BALANCED BUDGET MULTIPLIER
WITH INDIRECT TAXES
UNDER IMPERFECT COMPETITION

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Resumen: Se presentan dos contraejemplos a las propiedades keynesianas atri-
buidas a la competencia imperfecta en modelos de equilibrio general.
En particular, bajo los dos tipos habituales de impuestos indirectos, se
obtiene una relación no positiva y no creciente entre las magnitudes del
multiplicador con presupuesto equilibrado ý del bienestar con respecto
al poder de mercado.

Abstract: This paper presents two counter-examples to the Keynesian features
attributed to imperfect competition in general equilibrium models. In
particular, by considering indirect tax rates, a non possitive and mono-
tonically non-increasing relationship between the magnitude of both
the balanced budget and welfare multipliers and market-power is ob-
tained.

1. Introduction

General equilibrium models with imperfect competition have been used as an explanation of some Keynesian features with fully flexible prices. In this line, papers such as Hart (1982), Blanchard and Kiyotaki (1987), Dixon (1987) and, Mankiw (1988), among others, explore the effect of different market power settings on the macroeconomic multipliers, reaching a positive and monotonically increasing relationship between the balanced budget multiplier and the degree of market power. A common setup of these models is that the government can resort to profits or lump-sum taxation to balance its budget. In this framework imperfect competition works as the only source of inefficiency which generates a space for public intervention. This insight is supported on the basis that fiscal policy does not distort relative prices in the margin. This statement calls into question whether these Keynesian features of the multiplier remain unchanged under distortionary tax schemes. Within this trend, Molana and Moutos (1992), and Heijdra, Ligthart and Ploeg (1998) find non-positive multipliers for labor income tax rates, whereas Torregrosa (1998), for the same tax rates, proves that this multiplier can be monotonically decreasing with respect to market power.

Considering this point of departure, this paper deals with the relationship between the balanced budget multiplier and market power, for indirect (ad-valorem and excise) tax rate schemes, providing another counter-example to the Dixon-Mankiwś monotonicity result.

The paper is structured as follows. In section 2, the model is presented and both the output and the welfare multipliers are calculated in their general form. Sections 3 and 4 develop these multipliers for both the ad-valorem and excise tax rates respectively. Finally, section 5, summarizes with the final comments.

2. The Model and Multipliers

Let us consider an economy formed by two commodities: leisure (considered as the numèraire) and a composed commodity produced from labor; $n + 2$ independent agents: the representative consumer, the government and n non-competitive firms. The former two agents constitute the demand-side of the economy and the latter the supply-side, according to the following assumptions:

(i) Household preferences are represented by a separable utility function. On the one hand, a Cobb-Douglas sub-utility function over

consumption of the produced good (C) and leisure (L) and, on the other, a sub-utility function over the publicly-provided good (g)

$$u(C, L, g) = C^\alpha L^{1-\alpha} + \beta(g), \qquad (1)$$

where $\alpha \in (0, 1); \beta(0) = 0, Beta' > 0$ and $\beta'' \le 0$. Let us denote by T the endowment of time, p the price of the produced commodity, and π the total profits of the firms. The household budget constraint is given by

$$pC = T - L + \pi. \qquad (2)$$

Consumer's choice is related only to C and L. Thus, the solution for the maximization of (1) subject to (2) is

$$C = \frac{\alpha(T + \pi)}{p} \qquad (3)$$

$$L = (1 - \alpha)(T + \pi) \qquad (4)$$

(ii) The government's role is modeled in the usual Keynesian fashion: a (indirect) tax revenue R is used to finance the amount g of government purchases. Thus, given the price p, the government budget's constraint is

$$R = pg \equiv G \qquad (5)$$

Adding equations (3) and (5), the total expenditure in the economy is given by

$$Y = \alpha(T + \pi) + G, \qquad (6)$$

which represents the demand side of the economy.

(iii) The industry is formed by n non-competitive firms producing an amount q_j $(j = 1, 2, ...n)$ of output from labor. Without loss of generality, let us assume the simple constant returns technology $q_j = N_j$ $(N = \sum_1^n N_j)$. It is also assumed that the labor market is competitive and firms' choices are independent, although households are the firms' owners. Then the goal of the representative firm is to maximize

$$pq_j - q_j - R_j \qquad (7)$$

where R_j represents the amount of taxes levied on the jth firm and

$$R = \sum_1^n R_j, \qquad (8)$$

refers to total tax revenue. Section 3 is devoted to the ad-valorem tax rate case, where $R_j = tpq_j$ with $0 \leq t < 1$, while section 4 is concerned with the excise tax rate case $R_j = tq_j$ with $0 \leq t$.

The first order condition for equation (7) can be written as

$$p(1 - \mu) = 1 + \frac{dR_j}{dq_j}, \qquad (9)$$

where $\mu \in (0, 1)$ is interpreted as an ad-valorem measure of market power: when μ tends to one, industry behaves as monopolist (perfect collusion); when μ tends to zero, firms behave as Bertrand oligopolists (perfect competition); when μ equals $1/\varepsilon n$, where ε is elasticity of demand, firms behave "à la Cournot". Finally, given the better firm's choice q_j^* which fulfills equation (9), the supply-side of the economy is represented by total output

$$Q = \sum_1^n q_j^*, \qquad (10)$$

and aggregated profits in the economy equal

$$\pi = \sum_1^n \pi_j, \qquad (11)$$

and here $\pi_j = p(Q)q_j^* - q_j^* - R_j$ is the representative firm's profit in equilibrium. Finally, general equilibrium requires the usual market clearing condition

$$Y = pQ, \qquad (12)$$

which implies, according to equation (6), that

$$Q = g + \frac{\alpha(T + \pi)}{p}. \qquad (13)$$

Both π and p depend on g due to equations (5), (7) and (8). Then, differentiating equation (13) with respect to g, taking into account equation (3), the output balanced budget multiplier is

$$\frac{dQ}{dg} = 1 + \frac{\alpha}{p}\frac{d\pi}{dg} - \frac{C}{p}\frac{dp}{dg}. \tag{14}$$

The increase in output due to a raise in government purchases is affected, first, by an income effect through the change in profits and, second, by a price effect that results from an increase in the tax rate needed to finance the higher government purchases. Notice that equation (14) captures the main difference with the lump-sum taxation models quoted in section 1. In fact the first two terms of equation (14) are just the Dixon-Mankiw's multiplier. The last term adds the distortion due to indirect tax rates, whose effect is the opposite.

Finally, it is interesting to study the effect on welfare of this boost to the economy. Substituting equations (3) and (4) in equation (1) the indirect utility function is obtained

$$V(p, \pi, g) = \gamma(T + \pi)p^{-\alpha} + \beta(g),$$

where $\gamma = \alpha^{\alpha}(1 - \alpha)^{1-\alpha}$. Differentiating with respect to g and taking into account equation (3) we obtain

$$\frac{dV}{dg} = \gamma p^{-\alpha}\left(\frac{d\pi}{dg} - C\frac{dp}{dg}\right) + \beta', \tag{15}$$

which represents the impact of the balanced budget expansionary policy on welfare. As can be observed, the positive effect on welfare due to larger government purchases is diminished by the change in consumption. This change is motivated by a price increase and a decrease in profits both generated by the change in the tax rate. It is necessary to remark that this effect on welfare is the opposite to that predicted by Keynes. This is because, according to Keynes, the balanced budget expansionary policy should not cause changes in welfare.

The next sections are devoted to computing these effects for both the ad-valorem and the excise tax rates.

3. Balanced Budget Expansionary Policy Under Ad-valorem Tax Rates

In this case $R_j = tp(Q)q_j$ with $0 \leq t < 1$. Thus, according to equation (9), the equilibrium price is

$$p = \frac{1}{(1 - t)(1 - \mu)}. \tag{16}$$

Due to equations (5) and (8), equilibrium government purchases are given by

$$g = tQ,\qquad\qquad(17)$$

and aggregate profits by

$$\pi = \frac{\mu}{1-\mu}Q.\qquad\qquad(18)$$

In order to compute the balanced budget multiplier defined in equation (14), the variations on profits and price of the balanced budget expansionary policy must be calculated. First, from equation (18),

$$\frac{d\pi}{dg} = \frac{\mu}{1-\mu}\frac{dQ}{dg}.\qquad\qquad(19)$$

Second, to obtain the effect on price, let us start computing the effect of such an expansion on the tax rate consistent with the government's budget constraint given by equation (17), which is

$$\frac{dt}{dg} = \frac{1}{Q}\left(1 - t\frac{dQ}{dg}\right).\qquad\qquad(20)$$

Thus differentiating equation (16) with respect to g, and taking into account equation (20),

$$\frac{dp}{dg} = \frac{p}{(1-t)}\frac{1}{Q}\left(1 - t\frac{dQ}{dg}\right).\qquad\qquad(21)$$

Substituting equations (19) and (21) in equation (14), the output balanced budget multiplier equals zero, i.e.,

$$\frac{dQ}{dg} = 0.$$

This means that a balanced budget expansionary policy has no effects on output (employment). The explanation of this total crowding out effect, under ad-valorem tax rates, is that this boost to the economy increases both government's demand and prices, in such an amount that the decrease in consumption equals the increase in the government's demand. Hence, firms do not change either their output level or profits (substituting the result in equation (19), $\frac{d\pi}{dg} = 0$). The

effect on price is calculated differentiating equation (21) with respect to g, which is

$$\frac{dp}{dg} = \frac{p}{(1-t)}\frac{1}{Q} > 0.$$

This allows us to compute the effects of the balanced budget expansionary policy on the welfare. Then, substituting the multipliers in equation (15), taking into account equation (13) and (17) and operating, the following equality holds

$$\frac{dV}{dg} = \beta' - \gamma p^{1-\alpha}. \tag{22}$$

As can be seen, a balanced budget expansionary policy under ad-valorem tax rate affects welfare in two ways: a positive effect derived from the increase in government purchases, and a negative effect arising from the increase in price due to the increase in the tax rate. Finally, the direction in which the effect of the balanced budget expansionary policy on the welfare changes with respect to the degree of market power can be computed differentiating equation (22) with respect to μ

$$\frac{d\left(\frac{dV}{dg}\right)}{d\mu} = -\gamma(1-\alpha)p^{-\alpha}\frac{dp}{d\mu}.$$

Since, according to equation (16),

$$\frac{dp}{d\mu} = \frac{p}{1-\mu} > 0,$$

the effect of the balanced budget expansionary policy on welfare is monotonically decreasing with respect to the degree of market power.

4. Balanced Budget Expansionary Policy Under Excise Tax Rates

In this case $R_j = tq_j$ with $0 \le t < 1$.[1] Thus, according to equation (9), the equilibrium price is

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[1] Despite the fact that t can be greater than one, it is assumed that $t \le 1$ This is because the model is expressed in units of leisure. Thus, $t > 1$ would mean an excise tax rate higher than the current wage.This condition is also compatible with the fact that household expenditure is higher than public expenditure. Indeed,

$$p = \frac{1+t}{1-\mu}. \tag{23}$$

Due to equations (5) and (8), equilibrium government purchases are

$$g = \frac{tQ}{p} \tag{24}$$

and aggregate profits are

$$\pi = p\mu Q. \tag{25}$$

In order to calculate the balanced budget multiplier defined in equation (14), the variations on profits and price of the balanced budget expansionary policy must be calculated. First, from equation (25),

$$\frac{d\pi}{dg} = \mu \left(Q\frac{dp}{dg} + p\frac{dQ}{dg} \right). \tag{26}$$

Second, to obtain the effect on price, let us start computing the effect of such an expansion on the tax rate consistent with the government's budget constraint given by equation (24), that is

$$\frac{dt}{dg} = \frac{1}{Q} \left(g\frac{dp}{dg} + p - t\frac{dQ}{dg} \right). \tag{27}$$

Thus differentiating equation (23) with respect to g, and taking into account equation (27),

$$\frac{dp}{dg} = \frac{p}{Q} \left(p - t\frac{dQ}{dg} \right). \tag{28}$$

Substituting equations (26), (27) and (28) in equation (14), taking into account equation (24) and operating, the output balanced budget multiplier under excise tax rates can be written as

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the ratio between public expenditure and total expenditure is given, according to equations (12) and (17) by, $\frac{G}{Y} = \frac{t}{p}$ and the ratio between household expenditure and total expenditure is given, according to equations (2) and (6), by $\frac{pC}{Y} = 1 - \frac{t}{p}$. If $\forall \mu \in [0, 1)$ $pC > G$, then $1 - \frac{t}{p} > \frac{t}{p}$. Using equation (24), $\mu > \frac{t-1}{2t}$, but since $\mu \geq 0$, then $t \leq 1$.

$$\frac{dQ}{dg} = -\frac{(1-\alpha)\mu(1+t)^2}{(1-\mu)[1-(\alpha-(1-\alpha)t^2)\mu]} < 0, \qquad (29)$$

since $t \leq 1$ and $\mu < 1$. Equation (29) shows that the balanced budget expansionary policy has negative effects on output (employment). Let us show that this effect worsens when market power increases. Differentiating (29) with respect to μ and operating,

$$\frac{d(\frac{dQ}{dg})}{d\mu} = -\frac{(1-\alpha)(1+t)^2(1-(\alpha-(1-\alpha)t^2)\mu^2)}{(1-\mu)^2[1-(\alpha-(1-\alpha)t^2)\mu]^2} < 0. \qquad (30)$$

This result makes up a counter-example to the Dixon-Mankiw's monotonicity result, where, as competition in the goods market becomes less perfect, the balanced budget multiplier increases.

Finally let us calculate the effect of the balanced budget expansionary policy on welfare. Substituting equations (26), (27) and (28) in equation (14) and operating,

$$\frac{dV}{dg} = \gamma p^{-\alpha} \left((\mu + t)\frac{dQ}{dg} - 1 \right) + \beta', \qquad (31)$$

which shows two opposite effects, as was remarked upon in section 2. Differentiating equation (31) with respect to market power

$$\frac{d(\frac{dV}{dg})}{d\mu}$$

$$= \gamma \left((1-\alpha)p^{-\alpha}\frac{dp}{d\mu}[(\mu+t)\frac{dQ}{dg} - 1] + p^{1-\alpha}[\frac{dQ}{dg} + (\mu+t)\frac{d(\frac{dQ}{dg})}{d\mu}] \right)$$

$$< 0$$

since

$$\frac{dp}{d\mu} > 0, \quad \frac{dQ}{dg} < 0 \quad \text{and} \quad \frac{d(\frac{dQ}{dg})}{d\mu} < 0,$$

according to equations (23), (29) and (30). Thus, as in the ad-valorem tax rate case, the effect of the balanced budget expansionary policy on welfare is monotonically decreasing with respect to the degree of market power.

Conclusions

In this paper, both the monotonicity of the output multiplier and the effects on welfare of a balanced budget expansionary policy have been analyzed under the two main indirect taxes. The main contributions of this paper are: First, for the ad-valorem tax rate scheme, the output multiplier equals zero, which means that changes in public purchases have no effect on output, reaching a total crowding out effect independent of the degree of market power. The explanation of this crowding out effect is that the government's expansionary policy increases the price (through taxes) in such way that consumption falls in the same proportion as the increase in government purchases. With respect to the effect on welfare, this is monotonically decreasing with market power, which is opposite to that predicted by Keynes.

Secondly, in the case of an excise tax rate, the results are related to the market power in the opposite way to the conclusion reached by Dixon (1987) and Mankiw (1988) for non-distortionary taxation. The output balanced budget multiplier is negative and monotonically decreasing with respect to market power. The reason for this is that an increase in the excise tax rate distorts relative prices reducing output in greater proportion than the increase in public expenditure. This negative effect also increases as the degree of market power increases. Regarding welfare the effect of a government expansionary policy is monotonically decreasing with respect to market power as in the former case.

In conclusion, Keynesian features attributed to the general equilibrium models with fully flexible prices and imperfect competition depend almost entirely on the tax scheme considered. It is true that market power causes inefficiency but it is also true that some taxes might do so, too. In this sense, the government could be using an inefficient tool to amend a market failure.

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