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Abstract

This paper examines the inclusion of the dollar/euro exchange rate together with four important and highly traded commodities - aluminum, copper, gold and oil- in symmetric and asymmetric multivariate GARCH and DCC models. The inclusion of exchange rate increases the significant direct and indirect past shock and volatility effects on future volatility between the commodities in all the models. Model 2, which includes the business cycle industrial metal copper and not aluminum, displays more direct and indirect transmissions than does Model 3, which replaces the business cycle-sensitive copper with the highly energy-intensive aluminum. The asymmetric effects are the greatest in Model 3 because of the high interactions between oil and aluminum. Optimal portfolios should have more euro currency than commodities, and more copper and gold than oil.

JEL: C51, E27, Q43

Keywords: MGARCH; shocks; volatility; transmission; asymmetries; hedging

1. Introduction

Commodity and other asset markets have been highly volatile in recent years. Commodities like oil and copper have had significantly greater volatility than other commodities such as gold. Volatility brings risk and opportunity to traders and investors, and thereby should be examined. There are many reasons for volatility to occur in commodity markets. Market participants form different expectations of profitable opportunities, process information at different speeds, perform cross-market hedging across different asset classes and build and draw inventories at different levels. These factors contribute volatility to commodities over time, as well as volatility spillovers across commodity markets

Shocks or news can also create, transmit and exacerbate volatility in commodity markets. Shocks to the US dollar, for example, may exacerbate commodity fluctuations in the long-run equilibrium, and hence lead to volatility transmission across markets. Oil and gold are also more sensitive to changes in the dollar than are copper and aluminum. On the other hand, copper seems to be the most sensitive to the business cycle (Hammoudeh, Sari and Ewing, 2008). This heterogeneous sensitivity to news should also spawn and spill over different volatilities among commodities.

The tradability and liquidity of futures contracts usually affect commodity fluctuations. The more liquid are the contracts, the smoother will be commodity movements. Oil, gold, aluminum and copper are all exchange traded, but it is not known if they all have the same contract liquidity and similar fluctuations during trading. Even within global oil benchmarks which belong to "one common pool", liquidity, tradability and volatility can vary. For example, the contracts of the light crude benchmarks, WTI and Brent oil, are more liquid at NYMEX and ICE than their own contracts and the contracts of the medium crude benchmark Dubai/Oman at the Dubai Mercantile Exchange (DME). Moreover, WTI is less volatile than non-exchange traded Maya, the Mexican heavy crude benchmark (Hammoudeh, Ewing and Thompson, 2008). If gold contracts, for example, are more liquid than those of copper or aluminum, then gold should have less volatile fluctuations.

The same argument applies to the LME-traded copper, which is particularly sensitive to economic activity. Copper may be more volatile because its market participants do not significantly stockpile this metal, and do not speculate heavily relative to other metals because it is cheap, heavy and plentiful. On the other hand, the price of copper generally represents an accurate barometer of its demand in the real world, rather than an irrational bet on its future value.

Changes in, and the availability of, commodity inventories may also affect volatility, depending on whether the change will add to or subtract from inventories, and on the size of the build-up compared with their long-run averages. Moreover, owners of oil storage tankers can use their knowledge of the fullness or emptiness of the tanks to spread news to induce traders to act quickly on false information, and may affect the speed and direction of adjustments. Oil companies, for example, can use their information of future production to trade during positive and negative shocks. Varying inventories and the backwardation/contagion state of commodity markets may also affect volatility.

In this paper, we concentrate on representatives of four types of fuel and industrial commodity classes, namely aluminum, copper, gold and oil. Aluminum represents an energy-intensive commodity class, copper represents base metals, gold represents precious metals, and oil represents energy commodities. We also include a major macroeconomic variable, the dollar/euro exchange rate, as a link and policy variable. The dollar/euro exchange rate is widely used and recognized by both academics and practitioners as a mover of commodity markets. It is much more relevant representative of all exchange rates as far as commodities are concerned.[1] There are three recognized channels that link the dollar/euro exchange rate to the US dollar-priced commodities. They are the purchasing power and cost of the dollar-priced commodities in non-US dollar currencies, asset plays which makes commodities as an investment class more attractive than the dollar-denominated financial assets, and monetary easing outside the US in response to a sinking dollar which results in demand stimulus. At certain times, commodities dominate asset trading, have stronger linkages with the macro economy, and/or influence, or are influenced by, policy decisions.

As we are interested in volatility spillovers across commodities and the macroeconomy, we use multivariate symmetric and asymmetric GARCH models to estimate simultaneously the means and variances of the four commodity price and exchange rate returns to analyze volatility and its transmission mechanism. Asymmetry is relevant for commodities because positive and negative shocks of equal magnitude may have different impacts on commodity returns. Furthermore, we use the symmetric and asymmetric BEKK specifications which do not impose the restriction of constant conditional correlations across the commodity shocks. This procedure allows an

[1] In an MBA class experiment that included the major industrial commodities and seven measures of dollar exchange rates and indices, students found the dollar/euro exchange rate followed by the broad index to have the highest correlations with commodity prices.

examination of covariance spillovers across commodities, as well as a computation of hedge ratios. The BEKK model is more appealing than the more heavily restricted DCC model, which purports to estimate conditional correlations, but is unable to address spillovers. In this context, Caporin and McAleer (2008) evaluated the empirical performance of the scalar versions of BEKK and DCC, and found they were very similar.

Caporin and McAleer (2009) defined targeting as an aid in estimating matrices associated with large numbers of financial assets, analyzed the similarities and dissimilarities between alternative versions of BEKK and DCC, both with and without targeting, on the basis of structural derivation, the analytical forms of the sufficient conditions for the existence of moments, and the sufficient conditions for consistency and asymptotic normality, and computational tractability for very large numbers of financial assets, presented a consistent two step estimation method for the DCC model, and suggested that BEKK should be preferred in practical applications. However, we still use the symmetric and asymmetric DCC models as a diagnostic check of the results of the symmetric and asymmetric BEKK models. The DCC method also enables us to examine the conditional volatility and correlation cross-effects with meaningful estimated parameters and fewer computational complications that characterize alternative multivariate GARCH models.

This paper fills the empirical void in the literature on commodity volatility in four important areas. First, it uses multivariate conditional volatility models to determine volatility progression and transmission among the four commodities across different classes. Second, it uses symmetric and asymmetric models to gauge the sensitivity of the different commodities to positive and negative shocks. Third, it examines the bidirectional impacts between the exchange rate and commodities, taking into account flight to safety, asset reallocation and responsiveness to policy decisions. Fourth, it uses the volatility results to calculate dynamic hedge ratios and risk-minimizing optimal portfolio weights for two commodities, or for one commodity and the exchange rate.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 presents the empirical model and Section 4 discusses the data and descriptive statistics. Section 5 discusses the empirical results. Section 6 gives some concluding comments.

2. Review of the Literature

The literature on commodities has concentrated on their price co-movements and their roles in transmitting information on returns. The research on commodity volatility has been considerably less than on their counterparts in commodity prices and returns. This research has typically focused on volatility behavior for a single commodity over time, and not on volatility transmissions across commodities and over time, due to methodology complexities. The single commodity volatility research has used univariate models of conditional volatility (or GARCH) to examine the behavior of volatility over time, with a focus on own shocks and volatility dependencies over time, while ignoring volatility interdependencies across commodity markets and/or classes.

Bracker and Smith (1999) and Smith and Bracker (2003) apply the GARCH and EGARCH models to copper futures prices, and find these specifications to better explain volatility behavior for copper than do other models. McKenzie et al. (2001) explored the applicability of the univariate power ARCH (PARCH) model to precious metals futures contracts traded at the London Metal Exchange (LME), and found that asymmetric effects are not present, and the model did not provide an adequate explanation of the data. Tully and Lucey (2007) used the univariate asymmetric power GARCH (APGARCH) model to examine the asymmetric volatility of gold. They concluded that the exchange rate is the main macroeconomic variable that influences the volatility of gold, with few other macroeconomic variables having an impact.

Batten and Lucey (2007) studied the volatility of gold futures contracts traded on the Chicago Board of Trade (CBOT) using intraday and interday data. They used the univariate GARCH model to examine the volatility properties of the futures returns and the alternative nonparametric volatility static model of Garman and Klass (1980) to provide further insights into intraday and interday volatility dynamics of gold. The results of both measures provided significant variations within and between consecutive time intervals. They also found a low correlation between volatility and volume. Bhar et al. (2008) used the univariate GARCH model to examine the behavior of the short-run stationary components of four oil benchmarks

In terms of nonlinearity and chaotic structure, Yang and Brorsen (1993) concluded that palladium, platinum, copper and gold futures have chaotic structures. In contrast, Adrangi and Chatrath (2002) found that the nonlinearity in palladium and platinum is inconsistent with chaotic behavior. They concluded that ARCH-type models, with controls for seasonality and contractibility, explained the nonlinear dependence in their data for palladium and platinum.

In comparison with other studies on commodities, Plourde and Watkins (1998) compared the volatility in the prices of nine non-oil commodities to the volatility in oil prices. On the basis of several non-parametric and parametric tests, they found that oil prices tend to be more volatile than the prices of gold, silver, tin and wheat, and argued that the differences are more evident in the case of precious metals. Hammoudeh and Yuan (2008) used three different univariate GARCH models to investigate the volatility and leverage properties of two precious metals (gold and silver) and one base metal (copper). They found that in the standard univariate GARCH model, gold and silver have almost the same conditional volatility persistence, which is higher than that of the pro-cyclical copper. In the EGARCH model, they found that only copper has an asymmetric effect, and the transitory component of volatility converges to equilibrium faster for copper than for gold and silver in the CGARCH model. Using a rolling AR(1)-GARCH model, Watkins and McAleer (2008) showed that the conditional volatility for two nonferrous metals, namely aluminum and copper, is time-varying over a long horizon.

Finally, there are few studies that have used multivariate GARCH to examine volatility transmissions across commodities. Hammoudeh et al. (2004) use a trivariate BEKK model to examine the volatility between oil prices and oil industry equity indices. Ewing et al. (2002) employ a bivariate BEKK model for the oil and natural gas sectors to examine how volatility changes over time and across the two sectors. Moschini and Myers (2002) develop a different bivariate GARCH parameterization for cash and futures markets, with a flexible functional form for time-varying volatility that is suitable for testing whether the optimal hedge ratio is constant, and whether the time variations in the optimal hedge ratios are due solely to deterministic seasonality and time-to-maturity effects. Statistical tests reject both null hypotheses.

Thus, these studies, except for the last three, do not examine cross-volatility and shock effects between commodities using multivariate GARCH models. Even these three studies did not use a four variable GARCH model. This could be a major shortcoming when one considers that real world applications such as hedging, portfolio diversification and inter-commodity volatility predictions are conducted in multivariate settings. In this regard, we are interested in ascertaining to what extent commodity volatility interdependencies across markets and over time exist, and what role hedging and optimal portfolio formation play in mitigating their risks. Policy makers, traders and portfolio managers, as well as manufacturers, would be interested in this information because precious and industrial metals are investment assets, feed into inflation, and have important and diversified industrial uses in the jewelry, electronic and autocatalytic industries.

3. Empirical Models

In this section we present four different multivariate volatility models to achieve the four goals of the paper. The first two models are the symmetric and asymmetric BEKK models, while the second two models are the symmetric and asymmetric DCC models.

The commodities and the exchange rate in our empirical systems are indexed by i, and n is the total number of commodities and the exchange rate when the latter is included in the various models. Each system, whether all commodities or a combination of commodities and the dollar/euro exchange rate, has four variables, so that $n = 4$. The mean equation for commodity ***i*** (or the exchange rate) in this system is given as an AR(1) process, as follows:

$$R_{i,t} = a_i + b_i R_{i,t-1} + c_i D03 + \varepsilon_{i,t} \tag{1}$$

$$\varepsilon_{i,t} = H_t^{1/2}\eta_t\,, \quad \eta_t \sim \text{iid N(0,I)}$$

where $R_{i,t}$ is the return on the ith commodity (or exchange rate) of the *nx1* vector R_t , which is defined as a log difference. The innovation η_t is an *nx1* vector of *i.i.d.* random shocks, and H_t is the conditional covariance matrix of commodities (and exchange rate) at time *t.* D03 denotes the dummy variable for the 2003 Iraq War.

Commodities are affected by common macroeconomic variables and they also feed on themselves in terms of volatility. Therefore, we follow Engle and Kroner (1995) to form the evolution of the conditional covariance matrix as the multivariate BEKK model, which permits an examination of the cross-commodity effects. This specification is also more practicable than the VECH specification given in Bollerslev, Engle and Wooldridge (1988), which is highly over-parameterized. Commodity prices face both positive and negative shocks which may have different impacts on their volatilities. We will use both the symmetric and asymmetric versions of the BEKK model, which has the practical advantage that it restricts the estimated covariance matrix to be positive definite.

The symmetric BEKK model is given as:

$$H_{t+1} = C'C + A'\varepsilon_t \varepsilon_t' A + B' H_t B\,, \tag{2}$$

for which the coefficient matrices are given as:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad B=\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \quad C=\begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$

where C is a 4×4 lower triangular matrix with 10 parameters. The 4×4 matrices A and B represent the effects of past shocks and past conditional variances and covariances on their current counterparts of the various commodities/foreign exchange rate, respectively. The total number of estimated elements for the covariance equation (2) in the four-variable system is 42.

The interpretations of the basic estimated elements are not obvious. Ignoring the constant term, the conditional variance equations can be re-expressed as:

$$h_{ii,t+1}=\sum_{j=1}^{4}a_{ji}^{2}\varepsilon_{j,t}^{2}+\sum_{j=1}^{3}\sum_{k=j+1}^{4}2a_{ji}a_{ki}\varepsilon_{j,t}\varepsilon_{k,t}+\sum_{j=1}^{4}b_{ji}^{2}h_{jj,t}+\sum_{j=1}^{3}\sum_{k=j+1}^{4}2b_{ji}b_{ki}h_{jk,t} \qquad i=1,2,3,4 \tag{3}$$

Equation (3) shows how shocks and volatilities are transmitted across commodity/foreign exchange markets and over time.

The symmetric BEKK model assumes that negative and positive shocks of equal magnitude have identical effects on the conditional variance. An extension of the BEKK model that accommodates asymmetric effects of positive and negative shocks is the BEKK–AGARCH model. An extension of equation (2) that accommodates the shock asymmetries would include an asymmetric term in this equation. Define

$$v_t=\varepsilon_t\circ I_{\varepsilon_t<0}(\varepsilon_t)$$

where $\circ$ denotes the element-by-element Hadamard product of the vectors. Thus, v_t is a vector in which $\varepsilon_t = \varepsilon_t$ if ε_t <0 and ε_t =0 if $\varepsilon_t \geq 0$. Equation (2) then becomes

$$H_{t+1} = C'C + A'\varepsilon_t\varepsilon_t'A + B'H_tB + D'v_tv'_tD \qquad (4)$$

where the matrix D captures the asymmetric effects of negative shocks on volatilities.

We maximize the following likelihood function, assuming the errors are normally distributed:

$$L(\theta) = -T\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\ln\left|H_t\right| + \varepsilon_t'H_t^{-1}\varepsilon_t),$$

where T is the number of observations and θ is the estimated parameter vector. Numerical maximization techniques are used to maximize the non-linear log-likelihood function. Initial conditions are obtained by performing several initial iterations using the simplex algorithm, as recommended in Engle and Kroner (1995). The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm was then used to obtain the final estimate of the covariance matrix, with corresponding standard errors for the commodity/exchange rate models.

The assumption that the random shocks $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t,} \cdots, \varepsilon_{n,t}]'$ have a constant correlation matrix may not be well supported in commodity markets because of high uncertainty, structural changes and geopolitical events. Moreover, some researchers prefer to use an MGARCH model of multiple equations, in which each equation follows a univariate process and does not include any spillovers across variables. The results of this model can be used as diagnostic tests of BEKK-type models. Therefore, we use

Engle's (2002) DCC-MGARCH model to examine the time-varying conditional correlations among the commodities (or exchange rate). Furthermore, in contrast to the specification of the interdependent conditional variance in equation (2) of BEKK-MGARCH and equation (4) of BEKK-MAGARCH, the symmetric DCC-MGARCH model assumes that the conditional variance of each precious metal (or exchange rate) follows a univariate GARCH process:

$$h_{i,t} = c_i + \sum_{k=1}^{p} \alpha_{i,k} \varepsilon_{i,t-k}^2 + \sum_{s=1}^{q} \beta_{i,s} h_{j,t-s} \qquad (5)$$

where $\Sigma\alpha_{i,k}\varepsilon^2_{i,t-k}$ is the short-run persistence of precious metal (or exchange rate) ***i***'s own past shocks, and $\Sigma\beta_{i,s}h_{i,t-s}$ is the long-run persistence of the GARCH effects of past volatilities. It is worth noting that in equation (3) the conditional variances of precious metals (and exchange rate) are assumed to be independent of one another.

The estimation of the dynamic conditional covariance matrix of DCC-MGARCH entails two steps. First, the matrix Q_t used to calculate the dynamic conditional correlation is assumed to be time-varying and to be governed by two parameters, namely θ_1 and θ_2:

$$Q_t = (1-\theta_1-\theta_2)Q_0 + \theta_1\varepsilon_{t-1}\varepsilon'_{t-1} + \theta_2 Q_{t-1} \qquad (6)$$

where Q_0 is the conditional correlation matrix of ε_t, which is a consistent estimator of the conditional correlation matrix of the commodities, Q_t is a weighted average of positive definite and positive semidefinite matrices, which is used to provide the dynamic correlation matrix, and θ_1 and θ_2 are parameters. θ_1 represents the impact of past shocks on the current conditional correlation, while θ_2 captures the impact of the past conditional correlations. If the estimates of both θ_1 and θ_2 are statistically significant, then the conditional correlations are not constant. The dynamic conditional correlation coefficients ($\rho_{ij}(t)$) between commodities (or exchange rate) ***i*** and ***j*** are calculated by:

$$\rho_{ij}(t) = \frac{Q_{ij}(t)}{\sqrt{Q_{ii}(t)Q_{jj}(t)}} \tag{7}$$

Second, the sequence of dynamic conditional covariance matrices is then computed by $\rho_{ij}(t)$, and the estimated univariate conditional variances:

$$H_{ij}(t) = \rho_{ij}(t)\sqrt{H_{ii}(t)H_{jj}(t)} \tag{8}$$

$$H(t) = \begin{bmatrix} h_{11,t} & h_{12,t} & \cdots & h_{n1,t} \\ h_{21,t} & \ddots & \ddots & h_{2n,t} \\ \vdots & \ddots & \ddots & \vdots \\ h_{n1,t} & h_{n2,t} & \cdots & h_{nn,t} \end{bmatrix}$$

where $h_{ii,t}=h_{i,t}$ is for convenience of notation, which is estimated based on the univariate GARCH process, as shown in equation (3). The elements $h_{ii,t}$ and $h_{ij,t}$ are the estimated conditional variance and conditional covariance, respectively, at time t and $h_{ij,t} = h_{ji,t}$.

As the GARCH effects are assumed to follow a univariate process in the DCC-MGARCH model, the asymmetric effects are, therefore, directly incorporated in equation (5), as follows:

$$h_{i,t} = c_i + \sum_{k=1}^{p} \alpha_{i,k} \varepsilon_{i,t-k}^2 + \sum_{s=1}^{q} \beta_{i,s} h_{j,t-s} + \sum_{k=1}^{p} \gamma_{i,k} \varepsilon_{i,t-k}^2 \bullet I(\varepsilon_{i,t-k} < 0) \tag{8}$$

where the parameter γ captures the asymmetric effects in the DCC-MGARCH model.

4. Data Description

We use daily time series data (five working days per week) for the four commodity (aluminum, copper, gold and oil) closing spot prices and the US dollar/euro exchange rate for the period 4 January 1999 to 5 November 2007. The exchange rate is the value of the US dollar to one euro, suggesting that a rise in the rate implies devaluation of the dollar, and vice-versa. Aluminum, gold and oil are traded at COMEX in New York. Copper is traded at LME. Oil is represented by the benchmark West Texas Intermediate (WTI). The daily US dollar/euro exchange rate series is obtained from the database of the Federal Reserve Bank of Saint Louis. All commodity and exchange rate series are modeled in natural logarithms, and are depicted in **Figure 1**.

The ADF and PP unit root tests for both the drift and without drift specifications demonstrate that the commodity and exchange rate variables have unit roots with and without drift.[2] Therefore, we will examine and model the returns instead of the levels for the five variables. **Table 1** provides the descriptive statistics for the variables. Among the four commodities, oil followed by copper yielded the highest average return, while gold had the lowest return over the sample period. Oil also has the highest volatility, as defined by standard deviation, while gold has the lowest. It is not surprising that oil has the highest volatility because it is periodically managed by OPEC, and is also sensitive to

[2] The results are available from the authors upon request.

weather, frequent inventory changes and political tensions and military conflicts in the oil-producing countries.

Some studies have interpreted volatility as a proxy for information flow, in the sense that increases in information should translate into greater volatility (Lin and Chiang, 2005). Moreover, gold has been subdued due to low inflation during much of the sample period. All the series are leptukortic, that is, have fat tails, which requires testing the individual mean equations for ARCH effects. The results show that there are strong ARCH effects for the four commodities and the exchange rate, thereby warranting estimation of the GARCH model.

5. Empirical Results

We will estimate three sets of four empirical multivariate volatility models for three combinations of the four commodities and the exchange rate because of the well known convergence limitations of the BEKK models.[3] Each set of models includes the symmetric MGARCH, asymmetric MAGARCH, and DCC models. Model 1 will be considered as the basic model, and will include the four commodities, namely aluminum, copper, gold and oil. Model 2 consists of copper, gold, oil and the dollar/euro exchange rate. Finally, Model 3 is comprised of aluminum, gold, oil and the exchange rate. We included copper with the exchange rate in Model 2, and aluminum and exchange rate in Model 3, because copper is a base metal and aluminum is an industrial metal. Moreover,

[3] The BEKK model did not converge with five variables. We then estimated the DCC model for all five variables combined. The results show that the conditional correlation coefficients for the shocks are less than 1%, which implies that the DCC matrix converges to a constant matrix in the long run. We also estimated the VARMA-GARCH model of Ling and McAleer (2003), but did not obtain convergence.

aluminum is more energy-intensive compared with copper. Both commodities are included in Model 1.

Basic Models 1:

We will examine the statistically significant estimates in the four basic models for this group and the extent of past volatility and volatility interdependence effects. We first examine the results for the symmetric and asymmetric MGARCH models, and then for the symmetric and asymmetric DCC models for this group. The number of symmetric past volatility and volatility interdependence effects that are significant in each of two MGARCH models is seven, and those symmetric results for both models are similar. On the other hand, the number of asymmetric effects in MAGARCH that reflect negative shocks is only two, and both affect aluminum in this commodity setting, making the asymmetric MGARCH model the better of the two. We will first examine the symmetric part, which is similar in the two models, and followed by the asymmetric part. Finally, we use the results of the DCC model as a diagnostic check.

We start by examining the conditional variance (volatility), h_{11}, for aluminum in **Tables 2a and 2b**. This highly energy-intensive and industrial metal is significantly and positively affected by news (unexpected shocks), $\varepsilon^2{}_1$, from its own market without being affected by any news spillovers from the other three markets. In terms of sensitivity to own past volatility, h_{11}, aluminum is also significantly and positively affected by its past volatility. The aluminum ambivalence to news and volatility in both oil and copper is surprising, and may underline the different nature of this metal as both an industrial and energy-intensive metal, thereby placing it in a separate metal class from the others, even from the basic industrial metal copper.

Copper volatility, h_{22}, is significantly and positively affected by news or shocks generated in its own market, ε^2_2. In contrast to aluminum, the copper volatility is significantly and positively impacted by news in the gold market, $\varepsilon_2.\varepsilon_3$. Considering the effects of past volatility, copper is impacted only by its own shocks, as is the case of the aluminum market.

The volatility of gold, h_{33}, is much more heavily impacted by news from other markets than are the other three commodities. Specifically, it is significantly and positively affected by news from its own market, ε^2_3. The interaction term, ($\varepsilon_2.\varepsilon_3$), for shocks emanating from the copper and gold markets, significantly reduces the conditional volatility of the gold market. For example, news about power deficiency in major copper-producing countries, associated with news about explosions in a major gold mine, indirectly affects volatility in the gold market. This indirect impact is due to cross-market hedging, or sharing common information between the two markets. The volatility in the gold market is influenced by news because it is a safe haven in times of high risk and rising inflation. During bad times, investors dump copper and aluminum, and buy gold as part of a risk hedging asset reshuffling strategy.

When it comes to sensitivity to past volatilities, gold volatility is indirectly affected by the interaction of volatilities in the aluminum and copper markets, aluminum and own market, and copper and own market. It is also affected directly by its own market. It seems that gold volatility is impacted by other commodity volatility because traders and investors revert to it as a safe haven during times of high volatility in other markets. It is interesting that gold volatility is not impacted by volatility in the oil market, which is also involved in the flight to safety when the dollar exchange rate is impacted. Oil is, however, periodically managed by OPEC, and has its own trajectory. It is also possible that oil is overplayed by speculators.

The oil market volatility in this model seems to be independent of the volatility in the other three metals markets, where volatility is significantly and directly affected only by its own past shocks and volatility, as is the case with aluminum volatility. Oil has the highest unconditional volatility, as shown in **Table 1**, due to its manipulation by OPEC and sensitivity to its own fundamentals, speculators and the geopolitics of its supply.

The two significant asymmetric effects reflect the differential impacts of negative shocks relative to positive on current volatilities. There are small asymmetric effects of the interactions between aluminum and copper, and between aluminum and gold, on aluminum.

In summary, gold receives more symmetric shocks and volatility spillovers than any other commodity, with copper second. Moreover, gold and copper receive no asymmetric commodity shocks. On the other hand, aluminum and oil are explained by their own symmetric markets. However, negative shocks coming from the interactions with copper and gold have greater effects on aluminum than do positive shocks. Therefore, aluminum is more sensitive to negative than to positive shocks. Finally, there are limited volatility independencies among those commodities in both BEKK models.

The results of the two symmetric and asymmetric DCC models are intended as a diagnostic check on the symmetric and asymmetric MGARCH models as a constant conditional correlation matrix may not be well supported in commodity markets. There are 11 significant shock and volatility effects in the symmetric DCC model, while there are 10 symmetric and three asymmetric significant results in the two DCC models, making the latter the better model. Nevertheless, the shock, volatility and persistence results are very similar for both DCC models. The estimates mirror, to a large extent, those of the MGARCH models, underpinning the robustness of the MGARCH results. The ARCH (α) and GARCH (β), own past (unexpected) shocks and volatility effects for both DCC models, respectively, are significant. The degree of volatility persistence is the highest for copper, followed by aluminum, and the lowest is for gold, followed by oil. The asymmetry appears for aluminum, copper and gold, but not for oil, which is greater than in the asymmetric MGARCH model.

Figure 2 shows the variation in the estimated dynamic conditional correlations for the four commodities over time. It is clear that all six pairs, aluminum-copper, aluminum-gold, aluminum-oil, copper-gold, copper-oil and gold-oil, display marked variations over time. Five of the six pairs have conditional correlations that are both positive and negative, which could assist in formulating hedging strategies, and four of the six pairs have a large range of variation, with three having a range that exceeds one.

Models 2

These models contain copper, gold, oil and the dollar/euro exchange rate, with the highly energy-intensive industrial metal, aluminum, included in Model 3. As mentioned above, the exchange rate is included to account for a feedback mechanism between dollar-denominated metals and oil, and the exchange rate. The exchange rate is used as an accounting unit and a medium of exchange for trading commodities, and it also reflects expectations about future prices and speculation on commodities.

A comparison of the results of the two BEKK models reveals that the symmetric MGARCH model has 60 significant effects, while the asymmetric MAGARCH model has 51 such effects. The inclusion of the exchange rate weakens the symmetric impacts on oil in the MAGARCH model.

The symmetric MGARCH model shows that the inclusion of the exchange rate in this model increases substantially the direct and indirect effects of past shocks and volatilities on future volatility of the three commodities, compared with their effects in Model 1 (the basic model), as displayed in **Table 3.**

There are direct effects (ε_i^2) of news from and to own markets for all four commodities. Moreover, the direct news effects from the other markets on the own market are as follows: gold on copper, and vice-versa; oil on exchange rate, and vice-versa; exchange rate on gold, and vice versa; and gold on oil. It is interesting to find that news (shock) impacts are bidirectional between gold and the exchange rate, in lieu of the fact that gold, dollar and euro are used for foreign reserves. Furthermore, gold news unidirectionally affects oil volatility, despite the fact that gold and oil are dollar-denominated assets, and are considered safe havens and hedges against inflation and a depreciating dollar.

There are also indirect effects ($\varepsilon_i.\varepsilon_j$) from news interactions between markets on own markets. The most notable of these indirect effects is for the exchange rate and oil. There are not, however, as many indirect effects for copper and gold.

When we focus on the direct and indirect effects of past volatilities on future volatilities, we can see more significant relationships than in the shock effects, indicating that commodity volatility is predictable, even in a simultaneous setting. The results show that there are significant volatility effects (h_{ii}) on the own market volatility for all four markets in this model.

Direct volatility effects from other markets to the own market are: copper on three markets; exchange rate on three markets; gold on three markets; and oil on the exchange rate and gold markets, which is different from the case of shocks. These volatilities are affected simultaneously by fundamental forces, such as macroeconomic factors and cross-market hedging.

Finally, there are many indirect volatility transmissions, representing interactions of volatilities between markets. There are transmissions of volatility interactions in exchange rate and gold on all four markets; between exchange rate and oil on the foreign exchange, gold and oil markets; and between gold and oil on the foreign exchange, gold and oil markets. It seems that transmissions of indirect volatility interactions are the strongest among the exchange rate, gold and oil, and weakest for copper, which is more sensitive to the business cycle.

Some of the simultaneous direct results indicated above are consistent with those of the univariate GARCH model, which had an impact of the exchange rate on commodity volatility, particularly that of the exchange rate on gold (Tully and Lucey, 2006). Other effects are different from the univariate transmissions for oil, gold and copper (Hammoudeh and Yuan, 2006). These arise because of the inherent shortcoming of the univariate GARCH model, in that they block simultaneous feedbacks and spillovers.

The asymmetric items are five significant effects in the asymmetric MAGARCH Model 2 compared with two in the asymmetric Model 1, indicating greater responses to negative rather than positive shocks. While each commodity is more sensitive to one bad shock, gold is affected by two bad shocks as a result of interactions between copper and the exchange rate, and between copper and gold. Oil responds asymmetrically to interaction between copper and oil, while the exchange is sensitive to interaction between gold and oil.

The results of the two symmetric and asymmetric DCC models are also very similar, and are close to those of their MGARCH counterparts. The number of significant shock and volatility effects are eight in the symmetric DCC model, while there are seven in the symmetric part and two in the asymmetric part, making the asymmetric DCC model the better model. The degree of volatility persistence in these models is slightly higher than their counterparts in the other models. This has to do with the presence of the exchange rate, which has the highest volatility followed by aluminum. The lowest persistence is for gold by oil, as is the case in the previous models.

Models 3

The composition of the symmetric and asymmetric MGARCH and DCC Model 3, which replaces copper with aluminum but retains the other variables, differs from that of Model 2. The models examine the simultaneous interactions and transmissions when the business cycle-sensitive copper is replaced by a highly energy-intensive aluminum, which does not have the same economic interactions with the overall economy, as in the case of copper.

The results of the two MGARCH models reveal that there are 21 significant effects in the symmetric model, while there are 53 significant symmetries and 18 significant asymmetries in the asymmetric counterpart, making the MAGARCH the better model for this group. This should not be surprising because Model 1 reveals that aluminum is sensitive to asymmetric shocks from copper, gold and itself. Despite this, the simultaneous symmetric relationships are not as significant as in Model 2. Copper is known to have many more linkages with various economic sectors, and it is more directly sensitive to business cycles than is aluminum. Some economists call it Dr. Copper because of its ability to predict business cycles (Lahart, 2006). Copper also seems to share a greater sensitivity with gold and oil for common macroeconomic factors than with other commodities, including aluminum.

The empirical findings reveal that the direct shocks and volatility transmissions between the markets are still strong in this model compared with the all commodity model, but the indirect transmissions are much weaker than in Model 2. There are direct effects of news from and to own markets for the four markets in this model, as for Model 2. On the other hand, the direct news effects from other markets on own markets are evident only from the exchange rate to gold. Even in this direct news spillover case, there is no reciprocal news impact from gold to exchange rate as is the case in Model 2. The indirect effects from news interactions ($\varepsilon_i.\varepsilon_j$) between markets on own markets are also limited compared with the previous model. There are transmissions of (indirect) news interactions in the exchange rate and gold on the gold market, and between the exchange rate and oil on the oil market, as in Model 2.

The direct volatility transmissions from and to own markets are the same for all four markets, as in Model 2, but the direct volatility transmissions from other markets to own are concentrated primarily on the exchange rate and gold, and to a lesser extent on oil. This is largely due to cross hedging among these asset classes, but these transmissions are irrelevant for the aluminum market. The same analysis applies to indirect volatility transmissions.

There are 18 significant asymmetric effects in the asymmetric MAGARCH model, indicating greater responses to bad shocks in the asymmetric MGARCH Model 3 than in the asymmetric Model 2. While gold is the most sensitive to bad shocks in the previous asymmetric model, oil is the most responsive to asymmetric shocks, coming mostly from aluminum. This is also not surprising as aluminum is more energy-intensive than is copper.

The results of the two symmetric and asymmetric DCC models are also very similar, and are close to those of their MGARCH counterparts. However, the symmetric DCC model has 6 significant effects, while the asymmetric DCC has 9 significant symmetric and asymmetric effects. The patterns of volatility and volatility persistence are similar to what was observed in the DCC Model 2.

5. Implications for Portfolio Designs and Hedging Strategies

We now provide two examples using the estimates of the symmetric and asymmetric GARCH models (Model 2) for the copper, foreign exchange, gold and oil markets, and for the aluminum market and the others in the symmetric and asymmetric Model 3, to analyze portfolio design and hedging strategies. The results are virtually identical for both the symmetric and asymmetric models in group 2 and group 3.

5.1. Portfolio weights

The first example follows Kroner and Ng (1998) by considering a portfolio that minimizes risk without lowering expected returns. If we assume the expected returns to be zero, the optimal portfolio weight of one commodity (or asset) relative to the other in a two commodity (asset) portfolio is given by:

$$w_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}}$$

and

$$w_{12,t} = \begin{cases} 0, & if \quad w_{12,t} < 0 \\ w_{12,t}, & if \quad 0 \le w_{12,t} \le 1 \\ 1, & if \quad w_{12,t} > 1 \end{cases}$$

where $w_{12,t}$ is the portfolio weight for, say, commodity (asset) 1 relative to commodity (asset) 2 in one dollar portfolio of the two commodities (assets) 1 and 2 at time t, $h_{12,t}$ is the conditional covariance between commodity returns, or assets 1 and 2, and $h_{22,t}$ is the conditional variance of the commodity, or asset 2. The portfolio weight of the second commodity, or asset, in the one dollar portfolio is $1-w_{12,t}$.

The average values of $w_{12,t}$ for the commodities or assets in Model 2 are reported in **Table 5.** For instance, the average value of $w_{12,t}$ of a portfolio comprising copper and exchange rate is 0.14.[4] This suggests that the optimal holding of copper in one dollar of copper/euro portfolio in Model 2 is 14 cents, compared with 86 cents for the euro. Similar results are obtained for gold/euro and oil/ euro in Model 2, and for aluminum/euro in Model 3. These optimal portfolio weights suggest that investors should own more euro than commodities in their portfolios. For purely commodity portfolios, investors should hold more copper and gold than oil, and hold more gold than copper and aluminum in their portfolios.

5.2. Hedge ratios

As a second illustration, we follow the example given in Kroner and Sultan (1993) regarding risk-minimizing hedge ratios, and apply it to these markets. In order to minimize risk, a long position of one dollar taken in one commodity/asset market should be hedged by a short position of $\$\beta_t$ in another market at time t. The β_t is given by:

$$\beta_t = \frac{h_{12,t}}{h_{22,t}} \ ,$$

where β_t is the risk-minimizing hedge ratio for two commodities/assets, $h_{12,t}$ is the conditional covariance between markets 1 and 2, and $h_{22,t}$ is the conditional variance of the second market.

[4] Hassan and Malik (2007) used the BEKK model and estimated the average weight between the financial and technology sectors at 0.66, while the average risk-minimizing hedge ratio between these sectors was 0.64.

The second column of Table 5 reports the average values of β_t for the markets. By following this hedging strategy, one dollar long in the copper market, for example, should be shorted by 31 cents in the foreign exchange market, 34 cents in the gold market, and by 9 cents in the oil market. Similarly, one dollar long in the gold market should be shorted by 4 cents in the oil market. It seems that the most effective hedging is by shorting oil.

6. Conclusions

A significant amount of research has modeled simultaneous transmissions of *returns* among commodity markets using VARs. A growing number of studies have also examined the behavior of shocks and volatility of oil and industrial commodities using univariate versions of the GARCH family of volatility models. These studies did not examine the transmission of shocks and volatility shocks, shock asymmetries and hedging strategies for commodities in a simultaneous setting. Commodity markets employ cross-market hedging, share common information that affects future volatilities simultaneously, and have asymmetric sensitivity to positive and negative shocks. These markets lag behind stock markets in this regard. With the increasing globalization of the world's economies and commodity markets, analyzing commodity volatility spillovers, asymmetry to different shocks and hedging strategies is both important and useful. We have tried to fill these gaps for commodities in this paper.

While univariate volatility models examine the impacts arising from markets such as foreign exchange on another market, such as gold, the simultaneous commodity/foreign exchange multivariate volatility models found many direct and indirect shock and volatility transmissions, while confirming the direct impacts estimated in the univariate GARCH model, particularly between gold and the exchange rate.

Including the exchange rate in the commodity model increases the direct and indirect shocks and volatility transmissions, particularly between the exchange rate, gold and oil. Replacing the business cycle sensitive copper with the energy intensive aluminum diminished the transmission, but affected the spillovers between the exchange rate and gold, and oil to a lesser extent. Traders, investors and the policy market should be aware of the strong transmissions of shocks and volatilities between the exchange rate, gold and oil.

The industrial metals, copper and aluminum, have more asymmetric effects than do gold and oil. This makes them more volatile in a deep recession like the 2007-2009 Global Financial Crisis. The industrial metals also have greater volatility persistence than do oil and gold. Therefore, hedging is more warranted for industrial commodities than for precious metals like gold, and also oil. A dollar-based flexible exchange rate has more volatility persistence than industrial metals, gold and oil. The presence of this flexible exchange also increases the volatility persistence of both oil and gold.

In a two-asset portfolio, optimal portfolios hold a greater weight of the euro than of commodities, and more gold than aluminum, copper and oil. It would seem that the most effective way of hedging long positions with a shorting position is to short with oil.

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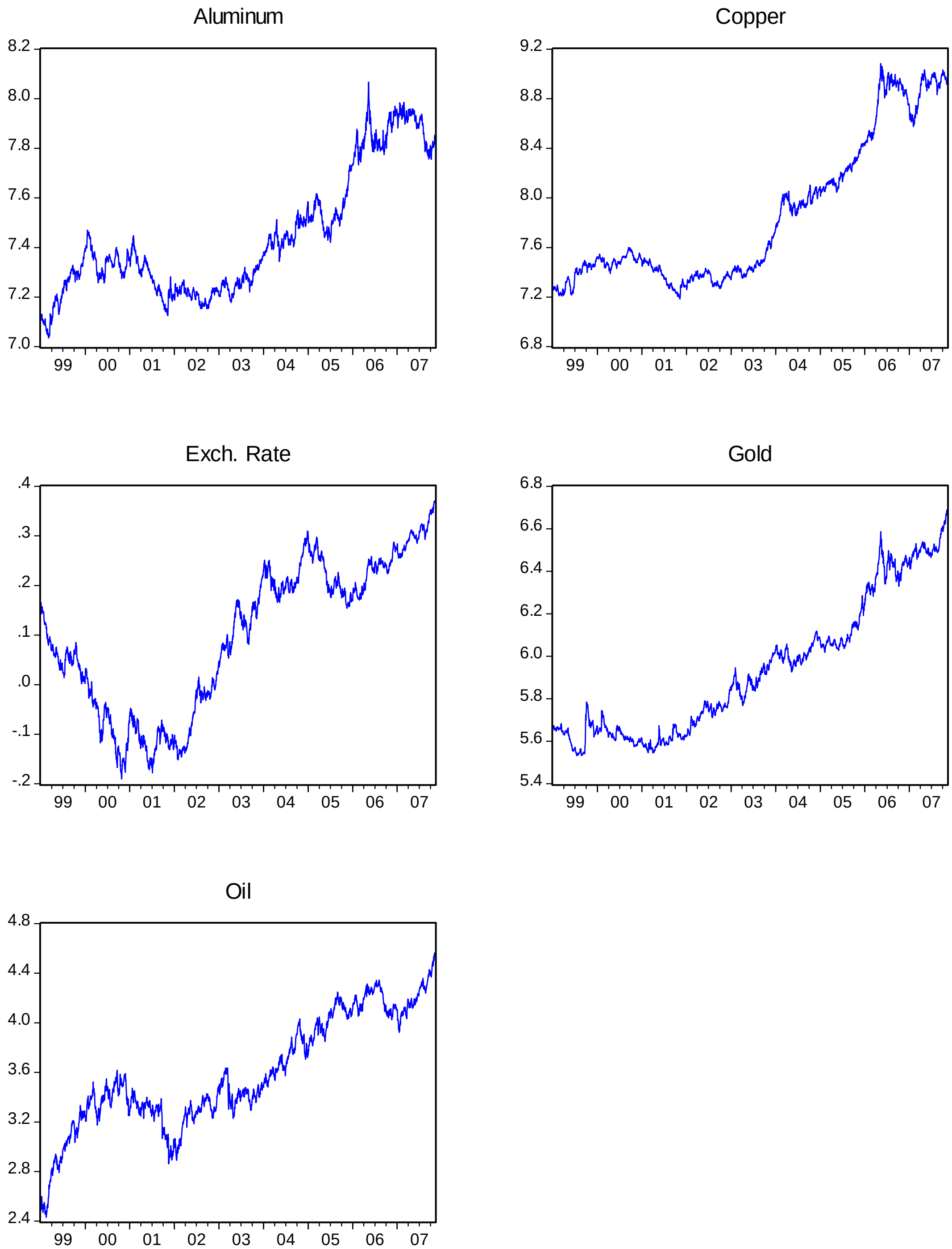
Figure 1. Historical Trajectories of the Four Commodities



Note: The graphs are for the log of the variables.

Figure 2. Dynamic Conditional Correlations for Model 1

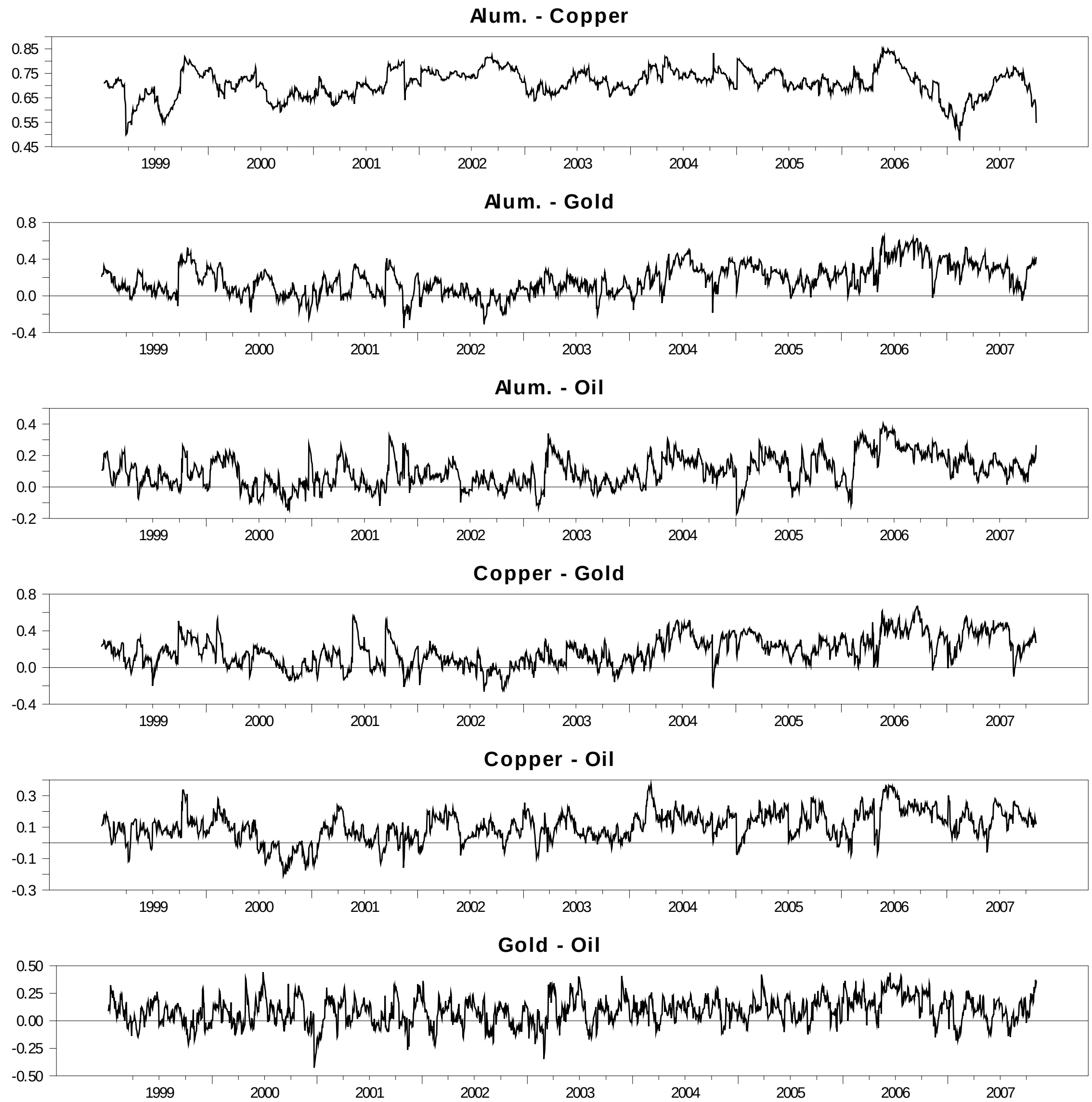


Table 1. Descriptive Statistics

Statistics	Aluminum	Copper	Exch. Rate	Gold	WTI Oil
Mean	0.0003	0.0007	0.0001	0.0004	0.0009
Median	0.0000	0.0000	0.0000	0.0001	0.0008
Maximum	0.0520	0.1155	0.0271	0.0701	0.1244
Minimum	-0.0826	-0.1036	-0.0247	-0.0625	-0.1709
Std. Dev.	0.0123	0.0153	0.0058	0.0098	0.0236
Skewness	-0.3288	-0.0957	0.0090	0.1160	-0.5517
Kurtosis	6.3808	8.1807	4.0321	8.9191	7.0413
Jarque-Bera	1139.294	2581.253	102.3437	3370.049	1685.484
Probability	0	0	0	0	0
ARCH Effect	11.75	18.09	18.4	4.05	8.38
Sum	0.7422	1.6463	0.2029	1.0305	2.0246
Sum Sq. Dev.	0.3509	0.5389	0.0782	0.2199	1.2802
Observations	2305	2305	2305	2305	2305

Notes: All commodity and dollar/euro exchange rate variables are log differences. The ARCH effect test was conducted on the AR(1) mean equations for up to 12 lags. The 5% critical value for this test is 1.75.

Table 2a. MGARCH Basic Model 1 for Aluminum, Copper, Gold and Oil

Independent Variable	$h_{11,t+1}$	$h_{22,t+1}$	$h_{33,t+1}$	$h_{44,t+1}$
$\varepsilon^2_{1,t}$	0.0198 [a]	2.00E-06	8.47E-04	5.20E-04
$\varepsilon_{1,t}\varepsilon_{2,t}$	-5.95E-04	1.65E-04	0.0020	-6.63E-04
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0043	8.40E-05	-0.0097	2.78E-04
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0011	-3.00E-06	-1.11E-04	0.0052
$\varepsilon^2_{2,t}$	1.80E-05	0.0173 [a]	0.0047	8.45E-04
$\varepsilon_{2,t}\varepsilon_{3,t}$	-1.28E-04	0.0088 [a]	-0.0229 [a]	-3.54E-04
$\varepsilon_{2,t}\varepsilon_{4,t}$	-3.30E-05	-3.39E-04	-2.64E-04	-0.0066
$\varepsilon^2_{3,t}$	9.17E-04	0.0045 [c]	0.1108 [a]	1.48E-04
$\varepsilon_{3,t}\varepsilon_{4,t}$	2.33E-04	-1.72E-04	0.0013	0.0028
$\varepsilon^2_{4,t}$	5.90E-05	7.00E-06	1.50E-05	0.0520 [a]
$h_{11,t}$	0.9615 [a]	7.30E-05	3.73E-04	8.60E-05
$h_{12,t}$	0.0036	-0.0085	3.23E-04 [a]	6.00E-05
$h_{13,t}$	0.0049	1.20E-04	0.0174 [b]	1.50E-05
$h_{14,t}$	-0.0017	-8.00E-06	-5.70E-05	0.0086
$h_{22,t}$	1.30E-05	0.9883 [a]	2.80E-04	4.20E-05
$h_{23,t}$	1.80E-05	-0.0139	0.0151 [b]	1.10E-05
$h_{24,t}$	-7.00E-06	0.0010	-5.00E-05	0.0061
$h_{33,t}$	2.50E-05	1.95E-04	0.8114 [a]	3.00E-06
$h_{34,t}$	-9.00E-06	-1.30E-05	-0.0027	0.0015
$h_{44,t}$	3.00E-06	1.00E-06	9.00E-06	0.8697 [a]
J.B. Stat	3074.2340 [a]	2888.5800 [a]	5156.0460 [a]	3252.5600 [a]
Breusch–Godfrey LM stat	0.0119	0.0765	0.0051	1.0294
Durbin–Watson stat	2.0002	1.9996	1.9993	2.0003
Log likelihood		27467.66		
AIC		-23.80		
#Obs.		2304		

Notes: Market subscripted by: 1 is aluminum, 2 is copper, 3 is gold, and 4 is oil. H_{ii} refers to the variance in market I, while *hij* is the covariance of market *i* in response to past volatility in market *j*. Shocks are defined similarly. The likelihood value for this model is 27500.74. a, b and refers to statistical significance at 1%, 5% and 10% levels.

Table 2b. MAGARCH Basic Model 1 for Aluminum, Copper, Gold and Oil

Independent Variable	$h_{11,t+1}$	$h_{22,t+1}$	$h_{33,t+1}$	$h_{44,t+1}$
$\varepsilon^2_{1,t}$	0.0142 [b]	2.26E-03	1.11E-04	7.26E-03
$\varepsilon_{1,t}\varepsilon_{2,t}$	4.99E-03 [b]	-8.44E-03	0.0009	8.81E-04
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0046 [c]	-2.59E-03	-0.0037	7.33E-04
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0000	9.60E-05	-1.10E-05	-0.0004
$\varepsilon^2_{2,t}$	1.75E-03	0.0315 [a]	0.0072 [b]	1.07E-04
$\varepsilon_{2,t}\varepsilon_{3,t}$	1.60E-03	0.0097 [a]	-0.0297 [a]	8.90E-05
$\varepsilon_{2,t}\varepsilon_{4,t}$	-8.00E-06	-3.57E-04	-8.90E-05	0.0000
$\varepsilon^2_{3,t}$	1.47E-03	0.0030	0.1233 [a]	7.40E-05
$\varepsilon_{3,t}\varepsilon_{4,t}$	-8.00E-06	-1.10E-04	0.0004	0.0000
$\varepsilon^2_{4,t}$	0.00E+00	4.00E-06	1.00E-06	0.0000
$h_{11,t}$	0.9595 [a]	0.00E+00	6.85E-04	7.70E-05
$h_{12,t}$	-0.0001	0.0002	4.19E-04 [a]	2.00E-06
$h_{13,t}$	-0.0065	-5.00E-06	0.0232 [a]	-1.33E-04
$h_{14,t}$	-0.0040 [c]	0.00E+00	1.40E-04	-0.0086
$h_{22,t}$	0.00E+00	0.9715 [a]	2.56E-04	0.00E+00
$h_{23,t}$	1.00E-06	-0.0185 [c]	0.0142 [a]	-4.00E-06
$h_{24,t}$	0.00E+00	-0.0008	8.60E-05	-0.0002
$h_{33,t}$	4.30E-05	3.51E-04	0.7842 [a]	2.30E-04
$h_{34,t}$	2.70E-05	1.50E-05	0.0047 [c]	0.0148
$h_{44,t}$	1.60E-05	1.00E-06	2.90E-05	0.9524 [a]
		Asymmetric $\varepsilon<0$		
$\varepsilon^2_{1,t}$	1.89E-02 [c]	2.69E-02 [b]	1.89E-03	0.0039
$\varepsilon_{1,t}\varepsilon_{2,t}$	-1.65E-02 [b]	-9.14E-03	-1.60E-03	0.0014
$\varepsilon_{1,t}\varepsilon_{3,t}$	-1.29E-02 [b]	-1.56E-02 [c]	-2.30E-03	-0.0004
$\varepsilon_{1,t}\varepsilon_{4,t}$	-9.03E-04	8.07E-04	-1.70E-04	-0.0138
$\varepsilon^2_{2,t}$	1.44E-02 [b]	3.10E-03	1.35E-03	0.0005
$\varepsilon_{2,t}\varepsilon_{3,t}$	1.13E-02 [a]	5.28E-03	1.94E-03	-0.0001
$\varepsilon_{2,t}\varepsilon_{4,t}$	7.88E-04	-2.74E-04	1.44E-04	-0.0049
$\varepsilon^2_{3,t}$	8.79E-03	8.98E-03	2.80E-03	0.0000
$\varepsilon_{3,t}\varepsilon_{4,t}$	6.16E-04	-4.66E-04	2.07E-04	0.0013
$\varepsilon^2_{4,t}$	4.30E-05	2.40E-05	1.50E-05	0.0497 [a]
J.B. Stat	2994.2820 [a]	3232.4320 [a]	4952.0300 [a]	3370.5660 [a]
Breusch–Godfrey LM stat	0.0380	0.1790	0.0379	0.6707
Durbin–Watson stat	2.0003	1.9994	1.9993	2.0005
Log likelihood		27522.56		
AIC		-23.83		
#Obs.		2304		

Notes: Market subscripted by: 1 is aluminum, 2 is copper, 3 is gold, and 4 is oil. H_{ii} refers to the variance in market I, while *hij* is the covariance of market *i* in response to past volatility in market *j*. Shocks are defined similarly. a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 2c. Symmetric DCC Basic Model 1 for Aluminum, Copper, Gold and Oil

	Aluminum	Copper	Gold	Oil
		Mean Equation		
C	0.0000	0.0000	0.0000	0.0011
AR(1)	-0.0355 [b]	-0.0131	0.0242	-0.0566 [b]
D03	0.0002	0.0009 [c]	0.0007 [b]	-0.0001
		Variance Equation		
C	1.00E-06 [a]	1.00E-06 [a]	6.00E-06 [a]	1.20E-05
ε^2(t-1)	0.0360 [a]	0.0337 [a]	0.1202 [a]	0.0400 [a]
h(t-1)	0.9564 [a]	0.9614 [a]	0.8168 [a]	0.9385 [a]
$\alpha+\beta$	0.9923	0.9951	0.9370	0.9785
		DCC Coefficients		
DDC(1)	0.01 [a]			
DDC(2)	0.99 [a]			
J.B. Stat	3358.7330 [a]	3144.3900 [a]	5277.9040 [a]	3280.5270 [a]
Breusch–Godfrey LM stat	0.2709	0.5226	0.1035	0.7965
Durbin–Watson stat	1.9990	1.9990	1.9996	2.0004
Log Likelihood		27497.28		
AIC		-23.84		
#Obs.		2304		

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 2d. Asymmetric DCC Basic Model 1 for Aluminum, Copper, Gold and Oil

	Aluminum	Copper	Gold	Oil
		Mean Equation		
C	0.0004	0.0004	0.0003	0.0015 [b]
AR(1)	-0.0392 [b]	-0.0161	0.0231	-0.0610 [a]
D03	-0.0003	0.0004	0.0003	-0.0010
		Variance Equation		
C	2.00E-06 [a]	1.00E-06 [a]	3.00E-06 [a]	1.50E-05
ε^2(t-1)	0.0586 [a]	0.0369 [a]	0.1523 [a]	0.0215
h(t-1)	0.9505 [a]	0.9658 [a]	0.8800 [a]	0.9354 [a]
$\varepsilon<0$	-0.0359 [a]	-0.0144 [b]	-0.1253 [a]	0.0305 [c]
$\alpha+\gamma/2+\beta$	0.9912	0.9955	0.9697	0.9721
		DCC Coefficients		
DDC(1)	0.01 [a]			
DDC(2)	0.99 [a]			
J.B. Stat	2856.3420 [a]	2936.3920 [a]	5109.9900 [a]	3353.8920 [a]
Breusch–Godfrey LM stat	0.4333	0.7165	0.0930	0.9954
Durbin–Watson stat	1.9988	1.9989	1.9996	2.0004
Log Likelihood		27530.09		
AIC		-23.84		
#Obs.		2304		

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 3a. Symmetric MGARCH Model 2 for Copper, Exchange Rate, Gold and Oil

Independent Variable	$h_{11,t+1}$		$h_{22,t+1}$		$h_{33,t+1}$		$h_{44,t+1}$	
$\varepsilon^2_{1,t}$	0.0218	a	2.00E-06		4.33E-04	b	0.00E+00	
$\varepsilon_{1,t}\varepsilon_{2,t}$	-0.0032	a	-1.35E-04		0.0044	a	1.08E-04	
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0084	a	-1.70E-05		-0.0072	a	3.50E-05	
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0006	c	4.00E-06		-3.40E-05		6.10E-05	
$\varepsilon^2_{2,t}$	4.72E-04		0.0115	a	0.0439	a	0.0234	a
$\varepsilon_{2,t}\varepsilon_{3,t}$	-0.0012	a	0.0014	a	-0.0723	a	0.0077	a
$\varepsilon_{2,t}\varepsilon_{4,t}$	-9.20E-05		-3.80E-04	a	-3.47E-04		0.0132	a
$\varepsilon^2_{3,t}$	0.0033	a	1.73E-04	a	0.1190	a	0.0025	b
$\varepsilon_{3,t}\varepsilon_{4,t}$	2.41E-04		-4.70E-05	a	5.71E-04		0.0043	a
$\varepsilon^2_{4,t}$	1.80E-05		1.30E-05	b	3.00E-06		0.0074	a
$h_{11,t}$	0.9750	a	1.00E-06	a	1.01E-04	a	2.80E-05	a
$h_{12,t}$	0.0064	a	7.23E-04	a	3.46E-04	a	-2.33E-04	a
$h_{13,t}$	-0.0130	a	-4.00E-06	a	0.0092	a	-1.37E-04	a
$h_{14,t}$	1.13E-04		2.00E-06	a	2.50E-05	a	0.0053	a
$h_{22,t}$	4.30E-05	a	0.9892	a	0.0012	a	0.0019	a
$h_{23,t}$	-8.60E-05	a	-0.0052	a	0.0317	a	0.0011	a
$h_{24,t}$	1.00E-06		0.0022	a	8.70E-05	a	-0.0435	a
$h_{33,t}$	1.74E-04	a	2.80E-05	a	0.8451	a	6.66E-04	a
$h_{34,t}$	-2.00E-06		-1.20E-05	a	0.0023	a	-0.0256	a
$h_{44,t}$	1.30E-08		5.00E-06	a	6.00E-06	a	0.9880	a
J.B. Stat	3290.9100	a	1372.3180	a	5232.2520	a	3133.8780	a
Breusch–Godfrey LM stat	0.4243		6.5452	a	0.0201		0.4847	
Durbin–Watson stat	1.9987		1.9998		1.9994		2.0004	
Log Likelihood	28915.49							
AIC	-25.05							
#Obs.	2304							

Notes: Market subscripted by: 1 is copper, 2 is dollar/euro foreign exchange,3 is gold, and 4 is oil. h_{ii} refers to the variance in market i, while *hij* is the covariance of market *i* in response to past volatility in market *j.* Shocks are defined similarly. a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 3b. MAGARCH Model 2 for Copper, Exchange Rate, Gold and Oil

Independent Variable	$h_{11,t+1}$		$h_{22,t+1}$		$h_{33,t+1}$		$h_{44,t+1}$	
$\varepsilon^2_{1,t}$	0.0209	a	3.00E-06		8.92E-04	a	2.92E-04	
$\varepsilon_{1,t}\varepsilon_{2,t}$	-4.63E-03	a	-2.05E-04	c	0.0041	a	3.64E-03	b
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0094	a	-1.90E-05	c	-0.0105	a	3.45E-04	
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0003		-2.00E-06		1.25E-04		0.0006	b
$\varepsilon^2_{2,t}$	1.03E-03	b	0.0138	a	0.0189	a	4.53E-02	a
$\varepsilon_{2,t}\varepsilon_{3,t}$	-2.09E-03	a	0.0013	a	-0.0484	a	4.30E-03	
$\varepsilon_{2,t}\varepsilon_{4,t}$	-5.90E-05		1.42E-04		5.76E-04		0.0069	a
$\varepsilon^2_{3,t}$	4.24E-03	a	0.0001	a	0.1241	a	4.09E-04	
$\varepsilon_{3,t}\varepsilon_{4,t}$	1.19E-04		1.30E-05		-0.0015		0.0007	
$\varepsilon^2_{4,t}$	3.00E-06		1.00E-06		1.80E-05		0.0010	
$h_{11,t}$	0.9745	a	1.00E-06	a	1.39E-04	a	1.00E-06	
$h_{12,t}$	0.0097	a	0.0008	a	4.44E-04	a	-3.50E-05	
$h_{13,t}$	-0.0150	a	-4.00E-06	a	0.0107	a	2.00E-06	
$h_{14,t}$	0.0009	b	1.00E-06	a	4.10E-05	a	0.0009	
$h_{22,t}$	9.70E-05	a	0.9845	a	1.42E-03	a	1.55E-03	a
$h_{23,t}$	-1.50E-04	a	-0.0046	a	0.0343	a	-8.50E-05	
$h_{24,t}$	9.00E-06	b	0.0006	a	1.33E-04	a	-0.0384	a
$h_{33,t}$	2.30E-04	a	2.10E-05	a	0.8283	a	5.00E-06	
$h_{34,t}$	-1.40E-05	b	-3.00E-06	a	0.0032	a	0.0021	
$h_{44,t}$	1.00E-06		0.00E+00	b	1.20E-05	b	0.9551	a
				Asymmetric $\varepsilon<0$				
$\varepsilon^2_{1,t}$	7.30E-05		4.60E-05		1.03E-02	a	0.0041	
$\varepsilon_{1,t}\varepsilon_{2,t}$	-2.14E-04		3.96E-04	c	-3.21E-02	a	-0.0102	b
$\varepsilon_{1,t}\varepsilon_{3,t}$	-8.98E-04		-8.60E-05		6.76E-03	a	0.0014	
$\varepsilon_{1,t}\varepsilon_{4,t}$	1.22E-04		-6.00E-06		1.09E-03	c	-0.0126	a
$\varepsilon^2_{2,t}$	6.27E-04		3.45E-03	a	1.00E-01	a	0.0254	a
$\varepsilon_{2,t}\varepsilon_{3,t}$	2.63E-03		-7.51E-04	b	-2.11E-02	a	-0.0034	
$\varepsilon_{2,t}\varepsilon_{4,t}$	-3.56E-04		-5.40E-05		-3.42E-03	c	0.0315	a
$\varepsilon^2_{3,t}$	1.10E-02	a	1.63E-04		4.45E-03		0.0005	
$\varepsilon_{3,t}\varepsilon_{4,t}$	-1.49E-03	a	1.20E-05		7.20E-04		-0.0042	
$\varepsilon^2_{4,t}$	2.02E-04	b	1.00E-06		1.17E-04		0.0390	a
J.B. Stat	3382.2970	a	1359.0920	a	4515.9870	a	3037.5550	a
Breusch–Godfrey LM stat	0.4136		5.7907	a	0.0444		0.6715	
Durbin–Watson stat	1.9987		1.9999		1.9995		2.0004	
Log Likelihood				28951.36				
AIC				-25.07				
#Obs.				2304				

Notes: Market subscripted by: 1 is copper, 2 is dollar/euro foreign exchange,3 is gold, and 4 is oil. h_{ii} refers to the variance in market i, while *hij* is the covariance of market *i* in response to past volatility in market *j*. Shocks are defined similarly. a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 3c. Symmetric DCC Model 2 for Copper, Exchange Rate, Gold and Oil

	Copper	Exchange Rate	Gold	Oil
		Mean Equation		
C	0.0002	-0.0001	-0.0001	0.0011 [c]
AR(1)	-0.0186	-0.0466 [b]	0.0281	-0.0577 [a]
D03	0.0010 [c]	0.0004 [c]	0.0008 [b]	-0.0001
		Variance Equation		
C	1.00E-06 [c]	0.00E+00	6.00E-06 [a]	1.40E-05
ε^2(t-1)	0.0375 [a]	0.0179 [a]	0.1235 [a]	0.0432 [a]
h(t-1)	0.9580 [a]	0.9812 [a]	0.8135 [a]	0.9325 [a]
$\alpha+\beta$	0.9955	0.9991	0.9370	0.9757
		DCC Coefficients		
DDC(1)	0.01 [a]			
DDC(2)	0.99 [a]			
J.B. Stat	3524.0620 [a]	1385.6650 [a]	5244.0990 [a]	3282.0560 [a]
Breusch–Godfrey LM stat	1.7719	6.0609 [a]	0.0681	0.9622
Durbin–Watson stat	1.9975	1.9998	1.9995	2.0003
Log Likelihood		28529.47		
AIC		-24.74		
#Obs.		2304		

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 3d. Asymmetric DCC Model 2 for Copper, Exchange Rate, Gold and Oil

	Copper		Exchange Rate		Gold		Oil	
			Mean Equation					
C	0.0004		0.0000		0.0002		0.0014	b
AR(1)	-0.0208		-0.0482	b	0.0276		-0.0621	a
D03	0.0007		0.0003		0.0005		-0.0008	
			Variance Equation					
C	1.00E-06	b	0.00E+00		3.00E-06	a	1.70E-05	
ε^2(t-1)	0.0400	a	0.0176	a	0.1523	a	0.0242	
h(t-1)	0.9655	a	0.9812	a	0.8799	a	0.9269	a
$\varepsilon<0$	-0.0183	b	0.0004		-0.1233	a	0.0330	c
$\alpha+\gamma/2+\beta$	0.9964		0.9988		0.9705		0.9676	
			DCC Coefficients					
DDC(1)	0.01	a						
DDC(2)	0.99	a						
J.B. Stat	2952.3270	a	1381.9830	a	5123.5450	a	3354.5930	a
Breusch–Godfrey LM stat	2.1688		6.4804	a	0.0582		1.1930	
Durbin–Watson stat	1.9973		1.9998		1.9996		2.0003	
Log Likelihood			28557.22					
AIC			-24.76					
#Obs.			2304					

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 4a. Symmetric MGARCH Model 3 for Aluminum, Exchange Rate, Gold and Oil

Independent Variable	$h_{11,t+1}$		$h_{22,t+1}$		$h_{33,t+1}$		$h_{44,t+1}$	
$\varepsilon^2_{1,t}$	0.0229	a	6.00E-06		3.29E-04		2.49E-04	
$\varepsilon_{1,t}\varepsilon_{2,t}$	2.32E-04		2.76E-04		0.0038		0.0029	
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0053	c	3.00E-05		-0.0063		5.70E-04	
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0014		-9.00E-06		3.70E-05		0.0015	
$\varepsilon^2_{2,t}$	2.00E-06		0.0118	a	0.0444	a	0.0347	
$\varepsilon_{2,t}\varepsilon_{3,t}$	5.40E-05		0.0013	c	-0.0733	a	0.0067	
$\varepsilon_{2,t}\varepsilon_{4,t}$	1.40E-05		-3.94E-04		4.33E-04		0.0173	b
$\varepsilon^2_{3,t}$	0.0012		1.44E-04		0.1211	a	0.0013	
$\varepsilon_{3,t}\varepsilon_{4,t}$	3.15E-04		-4.40E-05		-7.15E-04		0.0034	
$\varepsilon^2_{4,t}$	8.10E-05		1.30E-05		4.00E-06		0.0086	a
$h_{11,t}$	0.9655	a	3.58E-07		4.07E-04	c	4.00E-06	
$h_{12,t}$	0.0022		6.17E-04		7.68E-04	a	-9.20E-05	
$h_{13,t}$	-0.0028		-3.00E-06		0.0184	a	-3.60E-05	
$h_{14,t}$	-0.0018		1.00E-06		7.80E-05		0.0020	
$h_{22,t}$	5.00E-06		0.9895	a	0.0014	a	0.0021	b
$h_{23,t}$	-7.00E-06		-0.0053	b	0.0347	a	8.10E-04	
$h_{24,t}$	-4.00E-06		0.0021	a	1.47E-04		-0.0454	a
$h_{33,t}$	8.00E-06		2.90E-05		0.8312	a	3.13E-04	
$h_{34,t}$	5.00E-06		-1.10E-05	c	0.0035	c	-0.0176	
$h_{44,t}$	3.00E-06		5.00E-06	b	1.50E-05		0.9852	a
J.B. Stat	2937.2460	a	1366.9720	a	5270.5190	a	3222.5400	a
Breusch–Godfrey LM stat	0.5824		5.5120	a	0.0077		0.6051	
Durbin–Watson stat	1.9989		1.9998		1.9994		2.0003	
Log Likelihood					28528.88			
AIC					-24.71			
#Obs.					2304			

Notes: Market subscripted by: 1 is aluminum, 2 is dollar/euro foreign exchange, 3 is gold, and 4 is oil. h_{ii} refers to the variance in market i, while *hij* is the covariance of market *i* in response to past volatility in market *j*. Shocks are defined similarly. a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 4b. MAGARCH Model 3 for Aluminum, Exchange Rate, Gold and Oil

Independent Variable	$h_{11,t+1}$		$h_{22,t+1}$		$h_{33,t+1}$		$h_{44,t+1}$	
$\varepsilon^2_{1,t}$	0.0203	a	5.00E-06		3.55E-04		3.00E-03	
$\varepsilon_{1,t}\varepsilon_{2,t}$	3.87E-03	a	2.32E-04	c	0.0038	a	-9.87E-03	a
$\varepsilon_{1,t}\varepsilon_{3,t}$	0.0048	a	3.00E-05	c	-0.0066	a	-7.12E-04	
$\varepsilon_{1,t}\varepsilon_{4,t}$	0.0013	b	6.00E-06		9.10E-05		0.0046	a
$\varepsilon^2_{2,t}$	7.36E-04	b	0.0104	a	0.0403	a	3.24E-02	a
$\varepsilon_{2,t}\varepsilon_{3,t}$	9.07E-04	a	0.0013	a	-0.0699	a	2.34E-03	
$\varepsilon_{2,t}\varepsilon_{4,t}$	2.43E-04	c	2.48E-04		9.71E-04		-0.0150	a
$\varepsilon^2_{3,t}$	1.12E-03	a	0.0002	a	0.1214	a	1.69E-04	
$\varepsilon_{3,t}\varepsilon_{4,t}$	3.00E-04	b	3.20E-05		-0.0017		-0.0011	
$\varepsilon^2_{4,t}$	8.00E-05		6.00E-06		2.30E-05		0.0069	b
$h_{11,t}$	0.9678	a	1.00E-05	a	6.84E-04	a	2.51E-04	a
$h_{12,t}$	-0.0039	a	0.0031	a	9.00E-04	a	-9.89E-04	a
$h_{13,t}$	-0.0016		-1.40E-05	a	0.0237	a	5.56E-04	a
$h_{14,t}$	-0.0042	a	-1.90E-05	a	-5.50E-05	c	0.0141	a
$h_{22,t}$	1.60E-05	b	0.9805	a	1.19E-03	a	3.90E-03	a
$h_{23,t}$	6.00E-06		-0.0046	a	0.0312	a	-2.19E-03	a
$h_{24,t}$	1.70E-05	a	-0.0061	a	-7.20E-05	c	-0.0557	a
$h_{33,t}$	3.00E-06		2.10E-05	a	0.8205	a	1.23E-03	a
$h_{34,t}$	7.00E-06		2.90E-05	a	-0.0019	c	0.0313	a
$h_{44,t}$	1.80E-05	a	3.80E-05	a	4.00E-06		0.7951	a
			Asymmetric $\varepsilon<0$					
$\varepsilon^2_{1,t}$	2.74E-03	a	2.60E-05		1.24E-02	a	0.0129	
$\varepsilon_{1,t}\varepsilon_{2,t}$	-1.24E-04		3.46E-04		-1.52E-02	a	-0.1063	a
$\varepsilon_{1,t}\varepsilon_{3,t}$	2.56E-03	a	-5.20E-05		2.86E-03		0.0198	b
$\varepsilon_{1,t}\varepsilon_{4,t}$	-8.37E-04	a	2.50E-05		-6.00E-06		-0.0346	a
$\varepsilon^2_{2,t}$	6.00E-06		4.69E-03	a	1.85E-02	a	0.8725	a
$\varepsilon_{2,t}\varepsilon_{3,t}$	-1.16E-04		-7.08E-04		-3.50E-03		-0.1624	a
$\varepsilon_{2,t}\varepsilon_{4,t}$	3.80E-05		3.36E-04	a	7.00E-06		0.2842	a
$\varepsilon^2_{3,t}$	2.40E-03	b	1.07E-04		6.61E-04		0.0302	
$\varepsilon_{3,t}\varepsilon_{4,t}$	-7.82E-04	a	-5.10E-05		-1.00E-06		-0.0529	a
$\varepsilon^2_{4,t}$	2.55E-04	c	2.40E-05		0.00E+00		0.0926	a
J.B. Stat	2959.5220	a	1360.9660	a	4909.7820	a	2608.1460	a
Breusch–Godfrey LM stat	0.4916		5.6479	a	0.0721		1.2870	
Durbin–Watson stat	1.9990		1.9997		1.9996		2.0005	
Log Likelihood			28569.28					
AIC			-24.73					
#Obs.			2304					

Notes: Market subscripted by: 1 is aluminum, 2 is dollar/euro foreign exchange, 3 is gold, and 4 is oil. h_{ii} refers to the variance in market i, while *hij* is the covariance of market *i* in response to past volatility in market *j*. Shocks are defined similarly.

Table 4c. Symmetric DCC Model 3 for Aluminum, Exchange Rate, Gold and Oil

	Aluminum	Exchange Rate	Gold	Oil
		Mean Equation		
C	0.0001	-0.0002	-0.0001	0.0011
AR(1)	-0.0533[b]	-0.0427[b]	0.0302	-0.0583[a]
D03	0.0003	0.0004[c]	0.0008[b]	-0.0001
		Variance Equation		
C	2.00E-06[a]	4.20E-05[a]	6.00E-06[a]	1.30E-05[a]
ε^2(t-1)	0.0413[a]	-0.0041	0.1245[a]	0.0413[a]
h(t-1)	0.9482[a]	-0.2343	0.8107[a]	0.9354[a]
$\alpha+\beta$	0.9895		0.9352	0.9767
		DCC Coefficients		
DDC(1)	0.01[a]			
DDC(2)	0.99[a]			
J.B. Stat	3180.2570[a]	1384.5390[a]	5283.5860[a]	3277.4050[a]
Breusch–Godfrey LM stat	0.9710	5.4346[a]	0.0302	0.9120
Durbin–Watson stat	1.9987	1.9998	1.9995	2.0003
Log Likelihood		28837.23		
AIC		-25.01		
#Obs.		2304		

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 4d. Asymmetric DCC Model 3 for Aluminum, Exchange Rate, Gold and Oil

	Aluminum		Exchange Rate		Gold		Oil	
			Mean Equation					
C	0.0004		0.0000		0.0002		0.0015	b
AR(1)	-0.0589	b	-0.0511	b	0.0246		-0.0631	a
D03	0.0000		0.0003		0.0005		-0.0009	
			Variance Equation					
C	2.00E-06	a	0.00E+00		3.00E-06	a	1.50E-05	
ε^2(t-1)	0.0644	a	0.0174	a	0.1533	a	0.0214	
h(t-1)	0.9442	a	0.9813	a	0.8775	a	0.9343	a
$\varepsilon<0$	-0.0398	a	0.0005		-0.1239	a	0.0313	c
$\alpha+\gamma/2+\beta$	0.9888		0.9987		0.9689		0.9713	
			DCC Coefficients					
DDC(1)	0.01	a						
DDC(2)	0.99	a						
J.B. Stat	2942.3220	a	1381.5270	a	5151.0190	a	3333.4030	a
Breusch–Godfrey LM stat	1.0368		5.7998	a	0.0138		1.1084	
Durbin–Watson stat	1.9986		1.9998		1.9995		2.0004	
Log Likelihood			28946.91					
AIC			-25.10					
#Obs.			2304					

Notes: a, b and c refers to statistical significance at 1%, 5% and 10% levels.

Table 5a. Optimal Portfolio Weights and Hedge Ratios

Portfolio	Weight ($w_{12,t}$) of First Commodity/Asset in 1$ Portfolio (Kroner and Ng, 1998)	Short/Long Beta β_t (Kroner and Sultan,1993)
	Model 2	
Copper/Euro	0.14	0.31
Copper/Gold	0.27	0.32
Copper/Oil	0.72	0.09
Euro/Gold	0.78	0.22
Euro/Oil	0.95	0.01
Gold/Oil	0.87	0.04
	Model 3	
Aluminum/Euro	0.17	0.30
Aluminum/Gold	0.35	0.24
Aluminum/Oil	0.80	0.07

Notes: $w_{12,t}$ is the portfolio weight of commodity or asset 1 relative to commodity or asset 2 in a two-commodity/asset holding at time t, while average β_t is the risk-minimizing hedge ratio for the two commodities/assets.

Table 5b. Symmetric and Asymmetric MGARCH Optimal Portfolio Weights and Hedge Ratios

Portfolio	Weight ($w_{12,t}$) of First Commodity in 1$ Portfolio (Kroner and NG (1998)	Short/Long Beta β_t (Kroner and Sultan (1993)	Weight ($w_{12,t}$) of First Commodity in 1$ Portfolio (Kroner and NG (1998)	Short/Long Beta β_t (Kroner and Sultan (1993)
	Symmetric		Asymmetric	
		Model 1		
Aluminum/Copper	0.756	0.602	0.776	0.597
Aluminum/Gold	0.348	0.235	0.349	0.235
Aluminum/Oil	0.804	0.060	0.792	0.067
Copper/Gold	0.266	0.314	0.266	0.311
Copper/Oil	0.730	0.080	0.715	0.084
Gold/Oil	0.875	0.041	0.869	0.043
		Model 2		
Copper/EURO	0.140	0.311	0.144	0.319
Copper/Gold	0.266	0.317	0.264	0.314
Copper/Oil	0.723	0.088	0.719	0.084
EURO/Gold	0.776	0.223	0.772	0.227
EURO/Oil	0.950	0.010	0.946	0.009
Gold/Oil	0.869	0.042	0.870	0.046
		Model 3		
Aluminum/EURO	0.174	0.301	0.177	0.303
Aluminum/Gold	0.346	0.237	0.346	0.240
Aluminum/Oil	0.800	0.068	0.795	0.069
EURO/Gold	0.778	0.225	0.776	0.228
EURO/Oil	0.950	0.011	0.950	0.013
Gold/Oil	0.869	0.042	0.866	0.044

Notes: $w_{12,t}$ is the portfolio weight of commodity or asset 1 relative to commodity or asset 2 in a two-commodity/asset holding at time t, while average β_t is the risk-minimizing hedge ratio for the two commodities/assets.

Table 5c. Symmetric and Asymmetric DCC Optimal Portfolio Weights and Hedge Ratios

Portfolio	Weight ($w_{12,t}$) of First Commodity in 1$ Portfolio (Kroner and NG (1998)	Short/Long Beta β_t (Kroner and Sultan (1993)	Weight ($w_{12,t}$) of First Commodity in 1$ Portfolio (Kroner and NG (1998)	Short/Long Beta β_t (Kroner and Sultan (1993)
	Symmetric		Asymmetric	
		Model 1		
Aluminum/Copper	0.746	0.607	0.743	0.607
Aluminum/Gold	0.350	0.269	0.347	0.273
Aluminum/Oil	0.799	0.064	0.799	0.063
Copper/Gold	0.263	0.364	0.262	0.371
Copper/Oil	0.725	0.079	0.725	0.078
Gold/Oil	0.868	0.041	0.867	0.041
		Model 2		
Copper/EURO	0.146	0.309	0.146	0.305
Copper/Gold	0.262	0.354	0.260	0.361
Copper/Oil	0.724	0.084	0.724	0.084
EURO/Gold	0.780	0.203	0.771	0.206
EURO/Oil	0.947	0.008	0.947	0.008
Gold/Oil	0.870	0.043	0.870	0.043
		Model 3		
Aluminum/EURO	0.176	0.254	0.179	0.283
Aluminum/Gold	0.346	0.258	0.343	0.263
Aluminum/Oil	0.798	0.066	0.799	0.066
EURO/Gold	0.778	0.205	0.769	0.205
EURO/Oil	0.945	0.010	0.946	0.008
Gold/Oil	0.870	0.043	0.870	0.043

Notes: $w_{12,t}$ is the portfolio weight of commodity or asset 1 relative to commodity or asset 2 in a two-commodity/asset holding at time t, while average β_t is the risk-minimizing hedge ratio for the two commodities/assets.