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The Terms of Trade, Repudiation and Default on Sovereign  
Debt

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## Abstract

A poor country with volatile export prices borrows in international markets. When debt is denominated in foreign currency, there is a temptation to repudiate when export prices are low. Excusable partial defaults reduce this temptation, and thus help to support lending.

Keywords: debt, default, terms of trade

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## 1. Introduction

Sovereign debt may be supported by the threat of exclusion from future borrowing or by other sanctions in the event of default. Rose (2005) provides some evidence that default depresses international trade. A general problem in evaluating why countries pay their debts is that, assuming rational expectations, countries that would not pay do not have any debts in the first place, because lenders would refuse to lend to them. Arguably many of the poorest countries in the world are in this situation, since most of their debts are to official rather than private bodies, either because they have few assets to seize or because their time discount rate is high (so the threat of exclusion from future borrowing is ineffective). In practice sovereign defaults are quite frequent, and lending resumes after a period of negotiation over the extent of default (Manasse *et al.*, 2003). This has motivated the development of models in which excusable partial defaults occur in recognisably bad states of the world (Alfaro and Kanczuk, 2005; Grossman and Van Huyck, 1988). The empirical evidence suggests that the risk of such defaults is priced relatively accurately, in the sense that in the long run risky debt yields similar returns to safe debt (Klingen *et al.*, 2004; Lindert and Morton, 1989).

This paper focuses on an interesting interaction between excusable partial default and the incentive to repudiate debt altogether. By reducing debt repayment in bad states, it is possible that excusable partial default eliminates the incentive to repudiate, if the bad states are the critical ones in the sense that the incentive to repudiate is greatest. This depends on the nature of the world (whether it is the quantity or relative price of the borrowing country's output that is stochastic) and on the currency denomination of the

debt contract. The most relevant case is where the terms of trade are volatile and debt is denominated in foreign currency.

## 2. The Model

### 2.1. Assumptions

The model is based on that of Cole and Kehoe (1995), which is attractive because of its simplicity. In the original model there are two countries. One has a risk-neutral government which is a monopolist of an investment project but initially has no resources to exploit it. The other consists of risk-neutral bankers who compete to lend to this government. All agents have a discount factor of  $\beta$ , and competition amongst bankers forces the gross interest rate down to  $r = 1/\beta$ . The returns to the investment project are of the following pattern: in alternate periods they are zero, and in alternate periods an investment of one unit in the previous period produces output of a non-storable commodity of  $A > r$ . There is a maximum investment of one. The government has a linear per-period utility function in consumption:

$$U(c_t) = c_t \tag{1}$$

and a lifetime utility of

$$U = \sum_{t=0}^{\infty} \beta^t U(c_t). \tag{2}$$

Thus, if  $t$  is odd, the government borrows and invests one unit and consumes nothing. If  $t$  is even, the government has output of  $A$  from the previous period's investment, repays debt of  $r$ , borrows and invests nothing, and consumes  $A-r$ . Alternatively, the government could repudiate the debt, after which there is no further lending. As Cole and Kehoe (1995) show, repudiation is always preferable if the government is able to save the resources obtained and earn a gross return of  $r$  on them.<sup>1</sup> Therefore I assume that the government is unable to save, and that repudiation implies immediate consumption of the entire output, with zero output in all future periods.

This model is modified here in the following respects. First, there are two goods, one produced in the borrowing country and one in the lending country. Second, the output of the borrowing country is exchanged for imports from the lending country at an uncertain relative price ( $p$ ). The price follows a known probability distribution that is independent of the history of  $p$ , with a mean of one. Finally, utility is derived from consumption of both home goods ( $x$ ) and foreign goods ( $y$ ). Suppose that in period  $t$  the borrowing government has resources  $W_t$ , measured in terms of domestic output, and that its utility function is:

$$U(c_t) = \left( \frac{x_t}{1-\lambda} \right)^{1-\lambda} \left( \frac{y_t}{\lambda} \right)^\lambda. \quad (3)$$

Equation (2) is maximized subject to the constraint:

$$x_t + \frac{y_t}{p_t} = W_t \quad (4)$$

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<sup>1</sup> As Grossman and Han (1999) show, this is a consequence of the risk-neutrality of the borrower. With risk-averse borrowers and state-contingent debt repayments, repudiation is not necessarily optimal when the borrowers can save.

which yields:

$$U(c_t) = W_t p_t^\lambda. \quad (5)$$

Thus utility is greater, for a given quantity of the borrower's output, when the terms of trade are better and imports are cheaper.

With two goods, the units in which debt is denominated need to be specified. We consider three possibilities: that debt is indexed to the price of (1) the borrowing country's consumption bundle; (2) its export good; and (3) its import good. These cases are analogous to indexation of debt (1) to the borrowing country's GDP deflator; (2) to the borrowing country's consumer price index; and (3) to the lending country's GDP deflator (i.e. in foreign currency). The last case is particularly relevant in view of the fact that borrowing in foreign currency may be the only option available for many countries (Eichengreen and Hausmann, 1999). When export prices are low, the borrower's real exchange rate depreciates and debt denominated in foreign currency is particularly expensive to repay. These valuation effects may explain the severe output effects of currency crises in emerging markets (Frankel, 2005).

## *2.2. Indexation to consumer prices*

Debt can be supported if, whatever the realisation of the terms of trade, the benefits of future borrowing exceed the gains from repudiation. If the real exchange rate at time  $t$  is

$p_t$ , and  $t$  is an even number, then repudiation of the debt at time  $t$  yields discounted utility of

$$U(R) = Ap_t^\lambda. \quad (6)$$

Here  $R$  denotes repudiation. The expected utility of not repudiating the debt ( $N$ ) is:

$$E[U(N)] = Ap_t^\lambda - r + \frac{\beta^2}{1-\beta^2} [AE(p^\lambda) - r]. \quad (7)$$

The first two terms on the right-hand side represent utility in period  $t$ , and the last term represents the discounted expected utility of consumption in future even periods, when  $p$  is on average equal to one. Allowing for the fact that  $\beta = 1/r$ , debt will not be repudiated if  $UI \leq E[U(N)]$ , i.e. if

$$AE(p^\lambda) \geq r^3. \quad (8)$$

This differs only marginally from the one-good equivalent ( $A \geq r^3$ ).

### 2.3. Debt indexed to export prices

In this case  $U(R)$  is as given in (6), but  $E[U(N)]$  is

$$E[U(N)] = (A-r)p_t^\lambda + \frac{\beta^2}{1-\beta^2} [(A-r)E(p^\lambda)]. \quad (9)$$

Now that the debt is not indexed to the consumption bundle, the utility cost of repaying the debt depends on current relative prices. Yet the expected benefits of repayment are much the same, because they depend on the expected future utility cost of debt repayments. A high value of  $p_t$  makes present repayments particularly onerous. The condition for non-repudiation is that

$$p_t^\lambda \leq \frac{(A-r)E(p^\lambda)}{r(r^2-1)} \quad (10)$$

When the terms of trade are at their most favourable, this condition may not be met. Thus relative price volatility makes it harder for the threat of exclusion from future borrowing to support debt.

#### *2.4. Debt indexed to import prices*

In this case repudiation yields the same result as before, but repayment yields an expected utility of:

$$E[U(N)] = Ap_t^\lambda - \frac{r}{p_t^{1-\lambda}} + \frac{\beta^2}{1-\beta^2} \left( AE(p^\lambda) - rE\left(\frac{1}{p^{1-\lambda}}\right) \right) \quad (11)$$

and the condition that  $U(R) \leq E[U(N)]$  is



$$\frac{1}{p_t^{1-\lambda}} \leq \frac{1}{r^2 - 1} \left( AE(p^\lambda) - rE\left(\frac{1}{p^{1-\lambda}}\right) \right) \quad (12)$$

The critical circumstances in this case are when  $p$  is exceptionally low, since this is the case where debt is at its most burdensome.

### *2.5. Excusable default*

Now consider what happens if the lender can verify the borrower's output and is prepared to accept partial default on the debt if the borrower's current-period utility would otherwise fall below some threshold level,  $C$ . When default occurs, utility is just equal to  $C$ , and the borrower receives the surplus output. To compensate for this, the debt now bears a higher gross interest rate of  $R$  that incorporates a risk premium. This affects the incentive to repudiate debt. By making payments higher in good states and lower in bad states, it increases the incentive to repudiate if the critical value of  $p$  is high, and decreases it when the critical value of  $p$  is low. An example of the former is indexation of debt to export prices. With excusable default, the condition for non-repudiation is now:

$$p_t^\lambda \leq \frac{(A-R)E(p^\lambda)}{r(r^2 - 1)} \quad (13)$$

Since  $R > r$ , this is a more stringent condition than (10).

When debt is indexed to import prices, we get the opposite case. Define  $\hat{p}$  as the minimum price at which there is no default;  $\hat{p}$  is given by:

$$A\hat{p}^\lambda - \frac{R}{\hat{p}^{1-\lambda}} = C \quad (14)$$

Expected utility without repudiation is:

$$E[U(N)] = \max\left(Ap_t^\lambda - \frac{R}{p_t^{1-\lambda}}, C\right) + \frac{\beta^2}{1-\beta^2} \left( E\left(\max\left[AE(p^\lambda) - RE\left(\frac{1}{p^{1-\lambda}}\right), C\right]\right) \right) \quad (15)$$

$E[U(N)] - U(R)$  is increasing in  $p$  for  $p < \hat{p}$  and decreasing in  $p$  for  $p > \hat{p}$ , so the condition for lending to be supported by the threat of exclusion from future borrowing is that  $E[U(N)] \geq U(R)$  for  $p = \hat{p}$ . This requires that

$$\frac{R}{\hat{p}^{1-\lambda}} \leq \frac{1}{r^2 - 1} \left( E\left(\max\left[AE(p^\lambda) - RE\left(\frac{1}{p^{1-\lambda}}\right), C\right]\right) \right) \quad (16)$$

The right-hand side of (16) cannot be less than the right-hand side of (12). Equation (16) is therefore a less stringent condition than (12) if its left-hand side is smaller, i.e. if

$$\frac{R}{\hat{p}^{1-\lambda}} \leq \frac{r}{p_{\min}^{1-\lambda}} \quad (17)$$

where  $p_{\min}$  is the minimum value of  $p$ . Suppose that  $p_{\min} = (1-\beta)\hat{p}$ ; that the probability that  $p < p^*$  is  $\alpha$ , and that the average payment in the event of default is  $(1-\lambda\beta)R$ . Then

$$(1 - \alpha)R + \alpha(1 - \lambda\beta)R = r \quad (18)$$

so

$$(1 - \alpha\lambda\beta)R = r \quad (19)$$

It follows that (19) is a less stringent condition than (12) if

$$(1 - \beta)^{1-\lambda} < 1 - \alpha\lambda\beta \quad (20)$$

Since, for a unimodal distribution,  $\lambda \leq \frac{1}{2}$  (i.e. at worst all degrees of default are equally likely), and  $\alpha$  is likely to be a relatively small fraction, (19) is inevitably satisfied even for an unusually fat-tailed and truncated distribution.

This result implies that, with debt indexed to import prices, there can be circumstances where the threat of exclusion from future borrowing can support lending *only* if there is partial default in bad states, matched by higher payments in good states.

## 2.6. Stochastic output

Consider now the case where the terms of trade are fixed ( $p=1$ ) and instead the quantity of output  $A_t$  is stochastic, with a mean of  $A$ . Without excusable defaults, we have:

$$U(R) = A_t . \quad (20)$$

and:

$$E[U(N)] = A_t - r + \frac{\beta^2}{1 - \beta^2}(A - r) \quad (21)$$

Since the utility function is now linear, expected utility is  $(A-r)$ . Non-repudiation requires that

$$A \geq r^3. \quad (22)$$

If lenders are willing to guarantee a minimum utility level for the borrower and accept partial defaults on occasion, then in periods where there is no default and  $R$  is repaid, the first  $r$  in (21) must be replaced by  $R$ , which includes a risk premium, and the condition for non-repudiation becomes slightly more stringent:

$$A \geq (r^2 - 1)R + r. \quad (23)$$

Thus, in the case of stochastic quantities of output as opposed to prices, excusable default makes it slightly harder to support lending through the threat of exclusion from future borrowing.

### 3. Conclusions

Fluctuations in the terms of trade can undermine the threat of exclusion from future borrowing as a sanction against debt repudiation when debt is not indexed to the consumption bundle (i.e. denominated in domestic currency), because at one end of the price spectrum repayment is particularly costly relative to its future benefits. I have shown that a system of excusable defaults, which guarantee the borrower a minimum utility level in any period, help to support lending if debt is especially onerous in bad states, as in the case of indexation to import prices, which is analogous to the commonly observed case of denomination in foreign currency. It follows that, paradoxically, the partial defaults on foreign debt that are commonly observed may be a condition for lending to occur at all. This result applies only if it is the terms of trade rather than the volume of the borrower's output that is stochastic.

### References

- Alfaro, L., and F. Kanczuk (2005) "Sovereign Debt as a Contingent Claim: A Quantitative Approach" *Journal of International Economics* **65**, 297-314
- Cole, H.L., and P.J. Kehoe (1995) "The Role of Institutions in Reputation Models of Sovereign Debt" *Journal of Monetary Economics* **35**, 45-64
- Eichengreen, B., and R. Hausmann (1999) "Exchange Rates and Financial Fragility" *NBER Working Paper* no. 7418
- Frankel, J.A. (2005) "Contractionary Currency Crashes in Developing Countries" *IMF Staff Papers* **52**, 149-192

- Grossman, H.I., and T. Han (1999) "Sovereign Debt and Consumption Smoothing"  
*Journal of Monetary Economics* **44**, 149-158
- Grossman, H.I., and J.B. Van Huyck (1988). "Sovereign Debt as a Contingent Claim:  
Excusable Default, Repudiation, and Reputation" *American Economic Review* **78**,  
1088-1097
- Klingen, C., B. Weder and J. Zettelmeyer (2004) "How Private Creditors Fared in  
Emerging Debt Markets, 1970-2000" *IMF Working Paper* no. 04/13
- Lindert, P., and P.J. Morton (1989) "How Sovereign Debt Has Worked" in *Developing  
Country Debt and Economic Performance* by J. Sachs, Ed., University of Chicago  
Press: Chicago, 39-106
- Manasse, P., N. Roubini and A. Schimmelpfennig (2003) "Predicting Sovereign Debt  
Crises" *IMF Working Paper* no. 03/221
- Rose, A.K. (2005) "One Reason Countries Pay Their Debts: Renegotiation and  
International Trade" *Journal of Development Economics* **77**, 189-206