)$ is denoted by $u_i(s, \omega_i) = \sum_{\omega_{-i} \in \Omega_{-i}} u_i(g(s(\omega)), \omega) f_i(\omega_{-i} \mid \omega_i)$, where $f_i(\omega_{-i} \mid \omega_i) = \dfrac{f(\omega)}{\sum_{\omega'_{-i} \in \Omega_{-i}} f(\omega_i, \omega'_{-i})}$. Let $S_i^0 = S_i$. Recursively, for every $r \geq 1$, let $S_i^r \subset S_i$ denote the set of strategies $s_i \in S_i^{r-1}$ for agent i such that there exists no $s'_i \in S_i^{r-1}$ such that

$$u_i(s', \omega_i) \geq u_i(s_i, s'_{-i}, \omega_i) \text{ for all } (s'_{-i}, \omega_i) \in S_{-i}^{r-1} \times \Omega_i,$$

with strict inequality for some $(s'_{-i}, \omega_i) \in S_{-i}^{r-1} \times \Omega_i$, where we denote $S^r = \prod_{i=1}^{4n} S_i^r$ and $S_{-i}^r = \prod_{j \neq i} S_j^r$. Let $S_i^\infty = \lim_{r \to \infty} S_i^r$ and $S^\infty = \prod_{i=1}^{4n} S_i^\infty$. A strategy profile $s \in S$ is said to be *iteratively undominated* in $(G, (u_i)_{i=1}^{4n}, f)$ if $s \in S^\infty$. The set of iteratively undominated message profiles S^∞ in $(G, (u_i)_{i=1}^{4n}, f)$ is said to be *twice dominance solvable* if $S^\infty = S^2$. The set of iteratively undominated message profiles S^∞ in $(G, (u_i)_{i=1}^{4n}, f)$ is said to be *interchangeable* if every iteratively undominated strategy profile is a *Bayesian Nash equilibrium* in $(G, (u_i)_{i=1}^{4n}, f)$, i.e., for every $s \in S^\infty$, every $i \in \{1,...,4n\}$, and every $\omega_i \in \Omega_i$,

$$u_i(s, \omega_i) \geq u_i(s'_i, s_{-i}, \omega_i) \text{ for all } s_i \in S_i.$$

3.2. Auction Design

We consider the following mechanism $G^{**}=(M,g)$ where sellers and buyers are randomly divided into groups 1 and 2. Each seller announces a supply function, whereas each buyer announces a *triplicate* of messages, i.e., messages 1, 2, and 3. As message 1, she announces a demand function. As message 2, she announces a demand function conditional on the first messages of buyers in the same group as her. As message 3, she announces a demand function conditional on the first messages of all buyers. Almost certainly, the members of each group trade at the approximate market-clearing price vector based on the second messages of buyers in the other group.

Let Ξ denote the set of functions $\theta : D^n \to D$. Let W denote the set of functions $w : D^{2n} \to D$. For every seller $i \in \{1,...,2n\}$, let

$$M_i = D .$$

For every buyer $i \in \{2n+1,...,4n\}$, let

$$M_i = M_i^1 \times M_i^2 \times M_i^3 ,$$

$$M_i^1 = D,\ M_i^2 = \Xi,\ \text{and}\ M_i^3 = W .$$

We denote $m_i = (m_i^1, m_i^2, m_i^3) \in M_i^1 \times M_i^2 \times M_i^3$, and denote any strategy for each buyer $i \in \{2n+1,...,4n\}$ by $s_i = (s_i^1, s_i^2, s_i^3) \in S$, where $s_i^1 : \Omega_i \to D$, $s_i^1 : \Omega_i \to \Xi$, and $s_i^1 : \Omega_i \to W$. For every $m \in M$, let $m^{b,1} = (m_{2n+i}^1)_{i=1}^{2n} \in D^{2n}$ denote the first message profile of all buyers. For every $(\phi, m) \in \Phi \times M$, let $\hat{m}^{b,1,\phi} = (m_{\phi(2n+i)}^1)_{i=1}^{n} \in D^n$ denote the first message profile of buyers in group 1. For every $(\phi, m) \in \Phi \times M$, let $\tilde{m}^{b,1,\phi} = (m_{\phi(2n+i)}^1)_{i=n+1}^{2n} \in D^n$ denote the first message profile of buyers in group 2. Based on buyers' second messages, for every $(\phi, m) \in \Phi \times M$, we define

$$\hat{p}^{**}(\phi,m) \in \arg\min_{p\in P}[\max_{h\in\{1,...,k\}} \left| \sum_{i=1}^{n} \{ m_{\phi(i)}(p)(h) - m_{\phi(2n+i)}^2 (\hat{m}^{1,\phi})(p)(h) \} \right|],$$

and

$$\tilde{p}^{**}(\phi,m) \in \arg\min_{p\in P}[\max_{h\in\{1,...,k\}} \left| \sum_{i=n+1}^{2n} \{ m_{\phi(i)}(p)(h) - m_{\phi(2n+i)}^2 (\tilde{m}^{1,\phi})(p)(h) \} \right|],$$

where we assume that $\hat{p}^{**}(\phi,m)$ ($\tilde{p}^{**}(\phi,m)$) does not depend on the messages announced in group 2 (group 1, respectively).

Fix $\eta \in (0, \frac{1}{2})$ arbitrarily, which is close to zero. Fix $\phi \in \Phi$, $p \in P$, and $m \in M$ arbitrarily. With probability $\frac{\eta}{(2n!)^2 T}$, the mechanism G^{**} chooses $a^{**}(\phi, p, m, 1) = (x, q)$ where for every $i \in \{1,...,2n\}$,

$$q_{\phi(i)} = q_{\phi(2n+i)} = p \ \text{and}\ x_{\phi(2n+i)} = m_{\phi(2n+i)}^1 (q_{\phi(2n+i)}) .$$

Each buyer buys the same amounts as what she intends to demand according to her first message. With probability $\frac{\eta}{(2n!)^2 T}$, G^{**} chooses $a^{**}(\phi, p, m, 2) = (x, q)$ where for every $i \in \{1,...,2n\}$,

$$q_{\phi(i)} = q_{\phi(2n+i)} = p,$$

$$x_{\phi(2n+i)} = m^2_{\phi(2n+i)}(\hat{m}^{b,1,\phi})(q_{\phi(2n+i)}) \text{ if } i \leq n,$$

and

$$x_{\phi(2n+i)} = m^2_{\phi(2n+i)}(\tilde{m}^{b,1,\phi})(q_{\phi(2n+i)}) \text{ if } i \geq n+1.$$

Each buyer buys the same amounts as what she intends to demand according to her second message conditional on the first messages of buyers in the same group as her. With probability $\frac{\eta}{(2n!)^2 T}$, G^{**} chooses $a^{**}(\phi, p, m, 3) = (x, q)$ where for every $i \in \{1,...,2n\}$,

$$q_{\phi(i)} = q_{\phi(2n+i)} = p \text{ and } x_{\phi(2n+i)} = m^3_{\phi(2n+i)}(m^{b,1})(q_{\phi(2n+i)}).$$

Each buyer buys the same amounts as what she intends to demand according to her third message conditional on the first messages by all buyers. With probability $\frac{1-3\eta}{(2n!)^2 T}$, G^{**} chooses $a^{**}(\phi, p, m, 4) = (x, q)$ where for every,

$$q_{\phi(i)} = q_{\phi(2n+i)} = \tilde{p}^{**}(\phi, m) \text{ for all } i \in \{1,...,n\},$$

$$q_{\phi(i)} = q_{\phi(2n+i)} = \hat{p}^{**}(\phi, m) \text{ for all } i \in \{n+1,...,2n\},$$

and

$$x_{\phi(2n+i)} = m^3_{\phi(2n+i)}(m^{b,1})(q_{\phi(2n+i)}) \text{ for all } i \in \{1,...,2n\}.$$

Agents in group 1 (group 2) trade at the approximate market-clearing price vector in group 2 (group 1, respectively). Each buyer buys the same amounts as what she intends to demand according to her third message conditional on the first messages of all buyers.

We specify the selling amount $x_{\phi(i)}$ of each seller $\phi(i)$ in group 1 as follows, where $(x, q) = a^{**}(\phi, p, m, b)$, $b \in \{1,...,4\}$, and $i \in \{1,...,n\}$. Fix any commodity $h \in \{1,...,k\}$ arbitrarily. There exists $\hat{i} \in \{1,...,n\}$ satisfying the following properties. Suppose that there exists excess supply in group 1, i.e.,

$$\text{(7)} \qquad \sum_{j=1}^{n} m_{\phi(j)}(q_{\phi(j)})(h) \geq \sum_{j=1}^{n} x_{\phi(2n+j)}(h).$$

Then,

$$x_{\phi(i)}(h) = m_{\phi(i)}(q_{\phi(i)})(h) \text{ if } i < \hat{i},$$

$$x_{\phi(i)}(h) = \sum_{j=1}^{n} x_{\phi(2n+j)}(h) - \sum_{j=1}^{i-1} x_{\phi(j)}(h) \leq m_{\phi(i)}(q_{\phi(i)})(h) \text{ if } i = \hat{i},$$

and

$$x_{\phi(i)}(h) = 0 \text{ if } i > \hat{i}.$$

Suppose that inequality (7) does not hold. Then,

$$x_{\phi(i)}(h) = l \text{ if } i < \hat{i},$$

$$x_{\phi(i)}(h) = \sum_{j=1}^{n} x_{\phi(2n+j)}(h) - (i-1)l - \sum_{j=i+1}^{n} x_{\phi(j)}(h) \geq m_{\phi(i)}(q_{\phi(i)})(h) \text{ if } i = \hat{i},$$

and

$$x_{\phi(i)}(h) = m_{\phi(i)}(q_{\phi(i)})(h) \text{ if } i > \hat{i}.$$

If there exists excess supply in group 1, then, each seller before $\phi(\hat{i})$ can sell the same amount as what she intends to supply, whereas each seller after $\phi(\hat{i})$ sells zero amount. If there exists excess demand in group 1, then, each seller after $\phi(\hat{i})$ can sell the same amount as what she intends to supply, whereas each seller after $\phi(\hat{i})$ has to sell the maximal amount l. In the same way, we specify the selling amounts in group 2.

3.3. Iterative Dominance

We specify the model as follows. Each seller $i's$ production technology is described by $c_i : \{1,...,k\} \times \{1,....,l\} \times \Omega \to [0,\infty)$, where

$$c_i(h,1,\omega) \leq \cdots \leq c_i(h,l,\omega) \text{ for all } \omega \in \Omega \text{ and all } h \in \{1,...,k\}.$$

Hence, each seller's production technology has *no* externality among different commodities, and the unit cost for each commodity is *non-decreasing*. Let $c_i(x_i,\omega) = \sum_{h=1}^{k} \sum_{t=1}^{x_i(h)} c_i(h,t,\omega)$. Each seller $i's$ payoff for allocation (x,q) is described by

$$u_i(x,q,\omega) = \sum_{h=1}^{k} q_i(h) x_i(h) - c_i(x_i,\omega).$$

Each buyer $i's$ valuation is described by $v_i : \{0,...,l\}^k \times \Omega \to [0,\infty)$, where $v_i(0,...,0,\omega) = 0$. We allow any mixture of complements and substitutes among different commodities. Buyer $i's$ payoff is given by

$$u_i(x,q,\omega) = v_i(x_i,\omega) - \sum_{h=1}^{k} q_i(h) x_i(h).$$

For every subset $N \subset \{1,...,4n\}$, we denote $\omega_N = (\omega_j)_{j\in N}$, $-N = \{1,...,4n\} \setminus N$, $\omega_{-N} = (\omega_j)_{j\notin N}$, $f^N(\omega_{-N} \mid \omega_N) = \dfrac{f(\omega)}{\sum_{\omega'_{-N}\in\Omega_{-N}} f(\omega_N,\omega'_{-N})}$, $c_i(x_i,\omega_N) = \sum_{\omega_{-N}} c_i(x_i,\omega) f^N(\omega_{-N} \mid \omega_N)$, and $v_i(x_i,\omega_N) = \sum_{\omega_{-N}} v_i(x_i,\omega) f^N(\omega_{-N} \mid \omega_N)$. We denote $\omega_i = \omega_{\{i\}}$. For every seller $i \in \{1,...,2n\}$, we define $D_i(\omega_i) \subset D$ as the set of functions $d : P \to \{0,...,l\}^k$ such that for every $p \in P$,

$$d(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i, \omega_i)],$$

which implies seller $i's$ profit-maximizing supply when she is a price taker and receives private signal ω_i. For every buyer $i \in \{2n+1,...,4n\}$, we define $D_i(\omega_i) \subset D$ as the set of functions $d : P \to \{0,...,l\}^k$ such that for every $p \in P$,

$$d(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [v_i(x_i, \omega_i) - \sum_{h=1}^{k} p(h)x_i(h)],$$

which implies buyer $i's$ payoff-maximizing demand when she is a price taker and receives private signal ω_i.

The private signal structure satisfies *symmetry* in that there exist Ω_s and Ω_b such that

$\Omega_i = \Omega_s$ for all $i \in \{1,...,2n\}$,

$\Omega_i = \Omega_b$ for all $i \in \{2n+1,...,4n\}$,

and that for every $\omega \in \Omega$ and every $\omega' \in \Omega$,

$f(\omega) = f(\omega')$ if there exists $\phi \in \Phi$ such that $\omega'_i = \omega_{\phi(i)}$ for all $i \in \{2n+1,...,4n\}$.

We assume that for every buyer $i \in \{2n+1,...,4n\}$, every $\omega_i \in \Omega_i$, and every $\omega'_i \in \Omega_i \setminus \{\omega_i\}$,

(8) $\qquad D_i(\omega_i) \cap D_i(\omega'_i) = \phi$.

This implies that no buyer has the same payoff-maximizing demand function between different private signals. Based on this assumption, for every buyer $i \in \{2n+1,...,4n\}$ and every $\omega_i \in \Omega_i$, we define $\Xi_i(\omega_i) \subset \Xi$ as the set of functions $\theta : D^n \to D$ such that for every $(\phi, m) \in \Phi \times M$ and every $\omega_{-i} \in \Omega_{-i}$, if $i = \phi(2n+1)$, and

$m^1_{\phi(2n+j)} \in D_{\phi(2n+j)}(\omega_{\phi(2n+j)})$ for all $j \in \{1,...,n\}$,

then

$$\theta(\hat{m}^{b,1,\phi})(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [v_i(x_i, \hat{\omega}^{b,\phi}) - \sum_{h=1}^{k} p(h)x_i(h)],$$

where $\hat{\omega}^{b,\phi} = (\omega_{\phi(2n+j)})_{j=1}^n \in \Omega_b^n$ denote the private signal profile of buyers in group 1. Let $\widetilde{\omega}^{b,\phi} = (\omega_{\phi(3n+j)})_{j=1}^n \in \Omega_b^n$ denote the private signal profile of buyers in group 2. Because of symmetry, we can replace $j \in \{1,...,n\}$ and $\hat{\omega}^{b,\phi}$ with $j \in \{n+1,...,2n\}$ and $\widetilde{\omega}^{b,\phi}$, respectively. Any element of $\Xi_i(\omega_i)$ implies buyer $i's$ payoff-maximizing demand function when she is a price taker and receives full information about the private signals for all buyers in the same group as her. For every buyer $i \in \{2n+1,...,4n\}$ and every $\omega_i \in \Omega_i$, we define $W_i(\omega_i) \subset W$ as the set of functions $w : D^{2n} \to D$ such that for every $m \in M$ and every $\omega_{-i} \in \Omega_{-i}$, if

$m^1_j \in D_j(\omega_j)$ for all $j \in \{2n+1,...,4n\}$,

then

$$w(m^1)(p) \in \arg\max_{x_i \in \{0,...,l\}^k} [v_i(x_i, \omega^b) - \sum_{h=1}^{k} p(h) x_i(h)],$$

where $\omega^b = (\omega_{\phi(2n+j)})_{j=1}^{2n} \in \Omega_b^{2n}$ denotes the private signal profile of all buyers. Any element of $W_i(\omega_i)$ implies buyer $i's$ payoff-maximizing demand function when she is a price taker and receives full information about the private signals for all buyers.

Condition 2: For every $i \in \{1,...,2n\}$, $c_i(x_i, \omega)$ is independent of $\omega_{-i} \in \Omega_{-i}$. For every $i \in \{1,...,2n\}$ and every $j \in \{2n+1,...,4n\}$, $v_j(x_j, \omega)$ is independent of $\omega_j \in \Omega_j$.

The former part of Condition 2 implies that all sellers have only private values. The latter part implies that we allow interdependent values only in the buyers' side.

Theorem 4: *Suppose that Condition 2 holds. Then, a strategy profile $s \in S$ is iteratively undominated in $(G^{**}, (u_i)_{i=1}^{4n}, f)$ if and only if for every seller $i \in \{1,...,2n\}$ and every $\omega_i \in \Omega_i$,*

(9) $\qquad s_i(\omega_i) \in D_i(\omega_i)$

and for every buyer $i \in \{2n+1,...,4n\}$ and every $\omega_i \in \Omega_i$,

(10) $\qquad s_i^1(\omega_i) \in D_i(\omega_i)$, $s_i^2(\omega_i) \in \Xi_i(\omega_i)$, *and* $s_i^3(\omega_i) \in W_i(\omega_i)$.

The set of iteratively undominated message profiles in $(G^{**}, (u_i)_{i=1}^{4n}, f)$ is twice dominance solvable and interchangeable.

Proof: For every $(\phi, p, b) \in \Phi \times P \times \{1,...,4\}$, each seller $i \in \{1,...,2n\}$ receives the payoff given by $\sum_{h=1}^{k} q_i(h) x_i(h) - c_i(x_i, \omega_i)$, where $m = s(\omega)$ and $(x, q) = a^*(\phi, p, m, b)$. Note that m_i never influences q_i. Note from the former part of Condition 2 that q_i includes no information relevant to seller $i's$ cost condition. Without loss of generality, we assume that seller i belongs to group 1, i.e., $i = \phi(j)$ for some $j \in \{1,..., n\}$. (We can apply the same argument when she belongs to group 2.) Fix any commodity $h \in \{1,..., k\}$ arbitrarily.

Suppose that there exists excess supply in group 1, i.e., inequality (7) holds. If $j > \hat{i}$, then $x_i(h) = 0$, and seller i cannot change the selling amount by changing her message. If $j \le \hat{i}$, then

$$x_i(h) = \min[m_i(q_i)(h), r(h)], \text{ where } r(h) = \sum_{i'=1}^{n} x_{\phi(2n+i')}(h) - \sum_{i'=1}^{j-1} x_{\phi(i')}(h),$$

and she can change the selling amount into $\min[m_i'(q_i)(h), r(h)]$ by announcing any m_i' instead of m_i.

Next, suppose that inequality (7) does not hold. If $j < \hat{i}$, then $x_i(h) = l$, and she cannot change the selling amount by changing her message. If $j \geq \hat{i}$, then

$$x_i(h) = \max[\min[m_i(q_i)(h), r(h)], \bar{r}(h)],$$

where

$$\bar{r}(h) = \sum_{i'=1}^{n} x_{\phi(2n+i')}(h) - (j-1)l - \sum_{i'=j+1}^{n} x_{\phi(i')}(h),$$

$$r(h) = \sum_{i'=1}^{n} x_{\phi(2n+i')}(h) - \sum_{i'=1}^{j-1} x_{\phi(i')}(h),$$

and she can change the selling amount into $\max[\min[m_i'(q_i)(h), r(h)], \bar{r}(h)]$ by announcing any m_i' instead of m_i. Since seller $i's$ production technology satisfies no production externality and non-decreasingness, it follows from the above observations that a strategy s_i for seller i is dominant, i.e.,

$$u_i(g(s_i(\omega_i), m_{-i}), \omega_i) \geq u_i(g(m), \omega_i) \text{ for all } \omega_i \in \Omega_i \text{ and all } m \in M,$$

if and only if $m_i \in D_i(\omega_i)$ for all $\omega_i \in \Omega_i$.

Consider the case of $b = 1$. For every $(\phi, p) \in \Phi \times P$, each buyer $i \in \{2n+1, ..., 4n\}$ receives the payoff given by

$$(11) \qquad v_i(x_i, \omega) - \sum_{h=1}^{k} q_i(h) x_i(h),$$

where $m = s(\omega)$ and $(x, q) = a^*(\phi, p, m, 1)$. Note that m_i never influences q_i, $x_i = m_i^1(q_i)$, and buyer i can change the buying amount into $x_i = m_i'^1(q_i)$ by announcing any m_i' instead of m_i. Hence, buyer i can always maximize the expected value of (11) conditional on ω_i by announcing $s_i^1(\omega_i) \in D_i(\omega_i)$ for any $\omega_i \in \Omega_i$. Since the first message for buyer i is relevant to buyer $i's$ allocation only in the case of $b = 1$, it follows that if a strategy s_i for buyer i is included in S_i^1, then it must hold that $s_i^1(\omega_i) \in D_i(\omega_i)$ for all $\omega_i \in \Omega_i$.

Suppose that a strategy profile $s \in S$ satisfies that $s_i(\omega_i) \in D_i(\omega_i)$ for all sellers $i \in \{1, ..., 2n\}$, and $s_i^1(\omega_i) \in D_i(\omega_i)$ for all buyers $i \in \{2n+1, ..., 4n\}$. Fix $(\phi, p) \in \Phi \times P$ and a buyer $i \in \{2n+1, ..., 4n\}$ arbitrarily, where, without loss of generality, assume that buyer i belongs to group 1, i.e., $i = \phi(2n + j)$ for some $j \in \{1, ..., n\}$.

Consider the case of $b = 2$. Buyer i receives the payoff given by (11) where $m = s(\omega)$ and $(x, q) = a^*(\phi, p, m, 2)$. Note that m_i never influences q_i, and $x_i = s_i^2(\hat{m}^{b,1,\phi})(q_i)$. Equality (8) implies that $\hat{m}^{b,1,\phi}$ includes full information about $\hat{\omega}^{b,1,\phi}$. Hence, buyer i can always maximize the expected value of (11) conditional on $\hat{\omega}^{b,1,\phi}$ by announcing $s_i^2(\omega_i) \in \Xi_i(\omega_i)$ for any $\omega_i \in \Omega_i$. Since the second message for buyer i is relevant to buyer $i's$ allocation only in the case of $b = 2$, it follows that if a strategy s_i for buyer i is included in S_i^2, then it must hold that $s_i^2(\omega_i) \in \Xi_i(\omega_i)$ for all $\omega_i \in \Omega_i$.

Consider the case of either $b = 3$ or $b = 4$. Buyer i receives the payoff given by (11) where $m = s(\omega)$ and either $(x, q) = a^*(\phi, p, m, 3)$ or $(x, q) = a^*(\phi, p, m, 4)$. Note that m_i never influences q_i, and $x_i = s_i^2(m^{b,1,\phi})(q_i)$. Equalities (8) imply that $m^{b,1,\phi}$ includes full information about all buyers' private signals $\omega^{b,\phi}$. Note from the latter part of Condition 2 that q_i includes no additional information relevant to buyer $i's$ valuation, whenever she knows $\omega^{b,\phi}$. Hence, buyer i can always maximize the expected value of (11) conditional on $\omega^{b,\phi}$ by announcing $s_i^3(\omega_i) \in W_i(\omega_i)$ for any $\omega_i \in \Omega_i$. Since the third message for buyer i is relevant to buyer $i's$ allocation only in the cases of $b \in \{3,4\}$, it follows that if a strategy s_i for buyer i is included in S_i^2, then it must hold that $s_i^3(\omega_i) \in W_i(\omega_i)$ for all $\omega_i \in \Omega_i$.

The above arguments imply that for every agent $i \in \{1,...,2n\}$ and every $\omega_i \in \Omega_i$, any strategy included in S_i^2 is a best reply to every strategy profile for the other agents that is included in S_{-i}^2. This implies $S^\infty = S^2$, and every strategy profile in S^2 is a Bayesian Nash equilibrium.

Q.E.D.

3.4. Asymptotic Efficiency

We further specify the model as follows. We introduces an unobservable *macro shock* ω_0. Let Ω_0 denote the finite set of macro shocks. The probability of macro shock ω_0 and private signal profile ω occurring is given by $\bar{f}(\omega_0,\omega)\in[0,1]$, where $f(\omega)=\sum_{\omega_0\in\Omega_0}\bar{f}(\omega_0,\omega)$. Agents' private signals are conditionally independent in that $\bar{f}(\omega_0,\omega)=\bar{f}_0(\omega_0)\prod_{i=1}^{4n}\bar{f}_i(\omega_i\mid\omega_0)$ for all $(\omega_0,\omega)\in\Omega_0\times\Omega$, where $\bar{f}_0(\omega_0)=\sum_{\omega\in\Omega}\bar{f}(\omega_0,\omega)$ and $\bar{f}_i(\omega_i\mid\omega_0)=\dfrac{\sum_{\omega_{-i}\in\Omega_{-i}}\bar{f}(\omega_0,\omega)}{\bar{f}_0(\omega_0)}$. There exist $f_s(\cdot\mid\omega_0):\Omega_s\to[0,1]$ and $f_b(\cdot\mid\omega_0):\Omega_b\to[0,1]$ such that

$$\bar{f}_i(\cdot\mid\omega_0)=f_s(\cdot\mid\omega_0)\ \text{ for all }\ i\in\{1,...,2n\},$$

and

$$\bar{f}_i(\cdot\mid\omega_0)=f_b(\cdot\mid\omega_0)\ \text{ for all }\ i\in\{2n+1,...,4n\}.$$

There exist $c:\{1,...,k\}\times\{0,...,l\}\times\Omega_0\times\Omega_s\to[0,\infty)$ and $v:\{0,...,l\}^k\times\Omega_0\times\Omega_b\to[0,\infty)$ such that

$$c_i(h,t,\omega)=\sum_{\omega_0\in\Omega_0}c(h,t,\omega_0,\omega_i)\bar{f}(\omega_0\mid\omega)\ \text{ for all }\ i\in\{1,...,2n\},$$

and

$$v_i(x_i,\omega)=\sum_{\omega_0\in\Omega_0}v(x_i,\omega_0,\omega_i)\bar{f}(\omega_0\mid\omega)\ \text{ for all }\ i\in\{2n+1,...,4n\},$$

where $\bar{f}(\omega_0\mid\omega)=\dfrac{\bar{f}(\omega_0,\omega)}{f(\omega)}$. Hence, each agent's payoff depends on the other agent's private signals only through the macro shock. For every seller $i\in\{1,...,2n\}$, let

$$u_i(x,q,\omega_0,\omega_i)=\sum_{h=1}^{k}\{q_i(h)x_i(h)-\sum_{t=1}^{x_i(h)}c(h,t,\omega_0,\omega_i)\}.$$

For every buyer $i\in\{2n+1,...,4n\}$, let

$$u_i(x,q,\omega_0,\omega_i)=\sum_{h=1}^{k}\{\sum_{t=1}^{x_i(h)}v(h,t,\omega_0,\omega_i)-q_i(h)x_i(h)\}$$

We will show that when the number of agents is sufficiently large, the mechanism G^{**} satisfies approximate efficiency with *full* information about the macro shock and approximate uniform pricing. For every $\varepsilon>0$ that is close to zero, a strategy profile $s\in S$ is said to be $\varepsilon-$*efficient* in $(G,(u_i)_{i=1}^{4n},f)$ if for every $\omega_0\in\Omega_0$, the probability conditional on macro shock ω_0 of the mechanism G choosing any allocation $a\in A$ according to $s(\omega)\in M$ such that

$$\frac{\left|\max_{a'\in A}\sum_{i=1}^{4n}u_i(a',\omega_0,\omega_i)-\sum_{i=1}^{4n}u_i(a,\omega_0,\omega_i)\right|}{4n}\leq\varepsilon$$

is more than $1-\varepsilon$. For every $\varepsilon>0$ that is close to zero, a strategy profile $s\in S$ is said to be $\varepsilon-$*uniform pricing* in $(G,(u_i)_{i=1}^{4n},f)$ if for every $\omega\in\Omega$, the probability of the mechanism G choosing any allocation (x,q) according to $s(\omega)\in M$ such that for every $(i,h)\in\{1,...,4n\}\times\{1,...,k\}$ and every $j\in\{1,...,4n\}$,

$$\left|q_i(h)-q_j(h)\right|\leq\varepsilon$$

is more than $1-\varepsilon$.

The intuition of our arguments is as follows. For every $\varepsilon>0$, whenever n is sufficiently large, then the probability conditional on any macro shock $\omega_0\in\Omega_0$ that all buyers' private signal profile $\omega^b=(\omega_i)_{i=2n+1}^{4n}\in\Omega_b^{2n}$ satisfies

$$\left|\frac{\left|\{i\in\{2n+1,...,4n\}\mid\omega_i=\omega_b\}\right|}{2n}-f_b(\omega_b\mid\omega_0)\right|<\varepsilon\quad\text{for all}\quad\omega_b\in\Omega_b$$

is larger than $1-\varepsilon$. This holds true even if we replace ω^b with either $\hat{\omega}^{b,\phi}$ or $\tilde{\omega}^{b,\phi}$. We assume that for every $\omega_0\in\Omega_0$ and every $\omega_0'\in\Omega_0/\{\omega_0\}$,

(12) $$f_b(\cdot\mid\omega_0)\neq f_b(\cdot\mid\omega_0').$$

This implies that the probability distribution of each buyer's private signal occurring is different between distinct macro shocks. Hence, whenever n is sufficiently large, then, by observing either $\hat{\omega}^{b,\phi}$ or $\tilde{\omega}^{b,\phi}$, each buyer can receive almost full information about the unobservable macro shock ω_0. Note that whether each agent belongs to group 1 or group 2 is determined according to the uniform distribution on Φ. Remember equalities (8) implying that the first message announced by any buyer according to price-taking behavior includes full information about her private signal. Hence, it follows from inequalities (12) that whenever n is sufficiently large, then it is almost certain that both $\hat{p}^{**}(\phi,m)$ and $\tilde{p}^{**}(\phi,m)$ are approximated by the market-clearing price vector associated with all trades under full information about the macro shock.

Based on the above intuition, for every $\omega_0\in\Omega_0$, fix a continuous function $\rho^{**}(\omega_0):[0,1]^k\to R$ and $p^{**}(\omega_0)\in[0,1]^k$ arbitrarily, where $\rho(\omega_0)(p^{**}(\omega_0))=(0,...,0)$. Fix an infinite sequence $(\Omega^{(n)},f^{(n)},c^{(n)},v^{(n)},T^{(n)},\eta^{(n)})_{n=1}^{\infty}$ arbitrarily, where

$$\lim_{n\to\infty}T^{(n)}=\infty\quad\text{and}\quad\lim_{n\to\infty}\eta^{(n)}=0.$$

Hence, for sufficiently large n, the set of price vectors $P^{(n)}$ is approximated by $[0,1]^k$, and it is almost certain that agents trades at the price vector $\hat{p}^{**(n)}(\phi,m^{(n)})$ or $\tilde{p}^{**(n)}(\phi,m^{(n)})$. Here, the set of macro shocks Ω_0 does *not* depend on n.

We assume that inequalities (12) hold for all $n\geq 1$ in the strict sense, i.e., there exists

$\xi > 0$ such that for every $\omega_0 \in \Omega_0$, every $\omega_0' \in \Omega_0 / \{\omega_0\}$, and every $n \geq 1$,

(13) $$\sum_{\omega_b \in \Omega_b} \left| f_b^{(n)}(\omega_b \mid \omega_0) - f_b^{(n)}(\omega_b \mid \omega_0') \right| \geq \xi .$$

We also assume that for every infinite sequence of price vectors $(p^{(n)})_{n=1}^{\infty}$, every $p \in [0,1]^k$, and every infinite sequence of strategy profiles $(s^{(n)})_{n=1}^{\infty}$, whenever $s^{(n)}$ satisfies properties (9) and (10), implying price-taking behavior for all $n \geq 1$, and

$$\lim_{n \to \infty} p^{(n)} = p ,$$

then, for every $\varepsilon > 0$, there exists $\bar{n}$ such that for every $n \geq \bar{n}$, the probability conditional on any macro shock $\omega_0 \in \Omega_0$ that the realized private signal profile $\omega^{(n)} \in \Omega^{(n)}$ satisfies

$$\left| \frac{\sum_{i=1}^{2n} \{ s_i^{(n)}(\omega_i^{(n)})(p^{(n)}) - s_{2n+i}^{3(n)}(\omega_{2n+i}^{(n)})(m^1)(p^{(n)}) \}}{2n} - \rho^{**}(\omega_0)(p) \right| \leq \varepsilon$$

is larger than $1 - \varepsilon$, where $m^1 = (m_{2n+i}^1)_{i=1}^{2n}$ and $m_{2n+i}^1 = s_{2n+i}^{1(n)}(\omega_{2n+i}^{(n)})$. Hence, $\rho^{**}(\omega_0)(p)$ almost surely approximates the excessive supply based on all buyers' third messages when n is sufficiently large.

For every $n \geq 1$, every $s^{(n)} \in S^{(n)}$, and every $\omega^{(n)} \in \Omega^{(n)}$, we define

$$p^{**(n)}(s^{(n)}, \omega^{(n)}) \in \arg\inf_{p \in P^{(n)}} [\max_{h \in \{1,\dots,k\}} \left| \sum_{i=1}^{2n} \{ s_i^{(n)}(\omega_i^{(n)})(p)(h) - s_{2n+i}^{3(n)}(\omega_{2n+i}^{(n)})(m^1)(p)(h) \} \right|],$$

where $m^1 = (m_{2n+i}^1)_{i=1}^{2n}$ and $m_{2n+i}^1 = s_{2n+i}^{1(n)}(\omega_{2n+i}^{(n)})$. Hence, $p^{**(n)}(s^{(n)}, \omega^{(n)})$ approximates the market-clearing price vector associated with all trades based on all buyers' third messages. We assume that for every infinite sequence of strategy profiles $(s^{(n)})_{n=1}^{\infty}$, whenever $s^{(n)}$ satisfies properties (9) and (10) for all $n \geq 1$, then, for every $\varepsilon > 0$, there exists $\bar{n}$ such that for every $n \geq \bar{n}$, the probability conditional on any macro shock $\omega_0 \in \Omega_0$ that the realized private signal profile $\omega^{(n)} \in \Omega^{(n)}$ satisfies

$$\left| p^{(n)}(s^{(n)}, \omega^{(n)}) - p^{**}(\omega_0) \right| \leq \varepsilon$$

is larger than $1 - \varepsilon$. Hence, $p^{**}(\omega_0)$ almost surely approximates the market-clearing price vector associated with all trades based on all buyers' third messages when n is sufficiently large. From inequalities (13), it follows that $p^{**}(\omega_0)$ almost surely approximates the market-clearing price vector associated with all trades under full information about the macro shock when n is sufficiently large. For every sufficiently large n and for every strategy profile $s^{(n)}$ satisfying properties (9) and (10), it is almost certain that the maximal total surplus per capita $\dfrac{\max_{a^{(n)} \in A^{(n)}} \sum_{i=1}^{4n} u_i^{(n)}(a^{(n)}, \omega_0, \omega_i^{(n)})}{4n}$ conditional on $(\omega_0, \omega^{(n)})$ approximates the average payoff when all agents trade at the uniform price vector $p^{**}(\omega_0)$ as price takers with full information about the macro shock. Since whether each agent belongs to group 1 or group 2 is determined according to the uniform distribution on Φ, it follows that for every sufficiently large n and every strategy profile $s^{(n)}$, it is almost certain that both $\hat{p}^{**(n)}(\phi, s^{(n)}(\omega^{(n)}))$ and $\tilde{p}^{**(n)}(\phi, s^{(n)}(\omega^{(n)}))$ are approximated by $p^{**(n)}(s^{(n)}(\omega^{(n)}))$. Hence, whenever $s^{(n)}$ satisfies properties (9) and (10), then both $\hat{p}^{**(n)}(\phi, s^{(n)}(\omega^{(n)}))$ and $\tilde{p}^{**(n)}(\phi, s^{(n)}(\omega^{(n)}))$ are almost surely approximated by $p^{**}(\omega_0)$. From the above arguments, we have proved the following theorem.

Theorem 5: *For every $\varepsilon > 0$, there exists a positive integer n^{**} such that for every $n \geq n^{**}$, any strategy profile $s^{(n)}$ satisfying properties (9) and (10) is $\varepsilon-$efficient and $\varepsilon-$uniform pricing in $(G^{**(n)}, (u_i^{(n)})_{i=1}^{4n}, f^{(n)})$.*

We can show that any strategy profile with properties (9) and (10) satisfies *participation constraints* as follows. Suppose that for every n and every $\omega_s^{(n)} \in \Omega_s^{(n)}$, there exist $\omega_0 \in \Omega_0$ and $h \in \{1,...,k\}$ such that $c^{(n)}(h, l, \omega_s^{(n)}) < p^{**}(\omega_0)$. Then, every seller can earn a positive interim expected payoff whenever she trades at the market clearing price vector $p^{**}(\omega_0)$. When n is sufficiently large, it is almost certain that every seller trades at almost the same price vector as $p^{**}(\omega_0)$ and can sell the same amounts as what she intends to supply. (Note that any buyer always buys the same amounts as what she intends to demand.) Hence, it follows that for every sufficiently large n, any strategy profile $s = s^{(n)}$ with properties (9) and (10) satisfies participation constraints in the Bayesian game $(G, (u_i)_{i=1}^{4n}, f) = (G^{**(n)}, (u_i^{(n)})_{i=1}^{4n}, f^{(n)})$ in the sense that for every $i \in \{1,...,4n\}$ and every $\omega_i \in \Omega_i$,

$$\sum_{\omega_{-i} \in \Omega_{-i}} u_i(g(s(\omega)), \omega) f(\omega_{-i} \mid \omega_i) \geq 0.$$

3.5. Generalization

This subsection investigates the case where Condition 2 does not hold, and therefore, all sellers and buyers have interdependent values in the general sense. We modify the model by assuming that there exist $2rn$ agents, where $r \geq 3$ is an integer, the first rn agents are sellers, and the latter rn agents are buyers. We construct the following mechanism denoted by $G^+ = (M, g)$, where sellers and buyers are randomly divided into r distinct groups. We redefine Φ as the set of one-to-one mappings $\phi : \{1,...,2rn\} \rightarrow \{1,...,2rn\}$ where $\phi(i) \in \{1,...,rn\}$ for all $i \in \{1,...,rn\}$, and $\phi(i) \in \{rn+1,...,2rn\}$ for all $i \in \{rn+1,...,2rn\}$. For every $\beta \in \{1,...,r\}$, n sellers

$\phi((\beta-1)n+1),...,\phi(\beta n)$ and n buyers $\phi((r+\beta-1)n+1),...,\phi((r+\beta)n)$ belong to group β. For every agent $i \in \{1,...,2rn\}$, let

$$M_i = M_i^1 \times M_i^2 \times M_i^3,$$

$$M_i^1 = D,$$

M_i^2 is the set of functions $m_i^2 : D^{2n} \to D$,

and

M_i^3 is the set of functions $m_i^3 : D^{2(r-1)n} \to D$.

In contrast with G^{**}, not only buyers but also sellers announce triplicates of messages. We denote any strategy for each agent $i \in \{1,...,2rn\}$ by $s_i = (s_i^1, s_i^2, s_i^3) \in S$, where $s_i^1 : \Omega_i \to M_i^1$, $s_i^2 : \Omega_i \to M_i^2$, and $s_i^3 : \Omega_i \to M_i^3$. Let $m^1 = (m_i^1)_{i=1}^{2rn} \in D^{2rn}$ denote the first message profile of all agents. Let $m^{\beta,\theta} = (m_{\theta((\beta-1)n+i)}, m_{\theta((r+\beta-1)n+i)})_{i=1}^n$ denote the message profile of all agents in group $\beta \in \{1,...,r\}$. Let $m^{\beta,1,\phi} = (m^1_{\phi((\beta-1)n+i)}, m^1_{\phi((r+\beta-1)n+i)})_{i=1}^n$ denote the first message profile of all agents in group β. Let $m^{-\beta,1,\phi} = (m^{\beta',1,\phi})_{\beta' \in \{1,...,r\}\setminus\{\beta\}}$ denote the first message profile of all agents who do not belong to group β.

For every $(\phi, m) \in \Phi \times M$ and every $\beta \in \{1,...,r\}$, we define

$$p^+(\phi,\beta,m) \in \arg\inf_{p \in P} [\max_{h \in \{1,...,k\}} \left| \sum_{i=(\beta-1)n+1}^{\beta n} \{m^2_{\phi(i)}(m^{\beta+1,1,\phi})(p)(h) - m^2_{\phi(rn+i)}(m^{\beta+1,1,\phi})(p)(h)\} \right|],$$

where $p^+(\phi,\beta,m)$ does not depend on the messages $m^{\beta-1,\phi}$ announced in the precedent group $\beta-1$. (Here, we denote $1-1=r$ and $r+1=1$.) Hence, $p^+(\phi,\beta,m)$ approximates the market-clearing price vector based on the second messages announced by all agents in group β, where they are informed of the first messages announced in the subsequent group $\beta+1$.

Let $\eta \in (0,\frac{1}{2})$ denote an arbitrary positive real number that is close to zero. Fix $(\phi,m) \in \Phi \times M$ and $p \in P$ arbitrarily. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi,p,m,1) = (x,q)$ such that for every $i \in \{1,...,rn\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^1_{\phi(i)}(p).$$

Each seller sells the same amount of each commodity as what she intends to supply according to her first message, whereas each buyer $\phi(rn+i)$ has to buy the same amount of each commodity as what seller $\phi(i)$ intends to supply. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi,p,m,2) = (x,q)$ such that for every $i \in \{1,...,rn\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^1_{\phi(rn+i)}(p).$$

Each buyer buys the same amount of each commodity as what she intends to demand according to her *first* message, whereas each seller $\phi(i)$ has to sell the same amount of each commodity as what buyer $\phi(rn+i)$ intends to demand according to her first message. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi, p, m, 3) = (x, q)$ such that for every $\beta \in \{1,...,r\}$ and every $i \in \{(\beta-1)n+1,...,\beta n\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^2_{\phi(i)}(m^{\beta+1,1,\phi})(p).$$

Each seller sells the same amount of each commodity as what she intends to supply according to her second message conditional on the first messages of the agents in the subsequent group $\beta+1$, whereas each buyer $\phi(rn+i)$ has to buy the same amount of each commodity as what seller $\phi(i)$ intends to supply. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi, p, m, 4) = (x, q)$ such that for every $\beta \in \{1,...,r\}$ and every $i \in \{(\beta-1)n+1,...,\beta n\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^2_{\phi(rn+i)}(m^{\beta+1,1,\phi})(p).$$

Each buyer buys the same amount of each commodity as what she intends to demand according to her second message conditional on the first messages of the agents in the subsequent group $\beta+1$, whereas each seller $\phi(i)$ has to buy the same amount of each commodity as what buyer $\phi(rn+i)$ intends to demand. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi, p, m, 5) = (x, q)$ such that for every $\beta \in \{1,...,r\}$ and every $i \in \{(\beta-1)n+1,...,\beta n\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^3_{\phi(i)}(m^{-\beta,1,\phi})(p).$$

Each seller sells the same amount of each commodity as what she intends to supply according to her *third* message conditional on the first messages of all agents who no not belong to group β, whereas each buyer $\phi(rn+i)$ has to buy the same amount of each commodity as what seller $\phi(i)$ intends to supply. With probability $\frac{\eta}{(rn!)^2 T}$, G^+ chooses $a^+(\phi, p, m, 6) = (x, q)$ such that for every $\beta \in \{1,...,r\}$ and every $i \in \{(\beta-1)n+1,...,\beta n\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p \text{ and } x_{\phi(i)} = x_{\phi(rn+i)} = m^3_{\phi(rn+i)}(m^{-\beta,1,\phi})(p).$$

Each buyer buys the same amount of each commodity as what she intends to demand according to her *third* message conditional on the first messages of all agents who do not belong to group β, whereas each seller $\phi(i)$ has to buy the same amount of each commodity as what buyer $\phi(rn+i)$ intends to demand.

With probability $\frac{1-6\eta}{(2n!)^2 T}$, G^+ chooses $a^+(\phi, p, m, 7) = (x, q)$ such that for every $\beta \in \{1,...,r\}$ and every $i \in \{(\beta-1)n+1,...,\beta n\}$,

$$q_{\phi(i)} = q_{\phi(rn+i)} = p^{++}(\phi, m, \beta+1) \text{ and } x_{\phi(rn+i)} = m^3_{\phi(rn+i)}(m^{-\beta,1,\phi})(p^{++}(\phi, m, \beta+1)).$$

Agents in each group β trade at the approximate market-clearing price vector $p^{++}(\phi, m, \beta+1)$ in the subsequent group $\beta+1$. Each buyer buys the same amount of each commodity as what she intends to demand according to her third message conditional on the first messages of all agents who do not belong to group β.

In the case of $(x,q) = a^{+}(\phi, p, m, 7)$, we specify the selling amount $x_{\phi(i)}$ for each seller $\phi(i)$ as follows. Fix $\beta \in \{1,...,r\}$ and $h \in \{1,...,k\}$ arbitrarily. Suppose that seller $\phi(i)$ belongs to group β, i.e., $i \in \{(\beta-1)n+1,...,\beta n\}$. There exists $i^{+} = i^{+}(h,\beta) \in \{(\beta-1)n+1,...,\beta n\}$ satisfying the following properties. Suppose that there exists excess supply in group β, i.e.,

(14) $$\sum_{j=(\beta-1)n+1}^{\beta n} m^3_{\phi(j)}(m^{-\beta,1,\phi})(q_{\phi(j)})(h) \geq \sum_{j=1}^{n} x_{\phi((r+\beta-1)n+j)}(h).$$

Then,

$$x_{\phi(i)}(h) = m^3_{\phi(i)}(m^{-\beta,1,\phi})(q_{\phi(i)})(h) \text{ if } i < i^{+},$$

$$x_{\phi(i)}(h) = \sum_{j=1}^{n} x_{\phi((r+\beta-1)n+j)}(h) - \sum_{j=(\beta-1)n+1}^{i-1} x_{\phi(j)}(h)$$
$$\leq m^3_{\phi(i)}(m^{-\beta,1,\phi})(q_{\phi(i)})(h) \text{ if } i = i^{+},$$

and

$$x_{\phi(i)}(h) = 0 \text{ if } i > i^{+}.$$

Suppose that inequality (14) does not hold. Then,

$$x_{\phi(i)}(h) = l \text{ if } i < i^{+},$$

$$x_{\phi(i)}(h) = \sum_{j=1}^{n} x_{\phi((r+\beta-1)n+j)}(h) - (i-1)l - \sum_{j=i+1}^{\beta n} x_{\phi(j)}(h)$$
$$\geq m^3_{\phi(i)}(m^{-\beta,1,\phi})(q_{\phi(i)})(h) \text{ if } i = i^{++},$$

and

$$x_{\phi(i)}(h) = m^3_{\phi(i)}(m^{-\beta,1,\phi})(q_{\phi(i)})(h) \text{ if } i > i^{++}.$$

If there exists excess supply in group β, then, each seller before $\phi(\hat{i})$ can sell the same amount as what she intends to supply, whereas each seller after $\phi(\hat{i})$ sells zero amount. If there exists excess demand in group β, then, each seller after $\phi(\hat{i})$ can sell the same amount as what she intends to supply, whereas each seller after $\phi(\hat{i})$ has to sell the maximal amount l.

We assume that for every $i \in \{1,...,2rn\}$, every $\omega_i \in \Omega_i$, and every $\omega_i' \in \Omega_i \setminus \{\omega_i\}$,

(15) $$D_i(\omega_i) \cap D_i(\omega_i') = \phi.$$

This implies that no buyer has the same payoff-maximizing demand function between different private signals, and no seller has the same payoff-maximizing supply function between different private signals. Based on this assumption, for every buyer $i \in \{rn+1,...,2rn\}$ and every $\omega_i \in \Omega_i$, we define $M_i^{2+}(\omega_i) \in M_i^2$ as the set of functions $m_i^2 \in M_i^2$ such that for every $\omega \in \Omega$ and every $(\phi, m_{-i}) \in \Phi \times M_{-i}$, if $i = \theta(rn+1)$, and

$$m_{\phi(j)}^1 \in D_{\phi(j)}(\omega_{\phi(j)}) \text{ for all } j \in \{n+1,...,2n\} \cup \{(r+1)n+1,...,(r+2)n\},$$

then

$$m_i^2(m^{2,1,\phi})(p) \in \underset{x_i \in \{0,...,l\}^k}{\arg\max}[v_i(x_i, \omega_{N(2,\phi)}) - \sum_{h=1}^{k} p(h)x_i(h)],$$

where $N(\beta, \phi) \subset \{1,...,2rn\}$ is the set of members in group β, i.e.,

$$N(\beta, \phi) = \{\phi(n+1),...,\phi(2n), \phi((r+1)n+1),...,\phi((r+2)n)\}.$$

For every seller $i \in \{1,...,rn\}$ and every $\omega_i \in \Omega_i$, we define $M_i^{2+}(\omega_i) \in M_i^2$ as the set of functions $m_i^2 \in M_i^2$ such that for every $\omega_{-i} \in \Omega_{-i}$ and every $(\phi, m_{-i}) \in \Phi \times M_{-i}$, if $i = \theta(rn+1)$, and

$$m_{\phi(j)}^1 \in D_{\phi(j)}(\omega_{\phi(j)}) \text{ for all } j \in \{n+1,...,2n\} \cup \{(r+1)n+1,...,(r+2)n\},$$

then

$$m_i^2(m^{2,1,\phi})(p) \in \underset{x_i \in \{0,...,l\}^k}{\arg\max}[\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i, \omega_{N(2,\phi)})].$$

For every buyer $i \in \{rn+1,...,2rn\}$ and every $\omega_i \in \Omega_i$, we define $M_i^{3+}(\omega_i) \in M_i^3$ as the set of functions $m_i^3 \in M_i^3$ such that for every $\omega_{-i} \in \Omega_{-i}$ and every $(\phi, m_{-i}) \in \Phi \times M_{-i}$, if $i = \theta(rn+1)$, and

$$m_{\phi(j)}^1 \in D_{\phi(j)}(\omega_{\phi(j)}) \text{ for all } j \notin \{1,...,n\} \cup \{rn+1,...,(r+1)n\},$$

then

$$m_i^3(m^{-1,1,\phi})(p) \in \underset{x_i \in \{0,...,l\}^k}{\arg\max}[v_i(x_i, \omega_{\bigcup_{\beta \neq 1} N(\beta,\phi)}) - \sum_{h=1}^{k} p(h)x_i(h)].$$

For every seller $i \in \{1,...,rn\}$ and every $\omega_i \in \Omega_i$, we define $M_i^{3+}(\omega_i) \in M_i^3$ as the set of functions $m_i^3 \in M_i^3$ such that for every $\omega_{-i} \in \Omega_{-i}$ and every $(\phi, m_{-i}) \in \Phi \times M_{-i}$, if $i = \theta(rn+1)$, and

$$m_{\phi(j)}^1 \in D_{\phi(j)}(\omega_{\phi(j)}) \text{ for all } j \notin \{1,...,n\} \cup \{rn+1,...,(r+1)n\},$$

then

$$m_i^3(m^{-1,1,\phi})(p) \in \underset{x_i \in \{0,...,l\}^k}{\arg\max}[\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i, \omega_{\bigcup_{\beta \neq 1} N(\beta,\phi)})].$$

Condition 3: For every seller $i \in \{1,...,rn\}$, every $\omega \in \Omega$, every $\omega' \in \Omega$, and every $\phi \in \Phi$, if $\omega_i = \omega'_i$, $\phi(1) = i$, and $\omega_{\bigcup_{\beta \neq 1} B(\beta,\phi)} = \omega'_{\bigcup_{\beta \neq 1} B(\beta,\phi)}$, then for every $x_i \in \{0,...,l\}^k$, every $x'_i \in \{0,...,l\}^k$, and every $p \in P$,

$$[\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i,\omega) > \sum_{h=1}^{k} p(h)x_i'(h) - c_i(x_i',\omega)\,]$$
$$\Leftrightarrow [\sum_{h=1}^{k} p(h)x_i(h) - c_i(x_i,\omega') > \sum_{h=1}^{k} p(h)x_i'(h) - c_i(x_i',\omega')\,].$$

Condition 3 implies that each seller does not need to know what the private signals that the other agents in the same group as her possess are whenever she knows the private signals of all agents who do not belong to the same group as her. When r is sufficiently large, it might be natural to assume Condition 3.

The first message of each seller (buyer) influences her payoff only in the case that the mechanism G^+ chooses $a^+(\phi,p,m,1)$ ($a^+(\phi,p,m,2)$, respectively). Hence, it follows that if a strategy profile s is undominated, then it must hold that for every agent $i \in \{1,...,2rn\}$ and every $\omega_i \in \Omega_i$,

(16) $$s_i^1(\omega_i) \in D_i(\omega_i).$$

The second message of each seller (buyer) influences her payoff only in the case that the mechanism G^+ chooses $a^+(\theta,p,m,3)$ ($a^+(\theta,p,m,4)$, respectively). This, together with equalities (15), implies that if a strategy profile s is included in S^2, then it must hold that for every agent $i \in \{1,...,2rn\}$ and every $\omega_i \in \Omega_i$,

(17) $$s_i^2(\omega_i) \in M_i^{2+}(\omega_i).$$

The third message of each seller (buyer) influences her payoff only in the case that the mechanism G^+ chooses either $a^+(\phi,p,m,5)$ or $a^+(\phi,p,m,7)$ (either $a^+(\theta,p,m,6)$ or $a^+(\phi,p,m,7)$, respectively). In the case of $a^+(\phi,p,m,7)$, the residual total supply (demand) available to each buyer (seller, respectively) may include information about the private signals of the other agents of the same group as her. However, such information does not influence her choice of third message, because Condition 3 guarantees that she can maximize her payoff with full information about all agents' private signals by observing only the first messages of the agents who do not belong to the same group as her. This observation, together with the assumption of no production externality and decreasingness of sellers' const conditions, implies that if a strategy profile s is included in S^2, then it must hold that for every agent $i \in \{1,...,2rn\}$ and every $\omega_i \in \Omega_i$,

(18) $$s_i^3(\omega_i) \in M_i^{3+}(\omega_i).$$

From the above arguments, we can prove the following theorem in the same way as Theorem 4.

Theorem 6: *A strategy profile $s \in S$ is iteratively undominated in $(G^+,(u_i)_{i=1}^{4n},f)$ if and only if for every $i \in \{1,...,2rn\}$ and every $\omega_i \in \Omega_i$, properties (16), (17), and (18) hold. The set of iteratively undominated strategy profiles in $(G^+,(u_i)_{i=1}^{4n},f)$ is twice dominance solvable and interchangeable.*

Finally, we can apply the same argument as in Subsection 3.4 to the general interdependent value case, and show approximate efficiency, approximate uniform pricing, and participation constraints when the number of agents is sufficiently large.

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