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Glenn D. Rudebusch
Federal Reserve Bank of San Francisco

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Glenn D. Rudebusch*

Federal Reserve Bank of San Francisco

Abstract

During the past decade, much new research has combined elements of finance, monetary economics, and macroeconomics in order to study the relationship between the term structure of interest rates and the economy. In this survey, I describe three different strands of such interdisciplinary macro-finance term structure research. The first adds macroeconomic variables and structure to a canonical arbitrage-free finance representation of the yield curve. The second examines bond pricing and bond risk premiums in a canonical macroeconomic dynamic stochastic general equilibrium model. The third develops a new class of arbitrage-free term structure models that are empirically tractable and well suited to macro-finance investigations.

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1 Introduction

The evolution of economic ideas and models has often been altered by economic events. The Great Depression led to the widespread adoption of the Keynesian view that markets may not readily equilibrate. The Great Inflation highlighted the importance of aggregate supply shocks and spurred real business cycle research. The Great Disinflation fostered a New Keynesianism, which recognized the potency of monetary policy. The shallow recessions and relative calm of the Great Moderation helped solidify the dynamic stochastic general equilibrium (DSGE) model as a macroeconomic orthodoxy. Therefore, it also seems likely that the recent financial and economic crisis—the Great Panic and Recession of 2008 and 2009—will both rearrange the economic landscape and affect the focus of economic and financial research going forward.

A key feature of recent events has been the close feedback between the real economy and financial conditions. In many countries, the credit and housing boom that preceded the crisis went hand in hand with strong spending and production. Similarly, during the crash, deteriorating financial conditions helped cause the recession and were in turn exacerbated by the deep declines in economic activity. The starkest illustration of this linkage occurred in the fall of 2008, when the extraordinary financial market dislocations that followed the bankruptcy of Lehman Brothers coincided with a global macroeconomic free fall. Such macro-finance linkages pose a significant challenge to both macroeconomists and finance economists because of the long-standing separation between the two disciplines. In macro models, the entire financial sector is often represented by a single interest rate with no yield spreads for credit or liquidity risk and no role for financial intermediation or financial frictions. Similarly, finance models typically have no macroeconomic content, but instead focus on the consistency of asset prices across markets with little regard for the underlying economic fundamentals. In order to understand important aspects of the recent intertwined financial crisis and economic recession, a joint macro-finance perspective is likely necessary. In this article, I survey an area of macro-finance research that has examined the relationship between the term structure of interest rates and the economy in an interdisciplinary fashion.

The modeling of interest rates has long been a prime example of the disconnect between the macro and finance literatures. In the canonical finance model, the short-term interest rate is a simple linear function of a few unobserved factors, sometimes labeled "level, slope, and curvature," but with no economic interpretation. Long-term interest rates are related to those same factors, and movements in long-term yields are importantly determined by changes in risk premiums, which also depend on those latent factors. In contrast, in the macro literature, the short-term interest rate is set by the central bank according to macroeconomic stabilization goals. For example, the short rate may be determined by the deviations of inflation and output from targets set by the central bank. Furthermore, the macro literature commonly views long-term yields as largely determined by expectations of future short-term interest rates, which in turn depend on expectations of the macro variables; that is, possible changes in risk premiums are often ignored, and the expectations hypothesis of the term structure is employed.

Of course, differences between the finance and macro perspectives reflect in part different questions of interest and different avenues for exploration; however, it is striking that there is so little interchange or overlap between the two research literatures. At the very least, it suggests that there may be synergies from combining elements of each. From a finance perspective, the short rate is a fundamental building block for rates of other maturities because long yields are risk-adjusted averages of expected future short rates. From a macro perspective, the short rate is a key monetary policy instrument, which is adjusted by the central bank in order to achieve economic stabilization goals. Taken together, a joint macro-finance perspective would suggest that understanding the way central banks move the short rate in response to fundamental macroeconomic shocks should explain movements in the short end of the yield curve; furthermore, with the consistency between long and short rates enforced by the no-arbitrage assumption, expected future macroeconomic variation should account for movements farther out in the yield curve as well.

This survey considers three recent strands of macro-finance research that focus on the linkages between interest rates and the economy. The first of these, described in the next section, adds macro, in the form of macroeconomic variables or theoretical structure, to the canonical finance affine arbitrage-free term structure model. This analysis suggests that the latent factors from the standard finance term structure model do have macroeconomic underpinnings, and an explicit macro structure can provide insight into the behavior of the yield curve beyond what a pure finance model can suggest. In addition, this joint macro-finance perspective also illuminates various macroeconomic issues, since the additional term structure factors, which reflect expectations about the future dynamics of the economy, can help sharpen inference. The second strand of research, described in Section 3, examines the finance implications for bond pricing in a macroeconomic DSGE model. As a theoretical matter, asset prices and the macroeconomy are inextricably linked, as asset markets are the mechanism by which consumption and investment are allocated across time and states of nature. However, the importance of jointly modeling both macroeconomic variables and asset prices within a DSGE framework has only begun to be appreciated. Unfortunately, the standard DSGE framework appears woefully inadequate to account for bond prices, but there are some DSGE model modifications that promise better results. Finally, in Section 4, I describe the arbitrage-free Nelson-Siegel (AFNS) model. Practical computational difficulties in estimating affine arbitrage-free models have greatly hindered their extension in macro-finance applications. However, imposing the popular Nelson-Siegel factor structure on the canonical affine finance model provides a very useful framework for examining various macro-finance questions. Section 5 concludes.

2 Adding Macro to a Finance Model

Government securities of various maturities all trade simultaneously in active markets at prices that appear to preclude opportunities for financial arbitrage. Accordingly, the assumption that market bond prices allow no residual riskless arbitrage is central to an enormous finance literature that is devoted to the empirical analysis of the yield curve. This research typically models yields as linear functions of a few unobservable or latent factors with an arbitrage-free condition that requires the dynamic evolution of yields to be consistent with the cross section of yields of different maturities at any point in time (e.g., Duffie and Kan 1996 and Dai and Singleton 2000). However, while these popular finance models provide useful statistical descriptions of term structure dynamics, they offer little insight into the economic nature of the underlying latent factors or forces that drive changes in interest rates.

To provide insight into the fundamental drivers of the yield curve, macro variables and macro structure can be combined with the finance models. Of course, as discussed in Diebold, Piazzesi, and Rudebusch (2005), there are many ways in which macro and finance elements could be integrated. One decision faced in term structure modeling is how to summarize the price information at any point in time for a large number of nominal bonds. Fortunately, only a small number of sources of systematic risk appear to be relevant for bond pricing, so a large set of bond prices can be effectively summarized with just a few constructed variables or factors. Therefore, yield curve models invariably employ a small set of factors with associated factor loadings that relate yields of different maturities to those factors. For example, the factors could be the first few bond yield principal components. Indeed, the first three principal components account for much of the total variation in yields and are closely correlated with simple empirical proxies for level (e.g., the long rate), slope (e.g., a long rate minus a short rate), and curvature (e.g., a mid-maturity rate minus a short and long rate average). Another approach, which is popular among market and central bank practitioners, is a fitted Nelson-Siegel curve (introduced in Charles Nelson and Andrew Siegel, 1987) which can be extended as a dynamic factor model (Diebold and Li, 2006). A third approach uses the affine arbitrage-free canonical finance latent factor model.

The crucial issue in combining macro and finance then is how to connect the macroeconomic variables with the yield factors. Diebold, Rudebusch, and Aruoba (2006) provide a macroeconomic interpretation of the Diebold-Li (2006) dynamic Nelson-Siegel representation by combining it with a vector autoregression (VAR) representation for the macroeconomy. Their estimation extracts three latent factors (essentially level, slope, and curvature) from a set of 17 yields on US Treasury securities and simultaneously relates these factors to three observable macroeconomic variables. They find that the level factor is highly correlated with inflation, and the slope factor is highly correlated with real activity, but the curvature factor appears unrelated to the key macroeconomic variables. Related research also explores the linkage between macro variables and the yield curve using little or no macroeconomic structure, including, Kozicki and Tinsley (2001), Ang and Piazzesi (2003), Piazzesi (2005), Ang, Piazzesi, and Wei (2006), Dewachter and Lyrio (2006), Balfoussia and Wickens (2007), Wright (2009), and Joslin, Priebsch, and Singleton (2009). In contrast, other papers, such as Hördahl, Tristani, and Vestin (2006), and Rudebusch and Wu (2008), embed the yield factors within a macroeconomic structure. This additional structure facilitates the interpretation of a bidirectional feedback between the term structure factors and macro variables.

The remainder of this section describes one macro-finance term structure model in detail and considers two applications of that model.

2.1 Rudebusch-Wu Macro-Finance Model

The usual finance model decomposes the short-term interest rate into unobserved factors that are modeled as autoregressive time series that are unrelated to macroeconomic variation. In contrast, from a macro perspective, the short rate is determined by macroeconomic variables in the context of a monetary policy reaction function. The Rudebusch-Wu (2008) model reconciles these two views in a macro-finance framework that has term structure factors jointly estimated with macroeconomic relationships. In particular, this analysis combines an affine arbitrage-free term structure model with a small New Keynesian rational expectations macroeconomic model with the short-term interest rate related to macroeconomic fundamentals through a monetary policy reaction function. This combined macro-finance model is estimated from the data by maximum likelihood methods and demonstrates empirical fit and dynamics comparable to stand-alone finance or macro models. This new framework is able to interpret the latent factors of the yield curve in terms of macroeconomic variables, with the level factor identified as a perceived inflation target and the slope factor identified as a cyclical monetary policy response to the economy.

In the Rudebusch-Wu macro-finance model, a key point of intersection between the finance and macroeconomic specifications is the short-term interest rate. The short-term nominal interest rate, i_t, is a linear function of two latent term structure factors (as in the canonical finance model), so

$$i_t = \delta_0 + L_t + S_t, \tag{1}$$

where L_t and S_t are term structure factors usually identified as level and slope (and δ_0 is a constant). In contrast, the popular macroeconomic Taylor (1993) rule for monetary policy takes the form:

$$i_t = r^* + \pi_t^* + g_\pi(\pi_t - \pi_t^*) + g_y y_t, \tag{2}$$

where r^* is the equilibrium real rate, π_t^* is the central bank's inflation target, π_t is the annual inflation rate, and y_t is a measure of the output gap. This rule reflects the fact that the Federal Reserve sets the short rate in response to macroeconomic data in an attempt to achieve its goals of output and inflation stabilization.

To link these two representations of the short rate, level and slope are not simply modeled as pure autoregressive finance time series; instead, they form elements of a monetary policy reaction function. In particular, L_t is interpreted to be the medium-term inflation target of the central bank as perceived by private investors (say, over the next two to five years), so $\delta_0 + L_t$ is associated with $r^* + \pi_t^*$.[1] Investors are assumed to modify their views of this underlying rate of inflation slowly, as actual inflation, π_t, changes. Thus, L_t is linearly updated by news about inflation:

$$L_t = \rho_L L_{t-1} + (1 - \rho_L)\pi_t + \varepsilon_{L,t}. \tag{3}$$

The slope factor, S_t, captures the Fed's dual mandate to stabilize the real economy and keep inflation close to its medium-term target level, that is, S_t is identified with the term $g_\pi(\pi_t - \pi_t^*) + g_y y_t$. Specifically, S_t is modeled as the Fed's cyclical response to deviations of inflation from its target, $\pi_t - L_t$, and to deviations of output from its potential, y_t, with a very general specification of dynamics:

$$S_t = \rho_S S_{t-1} + (1 - \rho_S)[g_y y_t + g_\pi(\pi_t - L_t)] + u_{S,t} \tag{4}$$

$$u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}. \tag{5}$$

[1] The general identification of the overall level of interest rates with the perceived inflation goal of the central bank is a common theme in the recent macro-finance literature (notably, Kozicki and Tinsley, 2001, Gürkaynak, Sack, and Swanson, 2005, Dewachter and Lyrio, 2006, and Hördahl, Tristani, and Vestin, 2006).

The dynamices of S_t allow for both policy inertia and serially correlated elements not included in the simple static Taylor rule.[2]

The dynamics of the macroeconomic determinants of the short rate are then specified with equations for inflation and output that are motivated by New Keynesian models (adjusted to apply to monthly data):[3]

$$\pi_t = \mu_\pi L_t + (1 - \mu_\pi)[\alpha_{\pi_1}\pi_{t-1} + \alpha_{\pi_2}\pi_{t-2}] + \alpha_y y_{t-1} + \varepsilon_{\pi,t} \tag{6}$$

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y)[\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}] - \beta_r(i_{t-1} - L_{t-1}) + \varepsilon_{y,t} \ . \tag{7}$$

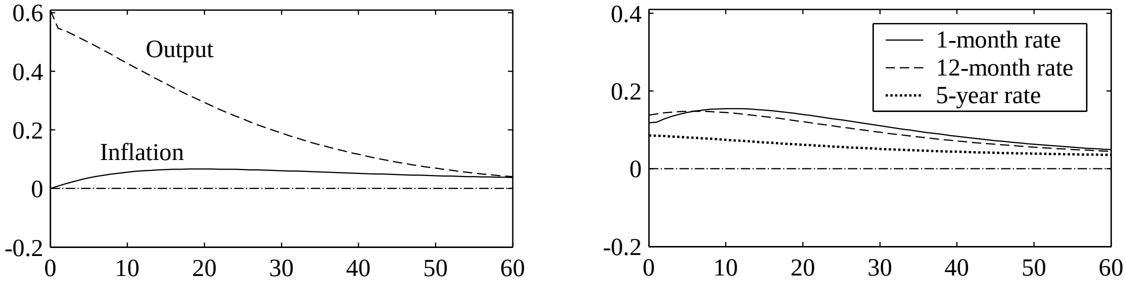
That is, inflation responds to the public's expectation of the medium-term inflation goal (L_t), two lags of inflation, and the output gap. Output depends on expected output, lags of output, and a real interest rate. A key inflation parameter is μ_π, which measures the relative importance of forward- versus backward-looking pricing behavior. Similarly, the parameter μ_y measures the relative importance of expected future output versus lagged output, where the latter term is crucial to account for real-world costs of adjustment and habit formation (e.g., Fuhrer and Rudebusch 2004).

The specification of long-term yields in this macro-finance model follows a standard no-arbitrage formulation. The state space of the combined macro-finance model can be expressed by a Gaussian VAR(1) process.[4] Some interesting empirical properties of this macro-finance model, estimated on US data, are illustrated in Figures 1 and 2. These figures display the impulse responses of macroeconomic variables and bond yields to a one standard deviation increase in two of the four structural shocks in the model. Each response is measured as a percentage point deviation from the steady state. Figure 1 displays the impulse responses to a positive output shock, which increases capacity utilization by .6 percentage point. The higher output gradually boosts inflation, and in response to higher output and inflation, the central bank increases the slope factor and interest rates. The interest rate responses are shown in the second panel. Bond yields of all maturities show similar increases and remain about 5 basis points higher than their initial levels even five years after the shock.

[2]If $\rho_u = 0$, the dynamics of S_t arise from monetary policy partial adjustment; conversely, if $\rho_S = 0$, the dynamics reflect the Fed's reaction to serially correlated information or events not captured by output and inflation. Rudebusch (2002, 2006) describes how the latter is often confused with the former in empirical applications.

[3]Much of the appeal of this specification is its theoretical foundation in a dynamic general equilibrium theory with temporary nominal rigidities.

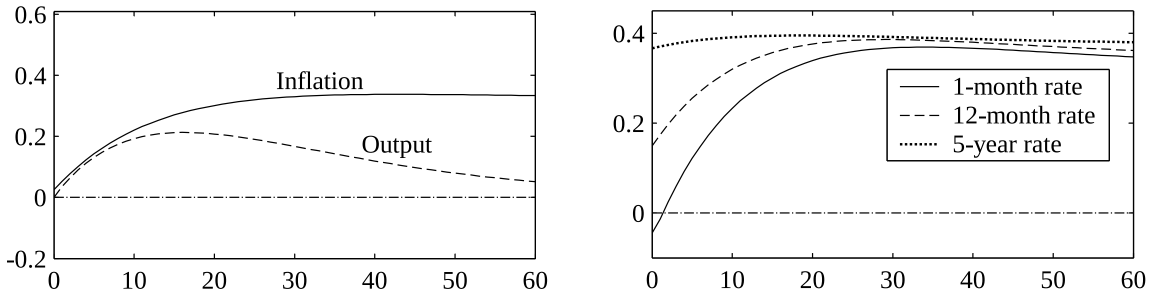
[4]There are four structural shocks, $\varepsilon_{\pi,t}$, $\varepsilon_{y,t}$, $\varepsilon_{L,t}$, and $\varepsilon_{S,t}$, which are assumed to be independently and normally distributed. The risk price associated with the structural shocks is assumed to be a linear function of only L_t and S_t. However, the macroeconomic shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{y,t}$ *are* able to affect the price of risk through their influence on π_t and y_t and, therefore, on the latent factors, L_t and S_t.



(a) Output and inflation response to output shock

(b) Interest rate response to output shock

Figure 1: **Impulse Responses to an Output Shock**
All responses are percentage point deviations from baseline. The time scale is in months.



(a) Output and inflation response to level shock

(b) Interest rate response to level shock

Figure 2: **Impulse Responses to a Level Shock**
All responses are percentage point deviations from baseline. The time scale is in months.

This persistence reflects the fact that the rise in inflation has passed through to the perceived inflation target L_t. One noteworthy feature of Figure 1 is how long-term interest rates respond to macroeconomic shocks. As stressed by Gürkaynak, Sack, and Swanson (2005), long rates do appear empirically to respond to news about macroeconomic variables; however, standard macroeconomic models generally cannot reproduce such movements because their variables revert to the steady state too quickly. By allowing for time variation in the inflation target, the macro-finance model can generate long-lasting macro effects and hence long rates that do respond to the macro shocks.

Figure 2 provides the responses of the variables to a perceived shift in the inflation target or level factor.[5] The first column displays the impulse responses to such a level shock, which increases the inflation target by 34 basis points—essentially on a permanent basis. In order to push inflation up to this higher target, the monetary authority must ease rates, so the slope factor and the 1-month rate fall immediately after the level shock. The short rate then

[5]Such a shift could reflect the imperfect transparency of an unchanged actual inflation goal in the United States or its imperfect credibility. Overall then, in important respects, this analysis improves on the usual monetary VAR, which contains a flawed specification of monetary policy (Rudebusch, 1998). In particular, the use of level, slope, and the funds rate allows a much more subtle and flexible description of monetary policy.

gradually rises to a long-run average that essentially matches the increase in the inflation target. The 12-month rate reaches the new long-run level more quickly, and the 5-year yield jumps up to that level immediately. The easing of monetary policy in real terms boosts output and inflation. Inflation converges to the new inflation target, but output returns to near its initial level.

2.2 Two Applications of the Rudebusch-Wu Model

Two applications of the Rudebusch-Wu model illustrate the range of issues that such a macro-finance model can address. The first of these is an exploration of the source of the Great Moderation—the period of reduced macroeconomic volatility from around 1985 to 2007. Several factors have been suggested as possible contributors to this reduction: better economic policy, a temporary run of smaller economic shocks, and structural changes such as improved inventory management. In any case, the factors underlying reduced macro volatility likely also affected the behavior of the term structure of interest rates, and especially the size and dynamics of risk premiums. Therefore, Rudebusch and Wu (2007) use their macro-finance model to consider whether the bond market's assessment of risk has shifted in such a way to shed light on the Great Moderation. Their analysis begins with a simple empirical characterization of the recent shift in the term structure of US interest rates using subsample regressions of the change in a long-term interest rate on the lagged spread between long and short rates.[6] The estimated regression coefficients do appear to have shifted in the mid-1980s, which suggests a change in the dynamics of bond pricing and risk premiums that coincided with the start of the Great Moderation.

These regression shifts can be modeled within an arbitrage-free model framework. Estimated subsample finance arbitrage-free models (without macro variables) can parse out whether the shift in term structure behavior reflects a change in underlying *factor dynamics* or a change in *risk pricing*. The results show that changes in pricing risk associated with the "level" factor are crucial for accounting for the shift in term structure behavior. The Rudebusch-Wu macro-finance model interprets the decline in the volatility of term premiums over time as reflecting declines in the conditional volatility and price of risk of the term structure level factor, which is linked in the model to investors' perceptions of the central bank's inflation target. The payoff from a macro-finance analysis is thus bidirectional. The macro contribution illuminates the nature of the shift in the behavior of the term structure, high-

[6] Following Campbell and Shiller (1991), such regressions have been used to test the expectations hypothesis of the term structure, but the regression evidence also provides a useful summary statistic of the changing behavior of the term structure.

lighting the importance of a shift in investors' views regarding the risk associated with the inflation goals of the monetary authority. The finance contribution suggests that more than just good luck was responsible for the quiescent macroeconomic period. Instead, a favorable change in economic dynamics, likely linked to a shift in the monetary policy environment, may have been an important element of the Great Moderation. Of course, the very recent period of financial panic, higher risk spreads, and greater macroeconomic volatility is at least a temporary lapse from the Great Moderation and may signal its end. From the perspective of Rudebusch and Wu (2007), such a change would be consistent with the greater fears of higher long-term inflation.

As a second application of the macro-finance model, Rudebusch, Swanson, and Wu (2006) examine the "conundrum" of surprisingly low long-term bond yields during the 2004-6 tightening of US monetary policy. While the Federal Reserve raised the federal funds rate from 1 percent in June 2004 to 5-1/4 percent in December 2006, the 10-year US Treasury yield actually edged down, on balance, from 4.7 percent to 4.6 percent over that same period. This directional divergence between short and long rates was at odds with historical precedent and appears even more unusual given other economic developments at the time, such as a solid economic expansion, a falling unemployment rate, rising energy prices, and a deteriorating federal fiscal situation, all of which have been associated with higher long-term interest rates in the past rather than lower. Of course, determining whether long-term interest rate movements represent a genuine puzzle requires a theoretical framework that takes into account the various factors that affect long-term rates, and a macro-finance perspective appears well-suited to such an investigation.

A summary of the Rudebusch-Wu model interpretation of the bond yield conundrum is shown in Figures 3 and 4. Figure 3 shows the 10-year zero-coupon US Treasury yield from 1984 through 2006 together with the model decomposition of that yield. The model-implied risk-neutral rate is the model's estimated yield on a riskless 10-year zero-coupon bond. The model-implied 10-year Treasury yield is the model's estimated yield on that same bond after accounting for risk. The model-implied term premium is the difference between these two lines. The model does not match the data perfectly, so the model's residuals—the difference between the model predictions taking into account risk and the data—are graphed in Figure 4. Despite the model's excellent fit to the data overall, the low 10-year yields during 2004 through 2006 is an episode that the model notably fails to fit. The model's residuals during this period averaged around 40 to 50 basis points. This large and persistent model deviation is consistent with a bond yield conundrum. Rudebusch, Swanson, and Wu (2006) also examined

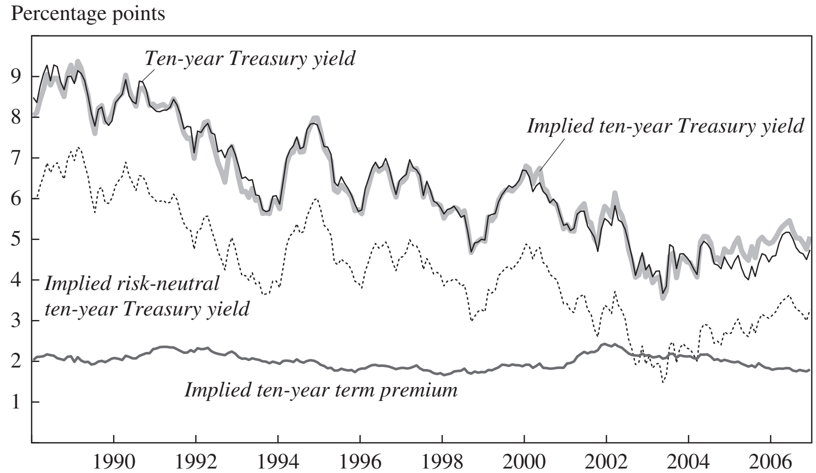


Figure 3: **Rudebusch-Wu Model Decomposition of Ten-Year Yield**
The ten-year US Treasury bond yield, the implied (or fitted) yield from the Rudebusch-Wu model, and the model decomposition of the yield into an expectations component (the risk-neutral rate) and a term premium.

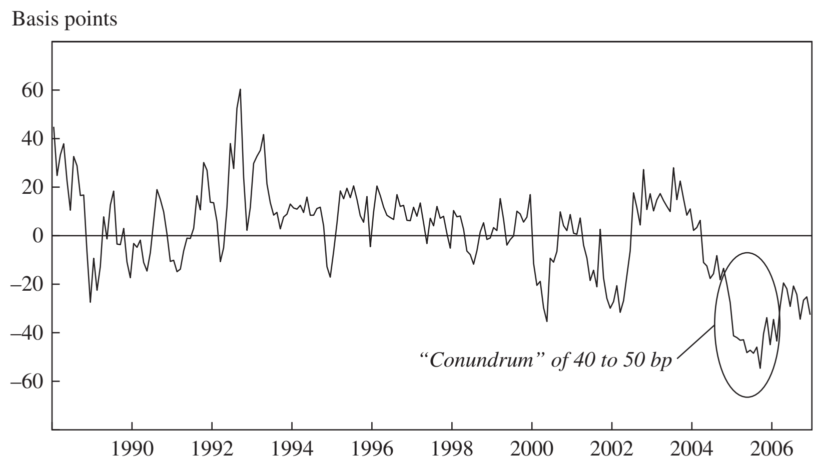


Figure 4: **Rudebusch-Wu Model Residuals for Ten-Year Yield**
The unexplained portion of the ten-year Treasury yield in the Rudebusch-Wu model.

several popular explanations for the conundrum by regressing the model's residuals on various proxies for uncertainty or volatility; however, the unusually low levels of long-term interest rates remained mostly unaccounted for in such an analysis. Of course, with the benefit of hindsight, it now appears that the bond yield conundrum was part of a broader global credit boom that was characterized by an underpricing of many types of risk, especially for fixed-income securities. Uncovering the source of that credit boom—the antecedent for the recent financial crisis—remains an important area of future research, and a macro-finance perspective is likely to be useful in that investigation.

3 Bond pricing in a DSGE Model

A second macro-finance term structure research direction has focused on the bond pricing implications of a standard macroeconomic model. Early work on bond pricing by Backus, Gregory, and Zin (1989) examined the bond premium using a consumption-based asset pricing model of an endowment economy. They found that "the representative agent model with additively separable preferences fails to account for the sign or the magnitude of risk premiums" and "cannot account for the variability of risk premiums" (p. 397). This basic inability of a standard theoretical model to generate a sufficiently large and variable nominal bond risk premium has been termed the "bond premium puzzle." Subsequently, Donaldson, Johnson, and Mehra (1990) and Den Haan (1995) showed that the bond premium puzzle is likewise present in standard real business cycle models with variable labor and capital and with or without simple nominal rigidities. Since these early studies, however, the "standard" theoretical model in macroeconomics has undergone dramatic changes and now includes a prominent role for habits in consumption and nominal rigidities that persist for several periods (such as staggered Taylor (1980) or Calvo (1983) price contracts), both of which may help the model account for the term premium.

Indeed, the bond premium puzzle has again attracted recent interest in the finance and macro literatures. Wachter (2006) and Piazzesi and Schneider (2006) have some success in resolving this puzzle within an endowment economy by using preferences that have been modified to include either an important role for habit, as in Campbell and Cochrane (1999), or "recursive utility," as in Epstein and Zin (1989). While such success in an endowment economy is encouraging, it is somewhat unsatisfying because the lack of structural relationships between the macroeconomic variables precludes studying many questions of interest. Accordingly, there has been interest in extending the endowment economy results to more fully specified DSGE models. Wu (2006), Bekaert, Cho, and Moreno (2005), Hördahl, Tristani, and Vestin (2007), and Doh (2006) use the stochastic discount factor from a standard DSGE model to study the term premium, but to solve the model, these authors have essentially assumed that the term premium is constant over time—that is, they have essentially assumed the expectations hypothesis. Assessing the variability as well as the level of the term premium, and the relationship between the term premium and the macroeconomy, requires a higher-order approximate solution method or a global nonlinear method, as in Ravenna and Seppälä (2006), Rudebusch, Sack, and Swanson (2007), Rudebusch and Swanson (2008, 2009), and

Gallmeyer, Hollifield, and Zin (2005). Still, it remains unclear whether the size and volatility of the bond premium can be replicated in a DSGE model without distorting its macroeconomic fit and stochastic moments.[7] The remainder of this section, which summarizes Rudebusch, Sack, and Swanson (2007), and Rudebusch and Swanson (2008, 2009) introduces a benchmark DSGE model and describes the implications of that model, and an alternative version with Epstein-Zin preferences, for matching both macroeconomic and financial moments in the data.

3.1 A Benchmark DSGE Model

The basic features of the simple benchmark DSGE model examined in Rudebusch and Swanson (2008) are as follows. Representative households are assumed to have preferences over consumption and labor streams given by:

$$\max \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - bc_{t-1})^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \right), \tag{8}$$

where β denotes the household's discount factor, c_t denotes consumption in period t, l_t denotes labor, bc_{t-1} denotes a predetermined stock of consumption habits, and γ, χ, χ_0, and b are parameters. There is no investment in physical capital in the model, but there is a one-period nominal risk-free bond and long-term default-free nominal bonds. The economy also contains a continuum of monopolistically competitive firms with fixed, firm-specific capital stocks that set prices according to Calvo contracts and hire labor competitively from households. The firms' output is subject to an aggregate technology shock. Furthermore, we assume there is a government that levies stochastic, lump-sum taxes on households and destroys the resources it collects. Finally, there is a monetary authority that sets the one-period nominal interest rate according to a Taylor-type policy rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[i^* + g_y(y_t - y_{t-1}) + g_\pi \pi_t\right] + \varepsilon_t^i, \tag{9}$$

where i^* denotes the steady-state nominal interest rate, y_t denotes output, π_t denotes the inflation rate, ε_t^i denotes a stochastic monetary policy shock, and ρ_i, g_y, and g_π are parameters.

In equilibrium, the representative household's optimal consumption choice satisfies the Euler equation:

$$(c_t - bc_{t-1})^{-\gamma} = \beta \exp(i_t) \mathrm{E}_t (c_{t+1} - bc_t)^{-\gamma} P_t / P_{t+1}, \tag{10}$$

[7] This work has a clear practical applications. For example, central banks around the world use the yield curve to help assess market expectations about future interest rates, but they have long recognized that such information can be obscured by time-varying risk premiums. In theory, the DSGE models also in use at central banks could be used to uncover the term premium component in bond yields.

where P_t denotes the dollar price of one unit of consumption in period t. The stochastic discount factor is given by:

$$m_{t+1} = \frac{\beta(c_{t+1} - bc_t)^{-\gamma}}{(c_t - bc_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}. \tag{11}$$

Bonds are priced via an arbitrage-free stochastic discounting relationship. Specifically, the price of a default-free n-period zero-coupon bond that pays one dollar at maturity, $p_t^{(n)}$, satisfies:

$$p_t^{(n)} = \mathrm{E}_t[m_{t+1} p_{t+1}^{(n-1)}], \tag{12}$$

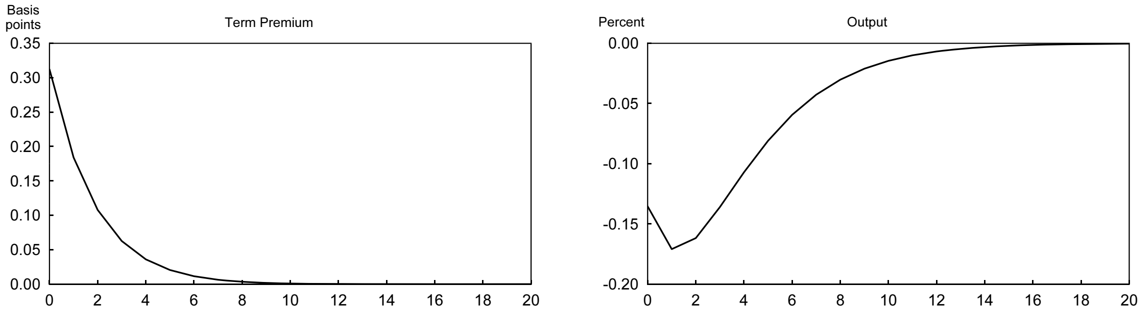
where $p_t^{(0)} = 1$ (the price of one dollar delivered at time t is one dollar). That is, the price of an n-period bond at time t equals the stochastically discounted price of an $n-1$-period bond in the following period.

The term premium can be defined as the difference between the yield on an n-period bond and the expected average short-term yield over the same n periods. Let $i_t^{(n)}$ denote the continuously compounded n-period bond yield (with $i_t \equiv i_t^{(1)}$); then the term premium, denoted $\psi_t^{(n)}$, can be computed from the stochastic discount factor in a straightforward manner:

$$\begin{aligned} i_t^{(n)} - \frac{1}{n}\mathrm{E}_t \sum_{j=0}^{n-1} i_{t+j} &= -\frac{1}{n}\log p_t^{(n)} + \frac{1}{n}\mathrm{E}_t \sum_{j=0}^{n-1} \log p_{t+j}^{(1)} \\ &= -\frac{1}{n}\log \mathrm{E}_t \left[\prod_{j=1}^{n} m_{t+j}\right] + \frac{1}{n}\mathrm{E}_t \sum_{j=1}^{n} \log \mathrm{E}_{t+j-1} m_{t+j}. \end{aligned} \tag{13}$$

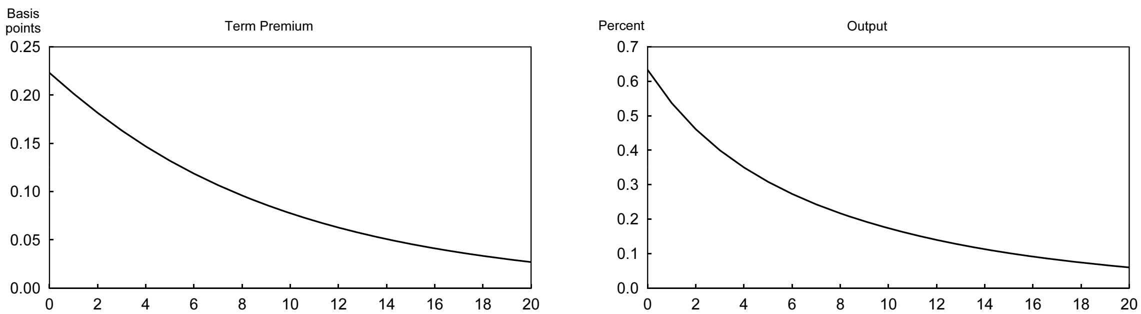
This equation highlights the endogeneity of the term premium. Movements in the term premium reflect changes in the stochastic discount factor, and in general, the stochastic discount factor, will respond to all of the various shocks affecting the economy, including innovations to monetary policy, technology, and government purchases.

Note that, even though the nominal bond in this model is default-free, it is still risky in the sense that its price can covary with the household's marginal utility of consumption. For example, when inflation is expected to be higher in the future, then the price of the bond generally falls because households discount its future nominal coupons more heavily. If times of high inflation are correlated with times of low output (as is the case for technology shocks in the model), then households regard the nominal bond as being very risky, because it loses value at exactly those times when the household values consumption the most. Alternatively, if inflation is not very correlated with output and consumption, then the bond is correspondingly less risky. In the former case, the bond would carry a substantial risk premium (its price



(a) Impulse response for term premium (b) Impulse response for output

Figure 5: **Impulse Responses to a Monetary Policy Shock**
The time scale is in quarters.



(a) Impulse response for term premium (b) Impulse response for output

Figure 6: **Impulse Responses to a Fiscal Spending Shock**
The time scale is in quarters.

would be lower than the risk-neutral price), while in the latter case, the risk premium would be smaller.

For a given set of standard parameters, this benchmark model can be solved and responses of the term premium and the other variables of the model to economic shocks can be computed. Figures 5 and 6 show the impulse response functions of the term premium and output to a monetary policy shock and a government purchases shock, respectively. These impulse responses demonstrate that the relationship between the term premium and output depends on the type of structural shock. For the monetary policy shock, a rise in the term premium is associated with current and future weakness in output. By contrast, for a shock to government purchases, a rise in the term premium is associated with current and future output strength. Thus, even the sign of the correlation between the term premium and output depends on the nature of the underlying shock that is hitting the economy.[8]

[8] Although there is no structural relationship running from the term premium to economic activity, Rudebusch, Sack, and Swanson (2007) also describe reduced-form empirical evidence that a decline in the term premium has typically been associated with stimulus to real economic activity, which is consistent with the view prevalent among market analysts and central bankers.

A second observation to draw from Figures 5 and 6 is that, in each case, the response of the term premium is very small, amounting to less than one-third of one basis point even at the peak of the response. Such minuscule responses raise serious questions about the ability of a benchmark DSGE model to match the nominal asset pricing facts. Indeed, standard DSGE models, even with nominal rigidities, labor market frictions, and consumption habits, appear to fall short of being able to price nominal bonds (Rudebusch and Swanson, 2008).

3.2 A DSGE Model with Epstein-Zin Preferences

The term premium on long-term nominal bonds compensates investors for inflation and consumption risks over the lifetime of the bond. A large finance literature finds that these risk premiums are substantial and vary significantly over time (e.g., Campbell and Shiller, 1991, Cochrane and Piazzesi, 2005); however, the economic forces that can justify such large and variable term premiums are less clear. The benchmark DSGE results—notably the insensitivity of bond premiums described above—are discouraging, but there may be modifications to the DSGE framework that allow it to match bond pricing facts. Piazzesi and Schneider (2006) provide some economic insight into the source of a large positive mean term premium in a consumption-based asset pricing model of an endowment economy with Epstein-Zin preferences. They show that investors require a premium for holding nominal bonds because a positive inflation surprise lowers a bond's value and is associated with lower future consumption growth. Using a similar structure—characterized by both Epstein-Zin preferences and reduced-form consumption and inflation empirics—Bansal and Shaliastovich (2007) also obtain significant time variation in the term premium. However, it is not certain that these endowment economy results will carry over to the DSGE setting. Therefore, Rudebusch and Swanson (2009) augment the standard DSGE model with Epstein-Zin preferences and evaluate the model on its ability to match both basic macroeconomic moments (e.g., the standard deviations of consumption and inflation) and basic bond pricing moments (e.g., the means and volatilities of the yield curve slope and bond excess holding period returns).[9]

As above, assume that a representative household chooses state-contingent plans for consumption, c, and labor, l, so as to maximize expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{14}$$

subject to an asset accumulation equation, where $\beta \in (0,1)$ is the household's discount factor

[9]Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2008) also price bonds in a DSGE model with Epstein-Zin preferences.

and the period utility kernel is $u(c_t, l_t)$. The maximand in this equation can be expressed in first-order recursive form as:

$$V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1}, \tag{15}$$

where the household's state-contingent plans at time t are chosen so as to maximize V_t.

This household value function can be generalized to an Epstein-Zin utility specification:

$$V_t \equiv u(c_t, l_t) + \beta \left(E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}, \tag{16}$$

where the parameter α can take on any real value. The key advantage of using an Epstein-Zin specification is that it breaks the equivalence between the inverse of the intertemporal elasticity of substitution and the coefficient of relative risk aversion that has long been noted in the literature regarding expected utility—see, e.g., Mehra and Prescott (1985) and Hall (1988). With Epstein-Zin preferences, the intertemporal elasticity of substitution over deterministic consumption paths remains the same, but now the household's risk aversion to uncertain lotteries over V_{t+1} can be amplified by the additional parameter α, a feature which is crucial for fitting both asset pricing and macroeconomic facts.

Indeed, while the term premium implied by the benchmark expected utility DSGE model is both too small and far too stable, Rudebusch and Swanson (2009) show that the DSGE model with Epstein-Zin preferences can produce a sizable and sufficiently variable term premium (as well as plausible yield curve slopes and excess holding period returns). Furthermore, the DSGE model with Epstein-Zin preferences fits all of the macroeconomic variables about as well as the standard utility version of the model. Even for relatively high levels of risk aversion, the dynamics of the macroeconomic variables implied by the model are largely unchanged, a finding that has also been noted by Tallarini (2000) and Backus, Routledge, and Zin (2007). Intuitively, the model is identical, up to first order, to standard macroeconomic DSGE representations because the first-order approximation to Epstein-Zin preferences is the same as the first-order approximation to standard expected utility preferences. Furthermore, the macroeconomic moments of the model are not very sensitive to the additional second- and higher-order terms introduced by Epstein-Zin preferences, while risk premiums are unaffected by first-order terms and completely determined by those second- and higher-order terms. Therefore, by varying the Epstein-Zin risk-aversion parameter while holding the other parameters of the model constant, the DSGE model is able to fit the asset pricing facts without compromising its ability to fit the macroeconomic data.

Although Epstein-Zin preferences appear useful in letting the DSGE model replicate certain bond pricing facts without compromising its ability to fit macroeconomic facts, the DSGE model financial sector remains far too rudimentary in terms of financial frictions and intermediation, and these remain important areas for future research.

4 The Arbitrage-Free Nelson-Siegel Model

Researchers have produced a vast literature of models of the yield curve. Many of these models are arbitrage-free latent factor models. Unfortunately, there are many technical difficulties involved with the estimation of AF latent factor models, which tend to be overparameterized and have numerous likelihood maxima that have essentially identical fit to the data but very different implications for economic behavior (Kim and Orphanides, 2005, Duffee, 2008, and Kim, 2009). The difficulties associated with simple, finance-only term structure models—multiple local optima, imprecise parameter estimates, and unknown small-sample distributions—are magnified when adding the greater complexity of macroeconomic interactions and have hindered their extension to macro-finance applications. For many finance researchers, the additional computational cost of adding serious macroeconomic relationships is too high. Similarly, for many macro researchers, the burden of modeling time varying term premiums is also too heavy. Therefore, an empirically tractable arbitrage-free term structure model would be a powerful tool that could potentially help illuminate many issues.

In this spirit, Christensen, Diebold, and Rudebusch (2007) introduce a new version of the arbitrage-free model that maintains the Nelson-Siegel factor loading structure for the yield curve. This arbitrage-free Nelson-Siegel (AFNS) model combines the best of two yield-curve modeling traditions. Although it maintains the theoretical restrictions of the affine AF modeling tradition, the Nelson-Siegel structure helps identify the latent yield-curve factors, so the AFNS model can be easily and robustly estimated. Furthermore, the AFNS model exhibits superior empirical forecasting performance. This section briefly describes the AFNS model and then provides two applications that illustrate its use for macro-finance investigations. In the first application, better measures of inflation expectations are obtained using an estimated AFNS model that captures the pricing of both nominal and real Treasury securities. In the second application, the effect of central bank liquidity facilities is determined in an estimated six-factor AFNS model of US Treasury yields, financial corporate bond yields, and term interbank rates.

4.1 The AFNS Term Structure Model

In contrast to the popular finance arbitrage-free models, many other researchers have employed representations that are empirically appealing but not well grounded in theory. Most notably, the Nelson-Siegel (1987) curve provides a remarkably good fit to the cross section of yields in many countries and has become a widely used specification among financial market practitioners and central banks (e.g., Svensson, 1995, Bank for International Settlements, 2005, and Gürkaynak, Sack, and Wright, 2007). Although for some purposes such a static representation is useful, a dynamic version is required to understand the evolution of bond prices over time. Hence, Diebold and Li (2006) develop a dynamic model based on the Nelson-Siegel curve and show that it corresponds exactly to a modern factor model, with yields that are affine in three latent factors, L_t, S_t, and C_t. In particular, the yield on a zero-coupon Treasury bond with maturity n at time t, $i_t(n)$, is given by:

$$i_t^{(n)} = L_t + S_t\Big(\frac{1-e^{-\lambda n}}{\lambda n}\Big) + C_t\Big(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\Big). \tag{17}$$

The factor loading for L_t is a constant one that does not decay with maturity. The factor loading for S_t starts at 1 and decays monotonically to 0. The factor loading for C_t starts at 0, increases, then decays to zero. These loadings ensure that L_t, S_t, and C_t have a standard interpretation of level, slope, and curvature. (The parameter λ determines the exact shape of these loadings.) Diebold and Li (2006) assume an autoregressive structure for the factors, which produces a fully dynamic Nelson-Siegel (DNS) specification.

A DNS model is easy to estimate and forecasts the yield curve quite well. Despite its good empirical performance, however, this model does not impose the presumably desirable theoretical restriction of absence of arbitrage (Diebold, Piazzesi, and Rudebusch, 2005). Indeed, the results of Filipović (1999) imply that *whatever* stochastic dynamics are chosen for the DNS factors, it is impossible to preclude arbitrage at the bond prices implicit in the resulting Nelson-Siegel yield curve. However, Christensen, Diebold, and Rudebusch (2007) show how to obtain the Nelson-Siegel factor loadings with just a small time-invariant adjustment term.[10] Specifically, nominal yields are assumed to depend on a state vector of the three nominal factors (i.e., level, slope, and curvature) denoted as $X_t = (L_t, S_t, C_t)$. The instantaneous risk-free rate is given by

$$i_t = L_t + S_t, \tag{18}$$

[10] Furthermore, Christensen, Diebold, and Rudebusch (2009) also provide generalizations of the AFNS model along the lines of the Svensson (1995) extension, which adds a second curvature term and is widely used at central banks.

while the dynamics of the three state variables under the risk-neutral (or Q) pricing measure are given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \begin{pmatrix} \sigma_L & 0 & 0 \\ 0 & \sigma_S & 0 \\ 0 & 0 & \sigma_C \end{pmatrix} \begin{pmatrix} dW_t^{Q,L} \\ dW_t^{Q,S} \\ dW_t^{Q,C} \end{pmatrix}, \tag{19}$$

where W^Q is a standard Brownian motion in $\mathbf{R}^3$. [11] Given this affine framework, Christensen, Diebold, and Rudebusch (2007) show that the yield on a zero-coupon Treasury bond with maturity n at time t, is given by

$$i_t^{(n)} = L_t + \left(\frac{1 - e^{-\lambda n}}{\lambda n}\right) S_t + \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) C_t + \frac{A(n)}{n}. \tag{20}$$

That is, the three factors are given exactly the same level, slope, and curvature factor loadings as in the Nelson-Siegel (1987) yield curve. A shock to L_t affects yields at all maturities uniformly; a shock to S_t affects yields at short maturities more than long ones; and a shock to C_t affects mid-range maturities most.[12] The yield function also contains a yield-adjustment term, $\frac{A(n)}{n}$, that is time-invariant and depends only on the maturity of the bond.

4.2 Two Applications of the AFNS Model

A first application of the AFNS model, in Christensen, Lopez, and Rudebusch (2008), produces estimates of the inflation expectations of financial market participants from prices of nominal and real bonds. While nominal bonds have a fixed notional principal, real bonds are directly indexed to overall price inflation. For example, the principal and coupon payments of US Treasury inflation-protected securities (TIPS) vary with changes in the consumer price index (CPI). Differences between comparable-maturity nominal and real yields are known as breakeven inflation (BEI) rates. However, BEI rates are imperfect measures of inflation expectations because they also include compensation for inflation risk. That is, a BEI rate could rise if future inflation uncertainty rose or if investors required greater compensation for that uncertainty, even if expectations for the future level of inflation remained unchanged.

[11] The diagonal volatility matrix is found to diminish out-of-sample forecast performance. The AFNS model dynamics under the Q-measure may appear restrictive, but coupled with general risk pricing, they provide a very flexible modeling structure. This model has also been generalized to allow for stochastic volatility in Christensen, Lopez, and Rudebusch (2010).

[12] Again, it is this identification of the general *role* of each factor, even though the factors themselves remain unobserved and the precise factor loadings depend on the estimated λ, that ensures the estimation of the AFNS model is straightforward and robust—unlike the maximally flexible affine arbitrage-free model.

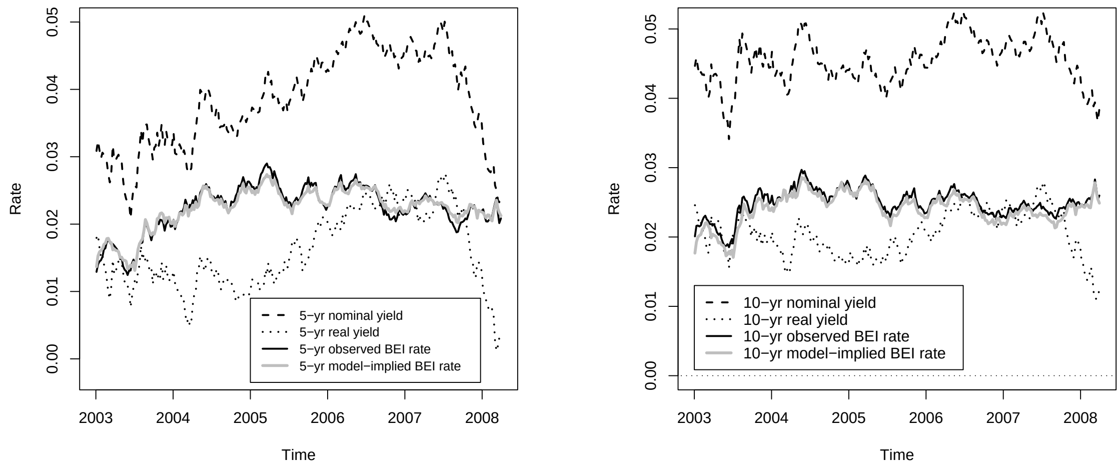
Obtaining a timely decomposition of BEI rates into inflation expectations and inflation risk premiums is of keen interest to market participants, researchers, and central bankers.

The decomposition of a BEI rate into inflation expectations and an inflation risk premium depends on the correlations between inflation and the unobserved stochastic discount factors of investors. Such a decomposition requires a model, and Christensen, Lopez, and Rudebusch (2008) use an affine four-factor AFNS model for this purpose. This model specifies the risk-neutral evolution of the underlying yield-curve factors as well as the dynamics of risk premiums. The resulting model describes the dynamics of the nominal and real stochastic discount factors and can decompose BEI rates of any maturity into inflation expectations and inflation risk premiums.[13] For parsimony—while still maintaining good fit—Christensen, Lopez and Rudebusch (2008) impose the assumption of a common slope factor across the nominal and real yields. Therefore, their joint model has four factors: a real level factor (L_t^R) that is specific to TIPS yields only; a nominal level factor (L_t^N) for nominal yields; and common slope and curvature factors. The joint four-factor AF model fits both the nominal and real yield curves quite well. Figure 7 shows the five- and ten-year nominal and real zero-coupon yields and their differences—i.e., the associated observed BEI rates, which have changed little on balance since 2004. Figure 7 also compares these observed BEI rates to comparable-maturity model-implied BEI rates, which are calculated as the differences between the fitted nominal and real yields from the estimated joint AFNS model. The small differences between the observed and model-implied BEI rates reflect the overall good fit of the model.

This joint AFNS model also can decompose the BEI rate into inflation expectations and the inflation risk premia at various horizons. Given the estimated model parameters and the estimated paths of the four state variables, the model-implied average five- and ten-year expected inflation series are illustrated in Figure 8. The model's estimates of inflation expectations were generated using only nominal and real yields without any data on inflation or inflation expectations. To provide some independent indication of accuracy, Figure 8 also plots survey-based measures of expectations of CPI inflation, which are obtained from the Blue Chip Consensus survey at the five-year horizon and from the Survey of Professional Forecasters at the ten-year horizon. The relatively close match between the model-implied and the survey-based measures of inflation expectations provides further support for the model's decomposition of the BEI rate.

A second macro-finance application of the AFNS model, provided in Christensen, Lopez

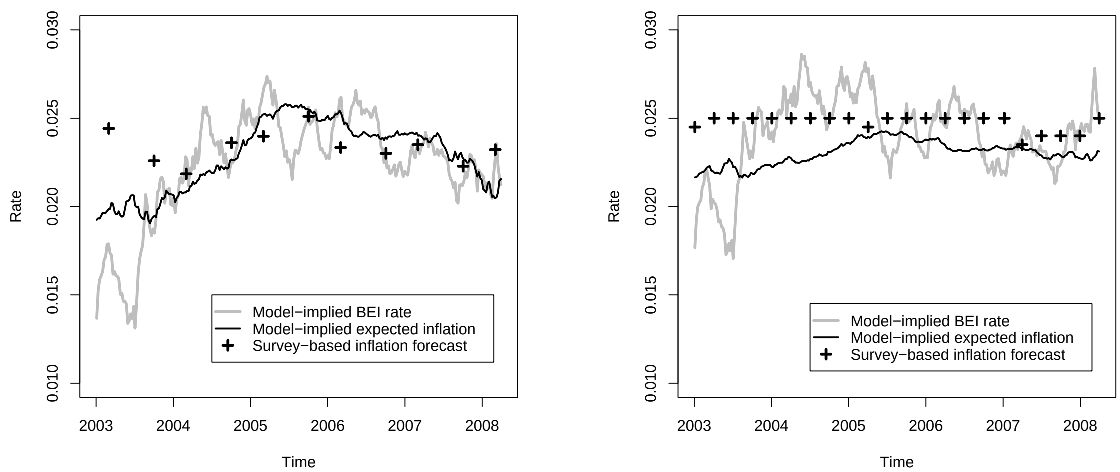
[13]Related research includes Ang, Bekaert, and Wei (2008), Chernov and Mueller (2008), Hördahl and Tristani (2008), D'Amico, Kim, and Wei (2008), Haubrich, Pennacchi, and Ritchken (2008), and Adrian and Wu (2008).



(a) Five-year maturity

(b) Ten-year maturity

Figure 7: **Nominal and Real Yields and BEI Rates**
Five- and ten-year nominal and real zero-coupon US Treasury yields with associated BEI rates and implied BEI rates from the joint AFNS model.



(a) Five-year horizon.

(b) Ten-year horizon.

Figure 8: **BEI Rates and Expected Inflation**
Five- and ten-year BEI rates, average expected inflation rates implied from the joint AFNS model, and survey-based measures of inflation expectations.

and Rudebusch (2009), investigates the effect of the new central bank liquidity facilities that were instituted during the recent financial crisis. In early August 2007, amidst declining prices and credit ratings for US mortgage-backed securities and other forms of structured credit, international money markets came under severe stress. Short-term funding rates in the interbank market rose sharply relative to yields on comparable-maturity government securities. For example, the three-month US dollar London interbank offered rate (LIBOR) jumped from only 20 basis points higher than the three-month US Treasury yield during the first seven months of 2007 to over 110 basis points higher during the final five months of the year. This enlarged spread was also remarkable for persisting into 2009.

LIBOR rates are widely used as reference rates in financial instruments, including derivatives contracts, variable-rate home mortgages, and corporate notes, so their unusually high levels appeared likely to have widespread adverse financial and macroeconomic repercussions. To limit these adverse effects, central banks around the world established an extraordinary set of lending facilities that were intended to increase financial market liquidity and ease strains in term interbank funding markets, especially at maturities of a few months or more. Specifically, on December 12, 2007, the Bank of Canada, the Bank of England, the European Central Bank (ECB), the Federal Reserve, and the Swiss National Bank jointly announced a set of measures designed to address elevated pressures in term funding markets. These measures included foreign exchange swap lines established between the Federal Reserve and the ECB and the Swiss National Bank to provide US dollar funding in Europe. The Federal Reserve also announced a new Term Auction Facility, or TAF, to provide depository institutions with a source of term funding. The TAF term loans were secured with various forms of collateral and distributed through an auction. These central bank actions were meant to improve the distribution of reserves and liquidity by targeting a narrow market-specific funding problem.

Christensen, Lopez and Rudebusch (2009) assess the effect of the establishment of these extraordinary central bank liquidity facilities on the interbank lending market and, in particular, on term LIBOR spreads over Treasury yields.[14] In theory, the provision of central bank liquidity could lower the liquidity premium on interbank debt through a variety of channels. On the supply side, banks that have a greater assurance of meeting their own unforeseen liquidity needs over time should be more willing to extend term loans to other banks. In addition, creditors should also be more willing to provide funding to banks that have easy and dependable access to funds, since there is a greater reassurance of timely repayment. On the demand side, with a central bank liquidity backstop, banks should be less inclined to borrow from other banks to satisfy any precautionary demand for liquid funds because their future idiosyncratic demands for liquidity over time can be met via the backstop. However, assessing the relative importance of these channels is difficult. Furthermore, judging the efficacy

[14]Related work includes Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008) and Wu (2009), who examine the effect of central bank liquidity facilities on the liquidity premium in LIBOR by controlling for movements in credit risk as measured by credit default swap prices for the borrowing banks in simple event-study regressions.

of central bank liquidity facilities in lowering the liquidity premium is complicated because LIBOR rates, which are for unsecured bank deposits, also include a credit risk premium for the possibility that the borrowing bank may default. The elevated LIBOR spreads during the financial crisis likely reflected both higher credit risk and liquidity premiums, so any assessment of the effect of the recent extraordinary central bank liquidity provisions must also control for fluctuations in bank credit risk.

To analyze the effectiveness of the central bank liquidity facilities in reducing interbank lending pressures, Christensen, Lopez, and Rudebusch (2009) estimate an affine arbitrage-free term structure representation of US Treasury yields, the yields on bonds issued by financial institutions, and term LIBOR rates using weekly data from 1995 to midyear 2008. The resulting six-factor AFNS representation provides arbitrage-free joint pricing of Treasury yields, financial corporate bond yields, and LIBOR rates. Three factors account for Treasury yields, two factors capture bank debt risk dynamics, and a third factor is specific to LIBOR rates. This structure can decompose movements in LIBOR rates into changes in bank debt risk premiums and changes in a factor specific to the interbank market that includes a liquidity premium. It also allows hypothesis testing and counterfactual analysis related to the introduction of the central bank liquidity facilities.

The model results support the view that the central bank liquidity facilities established in December 2007 helped lower LIBOR rates. Specifically, the parameters governing the term LIBOR factor within the model change after the introduction of the liquidity facilities. The hypothesis of constant parameters is overwhelmingly rejected, suggesting that the behavior of this factor, and thus of the LIBOR market, was directly affected by the introduction of central bank liquidity facilities. To quantify the impact that the introduction of the liquidity facilities had on the interbank market, Christensen, Lopez, and Rudebusch (2009) conduct a counterfactual analysis of what would have happened had they *not* been introduced. The full-sample model—without the regime switch—generates the actual and counterfactual paths for the 3-month LIBOR rate. The latter suggests what that spread *might* have been if it had been priced in accordance with prevailing conditions in the Treasury and corporate bond markets for U.S. financial firms.

Figure 9 illustrates the effect of the counterfactual path on the three-month LIBOR spread over the three-month Treasury rate since the beginning of 2007. Note that the model-implied three-month LIBOR spread is close to the observed spread over this period. From the start of the financial crisis—which was triggered by an August 9, 2007, announcement by the French bank BNP Paribas—until the TAF and joint central bank swap announcement in

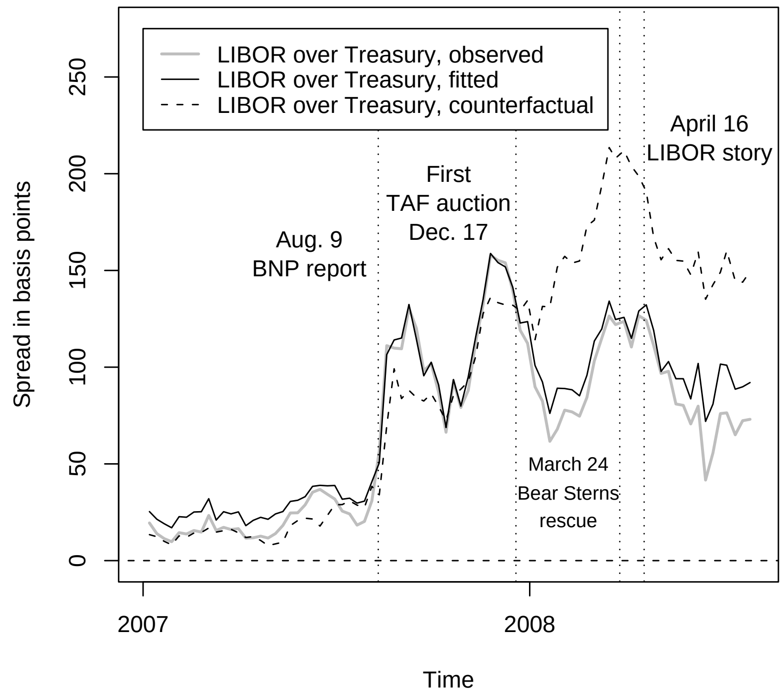


Figure 9: **Spread of LIBOR Rate over Treasury Yield**
Observed and fitted three-month LIBOR rate spread over the three-month Treasury yield in a six-factor model and a counterfactual model-based spread when the LIBOR-specific factor is fixed at its historical average prior to December 14, 2007.

mid-December 2007, the observed LIBOR rate averaged 8 basis points higher that the counterfactual rate. However, by the end of 2007, a significant wedge developed between the two. As of the end of the sample on July 25, 2008, the difference between the counterfactual spread and the observed three-month LIBOR spread was 82 basis points. Therefore, this analysis suggests that the three-month LIBOR rate would have been *higher* in the absence of the central bank liquidity facilities. Accordingly, the announcement of the central bank liquidity facilities on December 12, 2007, likely affected the interbank lending market in the intended way; that is, the increased provision of bank liquidity by central banks lowered LIBOR rates relative to where they might have been in the absence of these actions.

5 Conclusion

The macro-finance term structure literature is in its infancy with many important questions yet to answer. The importance of this research has only been heightened by the latest financial turmoil and economic recession. These recent events were triggered in part by a "fixed-income crisis" involving nominal bonds of various maturities and risk characteristics, which suggests that a better macro-finance understanding of bond pricing and risk premiums may be helpful in elucidating them. However, much of the research surveyed here predates the latest crisis episode and can form only part of a foundation for a broader research agenda to develop a better understanding of the relevant macro-finance linkages.

Indeed, a variety of new questions and issues have taken on a new urgency in the aftermath of the recent crisis. For example, in many countries, short-term interest rates have fallen to their zero lower bound. Furthermore, with inflation fairly well contained at low levels in many countries, the zero lower bound on nominal interest rates is likely to be a binding constraint going forward much more often than it has in the past. The zero bound has been largely ignored in the finance literature. In the future, developing versions of the affine arbitrage-free model that prevent interest rates from going negative will be a priority.[15] A second macro-finance issue highlighted in the recent crisis is the link between bond supply and the risk premium. As the short-term policy rates reached their effective lower bound, various central banks tried to lower longer-term yields by taking various unconventional balance sheet actions. Notably, the Bank of England and the Federal Reserve purchased significant amounts of longer-term securities in order to lower their bond yields.[16] However, existing models can provide little if any guidance to central banks about the link between bond supply, which is effectively reduced by the central bank purchases, and bond risk premiums. Understanding potential quantity effects on bond yields from a macro-finance perspective is also an important future research topic. Finally, the linkages between bond yields in different countries have also been highlighted during the latest crisis. Diebold, Li, and Yue (2008) provide a start, but much more work remains.

[15] Kim (2009) describes some models that respect the zero bound.

[16] McGough, Rudebusch, and Williams (2005) describe the policy rationale for such actions. Rudebusch (2009) provides some specifics about the Federal Reserve's unconventional policies. Krishnamurthy and Vissing-Jorgenson (2008) provide related empirical evidence.

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