

# OPTIMAL FINANCIAL CONTRACTING AND DEBT MATURITY STRUCTURE UNDER ADVERSE SELECTION

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*Resumen:* Analizamos un modelo en el que un país averso al riesgo financia su proyecto de desarrollo bajo información asimétrica. Antes de que este proyecto rinda sus frutos, se producen dos tipos de noticias, una de las cuales reduce la asimetría inicial de información entre el país y los inversionistas. Caracterizamos el contrato financiero óptimo bajo estas circunstancias, tanto en el caso en que se dispone de contratos financieros completos, como en el que el país solamente puede contratar deuda de corto y largo plazo.

*Abstract:* We analyze a model in which a risk-averse country finances its development project under asymmetric information. Before the project renders its fruits, two types of news will become available, one of which will reduce the asymmetry of information between the country and its investors. We characterize the optimal financial contract both when complete financial contracting is possible and when the country is restricted to using only short-term and long-term debt.

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## 1. Introduction

Recent experiences in Mexico and other developing countries have shown that international debt problems can arise not only as a result of large debts -properly measured in relative terms- but also as a result of its maturity profile. A given amount of debt may have very different consequences depending on its maturity structure. In particular, in the Mexican case, it has been argued that the extremely short maturity structure of the public debt by the end of 1994 was a major determinant of the economic crisis that erupted then.<sup>1</sup> Despite the dangers it encompasses, the fact that short-term debt is very commonly used calls for a search of its rationale based on first principles.

In this paper we examine the rational choice of the debt maturity structure within an optimal contracting framework when there are informational asymmetries in financial markets. The role of agency problems in financial markets has long been recognized as crucial in the understanding of widespread features -such as credit rationing- that differentiate these markets from other markets where textbook analysis of supply and demand works well. We consider a situation in which the country has initial private information about its development project, and there will be two types of news before the project matures. First, future news will reduce the asymmetry of information between the country and its investors. Second, international interest rates may change. Moreover, the situation we consider allows for certain adjustments in the country's economy before the project matures. These adjustments stop the course of the project as it was originally planned, and imply a partial loss of the initial investment. But, by stopping the project, they also avoid the realization of the worst-case scenario.

We carry out our analysis in two different stages. We first derive the optimal financial contract when the country has access to any financial contract that provides nonnegative returns to investors, i.e., we consider complete financial contracting. This allows us to describe the tasks that a financial contract must accomplish to optimize use of future information. To gain more insight, we consider separately cases in which only one type of information will arrive between the time investment takes place and the time the project matures. We first consider the case in which interest rates may rise, but there will

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<sup>1</sup> See for instance Cole and Kehoe (1996) for an analysis that emphasizes this view within a rigorous framework. Other related articles are Calvo and Mendoza (1996) and Sachs, Tornell and Velasco (1996).

be no more information regarding the country's project itself. We then analyze the cases in which there is no interest-rate risk, and in which there is uncertainty in both interest-rate movements and news about the country's project.

Our analysis of complete contracting yields insights into how financial contracts should be designed to optimize use of future information. First, when there is only interest-rate uncertainty, financial contracting should avoid any risk that the project will be canceled. It should also optimize risk sharing. This means that the country should not bear the risk of interest rate movements, which are not under its control.

When future news refers not only to interest rate movements, but also to information on the country's project, we have a very different picture. Optimal contracting may have to consider canceling the project if the news is sufficiently bad. Countries with good projects may optimally accept this risk in exchange for the smaller expected repayment it implies. Indeed, by intervening quickly when bad signals arrive, expected repayments from bad projects increase. Under competitive capital markets, this translates into smaller repayments from good projects. Thus, a country with a good project is willing to accept the risk of (inefficiently) suffering an adjustment in its project if there is bad news, because it is confident that the news will most likely be good, and the reduction in debt repayments this risk allows is substantial. Another feature that optimal contracting should exhibit is that repayments should also be contingent on future news about the country's project. Countries with good projects are willing to set higher repayments after bad news in exchange for smaller repayments under good news because they are less likely than countries with bad projects to get bad news. Thus, expected repayments from bad projects increase, which allows for a reduction in expected repayments from good projects.

In a second stage, we constrain the country to using only two different instruments. One is short-term debt, which comes due before the project matures, but after the change in interest rates and information about the country's project become known. The second is long-term debt that comes due after the project matures.

When there is only interest-rate risk, pure long term debt is optimal. It guarantees that the country will not need to come back to the capital markets before the project matures, and thus, there is no risk of the project having to be adjusted. It also ensures that the country will be isolated from movements in international interest rates. Thus, pure long-term debt replicates the optimal contract

under complete financing. When there will be news about the country's project, but there is no interest-rate risk, short-term debt plays a crucial role, and is critical in order to replicate the contract that is optimal under complete contracting. When the country finances with pure long-term debt, it does not allow for adjustments of the project after bad news. Thus, if such an adjustment is optimal, it will be better to use short-term debt. Then, the country will have to come back to the capital markets and, if these markets' assessment of the quality of its project is negative, the country will be unable to refinance its liabilities. This, in turn, will force the country to accept an adjustment in its project. Even when no such adjustment is optimal, short-term debt is useful because it makes repayments depend on future news. Indeed, with short-term debt, the country will refinance its debt at terms that will depend on future news.

Finally, when there is both risk of interest rate movements and news regarding the country's project, we find that the same forces driving the optimal debt maturity structure described above continue to make optimal the use of short-term debt. A mixture of short-term and long-term debt can no longer replicate the optimal financial contract under complete contracting.

The analysis of financial contracting under two different frameworks helps to enhance our understanding of the tradeoffs of different forms of financing. The complete contracts approach provides us with a clear picture of how optimal contracts should be designed to take advantage of different types of information accruing during the life of a project. It also provides us with a benchmark against which actual contracts can be compared. On the other hand, the debt markets approach is a better representation of the situation faced by a developing country. Indeed, financial contracts that make it possible to establish a different set of obligations for each possible realization of a set of random variables over a long period of time are simply not available to a developing country. We choose to capture this fact with a model in which there is the possibility of a hike in the interest rate, and there are no markets to insure against this risk. But, more generally, there may be other variables over which the country has no control and cannot be contracted upon. The international interest rate is but one of several such variables that provides a simple way to depart from the complete contracting framework. This variable, additionally, has indeed been at the heart of international debt crises: While nominal interest rates<sup>2</sup> –as measured by LIBOR– averaged 8

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<sup>2</sup> See, for instance, Dornbusch (1989).

percent in the 1970's, they were 14.4 percent in 1980, and reached 16.5 percent in 1981. Moreover, if one uses the rate of inflation in world trade (the industrial countries' unit export value) to deflate LIBOR, one obtains an average negative interest rate (-3.4 percent) for the 1970's, 1.4 percent in 1980 and 20.6 percent in 1981. There is widespread consensus that this increase in international interest rates played a major role in the onset of the 1982 international debt crisis.

From a formal point of view, our model is closest to Diamond (1991). Yet, both our setup and the focus of our analysis is different. First, with respect to the setup, in Diamond's model there is only one kind of intermediate news, while in ours there are news both about the country's project and a variable not directly related to its project -international interest rates. Second, in Diamond, borrowers are risk neutral, while here the country is risk-averse, which introduces risk-sharing issues. With respect to the focus of the analysis, Diamond focuses on the choice of either short-term or long-term debt for different prior probabilities of the borrower being of a good type -which he interprets as the borrower's rating. In contrast, we focus on the ability of debt contracts to accomplish the tasks performed by complete contracts under a risky environment and on the reasons why the use of (at least some) short-term debt may be optimal in such an environment.

Our paper is also related to the literature exploring conditions under which simple debt contracts are optimal, as in the costly state verification model of Gale and Hellwig (1985) in which the profits made by the firm can be observed only at a cost. The optimal contract resembles a debt contract, in that the investor inspects (and confiscates all of the firm's profits) only if profits are low. More recently, Krasa and Villamil (2000) analyze a model with limited commitment to payment and enforcement decisions, and find conditions under which simple debt is the optimal contract. Our model differs from this literature in that we assume the observability of income and the commitment to payment, and so there is no room for studying either costly state verification or enforcement decisions.

## 2. The model

In this section we present a model in the spirit of Diamond (1991, 1993). It is a model in which a country needs funds to undertake its development project, and has private information about the quality of such a project. Before the project matures, different types of news will arrive and it will be possible to alter the project.

2.1. *The project and the timing*

A country has access to a project needing an outlay of  $I$ . The project's life extends over three periods,  $t = 0, 1, 2$ . At  $t = 0$  the country raises  $I$  in a competitive credit market through a financial contract that provides nonnegative expected returns to investors. At  $t = 1$ , information arrives and (at least) some of the amounts invested can be safeguarded by adjusting the country's economy. This adjustment implies that the project may be partially or totally canceled. At  $t = 2$  the project matures and produces income.<sup>3</sup> With this timing, we capture the fact that after the project has been undertaken and new information has arrived, it is possible to ensure the recovery of (at least) some of the amounts invested by adjusting the country's project before it is completed.

There are two types of projects, and only the country knows which project it has. A country with a good project obtains an income of  $X > I$  at  $t = 2$ , while a country with a bad project obtains  $X$  with probability  $\pi$  and  $0$  otherwise, with  $\pi X < I$ . Thus, under symmetric information countries with bad projects would not receive financing. However, investors do not know if a country has a bad project. At date zero, they assign the country a probability  $f$  of having a good project. Therefore, there is a probability  $q = [f + (1 - f)\pi]$  that the country's date-two income will be  $X$ .

The project can be canceled or reduced before it matures. This eliminates (or at least reduces) the uncertainty in the country's income, from the investor's perspective. If a project is completely canceled, it yields  $L < I$  at date 2, whether it is good or bad. If  $100\phi$  percent of the project is canceled ( $0 < \phi < 1$ ), the project will yield  $\phi L$  for sure plus another return that depends on its type. A good project will yield an additional  $(1 - \phi)X$  for sure while a bad one will yield an additional  $(1 - \phi)X$  with probability  $\pi$  and  $0$  otherwise. Thus, cancellation of  $100\phi$  percent of the project still allows the remaining  $100(1 - \phi)$  percent of the project to be completed. Cancellation—even if it is only partial—is inefficient since it results in a loss of  $\phi(I - L)$ : Resources are wasted if a project is undertaken and shut down before it yields its full results.

Two kinds of news arrives prior to the maturity of the project, when it can still be adjusted: First, there is news about international risk-free interest rates. More precisely the one-period risk-free interest

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<sup>3</sup> In Diamond (1991, 1993) all projects also produce a "control" rent at  $t=2$  which accrues to the borrower -and is not transferable- if there is no cancellation at  $t = 1$ . Including this rent would complicate matters without altering our results.

rate will remain at zero with probability  $\lambda$ , and will increase to  $i > 0$  with probability  $1 - \lambda$ . Second, there is news about the project itself that reduces the asymmetry of information between the country and its outside investors: The performance of the country's economy is observed by investors, who learn something about the country's project. This second type of news can be good,  $s = u$  (an upgrade of the country's rating takes place) or bad,  $s = d$  (a downgrade takes place). Good borrowers receive bad news with probability  $e$ , and bad borrowers with probability  $r$ , with  $e < r$ .

The country maximizes its expected utility  $E u(Y)$ , where  $Y$  is date-two income, net of repayments to investors, and  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$ . Capital markets are competitive, and are willing to invest as long as the expected discounted sum of net repayments from the country equals (or is higher than) zero.

## 2.2. Updating beliefs

After observing the realization of  $s$ , investors update their beliefs about the country's type. They do this by applying Bayes' rule. Let  $f^d(f^u)$  be the updated probability—according to Bayes' rule—that the country is good given bad (good) news. We have:

$$f^d = \frac{ef}{fe + (1-f)r}, \quad (1)$$

$$f^u = \frac{(1-e)f}{f(1-e) + (1-f)(1-r)} \quad (2)$$

Likewise, denote by  $q^d(q^u)$  the conditional probability that date-two income—if there is no adjustment—will be  $X$  given bad (good) news.

## 3. Complete financial contracts

We will consider different types of financial contracts. First, as a benchmark, we will consider in this section optimal contracts when complete financial contracting is possible.

Under complete financial contracting, a financial contract specifies the repayment schedule—the repayments due under completion and cancellation of the project—and the percentage of cancellation, both as a function of the realization of date-one news. A contract

specifies  $\phi^{sn}$ ,  $L^{sn}$ , and  $R^{sn}$ , for  $s = d, u$ , and  $n = c, i$ , where  $\phi^{sn}$  denotes the fraction of liquidation,  $L^{sn}(R^{sn})$  the repayment due to investors under cancellation (completion) of the project, and the superscripts refer to date-one news. The first superscript indicates whether there are good ( $s = u$ ) or bad ( $s = d$ ) news about the country's project, and the second one whether interest rates remain unchanged ( $n = c$ ), or raise to  $i$  ( $n = h$ ).

We will also consider in the next section the case (in the spirit of Diamond 1991, 1993) when the country is restricted to raising funds, at  $t = 0$ , by issuing short-term debt that matures at  $t = 1$ , after the news is released, and long-term debt that comes due at the completion of the project (at  $t = 2$ ). Since short-term debt will come due before the project matures, the country will refinance this debt by borrowing again at the credit market.

When analyzing both complete financial contracts and short-term and long-term debt contracts, we will concentrate, as Flannery (1986) and Diamond (1991, 1993) do, on the contract preferred by good quality borrowers among the pooling equilibrium contracts. This means that we will study a situation in which all countries offer the same contract, which is the one preferred by good quality countries among those able to attract financing. Thus, in the case of complete financial contracting, the optimal contract solves

Program (1):

$$\begin{aligned} \text{Maximize } H = & e(1 - \lambda)u((1 - \phi^{dh})X - R^{dh} + \phi^{dh}L - L^{dh}) \\ & + e\lambda u((1 - \phi^{dc})X - R^{dc} + \phi^{dc}L - L^{dc}) \\ & + (1 - e)(1 - \lambda)u((1 - \phi^{uh})X - R^{uh} + \phi^{uh}L - L^{uh}) \\ & + (1 - c)\lambda u((1 - \phi^{uc})X - R^{uc} + \phi^{uc}L - L^{uc}) \end{aligned}$$

subject to

$$\begin{aligned} & [fe + (1 - f)r][(1 - \lambda)(q^d R^{dh} + L^{dh}) + \lambda(q^d R^{dc} + L^{dc})] \quad (3) \\ & + [f(1 - e) + (1 - f)(1 - r)][(1 - \lambda)(q^u R^{uh} + L^{uh}) + \lambda(q^u R^{uc} + L^{uc})] \\ & \geq [\lambda + (1 - \lambda)(1 + i)]I \end{aligned}$$

and

$$0 \leq R^{sn} \leq (1 - \phi^{sn})X, \quad 0 \leq L^{sn} \leq \phi^{sn}L, \quad 0 \leq \phi^{sn} \leq 1 \text{ for } s = d, u, \text{ and } n = c, h,$$



Let us interpret this program. Constraint (3) establishes that the contract is profitable for investors. With probability  $[fe + (1-f)r](1-\lambda)$  there will be bad news about the project ( $s = d$ ) and a hike in the interest rate ( $n = h$ ). In such a case, a fraction  $\phi^{dh}$  of the project is canceled and the country gets an income  $\phi^{dh}L$  with certainty plus an income  $(1 - \phi^{sn})X$  with probability  $q^d$ . Investors are repaid  $L^{dh}$  for sure and an additional  $R^{dh}$  with probability  $q^d$ . The amount  $L^{dh}$  is guaranteed because it is smaller than  $\phi^{dh}L$  and, therefore,  $L^{dh}$  can be thought of as the part of the income not subject to uncertainty ( $\phi^{dh}L$ ) that goes to investors. The amount  $R^{dh} \leq (1 - \phi^{dh})X$  is repaid only if there is a good realization of the country's income which, given the bad date-one news about the country's project, occurs with probability  $q^d$ . Thus, we may interpret  $R^{dh}$  as the part of the income subject to uncertainty that goes to investors. The three other terms in the left hand side of constraint (3) have a similar interpretation.

The objective function is the utility a good country obtains. With probability  $e(1-\lambda)$ , there will be bad news about the project and a hike in the interest rate. In such a case, a fraction  $\phi^{dh}$  of the project is canceled and the country's income is  $(1 - \phi^{dh})X + \phi^{dh}L$ , from which it has to subtract repayments  $R^{dh} + L^{dh}$ .<sup>4</sup> The three other terms in the objective function can be interpreted in an analogous way.

Given the existence of limited liability, if the country is able to attract financing it will find in its interest to carry out the project. To be able to attract financing, the country must find a contract that satisfies the constraints of the above program. To see if such a contract exists, set first scheduled repayments at their maximum level,  $R^{sn} = (1 - \phi^{sn})X$  and  $L^{sn} = \phi^{sn}L$  for all  $s$ , all  $n$ . Then null adjustment of the project after good news,  $\phi^{uh} = \phi^{uc} = 0$ . We can do this because if it were optimal to adjust the project even after good news, the project would surely not attract funds (because  $L \leq I$ ). Now maximizing the returns to investors (the left hand side of (3)) with respect to  $\phi^{dh}$  and  $\phi^{dc}$  we obtain that the project will attract financing if

$$\begin{aligned} \text{Max}\{[fe + (1-f)r]L + [f(1-e) + (1-f)(1-r)]q^u X, qX\} & \quad (4) \\ & \geq [\lambda + (1-\lambda)(1+i)]I \end{aligned}$$

The first term in the maximand obtains when the country's project is adjusted after bad news ( $\phi^{dh} = \phi^{dc} = 1$ ): with probability  $[fe + (1-f)r]$  bad news will indeed occur, in which case the

<sup>4</sup> Notice that, given a certain fraction of liquidation, a good country (privately) knows its (gross) income without any uncertainty.

project will be liquidated and repayment will equal  $L$ . With probability  $[f(1-e) + (1-f)(1-r)]$  there will be good news about the project, which will then be completed and yield expected repayments of  $q^u X$ . The second term in the maximand ( $qX$ ) obtains when the project is always completed, no matter what date-one news are received.

Note that even if  $qX < [\lambda + (1-\lambda)(1+i)]I$ , the project may still attract financing. In this case, if the project is undertaken and investors wait for its full completion, they will lose money -in expected value. Yet, if the project is carried out to its full completion only if there are good news, but is adjusted after bad news ( $\phi^{dn} = 1$ ), it will yield expected repayments sufficient to attract financing.

We now present the optimal financial contract for different sets of conditions about future information. First, we examine what happens when the only new information that will arrive prior to the maturity of the project is whether or not interest rates rise. Thus, the following proposition (whose proof is in the appendix) characterizes the optimal complete financial contract when future information will not reduce the asymmetry of information between the country and its creditors.

PROPOSITION 1. *When future news will not reduce the asymmetry of information between the country and its investors regarding the country's type ( $e = r$ ), the optimal financial contract establishes that:*

*i) The country's project suffers no adjustment, no matter what date-one information arrives,  $\phi^{sn} = 0$  for  $s = d, u$ ;  $n = h, c$ .*

*ii) The repayments due at the completion of the project do not depend on date-one information,  $R^{sn} = [\lambda + (1-\lambda)(1+i)]I/q$ , for  $s = d, u$ ;  $n = h, c$ .*

When the asymmetry of information between the country and its outside investors will not be reduced, the optimal form of financing should guarantee that the country's project is never adjusted, and that the repayments remain the same no matter what news is received in the future. The reason for the first feature is simply that since the project was worth undertaking in the first place, it remains so due to the lack of new information on its nature. With regard to the second feature, since the country is risk averse and investors risk neutral -presumably because they hold a diversified portfolio- it is optimal to have investors bear the whole risk of variables not under the control of the country. This results in constant repayments across all states of nature.

We now examine what happens when, in contrast to the case analyzed in proposition 1, future news reduces the asymmetry of information between the country and its investors. The following proposition (whose proof is in the appendix) characterizes the optimal complete

financial contract under such circumstances. It is worth noting for future reference, when we analyze debt financing, that this proposition characterizes the optimal contract under two different cases. First, it does so far the case where the only future information refers to the country's project because there is no risk of a rise in interest rates. This is a particular case of the result stated in the proposition, when  $i = 0$ . Second, it does so far the case where both interest rates may rise ( $i > 0$ ), and future relevant information about the country's project will arrive.

PROPOSITION 2. *When future news reduces the asymmetry of information between the country and its investors, the optimal financial contract establishes that:*

i) *After good news, the country's project suffers no adjustment,  $\phi^{uc} = \phi^{uh} = 0$ ,*

ii) *If  $q^d X > L$ , the project suffers no adjustment after bad news either,  $\phi^{dc} = \phi^{dh} = 0$ , and repayments at date 2 depend on whether good or bad news arrives, but not on whether there is an increase in the interest rate,  $R^{uh} = R^{uc} < R^{dh} = R^{dc} < X$ .*

iii) *If  $q^d X < L$ , the project is canceled after bad news ( $\phi^{dc} = \phi^{dh} = 1$ ), and the repayments are such that the country's net income (after debt repayments) is lower after bad news than after good news, but do not depend on whether there is an increase in the interest rate,  $0 < L - L^{dc} = L - L^{dh} < X - R^{uc} = X - R^{uh}$ .*

News about the country's project provides two benefits: i) It allows a decrease in expected repayments from good countries by increasing those from bad countries, and ii) It helps to increase the quality of the surviving (unadjusted) projects, by adjusting projects that are most likely to be bad. To take advantage of the news, the optimal contract exhibits the following features:

i) The country has to repay higher amounts if there is bad news. Countries with good projects accept this burden in exchange for reduced repayments following good news, because they are less likely than countries with bad projects to obtain bad news.

ii) If future news regarding the quality of the project is bad enough and the cost of adjusting the project is not so high ( $L > q^d X$ ), the project is adjusted after bad news. Countries with good projects accept this risk because they are confident that future news will most likely show that their projects are good, and that by taking this risk they substantially reduce their financing costs. These costs are reduced because bad projects yield higher returns when they are adjusted than when they are not. Under competitive capital markets, this increase in repayments from bad projects translates into lower

repayments from good projects. Moreover, under some parameter constellations, i.e.,  $qX < I$  the existence of accurate date-one information that triggers the project's adjustment when it is bad is a necessary condition for the project to attract financing.

#### 4. Short-term and long-term debt

Consider now the case in which the country has access to only two financial instruments to raise the necessary funds to undertake the project. These instruments are short-term debt and long-term debt, as in Diamond (1991, 1993). The country can issue short-term debt with nominal value  $S_1$  that matures at  $t = 1$ , after interim news is released, and long-term debt with nominal value  $D$ , that comes due after the project is completed at  $t = 2$ . How can  $S_1$  be repaid if the project has not yield any income? By raising funds again in the credit market. At  $t = 1$  the country has access again to a competitive credit market in which it can raise funds promising up to  $(X - D)$ , that is, the part of the resources that will not be needed to repay long-term debt. Thus, at  $t = 1$ , the country issues short-term debt with face value  $S_2$  that comes due at  $t = 2$ . If the country is unable to raise the necessary funds to repay  $S_1$ , the country's project is adjusted. Figure 1 illustrates the functioning of the credit market.

We now need to establish several relationships between the different types of debt. First, let us find how much debt the country must issue at date 1 to borrow the necessary funds to repay  $S_1$ . This depends on whether good ( $s = u$ ) or bad ( $s = d$ ) news about the quality of the project arrives, and whether the interest rate remains at zero ( $n = c$ ), or rises to  $i > 0$  ( $n = h$ ). To borrow  $S_1$  at  $t = 1$ , the country must issue debt with face value  $S_2$  satisfying:

$$S_2 = S_1(1 + i)/q^u \quad \text{if } s = u, n = h$$

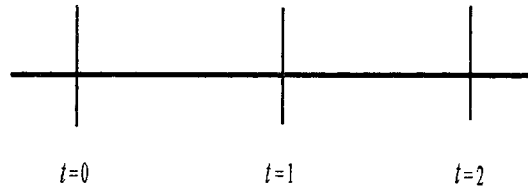
$$S_2 = S_1/q^u \quad \text{if } s = u, n = c$$

$$S_2 = S_1(1 + i)/q^d \quad \text{if } s = d, n = h$$

$$S_2 = S_1/q^d \quad \text{if } s = d, n = c$$

Figure 1

<i>Country borrows <math>I</math></i>	<i><math>S_1</math> matures</i>	<i><math>S_2</math> matures</i>
<i><math>S_1, D_2</math> issued</i>	<i><math>S_2</math> issued</i>	<i><math>D_2</math> matures</i>



To see why this is so, consider for instance the case  $s = u, n = h$  (the explanation for the other cases is similar). Since there is good news about the project ( $s = u$ ), financial markets expect that it will yield  $X$  with probability  $q^u$ . This is the probability with which the country will repay the debt  $S_2$ . Thus, the expected repayment from a debt with face value  $S_2$  is  $q^u S_2$ . Therefore, short-term debt  $S_2$  offers an expected (gross) rate of return of  $q^u S_2 / S_1$  which must equal the (gross) interest rate  $1 + i$ , from where  $S_2 = S_1(1 + i) / q^u$  follows.

Now, if default is avoided for all date-one news, financing through a mixture of short-term and long-term debt ( $S_1, D$ ) implies the following repayments at  $t = 2$ :

$$R^{uh} = D + S_1(1 + i) / q^u \quad (5)$$

$$R^{uc} = D + S_1 / q^u \quad (6)$$

$$R^{dh} = D + S_1(1 + i) / q^d \quad (7)$$

$$R^{dc} = D + S_1 / q^d \quad (8)$$

To see why consider, for instance, condition (5). The country must repay long-term debt with face value  $D$ , plus short-term debt with face value  $S_2$ . We obtain (5) by replacing  $S_2$  with the value it takes when  $s = u$  and  $n = h$ .

Let us now examine how the country should finance its investment using only short-term and long-term debt under different sets of conditions about future news. We will first consider the case in which adjustment of the project would be optimally avoided under complete contracting,  $L < q^d X$ . We will consider in subsection 4.4. the opposite case.

#### 4.1. *Financing when future news about the country's project is uninformative*

Consider the case in which future information will not reduce the asymmetry of information between the country and its creditors,  $e = r$ . We then have  $q^u = q^d = q$ .

Proposition 1 tells us that the optimum under complete contracting is achieved when financial liabilities guarantee that the project is never canceled and repayments are equal across all states of nature,  $R^{sn} = [\lambda + (1 - \lambda)(1 + i)]I/q$ , for  $s = d, u; n = h, c$ .

Thus, the optimum under complete contracting can be achieved with a mixture of short-term and long-term debt if a pair  $(S_1, D)$  exists, such that  $R^{sn} = [\lambda + (1 - \lambda)(1 + i)]I/q$  for all  $s$ , all  $n$ , where  $R^{sn}$  are given by conditions (5-8) and  $q^u = q^d = q$ . Thus, this optimum can be achieved (note that (7) and (8) become redundant) when:

$$D + S_1(1 + i)/q = [\lambda + (1 - \lambda)(1 + i)]I/q \quad (9)$$

$$D + S_1/q = [\lambda + (1 - \lambda)(1 + i)]I/q \quad (10)$$

which hold simultaneously for

$$D = [\lambda + (1 - \lambda)(1 + i)]I/q, \quad S_1 = 0. \quad (11)$$

Thus, even though only short-term and long-term debt are available, the country is able to replicate the optimal financial contract under complete contracting. The country accomplishes this by issuing only long-term debt.

It is worth comparing the interest rate on different kinds of financing. With long-term debt, the country raises  $I$  and promises to

pay  $[\lambda + (1 - \lambda)(1 + i)]I/q$ . Thus, the nominal interest rate on the country's long-term debt is  $[\lambda + (1 - \lambda)(1 + i)]/q - 1 > 0$ . Although the risk-free interest rate is zero for a short-term loan to be reimbursed at  $t = 1$ , the country has to pay an interest rate above zero on its long-term debt, for two reasons. First, the loan may not be repaid: This occurs when the project fails, which has a probability of  $(1 - q)$ , and accounts for the  $q$  term in the expression for the nominal interest rate. Second, interest rates may rise at  $t = 1$  to  $i$ , which occurs with probability  $(1 - \lambda)$  and explains the rest of the deviation of the country's long-term interest rate from zero.

Short-term debt is cheaper in the sense that a loan that is repaid for sure at date one pays an interest rate equal to zero, while a loan that will be repaid with certainty at  $t = 2$  pays an interest rate of  $(1 - \lambda)i$ . Yet, the proper comparison for the country is between issuing long-term debt and issuing short-term debt and refinancing it at  $t = 1$ . The country will pay zero interest rate on its short-term debt issued at date zero, but it will pay a (gross) interest rate of  $1/q$  with probability  $\lambda$  and  $(1 + i)/q$  with probability  $(1 - \lambda)$  on the debt it will issue at date one (to repay the debt maturing at this same date). Thus, the country will pay an expected net interest rate for short-term financing of  $\{[\lambda + (1 - \lambda)(1 + i)]/q\} - 1$ : the expected interest rate is the same for both short-term and long-term types of financing.

Summing up, although the interest rate on short-term debt is lower than that on long-term debt issued at the same date, the expected cost of both forms of financing are equal. The country thus optimally finances with pure long-term debt because it is risk-averse and, in contrast with short-term debt, long-term debt guarantees ensure completion of the project and a constant repayment schedule.

#### 4.2. *Financing without interest rate uncertainty*

Consider now the case in which future news will reduce the asymmetry of information between the country and its investors,  $r > e$ , but there is no interest rate uncertainty,  $i = 0$ . From proposition 2, we know that, under complete contracting, it would be optimal to set  $R^{uh} = R^{uc} < R^{ah} = R^{dc} < X$  if  $q^d X > L$ . This contract can be replicated with a mixture of short-term and long-term debt if there is a pair  $(S_1, D)$  that can produce the desired  $R^{sn}$  according to conditions (5-8). Under optimal complete contracting, date-two repayments should not depend on whether interest rates increase at date one. Let then

$R^{uh} = R^{uc} = R^u$ , and  $R^{dh} = R^{dc} = R^d$ . Conditions (5-8) then become:

$$D + S_1/q^u = R^u \quad (12)$$

$$D + S_1/q^d = R^d \quad (13)$$

These two equations have a unique solution in  $(S_1, D)$ , and this solution replicates the contract that is optimal under complete contracting. We have:

$$S_1 = \frac{R^d - R^u}{\frac{1}{q^d} - \frac{1}{q^u}} > 0, \quad (14)$$

$$D = \frac{q^u R^u - q^d R^d}{q^u - q^d} \quad (15)$$

This means that it is optimal to use short-term, refinanciable debt. The reason is that this debt, unlike long-term debt, makes the cost of funds depend on future news, and good borrowers know that they are more likely than bad borrowers to get good news. Thus, it is optimal to use short-term debt because it decreases expected repayments from good borrowers by raising expected repayments from bad borrowers.

#### 4.3. *Financing with two types of future information*

Consider now the case in which at date one new information regarding both the country's project and interest rates will arrive,  $r > e$  and  $i > 0$ . According to proposition 2, under complete financial contracting it is optimal not to make date-two repayments contingent on interest rate increases. We thus have  $R^{uh} = R^{uc} = R^u$ , and  $R^{dh} = R^{dc} = R^d$ . Equations (5-8) transform into

$$D + S_1(1+i)/q^u = R^u \quad (16)$$

$$D + S_1/q^u = R^u \quad (17)$$

$$D + S_1(1+i)/q^d = R^d \quad (18)$$

$$D + S_1/q^d = R^d \quad (19)$$

and there is no pair  $(S_1, D)$  that simultaneously solves these four equations. To see why, suppose the contrary. Then, (16) and (17)



imply  $S_1 = 0$ . Yet, substituting  $S_1 = 0$  in (17) and (19) we get  $R^u = R^d$ , a contradiction. So, a mixture of short-term and long-term debt, even if optimally designed, is not able to achieve what complete financial contracts can. We now look for the optimal debt contract. It is the pair  $(S_1, D)$  that solves

Program (2):

$$\begin{aligned} & \text{Maximize } e(1 - \lambda)u(X - S_1(1 + i)/q^d - D) + e\lambda u(X - S/q^d - D) \\ & + (1 - e)(1 - \lambda)u(X - S(1 - i)/q^u - D) + (1 - e)\lambda u(X - S/q^u - D) \\ & \text{s. t.} \\ & \quad D[f + (1 - f)\pi] + S_1[\lambda + (1 - \lambda)(1 + i)] \quad (20) \\ & \quad \geq [\lambda + (1 - \lambda)(1 + i)]I \end{aligned}$$

As is shown in the appendix, the solution of this program lies at  $S_1 > 0$ : It is optimal to finance with positive short-term debt. To see why, suppose that the country finances only with long-term debt. Then, the country retains the same income for all realizations of date-one news. As a result, the marginal utility of net income is the same across all realizations of date-one news. Yet, at this point, an increase in short-term debt increases the repayments a “good” country makes in bad states by less than what it reduces the repayments it makes in good states. The reason is, just as before, that short-term debt decreases expected repayments from good borrowers by raising expected repayments from bad borrowers. Therefore, it is optimal to finance (at least partially) with short-term debt.

#### 4.4. Financing when it is optimal to adjust the project

Up to now we have examined cases in which the mixture of short-term and long-term debt does not cause any adjustment in the project. Indeed, in all the cases studied so far, we have restricted our attention to situations in which the country is able to repay the short-term debt that matures at date one by issuing new short-term debt that matures at date two. Yet, as we know from proposition two, this is not necessarily optimal. We now examine what happens when it is optimal to adjust the project at date one. We know from proposition 2 that when complete contracting is possible and  $q^d X < L$ , the optimal contract cancels the project after bad news ( $\phi^{dc} = \phi^{dh} = 1$ ), and sets

repayments that do not depend on whether interest rates rise and that satisfy  $0 < L - L^{dc} = L - L^{dh} < X - R^{uc} = X - R^{uh}$ . Let then  $R^{uc} = R^{uh} = R^u$  and  $L^{dc} = L^{dh} = L^d$ . Under optimal complete contracting, when the date one news about the project is good, the project is not adjusted and repayments are  $R^u < X$  regardless of news about interest rates. On the other hand, when date-one news about the project is bad, the project is adjusted but, again, repayments  $L^d < L$  do not depend on interest rates news.

We now examine whether we can replicate the above contract with a mixture of short-term and long-term debt. We will assume throughout that when the debt that comes due at date one cannot be repaid, that is, when the state  $s$  in  $\{d, u\}$  occurs at date 1 and  $S_1, D$  are such that  $S_1 > q_s(X - D)$ , the project is adjusted,  $\phi = 1$ , and creditors recover at date 2  $\min\{S_1(1 + i) + D, L\}$ . Under this (polar case) assumption, creditors can enforce repayment of the country's debt up to the country's full income.

#### 4.4.1. Absence of interest rate uncertainty

Assume first that there is no interest rate uncertainty,  $i = 0$ . The condition

$$S_1 > q^d(X - D) \quad (21)$$

must hold for the project to be adjusted after bad news ( $s = d$ ), as required by the optimal contract under complete contracting. This condition states that the country will be unable to meet the repayments  $S_1$  at date one when bad news about its project arrives.

On the other hand, the condition

$$S_1 \leq q^u(X - D) \quad (22)$$

must hold to avoid the adjustment of the project after good news: The country should be able to meet its date-one repayments  $S_1$  when date-one news about its project is good.

Given that conditions (21) and (22) hold, the repayments prescribed by optimal complete contracting will be replicated if the following two conditions hold:

$$D + S_1/q^u = R^u \quad (23)$$

$$\text{Min}\{D + S_1, L\} = L^d \quad (24)$$

Condition (23) is the same as condition (12), because both conditions refer to a situation where the project should continue as originally planned. To interpret condition (24) notice first that, given condition (21), the country cannot raise enough money to meet its financial obligations after bad news and, therefore, the project is canceled. As a result, the country's income is  $L$ . Condition (24) reflects the assumption that creditors can collect as much as they need from the country's income to enforce repayment of  $(D + S_1)$ . Since the optimal contract under complete contracting shows  $L^d < L$ , condition (24) implies that  $D + S_1 < L$  and is equivalent to

$$D + S_1 = L^d \quad (24')$$

Thus, the optimal contract under complete contracting can be replicated by a mixture  $(S_1, D)$  of short-term and long-term debt if conditions (21), (22), (23) and (24') hold. To see if this is the case, we can solve for  $(S_1, D)$  in equations (23) and (24'). These two equations yield the following unique solution:

$$S_1 = \frac{q^u(R^u - L^d)}{1 - q^u} \quad (25)$$

$$D = \frac{L^d - q^u R^u}{(1 - q^u)} \quad (26)$$

Thus, the optimal contract under complete contracting will be replicated if the pair  $(S_1, D)$  in (25) and (26) satisfies conditions (21) and (22).

Substituting  $(S_1, D)$  from (25) and (26) in condition (21) yields

$$\frac{q^u(R^u - L^d)}{1 - q^u} > q^d \left( X - \frac{L^d - q^u R^u}{1 - q^u} \right)$$

which is equivalent to

$$q^u R^u (1 - q^d) > q^d (1 - q^u) X + L^d (q^u - q^d) \quad (27)$$

Condition (27) holds because i)  $q^u R^u > L$  -since otherwise the investors' participation constraint would not be met, ii)  $L > q^d X$  and iii)  $L > L^d$ , as simple algebra shows.

On the other hand, the fact that  $R^u < X$ , together with condition (23), implies that  $D + S_1/q^u < X$ , from which condition (22) follows.

Summing up, when there is no interest risk uncertainty, debt contracting can replicate optimal complete contracting.

#### 4.4.2. Interest rate uncertainty

When there is interest rate uncertainty ( $i > 0$ ), the optimal contract under complete contracting isolates the country from interest rate variations so that repayments remain constant across different realizations of interest rates:  $R^{uc} = R^{uh} = R^u$  and  $L^{dc} = L^{dh} = L^d$ . Thus, in addition to satisfying conditions (21) and (22) to ensure the optimal continuation/adjustment rule, we need

$$D + S_1/q^u = R^u \quad (28)$$

$$D + S_1(1 + i)/q^u = R^u \quad (29)$$

$$D + S_1 = L^d \quad (30)$$

$$D + S_1(1 + i) = L^d \quad (31)$$

Yet, there is no pair  $(D, S_1)$  that simultaneously solves (28)-(31), because (28) and (29) imply  $S_1 = 0$  which, together with (30) and (31), implies that  $i = 0$ .

If we restrict ourselves to a mixture of short-term and long-term debt we find that there are two reasons to recommend the use of short-term debt. First, if we set  $S_1 = 0$ , condition (21) will not be satisfied and the project will not be adjusted after bad news. Second, even in the class of debt contracts that produce no adjustment, it is not optimal to set  $S_1 = 0$ . The proof and the intuition here are identical to the case when continuation of the project is always optimal: By using only long-term debt, the country's income would be the same for all realizations of date-one news. In such a situation, however, an increase in short-term debt would benefit a country with a good project, because it would make repayments contingent on news about the country's project. Since countries with good projects are more likely to get good news, the rise in the repayments they make after bad news is more than compensated by the reduction in their repayments after good news.

The discussion in section 4 has analyzed how and why the country should finance its project when it is restricted to using a mixture of short-term and long-term debt. The following proposition does not repeat the whole argument, but only states the optimality of the use of short-term debt.

PROPOSITION 3. *If the country is constrained to finance its development project with a mixture of short-term and long-term debt  $(S_1, D)$ , then pure long-term debt is optimal if  $e = r$ . Otherwise, the optimal mixture of short-term and long-term debt exhibits  $S_1 > 0$ .*

### 5. Example

We now present a worked example to illustrate what the debt maturity structure may look like and how it might change under different circumstances, and to give an idea of the utility loss when complete contracts are not available.

Assume a utility function  $u(x) = \ln x$ , and the following parameter values:

$f = 1/2$ ,  $e = 1/4$ ,  $r = 1/2$ ,  $\pi = 1/2$ ,  $L = 1/8$ ,  $X = 15/8$ . These values imply  $q = 3/4$ ,  $q^d = 2/3$ , and  $q^u = 4/5$ .

i) Assume first that  $i = 0$ , so that there is no interest rate uncertainty.

Under complete contracting, the optimal financial contract establishes that the project should never be adjusted ( $\phi^d = \phi^u = 0$  and thus  $L^{dh} = L^{dc} = L^{uh} = L^{uc} = 0$ ), that repayments should not depend on interest rates, and that they should be higher after bad news about the project:  $R^{uh} = R^{uc} = 81/64 = 1.2652 < R^{dh} = R^{dc} = 47/32 = 1.4688 < X$ .

A mixture of short-term and long-term debt can replicate the above contract. It does so when  $S_1 = 13/16 = 0.8125$  and  $D = 1/4 = 0.25$ : Short-term debt is more than three quarters of total debt. This mixture of short-term and long-term debt replicates the optimal contract because it produces repayment obligations of:

$$D + S_1/q^u = 81/64 = 1.2652 \text{ after good news } (s = u), \text{ and}$$

$$D + S_1/q^d = 47/32 = 1.4688 \text{ after bad news } (s = d).$$

ii) Assume now that the interest rate will raise to  $i = 1/4$  with probability  $\lambda = 1/2$ .

Under complete contracting, the optimal financial contract sets again  $\phi^d = \phi^u = 0$  (and thus  $L^d = L^u = 0$ ), and  $R^{uh} = R^{uc} = 93/64 = 1.4531 < R^{dh} = R^{dc} = 51/32 = 1.59375 < X$ . Note that, even though there is interest rate uncertainty, the country's repayment obligations do not depend on interest-rate movements.

A mixture of short-term and long-term debt can no longer replicate the above contract. The optimal debt contract sets  $S_1 = 0.1991$  and  $D = 1.2014$ , with short-term debt representing about 14 percent of total debt. This mixture of short-term and long-term debt

produces repayment obligations that do depend on the behavior of interest rates:

$$R^{uh} = D + S_1(1+i)/q^u = 1.51243$$

$$R^{uc} = D + S_1/q^u = 1.4502$$

$$R^{dh} = D + S_1(1+i)/q^d = 1.5746$$

$$R^{dc} = D + S_1/q^d = 1.5$$

We can express the loss experienced by the country from having to rely on debt markets rather than on complete contracts as follows. Let  $H^{cc}(a)$  be the maximum utility the country can get when complete contracting is available and it has to guarantee investors an expected repayment of  $[\lambda + (1-\lambda)(1+i)]I + a$ . Let  $H^{DC}$  be the maximum utility the country can obtain with debt financing. This amount guarantees investors an expected repayment of  $[\lambda + (1-\lambda)(1+i)]I$ . Let  $a^*$  be such that  $H^{cc}(a^*) = H^{DC}$ . Then, in this example,  $a^* = 0.0028$ . So, the country would be willing to pay 0.0028, or 0.28 percent of the initial investment, to have access to a complete contract rather than to simple debt contracting. Thus, the welfare loss from not having access to a complete contract is not very big. On the other hand, the changes in the debt maturity structure are much bigger. While short-term debt is about three quarters of total debt when there is no uncertainty in interest rates, it is less than a sixth when interest rates may rise.

## 6. Conclusions

We have analyzed a model in which a risk-averse country needs to finance its development project under adverse selection. Before the project matures there will be two types of news. One of them refers to the country's project, and may reduce the asymmetry of information between the country and its investors. The other one refers to a variable that does not have a direct link to the country's project, nor is it under the control of the country, international interest rates. The project can be adjusted at a loss after the news just mentioned become known.

We have examined the optimal way to finance the project both under complete financial contracting, and when the country is restricted to using only short and long-term debt. We have found the characteristics that a financial contract should have to make optimal use of future information. When the country is limited to using only short and long-term debt, it is in general unable to replicate the contract that is optimal under complete contracting. In this situation, the country finds it in its interest to use short-term debt, except in the case in which future news is completely uninformative about the country's project.

We have identified three main reasons why a country should use short-term debt.

First, the country may simply not be able to attract long-term financing, and may instead have to resort to short-term debt. This occurs in our model when  $qX < I$ . In this case, investors perceive a small probability of the project succeeding, either because they assign too low a probability that the country has a good project, or because good projects do not have a high enough rate of return to compensate for the fact that the project may not be worth undertaking after all. Thus, if investors had to lend the amount required for the investment and wait until the completion of the project for their returns, they would expect to lose money, and therefore they do not finance the project. Yet, short-term debt raises the expected amount investors receive. This is because informative news will arrive at a time when it is possible to adjust the project in a way that avoids the worst-case scenario, and ensures the recovery of a large amount of the initial investment. We are dealing here with projects for which future news will be accurate and at the time they arrive adjustments that ensure the recovery of substantial amounts of the initial investment are available.

Second, even when the country is able to finance with pure long-term debt, short-term debt could still be optimal. By issuing short-term debt, the country effectively conditions the survival of its development project as originally planned on the realization of future news. If bad news arrives in the future, the country will not be able to refinance its debt and will have to accept some form of intervention that will result in an adjustment of its project so that it provides more certainty to investors, for instance, a stabilization policy. Countries with good projects accept the risk of suffering the loss due to the possible adjustment because they are confident that future news will most likely show that their projects are indeed good, and that they will thus be able to raise the cash needed to repay its short-term debt

and avoid any adjustment. The risk of adjustment is therefore small when compared with the substantial reduction in debt payments allowed by the use of short-term debt. This reduction occurs because bad projects yield better outcomes if they are intervened upon before their completion. This timely intervention translates into higher repayments from bad projects and, under competitive credit markets, ultimately translates into lower repayments from good borrowers.

Third, and closely related to the previous reason, a country should use short term debt because it allows the expected interest-rate to depend on future news about its project. Countries with good projects are confident that financial markets' future assessments of their projects will be good and that, therefore, they will be able to refinance their debt at better terms.

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## Appendix

### Proof of propositions 1 and 2

The optimal financial contract chooses  $\phi^{sn}$ ,  $R^{sn}$  and  $L^{sn}$ , for  $s = u, d$ , and  $n = c, h$ , so as to solve program (1):

$$\begin{aligned} \text{Max } H = & e(1 - \lambda)u((1 - \phi^{dh})X - R^{dh} + \phi^{dh}L - L^{dh}) \\ & + e\lambda u((1 - \phi^{dc})X - R^{dc} + \phi^{dc}L - L^{dc}) \\ & + (1 - e)(1 - \lambda)u((1 - \phi^{uh})X - R^{uh} + \phi^{uh}L - L^{uh}) \\ & + (1 - e)\lambda u((1 - \phi^{uc})X - R^{uc} + \phi^{uc}L - L^{uc}) \end{aligned}$$

subject to

$$[fe + (1 - f)r][(1 - \lambda)(q^d R^{dh} + L^{dh}) + \lambda(q^d R^{dc} + L^{dc})] \quad (3)$$

$$+[f(1 - e) + (1 - f)(1 - r)][(1 - \lambda)(q^u R^{uh} + L^{uh}) + \lambda(q^u R^{uc} + L^{uc})]$$

$$\geq [\lambda + (1 - \lambda)(1 + i)]I$$

and

$$0 \leq R^{sn} \leq (1 - \phi^{sn})X, \quad 0 \leq L^{sn} \leq \phi^{sn}L, \quad 0 \leq \phi^{sn} \leq 1 \text{ for } s = d, u, \text{ and } n = c, h.$$

i) At an optimum, constraint (3) holds with equality. Otherwise, we could decrease either  $R^{dh}$  or  $L^{dh}$ , still satisfy this constraint, and improve the objective function.

ii) If at an optimum  $\phi^{sn} < 1$ , then  $L^{sn} = \phi^{sn}L$ , for  $s = u, d$  and  $n = h, c$ . For ease of exposition, let us prove this assertion for  $s = u$  and  $n = h$  only. The proofs for the other cases are similar. Since at an optimum (3) holds with equality, solve then for  $R^{uh}$  as a

function of the rest of the decision variables, replace it in the objective function and ignore constraint (3). Call the new objective function (which does not have  $R^{uh}$  as an argument)  $V(\cdot)$ . We obtain:

$$\frac{\partial V}{\partial L^{uh}} = (1-e)(1-\lambda)u'(\cdot) \left( -1 + \frac{1}{q^u} \right) > 0.$$

Thus, the constraint  $L^{uh} \leq \phi^{uh}L$  is binding.

iii) At an optimum

$$\phi^{uh} = \phi^{uc} = 0.$$

To see why, set  $\phi^{uh} < 1$ , and replace  $L^{uh}$  by  $\phi^{uh}L$ . Let  $W$  be the function  $V(\cdot)$  when  $L^{uh}$  is replaced by  $\phi^{uh}L$ . We have

$$\frac{\partial W}{\partial \phi^{uh}} = (1-e)(1-\lambda)u'(\cdot) \left( -X + \frac{L}{q^u} \right) < 0$$

since  $L < q^u X$ . Thus,  $0 < \phi^{uh} < 1$  is not optimal. Simple computations show that  $\phi^{uh} = 1$  is not optimal either. Thus,  $\phi^{uh} = 0$  is optimal. The proof for  $\phi^{uc}$  is similar and thus omitted for brevity.

iv) At an optimum

$$\phi^{dh} = \phi^{dc} = 0 \text{ if } L < q^d X$$

$$\phi^{dh} = \phi^{dc} = 1 \text{ if } L > q^d X$$

The proof is similar to that of  $\phi^{uh} = \phi^{uc} = 0$ . But, since  $L$  may be higher or lower than  $q^d X$ , we may have  $\phi^{dh} = \phi^{dc}$  equal to 0 or 1.

v) If  $L < q^d X$ , then  $\phi^{sn} = 0$  is optimal for all  $s$ , all  $n$ . Replacing these values in program (1), we get  $L^{sn} = 0$  for all  $s$ , all  $n$ . Let  $R^{uh}$  be implicitly defined by (3). By using again the function  $V$  as defined above, we have the first-order condition

$$\begin{aligned} \frac{\partial V}{\partial R^{dh}} &= -e(1-\lambda)u'(X - R^{dh}) \\ &+ (1-e)(1-\lambda)u'(X - R^{uh}) \frac{fe + (1-f)r\pi}{f(1-e) + (1-f)(1-r)\pi} = 0 \end{aligned}$$

(some computations show that second-order conditions for a maximum are satisfied) from where

$$\frac{u'(X - R^{dh})}{u'(X - R^{uh})} = \frac{fe(1-e) + (1-f)r(1-e)\pi}{fe(1-e) + (1-f)e(1-r)\pi} = 0 \quad (A1)$$

Now, the left hand side of (A1) is greater than one when  $r > e$ , and is equal to one when  $r = e$ . Thus,

If  $r > e$ , then  $u'(X - R^{dh}) > u'(X - R^{uh})$ , which implies that  $R^{uh} < R^{dh}$ .

If  $r = e$ , then  $u'(X - R^{dh}) = u'(X - R^{uh})$ , and  $R^{uh} = R^{dh}$ .

On the other hand,

$$\begin{aligned} \frac{\partial V}{\partial R^{dc}} &= -e\lambda u'(X - R^{dc}) \\ &+ \frac{(1-e)(1-\lambda)u'(X - R^{uh})[fe + (1-f)r\pi]\lambda}{[f(1-e) + (1-f)(1-r)\pi][1-\lambda]} = 0 \end{aligned}$$

implies

$$\frac{u'(X - R^{dc})}{u'(X - R^{uh})} = \frac{fe(1-e) + (1-f)r(1-e)\pi}{fe(1-e) + (1-f)e(1-r)\pi} \quad (A2)$$

From (A1) and (A2), it follows that  $u'(X - R^{dc}) = u'(X - R^{dh})$ , which implies  $R^{dc} = R^{dh}$ .

We also have that

$$\begin{aligned} \frac{\partial V}{\partial R^{uc}} &= -(1-e)\lambda u'(X - R^{uc}) \\ &+ \frac{(1-e)(1-\lambda)u'(X - R^{uh})\lambda}{(1-\lambda)} = 0 \end{aligned} \quad (A3)$$

which implies  $u'(X - R^{uc}) = u'(X - R^{uh})$ , and thus  $R^{uc} = R^{uh}$ .

vi) Consider now the case  $L > q^d X$ . Then,  $\phi^{dh} = \phi^{dc} = 1$ ,  $\phi^{uh} = \phi^{uc} = 0$ ,  $R^{dh} = R^{dc} = 0$ ,  $L^{uh} = L^{uc} = 0$ , and using again the function  $V$  we have

$$\begin{aligned} \frac{\partial V}{\partial R^{uc}} &= -(1-e)\lambda u'(X - R^{uc}) \\ &+ \frac{(1-e)(1-\lambda)u'(X - R^{uh})\lambda}{(1-\lambda)} = 0 \end{aligned} \quad (A4)$$

which implies  $u'(X - R^{uc}) = u'(X - R^{uh})$ , and thus  $R^{uc} = R^{uh}$ .  
We also have

$$\frac{\partial V}{\partial L^{dc}} = -e\lambda u'(L - L^{dc}) + \frac{(1-e)(1-\lambda)u'(X - R^{uh})[fe + (1-f)r]\lambda}{[f(1-e) + (1-f)(1-r)\pi][1-\lambda]} = 0$$

from where

$$\frac{u'(L - L^{dc})}{u'(X - R^{uh})} = \frac{fe(1-e) + (1-f)(1-e)r}{fe(1-e) + (1-f)(1-r)e\pi} \quad (A5)$$

Now, the left hand side of (A5) is higher than one, because:

$$(1-e)r = r - re > (e-re)\pi = (1-r)e\pi.$$

Thus,  $u'(L - L^{dc}) > u'(X - R^{uh})$ , which implies  $L - L^{dc} < X - R^{uh}$

On the other hand,

$$\frac{\partial V}{\partial L^{dh}} = -e(1-\lambda)u'(L - L^{dh}) + \frac{(1-e)(1-\lambda)u'(X - R^{uh})[fe + (1-f)r]}{[f(1-e) + (1-f)(1-r)\pi]} = 0$$

which implies

$$\frac{u'(L - L^{dh})}{u'(X - R^{uh})} = \frac{fe(1-e) + (1-f)(1-e)r}{fe(1-e) + (1-f)(1-r)e\pi} \quad (A6)$$

(A5) and (A6) together imply that  $L^{dh} = L^{dc}$ . ■

**Proof that  $S_1 > 0$  in the optimal contract that solves program 2**

The optimal contract under debt financing chooses  $(S_1, D)$  so as to solve program 2:

$$\text{Max } G(S_1, D) = e(1-\lambda)u(X - S(1+I)/q^d - D)$$

$$+e\lambda u(X - S/q^d - D) + (1 - e)(1 - \lambda)u(X - S(1 - i)/q^u - D)$$

$$+(1 - e)\lambda u(X - S/q^u - D)$$

s. t.

$$D[f + (1 - f)\pi] + S_1[\lambda + (1 - \lambda)(1 + i)] \quad (20)$$

$$\geq [\lambda + (1 - \lambda)(1 + i)]I$$

Note first that at the optimum constraint (20) holds with equality. Otherwise, we could reduce either  $S_1$  or  $D$  and increase the objective function.

Thus, (20) implicitly defines  $D$  as a function of  $S_1$ . Let  $G(S_1)$  be the objective function when  $D$  is replaced by such a function. We have that, at  $S_1 = 0$ :

$$\frac{\partial G}{\partial S_1} = u'(X - D)[\lambda + (1 + \lambda)(1 + i)] \quad (A7)$$

$$\left[ \frac{1}{f + (1 - f)\pi} - \frac{e}{q^d} - \frac{(1 - e)}{q^u} \right]$$

(A7) is strictly positive if the last bracketed term is strictly positive. Now, when  $r = e$ ,  $q^d = q^u = f + (1 - f)\pi$ , and thus this term is zero. But, if  $r > e$ , then this term is positive, because it increases in  $r$ . Indeed,

$$\begin{aligned} \frac{\partial \left[ \frac{e}{q^d} + \frac{(1 - e)}{q^u} \right]}{\partial r} &= \frac{e^2 f(1 - f)(1 - \pi)}{[fe + (1 - f)r\pi]^2} \\ &\quad - \frac{f(1 - f)(1 - e)^2(1 - \pi)}{[f(1 - e) + (1 - f)(1 - r)\pi]^2} < 0 \end{aligned}$$

holds if  $e < r$ , as some algebra shows. Thus, the optimal mixture of  $(S_1, D)$  implies issuing a positive amount of short-term debt. ■