Granger Centre for time series econometrics

Granger Centre Discussion Paper Series

Testing for unit roots and the impact of quadratic trends, with an application to relative primary commodity prices

by

David I. Harvey, Stephen J. Leybourne and A. M. Robert Taylor

---

Granger Centre Discussion Paper No. 08/04

The University of Nottingham

Testing for Unit Roots and the Impact of Quadratic Trends, with an Application to Relative Primary Commodity Prices*

David I. Harvey, Stephen J. Leybourne and A. M. Robert Taylor
School of Economics and Granger Centre for Time Series Econometrics
University of Nottingham

June 2008

Abstract

In practice a degree of uncertainty will always exist concerning what specification to adopt for the deterministic trend function when running unit root tests. While most macroeconomic time series appear to display an underlying trend, it is often far from clear whether this component is best modelled as a simple linear trend (so that long-run growth rates are constant) or by a more complicated non-linear trend function which may, for instance, allow the deterministic trend component to evolve gradually over time. In this paper we consider the effects on unit root testing of allowing for a local quadratic trend, a simple yet very flexible example of the latter. Where a local quadratic trend is present but not modelled we show that the quasi-differenced detrended Dickey-Fuller-type test of Elliott *et al.* (1996) has both size and power which tend to zero asymptotically. An extension of the Elliott *et al.* (1996) approach to allow for a quadratic trend resolves this problem but is shown to result in large power losses relative to the standard detrended test when no quadratic trend is present. We consequently propose a simple and practical approach to dealing with this form of uncertainty based on a union of rejections-based decision rule whereby the unit root null is rejected whenever either of the detrended or quadratic detrended unit root tests rejects. A modification of this basic strategy is also suggested which further improves on the properties of the procedure. An application to relative primary commodity price data highlights the empirical relevance of the methods outlined in this paper. A by-product of our analysis is the development of a test for the presence of a quadratic trend which is robust to whether or not the data admit a unit root.

Keywords: Unit root test; trend uncertainty; quadratic trends; asymptotic power; union of rejections decision rule.

JEL Classification: C22.

*Correspondence to: David Harvey, School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, U.K. Email: dave.harvey@nottingham.ac.uk

1 Introduction

Testing for the presence of an autoregressive unit root has been an issue at the core of applied time series analysis for almost thirty years, since the publication of the seminal article by Dickey and Fuller (1979). Where economic data are concerned, the stochastic autoregressive time series process of interest is generally not considered to be observed directly, but is instead assumed to be observed subject to some additive deterministic component of unknown form, and legitimate inference on the properties of the underlying stochastic process cannot be made until the confounding effects of such deterministic components are removed. It is therefore common practice to apply unit root test procedures that are invariant to certain forms of deterministics and this is typically carried out by including a deterministic specification that is felt appropriate in the fitted regression from which the unit root test is calculated. Two considerations then arise. On the one hand, underfitting the deterministic component will almost always result in the unit root test procedure possessing little or no power to detect a stationary autoregressive root. On the other hand, overfitting the deterministic component will inevitably compromise the power of the procedure relative to that obtainable if the correct deterministic component were specified.

In the macroeconomic context, taking a constant term as given, the deterministic component open to question is often a linear trend term and in a recent paper, Harvey *et al.* (2008) (HLT) consider unit root testing under uncertainty regarding this deterministic component in the data. It is well-known that under certain conditions, the quasi-differenced (QD) demeaned and detrended augmented Dickey-Fuller (ADF) unit root tests of Elliott *et al.* (1996) are efficient relative to their ordinary least squares (OLS) demeaned and detrended counterparts. HLT therefore examine a strategy based on the union of rejections of the QD demeaned and detrended tests. The union principle exploits the fact that when a trend is absent, both QD tests are correctly sized under the unit root null but under the (locally) stationary alternative the demeaned test is the more likely of the two to signal a rejection of the unit root null in favour of stationarity (around a mean) since its power is not compromised by the inclusion of an irrelevant trend term. When a linear trend is present, the demeaned test becomes undersized and has trivially low power, to the extent that it is unlikely ever to reject the unit root null. Unit root inference then essentially becomes contingent on the detrended test alone, whose size and power to reject the unit root null in favour of stationarity (around a trend) are unchanged due to its invariance to a linear trend. Despite its simplicity, this union of rejections procedure was shown to be generally at least as powerful as competing procedures involving some form of pre-testing for the presence of the linear trend as a method to select between the QD demeaned or detrended unit root tests.

Of course, for many macroeconomic time series, a linear trend, rather than a constant, deterministic component might be considered appropriate as the default specification. This would be case for series such as, for example, real GDP, industrial production, money supply and consumer or commodity prices, where trending (or drift) behaviour appears evident. The question then arises as to whether the linear trend is sufficiently general to capture the actual deterministic behaviour or whether some more flexible specification for the trend function ought to be considered.

Once we entertain this possibility and move outside the linear trend environment, however, the range of choice is virtually endless. Polynomial trend functions, trigonometric functions and single or multiple structural breaks in level or trend represent just some of the nonlinear deterministic specifications that have been suggested; see, *inter alia*, Ouliaris *et al.* (1989), Bierens (1997), Lee *et al.* (2008), Perron (1989) and Perron and Rodríguez (2003).[1] There is, however, little in the way of consensus as to which approach is the most appropriate and, since all such approaches are capable of mimicking each other to a reasonable degree of approximation, this issue is unlikely to be resolved either quickly or easily. There is also a matter of interpretation here; if we allow a stochastic component of a series to evolve around a deterministic component of almost unlimited flexibility, exactly what pertinent information does a rejection (or non-rejection) of the unit root hypothesis then actually convey to a practitioner? This information would seem really rather inadequate, and the subsequent implications for any forecasting exercise are even less attractive. Intuition also suggests that it becomes more and more difficult to identify any (non-) stationarity present in the stochastic component of a time series if we continually increase the generality of the allowable deterministic specification; see, in particular, Phillips (1998).

For these reasons, in this paper we generalize the linear trend specification along the direction of allowing for a local quadratic trend term. This is arguably the simplest departure from trend linearity that is able to model a reasonable degree of local nonlinearity in the deterministic trend function. In particular, as noted in Ayat and Burridge (2000), a quadratic trend provides a simple means of proxying a linear trend which undergoes a break at some unknown point, or even repeated shifts in the deterministic level of the process. We first demonstrate theoretically the effects of an unattended local quadratic trend term on the Elliott *et al.* (1996) unit root test that allows for a linear trend, denoted $DF\text{-}QD^{\tau}$. We show that this test has size and local power which rapidly converge towards zero as the magnitude of the local quadratic trend increases. Secondly, we consider an Elliott *et al.* (1996)-based unit root test that allows for a quadratic trend, which we denote by $DF\text{-}QD^{q}$. By construction, $DF\text{-}QD^{q}$ is invariant to the magnitude of the quadratic trend. We derive its asymptotic local power function and tabulate its asymptotic null distribution. This allows us to characterize the sacrifices in asymptotic local power that arise from application of $DF\text{-}QD^{q}$ instead of $DF\text{-}QD^{\tau}$ when no quadratic term is present.

On the basis of these results, when uncertainty exists over the presence or otherwise of a quadratic trend, so that one is uncertain over which of $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$ to apply, we parallel the approach of HLT and suggest a test procedure based on a simple conservative union of rejections decision rule: reject the unit root null if either of the individual tests $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$ rejects. Such an approach capitalizes on both the far superior size and power properties of $DF\text{-}QD^{q}$ when a local quadratic trend does exist, and the relatively higher power of $DF\text{-}QD^{\tau}$ otherwise. Our asymptotic and finite sample simulation results show that this simple procedure works rather well in

[1]For a wider discussion of the possibility of nonlinear deterministics in the macroeconomic context see, for example, Murray and Nelson (2000) and Stock and Watson (1999).

practice.

We subsequently show that it is possible to modify the simple (conservative) union of rejections procedure to improve its power when a quadratic trend is present. The modification is based on using information from auxiliary test statistics designed to detect the presence of a quadratic trend. Tests based on these statistics represent extensions of the Harvey *et al.* (2007) linear trend analysis to the quadratic trend case, and are constructed so as to yield robust inference on the quadratic term; that is, they provide inference which (asymptotically) does not depend on whether the autoregressive unit root is present or not. In our context, these auxiliary statistics are not deployed as traditional pre-tests; that is, their outcome is not used to decide whether to apply $DF\text{-}QD^{\tau}$ or $DF\text{-}QD^{q}$, because these pre-tests can have relatively low power to detect quadratic trends which are small in magnitude, but are nonetheless sufficiently large to seriously bias $DF\text{-}QD^{\tau}$ towards non-rejection. Rather, their outcome is simply used to select between conservative and non-conservative critical values for the union of rejections of $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$. Asymptotic and finite sample simulation evidence shows very worthwhile power gains arise from using the modified approach.

We apply the unit root test procedures developed above to a set of 24 relative primary commodity annual price series, covering the period 1900–2003. This is an updated version of the well-known Grilli and Yang (1988) dataset. These data are usually studied for evidence of the Prebisch-Singer hypothesis which specifies a downward linear trend in relative commodity prices. In contrast, we focus our attention on identifying the order of integration of the series allowing for the possibility of quadratic deterministic trends via the union of rejections approach. In brief, almost without exception, whenever either or both $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$ reject the unit root, so does the union of rejections strategy, rejecting the unit root in almost two-thirds of the series considered. Importantly, almost half of these rejections arise by virtue of the quadratic trend unit root test in the union, but *not* its linear trend counterpart, thereby rather transparently demonstrating the genuine practical potential of allowing for quadratic trend deterministics when examining economic series for unit roots.

The plan of the paper is as follows. Section 2 sets out the model and underlying assumptions and defines $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$. In Section 3 we establish the asymptotic properties of $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$ in the local quadratic trend model. The union of rejections strategy is introduced in section 4, while its modified variant, based on the auxiliary quadratic trend tests, is discussed and compared in section 5. Finite sample simulations are presented in section 6. Our application to primary commodity prices is given in section 7, while some conclusions are offered in section 8. Proofs of the main technical results in this paper are given in an appendix. In the sequel we use the following notation: '$x := y$' ('$x =: y$') to indicate that x is defined by y (y is defined by x); $\lfloor \cdot \rfloor$ to denote the integer part of its argument, and '$\xrightarrow{d}$' to denote weak convergence.

2 The Model and Unit Root Tests

Consider the case where we have a sample of T observations generated according to the data generating process (DGP):

$$y_t = \mu + \beta t + \gamma t^2 + u_t, \quad t = 1, ..., T \tag{1}$$
$$u_t = \rho_T u_{t-1} + \varepsilon_t, \quad t = 2, ..., T. \tag{2}$$

Within (2), we assume that the initial condition u_1 satisfies $T^{-1/2}u_1 \xrightarrow{p} 0$, and we set $\rho_T = 1 - c/T$ for $0 \leq c < \infty$. Here $c = 0$ corresponds to the unit root, or $I(1)$, case; and $c > 0$ the local alternative.

The innovation process $\{\varepsilon_t\}$ of (2) is taken to satisfy the following conventional (cf. Chang and Park, 2002, and Phillips and Solo, 1992, *inter alia*) stable and invertible linear process-type assumption:

Assumption 1 *The stochastic process* $\{\varepsilon_t\}$ *is such that*

$$\varepsilon_t = C(L)e_t, \; C(L) := \sum_{i=0}^{\infty} C_i L^i, \; C_0 := 1$$

with $C(z) \neq 0$ *for all* $|z| \leq 1$ *and* $\sum_{i=0}^{\infty} i|C_i| < \infty$, *and where* $\{e_t\}$ *is a martingale difference sequence with conditional variance* σ^2 *and* $\sup_t E(e_t^4) < \infty$. *We also define* $\sigma_\varepsilon^2 := E(\varepsilon_t^2)$ *and* $\omega_\varepsilon^2 := \lim_{T\to\infty} T^{-1}E(\sum_{t=1}^{T} \varepsilon_t)^2 = \sigma^2 C(1)^2$.

Any process which satisfies Assumption 1 will be referred to as $I(0)$ in what follows.

Our focus is on testing the null hypothesis of a unit root, $H_0 : \rho_T = 1$, against the local alternative $H_1 : \rho_T = 1 - c/T$, $0 < c < \infty$, in situations where uncertainty surrounds whether or not a quadratic trend should be included in the deterministic component of the model. We assume that in these scenarios, it is unlikely that there is also doubt as to whether or not to include a linear trend, thus we restrict our attention to models where the decision for the deterministic component is between $\mu + \beta t$ and $\mu + \beta t + \gamma t^2$.

If it is known that a linear, but not quadratic, trend is present in the data (i.e. $\gamma = 0$), the near efficient unit root test is the Dickey and Fuller (1979)-type test proposed by Elliott *et al.* (1996) based on QD linear detrending. This test, $DF\text{-}QD^\tau$, rejects for large negative values of the t-statistic for $\rho = 1$ in the fitted regression equation

$$\tilde{u}_t = \rho \tilde{u}_{t-1} + \sum_{j=1}^{p} \phi_j \Delta \tilde{u}_{t-j} + e_t, \quad t = p + 2, ..., T \tag{3}$$

where, on setting $\bar{\rho} := 1 - \bar{c}_\tau/T$, $\tilde{u}_t := y_t - z_t'\tilde{\theta}$, with $\tilde{\theta}$ obtained from the QD regression of $\mathbf{y}_{\bar{c}} := (y_1, y_2 - \bar{\rho}y_1, ..., y_T - \bar{\rho}y_{T-1})'$ on $\mathbf{Z}_{\bar{c}} := (z_1, z_2 - \bar{\rho}z_1, ..., z_T - \bar{\rho}z_{T-1})'$, where $z_t := (1, \; t)'$. Elliott *et al.* (1996) suggest the value $\bar{c} = 13.5$, based on the fact that when $\bar{c}_\tau = 13.5$, the asymptotic Gaussian power envelope is at 0.50. It is assumed that p is chosen according to some consistent model selection procedure, such as the MAIC procedure of Ng and Perron (2001) and Perron and Qu (2007).

If, on the other hand, a quadratic trend is present (i.e. $\gamma \neq 0$), the near efficient test, $DF\text{-}QD^q$, takes the same form as $DF\text{-}QD^\tau$, but with $z_t := (1,\ t,\ t^2)'$, thereby incorporating a quadratic trend component in the QD detrending function. In unreported simulations, we found that the Gaussian power envelope for this case is at 0.50 for a value of $c = 18.5$, thus we advocate the use of $\bar{c}_q = 18.5$ (in place of $\bar{c}_\tau$) in the implementation of $DF\text{-}QD^q$. This coincides with the recommendation of Ayat and Burridge (2000), who conducted large sample simulations of $DF\text{-}QD^q$ directly for different $\bar{c}_q$.

3 Asymptotic Behaviour Under Quadratic Trends

In this section we consider the asymptotic behaviour of the unit root tests in the situation of uncertainty over the presence of a quadratic trend. We consider the magnitude of the quadratic trend to be a decreasing function of the sample size (i.e. local to zero), so as to prevent the deterministic quadratic from completely dominating the stochastic component of the series in the limit. This approach provides a sensible asymptotic model of possible quadratic trend behaviour in economic time series, and also accords well with our focus on cases where it is uncertain whether or not a quadratic trend actually exists in the model for y_t.

Under the local-to-unity alternative $H_1 : \rho_T = 1 - c/T$, the relevant Pitman drift on the quadratic trend coefficient, γ, is given by $\gamma = \kappa T^{-3/2}$, thus in this subsection we consider the impact of such quadratic trends on the unit root tests $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ introduced in the previous section. The asymptotic behaviour of the unit root tests are summarized in the following lemma.

Lemma 1 *Let $\{y_t\}$ be generated according to (1)-(2) and Assumption 1, with $\rho_T = 1 - c/T$, $0 \leq c < \infty$ and $\gamma = \kappa T^{-3/2}$. Then*

$$DF\text{-}QD^\tau \xrightarrow{d} \frac{J_c^{\tau,\bar{c}_\tau}(1)^2 - 1}{2\sqrt{\int_0^1 J_c^{\tau,\bar{c}_\tau}(r)^2 dr}} =: \tau_\tau \tag{4}$$

$$DF\text{-}QD^q \xrightarrow{d} \frac{J_c^{q,\bar{c}_q}(1)^2 - 1}{2\sqrt{\int_0^1 J_c^{q,\bar{c}_q}(r)^2 dr}} =: \tau_q \tag{5}$$

where

$$\begin{aligned} J_c^{\tau,\bar{c}_\tau}(r) &:= W_c(r) - \pi_{1,\bar{c}_\tau}^{-1} M_{1,\bar{c}_\tau} r + \kappa^*(r^2 - \pi_{1,\bar{c}_\tau}^{-1} \pi_{2,\bar{c}_\tau} r) \\ J_c^{q,\bar{c}_q}(r) &:= W_c(r) - d_{\bar{c}_q}^{-1}(\pi_{3,\bar{c}_q} M_{1,\bar{c}_q} - \pi_{2,\bar{c}_q} M_{2,\bar{c}_q}) r - d_{\bar{c}_q}^{-1}(\pi_{1,\bar{c}_q} M_{2,\bar{c}_q} - \pi_{2,\bar{c}_q} M_{1,\bar{c}_q}) r^2 \\ W_c(r) &:= \int_0^r e^{-(r-s)c} dW(s) \end{aligned}$$

with $\kappa^* := \kappa/\omega_\varepsilon$, $W(r)$ *a standard Brownian motion process, and*

$$\begin{aligned}
\pi_{1,\bar{c}_i} &:= 1 + \bar{c}_i + \bar{c}_i^2/3, \qquad i = \tau, q \\
\pi_{2,\bar{c}_i} &:= 1 + \bar{c}_i + \bar{c}_i^2/4, \qquad i = \tau, q \\
\pi_{3,\bar{c}_q} &:= 4/3 + \bar{c}_q + \bar{c}_q^2/5, \\
d_{\bar{c}_q} &:= \pi_{1,\bar{c}_q}\pi_{3,\bar{c}_q} - \pi_{2,\bar{c}_q}^2 \\
M_{1,\bar{c}_i} &:= (1+\bar{c}_i)W_c(1) + \bar{c}_i^2 \int_0^1 sW_c(s)ds, \qquad i = \tau, q \\
M_{2,\bar{c}_q} &:= (2+\bar{c}_q)W_c(1) - 2\int_0^1 W_c(s)ds + \bar{c}_q^2 \int_0^1 s^2 W_c(s)ds.
\end{aligned}$$

Remark 1 Due to the neglected quadratic trend, the asymptotic distribution of *DF-QD*$^\tau$ depends on the magnitude of κ^*, while the limit of *DF-QD*^{q} does not due to its invariance to the quadratic trend component. Note also that neither of the limit distributions depend on the parameters σ^2 and σ_ε^2; in the case of *DF-QD*$^\tau$ this arises since the implied estimators of these quantities remain consistent in the presence of a neglected local-to-zero quadratic trend.

Figure 1 shows the asymptotic power functions of the two tests across $c = \{0, 1, 2, ..., 40\}$, for the local quadratic trend parameter values $\kappa^* = \{0, 1, 2, 3, 4, 5\}$.[2] The results were obtained by direct simulation of the limiting distributions in Lemma 1, approximating the Wiener processes using $NIID(0, 1)$ random variates, and with the integrals approximated by normalized sums of 1000 steps. The tests are conducted using asymptotic critical values appropriate for the nominal 0.05 significance level for a correctly specified model, i.e. $\kappa^* = 0$ for *DF-QD*$^\tau$.[3] Here and throughout the paper, the reported simulations were programmed in Gauss 7.0 using 50,000 Monte Carlo replications.

Figure 1(a) shows the results for the case where $\kappa^* = 0$, and clearly highlights the asymptotic power gains that can be achieved by excluding a quadratic trend term when none is present in the data. Indeed, *DF-QD*$^\tau$ displays rejection frequencies up to 0.28 higher than that associated with *DF-QD*^{q}. The preferred test in this context is obviously *DF-QD*$^\tau$, in line with its aforementioned property of near-efficiency for this DGP. In contrast, Figures 1(b)-1(f) show that when $\kappa^* \neq 0$, *DF-QD*^{q} dominates *DF-QD*$^\tau$ in terms of power. Indeed, while the power of *DF-QD*^{q} is unaffected by the presence of a local quadratic trend, it is only for the smallest (non-zero) value of the quadratic trend ($\kappa^* = 1$) that *DF-QD*$^\tau$ has asymptotic power greater than size. As κ^* increases, both the size and power of *DF-QD*$^\tau$ converge to zero, so that by the time $\kappa^* = 5$, *DF-QD*$^\tau$ records almost no rejections of the unit root null.

These results highlight that while *DF-QD*$^\tau$ is the near efficient test when $\kappa^* = 0$, severe power losses arise relative to *DF-QD*^{q} when a local quadratic trend is present;

---

[2] Observing that $W(r)$ and $-W(r)$ are equal in distribution it is straightforwardly seen from (4) that the asymptotic size and local power function of *DF-QD*$^\tau$ are invariant to the sign of κ^*. As a consequence, we only report results for $\kappa^* \geq 0$.

[3] The asymptotic 0.10, 0.05 and 0.01 level critical values for *DF-QD*$^\tau$ are, respectively, -2.56, -2.85 and -3.41; those for *DF-QD*^{q} are, respectively, -3.15, -3.43 and -3.97.

conversely, $DF\text{-}QD^q$ is near efficient in the case of $\kappa^* \neq 0$, but lacks power relative to $DF\text{-}QD^\tau$ when the DGP contains only a linear trend. Thus, we would wish to employ $DF\text{-}QD^\tau$ to test the unit root hypothesis when $\kappa^* = 0$, but to base inference instead on $DF\text{-}QD^q$ when $\kappa^* \neq 0$. However, given that the value of κ^* is in practice unknown, such a procedure would be infeasible. Instead, in the next section we consider a feasible testing strategy that attempts to capture both the relatively superior power performance of $DF\text{-}QD^q$ when a local quadratic trend is present, and the power gains of $DF\text{-}QD^\tau$ otherwise.

4 A Union of Rejections Strategy

The results of the previous section demonstrate that when a quadratic trend is present, one would most certainly want to conduct a unit root test that accounts for a quadratic term in the deterministics; on the other hand, if only a linear trend exists, applying such a quadratic trend-based unit root test would involve a substantial loss of power relative to the efficient test that assumes a linear deterministic trend alone. We now turn our attention to the practical situation where a lack of knowledge pertains as to the presence or otherwise of a quadratic trend component, so that one is uncertain over which of $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ should be applied. Given this uncertainty over the presence of a quadratic trend, it is worthwhile to consider procedures which attempt to capitalize on both the relatively high power of $DF\text{-}QD^q$ when a local quadratic trend does exist, and the relatively high power of $DF\text{-}QD^\tau$ otherwise.

HLT consider a parallel problem where testing for a unit root is conducted in the presence of uncertainty surrounding whether or not a *linear* trend exists in the data. The efficient testing approach in this context is to apply $DF\text{-}QD^\tau$ if a trend is present, and the equivalent test that excludes a deterministic trend (denoted $DF\text{-}QD^\mu$) if no trend in fact exists. Of a number of methods that could be employed to choose between these tests when the presence of the trend is uncertain, these authors find that a simple union of rejections decision rule performs very well, rejecting the null if either of the individual tests $DF\text{-}QD^\mu$ and $DF\text{-}QD^\tau$ reject. This approach was found to outperform pre-testing for the presence of a trend, in part because, particularly in a local-to-unit root setting, the pre-tests have low power for trends of small to moderate magnitude, while at the same time such trends are still large enough to cause the power of $DF\text{-}QD^\mu$ to fall towards zero. In the present situation of testing for a unit root when uncertain over a *quadratic* trend, one possible approach would be to pre-test for the presence of a quadratic trend component, and then apply either $DF\text{-}QD^\tau$ or $DF\text{-}QD^q$ conditional on the outcome. However, it is of course to be fully expected that the inherent problems with the pre-testing approach observed in the linear trend context would also apply here, and some evidence of this is observed in the application to relative primary commodity prices presented in section 7. As a consequence, following HLT, we consider the performance of a unit root testing strategy based on a union of rejections of the individual tests $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$.

Denoting the asymptotic ξ level critical values of $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ by cv_ξ^τ

and cv_ξ^q, respectively, we define the simple union of rejections strategy, UR, as

$$UR\text{:}\quad \text{Reject } H_0 \text{ if } \{DF\text{-}QD^\tau < cv_\xi^\tau \text{ or } DF\text{-}QD^q < cv_\xi^q\}.$$

Following the rejoinder to HLT, an alternative way of representing this decision rule makes use of a single test statistic t_{UR} as follows

$$UR\text{:}\quad \text{Reject } H_0 \text{ if } \left\{ t_{UR} = \min\left(DF\text{-}QD^\tau, \frac{cv_\xi^\tau}{cv_\xi^q} DF\text{-}QD^q \right) < cv_\xi^\tau \right\}.$$

An application of the continuous mapping theorem (CMT) coupled with the results in Lemma 1 allows us to immediately obtain the asymptotic distribution of t_{UR} as

$$t_{UR} \xrightarrow{d} \min\left(\tau_\tau, \frac{cv_\xi^\tau}{cv_\xi^q} \tau_q \right).$$

The asymptotic behaviour of this simple testing strategy at the 0.05 significance level can be seen in Figure 1. Since the strategy simply involves rejection of the null when either of the individual tests reject, the power curve lies on or outside that of $DF\text{-}QD^\tau$ when $\kappa^* = 0$, and that of $DF\text{-}QD^q$ when $\kappa^* \neq 0$. While this strategy clearly achieves the desired aim of capitalizing on both the high power of $DF\text{-}QD^q$ when a local quadratic trend is present, and the high power of $DF\text{-}QD^\tau$ in the absence of a quadratic component, it clearly lacks asymptotic size control across all values of κ^*. The size of the procedure is at a maximum when $\kappa^* = 0$, where asymptotic size at the nominal 0.05 significance level is 0.080 (though somewhat below the Bonferroni upper bound of 0.10 for this union of rejections).

If this degree of size distortion is deemed unacceptable, a simple correction can be applied to ensure that UR is conservative in the limit. Consider adjusting the critical values of the individual unit root tests through multiplication by a common constant, ψ_ξ, chosen so that the resulting decision rule yields asymptotic size of ξ when $\kappa^* = 0$ for tests run at the ξ significance level. Such a modified decision rule will then be correctly sized in the limit for $\kappa^* = 0$, and conservative for $\kappa^* \neq 0$. This conservative UR decision rule, now denoted UR^c to distinguish it from uncorrected UR, can be written as

$$UR^c\text{:}\quad \text{Reject } H_0 \text{ if } \{DF\text{-}QD^\tau < \psi_\xi cv_\xi^\tau \text{ or } DF\text{-}QD^q < \psi_\xi cv_\xi^q\}$$

or, alternatively,

$$UR^c\text{:}\quad \text{Reject } H_0 \text{ if } \left\{ t_{UR} = \min\left(DF\text{-}QD^\tau, \frac{cv_\xi^\tau}{cv_\xi^q} DF\text{-}QD^q \right) < \psi_\xi cv_\xi^\tau \right\}.$$

The appropriate constant ψ_ξ can be determined by simulating the limit distribution of t_{UR}, calculating the ξ-level critical value for this empirical distribution, say cv_ξ^m, and then computing $\psi_\xi := cv_\xi^m / cv_\xi^\tau$.[4]

[4]The asymptotic 0.10, 0.05 and 0.01 level ψ_ξ values are, respectively, 1.069, 1.058 and 1.043.

Figure 1 also shows the asymptotic performance of the UR^c strategy. In comparison with the individual tests $DF\text{-}QD^{\tau}$ and $DF\text{-}QD^{q}$, the robust nature of this size-controlled procedure can be clearly seen. When $\kappa^* \neq 0$, UR^c avoids the severe power losses associated with $DF\text{-}QD^{\tau}$, exhibiting power a little below that of $DF\text{-}QD^{q}$. When $\kappa^* = 0$, UR^c then considerably outperforms $DF\text{-}QD^{q}$ in terms of local asymptotic power, capturing most of the relative power gains displayed by $DF\text{-}QD^{\tau}$. The union of rejections approach therefore leads to a unit root testing strategy that has very attractive asymptotic size and local power behaviour, and should prove very useful in practical applications when there is uncertainty regarding the presence of a quadratic component in the deterministic time path of the series.

5 A Modified Union of Rejections Strategy

The results of the previous section show that the UR^c strategy achieves correct asymptotic size and good overall power, both in cases where a local quadratic trend is present and also when the trend specification is simply linear. However, for large magnitudes of the local quadratic trend, Figure 1 shows that the uncorrected UR strategy is approximately correctly sized in the limit (since the size of $DF\text{-}QD^{\tau}$ converges to zero) and has superior power performance in comparison with UR^c; one would therefore wish to apply UR in these circumstances rather than the conservative UR^c. It is consequently worthwhile considering a modified strategy that makes use of auxiliary information regarding the presence or otherwise of a quadratic trend to decide between the application of either UR or UR^c, following the approach advocated in Breitung's commentary on HLT and the latter authors' rejoinder, for the parallel problem of unit root testing in the presence of uncertainty over the presence of a linear trend.

Let $|t_\gamma|$ denote generically a statistic for testing the null hypothesis that $\gamma = 0$, i.e. no quadratic trend, against a two-sided alternative $\gamma \neq 0$, with the corresponding asymptotic ξ level critical value denoted by cv_ξ^γ. Given a suitable choice of quadratic trend test statistic, we can then define the modified union of rejections testing strategy $UR(|t_\gamma|)$ as follows.

$$\begin{array}{ll} \text{If } |t_\gamma| > cv_\xi^\gamma: & UR(|t_\gamma|) := UR \\ \text{If } |t_\gamma| \leq cv_\xi^\gamma: & UR(|t_\gamma|) := UR^c. \end{array}$$

Notice that this strategy is quite different from one that applies either $DF\text{-}QD^{\tau}$ or $DF\text{-}QD^{q}$ according to the outcome of a pre-test, since here a union of rejections is always conducted. The potential pitfalls of a pre-testing strategy discussed in the previous section (namely the erroneous application of $DF\text{-}QD^{\tau}$ when a local quadratic trend is present, due to low pre-test power) are therefore avoided.

As regards testing for a quadratic trend, it is desirable to employ a test that is robust to the order of integration of the series, since otherwise an *ex ante* assumption would be required, specifying whether or not a unit root is present in the series; such an assumption would be wholly inappropriate, given the ultimate purpose of testing for a unit root. Harvey *et al.* (2007) develop powerful tests for a linear trend that are robust to the order of integration, and here we adapt these procedures to the current problem of testing for a quadratic deterministic trend component.

The quadratic trend test equivalents of the procedures recommended by Harvey *et al.* (2007) take the following form. First, define by t_0 the autocorrelation-corrected OLS-based t-ratio for testing $\gamma = 0$ in the regression (1), i.e.

$$t_0 := \frac{\hat{\gamma}}{\sqrt{\hat{\omega}_u^2[(\sum_{t=1}^T z_t z_t')^{-1}]_{33}}}$$

where $\hat{\gamma}$ denotes the OLS estimator of γ in (1), $\hat{\omega}_u^2$ is a long run variance estimator formed using $\hat{u}_t := y_t - \hat{\mu} - \hat{\beta}t - \hat{\gamma}t^2$, $[.]_{33}$ denotes the $(3,3)$ element of $[.]$, and $z_t := (1, t, t^2)'$. Now consider applying first differences to (1). This yields the regression equation

$$\Delta y_t = \delta_1 + \delta_2 t + v_t, \quad t = 2, ..., T \tag{6}$$

where $\delta_1 := \beta - \gamma$, $\delta_2 := 2\gamma$ and $v_t := \Delta u_t$. Define by t_1 the autocorrelation-corrected OLS-based t-ratio for testing $\delta_2 = 0$ in (6), i.e.

$$t_1 := \frac{\hat{\delta}_2}{\sqrt{\hat{\omega}_v^2[(\sum_{t=1}^T z_t z_t')^{-1}]_{22}}}$$

where $\hat{\delta}_2$ denotes the OLS estimator of δ_2 in (6), $\hat{\omega}_v^2$ is a long run variance estimator based on $\hat{v}_t := \Delta y_t - \hat{\delta}_1 - \hat{\delta}_2 t$, $[.]_{22}$ denotes the $(2,2)$ element of $[.]$, and where now $z_t := (1, t)'$. The long run variance estimators $\hat{\omega}_u^2$ and $\hat{\omega}_v^2$ are computed using the quadratic spectral kernel with Newey and West (1994) automatic bandwidth selection adopting a non-stochastic prior bandwidth of $\lfloor 4(T/100)^{2/25} \rfloor$. The individual tests based on t_0, and t_1, attain the Gaussian asymptotic local power envelope for testing $\gamma = 0$ when u_t in (1) is $I(0)$, and $I(1)$, respectively, and both are asymptotically standard normal under the null hypothesis $\gamma = 0$.

The Harvey *et al.* (2007) robust testing approach then involves taking a weighted average of the two statistics t_0 and t_1, with the weight specified such that the hybrid statistic reduces to t_0 when u_t is $I(0)$, and t_1 when u_t is $I(1)$; i.e.

$$t_\lambda := (1 - \lambda)t_0 + \lambda t_1$$

with

$$\lambda := \exp\left(-g\left(\frac{DF\text{-}QD^q}{KPSS^q}\right)^2\right) \tag{7}$$

where g is some positive constant and $KPSS^q$ is a quadratic trend-based version of the Kwiatkowski *et al.* (1992) stationarity test statistic, i.e.

$$KPSS^q := \frac{\sum_{t=1}^T \left(\sum_{i=1}^t \hat{u}_i\right)^2}{T^2 \hat{\omega}_u^2}.$$

It is straightforward to show that the weight function λ possesses the properties that $\lambda \xrightarrow{p} 0$ when u_t is $I(0)$, and $\lambda \xrightarrow{p} 1$ when u_t is $I(1)$.

We also consider a quadratic trend test equivalent of the t_λ^{m2} test of Harvey *et al.* (2007), which would be expected to provide more power when u_t is a local-to-unit root process. This involves replacing t_1 with $t_1^{m\,2} := \eta_\xi R_2 t_1$ where

$$R_2 := \left(\frac{\hat{\omega}_v^2}{T^{-1}\hat{\sigma}_u^2} \right)^2$$

and $\hat{\sigma}_u^2 := (T-3)^{-1} \sum_{t=1}^{T} \hat{u}_t^2$. Here, η_ξ is a constant chosen so that, at a given significance level ξ, t_λ^{m2} has an asymptotic standard normal critical value regardless of whether u_t is $I(1)$ or $I(0)$.[5]

The asymptotic behaviour of the quadratic trend tests t_λ and t_λ^{m2} is given in the following lemma.

Lemma 2 *Let $\{y_t\}$ be generated according to (1)-(2) and Assumption 1, with $\rho_T = 1 - c/T$, $0 \leq c < \infty$ and $\gamma = \kappa T^{-3/2}$. Then*

$$t_\lambda \xrightarrow{d} \frac{\kappa^*}{\sqrt{3}} + \sqrt{12}\left\{ \tfrac{1}{2} W_c(1) - \int_0^1 W_c(r)dr \right\}$$

$$t_\lambda^{m2} \xrightarrow{d} \eta_\xi \left\{ \int_0^1 N_c(r)^2 dr \right\}^{-2} \left[\frac{\kappa^*}{\sqrt{3}} + \sqrt{12}\left\{ \tfrac{1}{2} W_c(1) - \int_0^1 W_c(r)dr \right\} \right]$$

where $N_c(r)$ denotes the continuous time projection of $W_c(r)$ onto the space spanned by $\{1, r, r^2\}$, and where $W_c(r)$ is as defined in Lemma 1.

Note that the constant g in (7) has no impact in the limit, and as in Harvey *et al.* (2007) we calibrated its value on the basis of unreported finite sample size and power simulations. We found that $g = 0.00001$ for t_λ, and $g = 0.00015$ for t_λ^{m2}, gave an appealing finite sample size/power trade-off, and thus we employ these values throughout the remainder of the paper.

We denote the modified union of rejections strategies that make use of t_λ and t_λ^{m2} by $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ respectively. These modified strategies do not guarantee asymptotic size control, since they select between the conservative strategy UR^c and the potentially over-sized strategy UR. However, simulations of the limit distributions show that the maximum asymptotic sizes of $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ are, respectively, 0.051 and 0.057 (both occurring when $\kappa^* = 0$), thus the size distortions are almost inconsequential, particularly so in the case of $UR(|t_\lambda|)$.

The asymptotic behaviour of $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ is shown in Figure 1. When $\kappa^* = 0$, the quadratic trend tests do not reject the null of $\gamma = 0$ (aside from the usual Type I error), and so $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ are almost identical to UR^c. However, when $\kappa^* \neq 0$, relative power gains over UR^c can be observed. Specifically, when $\kappa^* = 1$ and $\kappa^* = 2$, the $UR(|t_\lambda^{m2}|)$ strategy begins to select UR rather than UR^c, resulting in power improvements; indeed, in the case of $\kappa^* = 2$, power gains of up to 0.10 are

[5] For two-tailed t_λ^{m2} tests conducted at the 0.10, 0.05 and 0.01 levels, the η_ξ values are, respectively, 0.000801, 0.000647 and 0.000427.

observed over UR^c, markedly closing the gap between UR^c and the best performing test in this setting, $DF\text{-}QD^q$. For $\kappa^* = 3$, $UR(|t_\lambda^{m2}|)$ has power almost equal to that of $DF\text{-}QD^q$, while the power curve of $UR(|t_\lambda|)$ also now begins to move away from UR^c towards UR. By the time $\kappa^* \geq 4$, little difference is observed between $UR(|t_\lambda|)$, $UR(|t_\lambda^{m2}|)$ and $DF\text{-}QD^q$, with the modified union of rejections strategies displaying decent power gains over the relatively simple conservative strategy, UR^c. Thus, for the modest additional computational requirement involved in calculating a robust test statistic for a quadratic trend, a modified union of rejections testing strategy can be employed, which delivers worthwhile power gains in certain circumstances.

6 Finite Sample Simulations

We now consider a set of finite sample simulations based on the DGP (1)-(2) with $\varepsilon_t \sim NIID(0,1)$, such that $\kappa^* = \kappa$, and a sample size of $T = 150$. We set $\mu = \beta = 0$ without loss of generality and consider $\gamma = \kappa T^{-3/2}$ with $\kappa = \kappa^* = \{0,1,2,3,4,5\}$, so that the values of γ correspond to the local quadratic trend settings considered in the asymptotic analysis. Similarly, we set $\rho_T = 1 - c/T$ with $c = \{0,1,2,...,40\}$ for comparability with the asymptotic simulations. The individual unit root tests and the union of rejections strategies are conducted at the nominal (asymptotic) 0.05 significance level, using the asymptotic critical values and scaling constants reported in previous sections. The number of lagged difference terms included in the Dickey-Fuller regressions, p, is determined by application of the MAIC procedure of Ng and Perron (2001) with maximum lag length set at $p_{\max} = \lfloor 12(T/100)^{1/4} \rfloor$, using the modification suggested by Perron and Qu (2007).

Figure 2 reports the finite sample sizes and powers of $DF\text{-}QD^q$, $DF\text{-}QD^\tau$, UR, UR^c, $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ for these settings. Turning first to empirical size, it can be seen that, with the exception of UR, all three union of rejections strategies have almost no discernible finite sample over-size. Across the values of γ considered, the conservative strategy UR^c has size in the range 0.025–0.044, while the sizes of the modified strategies $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ are in the ranges 0.042–0.047 and 0.043–0.052, respectively. That the robust size behaviour observed in the limit also carries over to good finite sample size control is encouraging and adds to the value of these unit root testing strategies for practical applications.

The relative finite sample power performance of the unit root testing strategies largely mirrors that observed in the limit. The UR^c strategy is again seen to capture most of the power gains of $DF\text{-}QD^\tau$ over $DF\text{-}QD^q$ when $\gamma = 0$, but avoids the very low power associated with $DF\text{-}QD^\tau$ when $\gamma \neq 0$, instead displaying a power curve relatively close to that of $DF\text{-}QD^q$. The modified union of rejections strategies $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ have even more attractive power profiles, being closer to the $DF\text{-}QD^q$ power curve when a quadratic trend is present, and almost identical to UR^c when only a linear trend is present in the DGP. Of the two procedures, $UR(|t_\lambda^{m2}|)$ has the marginally better power, moving above the UR^c curve for smaller values of γ than occurs for $UR(|t_\lambda|)$. It is interesting to note, however, that for moderate non-zero quadratic trend magnitudes (i.e. $\kappa = 2$ and $\kappa = 3$ in Figure 2), $UR(|t_\lambda|)$ has relatively better power performance than is predicted by the asymptotic results, with t_λ detecting the quadratic trend at smaller magnitudes than would be expected. This result parallels the finding in the rejoinder to HLT in the context of uncertainty regarding the presence of a linear trend, and might be expected given the local-to-unity results of Harvey *et al.* (2007) where the t_λ-type linear trend test is found to perform relatively better (in comparison to the corresponding t_λ^{m2}-type test) in finite samples than was observed in the limit.

The asymptotic and finite sample results taken together show that the union of rejections strategies proposed in this paper, and in particular $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$, provide robust, size controlled and powerful procedures for testing for a unit root when uncertainty exists regarding the presence of a quadratic component in the deterministic trend. The strategies are relatively easy to implement and we hope should appeal to practitioners.

7 Application to Primary Commodity Prices

In this section we apply the individual unit root tests $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$, and the union of rejections strategies UR^c, $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$, to a set of 24 relative primary commodity price series. The data are indices of primary commodity prices relative to the price of manufactures, observed annually over the period 1900–2003 (104 observations) and measured in logarithms. Plots of the series are presented in Figures 3a and 3b. The dataset is that compiled by Pfaffenzeller *et al.* (2007), which updated the widely used dataset of Grilli and Yang (1988). These data have been studied extensively to assess the Prebisch-Singer hypothesis (Prebisch, 1950, and Singer, 1950), examining evidence for downward trends in the relative commodity prices; our main interest here, however, concerns the integration properties of the series, and thus complements studies whose primary focus is on underlying trend behaviour. The unit root tests are conducted at the nominal asymptotic 0.10, 0.05 and 0.01 significance levels, and, as in the previous section, lag augmentation was performed using the MAIC approach of Ng and Perron (2001) and Perron and Qu (2007), with $p_{\max} = \lfloor 12(T/100)^{1/4} \rfloor$.

The results are reported in Table 1. We find that for four of the commodities considered (aluminium, rubber, timber and zinc), the null hypothesis of a unit root is rejected (at least at the 0.10 level) by both $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$; moreover, in each of these four cases, the three union of rejections strategies also indicate rejection of the null. For eight commodity price series (banana, cocoa, cotton, hides, jute, silver, tea and tin), no rejections are found at conventional significance levels by either of the individual tests $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$, thus no evidence against a unit root appears to be present for these commodities. The inferences from UR^c, $UR(|t_\lambda|)$ and $UR(|t_\lambda^{m2}|)$ are again consistent with those obtained from $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ for these series, which obviously follows since if neither of the individual unit root tests reject, the union of rejections strategies cannot generate rejections of the null.

Of particular interest are the commodity series for which one, but not both, of the individual tests $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ results in rejection of the null. For the beef, copper, lamb and sugar series, rejections are only obtained by the $DF\text{-}QD^\tau$ test. Given the analytical and simulation results, this pattern of rejections is suggestive of these series being stationary around a linear trend, with the non-rejections of *DF-QD*^{q} arising due to a lack of relative power. With the exception of sugar, rejections of the null are also obtained through application of the union of rejections strategies, again confirming the robust performance of these approaches. For the remaining eight commodities (coffee, lead, maize, palmoil, rice, tobacco, wheat and wool), the *DF-QD*^{q} test results in rejection of the unit root null, in direct contrast to inference gleaned from *DF-QD*$^\tau$. Recalling again the results of previous sections, it appears that these series are best modelled by a stationary process around a quadratic trend, with the test based on only a linear trend specification lacking power to reject the unit root null. Turning to the union of rejections strategies for these eight commodities, we find that rejections of the null are indicated in every case.

Figures 3a and 3b also present plots of the fitted deterministic components appropriate for series where rejections of the unit root null were obtained. Specifically, if the *DF-QD*$^\tau$ and/or *DF-QD*^{q} tests reject for a given series, we super-impose the corresponding fitted linear and quadratic trends, obtained from the QD estimates of the trend parameters so as to be consistent with the QD procedure involved in the unit root tests. In the cases where only *DF-QD*^{q} rejects, the fitted linear trends are clearly seen to be insufficiently rich deterministic specifications, with the fitted quadratic trend components displaying substantial nonlinearity. The quadratic specification allows for a smooth evolution of the deterministic component from an upward (nonlinear) trend at the beginning of the time series to a downward trend later in the sample period. One possible interpretation is that the quadratic trends in these cases are acting as a local approximation to one or more breaks in level or linear trend for these series, as studied by, for example, Leon and Soto (1997) and Kellard and Wohar (2006). However, it should be stressed that the underlying DGP is unknown, and the quadratic trend could be acting as a proxy for a variety of different deterministic generating processes, as indeed a segmented trend function might. For the series where both *DF-QD*$^\tau$ and *DF-QD*^{q} reject, the nonlinearity in the fitted quadratic trend is markedly less pronounced, with the quadratic fit generally being close to that obtained from estimating a linear trend alone, as might be expected in these cases. This pattern is even more apparent for series where only *DF-QD*$^\tau$ rejects, where the quadratic fits very closely approximate the fitted linear trends in most cases.

It is also of interest to consider how a unit root testing approach based on pre-testing for a quadratic trend would perform for these series. Table 1 also records rejections of the null of no quadratic trend, against a two-sided alternative, by the two trend tests introduced in section 4: $|t_\lambda|$ and $|t_\lambda^{m2}|$. Consider then a strategy of conducting tests for a quadratic trend using either $|t_\lambda|$ or $|t_\lambda^{m2}|$, and then applying *DF-QD*^{q} upon rejection of the null of no quadratic trend, and *DF-QD*$^\tau$ otherwise. Of course, pre-testing can only affect the unit root test outcome when the individual unit root tests differ in the conclusions they imply, hence we focus on the twelve series discussed in the previous paragraph. We find that on the occasions where only *DF-QD*$^\tau$ rejects (beef, copper, lamb and sugar), neither of the pre-tests reject, thus both pre-test strategies would result in rejections of the unit root null. However, of the eight series where rejections are only obtained by *DF-QD*^{q} (coffee, lead, maize, palmoil, rice, tobacco, wheat and wool), pre-testing on the basis of t_λ would result in using *DF-QD*^{q} for just five series, thereby resulting in non-rejections of the unit root null in the remaining three cases (coffee, palmoil and wool). Similarly, pre-testing on the basis of t_{λ}^{m2} would give rise to non-rejection of the unit root for four series (coffee, lead, rice and wheat). These results are consistent with the anticipated problems associated with pre-testing discussed in section 4, namely that for quadratic trends of small magnitude, the pre-tests would be expected to have low power, but the quadratic trend (or the nonlinear trend effect which it proxies) could still be large enough to drastically reduce the power of $DF\text{-}QD^{\tau}$. Given the more robust results of UR^{c}, $UR(|t_{\lambda}|)$ and $UR(|t_{\lambda}^{m2}|)$, evidence appears to exist in this dataset in favour of union of rejections strategies in preference to pre-testing approaches. Moreover, unit root testing strategies based on pre-tests conducted in this rudimentary manner lack proper asymptotic size control, and size correction will further reduce the power of the overall strategies.

In summary, the attractive performance of the union of rejections strategies displayed in the Monte Carlo simulations is borne out in this empirical application, highlighting the potential advantages of such a unit root testing approach. With only one exception (sugar), whenever either or both of the individual unit root tests reject, the union of rejections strategies also suggest rejection of the null. If reliance was placed purely on the $DF\text{-}QD^{\tau}$ test, rejections of the null would be obtained for a total of only eight series, due to the neglected quadratic trend component that appears to be important in several cases, at least as a local approximation for the determinstic component of those series. On the other hand, application of $DF\text{-}QD^{q}$ alone would lead to a greater total number of rejections (twelve), but would fail to detect stationarity for four of the commodities for which the linear trend-based test rejected. In contrast, the union of rejections strategies detect departures from the unit root null for a total of fifteen relative commodity price series, capturing all but one (0.10 level) rejection implied by the two individual unit root tests taken together. We can also note that in this application, the three different union of rejections strategies UR^{c}, $UR(|t_{\lambda}|)$ and $UR(|t_{\lambda}^{m2}|)$ give almost identical inference to each other across the different series considered. Only for tobacco do the results differ, where we find that $UR(|t_{\lambda}|)$ and $UR(|t_{\lambda}^{m2}|)$ indicate rejections at the 0.01 level, whereas UR^{c} implies rejection at only the 0.05 level. The more emphatic rejections suggested by $UR(|t_{\lambda}|)$ and $UR(|t_{\lambda}^{m2}|)$ are in line with our theoretical results, and this result supports our recommendation to use either $UR(|t_{\lambda}|)$ or $UR(|t_{\lambda}^{m2}|)$ in practice.

Finally, then, it appears that for roughly two-thirds of the relative price commodity series considered, evidence of stationarity about some deterministic component is detected. However, for many of these series, such evidence is only forthcoming when allowance is made for a more flexible deterministic specification than is admitted by a purely linear trend. Of course, a quadratic trend in these series may be unlikely to persist into the long run, but it nonetheless appears to act well as a local approximation to whatever (nonlinear) deterministic component is actually present in the unknown data generating process, and does so without recourse to highly involved procedures for estimating and dating complex models of deterministic structural change, as would be required with, for example, multiple level or trend break models.

8 Conclusions

In this paper we have investigated the impact that uncertainty over the presence or otherwise of a local quadratic trend in the underlying data generating process has on the quasi-differenced detrended Dickey-Fuller-type tests of Elliott *et al.* (1996), and investigated new procedures which attempt to retain good power properties in the presence of such uncertainty. We found that the quadratic detrended test, $DF\text{-}QD^q$, was much less powerful than the corresponding quasi-differenced linear detrended test, $DF\text{-}QD^\tau$, in the absence of a quadratic trend, as would be expected. However, where a non-trivial local quadratic trend was present, the $DF\text{-}QD^\tau$ test was shown to have negligible power. We consequently investigated a simple union of rejections based decision rule whereby the unit root null is rejected if either $DF\text{-}QD^q$ or $DF\text{-}QD^\tau$ yields a rejection. For individual tests each run at a given significance level, this simple union test was shown to be somewhat over-sized for small values of the local quadratic trend parameter, and consequently we also developed a conservative version of this test using a modified decision rule. We then showed that the power of the conservative procedure could be further improved upon by using auxiliary information from a test statistic for the presence of a quadratic trend to choose between the conservative and original critical values when forming a decision rule for the union of rejections procedure. We reported asymptotic and finite sample evidence which suggested that our simple union of rejections decision rule performs very well in practice. It has the added advantage of being very easy for practitioners to compute, and, with only a little extra computational effort, can be further improved upon.

We subsequently employed the unit root tests and union of rejections strategies to examine the integration properties of a set of relative primary commodity price series. The additional flexibility afforded by the quadratic trend formulation was shown to allow considerably more rejections of the unit root hypothesis than were obtained under a simpler linear trend formulation, highlighting the potential value of local quadratic trend models as approximations to the unknown, potentially nonlinear, deterministic component. Further, with only one exception, when either or both of $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ rejected the unit root null, so did the recommended union of rejections strategies, illustrating the usefulness of these new robust procedures. Overall, we find evidence of stationarity about a possibly nonlinear trend for about two-thirds of the commodities considered.

Appendix

Proof of Lemma 1

In what follows, we may set $\mu = \beta = 0$ in the DGP due to invariance of $DF\text{-}QD^\tau$ and $DF\text{-}QD^q$ to these parameters, i.e. $y_t = \kappa T^{-3/2}t^2 + u_t$. In the following algebra, nothing of asymptotic consequence is lost if we make the simplifying assumption that $\varepsilon_t = e_t$, so that $\omega_\varepsilon^2 = \sigma_\varepsilon^2 = \sigma^2$, allowing us to impose $p = 0$ in the Dickey-Fuller regressions.

Limit distribution of DF-QD$^\tau$

First observe that the scaled limit of the QD-detrended residuals, $\tilde{u}_t := y_t - \tilde{\mu} - \tilde{\beta}t$ satisfy

$$T^{-1/2}\tilde{u}_{\lfloor rT\rfloor} = T^{-1/2}u_{\lfloor rT\rfloor} + \kappa r^2 - T^{-1/2}\tilde{\mu} - T^{1/2}\tilde{\beta}r.$$

In order to establish the large sample of $T^{-1/2}\tilde{\mu}$ and $T^{1/2}\tilde{\beta}$ notice that

$$\begin{bmatrix} \tilde{\mu} \\ \tilde{\beta} \end{bmatrix} := \begin{bmatrix} 1+(1-\bar{\rho})^2(T-1) & 1+(1-\bar{\rho})\sum_{t=2}^T\{t-\bar{\rho}(t-1)\} \\ 1+(1-\bar{\rho})\sum_{t=2}^T\{t-\bar{\rho}(t-1)\} & 1+\sum_{t=2}^T\{t-\bar{\rho}(t-1)\}^2 \end{bmatrix}$$
$$\begin{matrix} 1+(1-\bar{\rho})\sum_{t=2}^T\{t^2-\bar{\rho}(t-1)^2\} \\ 1+\sum_{t=2}^T\{t-\bar{\rho}(t-1)\}^2 \end{matrix}\Bigg]^{-1} \begin{bmatrix} y_1+(1-\bar{\rho})\sum_{t=2}^T(y_t-\bar{\rho}y_{t-1}) \\ y_1+\sum_{t=2}^T(y_t-\bar{\rho}y_{t-1})\{t-\bar{\rho}(t-1)\} \end{bmatrix}.$$

Denote the right hand side of the above expression by $A^{-1}B$, with a_{ij} denoting the (i,j) element of A, and b_i the ith element of B. The asymptotic behaviour of the terms comprising A are as follows:

$$\begin{aligned} a_{11} &= 1+\bar{c}_\tau^2T^{-2}(T-1)\to 1 \\ a_{12} &= 1+\bar{c}_\tau T^{-1}(T-1)+\bar{c}_\tau^2T^{-2}\textstyle\sum_{t=2}^T t+o(1)\to 1+\bar{c}_\tau+\bar{c}_\tau^2/2 \\ a_{22} &= T+2\bar{c}_\tau T^{-1}\textstyle\sum_{t=2}^T t+\bar{c}_\tau^2T^{-2}\sum_{t=2}^T t^2+o(T) \end{aligned}$$

so that $T^{-1}a_{22}\to 1+\bar{c}_\tau+\bar{c}_\tau^2/3 =: \pi_{1,\bar{c}_\tau}$. The corresponding results for the elements of B are as follows:

$$b_1 = y_1+\bar{c}_\tau T^{-1}(y_T-y_1)+\bar{c}_\tau^2T^{-2}\textstyle\sum_{t=2}^T y_{t-1} = u_1+o_p(1)$$

so that $T^{-1/2}b_1 \xrightarrow{p} 0$; next, since $b_2 = y_T+\bar{c}_\tau T^{-1}\sum_{t=2}^T y_{t-1}+\bar{c}_\tau T^{-1}\sum_{t=2}^T t\Delta y_t + \bar{c}_\tau^2T^{-2}\sum_{t=2}^T ty_{t-1}+o_p(T^{1/2})$, we have that

$$\begin{aligned} T^{-1/2}b_2 &= \kappa+T^{-1/2}u_T+\bar{c}_\tau T^{-3/2}\textstyle\sum_{t=2}^T u_{t-1}+3\bar{c}_\tau\kappa T^{-3}\sum_{t=2}^T t^2+\bar{c}_\tau T^{-3/2}\sum_{t=2}^T t\Delta u_t \\ &\quad +\bar{c}_\tau^2T^{-5/2}\textstyle\sum_{t=2}^T tu_{t-1}+\bar{c}_\tau^2\kappa T^{-4}\sum_{t=2}^T t^3+o_p(1) \\ &\xrightarrow{d} \kappa+\sigma W_c(1)+\bar{c}_\tau\sigma\int_0^1 W_c(s)ds+\bar{c}_\tau\kappa+\bar{c}_\tau\sigma\left\{W_c(1)-\int_0^1 W_c(s)ds\right\} \\ &\quad +\bar{c}_\tau^2\sigma\int_0^1 sW_c(s)ds+\bar{c}_\tau^2\kappa/4 \\ &= \sigma\left\{(1+\bar{c}_\tau)W_c(1)+\bar{c}_\tau^2\int_0^1 sW_c(s)ds+\kappa^*(1+\bar{c}+\bar{c}^2/4)\right\} = \sigma\{M_{1,\bar{c}_\tau}+\kappa^*\pi_{2,\bar{c}_\tau}\}. \end{aligned}$$

Consequently we obtain that

$$\begin{aligned} \begin{bmatrix} T^{-1/2}\tilde{\mu} \\ T^{1/2}\tilde{\beta} \end{bmatrix} &= \begin{bmatrix} a_{11} & T^{-1}a_{12} \\ a_{12} & T^{-1}a_{22} \end{bmatrix}^{-1} \begin{bmatrix} T^{-1/2}b_1 \\ T^{-1/2}b_2 \end{bmatrix} \\ &\xrightarrow{d} \begin{bmatrix} 1 & 0 \\ 1+\bar{c}_\tau+\bar{c}_\tau^2/2 & \pi_{1,\bar{c}_\tau} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \sigma\{M_{1,\bar{c}_\tau}+\kappa^*\pi_{2,\bar{c}_\tau}\} \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ \sigma\pi_{1,\bar{c}_\tau}^{-1}\{M_{1,\bar{c}_\tau}+\kappa^*\pi_{2,\bar{c}_\tau}\} \end{bmatrix}. \end{aligned}$$

Using the foregoing results and applications of the CMT we therefore obtain that

$$\begin{aligned} T^{-1/2}\tilde{u}_{\lfloor rT \rfloor} &\xrightarrow{d} \sigma W_c(r) + \kappa r^2 - \sigma \pi_{1,\bar{c}_\tau}^{-1}\{M_{1,\bar{c}_\tau} + \kappa^* \pi_{2,\bar{c}_\tau}\} r \\ &= \sigma\{W_c(r) - \pi_{1,\bar{c}_\tau}^{-1} M_{1,\bar{c}_\tau} r + \kappa^*(r^2 - \pi_{1,\bar{c}_\tau}^{-1}\pi_{2,\bar{c}_\tau} r)\} =: \sigma J_c^{\tau,\bar{c}_\tau}(r). \end{aligned}$$

Noting that $J_c^{\tau,\bar{c}_\tau}(0) = 0$, together with the fact that the error variance estimator from (3) converges in probability to σ^2, the stated result for *DF-QD*$^\tau$ in (4) then follows using standard arguments.

Limit distribution of DF-QD^q

The theoretical results presented by Elliott *et al.* (1996) allow for polynomial trends in the deterministic component, and a generic limit distribution for the point-optimal likelihood ratio unit root test is provided in their appendix. Here, we explicitly derive the limit distribution of the QD-detrended Dickey-Fuller-type test statistic for the specific case of a quadratic trend. Due to the invariance of *DF-QD*^{q} to γ, we can additionally set $\gamma = 0$ in the DGP without loss of generality, so that $y_t = u_t$. The proof then follows along similar lines as that for *DF-QD*$^\tau$. The QD-detrended residuals are in this case given by $\tilde{u}_t = y_t - \tilde{\mu} - \tilde{\beta} t - \tilde{\gamma} t^2$, so that

$$T^{-1/2}\tilde{u}_{\lfloor rT \rfloor} = T^{-1/2} u_{\lfloor rT \rfloor} - T^{-1/2}\tilde{\mu} - T^{1/2}\tilde{\beta} r - T^{3/2}\tilde{\gamma} r^2.$$

The large sample behaviour of $T^{-1/2}\tilde{\mu}$, $T^{1/2}\tilde{\beta}$ and $T^{3/2}\tilde{\gamma}$ can be obtained as follows

$$\begin{bmatrix} \tilde{\mu} \\ \tilde{\beta} \\ \tilde{\gamma} \end{bmatrix} := \begin{bmatrix} 1 + (1-\bar{\rho})^2(T-1) & 1 + (1-\bar{\rho})\sum_{t=2}^T\{t - \bar{\rho}(t-1)\} \\ 1 + (1-\bar{\rho})\sum_{t=2}^T\{t - \bar{\rho}(t-1)\} & 1 + \sum_{t=2}^T\{t - \bar{\rho}(t-1)\}^2 \\ 1 + (1-\bar{\rho})\sum_{t=2}^T\{t^2 - \bar{\rho}(t-1)^2\} & 1 + \sum_{t=2}^T\{t - \bar{\rho}(t-1)\}\{t^2 - \bar{\rho}(t-1)^2\} \end{bmatrix.$$
$$\left.\begin{matrix} 1 + (1-\bar{\rho})\sum_{t=2}^T\{t^2 - \bar{\rho}(t-1)^2\} \\ 1 + \sum_{t=2}^T\{t - \bar{\rho}(t-1)\}\{t^2 - \bar{\rho}(t-1)^2\} \\ 1 + \sum_{t=2}^T\{t^2 - \bar{\rho}(t-1)^2\}^2 \end{matrix}\right]^{-1} \begin{bmatrix} y_1 + (1-\bar{\rho})\sum_{t=2}^T(y_t - \bar{\rho} y_{t-1}) \\ y_1 + \sum_{t=2}^T(y_t - \bar{\rho} y_{t-1})\{t - \bar{\rho}(t-1)\} \\ y_1 + \sum_{t=2}^T(y_t - \bar{\rho} y_{t-1})\{t^2 - \bar{\rho}(t-1)^2\} \end{bmatrix}$$

As before, denote the right hand side of the above expression by $A^{-1}B$. The limits of a_{11}, a_{12} and a_{22} are as for the *DF-QD*$^\tau$ proof above, but with $\bar{c}_\tau$ replaced by $\bar{c}_q$. The limits of the remaining terms in A are

$$\begin{aligned} a_{13} &= 2\bar{c}_q T^{-1}\sum_{t=2}^T t + \bar{c}_q^2 T^{-2}\sum_{t=2}^T t^2 + o(T) \\ a_{23} &= 2\sum_{t=2}^T t + 3\bar{c}_q T^{-1}\sum_{t=2}^T t^2 + \bar{c}_q^2 T^{-2}\sum_{t=2}^T t^3 + o(T^2) \\ a_{33} &= 4\sum_{t=2}^T t^2 + 4\bar{c}_q T^{-1}\sum_{t=2}^T t^3 + \bar{c}_q^2 T^{-2}\sum_{t=2}^T t^4 + o(T^3) \end{aligned}$$

so that $T^{-1}a_{13} \to \bar{c}_q + \bar{c}_q^2/3$, $T^{-2}a_{23} \to 1 + \bar{c}_q + \bar{c}_q^2/4 =: \pi_{2,\bar{c}_q}$, and $T^{-3}a_{33} \to 4/3 + \bar{c}_q + \bar{c}_q^2/5 =: \pi_{3,\bar{c}_q}$, respectively. The limits of b_1 and b_2 are also as for the *DF-QD*$^\tau$ proof, with $\bar{c}_\tau$ replaced by $\bar{c}_q$ and $\kappa = 0$, i.e. $T^{-1/2}b_1 \xrightarrow{p} 0$, $T^{-1/2}b_2 \xrightarrow{d} \sigma M_{1,\bar{c}_q}$. Turning to b_{33}, we have that

$$\begin{aligned} b_3 = \; & 2\sum_{t=2}^T t\Delta y_t + 2\bar{c}_q T^{-1}\sum_{t=2}^T t y_{t-1} + \bar{c}_q T^{-1}\sum_{t=2}^T t^2 \Delta y_t \\ & + \bar{c}_q^2 T^{-2}\sum_{t=2}^T t^2 y_{t-1} + o_p(T^{3/2}) \end{aligned}$$

so that

$$\begin{aligned}
T^{-3/2}b_3 &= 2T^{-3/2}\textstyle\sum_{t=2}^{T} t\Delta u_t + 2\bar{c}_q T^{-5/2}\sum_{t=2}^{T} t u_{t-1} + \bar{c}_q T^{-7/2}... \\
\end{aligned}$$

$$\begin{aligned}
T^{-3/2}b_3 &= 2T^{-3/2}\textstyle\sum_{t=2}^{T} t\Delta u_t + 2\bar{c}_q T^{-5/2}\sum_{t=2}^{T} t u_{t-1} + \bar{c}_q T^{-5/2}\sum_{t=2}^{T} t^2\Delta u_t \\
&\quad +\bar{c}_q^2 T^{-7/2}\textstyle\sum_{t=2}^{T} t^2 u_{t-1} + o_p(1) \\
&\xrightarrow{d} 2\sigma\left\{W_c(1) - \int_0^1 W_c(s)ds\right\} + 2\bar{c}_q\int_0^1 sW_c(s)ds + \bar{c}_q\left\{W_c(1) - 2\int_0^1 sW_c(s)ds\right\} \\
&\quad +\bar{c}_q^2\int_0^1 s^2 W_c(s)ds \\
&= \sigma\left\{(2+\bar{c}_q)W_c(1) - 2\int_0^1 W_c(s)ds + \bar{c}_q^2\int_0^1 s^2W_c(s)ds\right\} = \sigma M_{2,\bar{c}_q}.
\end{aligned}$$

Collecting results we therefore have, using the CMT, that

$$\begin{aligned}
\begin{bmatrix} T^{-1/2}\tilde{\mu} \\ T^{1/2}\tilde{\beta} \\ T^{3/2}\tilde{\gamma} \end{bmatrix} &= \begin{bmatrix} a_{11} & T^{-1}a_{12} & T^{-2}a_{13} \\ a_{12} & T^{-1}a_{22} & T^{-2}a_{23} \\ T^{-1}a_{13} & T^{-2}a_{23} & T^{-3}a_{33} \end{bmatrix}^{-1} \begin{bmatrix} T^{-1/2}b_1 \\ T^{-1/2}b_2 \\ T^{-3/2}b_3 \end{bmatrix} \\
&\xrightarrow{d} \begin{bmatrix} 1 & 0 & 0 \\ 1+\bar{c}_\tau+\bar{c}_\tau^2/2 & \pi_{1,\bar{c}_q} & \pi_{2,\bar{c}_q} \\ \bar{c}_q+\bar{c}_q^2/3 & \pi_{2,\bar{c}_q} & \pi_{3,\bar{c}_q} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \sigma M_{1,\bar{c}_q} \\ \sigma M_{2,\bar{c}_q} \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ \sigma d_{\bar{c}_q}^{-1}(\pi_{3,\bar{c}_q}M_{1,\bar{c}_q} - \pi_{2,\bar{c}_q}M_{2,\bar{c}_q}) \\ \sigma d_{\bar{c}_q}^{-1}(\pi_{1,\bar{c}_q}M_{2,\bar{c}_q} - \pi_{2,\bar{c}_q}M_{1,\bar{c}_q}) \end{bmatrix}.
\end{aligned}$$

Using the CMT, it then follows immediately that

$$T^{-1/2}\tilde{u}_{\lfloor rT\rfloor} \xrightarrow{d} \sigma\{W_c(r) - d_{\bar{c}_q}^{-1}(\pi_{3,\bar{c}_q}M_{1,\bar{c}_q} - \pi_{2,\bar{c}_q}M_{2,\bar{c}_q})r - d_{\bar{c}_q}^{-1}(\pi_{1,\bar{c}_q}M_{2,\bar{c}_q} - \pi_{2,\bar{c}_q}M_{1,\bar{c}_q})r^2\} =: \sigma J_c^{q,\bar{c}_q}(r).$$

Noting that $J_c^{q,\bar{c}_q}(0) = 0$, and that the error variance estimator from (3) converges in probability to σ^2, the stated result for *DF-QD*^{q} in (5) again follows using standard arguments.

Proof of Lemma 2

Since $\rho_T = 1 - c/T$, $0 \leq c < \infty$, it follows from straightforward generalizations of the results in Harvey *et al.* (1997) that $t_0 = o_p(T^{1/2})$ and that $\lambda = 1 + o_p(T^{-1/2})$, so that $t_\lambda = t_1 + o_p(1)$ and $t_\lambda^{m2} = \eta_\xi R_2 t_1 + o_p(1)$. To find the limiting distribution of t_λ, we therefore need only in this case establish the limiting distribution of t_1. The numerator of t_1 is given by

$$\begin{aligned}
\hat{\delta}_2 &= \delta_2 + \frac{\sum_{t=2}^{T}\left\{t - (T-1)^{-1}\sum_{s=2}^{T} s\right\} v_t}{\sum_{t=2}^{T}\left\{t - (T-1)^{-1}\sum_{s=2}^{T} s\right\}^2} \\
&= 2\kappa T^{-3/2} + \frac{\sum_{t=2}^{T} t\Delta u_t - (T-1)^{-1}\sum_{t=2}^{T} t\sum_{t=2}^{T}\Delta u_t}{\sum_{t=2}^{T} t^2 - (T-1)^{-1}\left(\sum_{t=2}^{T} t\right)^2}
\end{aligned}$$

so that

$$
\begin{aligned}
T^{3/2}\hat{\delta}_2 &= 2\kappa + \frac{T^{-3/2}\sum_{t=2}^{T} t\Delta u_t - T^{-2}\sum_{t=2}^{T} t.T^{-1/2}u_T}{T^{-3}\sum_{t=2}^{T} t^2 - \left(T^{-2}\sum_{t=2}^{T} t\right)^2} + o_p(1) \\
&\xrightarrow{d} 2\kappa + 12\left\{\frac{1}{2}\omega_\varepsilon W_c(1) - \omega_\varepsilon \int_0^1 W_c(r)dr\right\}.
\end{aligned}
$$

On scaling by $T^{3/2}$, the denominator of t_1 is given by

$$
\begin{aligned}
T^{3/2}\sqrt{\hat{\omega}_v^2[(\sum_{t=1}^{T} z_t z_t')^{-1}]_{22}} &= \sqrt{\hat{\omega}_v^2\left[T^{-3}\sum_{t=2}^{T} t^2 - \left(T^{-2}\sum_{t=2}^{T} t\right)^2\right]^{-1} + o_p(1)} \\
&\xrightarrow{p} \sqrt{12\omega_\varepsilon^2}.
\end{aligned}
$$

since $\hat{\omega}_v^2 \xrightarrow{p} \omega_v^2 = \omega_\varepsilon^2$ under the local-to-unit root specification for u_t. The limiting distribution of t_1, and hence of t_λ, then obtains using the CMT as

$$
\begin{aligned}
t_\lambda &\xrightarrow{d} \frac{2\kappa + 12\left\{\frac{1}{2}\omega_\varepsilon W_c(1) - \omega_\varepsilon \int_0^1 W_c(r)dr\right\}}{\sqrt{12\omega_\varepsilon^2}} \\
&= \frac{\kappa^*}{\sqrt{3}} + \sqrt{12}\left\{\tfrac{1}{2}W_c(1) - \int_0^1 W_c(r)dr\right\}.
\end{aligned}
$$

Given that the limiting distributions of t_1 and t_λ coincide, in deriving the limit of t_λ^{m2} we need only establish the asymptotic behaviour of R_2. It is entirely straightforward to show that

$$
T^{-1}\hat{\sigma}_u^2 \xrightarrow{d} \omega_\varepsilon^2 \int_0^1 N_c(r)^2 dr
$$

and, since $\hat{\omega}_v^2 \xrightarrow{p} \omega_\varepsilon^2$, we therefore have that

$$
R_2 \xrightarrow{d} \left\{\int_0^1 N_c(r)^2 dr\right\}^{-2}.
$$

An application of the CMT then yields the result that

$$
t_\lambda^{m2} \xrightarrow{d} \eta_\xi \left\{\int_0^1 N_c(r)^2 dr\right\}^{-2}\left[\frac{\kappa^*}{\sqrt{3}} + \sqrt{12}\left\{\tfrac{1}{2}W_c(1) - \int_0^1 W_c(r)dr\right\}\right]
$$

completing the proof.

References

Ayat, L. and Burridge, P. (2000). Unit root tests in the presence of uncertainty about the non-stochastic trend. Journal of Econometrics 95, 71–96.

Bierens, H.J. (1997). Testing the unit root with drift hypothesis against nonlinear trend stationarity, with an application to the US price level and interest rate. Journal of Econometrics 81, 29–64.

Chang, Y. and Park, Y.J. (2002). On the asymptotics of ADF tests for unit roots. Econometric Reviews 21, 431–47.

Dickey, D.A. and Fuller, W.A. (1979). Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74, 427–31.

Elliott, G., Rothenberg, T.J. and Stock, J.H. (1996). Efficient tests for an autoregressive unit root. Econometrica 64, 813–36.

Grilli, R.E. and Yang, M.C. (1988). Commodity prices, manufactured goods prices, and the terms of trade of developing countries. World Bank Economic Review 2, 1–48.

Harvey, D.I., Leybourne, S.J. and Taylor, A.M.R. (2007). A simple, robust and powerful test of the trend hypothesis. Journal of Econometrics 141, 1302–30.

Harvey, D.I., Leybourne, S.J. and Taylor, A.M.R. (2008). Unit Root Testing in Practice: Dealing with Uncertainty over the Trend and Initial Condition (with commentaries and rejoinder). Econometric Theory, forthcoming.

Kellard, N. and Wohar, M.E. (2006). On the prevalence of trends in primary commodity prices. Journal of Development Economics 79, 146–67.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root? Journal of Econometrics 54, 159–78.

Lee, J., Becker, R. and Enders, W. (2008). Stationarity tests with unattended nonlinearity. Journal of Time Series Analysis, forthcoming.

Leon, J. and Soto, R. (1997). Structural breaks and long-run trends in commodity prices. Journal of International Development 9, 347–66.

Müller, U.K. and Elliott, G. (2003). Tests for unit roots and the initial condition. Econometrica 71, 1269–86.

Murray, C.J. and Nelson, C.R. (2000). The uncertain trend in U.S. GDP. Journal of Monetary Economics 46, 79-95.

Ng, S. and Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. Econometrica 69, 1519–54.

Ouliaris, S., Park, J.Y. and Phillips, P.C.B. (1989). Testing for a unit root in the presence of a maintained trend. In Ray, B. (Ed.), Advances in Econometrics and Modelling. Kluwer, Dordecht, 6-28.

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. Econometrica 57, 1361-1401.

Perron, P. and Qu, Z. (2007). A simple modification to improve the finite sample properties of Ng and Perron's unit root tests. Economics Letters 94, 12–19.

Perron, P. and Rodríguez, G. (2003). GLS detrending, efficient unit root tests and structural change. Journal of Econometrics 115, 1-27.

Pfaffenzeller, S., Newbold, P. and Rayner, A.J. (2007). A short note on updating the Grilli and Yang commodity price index. World Bank Economic Review 21, 151–63.

Phillips P.C.B. (1998). New tools for understanding spurious regressions. Econometrica 66, 1299-1325.

Phillips P.C.B. and Solo, V. (1992). Asymptotics for linear processes. Annals of Statistics 20, 971–1001.

Prebisch, R. (1950). The economic development of Latin America and its principal problems. Economic Bulletin For Latin America 7, 1–12.

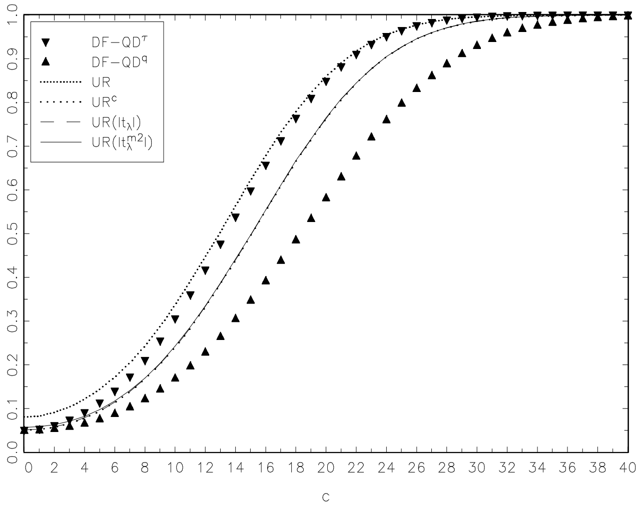
Singer, H. (1950). Comments to the terms of trade and economic development. Review of Economics and Statistics 40, 84–9.

Stock, J.H. and Watson, M.W. (1999). Business cycle fluctuations in U.S. macroeconomic time series. In Taylor, J.B. and Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier, Amsterdam, 3-64.

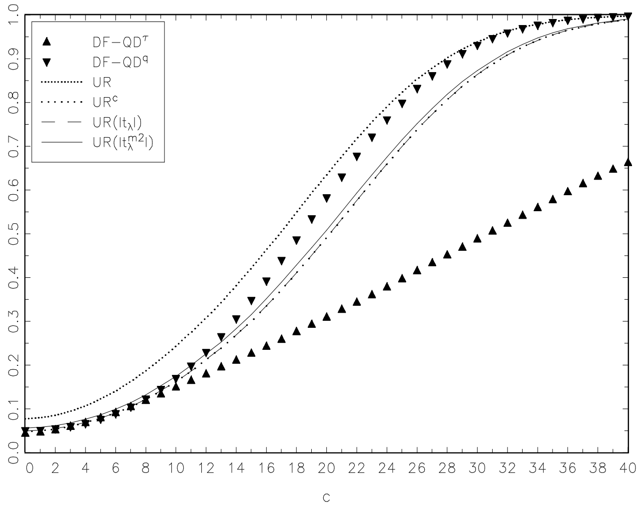
Table 1. Application of unit root tests to relative primary commodity prices: 1900–2003

	$DF\text{-}QD^{\tau}$	$DF\text{-}QD^{q}$	UR^{c}	$UR(\lvert t_{\lambda}\rvert)$	$UR(\lvert t_{\lambda}^{m2}\rvert)$	$\lvert t_{\lambda}\rvert$	$\lvert t_{\lambda}^{m2}\rvert$
Aluminium	−2.902**	−3.326*	*	*	*		
Banana	−1.325	−2.957				**	*
Beef	−2.882**	−2.896	*	*	*		
Cocoa	−2.036	−2.071					
Coffee	−2.273	−3.413*	*	*	*		
Copper	−2.843*	−2.897	*	*	*		
Cotton	−0.868	−2.343				***	
Hides	−1.435	−1.479					
Jute	−0.739	−1.590				***	
Lamb	−3.006**	−3.010	*	*	*		
Lead	−1.344	−3.916**	**	**	**	**	
Maize	−0.535	−5.301***	***	***	***	***	***
Palmoil	−1.448	−4.547***	***	***	***		**
Rice	−0.931	−3.777**	**	**	**	***	
Rubber	−3.216**	−3.539**	**	**	**		
Silver	−1.777	−1.844					
Sugar	−2.652*	−2.659					
Tea	−2.068	−2.941					
Timber	−3.382**	−3.746**	**	**	**		
Tin	−2.100	−2.607					
Tobacco	−0.710	−4.069***	**	***	***	***	***
Wheat	−1.144	−4.112***	**	**	**	*	
Wool	−0.825	−4.553***	***	***	***		***
Zinc	−4.621***	−4.682***	***	***	***		

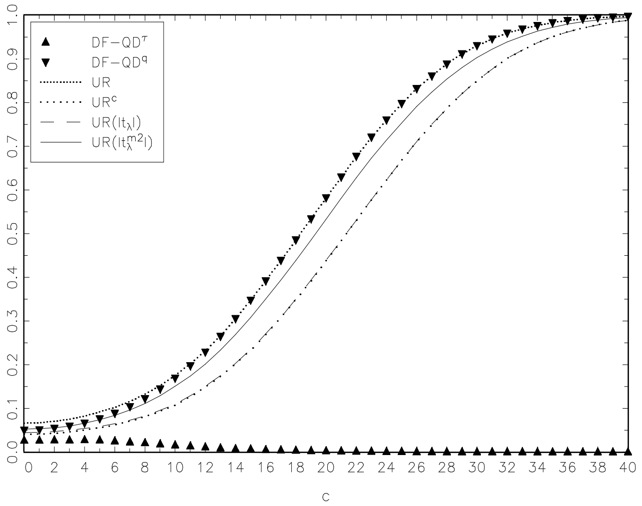
Note: *, ** and *** denote rejection of the relevant null hypothesis at the 0.10-, 0.05- and 0.01-levels respectively.



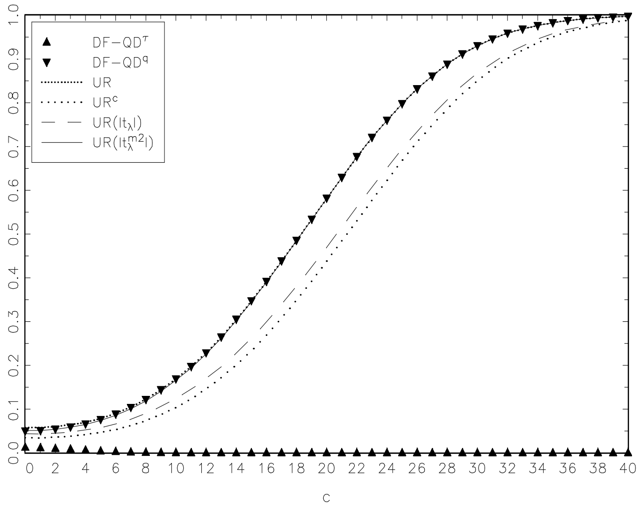
(a) $\kappa^* = 0$



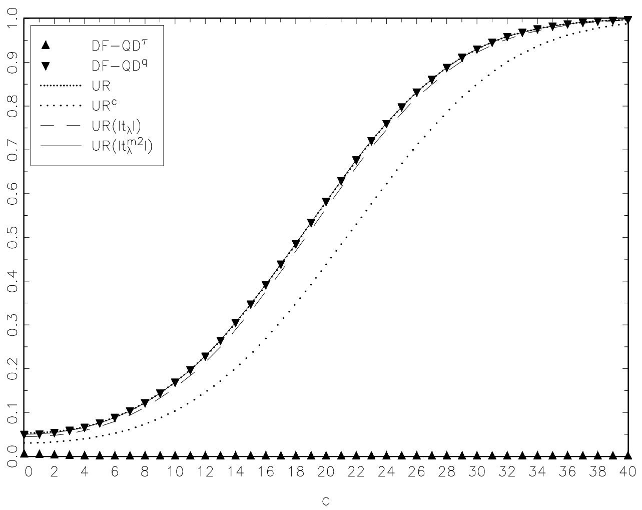
(b) $\kappa^* = 1$



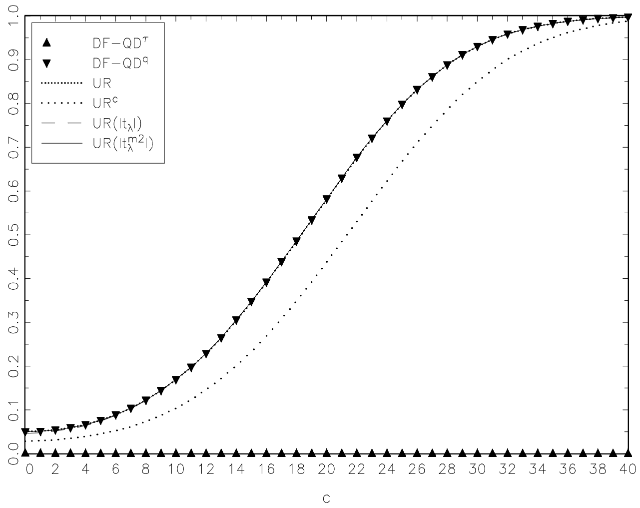
(c) $\kappa^* = 2$



(d) $\kappa^* = 3$

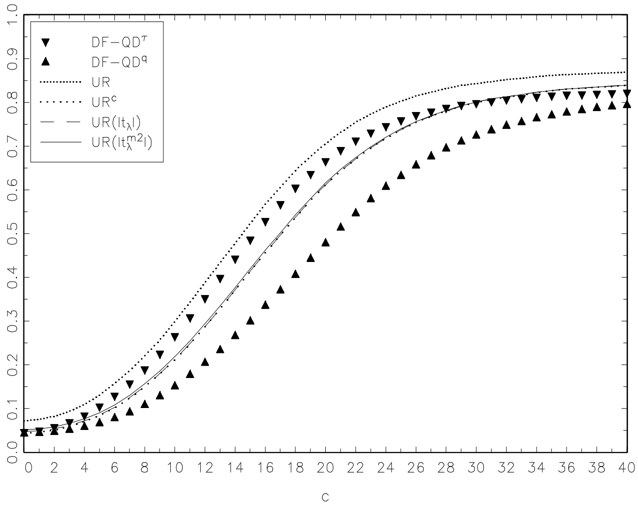


(e) $\kappa^* = 4$

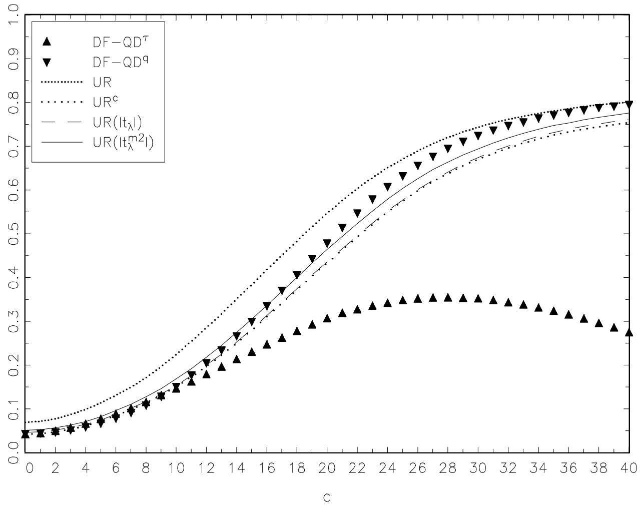


(f) $\kappa^* = 5$

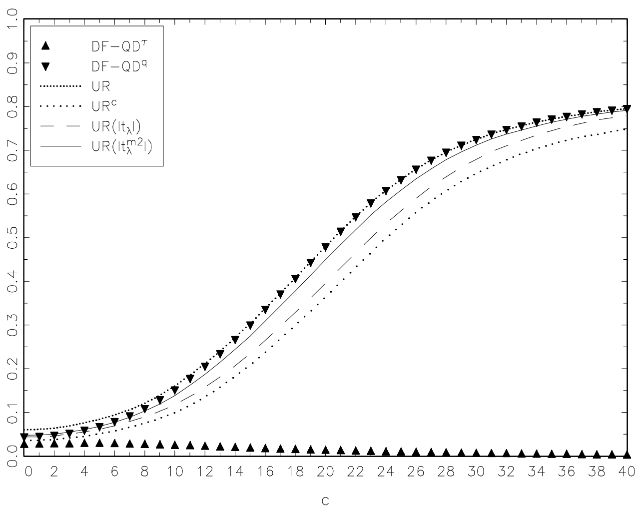
Figure 1. Asymptotic size and local power: $\gamma = \kappa T^{-3/2}$



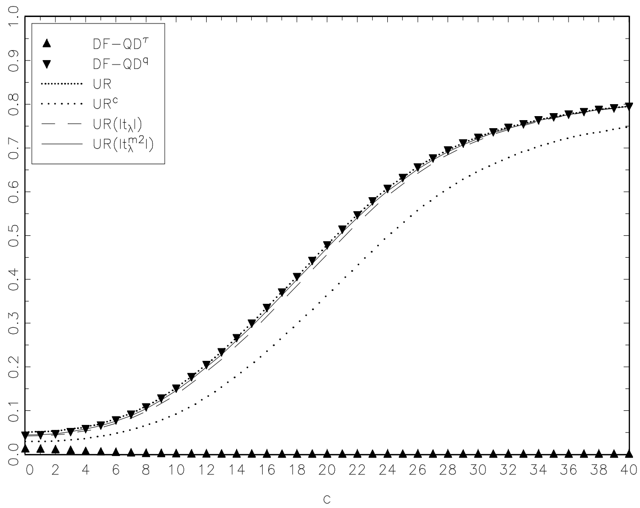
(a) $\kappa = 0$



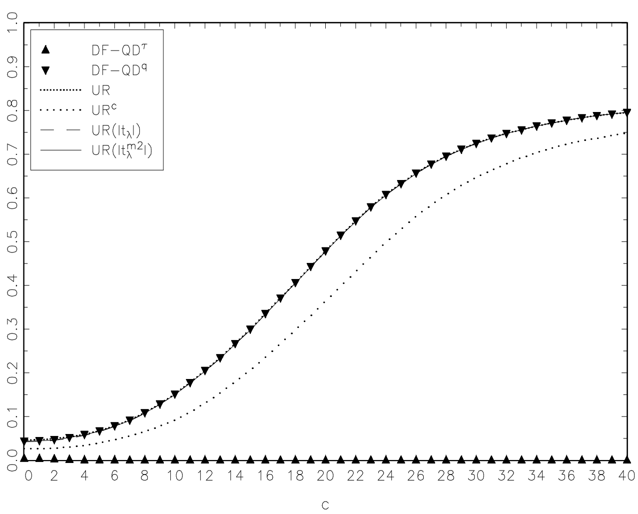
(b) $\kappa = 1$



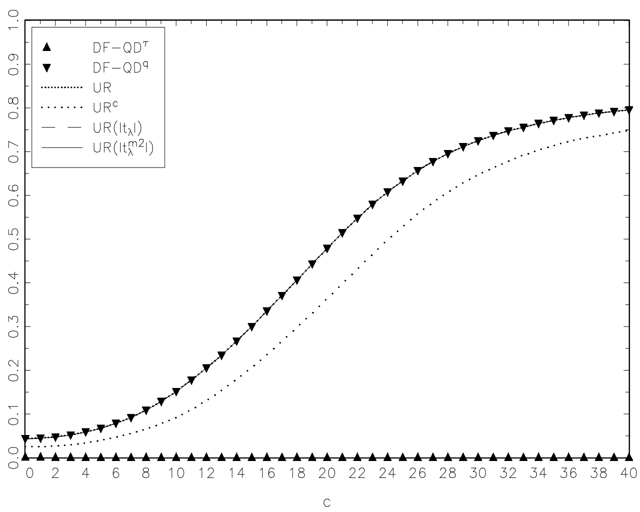
(c) $\kappa = 2$



(d) $\kappa = 3$

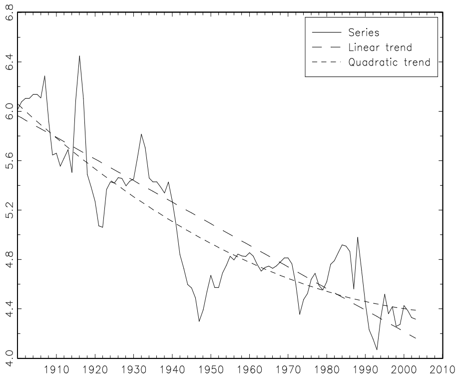


(e) $\kappa = 4$

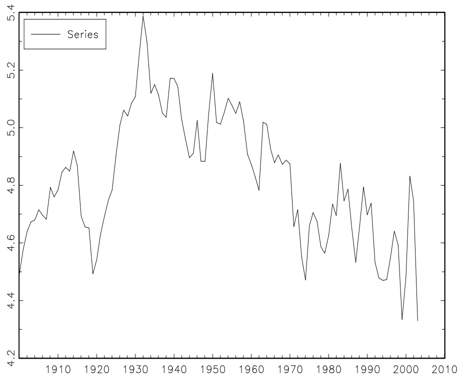


(f) $\kappa = 5$

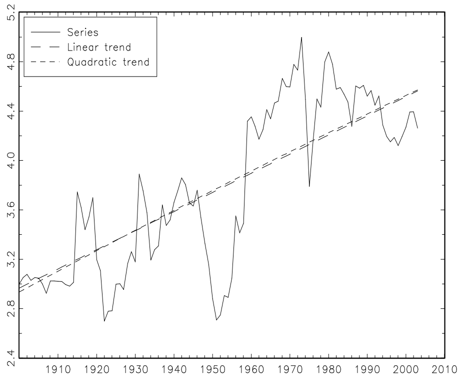
Figure 2. Finite sample and power: $T = 150$, $\gamma = \kappa T^{-3/2}$



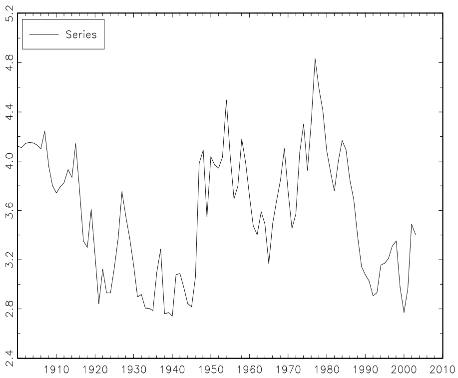
Aluminium



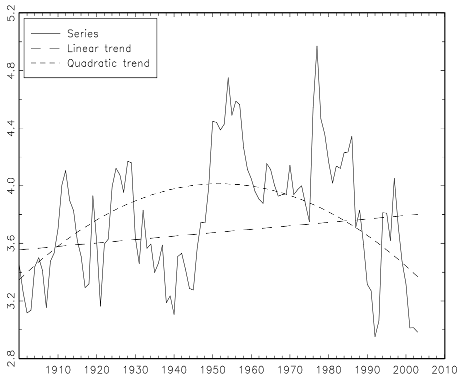
Banana



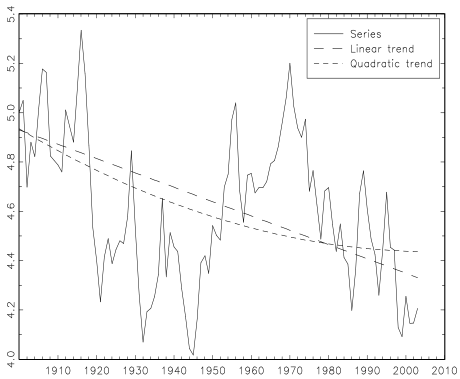
Beef



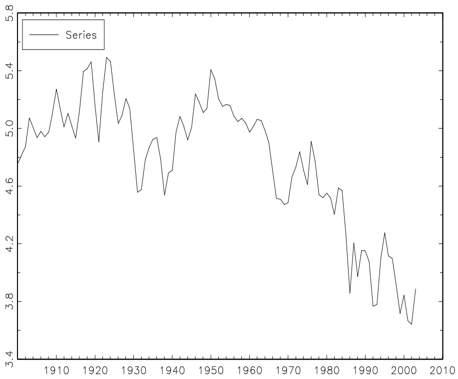
Cocoa



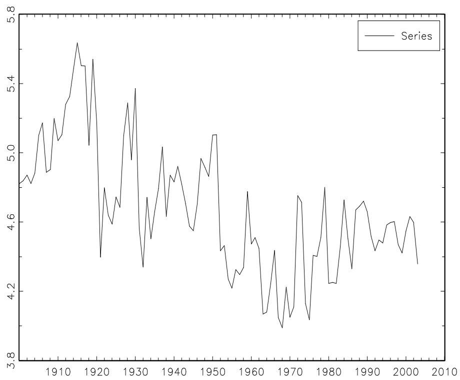
Coffee



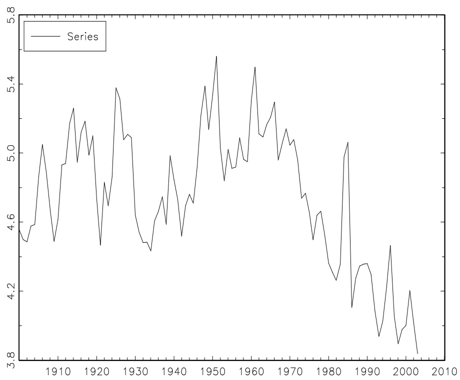
Copper



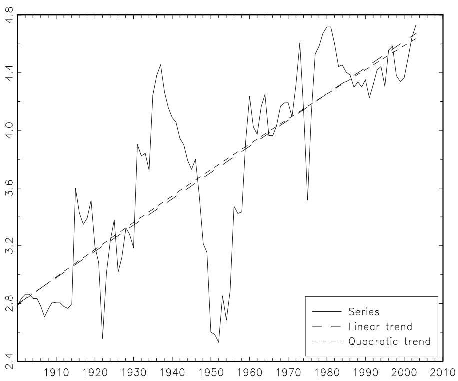
Cotton



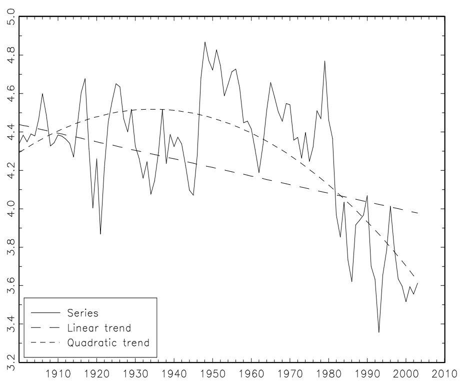
Hides



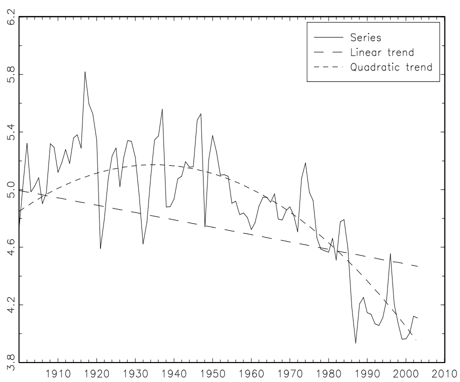
Jute



Lamb



Lead



Maize

Figure 3a. Relative primary commodity price series and fitted deterministic components

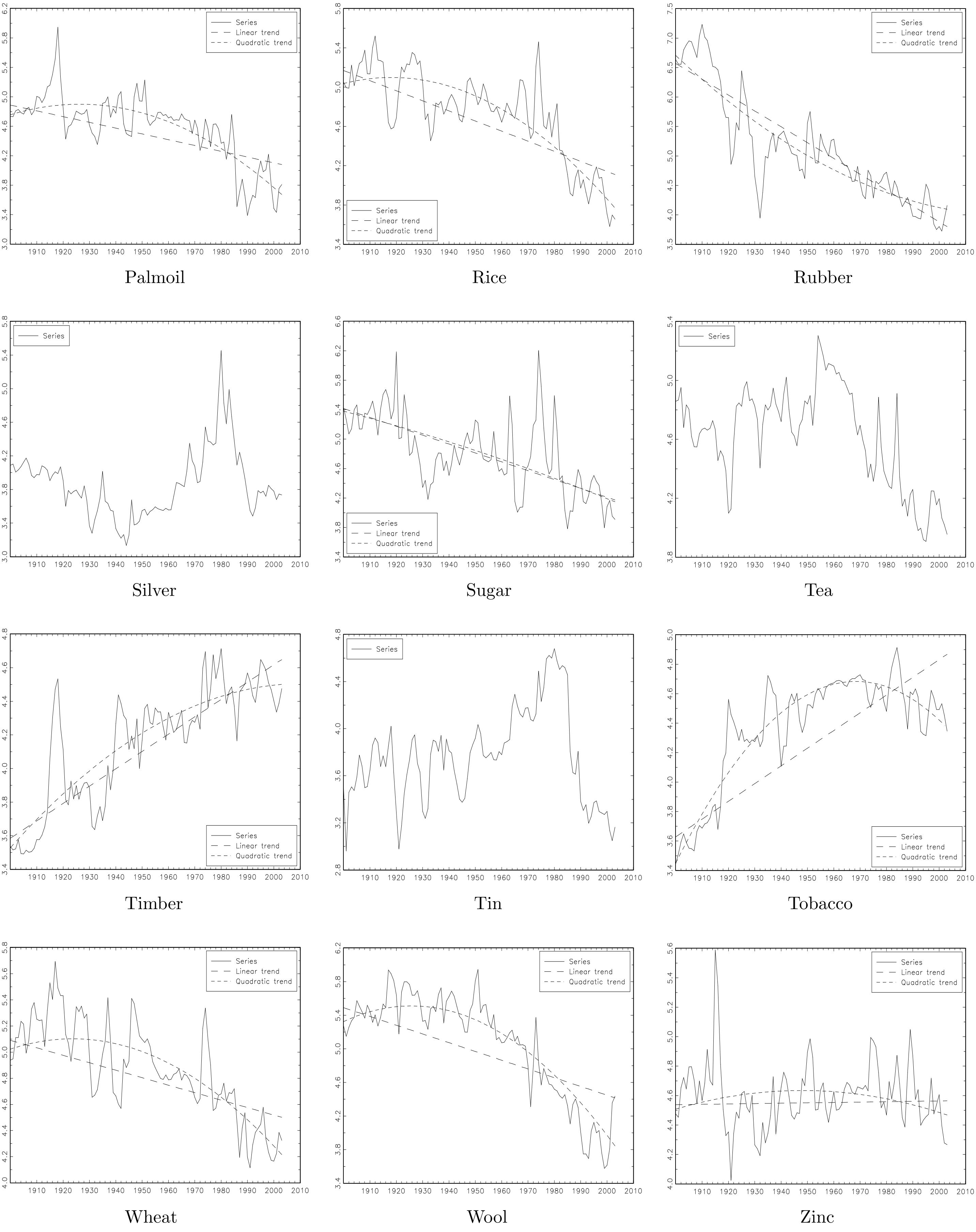


Figure 3b. Relative primary commodity price series and fitted deterministic components