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Asset Pricing With Multiplicative Habit and Power-Expo Preferences

by

William T. Smith* and Qiang Zhang*

Abstract

Multiplicative habit introduces an additional consumption risk as a determinant of equity premium, and allows time preference and habit strength, in addition to risk aversion, to affect "price of risk". A model combining multiplicative habit and power-expo preferences cannot be rejected.

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Two formulations of habit formation have been used in macroeconomics and finance. One is the subtractive habit, popularized by Constantinides (1990). The other is the multiplicative habit proposed by Abel (1990). Carroll (2000) has especially argued for the multiplicative habit model.

Empirical analyses of habit formation have so far concentrated on subtractive habit [Ferson and Constantinides (1991), Heaton (1995), Campbell and Cochrane (1999), Otrok, Ravikumar and Whiteman (1999), Dynan (2000)]. Although there have been analytical and calibration studies of multiplicative habit [Abel (1999), Chan and Kogan (2002)], we are not aware of any econometric study that confronts it with data. We fill this gap by testing first the original Abel (1990) model and then an extended version that we develop below. In addition, we derive expressions for the equity premium to flesh out the asset pricing implications of both models.

Our extension of this model relies on a novel utility function called "power-expo" preferences. Introduced by Saha (1993), and initially estimated by Saha, Shumway, and Talpaz (1994, SST), these preferences have found applications in experiments [Holt and Laury (2002), HL] and in growth [Xie (2000)]. However, they have not been exploited in the asset pricing literature.[1] The virtue of power-expo utility is that it allows relative risk aversion (RRA) to be flexible. This is important since there is empirical evidence for both increasing RRA [HL (2002), SST (1994)] and decreasing RRA (DRRA) [Ogaki and Zhang (2001), Zhang and Ogaki (2004)].

1. Preferences

Let c_t denote the consumption of the representative agent at period *t*. He is endowed with the following period felicity function

$$u(c_t, c_{t-1}) = \frac{1 - e^{-a\frac{\left(c_t / c_{t-1}^{\alpha}\right)^{1-\gamma} - 1}{1-\gamma}}}{a}. \tag{1}$$

This subsumes two important special cases.

[1] Guiso and Paiella (2000) used a closely related utility function.

When $\alpha > 0$ and $a = 0$, (1) reduces to the multiplicative habit model [Abel (1990)]

$$u(c_t, c_{t-1}) = \frac{\left(c_t / c_{t-1}^{\alpha}\right)^{1-\gamma} - 1}{1-\gamma}. \tag{2}$$

Here RRA for given habit is the constant γ.

When $\alpha = 0$ and $a > 0$, (1) reduces to the power-expo utility function [Saha (1993)]

$$u(c_t) = \frac{1 - e^{-a\frac{c_t^{1-\gamma} - 1}{1-\gamma}}}{a}, \tag{3}$$

where $\gamma \geq 0.$ RRA for this class of preferences is $R(c_t) = \gamma + a c_t^{1-\gamma}$, i.e. non-constant.

Similarly, RRA implied by (1) for given habit is

$$R(c_t, c_{t-1}) = \gamma + a\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}. \tag{4}$$

2. Asset-Pricing

The representative agent can invest in n risky assets with net returns $r_{i,t+1}$, $i = 1,...,n,$ and a riskless asset with net return r_f. Denote the conditional expectation based on information available at t by E_t. The Euler equation for the i^{th} asset is[2]

$$\begin{aligned} &E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] = \\ &\qquad \beta E_t\left[\left(\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right)\frac{c_t}{c_{t+1}}\left(1 + r_{i,t+1}\right)\right]. \end{aligned} \tag{5}$$

Here β is the time discount factor. To shed some light on what (5) means for equity premium it is useful to consider our two special cases.

[2] All derivations are available upon request.

First, suppose that there is power-expo utility without habit formation. In this case, the equity premium is approximately

$$E_t r_{i,t+1} - r_f \approx \left(\gamma + a c_t^{1-\gamma}\right) \cdot \mathrm{cov}_t\left(r_{i,t+1}, \Delta c_t / c_t\right). \tag{6}$$

Meyer and Meyer (2005) suggest that DRRA may be sufficient to explain the equity premium. However, (6) shows why this cannot be the true. If RRA is not constant, the equity premium should move systematically with the *level* of consumption. This is blatantly counterfactual. As Cochrane (2001, p. 467) asserts, "We cannot tie risk aversion to the *level* of consumption and wealth, since that increases over time while equity premia have not declined."

Next suppose that there is habit formation with constant RRA, as in Abel (1990). We suggest a novel method to see how habit may help explain the equity premium:

$$E_t r_{i,t+1} - r_f \approx \left[\gamma - \frac{\beta\alpha^2(1-\gamma)}{1-\beta\alpha}\right]\mathrm{cov}_t\left(r_{i,t+1}, \frac{\Delta c_{t+1}}{c_t}\right) + \beta\alpha\frac{1-\gamma}{1-\beta\alpha}\mathrm{cov}_t\left(r_{i,t+1}, \frac{\Delta c_{t+2}}{c_{t+1}}\right). \tag{7}$$

The premium now depends upon *both* the contemporaneous covariance between equity return and consumption growth *and* the covariance between the return and consumption growth next period. This suggests why the multiplicative habit model might perform better than the standard consumption-CAPM based on constant RRA preferences: not only there is now a new risk factor but also the factor weights depend on time preference and habit strength, in addition to RRA.

Now consider the equity premium for the general case with both habit formation and non-constant RRA:

$$E_t r_{i,t+1} - r_f = \frac{-\mathrm{cov}_t\left(r_{i,t+1}, \left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right) + \beta\alpha\,\mathrm{cov}_t\left(r_{i,t+1}, \left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right)}{E_t\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}}}. \tag{8}$$

As in Abel's (1990) habit model, there are again two risks to holding equity. Power-expo preferences affect the equity premium through the exponential terms in (8). They can potentially raise the covariances between equity return and consumption by making the consumption terms in (8) more volatile.

3. Empirical Results

Hansen, Heaton and Yaron (1996) report that the continuously updated GMM estimator (CUE) produces a model specification test with much smaller size distortion in the finite sample than the 2-step GMM. We hence use the CUE in our tests of the models above.

It is well-known that habit introduces a moving average structure, and therefore serial correlation, to the Euler equation disturbance. In our case, it is a first-order serial correlation. However, allowing for first-order serial correlation in our estimation of (5) always led to negative-definite weighting matrices in the CUE criterion function. This is a well-known problem in applications of GMM. Therefore, we adopt a solution to this problem described in Ogaki (1993, p. 468): Newey and West's (1987) Bartlett kernel estimator that does *not* impose zero weights on autocovariances of orders higher than 1. Specifically, we have used bandwidth values 3, 4, 5, 9, 13, and 17 in the Bartlett kernel. We find that for all these values except 3, the negative-definite weighting matrix problem disappears. Furthermore, the estimation and test results are very similar across these bandwidth values. Our empirical results below are based on setting this parameter to 9.

Our data are quarterly U.S. aggregate data on nondurable goods and services consumption, the value-weighted return on New York Stock Exchange stocks, and the return on U.S Treasury bills from 1958:IV to 2001:IV. All data are deflated by the Consumer Price Index. See the notes to Table 1 for the instruments that we use. Time aggregation bias, if it exists, should affect the tests on Abel's (1990) model and the general model the same way. But our test results on them are dramatically different, suggesting that such bias may not be a serious problem.

Table 1 presents the empirical results on (5). The top panel of this table is for the special case of the Abel (1990) model, which requires $a = 0$. Here all the three parameters are precisely estimated, but the chi-square test of model specification rejects Abel's model decisively: the *p*-value of the test statistic is virtually zero.

A different set of results emerge in the next panel for the general model with multiplicative habit and power-expo preferences. First, the chi-square test no longer rejects model specification at conventional significance levels: the *p*-value is now a comfortable 54.3%. Thus introducing power-expo preferences has substantially improved the model's fit with the data. Second, all the parameters except γ are estimated to be significantly different from 0.[3] The discount factor β is reasonably estimated to be 0.977. The α estimate, being 1.023 with a standard error of 0.020, is significantly different from 0, implying that the power-expo model without habit formation (which requires $\alpha = 0$) is rejected.[4] This is consistent with our explanation above on why the original power-expo model cannot accommodate the historical equity premium. Lastly, the significant *a* estimate of 1.203 (with a standard error of 0.453) indicates rejection of Abel's (1990) habit model. This result confirms the rejection of that model in the top panel of the table.

Since our results show that each element alone, power-expo preferences or habit, is inadequate in explaining the equity return and T bill return at the same time, the non-rejection of the general model we have seen above must be due to the *combination* of both elements.

Given that the α estimate is insignificantly different from 1, we impose the restriction $\alpha = 1$ to re-estimate the general model. The model still cannot be rejected at conventional significance levels by the chi-square test: the *p*-value is 32.3%. The second column of this table reports a chi-square, likelihood-ratio type, test of the restriction $\alpha = 1$ using the difference between the two chi-square statistics in the first column. The *p*-value for this test is 13.3%, thereby not rejecting

[3] It is well-known that to obtain precise estimate for the curvature parameter that controls RRA (e.g. γ in the present paper) is difficult even in the standard C-CAPM featuring the power utility function.

[4] The power-expo model without habit is not explicitly tested due to the non-stationarity of Euler equation terms.

$\alpha = 1$ at conventional significance levels. This result is important because it ensures stationarity, a key assumption underlying GMM, of all the terms in (5).

In addition, this result implies that consumers derive utility from consumption *growth*, and according to (4), RRA increases with consumption growth for estimated values of model parameters. However, since consumption growth fluctuates mildly over time with a mean of 1.005, RRA bounces somewhat tightly around about 1.

4. Conclusions

We find favorable empirical results for the general model with both multiplicative habit formation and non-constant RRA. This contrasts with Meyer and Meyer's (2005) claim that DRRA is sufficient for solving the equity premium puzzle, but habit formation is dispensable. The model's implication on risk aversion is also sharply different from that of Campbell and Cochrane's (1999) subtractive habit model. There consumers become substantially more risk averse as consumption falls close to habit. Here they are slightly more risk averse when consumption rises relative to habit.

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Table 1 Continuously Updated GMM Estimation of (5)

Using Stock and T Bill Returns

Specification Test	Restriction Test	β	γ	α	a
		Abel (1990) Habit Model			
43.65	--	0.990	1.099	1.010	--
(0.000)	--	(0.142)	(0.035)	(0.145)	--
		General Model with Habit and Power-Expo Preferences			
6.940	--	0.977	0.012	1.023	1.203
(0.543)	--	(0.019)	(0.228)	(0.020)	(0.453)
9.196	2.256	1.000	0.011	1	0.982
(0.326)	(0.133)	(.000)	(0.162)	--	(0.160)

Notes: 1. The first and second columns report chi-square statistics and associated *p*-values (in parentheses below the statistics). 2. The standard errors are in parentheses below parameter estimates. 3. The results here are based on using six instruments: 1, consumption growth, stock return, dividend yield, T bill return and the term premium on Treasury bonds, all lagged and in real terms. Results based on the first four instruments alone are similar.

Appendices

I. The Euler Equation

Lifetime utility is

$$U_t = \sum_{j=0}^{\infty} \frac{\beta^j}{a}\left[1 - e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+j}}{c_{t+j-1}^{\alpha}}\right)^{1-\gamma}}\right]. \tag{A.1}$$

The lifetime marginal utility of c_t is then

$$\frac{\partial U_t}{\partial c_t} = \frac{c_t^{-\gamma}}{c_{t-1}^{\alpha(1-\gamma)}} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha \frac{c_{t+1}^{1-\gamma}}{c_t^{\alpha(1-\gamma)+1}} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}. \tag{A.2}$$

Define $R_{i,t+1} = 1 + r_{i,t+1}$, and $R_f = 1 + r_f$. Following Abel (1990), the Euler equation for asset i is

$$E_t \frac{\partial U_t}{\partial c_t} = \beta E_t \frac{\partial U_{t+1}}{\partial c_{t+1}} R_{i,t+1}. \tag{A.3}$$

Evaluating this using Equation (A.2) yields

$$\begin{aligned} &E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] = \\ &\beta E_t\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}} R_{i,t+1} \end{aligned} \tag{A.4}$$

Since Equation (A.4) holds for all assets, including the risk-free asset, it follows that

$$\begin{aligned} &E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] = \\ &\beta E_t\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}} R_f \end{aligned} \tag{A.5}$$

Combining Equations (A.4) and (A.5) yields

$$E_t\left[\frac{c_{t+1}^{-\gamma}}{c_t^{\alpha(1-\gamma)-1}}e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\frac{c_{t+2}^{1-\gamma}}{c_{t+1}^{\alpha(1-\gamma)+1}}e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\left(R_{i,t+1}-R_f\right)=0. \tag{A.6}$$

II. Equity Premia

The paper reports expressions for the equity premium in three cases.

II.A Power-Expo Utility without Habit Formation

When $\alpha = 0$ Equation (A.6) reduces to

$$E_t c_{t+1}^{-\gamma} e^{-\frac{a}{1-\gamma}c_{t+1}^{1-\gamma}}\left(R_{i,t+1}-R_f\right)=0. \tag{A.7}$$

Define $g_{t+1} = \left(c_{t+1} - c_t\right)/c_t$. Equation (A.7) can then be expressed as

$$E_t\left(1+g_{t+1}\right)^{-\gamma} e^{-\frac{a}{1-\gamma}\left(1+g_{t+1}\right)^{1-\gamma}c_t^{1-\gamma}}\left(r_{i,t+1}-r_f\right)=0. \tag{A.8}$$

Taking a Taylor series around $g_{t+1} = r_{i,t+1} = r_f = 0$ yields Equation (6) in the text.

II.B Habit Formation without Power-Expo Utility

When $a = 0$ Equation (A.6) reduces to the Euler equation in Abel (1990):

$$E_t\left[\frac{c_{t+1}^{-\gamma}}{c_t^{\alpha(1-\gamma)-1}} - \beta\alpha\frac{c_{t+2}^{1-\gamma}}{c_{t+1}^{\alpha(1-\gamma)+1}}\right]\left(R_{i,t+1}-R_f\right)=0. \tag{A.9}$$

Define $g_{t+2} = \left(c_{t+2} - c_{t+1}\right)/c_{t+1}$. Taking a Taylor series around $g_{t+1} = g_{t+2} = r_{i,t+1} = r_f = 0$

yields Equation (7) in the text.

II.C Habit Formation with Power-Expo Utility

The expression of the equity premium for this case, Equation (8) in the text, is obtained by rewriting Equation (A.6) using the definition of covariance.