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Equilibrium Layoff as Termination of a Dynamic Contract

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Abstract

In a dynamic model of the labor market with moral hazard, equilibrium layoff is modeled as termination of an optimal long-term contract. Termination, together with compensation (current and future), is used as an incentive device to induce worker efforts. I then use the model to study analytically the effects of a firing tax on termination and worker compensation and utility. There are three layers to the impact of a firing tax on layoff and worker utility. A higher firing tax could either reduce aggregate termination and increase worker utility, or increase aggregate termination and reduce worker utility, depending on the structure of the environment.

1 Introduction

In this paper, I construct an equilibrium model of the labor market where layoff is modelled as termination of an optimal long-term contract. I then use the model to study specifically the effects of a firing tax on worker utility, labor turnover, and employment.

In the model, the process of employment contracting is complicated by the unobservability of worker efforts. In order to mitigate moral hazard, firms and workers enter into long-term relationships, where termination is used as an incentive instrument, along with an optimally designed structure for the worker's current and future compensation. In the model, terminated workers are allowed to go back to the labor market to seek new matches and enter into new contracts. Firms, on the other hand, are allowed to hire a replacement, immediately after terminating the old worker. The operation of the optimal contract thus induces positive equilibrium labor market flows, both into and out of employment.

Termination as an incentive device is a natural way to think about layoff. This idea was first successfully explored in the literature of efficiency wages (e.g., Shapiro and Stiglitz (1984)) to model involuntary unemployment. Unemployment is also involuntary in my model. Compared to the efficiency wage models though, my model offers at least two advantages. First, efficiency wage models are often criticized because the employment contracts in these models are not fully optimal. In Shapiro and Stiglitz, for example, because wages are constant, termination (layoff) is the only incentive device that firms have available to prevent workers from shirking. In the model here, workers and firms enter into fully optimal contracts where compensation is determined optimally upon the worker's performance history. Second, in the existing efficiency wage models, in equilibrium *no* workers are actually fired because of shirking. The contract makes effort-making incentive compatible so no one is caught shirking in equilibrium, and the unemployed are just a rotating pool of workers who quit for reasons that are exogenous to the model. In the model here, workers are actually fired (laid off) involuntarily from their jobs, firing is part of the model's equilibrium path.

Layoff as termination of an optimal long-term contract under moral hazard was first formally modelled by Stiglitz and Weiss (1983). They study a two-period closed principal-agent relationship where the terminated worker cannot be replaced. Spear and Wang (2005) study optimal termination by adding an external labor market to an otherwise standard dynamic principal-agent model. In their model, the terminated worker is immediately replaced by a new worker that the firm hires from the labor market, but the terminated worker is never to be employed again. Spear and Wang also characterize the properties of the optimal termination policy in their two-period and infinite horizon settings. In Wang (2005), layoff as termination of an optimal dynamic contract was built into a fully dynamic equilibrium environment to model involuntary layoffs and voluntary retirements simultaneously, both as outcomes of

an optimal dynamic contract. In Wang (2005), endogenous layoffs and retirements generate positive flows in the labor market, into and out of unemployment, and into retirement.

In this paper, I modify the model of Wang (2005) to construct a model economy with overlapping generations of two-period lived, risk neutral workers. This modification allows me to focus on the idea of termination as involuntary layoff. It also allows me to obtain an analytical treatment of the model's stationary equilibria, making the model a convenient vehicle for policy analysis. With the analytical tractability in hand, I then open up the model to allow entry and exit of firms, and to let the number of jobs in the economy determined endogenously. The level of employment is fixed in Wang (2005).

I then use the model to study the effects of a firing tax on the labor market. The labor market impact of firing taxes is an extensively researched topic, motivated partly by the U.S. and European comparison in labor market institutions and experiences. The literature offers two sets of theoretical conclusions. First, higher firing taxes reduce labor turnover and increase unemployment durations, with an ambiguous impact on unemployment rate (e.g., Hopenhayn and Rogerson (1993), Alvarez and Veracierto (1999), Blanchard and Portugal (2001).) Second, higher firing taxes increase equilibrium wages and reduce job creation (e.g., Millard and Mortensen (1997), Mortensen and Pissarides (1998).)

Most models in the above literature follow Hopenhayn and Rogerson (1993) and Mortensen and Pissarides (1994) to use a broadly defined search-matching framework where firms trade off between gains in matching productivity and the costs of firing/hiring to decide whether a worker should be retained or terminated. Typically a bargaining mechanism is used for wage determination. So a higher tax on firing leads firms to fire and hire less frequently. A higher firing tax also gives workers more bargaining power and hence higher wages.

This paper takes a different approach to the same set of questions and offers a set of new insights and conclusions. I do not rely on any of the essential elements for building the existing models for the construction of my model. In my model, workers are homogenous and their productivity remains constant over time. I assume a competitive supply of workers in the labor market to give them zero bargaining power in employment contract negotiations. Compensation is determined by moral hazard and incentive considerations, not through equating demand and supply in the labor market, or through bargaining between the matched firm and worker. I choose to enter into the structure of the optimal contract to look for the mechanism that generates the effects of a firing tax.

At the heart of my analysis is a trade-off between the uses of two costly incentive devices, termination and compensation, for achieving efficiency in employment contracting. Two variables are key to this trade-off: the firing tax, denoted γ , which is the policy instrument of interest in this paper; and the utility of the worker upon termination, denoted w_* . This w_* depends on two variables: the probability of transition

to employment for the unemployed worker, and the utility that a new job offers.

A higher firing tax makes termination more costly to the firm. This forces the firm to rely more on compensation for incentives, reducing the probability of worker termination. On the other hand, the effects of a change in w_* on termination could go both ways. There is a “wealth effect”: A higher w_* makes termination a “softer” punishment to the worker and hence a less effective incentive device for the firm. So with a higher w_* , in order to obtain a given amount of incentives through termination, termination must be used more often. There is also a “substitution effect”: A higher w_* that makes termination a less effective incentive device could induce the firm to shift away from the use of termination to the use of compensation for incentives. In the model, the substitution effect dominates.

A higher firing tax affects termination and layoff at three different levels. First is the cost effect I have just described: other things equal, a higher firing tax reduces termination by making it a more costly incentive device. The second effect of a higher firing tax on termination works through a lower w_* . Other things especially the number of jobs in the economy equal, a higher firing tax lowers the probability for a worker who is laid off to transition back to employment, because of the fewer vacancies available (which in turn is because firms are firing less frequently, due to the first effect I have just described). This lowers the value of w_* and increases the firm’s use of termination for incentives. The third effect is an even deeper equilibrium effect. Since a higher firing tax reduces the value of the firm in the market, it reduces the equilibrium number of jobs in the economy. This in turn reduces the probability for the laid-off worker to obtain new employment, leads to a further decrease in w_* . The substitution effect of w_* on termination works for the second time to increase the use of termination for incentives.

Parallel to the effects of a firing tax on termination, the effects of a firing tax on worker compensation and utility also have three layers in its functioning. Termination, which puts the worker into involuntary unemployment in the state of low output, provides downward incentives only. Compensation, on the other hand, can be used both to punish (in the low output state) and to reward (in the high output state) the worker. Suppose there is an increase in the firing tax to make termination more costly. This induces the contract to shift away from termination and to relying more on the use the compensation for incentives. With the equilibrium contract, because of a binding limited liability constraint on compensation in the low output state, the firm is forced to increase the worker’s compensation in the high output state for incentives. This increases the worker’s expected compensation and utility.

The second effect of a firing tax on worker utility works through w_* . A higher firing tax lowers w_* , as I have already described. This in turn makes termination a more efficient incentive device, as it now implements a more severe punishment to the worker, reducing the burden on compensation for incentives, allowing the firm to offer a lower expected utility to the worker. The third effect of a higher firing tax on worker utility also works through w_* . A higher firing tax reduces the equilibrium

number of firms in the market, lowers w_* and the worker's expected utility.

Which effects dominate? Suppose the supply of jobs in the economy is fixed, and hence the third effect of a higher firing tax on termination and compensation is absent. A higher firing tax always reduces aggregate termination, so the first effect on termination dominates the second. The effects of a higher firing tax on worker utility is not monotonic. A higher firing tax could make some workers worse off (where the second effect dominates) and some better off (where the first effect dominates), but a sufficiently large increase in firing tax from a sufficiently low initial level always increases worker utility.

When the economy's supply of jobs is determined endogenously, the model does not even make an unambiguous prediction on the effects of a higher firing tax on termination. Which effect dominates depends on the structure of the environment. Under a specific structure of the environment I assume in the paper, the net effect of a higher firing tax is reduced termination and increased worker utility. However, a simple extension of the model suggests scenarios in which completely opposite net effects of a firing tax on aggregate termination and worker utility could obtain. Depending on a cost parameter, a higher firing tax could cause aggregate termination to go up and worker utility to go down.

An interesting feature of the model is, depending on the level of the firing tax, in equilibrium identical firms can follow different termination policies and offer different starting utilities to the otherwise homogeneous workers. This feature of the model results from how firms interact strategically in their use of termination for incentives. To see this, suppose other firms use termination more frequently so there is higher turnover in the market. Then the probability with which an unemployed worker finds new employment is higher. This makes termination less effective as an incentive device to punish low outputs. In turn, this pushes an individual firm away from using termination for incentives. Similarly, suppose fewer firms in the market are using termination for incentives. Then termination becomes a more efficient incentive device, individual firms would like to use termination more often. It is this mechanism and the linearity in the firm's value function that give rise to the mix in the firms' termination policies.

Sections 2 and 3 describe the model and the equilibrium. Section 4 offers the solution to the problem of optimal contracting and then provides a characterization for the model's stationary equilibrium. Section 4 also discusses the roles of the model's two incentive instruments: termination and compensation. Section 5 studies the effects of a firing tax. Section 6 studies the effects of a change in the number of firms. Section 7 allows the number of firms (jobs) to be determined endogenously. Section 8 concludes the paper.

2 Model

Let t denote time: $t = 1, 2, \dots$. There is one perishable consumption good in each period. There is a sequence of overlapping generations of workers, each generation containing $1/2$ units of mass. Workers live for two periods, are young in the first and old in the second. The preferences of the workers who are alive in periods t and $t + 1$ are given by:

$$E \{(1 - \beta)H(c_t, a_t) + \beta H(c_{t+1}, a_{t+1})\},$$

where c_t (c_{t+1}) and a_t (a_{t+1}) are period t ($t + 1$) consumption and effort, respectively, and $\beta \in [0, 1)$ is the discount factor. The workers are risk neutral and have $H(c, a) = c - a$, where $c \geq 0$, $a \in \{0, \psi\}$ with $\psi > 0$.

There are $\eta \in (0, 1)$ units of firms in the model. In the next four sections, I will treat η as an exogenous variable, then in Section 7, η will be determined endogenously. Firms live forever and maximize expected discounted net profits. Firms use the same discount rate β to discount future profits.¹ Each firm can employ in each period one worker to produce. The effort that the employed worker makes is observed by the worker himself only. By choosing effort a_t in period t , the worker produces a random output in period t that is a function of a_t . Let $\theta^t \in \Theta$ denote the realization of this random output, where $\Theta = \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$. Let $x_i = \{\theta = \theta_i | a = \psi\}$ and let $x'_i = \{\theta = \theta_i | a = 0\}$. That is, x_i is the probability of output θ_i if the worker works, x'_i is the probability of output θ_i if the worker shirks.

At the beginning of each period, a labor market opens in which unemployed workers and vacant firms are randomly matched. To make the market structure of the model simple, we assume that an unemployed worker has in each period at most one opportunity to be matched with a firm. Since there are more workers than firms in the model, I assume the opportunities to match with a firm are equally distributed among the unemployed.

Now if a firm is matched with an old worker, they sign an one period contract. If a firm is matched with a young worker, they have the opportunity to enter into a long-term contract.² This long-term contract, which can potentially last for two periods, may specify a condition under which the firm fires the worker and replaces him with a new worker at the end of the first period of the contract. When a worker is fired, he is free to go back to the labor market to look for new employment opportunities. As part of the physical environment, I assume that once the worker is fired (i.e., once the contract is terminated), the interaction between the worker and the firm ends. In particular, if a worker is fired by a firm at the end of period t , he will not be able to receive payments from the firm in period $t + 1$.

¹We do not model the capital market to endogenize interest rates.

²I assume in this paper that age discrimination is either infeasible, or excessively costly to the firm, due perhaps to legal restrictions that I do not model. An obvious extension of the model can be constructed to study the effects of age discrimination.

Each time a firm fires a worker, it must incur a fixed cost of $\gamma \geq 0$, in units of consumption. This fixed cost is interpreted as a firing tax levied by the government. In this paper, I do not model how the government makes use of the tax proceeds γ . I simply assume that γ is consumed by the government after it is collected.

3 Equilibrium

In this section, I first define a contract, taking as given the labor market in which the contract must operate; I then define what a labor market equilibrium is.

Let L denote the *equilibrium* number of workers that are unemployed and looking for a job at the beginning of a period. Note that among these workers, $1/2$ units are young workers who have just entered the labor market, the rest are old workers who either never worked or were laid off in the previous period. Let α be the *equilibrium* fraction of the unemployed workers to obtain employment during the period. (That is, α is the probability to transition from unemployment to employment for the unemployed workers.) Let w_* be the *equilibrium* expected utility of an unemployed worker at the beginning of his old age, before the labor market opens. Note that given our assumptions that the consumption good is perishable, and that the firm and the worker cannot maintain a relationship once the worker is fired, all old and unemployed workers are the same, whether or not they were employed at young age.

Firms take the aggregate variables L , α , and w_* as given. In each period, among the η units of firms, some are paired with a young worker, some with an old. The value of the firm depends on the type of the worker (young or old) it is paired with and on the expected utility that the firm must promise to the worker.

Consider first the problem of a firm that is matched with an old worker. A contract is written $\{c_1, c_2\}$, where c_i is the worker's compensation in output state i .³ Let $V_o(w)$ denote the value of this firm where w is the worker's promised expected utility. Then

$$V_o(w) = \max_{c_1, c_2} \left\{ (1 - \beta) \sum x_i (\theta_i - c_i) + \beta \bar{V} \right\} \quad (1)$$

subject to

$$c_1, c_2 \geq 0, \quad (2)$$

$$x_1 c_1 + x_2 c_2 - \psi \geq x'_1 c_1 + x'_2 c_2, \quad (3)$$

$$x_1 c_1 + x_2 c_2 - \psi = w, \quad (4)$$

where \bar{V} denotes the expected value of the firm at the beginning of the next period when it is free to hire a new worker, (2) is a limited liability constraint on compensation, (3) is the incentive constraint, (4) is a promise-keeping constraint.

³This contract can either be viewed as a contract that the firm offers to a newly matched old worker, or it can be viewed as the continuation of a long-term contract for a young worker in the last period and the worker is retained and is now old in the current period.

Next, consider the optimization problem of a firm that is just matched with a young worker. The contract takes the form of

$$\sigma_y = \{c_{ik}, w_{ik}, p_i, i = 1, 2, k = r, f\}$$

where p_i is the probability with which the worker is fired at the end of period 2 if output in period 1 is θ_i ; c_{ik} is the worker's period-1 compensation if the period-1 output is θ_i and that his next period employment state at the firm is k , $k = r, f$, where $k = r(f)$ indicates the worker is retained (fired); Finally, w_{ik} is the worker's expected utility at the beginning of the next period in state ik . Obviously, $w_{if} = w_*$, given that all interactions between the firm and the worker end after the worker is fired.

Now let $V_y(w)$ denote the firm's value conditional on the young worker's promised expected utility being w . I have

$$V_y(w) = \max_{\{c_{ik}, w_{ir}, p_i\}} \sum x_i \left\{ p_i [(1 - \beta)(\theta_i - c_{if} - \gamma) + \beta\bar{V}] + (1 - p_i) [(1 - \beta)(\theta_i - c_{ir}) + \beta V_o(w_{ir})] \right\} \quad (5)$$

subject to

$$c_{ik}, w_{ir} \geq 0, 0 \leq p_i \leq 1, i = 1, 2, k = r, f, \quad (6)$$

$$\begin{aligned} & \sum x_i [(1 - p_i)((1 - \beta)c_{ir} + \beta w_{ir}) + p_i((1 - \beta)c_{if} + \beta w_*)] - \psi \\ & \geq \sum x'_i [(1 - p_i)((1 - \beta)c_{ir} + \beta w_{ir}) + p_i((1 - \beta)c_{if} + \beta w_*)], \end{aligned} \quad (7)$$

$$\sum x_i [(1 - p_i)((1 - \beta)c_{ir} + \beta w_{ir}) + p_i((1 - \beta)c_{if} + \beta w_*)] - \psi = w. \quad (8)$$

The formulation of the firm's problem is completed with

$$\hat{w} = \max \left\{ \arg \max_{w' \geq \beta w_*} V_y(w') \right\}, \quad (9)$$

$$\tilde{w} = \max \left\{ \arg \max_{w' \geq 0} V_o(w') \right\}, \quad (10)$$

$$\bar{V} \equiv \frac{1}{2L} V_y(\hat{w}) + (1 - \frac{1}{2L}) V_o(\tilde{w}), \quad (11)$$

where equation (9) says that the firm chooses \hat{w} , the optimal expected utility of the young worker, to maximize the firm's value subject to the the worker's participation. Note that the participation of the worker requires the worker's expected utility be greater than or equal to βw_* : the worker's expected utility if he turns down the

firm's offer and be unemployed in the first period of his life. Equation (9) also says that if the firm is indifferent between two different levels of the new worker's starting expected utility, then it chooses the \hat{w} to maximize the worker's expected utility.

Similarly, if the firm is matched with an old worker, (10) says that it chooses \tilde{w} as the worker's promised expected utility to maximize the firm's value subject to the participation of the worker. Here, if the old worker turns down the firm's offer, he will be unemployed and his expected utility will be zero.

Finally, equation (11) gives the Bellman equation for \bar{V} . The new worker will be a young worker with probability $(1/2)/L$, and an old worker with probability $1 - (1/2)/L$.

Let $\{(V_o, V_y, \bar{V}), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*; \hat{w}, \tilde{w}\}$ denote a solution to the firm's problem (1)-(11), here $(V_o, V_y), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*$ are all functions of w .

Having described the firm's optimization problem, I now proceed to describe a labor market equilibrium.

Let $\hat{p}_1^* = p_i^*(\hat{w})$. This is the probability with which a young worker is fired in output state i . In steady state it then must hold that

$$L \equiv 1 - \frac{1}{2}\alpha[1 - (x_1\hat{p}_1^* + x_2\hat{p}_2^*)] \quad (12)$$

where $\frac{1}{2}\alpha$ is the measure of the young workers employed in a period, and hence $\frac{1}{2}\alpha[1 - (x_1\hat{p}_1^* + x_2\hat{p}_2^*)]$ is the measure of the workers who are retained (not fired) at the beginning (or end) of a period.

Now among the L units of young and old workers looking for employment at the beginning of a period, a fraction $\frac{\eta - (\alpha/2)[1 - (x_1\hat{p}_1^* + x_2\hat{p}_2^*)]}{1 - (\alpha/2)[1 - (x_1\hat{p}_1^* + x_2\hat{p}_2^*)]}$ will obtain employment. So in steady state it must hold that

$$\alpha = \frac{\eta - (\alpha/2)[1 - (x_1\hat{p}_1^* + x_2\hat{p}_2^*)]}{L} = 1 - \frac{1 - \eta}{L}. \quad (13)$$

Finally, in equilibrium it must also hold that

$$w_* = \alpha\tilde{w}. \quad (14)$$

Definition 1 A stationary equilibrium is a vector

$$\{L, \alpha, w_*; (V_o, V_y, \bar{V}), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*, \hat{w}, \tilde{w}\}$$

where

- (i) $\{(V_o, V_y), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*, \hat{w}, \tilde{w}\}$ solves equations (1)-(11), given L, α , and w_* ;
- (ii) $\{L, \alpha, w_*\}$ satisfy the equilibrium conditions (12)-(14).

4 Solution

I start by considering the firm's optimization problem with an old worker (equations (1)-(4)), taking \bar{V} as given. Here the incentive constraint requires that $c_2 - c_1 \geq \frac{\psi}{x_2 - x'_2}$ which, given the non-negativity of c_1 and the promise-keeping constraint, implies

$$w \geq \underline{w}, \quad \text{where } \underline{w} \equiv \frac{x'_2}{x_2 - x'_2} \psi > 0. \quad (15)$$

Thus, if $w \geq \underline{w}$ then $a^* = \psi$ can be implemented, and an optimal contract is $c_1^*(w) = 0$, $c_2^*(w) = \frac{w + \psi}{x_2}$, and

$$V_o(w) = (1 - \beta)(\bar{\theta} - \psi - w) + \beta\bar{V}, \quad \forall w \geq \underline{w}, \quad (16)$$

where $\bar{\theta} \equiv \sum x_i \theta_i$. If $w < \underline{w}$ then the effort $a = \psi$ cannot be implemented, and the firm's value is $V_o(w) = (1 - \beta)(\underline{\theta} - w) + \beta\bar{V}$, where $\underline{\theta} \equiv \sum x'_i \theta_i$. Throughout the paper I assume that the difference $\bar{\theta} - \underline{\theta}$ is large enough that the low effort $a = 0$ is never desirable. This in turn implies that the optimal expected utility that the firm should promise to a newly matched old worker is simply \underline{w} :

$$\tilde{w} = \underline{w}. \quad (17)$$

and hence

$$w_* = \alpha \tilde{w} < \tilde{w}.$$

In other words, all unemployment is involuntary in the model.

I now characterize the problem that defines V_y . Let

$$w_A \equiv \beta w_* + \underline{w}, \quad w_B \equiv (1 + \beta)\underline{w}.$$

Clearly, $w_A < w_B$ since $w_* = \alpha \underline{w}$ and $\alpha \in (0, 1)$.

Assumption 1 $\bar{V} + w_* - (\bar{\theta} - \psi) - \gamma/\beta < 0$.

As will be clear from the Appendix, Assumption 1 implies that in equilibrium firms seek to minimize, rather than maximize, the probability of firing. A difficulty about this assumption though is that it is about \bar{V} and w_* , both endogenous variables of the model. My strategy is to solve for the optimal contracts under Assumption 1, and then verify that this assumption is indeed satisfied in equilibrium.

Proposition 1 Suppose Assumption 1 holds. Then the firm's optimal contract for a young worker is as follows.

- (i) If $w < w_A$, then w is not attained by any incentive compatible contract.
- (ii) If $w \in [w_A, w_B)$, then the optimal contract has

$$p_1^*(w) = \frac{(1 + \beta)\underline{w} - w}{\beta(\underline{w} - w_*)}, \quad p_2^*(w) = 0, \quad (18)$$

$$c_{1r}^*(w) = c_{1f}^*(w) = c_{2r}^* = 0, \quad (19)$$

$$w_{1r}^*(w) = \underline{w}, \quad w_{2r}^*(w) = \left(w - \underline{w} + \frac{\psi}{x_2 - x_2'} \right) / \beta. \quad (20)$$

(iii) If $w \geq w_B$, then

$$p_1^*(w) = p_2^*(w) = 0, \quad (21)$$

$$c_{1r}^* = c_{2r}^* = 0 \quad (22)$$

and $\{w_{1r}^*, w_{2r}^*\}$ can be any pair of $\{w_{1r}, w_{2r}\}$ that satisfies the following conditions:

$$w_{1r}, w_{2r} \geq \underline{w}, \quad w_{2r} - w_{1r} \geq \frac{\psi}{(x_2 - x_2')\beta}, \quad x_1 w_{1r} + x_2 w_{2r} = (w + \psi) / \beta. \quad (23)$$

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The propositions in the paper are all proved in the appendix.

Notice that for all $w \in (w_A, w_B)$, it holds that $p_2^*(w_B) = 0 < p_1^*(w) < 1 = p_2^*(w_B)$, where $p_1^*(w)$ is decreasing in w . So the probability of firing is a decreasing function of the worker's expected utility. Notice also that the probability of termination p_1^* is an increasing function of w_* . Finally, notice that the form of the optimal contract does not depend directly on the value of γ . This simplifies analysis, a technical advantage offered by the assumption of linear utilities.

Given Proposition 1, it is straightforward to calculate that for all $w \in [w_A, w_B)$

$$V_y(w) = k_0 - kw$$

where k_0 and k are constants and

$$\begin{aligned} k &= x_1 \frac{\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2 \\ &\equiv \frac{x_1 s}{\underline{w} - w_*} + (1 - \beta)x_2; \end{aligned} \quad (24)$$

and for all $w \in [w_B, \infty)$,

$$\begin{aligned} V_y(w) &= (1 - \beta)\bar{\theta} + \beta x_1 V_o(w_{1r}^*) + \beta x_2 V_o(w_{2r}^*) \\ &= (1 - \beta^2)(\bar{\theta} - \psi) + \beta^2 \bar{V} - (1 - \beta)w. \end{aligned}$$

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Notice the value function $V_y(w)$ can be upward or downward sloping over the interval $[w_A, w_B]$, depending on the value of k . In equilibrium if $k > 0$, then $\hat{w} = w_A$

⁴One specific solution can be obtained by setting $w_{2r} - w_{1r} = \frac{\psi}{(x_2 - x_2')\beta}$. In this case, $w_{1r}^*(w) = (w - \underline{w})/\beta$, $w_{2r}^*(w) = (w - \underline{w} + \frac{\psi}{x_2 - x_2'})/\beta$.

⁵Use equations (16) and (23) and collect terms.

and the firm's optimal termination policy is to fire the worker with probability one in the low output state; if $k \leq 0$, then $\hat{w} = w_B$ and the firm's optimal policy is to always retain the worker.

Equation (24) is important for understanding what determines an individual firm's optimal termination policy, given the exogenous and endogenous variables of the model that the firm takes as given. These variables include the firing tax γ , and what the rest of the market is doing in terms termination, as summarized partially by w_* .

Consider the effects of an increase in γ on termination. I first describe what I called in the introduction the first effect of a firing tax on termination. Holding the values of all other variables constant,⁶ an increase in γ reduces the value of k . Moreover, since $\bar{V} \geq V_o(\underline{w})$, k is positive if γ is sufficiently small. On the other hand, k will become negative as γ gets sufficiently large. So for γ small, the slope of the firm's value function is negative for all $w \geq w_A$. This in turn implies that the firm's optimal value is attained at $w = w_A$, where it is optimal to fire the worker in the low output state according to Proposition 1. On the other hand, for γ large enough, the firm's value $V_y(w)$ is increasing in w for $w \in [w_A, w_B]$ and decreasing in w for all $w \geq w_B$. This implies that the firm's optimal value is attained at $w = w_B > w_A$, where the contract involves no termination, with all the incentives created through giving the worker more rewards in the high output state.

Next, use (24) to consider the effects of a change in w_* on an individual firm's optimal termination policy. Note an increase in w_* implies an increase in α which in turn implies more turnover in the aggregate: other firms are firing more frequently.

Suppose initially termination is optimal: $k > 0$ and $\hat{w} = w_A = w_* + \beta$. Then $V_y(\hat{w}) = k_0 - k(w_* + \beta\underline{w})$. Substitute this and $V_o(\underline{w}) = \tilde{V}_o + \beta\bar{V}$, where $\tilde{V}_o = (1 - \beta)(\bar{\theta} - \psi - \underline{w})$, into equation (11) and collect terms to obtain

$$(\beta\underline{w} + w_*)k = k_0 + (2L - 1)\tilde{V}_o - [(1 - \beta)2L + \beta]\bar{V},$$

where

$$k_0 = (1 + \beta)\underline{w}k + (1 + \beta)\tilde{V}_o + \beta^2\bar{V}.$$

⁷ Now substitute $V_o(\underline{w}) = \tilde{V}_o + \beta\bar{V}$ into equation (24) to get

$$k = x_1 \frac{(1 - \beta)\bar{V} - \tilde{V}_o - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2.$$

Solve the above equations to obtain

$$\frac{1}{1 - \beta} \left(\frac{1}{x_1} - \frac{1}{2L + \beta} \right) k = -\frac{\gamma/\beta}{\underline{w} - w_*} + \frac{x_2}{x_1}.$$

⁶Other variables will be allowed to change when I consider the equilibrium effects of a firing tax in Proposition 2.

⁷This is derived from $k_0 - kw_B = V_y(w_B)$ where $V_y(w_B) = (1 - \beta^2)(\bar{\theta} - \psi - \underline{w}) + \beta^2\bar{V}$.

Clearly then, an increase in w_* causes k to decrease.⁸ Moreover, for w_* sufficiently high (sufficiently close to \underline{w}), k must be negative. When this occurs, the firm shifts its termination policy from a positive probability of termination to no termination.

To summarize, as w_* increases, the optimal response of an individual firm is either to keep its existing termination policy, or to reduce termination. In other words, if the market moves to higher turnover, then the individual firm prefers to go in the opposite direction: to (weakly) lower turnover.

The logic for this seemingly counterintuitive result is that when there is more turnover in the market, the probability with which an unemployed worker finds new employment is higher, making termination less effective as an incentive device (for punishing the non-performing worker), and the firm chooses to use less of it.

Note, however, there does exist a “wealth effect” in the sense that since higher turnover in the rest of the economy makes termination less efficient as an incentive device, it forces the individual firm to use more termination in order to obtain a given amount of incentives. To see this, fix $w \in (w_A, w_B)$ and observe $\partial p_1^*(w)/\partial w^* > 0$. In equilibrium, though, this “wealth effect” is dominated by the “substitution effect” described earlier.

The above discussion has prepared us for the following proposition that formally describes the model’s stationary equilibria.

Proposition 2

(i) Suppose $\gamma < \bar{\gamma}_A$, where $\bar{\gamma}_A$ is given by equation (29). Then the model has a unique equilibrium, a type-A equilibrium, where all firm use the same termination policy with which young workers are terminated with a positive probability, the optimal contract is described by Proposition 1(ii), and the equilibrium values of α , L , w_* , \hat{w} , k , and the value of $\bar{\gamma}_A$ are given by

$$\alpha = 1 - \frac{1 - \eta}{1 - \alpha x_2/2} \tag{25}$$

$$L = 1 - \alpha x_2/2 \tag{26}$$

$$w_* = \alpha \underline{w}, \hat{w} = w_A = \beta w_* + \underline{w} \tag{27}$$

$$k = x_1 \frac{\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2 > 0 \tag{28}$$

$$\bar{\gamma}_A \equiv \frac{\beta x_2}{x_1} (\underline{w} - w_*) \tag{29}$$

⁸Since $L > 1/2$, $\frac{1}{x_1} - \frac{1}{2L+\beta} > 0$.

(ii) Suppose $\gamma > \bar{\gamma}_B$, where $\bar{\gamma}_B (> \bar{\gamma}_A)$ is given by (34). Then the model has a unique equilibrium, a type-B equilibrium, where there is no termination, the optimal contract is described by Proposition 1, and the equilibrium α , L , w_* , \hat{w} , and $\bar{\gamma}_B$ are given by

$$\alpha = 1 - \frac{1 - \eta}{1 - \alpha/2} \quad (30)$$

$$L = 1 - \alpha/2 \quad (31)$$

$$w_* = \alpha \underline{w}, \quad \hat{w} = w_B = (1 + \beta) \underline{w}. \quad (32)$$

$$k = x_1 \frac{\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2 < 0 \quad (33)$$

$$\bar{\gamma}_B \equiv \frac{\beta x_2}{x_1} (\underline{w} - w_*). \quad (34)$$

(iii) Suppose $\gamma \in [\bar{\gamma}_A, \bar{\gamma}_B]$. Then the model has a unique equilibrium, a type-AB equilibrium, where a fraction $\delta \in (0, 1)$ of the firms, type-A firms, start their young workers with expected utility $\hat{w} = w_A$ and fire them with probability one if they produce a low output; and a fraction $(1 - \delta)$ of the firms, type-B firms, start their young workers with expected utility $\hat{w} = w_B$ and will fire them with probability zero, even in the state of low output. Let Z_y^i denote the measure of young workers employed in type i firms, $i = A, B$. The equilibrium has

$$Z_y^A = \frac{1/2}{L} (\delta\eta - x_2 Z_y^A), \quad (35)$$

$$Z_y^B = \frac{1/2}{L} [(1 - \delta)\eta - Z_y^B], \quad (36)$$

$$L = 1 - x_2 Z_y^A - Z_y^B, \quad (37)$$

$$\alpha = \frac{\eta - x_2 Z_y^A - Z_y^B}{L}, \quad (38)$$

$$k = x_1 \frac{\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2 = 0, \quad (39)$$

$$w_* = \alpha \underline{w} \quad (40)$$

$$w_A = \beta w_* + \underline{w}, \quad w_B = (1 + \beta) \underline{w}. \quad (41)$$

In (iii) of the proposition, $x_2 Z_y^A$ is the measure of all retained workers in type-A firms, and $\delta \eta - x_2 Z_y^A$ is thus the number of all vacancies in type-A firms, and among these vacancies a fraction $1/(2L)$ will be filled with young workers. This explains why (35) must hold for type-A firms in a stationary equilibrium. Similarly, equation (36) must hold for type-B firms.

In a type-AB equilibrium, the function $V_y(w)$ is horizontal over the interval $[w_A, w_B]$ and the two types of firms are located at the two ends of this interval. Notice that, in equilibrium, firms are not exactly indifferent between being type-A and being type-B. Indeed, type-A firms would like to move from w_A to w_B so that they can promise a higher expected utility to their workers while keeping the value of the firm constant. But this will not break the equilibrium. The reason is, once a positive measure of type-A firms have moved from w_A to w_B , α and w_* will be lower, k then turns to negative, implying $U(w_A) > U(w_B)$, and hence type-B firms will all want to be a type-A firm. The equilibrium is restored until the fraction of type-A firms goes back to δ .

The equilibrium conditions for a type-AB equilibrium can be consolidated to obtain:

$$\gamma = \frac{\beta x_2}{x_1} (1 - \alpha) \underline{w}, \quad (42)$$

$$\alpha = 1 - \frac{1 - \eta}{L} \quad (43)$$

$$L + \frac{x_2 \delta \eta}{2L + x_2} + \frac{(1 - \delta) \eta}{2L + 1} = 1. \quad (44)$$

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⁹Since $V(\hat{w}_A) = V(\hat{w}_B)$, we have

$$\bar{V} = \frac{1}{2L} V_y(w_B) + (1 - \frac{1}{2L}) V_o(\underline{w}),$$

or $2L [\bar{V} - V_o(\underline{w})] = V_y(w_B) - V_o(\underline{w})$. Next, use $V_y(w_B) = (1 - \beta^2)(\bar{\theta} - \psi) + \beta^2 \bar{V} - (1 - \beta)(1 + \beta) \underline{w}$ and $V_o(\underline{w}) = (1 - \beta)(\bar{\theta} - \psi - \underline{w}) + \beta \bar{V}$, we can show that $V_y(w_B) - V_o(\underline{w}) = -\beta [\bar{V} - V_o(\underline{w})]$. We therefore have $\bar{V} - V_o(\underline{w}) = 0$. This in turn implies that we can rewrite equation (39) as $\frac{x_1 \gamma}{x_2 \beta} = \underline{w} - w_*$ and hence (42) holds. Next, use equations (37)-(38) to derive equation (43), and use equations (35)-(37) to derive equation (44). The following is easy to verify. Set $\delta = 1$, then (43) and (44) are just (25) and (26). Set $\delta = 0$, then (43) and (44) are just (30),(31).

In the proof of Proposition 2, Lemma 5, it is shown that $\bar{\gamma}_B > \bar{\gamma}_A > 0$. It is also shown that in each of the equilibrium types, $\alpha \in (0, 1)$, with $\alpha_A > \alpha_B$ and $L_A > L_B$, where α_A and L_A solve conditions (25) and (26), and α_B and L_B solve conditions (30) and (31). So the probability to transition from unemployment to employment, α , is greater in a type-A equilibrium than in a type-B equilibrium. If employed workers are fired with higher probabilities, then unemployed workers are hired with higher probabilities. There is more activity in the labor market: more workers are looking for jobs, and a greater fraction of them will become employed.

By Proposition 2 then, the level of the firing tax, γ , is critical for the type of the equilibrium that is obtained. If $\gamma < \bar{\gamma}_A$, the slope of the firm's value function $V_y(w)$ is negative over $[w_A, w_B]$, young workers then enter into contracts with which they are never fired. If $\gamma > \bar{\gamma}_A$, the slope of the firm's value function $V_y(w)$ is positive over $[w_A, w_B]$, and the young workers then enter into contracts where they are fired whenever a low output is produced. If $\gamma \in [\bar{\gamma}_A, \bar{\gamma}_B]$, then the firm's value function $V_y(w)$ is flat over the interval $[w_A, w_B]$, and in equilibrium some young workers enter into contracts where they are never fired, some into contracts where they are fired after producing a low output. To summarize, a higher γ reduces equilibrium termination.

Observe that $\bar{\gamma}_A$ and $\bar{\gamma}_B$ are both decreasing functions of w_* . That is, a higher w_* reduces termination. This demonstrates the “substitution effect” of a higher w_* on termination. A higher w_* implies a “softer” punishment for the worker in the case of layoff, making termination less effective as an incentive device. This leads the firms to shift away from using termination for incentives.

Corollary 1 In the equilibrium where young workers have a lower (higher) probability to be fired, they also have a higher (lower) starting expected utility. That is, \hat{w} is lower in a type-A equilibrium than in a type-B equilibrium.

This corollary reveals the critical termination/compensation trade off that is at the heart of my analysis. In the model, there is a fixed amount of incentives that the contract must provide for the young worker. The contract has available two devices, termination and compensation, for creating these incentives. Termination provides only downward incentives, by putting the worker into involuntary unemployment in the state of low output.¹⁰ Compensation, on the other hand, can be used both to punish the worker in the low output state and to reward him in the high output state.

Both devices are costly. That compensation is a costly incentive device is due to the non-negativity (limited-liability) constraint imposed on the worker's compensation. Suppose compensation need not be non-negative. Then incentives are free to the firm. Specifically, incentives could be obtained by making the difference between the levels of compensation in the low and high states sufficient large, while maintaining a constant amount of expected compensation to the worker. Given risk neutrality, this would not reduce the value of the firm, no matter how much incentives that are

¹⁰Remember a retained worker gets at least \underline{w} , a terminated worker gets $w_* < \underline{w}$.

needed. Under a binding non-negativity constraint on compensation though, the firm would be forced to increase the reward to the worker in the high output state in order to maintain incentive compatibility. This costs the firm, but increases the worker's expected utility.

So the optimal contract must achieve an efficient trade off between the two devices. Suppose there is now an increase in γ . This induces the contract to shift away from termination and to relying more on compensation for incentives. With the non-negativity constraint on compensation in the low output state binding, the additional incentives must then be obtained from increasing the worker's compensation in the high output state. This increases the starting expected compensation of the worker. In other words, the firm is forced to pay the worker more if it must reduce the use of termination for incentives.

Observe that the trade off between termination and compensation is essentially a choice between downward incentives (punishments) and upward incentives (rewards). Since the firm is constrained in using compensation for downward incentives, a higher firing tax that increases the cost of downward incentives through termination leads to a substitution to upward incentives. This in turn implies a higher utility for the worker.

5 Effects of a Firing Tax

In this section, I measure the effects of a higher firing tax on the economy's equilibrium labor turnover, worker utility, and firm welfare. I start with a set of properties of the equilibria that are characterized in Proposition 2.

Proposition 3 (i) The equilibrium α , L and δ are all continuous functions of γ .
(ii) Over the interval $[\bar{\gamma}_A, \bar{\gamma}_B]$, α , L and δ are strictly decreasing in γ .

So a higher firing tax reduces termination. A higher firing tax reduces the size of the pool of the unemployed workers at the beginning of a period, it also reduces the unemployed workers' probability of transition to employment.

I now consider the effects of a higher γ on worker utility. The utility of the newly employed old workers is \underline{w} in all equilibrium types and is constant in γ . So I need only consider the utility of the newly employed young workers, that is, \hat{w} .

Obviously, \hat{w} is strictly lower over the interval $[0, \bar{\gamma}_A]$ than over the interval $[\bar{\gamma}_B, \infty)$. So a sufficiently large increase in γ always increases the utility the young worker.

Over the interval $[\bar{\gamma}_A, \bar{\gamma}_B]$, though, the effects of a higher γ are not monotonic. In a type-AB equilibrium, the utility of the young worker who is employed in a type-B firm is equal to what he would get in a type-B equilibrium and is hence constant in γ . For the young worker who is employed in a type-A firm (which adopts an active

termination policy), his utility is $(\beta\alpha + 1)\underline{w}$, where α solves (42) and decreases as γ increases.

Since α is strictly lower in a type-AB equilibrium than in a type-A equilibrium, young workers employed at a type-A firm in a type-AB equilibrium are strictly worse off than the young workers in a type-A equilibrium. The lower α , which implies a lower w_* , makes termination a more efficient incentive device, as it now implements a more severe punishment to the worker. This allows the firm to rely less on compensation for incentives, which in turn allows the firm to promise a lower expected utility to the worker. Moreover, as γ increases over the interval $[\bar{\gamma}_A, \bar{\gamma}_B]$, the utility of the young workers employed at type-A firms falls while the measure of these workers shrinks.¹¹

The overall effects of a higher γ on the welfare of the workers could also be measured, by comparing the expected utilities of the new labor market entrants across different equilibrium types. Let me compare worker welfare between a type-A equilibrium and a type-B equilibrium. There are three effects to be considered. First, because of the lower labor turnover in the type-B equilibrium, workers who are unemployed at the beginning of old age are better off in a type-A equilibrium than in a type-B equilibrium. Second, for the same reason, young workers have a higher probability to be employed in a type-A equilibrium than in a type-B equilibrium. Third, since $w_A < w_B$, young workers receive a lower expected utility in a type-A equilibrium than in a type-B equilibrium.

Let U_A and U_B denote the equilibrium expected utilities of the new labor market entrants in the equilibrium types A and B, respectively.

$$U_A = \alpha_A w_A + (1 - \alpha_A)\beta\alpha_A \underline{w} = \alpha_A(1 + \beta)\underline{w},$$

$$U_B = \alpha_B w_B + (1 - \alpha_B)\beta\alpha_B \underline{w} = \alpha_B[1 + \beta(2 - \alpha_B)]\underline{w},$$

where α_A is given by (25) and α_B by (30).

Obviously, the model does not make an unambiguous prediction on the overall effects of a higher firing tax on worker welfare. If β is sufficiently small, then $U_A > U_B$. If x_2 is sufficiently close to one and hence α_A is sufficiently close to α_B (see (25) and (30)), then $U_A < U_B$.

Last, consider the effects of a higher firing tax on firm welfare. Use \bar{V} as a measure of firm welfare and write it as $\bar{V}(\gamma)$ to indicate that its value depends on γ , the firing tax.

Proposition 4 The firm's value $\bar{V}(\gamma)$ is strictly decreasing in γ over the interval $[0, \bar{\gamma}_A)$ and constant over the interval $[\bar{\gamma}_A, \infty)$.

¹¹The average utility of the newly employed young workers is

$$\delta(1 + \beta\alpha)\underline{w} + (1 - \delta)(1 + \beta)\underline{w}.$$

Analytically, it is not clear whether this value is monotonically increasing in γ .

So a higher firing tax reduces the value of the firm. A lower firing tax gives firms a more efficient incentive instrument to use and makes them better off.

6 Effects of an Increase in η

In this section, I study the effects of a change in η on the model's endogenous variables. This will prepare me for the analysis in the next section where the number of jobs η is determined endogenously.

Start by observing that conditional on the type of the equilibrium that is obtained in the model, I have

$$\frac{\partial \alpha}{\partial \eta} > 0, \quad \frac{\partial L}{\partial \eta} < 0, \quad \frac{\partial w_*}{\partial \eta} > 0. \quad (45)$$

Moreover, conditional on the equilibrium being type-A, workers are strictly better off with a higher η ($\hat{w} = \beta w_* + \underline{w}$ is higher as η increases).

Next, observe that

$$\frac{\partial \bar{\gamma}_A}{\partial \eta} < 0, \quad \frac{\partial \bar{\gamma}_B}{\partial \eta} < 0. \quad (46)$$

So if there are more firms in the economy, workers will be terminated less often. A higher η implies a higher w_* , the contract could then rely more on the use of compensation and less on the use of termination for incentives.

In the following, I will write $\bar{\gamma}_A$, $\bar{\gamma}_B$ and \bar{V} and $\bar{\gamma}_A(\eta)$, $\bar{\gamma}_B(\eta)$, and $\bar{V}(\eta)$ to indicate respectively their dependence on η .

Now imagine η increases from zero to one. Observe that as η increases, the type of the equilibrium that is obtained in the economy changes. There are three cases, as depicted in Figure 1.¹²

Case (I): $\gamma < \bar{\gamma}_A(0)$. In this case, as η increases from 0 to 1, the economy first has a type-A equilibrium, then a type-AB equilibrium, and finally a type-B equilibrium, with diminishing termination activities.

Case (II): $\bar{\gamma}_A(0) < \gamma < \bar{\gamma}_B(0)$. In this case, as η increases, the economy first has a type-AB equilibrium, then a type-B equilibrium.

Case (III): $\gamma > \bar{\gamma}_B(0)$. In this case, the economy always has a type-B equilibrium.

Next, I compute the equilibrium value of the firm as a function of η .

¹²In Figure 1,

$$\bar{\gamma}_A(0) = \lim_{\eta \rightarrow 0} \bar{\gamma}_A(\eta), \quad \bar{\gamma}_B(0) = \lim_{\eta \rightarrow 0} \bar{\gamma}_B(\eta).$$

$$\bar{\gamma}_A(1) = \lim_{\eta \rightarrow 1} \bar{\gamma}_A(\eta), \quad \bar{\gamma}_B(1) = \lim_{\eta \rightarrow 1} \bar{\gamma}_B(\eta).$$

Clearly, $0 < \bar{\gamma}_A(0) < \bar{\gamma}_B(0)$ and $\bar{\gamma}_A(1) = \bar{\gamma}_B(1) = 0$.

In the proof of Proposition 2 in the appendix, I show that for all $\eta < \eta_A(\gamma)$,

$$\bar{V}(\eta) = \frac{\bar{\theta} + x_2\beta(\bar{\theta} - \psi - w_* - m/\beta - x_1\gamma) + (2L - 1)(\bar{\theta} - \psi - \underline{w})}{2L + x_2\beta},$$

and for all $\eta \geq \eta_A(\gamma)$,

$$\bar{V}(\eta) = \bar{V}_B \equiv \bar{\theta} - \psi - \underline{w}. \quad (47)$$

where $\eta_A(\gamma)$ and $\eta_B(\gamma)$ are defined by

$$\bar{\gamma}_A(\eta_A(\gamma)) = \gamma, \quad \bar{\gamma}_B(\eta_B(\gamma)) = \gamma.$$

That is, for any given γ , $\eta_A(\gamma)$ [$\eta_B(\gamma)$] is the level of η at which the value of $\bar{\gamma}_A$ $\bar{\gamma}_B$ is just equal to γ (Figure 1). Obviously,

$$\eta'_A(\gamma), \eta'_B(\gamma) < 0.$$

Proposition 5 (i) Equilibrium termination decreases (weakly) as the economy's number of firms η increases. (ii) The equilibrium value of the firm $\bar{V}(\eta)$ is a continuous function of η , it is strictly decreasing over the interval $(0, \eta_A]$ and constant over $[\eta_A, 1)$.

7 Endogenous Employment

So far, I have assumed that the number of the firms in the economy is fixed, so the number of workers employed is fixed, and hence the rate of unemployment is fixed. In this section, I open up the model to let η , the number of firms in the economy, be endogenously determined.

Specifically, I assume in this section that there is a competitive supply of potential firms in the economy. These potential firms are free to enter the market, if it is profitable for them to do so. On the other hand, the incumbent firms are free to exit the market, if it is profitable for them to do so. To close the model, I assume in order to stay in the market, the firm must incur a discounted cost equal to C_0 . This is interpreted as the discounted value of all current and future costs that the firm incurs to maintain its daily operations, C_0 is not included in the values of θ . Clearly then, if $\bar{V} < C_0$, firms will exit the market; if $\bar{V} > C_0$, firms will enter. To ensure the existence of an equilibrium with unemployment, I assume $C_0 > \bar{V}_B$, \bar{V}_B defined by equation (47). That is, firms would not be profitable if they were not allowed to fire their workers.

Definition 2 With a competitive supply of firms, a stationary equilibrium is a vector

$$\{\eta, L, \alpha, w_*; (V_o, V_y, \bar{V}), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*, \hat{w}, \tilde{w}\}$$

where

- (i) $\{(V_o, V_y), (c_1^*, c_2^*), (c_{ik}^*, w_{ir}^*), p_i^*, \hat{w}, \tilde{w}\}$ solves equations (1)-(11), given η, L, α , and w_* ;
- (ii) $\{\eta, L, \alpha, w_*, \bar{V}\}$ satisfy the equilibrium conditions (12)-(14) and

$$\bar{V} = C_0. \quad (48)$$

Equation (48) is a zero profit condition. Under the assumption of $C_0 > \bar{V}_B$, and given that the firm's value \bar{V} is monotone decreasing in η by Proposition 6, the model has a unique stationary equilibrium, and the determination of the equilibrium number of firms, denoted η^* , is depicted in Figure 2. Suppose $\eta < \eta^*$, more firms will enter until the net value of entering, $\bar{V} - C_0$, is reduced to zero as η goes to η^* . On the other hand, if $\eta > \eta^*$, then firms will exit until $\bar{V} = C_0$ is restored.

Imagine an increase in γ . Other exogenous variables of the model constant, this shifts down the function $\bar{V}(\eta)$ over the range of $(0, \eta_A)$, as depicted in Figure 2. In turn, this leads to a decrease in the equilibrium number of firms in the market, η^* , and therefore less employment. Aggregate termination is lower.¹³

What happens to the equilibrium worker utility \hat{w} ? Notice that the economy is in a type-A equilibrium. As γ increases and the number of jobs η falls, the probability for an unemployed worker to obtain employment α falls, leading to a decrease in w_* and hence a decrease in $\hat{w} = w_A = \beta w_* + \underline{w}$.

This process is natural. As γ increases to cause w_* to decrease, termination becomes a more efficient incentive device, as it now implements a more severe punishment to the worker. This in turn reduces the burden on compensation as an incentive device for the firm, allowing the firm to offer a lower expected utility to the young worker.

Observe that without the assumption $C_0 > \bar{V}_B$, the model may not have an equilibrium with unemployment. This happens because the firm's value function $\bar{V}(\eta)$ is flat over the range $[\eta_A, 1)$. This is partly due to our assumption on the model's market structure, which gives the firm all the bargaining power in contract negotiations, whenever there is unemployment. This would not be the case if some matching frictions are built into the model to make the value of the firm go down as the market gets tighter. With a strictly decreasing value function $\bar{V}(\eta)$ over the range of $\eta \in (0, 1)$, the model would allow for equilibria where at least some firms do not pursue an active termination policy, depending on the value of C_0 .

One specific way to avoid the flat portion in the value function $\bar{V}(\eta)$ is to introduce an additional cost to the firm. This cost is independent of all interactions and transactions that I have already modeled. It depends on the variable η only, and takes the general form of

$$C(\eta), \text{ where } C(\eta) \leq 0, C'(\eta) > 0, \forall \eta.$$

¹³Here, a higher γ does not change the probability a new worker is fired, but it does result in fewer firms in the market.

A simple way to interpret the cost function C is to think of it as representing the effects of market tightness. A higher η represented a tighter labor and/or product market condition which imposes a cost on the firm.

With the above qualifications, the firm's value as a function of η is then written

$$\tilde{V}(\eta) \equiv \bar{V}(\eta) - C(\eta).$$

Suppose C is such that $\tilde{V}(1) < 0$. Then the model determines a unique equilibrium η for all $C_0 \geq 0$. Suppose next that C is such that the equilibrium η is determined for different values of γ as in Figure 3. Then a higher firing tax could result in more termination and lower utility for the workers, the opposite of what happens when $C = 0$. The higher firing tax γ' induces a switch of dominance between the first and the third effects of a higher firing tax, on both termination and worker expected utility. Note that adding $C(\eta)$ to the model does not change the values of η_A and η_B .

8 Conclusion

This paper presents an analytical study of the effects of a firing tax on layoffs and worker utility. I have maintained a minimum structure of the model that is necessary for capturing the essentials of an environment where layoff as termination of a dynamic contract is used as an incentive device. The model though is rich enough for the analysis of perhaps a number of interesting problems, besides the effects of a firing tax. For instance, assuming risk aversion for the workers, the model can then be used to consider the effects of unemployment insurance on worker termination and employment.

The model identifies three effects of a firing tax on layoffs and worker compensation and utility. The model does not offer unambiguous conclusions on whether a firing tax leads to higher or lower turnover and worker utility. This calls for the construction of a quantitative version of the model that is calibrated to data to draw definitive conclusions on which effects dominate in a given environment.

9 Appendix

9.1 Proof of Proposition 1

Lemma 1. (1) It is optimal to set $c_{ir} = 0$.

Proof. Suppose $c_{ir} = \Delta > 0$. Then we can always set $c_{ir} = c_{ir} - \Delta = 0$ and set $\beta w_{ir} = \beta w_{ir} + \Delta$. This would not violate the constraints, and the firm is indifferent (given that V_o is linear in w). This proves the lemma.

Given Lemma 1, the incentive constraint can be written $[(1 - p_2)\beta w_{2r} + p_2((1 - \beta)c_{2f} + \beta w_*)] - [(1 - p_1)\beta w_{1r} + p_1((1 - \beta)c_{1f} + \beta w_*)] \geq \frac{\psi}{x_2 - x_2'}$.

Lemma 2. It is optimal to set $c_{if} = 0$ if $p_i < 1$.

The proof of this lemma is also easy and is left for the reader. In addition, we can always set $c_{if} = 0$ if $p_i = 0$. These imply that we can focus on contracts that satisfy

$$p_i c_{if} = c_{if}. \quad (49)$$

Now for each i , consider the following problem which is a component of the problem (5)-(8) subject to an additional constraint (19).

$$\max_{\{c_{if}, w_{ir}, p_i\}} \left\{ p_i \left[-(1 - \beta)(c_{if} + \gamma) + \beta \bar{V} \right] + (1 - p_i) \beta V_o(w_{ir}) \right\} \quad (50)$$

subject to

$$c_{if} \geq 0, \quad w_{ir} \geq \underline{w}, \quad 0 \leq p_i \leq 1, \quad (51)$$

$$(1 - p_i) \beta w_{ir} + p_i ((1 - \beta) c_{if} + \beta w_*) - \psi = \Delta_i, \quad (52)$$

$$p_i c_{if} = c_{if}, \quad (53)$$

where $\Delta_i > 0$ is a given constant.

To solve the above optimization problem, we use the following strategy. We first ignore constraint (53) and solve for the optimal solution to the resulting optimization problem. We then verify that the optimal solution satisfies (53).

So now ignore constraint (53) and consider the optimization problem (50)-(52). Fix c_{if} at its optimal level. We then must solve the following problem

$$\max_{w_{ir}, p_i} \left[p_i \bar{V} + (1 - p_i) V_o(w_{ir}) \right] \beta - p_i \gamma (1 - \beta)$$

subject to

$$w_{ir} \geq \underline{w}, \quad 0 \leq p_i \leq 1,$$

$$(1 - p_i) w_{ir} + p_i w_* = \delta_i$$

where $\delta_i \equiv (\Delta_i + \psi - (1 - \beta) c_{if}) / \beta > 0$. Substituting $V_o(w) = (1 - \beta)(\bar{\theta} - \psi - w) + \beta \bar{V}$ into the above problem to rewrite it as

$$\max_{\{w_{ir}, p_i\}} \left\{ (1 - \beta) p_i \left[\bar{V} - (\bar{\theta} - \psi - w_*) - \gamma / \beta \right] + \left[(1 - \beta)(\bar{\theta} - \psi) + \beta \bar{V} - (1 - \beta) \delta_i \right] \right\} \beta$$

subject to

$$w_{ir} \geq \underline{w}, \quad 0 \leq p_i \leq 1, \quad (54)$$

$$(1 - p_i)w_{ir} + p_iw_* = \delta_i. \quad (55)$$

We now solve this problem. We first solve it under Assumption 1. This assumption implies our task is to minimize p_i subject to the constraints (54), (55).

Notice first that $\delta_i \geq (1 - p_i)\underline{w} + p_iw_* \geq w_*$.

Step 1. Suppose $\delta_i \geq \underline{w}$. Then clearly $p_i^* = 0$.

Step 2. Suppose $\delta_i \in (w_*, \underline{w})$. Clearly, $p_i^* < 1$, for otherwise $\delta_i = w_*$. Next, $p_i^* > 0$, for otherwise $\delta_i > \underline{w}$. Third, $w_{ir}^* = \underline{w}$, for otherwise w_{ir} and p_i can both be reduced. Therefore we have if $\delta_i \in (w_*, \underline{w})$, then p^* solves

$$(1 - p_i)\underline{w} + p_iw_* = \delta_i.$$

Step 3. Suppose $\delta_i = w_*$. Then $p_i^* = 1$.

Lemma 3. $p_2^*(w) = 0$ for all w .

Proof. We first show $p_2^*(w) < 1$ for all w . Suppose $p_2^*(w) = 1$ for some given w . Then the incentive constraint can be written

$$[(1 - \beta)c_{2f} + \beta w_*] - \{(1 - p_1)\beta w_{1r} + p_1[(1 - \beta)c_{1f} + \beta w_*]\} \geq \frac{\psi}{x_2 - x_2'}.$$

This implies $c_{2f} > 0$, given that $c_{1f} \geq 0$ and $w_{1r} > w_*$. Now set $w_{2r} = \underline{w}$. Reduce c_{2f} by Δ , Δ being a positive but sufficiently small number. Set $c_{2r} = c_{2f}$. Reduce p_2 from 1 to p_2' so that

$$(1 - p_2')[(1 - \beta)c_{2f} + \beta \underline{w}] + p_2'[(1 - \beta)(c_{2f} - \Delta) + \beta w_*] = (1 - \beta)c_{2f} + \beta w_*,$$

or equivalently

$$\left(\frac{1}{p_2'} - 1\right)\beta(\underline{w} - w_*) = (1 - \beta)\Delta. \quad (56)$$

Under the above deviation from the initial contract, the firm obtains a net gain equal to

$$\xi \equiv (1 - \beta)p_2'\Delta - (1 - p_2')\beta[\bar{V} - V_o(\underline{w})] + (1 - p_2')(1 - \beta)\gamma.$$

We show that $\xi > 0$. We need only show

$$(1 - \beta)\Delta > \left(\frac{1}{p_2'} - 1\right)\{\beta[\bar{V} - V_o(\underline{w})] - (1 - \beta)\gamma\}.$$

Given (56), in turn we need only show

$$\underline{w} - w_* > \bar{V} - V_o(\underline{w}) - \frac{1 - \beta}{\beta}\gamma.$$

But for any $w \geq \underline{w}$,

$$\begin{aligned}
\bar{V} - V_o(w) - \frac{1-\beta}{\beta}\gamma &= \bar{V} - (1-\beta)(\bar{\theta} - \psi - \gamma_0 - w) - \beta\bar{V}] - \frac{1-\beta}{\beta}\gamma \\
&= (1-\beta) \left\{ \bar{V} - (\bar{\theta} - \psi - \gamma_0 - w) - \gamma/\beta \right\} \\
&= (1-\beta) \left\{ \bar{V} - (\bar{\theta} - \psi - \gamma_0 - w_*) - \gamma/\beta + (w - w_*) \right\} \\
&< (1-\beta) \{0 + (w - w_*)\} \\
&< w - w_*.
\end{aligned}$$

So we have show $\xi > 0$ and hence $p_2^*(w) < 1$. But given $p_2^*(w) < 1$, it is then optimal to have $c_{2f} = 0$. So from the incentive constraint we have

$$(1-p_2)\beta w_{2r} + p_2\beta w_* > \frac{\psi}{x_2 - x'_2}$$

and therefore

$$(1-p_2)w_{2r} + p_2w_* > \frac{\psi}{x_2 - x'_2} \frac{1}{\beta} > \underline{w},$$

and thus $p_2^*(w) = 0$. The lemma is proven.

Lemma 4. $c_{1f}^*(w) = 0$ for all w .

Suppose $c_{1f} > 0$. Then we can reduce c_{1f} and increase w_{2r} to make the worker indifferent but the firm strictly better off.

Note that Lemmas 3, 4 imply $c_{if} = p_i c_{if}$. This shows that that it was legitimate for us to ignore constraint (53).

Given the lemmas, we can rewrite the problem that defines V_y as

$$V_y(w) = \max_{w_{1r}, w_{2r}, p_1} \left\{ (1-\beta)\bar{\theta} + x_1[p_1\beta\bar{V} + (1-p_1)\beta V_o(w_{1r}) - p_1(1-\beta)\gamma] + x_2\beta V_o(w_{2r}) \right\}$$

subject to $c_{if} \geq 0$, $w_{ir} \geq \underline{w}$,

$$\beta w_{2r} - [(1-p_1)\beta w_{1r} + p_1\beta w_*] \geq m$$

$$w = x_1[(1-p_1)\beta w_{1r} + p_1\beta w_*] + x_2\beta w_{2r} - \psi,$$

where $m \equiv \frac{\psi}{x_2 - x'_2}$.

Now the incentive constraint requires $\beta w_{2r} \geq (1-p_1)\beta w_{1r} + p_1\beta w_* + m$. Substitute this into the promise-keeping constraint, we have

$$w \geq (1-p_1)\beta w_{1r} + p_1\beta w_* + x_2m - \psi.$$

Now $x_2m - \psi = \underline{w}$, and $\min_{p_1 \in [0,1]} [(1-p_1)\beta w_{1r} + p_1\beta w_*] = \beta w^*$.¹⁴ Let

$$w_A \equiv \beta w_* + \underline{w},$$

w_A is the minimum of w attainable by an incentive compatible contract. Moreover, w_A is attainable if and only if we set

$$p_1(w_A) = 1,$$

$$w_{2r}(w_A) = w_* + m/\beta$$

Let

$$w_B \equiv \beta \underline{w} + m - \psi = (1 + \beta)\underline{w} > w_A$$

Here the last inequality holds because $\underline{w} > w_*$.

Proof of Proposition 1

Step 1. We first show that $p_1^*(w) = 0$ if and only if $w \geq w_B$.

Suppose $p_1(w) = 0$. Then substitute the incentive constraint into the promise-keeping constraint to have

$$w \geq \beta w_{1r} + x_2m - \psi \geq \beta \underline{w} + x_2m - \psi = w_B.$$

Suppose $w \geq w_B$. We show it must hold that $p_1(w) = 0$ at the optimum. Suppose not. That is, suppose $p_1(w) > 0$. Then change p_1 and w_{2r} by $dp_1 < 0$ and $dw_{2r} < 0$ respectively so that the worker's expected utility remains unchanged, i.e.,

$$x_1(w_{1r} - w_*)dp_1 = x_2dw_{2r}.$$

Now this will make the firm's expected utility change by

$$dV_y = x_1[\beta \bar{V} - \beta V_o(w_{1r}) - (1 - \beta)\gamma]dp_1 + x_2\beta V'_o(w_{2r})dw_{2r}$$

Now $V'_o(w_{2r}) = -(1 - \beta)$. This, plus $x_2dw_{2r} = x_1(w_{1r} - w_*)dp_1$, gives us

$$\begin{aligned} dV_y &= \beta x_1[\bar{V} - V_o(w_{1r}) - (1 - \beta)\gamma/\beta - (1 - \beta)(w_{1r} - w_*)]dp_1 \\ &= \beta x_1[\bar{V} - (1 - \beta)(\bar{\theta} - \psi) - \beta \bar{V} + (1 - \beta)w_{1r} - (1 - \beta)w_{1r} \\ &\quad - (1 - \beta)\gamma/\beta + (1 - \beta)w_*]dp_1 \\ &= (1 - \beta)\beta x_1[\bar{V} - (\bar{\theta} - \psi) - \gamma/\beta + w_*]dp_1 \\ &> 0, \end{aligned}$$

where the last inequality holds because of Assumption 1 and $dp_1 < 0$. Therefore we have shown that if $w \geq w_B$ then it must hold that $p_1(w) = 0$ for optimality. Therefore we have shown $p_1^*(w) = 0$ if and only if $w \geq w_B$.

¹⁴This is because the minimization problem $\min_{p_1 \in [0,1]} [(1-p_1)\beta w_{1r} + p_1\beta w_*]$ has a unique solution $p_1 = 1$, given $w_{1r} \geq \underline{w} > w_*$.

Step 2. We show that if $w_A \leq w < w_B$, then the optimal contract must have $w_{1r}^* = \underline{w}$. Suppose $w \in [w_A, w_B)$ and $w_{1r} > \underline{w}$. Then we can always reduce p_1 and w_{1r} to make the worker indifferent but the firm strictly better off. Specifically, set $dp_1 < 0$, $dw_{1r} < 0$, and

$$d[(1 - p_1)w_{1r} + p_1w_*] = 0.$$

This implies

$$\frac{dw_{1r}}{dp_1} = \frac{w_{1r} - w_*}{1 - p_1} > 0.$$

With the above designed dp_1 and dw_{1r} , and with $V'_o = -(1 - \beta)$, $V_o(w_{1r}) = (1 - \beta)(\bar{\theta} - \psi - w) + \beta\bar{V}$, the firm's expected utility will change by

$$\begin{aligned} dV_y &= \beta x_1 d[p_1\bar{V} + (1 - p_1)V_o(w_{1r}) - p_1(1 - \beta)\gamma/\beta] \\ &= \beta x_1 [\bar{V}dp_1 - V_o(w_{1r})dp_1 - (1 - p_1)(1 - \beta)dw_{1r} - (1 - \beta)\gamma dp_1/\beta] \\ &= \beta(1 - \beta)x_1 [\bar{V} - (\bar{\theta} - \psi) + w_* - \gamma/\beta] dp_1 \\ &> 0. \end{aligned}$$

Step 3. Again let $w_A \leq w < w_B$, then given $w_{1r} = \underline{w}$, constraints (33) and (34) can be rewritten as

$$w_{2r} - [(1 - p_1)\underline{w} + p_1w_*] \geq m/\beta, \quad (57)$$

$$w = x_1[(1 - p_1)\beta\underline{w} + p_1\beta w_*] + x_2\beta w_{2r} - \psi. \quad (58)$$

We now claim that in order for the contract to be optimal, the incentive constraint must be binding. We show this. Suppose (59) is a strict inequality. We can reduce both p_1 and w_{2r} to make the worker indifferent but the firm strictly better off while holding the incentive constraint satisfied. Specifically, set $dp_1 < 0$, $dw_{2r} < 0$, and let $|dp_1|$ and $|dw_{2r}|$ be sufficiently small so that the incentive constraint is satisfied and

$$d[(1 - p_1)w_{1r} + p_1w_*] = 0,$$

where the last equation implies

$$\frac{dw_{2r}}{dp_1} = \frac{x_1(w_{1r} - w_*)}{x_2} > 0.$$

With the above designed dp_1 and dw_{1r} , the firm's expected utility will change by

$$\begin{aligned} dV_y &= \beta x_1 [\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta] dp_1 + \beta x_2 V'_o(w_{2r}) dw_{2r} \\ &= \beta x_1 [\bar{V} - (1 - \beta)(\bar{\theta} - \psi - \underline{w}) - \beta\bar{V} - (1 - \beta)(\underline{w} - w_*) - (1 - \beta)\gamma/\beta] dp_1 \\ &= \beta(1 - \beta)x_1 [\bar{V} - (\bar{\theta} - \psi) + w_* - \gamma/\beta] dp_1 \\ &> 0. \end{aligned}$$

Step 4. Following Step 3, there is a unique pair of (p_1, w_{1r}) that satisfies equations (57), (59) which can be solved to obtain

$$p_1^*(w) = \frac{(1 + \beta)\underline{w} - w}{\beta(\underline{w} - w_*)},$$

$$w_{2r}^*(w) = \frac{w - \underline{w} + m}{\beta},$$

Clearly $p_1^* \in [0, 1]$ and $w_{2r}^* \geq \bar{w}$ for all w . This completes the proof.

9.2 Proof of Proposition 2

Part (i). In order to show that Part (i) of Proposition 2 describes an equilibrium, we need to show conditions (9)-(10), (18) are satisfied. That (10) is satisfied is obvious.

We first show that condition (9) is satisfied. We need only show that the value function $V_y(w)$ is downward sloping over the range $w \in [w_A, \infty)$. In turn, we need only show that the slop of $V_y(w)$ over the range $[w_A, w_B]$ is negative. That is, we need show that

$$x_1 \frac{\bar{V} - V_o(\underline{w}) - (1 - \beta)\gamma/\beta}{\underline{w} - w_*} + (1 - \beta)x_2 < 0.$$

or equivalently

$$\frac{\gamma}{\beta} < \bar{V} - (\bar{\theta} - \psi - \underline{w}) + \frac{x_2}{x_1}(\underline{w} - w_*). \quad (59)$$

We show that this is guaranteed by $\gamma < \bar{\gamma}_A$.

We first derive $\bar{\gamma}$. We use equations (11), (43), and the following equations:

$$V_o(\tilde{w}) = V_o(\underline{w}) = (1 - \beta)(\bar{\theta} - \psi - \underline{w}) + \beta\bar{V},$$

$$V_y(\hat{w}) = (1 - \beta)\bar{\theta} + x_1\beta\bar{V} + x_2\beta[(1 - \beta)(\bar{\theta} - \psi - w_{2r}^*(\hat{w})) + \beta\bar{V}] - x_1(1 - \beta)\gamma.$$

By substituting the above two equations into (11) and dividing both sides by $(1 - \beta)$, we obtain

$$(2L + \beta x_2)\bar{V} = \bar{\theta} + (x_2\beta + 2L - 1)(\bar{\theta} - \psi) - x_2(\beta w_* + m) - (2k - 1)\underline{w} - x_1\gamma.$$

Now substitute the above equation into (59), equation (59) is then equivalent to

$$\begin{aligned} \left(1 + \frac{2L}{\beta}\right)\gamma &< \psi + (1 + \beta x_2)\underline{w} + (x_2\beta + 2k)\frac{x_2}{x_1}(\underline{w} - w_*) - x_2(\beta w_* + m) \\ &= \left(1 + \frac{2L}{\beta}\right)\frac{\beta x_2}{x_1}(\underline{w} - w_*), \end{aligned}$$

or $\gamma < \bar{\gamma}_A$, which holds.

Next, we show that Assumption 1 holds. Given Proposition 1, we have

$$V_o(\tilde{w}) = (1 - \beta)(\bar{\theta} - \psi - \underline{w}) + \beta\bar{V},$$

$$\begin{aligned} V_y(\hat{w}) &= (1 - \beta)\bar{\theta} + x_1\beta\bar{V} + x_2\beta V_o(w_{2r}^*(\hat{w})) - x_1(1 - \beta)\gamma \\ &= (1 - \beta)\bar{\theta} + x_1\beta\bar{V} + x_2\beta[(1 - \beta)(\bar{\theta} - \psi - w_{2r}^*(\hat{w})) + \beta\bar{V}] - x_1(1 - \beta)\gamma, \end{aligned}$$

where $w_{2r}^*(\hat{w}) = w_* + m/\beta$.

Substitute the above equations into equation (11) and solve for \bar{V} to obtain

$$\bar{V} = \bar{V}_A \equiv \frac{f(\gamma)}{1 + \frac{1}{2L}x_2\beta} \quad (60)$$

where

$$f(\gamma) = \frac{1}{2L} \left[\bar{\theta} + x_2\beta \left(\bar{\theta} - \psi - w_* - \frac{m}{\beta} \right) - x_1\gamma \right] + \left(1 - \frac{1}{2L} \right) [\bar{\theta} - \psi - \gamma_0 - \underline{w}].$$

We need to show $\bar{V} < \bar{\theta} - \psi - \gamma_0 - w_* + \gamma/\beta$. It is sufficient to show

$$\bar{\theta} + x_2(\bar{\theta} - \psi - w_* - m/\beta) + (2L - 1)[\bar{\theta} - \psi - \underline{w}] < (2L + x_2\beta)[\bar{\theta} - \psi - w_*]$$

or

$$\bar{\theta} - x_2m + (2L - 1)[\bar{\theta} - \psi - \underline{w}] - 2L[\bar{\theta} - \psi - w_*] < 0$$

or

$$0 < 2L(\underline{w} - w_*)$$

which holds. This concludes the proof of Part (i) of Proposition 2.

Parts (ii). We need only show that Assumption 1 is satisfied. Given $\hat{w} = w_B = (1 + \beta)\underline{w}$, we have

$$\begin{aligned} \bar{V} &= \frac{1}{2L}V_y(\hat{w}) + \left(1 - \frac{1}{2L}\right)V_o(\tilde{w}) \\ &= \frac{1}{2L} \left\{ (1 - \beta)\bar{\theta} + \beta \left[(1 - \beta)(\bar{\theta} - \psi) + \beta\bar{V} \right] - (1 - \beta)(w_B + \psi) \right\} \\ &\quad + \left(1 - \frac{1}{2L}\right) \left\{ (1 - \beta)(\bar{\theta} - \psi - \underline{w}) + \beta\bar{V} \right\} \end{aligned}$$

Solve for \bar{V} from the above equation and collect terms to obtain

$$\bar{V} = \bar{V}_B \equiv \bar{\theta} - \psi - \underline{w}. \quad (61)$$

Now

$$\bar{V} + w_* - (\bar{\theta} - \psi) - \gamma/\beta = -(\underline{w} - w_*) - \gamma/\beta < 0.$$

So Assumption 1 is satisfied.

Part (iii). We need only show that Assumption 1 is satisfied. The proof is the same as that in Part (ii). This concludes the proof of the proposition.

Lemma 5 (i) In each of the equilibrium types, $\alpha \in (0, 1)$. (ii) $\alpha_A > \alpha_B$ and $L_A > L_B$. (iii) $\bar{\gamma}_A < \bar{\gamma}_B$.

Proof. Let

$$f(\alpha; x) = 1 - \frac{1 - \eta}{1 - (\alpha/2)(1 - x)},$$

for $\alpha \in [0, 1]$ and $x \in [0, 1]$. We have, for all x , $f'_1(\alpha; x) < 0$ for all $\alpha \in [0, 1]$; in addition, $f(0, x) > 0$, $f(1, x) < 1$. These imply that, for any given x , the equation $\alpha = f(\alpha; x)$ has a unique solution for α . Moreover, $f(\alpha; x)$ is strictly increasing in x . Therefore the value of α that solves $\alpha = f(\alpha, x)$ increases as x increases. In other words, $\alpha_A > \alpha_B$. That $L_A > L_B$ follows immediately.

9.3 Proof of Proposition 3

That α and L are decreasing in α follows directly from (42) and (43). Let $F(L, \delta)$ denote the left hand side of equation (44). Then

$$\frac{\partial F}{\partial \delta} = -\frac{2L(1 - x_2)}{(2L + x_2)(2L + 1)} < 0,$$

$$\frac{\partial F}{\partial L} = 1 - \frac{2\delta x_2 \eta}{(2L + x_2)^2} - \frac{2\delta \eta}{(2L + 1)^2} > 1 - \frac{4\delta \eta}{(2L + 1)^2} > 0.$$

In the equation above, the first inequality holds because the function $g(x) = x/(2L + x)^2$ is strictly increasing in x over $(0, 1)$, for $g'(x) = (2L + x)^{-3}(2L - x) > 0$ where remember $2L > 1$. The second inequality holds because $2L + 1 > 2$. I therefore conclude that an increase in γ results in a decrease in L , which, in turn, must result in a decrease in δ in order for equation (44) to hold. This proves (ii). I leave it for the reader to verify part (ii) of the proposition.

9.4 Proof of Proposition 4

The proposition is proved by observing equations (60) and (61), and the fact that

$$\bar{V}(\bar{\gamma}_A) = \bar{V}(\gamma) = \bar{V}(\bar{\gamma}_B), \quad \forall \gamma \in (\bar{\gamma}_A, \bar{\gamma}_B).$$

To show that the above holds, first substitute $V_o(\underline{w}) = (1 - \beta)(\bar{\theta} - \psi - \underline{w}) + \beta \bar{V}$ into condition (39) and then use equation (42). This proves $\bar{V}(\gamma) = \bar{V}(\bar{\gamma}_B)$, $\forall \gamma \in (\bar{\gamma}_A, \bar{\gamma}_B)$.

Next, I show $\bar{V}(\bar{\gamma}_A) = \bar{V}(\bar{\gamma}_B)$. Using (60) and (61), this is equivalent to showing

$$\begin{aligned} \left[\bar{\theta} + x_2\beta(\bar{\theta} - \psi - w_* - m/\beta) - x_1\bar{\gamma}_A \right] + (2L - 1)(\bar{\theta} - \psi - \underline{w}) \\ = (2L + x_2\beta)(\bar{\theta} - \psi - \underline{w}), \end{aligned}$$

or

$$\psi + \underline{w} + x_2\beta(\underline{w} - w_*) - \frac{x_2\psi}{x_2 - x'_2} - x_1\bar{\gamma}_A = 0,$$

which holds, given $w_* = \alpha\underline{w}$, $\underline{w} = \frac{x_2\psi}{x_2 - x'_2}$, and equation (29).

9.5 Proof of Proposition 5

I need only prove (ii). Let $\eta \in (0, \eta]$. Let $C \equiv \bar{\theta} + x_2\beta(\bar{\theta} - \psi - w_* - m/\beta) - x_1\gamma$ and $D \equiv \bar{\theta} - \psi - \gamma_0 - \underline{w}$. Then I can write $\bar{V}(\eta)$ as

$$\begin{aligned} \bar{V}(\eta) &\equiv \frac{C - (1 + x_2\beta)D}{2L + x_2\beta} + D \\ &= \frac{(1 - \alpha)x_2\beta\underline{w} - x_1\gamma}{2 - \alpha x_2} \equiv f(\alpha) + D, \end{aligned}$$

where I have used the equilibrium relationship $L = 1 - \alpha x_2/2$ (equation (26)) and $w_* = \alpha\underline{w}$. Now it is straightforward to show that $f'(\alpha) < 0$. Now by the equilibrium condition (25), α is a strictly increasing function of η . Therefore we have shown that $\bar{V}(\eta)$ is strictly decreasing in η over $(0, \eta_A)$. Last, observe that at $\eta = \eta_A$, $\bar{V}(\eta) = D$. This proves that the function $\bar{V}(\eta)$ is continuous at $\eta = \eta_A$.

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Figure -1

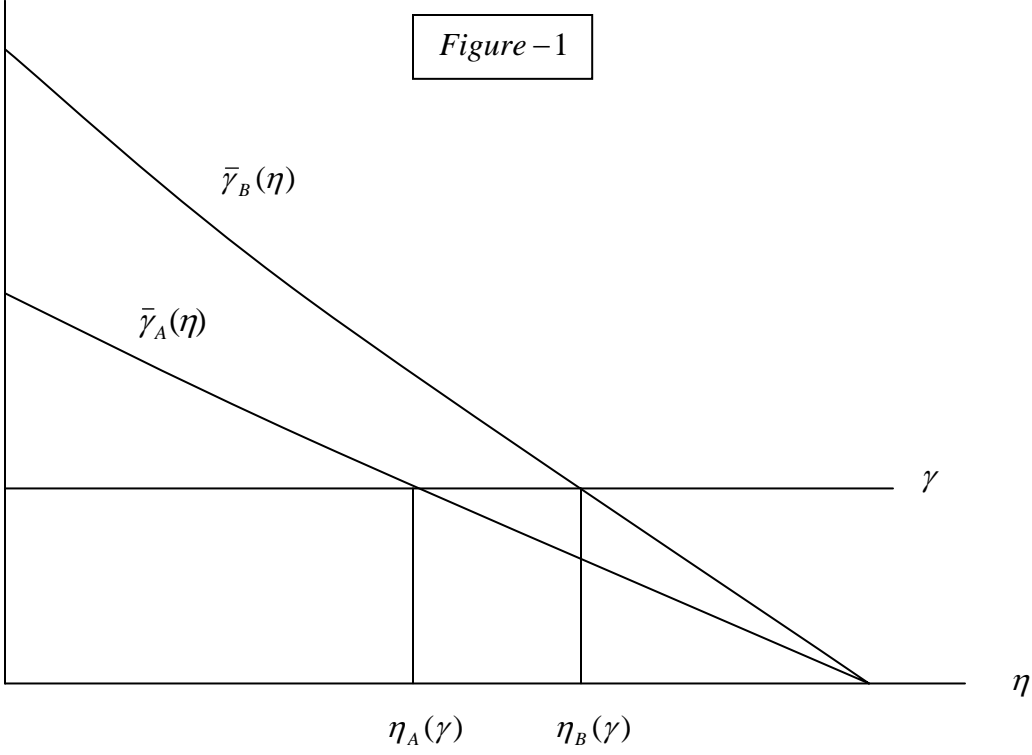


Figure - 2

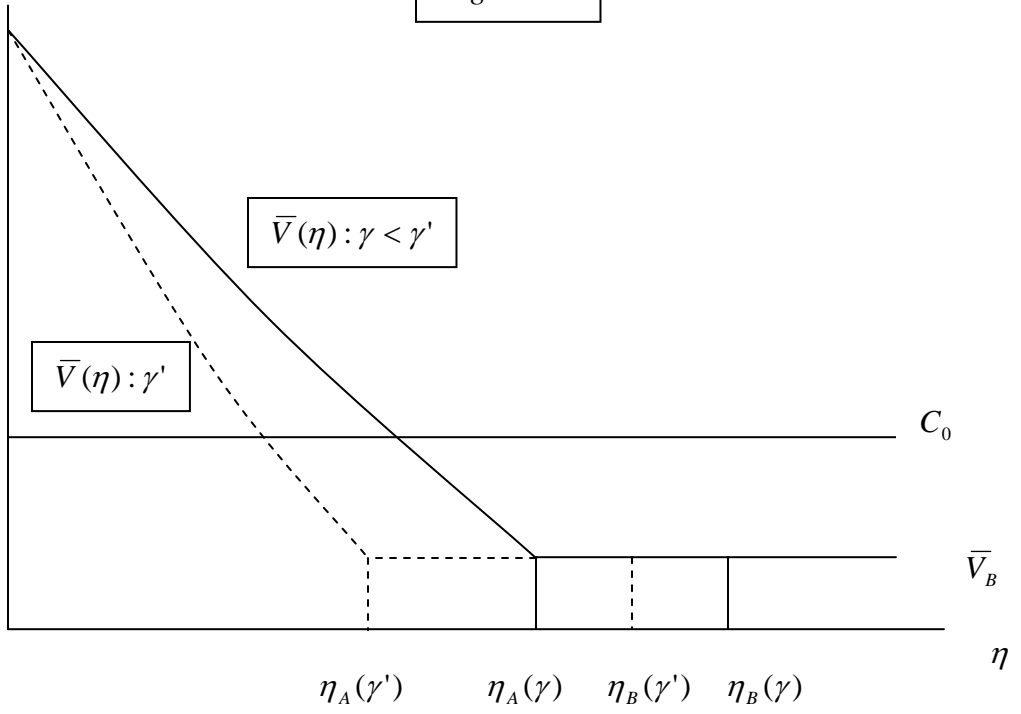


Figure - 3

