Minimax Regret and Efficient Bargaining Under Uncertainty*

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Abstract

Bargaining under uncertainty is modeled by the assumption that there are several possible states of nature, each of which is identified with a bargaining problem. We characterize bargaining solutions which generate *ex ante* efficient combinations of outcomes under the assumption that the bargainers have minimax regret preferences. For the case of two bargainers a class of monotone utopia-path solutions is characterized by the efficiency criterion, but for more than two bargainers only dictatorial solutions are efficient. By incorporating scale covariance into the minimax regret preferences a possibility result is obtained for the general case.

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1 Introduction

An n-person bargaining problem is a set of feasible utility n-tuples together with a prespecified n-tuple, the disagreement outcome (cf. Nash, 1950). The utilities represent the preferences of the bargainers. These may vary from preferences over lottery sets, represented by von Neumann-Morgenstern utility functions, to profit functions of firms in an oligopoly situation. Depending on the situation, there may be uncertainty concerning the exact shape of the bargaining problem. For instance, demand functions in an oligopoly situation may be subject to stochastic influences. A common way to model such uncertainty is to assume that there are several states of nature, exactly one of which will be realized as the true state. In the present context, a state of nature can be identified with a specific bargaining problem.

A bargaining solution (Nash, 1950) assigns a feasible utility n-tuple to every bargaining problem. Thus, implicitly, a bargaining solution aggregates the individual preferences of the bargainers into a collective outcome. In the case of uncertainty, a bargaining solution assigns a utility n-tuple to each possible bargaining problem, i.e., to each possible state of nature. Given a specific bargaining solution to be employed, each bargainer *ex ante* faces a list of possible outcomes, exactly one of which will be realised. We assume that, in order to evaluate different lists of possible outcomes, each bargainer has a preference relation over such lists. In more economic terms, each bargainer has a criterion to decide between several contingent contracts. Although most bargaining solutions in the literature are *ex post* efficient, this does not imply that they are also *ex ante* efficient. This raises the following questions. For a given criterion for decision making under uncertainty, which bargaining solution(s) lead(s) to *ex ante* efficient contingent contracts and, conversely, for a given bargaining solution, does there exist a criterion for decision making under uncertainty according to which a contingent contract prescribed by that bargaining solution is *ex ante* efficient?

By answering these questions, we obtain a new view on bargaining solutions, as aggregators of individual preferences under uncertainty. Furthermore, the approach leads to characterizations of bargaining solutions that are different from the usual axiomatizations and from noncooperative implementations, both of which were initiated by Nash (1950, 1953).

The first paper concerned with this approach is Bossert *et al.* (1996) in which—as the main result—a class of strictly monotone path solutions is characterized by imposing *ex ante* efficiency with respect to the maximin criterion. According to this criterion the minimal gains with respect to the disagreement point should be maximized.

In the present paper we consider the minimax regret criterion, where regret is measured with respect to the utopia payoffs, i.e., the maximal attainable payoffs. In particular, we study solutions that are *ex ante* efficient with respect to this criterion. For the two-person case, we show that this criterion determines a class of monotone paths, originating from the utopia point, of which the intersection with the Pareto optimal boundary is the solution point. For the n-person case ($n > 2$) it turns out that only the dictatorial solutions are *ex ante* efficient with respect to the minimax regret criterion. Modifying the criterion such that it is compatible with scale covariance, however, leads to a characterization of solutions determined by a monotone path between the normalized utopia and disagreement points. As will be indicated, these solutions can alternatively be characterized by *ex ante* efficiency with respect to a version of the maximin criterion that is compatible with scale covariance. They include the Raiffa-Kalai-Smorodinsky solution (Raiffa, 1953; Kalai and Smorodinsky, 1975).

The organization of the paper is as follows. Section 2 introduces the model and main definitions. Section 3 deals with the minimax regret criterion and presents the positive result for $n = 2$ and the negative one for $n > 2$. In Section 4 the minimax regret criterion is normalized in a way compatible with scale covariance, leading to the characterization announced earlier. Section 5 concludes.

2 Model and main definitions

An *n-person (bargaining) problem* is a set $S \subset I\!R^n$ such that there is a point $\bar{d} \in S$ with $S \subset \bar{d} + I\!R^n_+$, S is compact, contains a vector $x > \bar{d}$, and is *strictly comprehensive*, that is, for all $x \in S$ and $y \in I\!R^n$, if $\bar{d} \leq y \leq x$ with $y \neq x$, then $y \in S$ and there is a $z \in S$ with $z > y$.[1] Elements of

[1]The set of all nonnegative vectors in $I\!R^n$ is denoted by $I\!R^n_+$, and $I\!R^n_-$ is the set of all nonpositive vectors. The set of all strictly positive vectors in $I\!R^n$ is denoted by $I\!R^n_{++}$. A sum like $\bar{d} + I\!R^n_+$ denotes the usual vector addition, that is, $\bar{d} + I\!R^n_+ := \{\bar{d} + x \mid x \in I\!R^n_+\}$. For any two vectors $x, y \in I\!R^n$, $x > (\geq)\, y$ means $x_i > (\geq)\, y_i$ for all $i = 1, \ldots, n$. The inequalities $<$ and $\leq$ are defined analogously.

S are called *feasible outcomes*, while $\bar{d}$ is called the *disagreement outcome*. The interpretation is that $\bar{d}$ results if the bargainers fail to reach some other outcome $x \in S$. Note that $\bar{d}$ is uniquely determined by S, so that we can write $\bar{d} = d(S)$. The i-th coordinate of an outcome represents the utility to bargainer i. Throughout, we assume $n \geq 2$ to be fixed. The set of bargainers is denoted by $N := \{1, \ldots, n\}$, and $\mathcal{B}$ denotes the class of all n-person bargaining problems.

Compactness is a standard condition in bargaining theory. By the requirement $S \subset d(S) + \mathbb{R}^n_+$ we exclude nonindividually rational outcomes from consideration. Comprehensiveness can be interpreted as disposability of utility. Strict comprehensiveness additionally implies that every weakly Pareto optimal outcome is also strongly Pareto optimal, so that it suffices to define, for a bargaining problem S,

$$P(S) := \{x \in S \mid \forall y \in S \; [y \geq x \Rightarrow y = x]\},$$

the *Pareto set of* S. Restricting attention to strictly comprehensive problems facilitates the exposition, see also Section 5.[2]

A *(bargaining) solution* is a mapping $F : \mathcal{B} \to \mathbb{R}^n$ with $F(S) \in S$ for all $S \in \mathcal{B}$. A solution F is called *Pareto optimal* if $F(S) \in P(S)$ for all $S \in \mathcal{B}$.[3]

In order to introduce uncertainty into the model we assume that there are a finite number of *states of nature*, exactly one of which will be realised. It is without loss of generality and easier for the exposition of the results to assume that there are only two states. A *(bargaining) problem under uncertainty* is, thus, defined by a pair $(S, S') \in \mathcal{B} \times \mathcal{B}$.

For each problem under uncertainty (S, S'), we assume that each bargainer i has a preference relation (i.e., a complete and transitive binary relation) $\succeq_i$ over pairs $(x, x') \in S \times S'$ which depends only on the i-th coordinates (x_i, x'_i) and which is weakly monotonic. The latter means that $(x, x') \succ_i (y, y')$ whenever $x_i > y_i$ and $x'_i > y'_i$, where $\succ_i$ denotes the asymmetric part of $\succeq_i$. A preference relation with these properties is called *regular*. Note that, by definition, such a preference relation depends on the problem under uncertainty (S, S'). We will, however, also use a notation like $\succeq_i$ to

[2] Observe that we do not impose the usual convexity assumption on the bargaining problem. All our results, however, would go through without modification if this assumption were added.

[3] We use the expression "Pareto optimal" rather than "efficient" to distinguish this property from the efficiency condition related to uncertainty, to be introduced below.

denote bargainer i's preferences for every problem in $\mathcal{B} \times \mathcal{B}$; if all these preferences are regular, then also $\succeq_i$ is called regular.

Let F be a bargaining solution, and let $\succeq = (\succeq_1, \ldots, \succeq_n)$, where $\succeq_i$ is regular for every $i \in N$. We call F *efficient with respect to* $\succeq$ if for all $(S, S') \in \mathcal{B} \times \mathcal{B}$ and all $(x, x') \in S \times S'$ there is a bargainer i with $(F(S), F(S')) \succeq_i (x, x')$. In other words, there is no "contingent contract" which is strictly preferred by all bargainers to the contract assigned by F. The following lemma shows that efficiency with respect to a profile of regular preferences implies Pareto optimality.

Lemma 1 *Let F be a bargaining solution, and let $\succeq = (\succeq_1, \ldots, \succeq_n)$, with $\succeq_i$ regular for every $i \in N$. Let F be efficient with respect to $\succeq$. Then F is Pareto optimal.*

Proof Let $S \in \mathcal{B}$ and $x \in S \backslash P(S)$. By the strict comprehensiveness of S, there is a $y \in S$ with $y > x$. It follows that $F(S) \neq x$, for otherwise $(y, y) \succ_i (F(S), F(S))$ in (S, S) by regularity of $\succeq_i$ for all i, violating efficiency of F with respect to $\succeq$. This completes the proof. $\square$

In this paper we only consider regular preferences. In particular, we exclude (possibly interesting) preferences which do not only depend on a bargainer i's own coordinates.

3 Minimax regret

The *utopia point* $u(S)$ of a problem $S \in \mathcal{B}$ is defined by

$$u_i(S) := \max_{x \in S} x_i$$

for every $i \in N$. Bargainer i's *minimax regret preference* $\succeq_i^u$ is defined as follows. For all $(S, S') \in \mathcal{B} \times \mathcal{B}$ and all $(x, x'), (y, y') \in S \times S'$, $(x, x') \succeq_i^u (y, y')$ if

$$\max\{u_i(S) - x_i, u_i(S') - x_i'\} \leq \max\{u_i(S) - y_i, u_i(S') - y_i'\}.$$

Note that $\succeq_i^u$ is regular for every bargainer i.

In order to study bargaining solutions that are efficient with respect to minimax regret preferences we first show that this condition implies a monotonicity condition, as specified by the following lemma. The lemma is an adaptation of Lemma 2 in Bossert *et al.* (1996).

Lemma 2 *Let the bargaining solution F be efficient with respect to $\succeq^u = (\succeq^u_1, \ldots, \succeq^u_n)$. Let $S, T \in \mathcal{B}$. Let $x = F(S)$ and $y = F(T)$. Then $u(S) - x \geq u(T) - y$ or $u(S) - x \leq u(T) - y$.*

Proof Suppose not. Then there are $i, j \in N$ with $u_i(S) - x_i > u_i(T) - y_i$ whereas $u_j(S) - x_j < u_j(T) - y_j$. Let $I := \{k \in N \mid u_k(S) - x_k > u_k(T) - y_k\}$ and $J := \{k \in N \mid u_k(S) - x_k < u_k(T) - y_k\}$. Note that $I, J \neq \emptyset$ and that $u_k(S) - x_k > 0$ for all $k \in I$ and $u_k(T) - y_k > 0$ for all $k \in J$. By this and strict comprehensiveness we can find $x' \in S$ and $y' \in T$ with

$$\begin{aligned} \forall k \in I: &\quad u_k(S) - x_k > u_k(S) - x'_k > u_k(T) - y_k \\ \forall k \in N \backslash I: &\quad u_k(S) - x_k < u_k(S) - x'_k \\ \forall k \in J: &\quad u_k(T) - y_k > u_k(T) - y'_k > u_k(S) - x_k \\ \forall k \in N \backslash J: &\quad u_k(T) - y_k < u_k(T) - y'_k. \end{aligned}$$

By construction, $(x', y') \succ^u_k (x, y)$ for all $k \in N$, which is a violation of efficiency of F with respect to $\succeq^u$. □

Note that Lemmas 1 and 2 in particular imply *translation covariance* of the solution F under consideration, i.e., $F(S) + b = F(S + b)$ for all $S \in \mathcal{B}$ and $b \in \mathbb{R}^n$ (this follows from taking $T = S + b$ in Lemma 2).

For every $i \in N$ let D^i denote the *dictatorial solution for player i*. This solution assigns to every $S \in \mathcal{B}$ the point with i-th coordinate equal to $u_i(S)$ and every other coordinate j equal to $d_j(S)$. A solution F is a *dictatorial solution* if there exists $i \in N$ such that $F = D^i$. Our first theorem is an impossibility result: if $n > 2$, there exists no efficient and nondictatorial solution.

Theorem 1 *Let $n > 2$, and let F be a bargaining solution. Then F is efficient with respect to $\succeq^u = (\succeq^u_1, \ldots, \succeq^u_n)$ if, and only if, F is a dictatorial solution.*

Proof The if-part is left to the reader. For the only-if part it is, in view of Lemma 2, sufficient to prove that for every $S \in \mathcal{B}$ there is an $i \in N$ with $F(S) = D^i(S)$. Suppose this is not true, and let $S \in \mathcal{B}$ be such that $x := F(S) < u(S)$. By translation covariance (see the remark following the proof of Lemma 2) we may assume without loss of generality that $u(S) = (1, 1, \ldots, 1)$. Choose $\lambda \in \mathbb{R}$ with $\lambda < x_1 + x_2 - 1$, hence $x_1 + x_2 > \lambda + 1$ and

$\lambda < 1$. Choose $\mu \in I\!R$ with $x_3 < \mu < 1$, and let

$$L := \text{convex hull of } \{(\lambda, \lambda, \mu, \lambda, \ldots, \lambda), (\lambda, \lambda, 1, \lambda, \ldots, \lambda), z^i \mid i \in N \backslash \{3\}\},$$

where z^i has i-th coordinate equal to 1, third coordinate equal to μ, and all other coordinates equal to λ. Observe that $u(L) = (1, 1, \ldots, 1) = u(S)$. In view of Lemma 2, $x_3 < \mu$, and the fact that every point in L has third coordinate at least equal to μ, it follows that $F(L) \geq x$. This, however, is impossible because $x_1 + x_2 > \lambda + 1 > y_1 + y_2$ for every $y \in L$ by the choice of λ. Thus, we have a contradiction, and the proof is complete. $\square$

This impossibility result does not extend to the case $n = 2$: there, we can find nondictatorial solutions that are efficient with respect to maximin regret preferences. We start the analysis by defining a monotone u-path.[4] A *monotone u-path* is a function $w : (-\infty, 0] \to I\!R^2_-$ satisfying for all $s, t \in (-\infty, 0]$ with $s \leq t$:

(i) $w_1(s) + w_2(s) = s$

(ii) $w(s) \leq w(t)$.

Let W denote the collection of all monotone u-paths. With each $w \in W$ we associate a *monotone u-path bargaining solution* F^w, defined as follows. For a two-person bargaining problem S,

$$\{F^w(S)\} = P(S) \cap \{u(S) + w(s) \mid s \in (-\infty, 0]\}.$$

It is easy to see that F^w is well-defined. Observe that this definition cannot straightforwardly be extended to more than two players: the set on the right-hand side of the equation could be empty. We have the following result.

Theorem 2 *Let $n = 2$, and let F be a bargaining solution. Then F is efficient with respect to $\succeq^u = (\succeq^u_1, \ldots, \succeq^u_n)$ if, and only if, F is a monotone u-path solution.*

Proof We leave verification of the if-part to the reader. For the only-if part, let F be a bargaining solution that is efficient with respect to $\succeq^u$.

[4] See Thomson and Myerson (1980) for a study of solutions defined with the aid of monotone paths.

First, for every $-\infty < t < 0$ define the set $V_t := \{x \in \mathbb{R}^2_- \mid x_1 + x_2 \leq t,\ x_1 \geq t,\ x_2 \geq t\}$. Define $w : (-\infty, 0] \to \mathbb{R}^2_-$ by $w(0) := 0$ and $w(t) := F(V_t)$ for every $t < 0$. By Lemma 1, w satisfies property (i), and by Lemma 2, w satisfies property (ii) of a monotone u-path. Hence, w is a monotone u-path.

The proof is completed by showing that $F = F^w$. By construction, $F(V_t) = F^w(V_t)$ for every $t < 0$. Let $S \in \mathcal{B}$ with $u(S) = 0$, and let $t := F^w_1(S) + F^w_2(S)$. Then $F^w(S) = F^w(V_t) = F(V_t)$. Because $F(V_t) \in P(S)$, Lemma 2 implies $F(S) = F(V_t)$, hence $F(S) = F^w(S)$.

Finally, consider $S \in \mathcal{B}$ arbitrary. Then $F(S) = F^w(S)$ by the previous part of the proof and translation covariance of F, see the remark following the proof of Lemma 2. This completes the proof. □

A well-known example of a monotone u-path solution is the *equal-loss solution*, described by the monotone u-path w with $w_i(t) = w_j(t)$ for all $t < 0$ and all $i, j \in N$. This may be singled out by adding an axiom of anonymity or symmetry. See also Chun (1988).

4 Minimax regret and scale covariance

In this section we modify the minimax regret preference relation in order to accommodate the scale covariance property. At the same time an impossibility result as in the previous section will be avoided.

A solution F is called *scale covariant* if $aF(S) + b = F(aS + b)$ for all $a \in \mathbb{R}^n_{++}$ and $b \in \mathbb{R}^n$.[5] Bargainer i's minimax regret preference may be normalized to a preference $\tilde{\succeq}^u_i$ as follows. For all $(S, S') \in \mathcal{B} \times \mathcal{B}$ and all $(x, x'), (y, y') \in S \times S'$, we have $(x, x') \tilde{\succeq}^u_i (y, y')$ if

$$\max\left\{\frac{u_i(S) - x_i}{u_i(S) - d_i(S)}, \frac{u_i(S') - x'_i}{u_i(S') - d_i(S')}\right\} \leq \max\left\{\frac{u_i(S) - y_i}{u_i(S) - d_i(S)}, \frac{u_i(S') - y'_i}{u_i(S') - d_i(S')}\right\}.$$

Observe that this preference—called *normalized minimax regret preference*—is regular, so that Lemma 1 still applies. Instead of Lemma 2 we have the following lemma, which we state without proof.

[5] Here we use the notation $ax := (a_1x_1, \ldots, a_nx_n)$ and $aS := \{y \in \mathbb{R}^n \mid y = ax \text{ for some } x \in S\}$, for all $a, x \in \mathbb{R}^n$ and $S \subseteq \mathbb{R}^n$.

Lemma 3 *Let the bargaining solution F be efficient with respect to $\tilde{\succeq}^u = (\tilde{\succeq}_1^u, \ldots, \tilde{\succeq}_n^u)$. Let $S, T \in \mathcal{B}$. Let $x = F(S)$ and $y = F(T)$. Then*

$$\frac{u_i(S) - x_i}{u_i(S) - d_i(S)} \geq \frac{u_i(T) - y_i}{u_i(T) - d_i(T)} \text{ for every } i \in N$$

or

$$\frac{u_i(S) - x_i}{u_i(S) - d_i(S)} \leq \frac{u_i(T) - y_i}{u_i(T) - d_i(T)} \text{ for every } i \in N.$$

Together with Lemma 1, Lemma 3 implies in particular that such an F is scale covariant: take $T = aS + b$ with a and b as in the definition of scale covariance.

We will characterize all solutions that are efficient with respect to the normalized minimax regret preferences by considering *normalized monotone u-paths*. Such a normalized monotone u-path is a function $z : [1, n] \to \mathbb{R}_+^n$ satisfying for all $1 \leq s \leq t \leq n$:

(a) $\sum_{i=1}^n z_i(s) = s$

(b) $z(s) \leq z(t)$

(c) $z(s) \leq (1, 1, \ldots, 1)$.

Let Z denote the collection of all normalized monotone u-paths. With each $z \in Z$ we associate a *normalized montone u-path bargaining solution* F^z, defined as follows. For a bargaining problem $S \in \mathcal{B}$ with $d(S) = (0, 0, \ldots, 0)$ and $u(S) = (1, 1, \ldots, 1)$ let

$$\{F^z(S)\} = P(S) \cap \{z(s) \mid s \in [1, n]\}.$$

For an arbitrary S define $F^z(S) = aF^z(a'(S - d(S))) + d(S)$, where $a := u(S) - d(S)$ and $a' \in \mathbb{R}^n$ is defined by $a'_i := (u_i(S) - d_i(S))^{-1}$. Observe that F^z is well defined in particular by strict comprehensiveness of S, and that F^z is scale covariant by definition.

Theorem 3 *Let $n \geq 2$, and let F be a bargaining solution. Then F is efficient with respect to $\tilde{\succeq}^u = (\tilde{\succeq}_1^u, \ldots, \tilde{\succeq}_n^u)$ if, and only if, F is a normalized monotone u-path solution.*

Proof The if-part is left to the reader. For the only-if part, let F be a bargaining solution that is efficient with respect to $\tilde{\succeq}^u$.

Let $V_1 := \{x \in I\!R^n_+ \mid \sum_{i=1}^n x_i = 1\}$ and for every $1 < t < n$ and $0 < \epsilon < (n-t)/n$ let

$$V_t^\epsilon := \text{convex hull of } V_1 \cup \{x \in I\!R^n_+ \mid \sum_{i=1}^n x_i = t,\ x_i \leq \epsilon \text{ for all } i \in N\}.$$

Observe that every such V_t^ϵ is a well-defined bargaining problem, that is, $V_t^\epsilon \in \mathcal{B}$. Define $z : [1,n] \to I\!R^n_+$ as follows. Let $z(1) := F(V_1)$ and $z(n) := (1,1,\ldots,1)$. Take $1 < t < n$. If there is an ϵ such that $\sum_{i=1}^n F_i(V_t^\epsilon) = t$ then let $z(t)$ be equal to this point $F(V_t^\epsilon)$. This construction is independent of ϵ in view of the dominance property established in Lemma 3. The same lemma also implies that, if such an ϵ exists for t then it also exists for all $1 < t' < t$. Let $\bar{t} \leq n$ be the supremum of all t for which such an ϵ exists. Define $z(\bar{t}) := \lim_{t \to \bar{t}} z(t)$, and for $\bar{t} < t \leq n$ let $z(t)$ be the point on the line segment connecting $z(\bar{t})$ and $z(n)$ with sum of the coordinates equal to t. It is easily seen that z is a normalized monotone u-path.

Let $S \in \mathcal{B}$. It is sufficient to show that $F(S) = F^z(S)$. In view of scale covariance of F^z and F we may assume without loss of generality that $d(S) = (0,0,\ldots,0)$ and $u(S) = (1,1,\ldots,1)$. Let $1 \leq t < n$ with $t = \sum_{i=1}^n F_i(S)$. There is an ϵ with $F^z(S) = F^z(V_t^\epsilon) = F(V_t^\epsilon)$ by construction of z and definition of F^z. Because $F(V_t^\epsilon) \in P(S)$, Lemma 3 implies $F(S) = F(V_\epsilon^t)$, hence $F(S) = F^z(S)$. This completes the proof. □

A well-known example of a normalized monotone u-path solution is the Raiffa-Kalai-Smorodinsky solution (Raiffa, 1953; Kalai and Smorodinsky, 1975), described by the path z with $z_i(t) = z_j(t)$ for all $i, j \in N$ and $1 \leq t \leq n$. This solution may be characterized by adding an axiom of anonymity or symmetry.

Normalized monotone u-path solutions may be characterized, alternatively, by requiring efficiency with respect to a (normalized version of) the maximin criterion, namely the preference $\tilde{\succeq}_i^d$ for bargainer i defined as follows. For all $(S, S') \in \mathcal{B} \times \mathcal{B}$ and all $(x, x'), (y, y') \in S \times S'$, we have $(x, x')\tilde{\succeq}_i^d(y, y')$ if

$$\min\left\{\frac{x_i - d_i(S)}{u_i(S) - d_i(S)}, \frac{x'_i - d_i(S')}{u_i(S') - d_i(S')}\right\} \geq \min\left\{\frac{y_i - d_i(S)}{u_i(S) - d_i(S)}, \frac{y'_i - d_i(S')}{u_i(S') - d_i(S')}\right\}.$$

A proof of the suggested characterization is left to the reader. It should be noted that such a "dual" characterization does *not* hold for the non-normalized, non-scale-covariant versions, as follows from comparing our results with those in Bossert *et al.* (1996). In the latter paper strict comprehensiveness is not imposed. This leads to some technical complications and the necessity to impose an additional axiom of continuity on the bargaining solutions. For (non-normalized) maximin preferences, however, Bossert *et al.* (1996) obtain a class of monotone path solutions, and this result can easily be adapted to our framework. In contrast, for (non-normalized) minimax regret preferences, we have an impossibility result in the case of more than two players (see Theorem 1).

5 Concluding remarks

The approach followed in Bossert *et al.* (1996) and in this paper is essentially based on the idea that a rich underlying structure with respect to individual decision making may be used to derive implications for collective decision making while only imposing relatively mild additional requirements. The individual decision criteria used in this paper are minimax regret and a normalized version thereof. In Bossert and Peters (1998), this idea is applied to multi-attribute individual and collective decision making in an expected-utility framework. Interpreted in a bargaining context, the results obtained there lead to generalized (nonsymmetric) Nash and utilitarian solutions.

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