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Finitely Repeated Prisoners' Dilemma with Small Fines: Penance Contract[+]

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Abstract

We investigate the finitely repeated prisoners' dilemma with explicit contractual devices. We show that full collusion can be achieved by incentivizing the players' final period of play with small fines. Our incentivizing modality is the penance contract, by which a player is penalized if (and only if) he deviates from the penance strategy in the final period. We show that using this contractual agreement brings the penance strategy profile into unique subgame perfect equilibrium and achieves full collusion without being overturned by renegotiation.

Key Words: Finitely Repeated Prisoners' Dilemma, Small Fines, Penance Contracts, Full Collusion, Uniqueness, Renegotiation-Proofness.
JEL Classification Numbers: C72, C73, D74, L14.

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1. Introduction

This paper investigates the *finitely repeated prisoners' dilemma* played by players 1 and 2, wherein we allow *side payments* between them. The players agree on a side payment contract at the beginning of play and make their payments according to the terms of the contract at the end of the game. Using this contractual device demonstrates the fact that *full collusion* is sustained by the *unique* subgame perfect equilibrium and that the players' contractual agreement is robust to *renegotiation*.

The repeated-game literature makes clear that full collusion is impossible to achieve in a class of component games such as the prisoners' dilemma when the repetitions are finite.[1] The backward induction technique prevents the players from selecting the cooperative action as long as they remain unincentivized by an explicit device in their final period of play. This paper shows that full collusion before the final period can be sustained by the history-contingent continuation payoffs once the players' final period of play is incentivized by an explicit device of side-payment contract; in this case, full collusion is enforceable, even if the player's liability is severely limited in value.

In order to show this permissive result, we employ a side-payment contract named the *penance contract*, according to which players are required to pay a small fine to their opponents if (and only if) they fail to follow a strategy named the *penance strategy* in the final period. While the basic concept of the penance strategy was introduced by Farrell and Maskin (1989) and van Damme (1989) in the literature of repeated games with renegotiation, this paper modifies their concept slightly by treating the two players asymmetrically: according to the penance strategy profile, the players continue to select the cooperative action profile such that player 1 pays for any deviation from the penance strategy by selecting the cooperative action in the next period, while player 2 simultaneously retaliates against that deviation by selecting the defective action; in contrast, player 2 pays penance for deviation and suffers retaliation only if that deviation occurs unilaterally. This paper shows that the penance strategy profile is the

[1] For general surveys on repeated games, see Pearce (1992), Fudenberg and Tirole (1995, Chapter 5), and Mailath and Samuelson (2006).

unique subgame perfect equilibrium when the players agree on the penance contract at the beginning of their repeated play. Hence, we can conclude that small fines assure full collusion within the unique subgame perfect equilibrium, even if play is finite. Moreover, uniqueness holds in every subgame, suggesting that the equilibrium restriction of Pareto perfection is automatically satisfied.[2]

We argue that the penance contract is robust to renegotiation-proofness: in every period and every subgame, the players have no incentive to agree to replace the penance contract with any other combination of contract and strategy profile at the expense of a tiny renegotiation cost. Accordingly, the penance contract is superior to other contractual agreements. For instance, we introduce an alternative contract, the *trigger contract*, by which the first player to select the defective action is penalized. Flowing from the trigger contract, the trigger strategy profile is the unique subgame perfect equilibrium, achieving full collusion. However, since this profile pre-determines the identity of the penalized player before the final period, the equilibrium play in the subgame after a deviation is merely the repetition of the choice of defective action: it is inevitably overturned by renegotiation. In contrast, the penance contract never determines the player who is to be penalized before reaching the final period; thus, since any deviant will immediately rebuild his trust, the continuation payoffs maintain their efficiency, even when off the equilibrium path.

Several works explore the possibility that full collusion is enforceable even with finite play and in the absence of explicit devices. For instance, Benoit and Krishna (1985) and Friedman (1985) investigate an alternative class of component games that have multiple inefficient Nash equilibria and show that full collusion can be approximately sustained by a subgame perfect equilibrium. Radner (1980) and Lipman and Wang (2000) show that full collusion is enforceable in the finitely repeated prisoners' dilemma when players are irrational or when there is a switching cost. For further elucidation of collusive phenomena in finitely repeated games, see Sobel (1985)

[2] Owing to multiplicity, several works, such as Bernheim and Ray (1989), Farrell and Maskin (1989), van Damme (1989), Pearce (1987), and Benoit and Krishna (1993), have used many different definitions of Pareto perfection in repeated games.

and Neyman (1985, 1999).[3] These works typically depend on the multiplicity of equilibria and do not investigate renegotiation-proofness. The points of departure from these works are that full collusion is achievable if we incentivize only the final period of play and that the players can solve both the uniqueness and renegotiation-proofness by agreeing on the penance contract.

There is considerable scholarship on implementation in moral hazard which clarifies whether a single-period relationship attains the first-best through explicit contracting. See Dutta and Radner (1994) and Salanie (1997). In particular, Rey and Salanie (1990) and Fudenberg et al. (1990) show that renegotiable short-term contracts implement efficiency in a long-term relationship. These works crucially depend on the assumption that large side payments are available. In contrast to these works, this paper uses only small side payments; obviously, the establishment of a long-term relationship dramatically economizes on monetary transfers without harming players' incentive to collude.

The remainder of this paper is organized as follows. Section 2 defines the model. Section 3 specifies the penance-strategy profile and the penance contract, showing that full collusion is uniquely enforceable. Subsections 4.1 and 4.2 employ the trigger contract alternative and, after comparing it with the penance contract, demonstrate that the penance contract satisfies a strong notion of renegotiation-proofness through its small fines, whereas the trigger contract does not. Subsection 4.3 applies the findings of this paper to more general component games.

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[3] See also the experimental work by Andreoni and Varian (1999).

2. The Model

Arbitrarily fix a positive integer $T \geq 2$. Let us investigate a situation in which players 1 and 2 T times repeatedly play the component game given by $G \equiv (A_1, A_2, u_1, u_2)$. A_i denotes the set of all actions for each player $i \in \{1,2\}$, $A \equiv A_1 \times A_2$, and $u_i : A \to R$ denotes the payoff function for each player i. Let us confine our attention to the prisoners' dilemma game specified by $A_1 = A_2 = \{c, d\}$,

$$u_1(c,c) = u_2(c,c) = 1,\ \ u_1(d,c) = u_2(c,d) = M+1,$$

$$u_1(c,d) = u_2(d,c) = -L, \text{ and } \ u_1(d,d) = u_2(d,d) = 0,$$

where $L > 0$, $M > 0$, and $L+1 > M$. Let us call c and d the *cooperative* action and the *defective* action respectively. Note that the payoff vector $(1,1)$ that is induced by the cooperative action profile (c,c) is efficient, that the defective action d is dominant for each player, and that

$$u_1(c,c) + u_2(c,c) > u_1(a) + u_2(a) \ \text{ for all } \ a \in A / \{(c,c)\},$$

which was implied by the inequality of $L+1 > M$. Subsection 4.3 investigates more general two-player games.

For every period $t \in \{1,...,T\}$, let us denote by $h(t) = (a(\tau))_{\tau=1}^{t} \in A^t$ a history up to period t, where $a(\tau) = (a_1(\tau), a_2(\tau)) \in A$ implies the action profile selected in period $\tau \in \{1,...,t\}$. Let $H(t)$ denote the set of all histories up to period t. Let us call $h(T) \in H(T)$ a *complete* history. We denote by $h(0)$ the *null* history.

Let us define a *contract* $y = (y_1, y_2)$ as

$$y_i : H(T) \to R \ \text{ for all } \ i \in \{1,2\},$$

where we assume *balanced budgets* in that

(1) $$y_1(h(T)) + y_2(h(T)) = 0 \ \text{ for all } \ h(T) \in H(T).$$

Let Y denote the set of all contracts satisfying balanced budgets. At the beginning of their repeated play, the players agree on a contract $y \in Y$; at the end of their repeated play, each player i receives the side payment given by $y_i(h(T)) \in R$ when the complete history $h(T) \in H(T)$ is realized. The payoff for each player i induced by the complete history $h(T) \in H(T)$ is defined as the summation of instantaneous payoffs and side payments, thus, $\sum_{\tau=1}^{T} u_i(a(\tau)) + y_i(h(T))$.

Let us denote by (G, T, y) the T times finitely repeated game associated with the contract of $y \in Y$. A pure strategy, or shortly a strategy, for each player i is defined as $\sigma_i : \bigcup_{t=0}^{T-1} H(t) \to \{c, d\}$. The player selects the action $a_i(t) = \sigma_i(h(t-1)) \in A_i$ in each period $t \in \{1, ..., T\}$ when the history $h(t-1) \in H(t-1)$ up to period $t-1$ is realized. Let Σ_i denote the set of strategies for player i.[4] Let $\sigma = (\sigma_1, \sigma_2)$ denote a strategy profile. Let $\Sigma \equiv \Sigma_1 \times \Sigma_2$ denote the set of all strategy profiles.

We define the payoff for each player i induced by any strategy profile $\sigma \in \Sigma$ as

$$v_i(\sigma, y) = \sum_{\tau=1}^{T} u_i(a(\tau)) + y_i(h(T)),$$

where $\sigma(h(t-1)) = (\sigma_1(h(t-1)), \sigma_2(h(t-1)) \in A$ and $a(t) = \sigma(h(t-1))$ for all $t \in \{1, ..., T\}$. For every $t \in \{1, ..., T\}$ and every $h(t-1) \in H(t-1)$, we define the payoff for each player i induced by $\sigma \in \Sigma$ after period $t-1$ when the history $h(t-1) \in H(t-1)$ up to period $t-1$ is realized as

$$v_i(\sigma, y, h(t-1)) = \sum_{\tau=t}^{T} u_i(a(\tau)) + y_i(h(T)),$$

where $h(T) = (h(t-1), a(t), ..., a(T))$, and $a(\tau) = \sigma(h(\tau-1))$ for all $\tau \in \{t, ..., T\}$. A strategy profile $\sigma \in \Sigma$ is said to be a *subgame perfect equilibrium* in (G, T, y) if, for every $t \in \{1, ..., T\}$, every $h(t-1) \in H(t-1)$, and every $i \in \{1, 2\}$,

$$v_i(\sigma, y, h(t-1)) \geq v_i(\sigma_i', \sigma_j, y, h(t-1)) \quad \text{for all } \sigma_i' \in \Sigma_i,$$

where $j \neq i$.

[4] For logical convenience, we confine our discussion to pure strategies. We can apply our premises, however, to more general classes of behavioral strategies.

3. Penance Strategies and Penance Contract

Let us specify $\sigma^* \in \Sigma$ as the *penance strategy profile*;

$$\sigma^*(h(0)) = (c,c),$$

for every $t \in \{2,...,T\}$ and every $h(t-1) \in H(t-1)$,

$$\sigma^*(h(t-1)) = (c,c) \quad \text{if } a(t-1) = \sigma^*(h(t-2)),$$

$$\sigma^*(h(t-1)) = (c,d) \quad \text{if } a_1(t-1) \neq \sigma_1^*(h(t-2)),$$

and

$$\sigma^*(h(t-1)) = (d,c) \quad \text{if } a_1(t-1) = \sigma_1^*(h(t-2)) \text{ and } a_2(t-1) \neq \sigma_2^*(h(t-2)).$$

According to the penance strategy profile σ^*, the players select the cooperative action profile (c,c) in the initial period. In every period and every subgame, players select (c,c) in the next period if neither one has deviated from σ^*. In every period and every subgame, if player 1 deviates from σ^*, he penances for his deviation by selecting the cooperative action c in the next period, and player 2 retaliates against that deviation by selecting the defective action d at the same time. In every period and every subgame, if player 2 unilaterally deviates from σ^*, player 1 retaliates by selecting d in the next period, and player 2 penances by selecting c at the same time. In contrast to the definitions of penance strategy profile occurring in works such as Farrell and Maskin (1989) and van Damme (1989), this paper's definition treats the players asymmetrically: if both players deviate simultaneously, player 1 does penance, but player 2 never does.

According to the penance strategy profile, the players continue to choose (c,c), thereby achieving full cooperation. A deviant is penalized *only* in the next period; the opponent retaliates with the defective action choice d while the deviant also being required to penance by selecting the cooperative action c, a more costly choice than the defective action. They can return to full cooperation two periods later as long as they follow the penance strategy profile immediately after the deviation.

Arbitrarily fix a positive real number $X > 0$, which is regarded as the fine, where we assume that

(2) $X > \max[L, M]$.

Let us specify $y^* \in Y$ as the *penance contract*; for every $i \in \{1,2\}$ and every $h(T) \in H(T)$,

$$y_i^*(h(T)) = 0 \quad \text{if } a(T) = \sigma^*(h(T-1)),$$

$$y_i^*(h(T)) = 0 \quad \text{if } a_1(T) \neq \sigma_1^*(h(T-1)) \text{ and } a_2(T) \neq \sigma_2^*(h(T-1)),$$

$$y_i^*(h(T)) = -X \quad \text{if } a_i(T) \neq \sigma_i^*(h(T-1)) \text{ and } a_j(T) = \sigma_j^*(h(T-1)),$$

and

$$y_i^*(h(T)) = X \quad \text{if } a_i(T) = \sigma_i^*(h(T-1)) \text{ and } a_j(T) \neq \sigma_j^*(h(T-1)),$$

where $j \neq i$. The penance contract y^* is contingent only on the players' final period of play, depending on whether they follow the penance strategy profile σ^* in the final period T. A player is penalized by paying the fine of X to the opponent if (and only if) he deviates from σ^* in the final period T. Note that a player who follows σ^* in the final period is never penalized, even if he deviates from σ^* before the final period. If both players jointly deviate in the final period, their payments are cancelled out.

Theorem 1: *The penance strategy profile* σ^* *is the unique subgame perfect equilibrium in* (G, T, y^*).

Proof: No player $i \in \{1,2\}$ is ever required to pay the fine of X after selecting the action $\sigma_i^*(h(T-1))$ in the final period T; any other action triggers a fine. Since the instantaneous gain from selecting $a_i(T) \neq \sigma_i^*(h(T-1))$ is at most $\max[L, M]$, it follows from (2) that each player i selects $a_i(T) = \sigma_i^*(h(T-1))$.

Arbitrarily fix any period $t \in \{1, ..., T-1\}$ and a history $h(t-1) \in H(t-1)$ up to period $t-1$. Assume that both players follow σ^* after period t. If player 1 selects $\sigma_1^*(h(t-1))$ in period t, his future payoff after period t is at least $T-t$. If he does not select $\sigma_1^*(h(t-1))$ in period t, his future payoff equals $T-t-1-L$. Hence, the difference in future payoff is at least $L+1$. Since $L+1>M$, this difference is greater than the maximum difference in instantaneous payoffs (thus, $\max[L,M]$). Hence, player 1 selects $a_1(t)=\sigma_1^*(h(t-1))$ in period t.

Suppose that player 1 selects $a_1(t)=\sigma_1^*(h(t-1))$ in period t.[5] If player 2 selects $\sigma_2^*(h(t-1))$ in period t, his future payoff after period t equals $T-t$. If player 2 does not select $\sigma_2^*(h(t-1))$, his future payoff equals $T-t-1-L$. Hence, the difference in future payoffs equals $L+1$. Since $L+1>M$, this difference is greater than the maximum difference in instantaneous payoffs (thus, $\max[L,M]$). Hence, player 2 selects $a_2(t)=\sigma_2^*(h(t-1))$ in period t.

From the above observations, we have proved that σ^* is the unique subgame perfect equilibrium in (G,T,y^*).

Q.E.D.

Theorem 1 implies that full cooperation can be sustained by the unique subgame perfect equilibrium, where no side payment takes place on the equilibrium path. Incentivizing only the final period of play is sufficient to achieve full cooperation.

Given $M>1$, using the basic concept of penance contract and penance strategy profile becomes inevitable, at least in the final two periods: the opponent's retaliatory selection of the defective action in the final period is insufficient to incentivize a player to select the cooperative action in period $T-1$. In addition, we have to urge any deviant to pay penance for his deviation by selecting the cooperative action in the final period, which costs him more than would the defective action.

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[5] This supposition is crucial to this proof because of the asymmetric treatment in the specification of σ^*.

4. Discussions

4.1. Trigger Strategy and Trigger Contract

Since the penance contract y^* incentivizes only the final period of play, we need only a small fine—negligible compared to the entire payoff—provided the time length T is sufficiently large. We can find alternative contracts with small fines to eliminate unwanted equilibria once we survey a range of contracts that do not necessarily incentivize only the last period of play.

For instance, let us specify $\hat{y} \in Y$ as the *trigger contract*. For every $i \in \{1,2\}$, every $t \in \{1,...,T\}$, and every complete history $h(T) \in H(T)$ such that $a(\tau) = (c,c)$ for all $\tau \in \{1,...,t-1\}$, and $a(t) \neq (c,c)$,

$$\hat{y}_i(h(T)) = 0 \qquad \text{if } a(t) = (d,d),$$

$$\hat{y}_i(h(T)) = -X \qquad \text{if } a_i(t) = d \text{ and } a_j(t) = c,$$

and

$$\hat{y}_i(h(T)) = X \qquad \text{if } a_i(t) = c \text{ and } a_j(t) = d.$$

For the complete history $h(T) \in H(T)$ such that $a(t) = (c,c)$ for all $t \in \{1,...,T\}$,

$$\hat{y}_i(h(T)) = 0.$$

According to the trigger contract $\hat{y}$, any first player to select the defective action d is penalized by paying the fine of X to the opponent.

Let us specify $\hat{\sigma} \in \Sigma$ as the *trigger strategy profile*;

$$\sigma^*(h(0)) = (c,c),$$

for every $t \in \{2,...,T\}$ and every $h(t-1) \in H(t-1)$,

$$\sigma^*(h(t-1)) = (c,c) \qquad \text{if } a(\tau) = \sigma^*(h(\tau-1)) \text{ for all } \tau \in \{1,...,t-1\},$$

and

$$\sigma^*(h(t-1)) = (d,d) \qquad \text{otherwise.}$$

According to the trigger strategy profile $\hat{\sigma}$, the players select (c,c) in the initial period. In every period and every subgame, the players select (c,c) if neither one selected d in any previous period; they select (d,d) otherwise. Hence, they continue to choose (c,c), thereby achieving full cooperation.

The basic concept of trigger contract is inspired by the Abreu-Matsushima mechanism (Abreu and Matsushima [1992, 1994]) in the implementation theory literature: a "tail-chasing" competition *à la* Abreu-Matsushima motivates each player to wish to avoid being the first one to select the defective action, rendering the occurrence of such a selection virtually impossible.

Proposition 2: *The trigger strategy profile* $\hat{\sigma}$ *is the unique subgame perfect equilibrium in* $(G,T,\hat{y})$, *where we assume the inequality of (2).*

Proof: Given that no player has selected d before, by selecting d instead of c in the current period, a player earns the additional instantaneous payoff that is at most $\max[L,M]$ but is also required to pay the fine of X, which, as the inequality of (2) implies, is greater than $\max[L,M]$. From the next period onwards, the player must continue to select (d,d), since the continuation of (d,d) is the unique subgame perfect equilibrium in the corresponding subgame. Hence, no player wishes to be the first one to select d, a fact which induces a tail-chasing competition *à la* Abreu-Matsushima between the players. Hence, $\hat{\sigma}$ is the unique subgame perfect equilibrium in $(G,T,\hat{y})$.

Q.E.D.

4.2. Renegotiation-Proofness

The penance contract has an advantage over the trigger contract: while the trigger contract is vulnerable to renegotiation, the penance contract is not. Arbitrarily fix a positive real number $\eta > 0$ as the *renegotiation cost*. In every period and every subgame, let us allow the players to replace the original agreement concerning strategy profile and contract with any other agreement. To do this, each player must spend the renegotiation cost $\eta > 0$. A combination of strategy profile and contract $(\sigma, y) \in \Sigma \times Y$

is said to be *renegotiation-proof* if for every $t \in \{1,...,T\}$ and every $h(t-1) \in H(t-1)$, there exists no other combination $(\sigma', y') \in \Sigma \times Y / \{(\sigma, y)\}$ such that

$$v_i(\sigma', h(t-1), y') - \eta \geq v_i(\sigma, h(t-1), y) \text{ for all } i \in \{1,2\}.$$

The greater the renegotiation cost η, the less restrictive is the renegotiation-proofness. From the specification of the prisoners' dilemma game, it follows that any combination $(\sigma, y) \in \Sigma \times Y$ is renegotiation-proof if (and only if) for every $t \in \{1,...,T\}$ and every $h(t-1) \in H(t-1)$,

(3) $$v_1(\sigma, h(t-1), y) + v_2(\sigma, h(t-1), y) > 2(T-t+1) - 2\eta.$$

The following proposition shows that the penance contract needs only a small renegotiation cost to satisfy renegotiation-proofness, whereas the trigger contract needs a large one.

Proposition 3: *The combination of penance strategy profile and penance contract* (σ^*, y^*) *is renegotiation-proof if and only if*

(4) $$\eta \geq \frac{1+L-M}{2}.$$

The combination of trigger strategy profile and trigger contract $(\hat{\sigma}, \hat{y})$ *is renegotiation-proof if and only if*

(5) $$\eta \geq T-1.$$

Proof: Note that $v_1(\sigma^*, h(t-1), y^*) + v_2(\sigma^*, h(t-1), y^*)$ is equivalent to

$$\text{either } 2(T-t+1) \text{ or } 2(T-t)+1+M-L.$$

Note also that, for every $t \in \{2,...,T\}$, there exists $h(t-1) \in H(t-1)$ such that

$$v_1(\sigma^*, h(t-1), y^*) + v_2(\sigma^*, h(t-1), y^*) = 2(T-t)+1+M-L.$$

Hence, it follows from (3) that the necessary and sufficient condition for (σ^*, y^*) to be renegotiation-proof is

$$2(T-t)+1+M-L > 2(T-t+1) - 2\eta \text{ for all } t \in \{2,...,T\},$$

which is the same as the inequality (4).

Note that $v_1(\hat{\sigma}, h(t-1), \hat{y}) + v_2(\hat{\sigma}, h(t-1), \hat{y})$ is equivalent to either $2(T-t+1)$ or 0.

Note that, for every $t \in \{2,...,T\}$, there exists $h(t-1) \in H(t-1)$ such that

$$v_1(\sigma^*, h(t-1), y^*) + v_2(\sigma^*, h(t-1), y^*) = 0.$$

Hence, it follows from (3) that the necessary and sufficient condition for $(\hat{\sigma}, \hat{y})$ to be renegotiation-proof is

$$0 > 2(T-t+1) - 2\eta \quad \text{for all} \quad t \in \{2,...,T\},$$

that is, $0 > 2(T-1) - 2\eta$, which is equivalent to the inequality of (5).

Q.E.D.

Proposition 2 implies that the penance contract satisfies renegotiation-proofness much more easily than does the trigger contract. Note from the inequality of (4) that, given a sufficiently large T, (σ^*, y^*) is renegotiation-proof even if the renegotiation cost is negligible compared to the entire payoff. However, it is clear from the inequality of (5) that $(\hat{\sigma}, \hat{y})$ is renegotiation-proof only if the renegotiation cost is high relative to the entire payoff.

Under the penance contract, the fact that penalization is never determined before the end of play is crucial in its renegotiation-proofness. Let us consider any agreement by the terms of which side payments are determined before reaching the final period T. Note that, in any subgame after this determination, the repetition of defective action profile (d, d) is the only possible subgame perfect equilibrium: the players are willing to replace their original agreement with a more cooperative one, resulting in a combination of penance contract and penance strategy profile, or (σ^*, y^*) in this case.

The requirement of renegotiation-proofness in this subsection might be too restrictive: the players are allowed to replace the original agreement by any agreement that does not necessarily meet their incentive compatibility. Nevertheless, we cannot expect the combination of trigger-strategy profile and trigger contract $(\hat{\sigma}, \hat{y})$ to satisfy a less demanding version of renegotiation-proofness. Whenever any player deviates, they are both willing to replace $(\hat{\sigma}, \hat{y})$ by (σ^*, y^*), which meets a more restrictive standard of renegotiation-proofness.

4.3. Generalization

We have focused on the prisoners' dilemma game. This subsection extends our arguments to more general two-player games. For each $i \in \{1,2\}$, let us select three actions, $c_i \in A_i$, $d_i \in A_i$, and $e_i \in A_i$, which satisfy that for every $i \in \{1,2\}$,

$$u_i(c_i, c_j) - u_i(e_i, d_j) > \max_{a_i \in A_i} u_i(a_i, c_j) - u_i(c_i, c_j),$$

$$u_i(c_i, c_j) - u_i(e_i, d_j) > \max_{a_i \in A_i} u_i(a_i, d_j) - u_i(e_i, d_j),$$

and

$$u_1(c_1, c_2) + u_2(c_1, c_2) \geq u_1(a) + u_2(a) \quad \text{for all} \quad a \in A,$$

where $j \neq i$. Let us call $c_i \in A_i$, $d_i \in A_i$, and $e_i \in A_i$ the *cooperative* action, the *defective* action, and the *costly* action for each $i \in N$ respectively. The case of the prisoners' dilemma game implied that $c_i = e_i = c$ and $d_i = d$ for all $i \in \{1,2\}$.

Let us specify $\sigma^{**} \in \Sigma$ as the *generalized penance strategy profile*;

$$\sigma^{**}(h(0)) = (c_1, c_2),$$

for every $t \in \{2,...,T\}$ and every $h(t-1) \in H(t-1)$,

$$\sigma^{**}(h(t-1)) = (c_1, c_2) \quad \text{if} \quad a(t-1) = \sigma^{**}(h(t-2)),$$

$$\sigma^{**}(h(t-1)) = (e_1, d_2) \quad \text{if} \quad a_1(t-1) \neq \sigma_1^{**}(h(t-2)),$$

and

$$\sigma^{**}(h(t-1)) = (d_1, e_1) \quad \text{if} \quad a_1(t-1) = \sigma_1^{**}(h(t-2)) \quad \text{and}$$

$$a_2(t-1) \neq \sigma_2^{**}(h(t-2)).$$

According to σ^{**}, the players select the cooperative action profile (c_1, c_2) in the initial period. In every period and every subgame, if no player deviates from σ^*, they select (c_1, c_2) in the next period. In every period and every subgame, if player 1 deviates from σ^*, he selects the costly action e_1, and player 2 selects the defective action d_2 in the next period. In every period and every subgame, if player 2 unilaterally deviates from σ^*, player 1 selects d_1, and player 2 selects e_2 in the next period.

Let us specify y^{**} as the *generalized penance contract*; for every $i \in \{1,2\}$ and every $h(T) \in H(T)$,

$$y_i^{**}(h(T)) = 0 \qquad \text{if } a(T) = \sigma^{**}(h(T-1)),$$

$$y_i^{**}(h(T)) = 0 \qquad \text{if } a_1(T) \neq \sigma_1^{**}(h(T-1)) \text{ and } a_2(T) \neq \sigma_2^{**}(h(T-1)),$$

$$y_i^{**}(h(T)) = -X \qquad \text{if } a_i(T) \neq \sigma_i^{**}(h(T-1)) \text{ and } a_j(T) = \sigma_j^{**}(h(T-1)),$$

and

$$y_i^{**}(h(T)) = X \qquad \text{if } a_i(T) = \sigma_i^{**}(h(T-1)) \text{ and } a_j(T) \neq \sigma_j^{**}(h(T-1)),$$

where $j \neq i$. According to y^{**}, each player is penalized by paying the fine of X if (and only if) he deviates from σ^{**} in the final period T.

Let us assume that

$$X > \max_{a_i \in A_i} u_i(a_i, c_j) - u_i(c_i, c_j) \text{ and } X > \max_{a_i \in A_i} u_i(a_i, d_j) - u_i(e_i, d_j).$$

In the case of the prisoners' dilemma game, this assumption corresponds to the inequality of (2). As with Theorem 1, we can prove that *the generalized penance strategy profile* σ^{**} *is the unique subgame perfect equilibrium in* (G, T, y^{**}).

Let us assume also that

$$u_1(e_1, d_2) + u_2(e_1, d_2) > u_1(c_1, c_2) + u_2(c_1, c_2) - 2\eta,$$

and

$$u_1(d_1, e_2) + u_2(d_1, e_2) > u_1(c_1, c_2) + u_2(c_1, c_2) - 2\eta.$$

In the case of the prisoners' dilemma game, this assumption corresponds to the inequality of (4). As with Proposition 3, we can prove that *the combination of the generalized penance strategy profile and the generalized penance contract* (σ^{**}, y^{**}) *is renegotiation-proof.*

References

Abreu, D. and H. Matsushima (1992): “Virtual Implementation in Iteratively Undominated Strategies: Complete Information,” *Econometrica* 60, 993–1008.

Abreu, D. and H. Matsushima (1994): “Exact Implementation,” *Journal of Economic Theory* 64, 1–19.

Andreoni, J. and H. Varian (1999): “Preplay Contracting in the Prisoners' Dilemma,” *Proceeding of National Academic Science* 96, 10933–10938.

Benoit, J. and V. Krishna (1985): “Finitely Repeated Games,” *Econometrica* 53, 905–922.

Benoit, J. and V. Krishna (1993): “Renegotiation in Finitely Repeated Games,” *Econometrica* 61, 303–323.

Bernheim, D. and D. Ray (1989): “Collective Dynamic Consistency in Repeated Games,” *Games and Economic Behavior* 1, 295–326.

Dutta, P. and R. Radner (1994): “Moral Hazard,” in *Handbook of Game Theory, Volume 2*, ed. by R. Aumann and S. Hart. Elsevier Science B. V.

Farrell J. and E. Maskin (1989): “Renegotiation in Repeated Games,” *Games and Economic Behavior* 1, 327–360.

Friedman, J. (1985): “Cooperative Equilibria in Finite Horizon Noncooperative Supergames,” *Journal of Economic Theory* 35, 390–398.

Fudenberg, D., B. Holmstrom, and P. Milgrom (1990): “Short-Term Contracts and Long-term Agency Relationships,” *Journal of Economic Theory* 51, 1–31.

Fudenberg, D. and J. Tirole (1995): *Game Theory*, MIT Press.

Lipman, B. and R. Wang (2000): “Switching Cost in Frequently Repeated Games,” *Journal of Economic Theory* 93, 149–190.

Mailath, G. and L. Samuelson (2006): *Repeated Games and Reputation: Long-Run Relationships*, Oxford Univ. Press.

Neyman, A. (1985): “Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoners' Dilemma,” *Economics Letters* 19, 227-229.

Neyman, A. (1999): “Cooperation in Repeated Games When the Number of Stages Is

Not Commonly Known," *Econometrica* 67, 45-64.

Pearce, D. (1987): "Renegotiation-Proof Equilibria: Collective Rationality and Intertemporal Cooperation," Cowles Foundation Discussion Paper N. 855, Yale University.

Pearce, D. (1992): "Repeated Games: Cooperation and Rationality," in *Advances in Economic Theory: Sixth World Congress*, ed. by J.-J. Laffont. Cambridge University Press.

Radner, R. (1980): "Collusive Behavior in Noncooperative Epsilon-Equilibria of Oligopolies with Long but Finite Lives," *Journal of Economic Theory* 22, 136–154.

Rey, P. and B. Salanie (1990): "Long-Term, Short-Term and Renegotiation: On the Value of Commitment in Contracting," *Econometrica* 58, 597–619.

Salanie, B. (1997): *The Economics of Contracts*, MIT Press.

Sobel, J. (1985): "A Theory of Credibility," *Review of Economic Studies* 52, 557-573.

Van Damme, E. (1989): "Renegotiation-Proof Equilibria in Repeated Prisoners' Dilemma," *Journal of Economic Theory* 47, 206–207.