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The Evolutionary Processes for the Populations of Firms and Workers

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Abstract

This paper analyzes the cultural evolution of firms and workers. Following an imitation rule, each firm and worker decides whether to be innovative (or not) and skilled (or unskilled). We apply evolutionary game theory to find the system of replicator dynamics, and characterize the low-level and high-level equilibria as Evolutionarily Stable Strategies (ESS) "against the field." Hence, we study how a persistent state of underdevelopment can arise in strategic environments in which players are imitative rather than rational maximizers. We show that when the current state of the economy is in the basin of attraction of the poverty trap, players should play against the field if they want to change their status quo. The threshold level to overcome the poverty trap can be lowered if there is an appropriate policy using income taxes, education costs and skill premia. Hence, we study the replicator dynamics with a subsidy and payoff taxation to overcome the poverty trap.

Keywords: Imitative behavior, conformism, poverty traps, skill premium, strategic complementarities.
JEL Classification: C72, C79, D83, O12.

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Resumen

En este trabajo se analiza la evolución cultural de trabajadores y firmas, como determinante de la evolución social de una economía dada. Siguiendo una conducta imitativa firmas y trabajadores optan por ser calificados (o no) o ser innovadores (o no). Las reglas implícitas en este proceso son modeladas mediante un sistema dinámico cuyas soluciones, dependiendo de condiciones iniciales, son las que determinan qué imitar y definen la evolución futura de la sociedad. Dicha evolución puede conducir a un equilibrio bajo y por una senda de bajo crecimiento, donde los trabajadores prefieren no ser calificados y las firmas, no ser innovadoras; o a un equilibrio alto, donde puede coexistir un porcentaje alto de trabajadores calificados y firmas innovadoras con un porcentaje de trabajadores no calificados y firmas no innovadoras. La senda que se transite depende de las condiciones iniciales existentes en la economía. El primero de los equilibrios es una trampa de pobreza, que aparece como un atractor local del sistema dinámico. Las trayectorias que definen la evolución, quedan identificadas por pares estratégicos sobre las mejores respuestas de las firmas y/o trabajadores en la elección del otro, conformando un proceso evolutivo de estrategias complementarias. Se introduce el concepto de estrategias evolutivamente estables (ESS) contra el campo, para analizar la persistencia de determinadas respuestas ante modificaciones en las condiciones estructurales de la economía. Finalmente, se muestra el papel del planificador central para ubicar a la economía en condiciones iniciales que la lleven, por sus propias leyes, a un equilibrio alto.

Palabras Clave: Comportamiento imitativo, trampas de pobreza, estrategias complementarias, premios al desempeño.
Clasificación JEL: C72, C79, D83, O12.

Introduction

The notion of strategic complementarities is widely studied. It is well understood and accepted that the complementarity between R&D (innovative firms) and human capital accumulation (skilled workers) is an engine of economic growth (Lucas, 1988). Early research about the complementarity of R&D and human capital was carried out by Nelson and Phelps (1966) and Schultz (1975), who pointed out that the major role of education is to adapt, and to generate new technologies; that is, to adapt technological changes generated by innovative firms. Redding (1996) formalized this idea using an R&D-based growth model developed by Aghion and Howitt (1999). He showed the existence of strategic complementarities between workers' investment in education and firms' investment in R&D, and then demonstrated how a development trap would result when both types of investment are inactive. More recent studies develop different models to prove that high-skilled labor and high-technology firms are complements; they require this proof in order to prove the existence of a high-level equilibrium, particularly see Acemoglu (1997, 1998). In fact, it is generally thought that new technologies reduce the demand for unskilled workers and increase the demand for skilled workers, since skilled workers adapt more easily to technological change. This is the well-known notion of "Skill-Biased Technological Change" which implies a shift in the production technology that favors skilled over unskilled labor by increasing skilled labor's relative productivity and, therefore, its relative demand (see Acemoglu, 2002; Aghion, 2006; Hornstein *et al*., 2005).

Moreover, a number of recent contributions have emphasized the role of skill resources as a crucial constraint on selecting the technological profile to implement in developing economies. Greenwood and Yorukoglu (1997), for instance, consider that adopting investment-specific technical change requires specific human capital, physical capital, and an increase in skilled labor. Hendricks (2000) develops a model of growth through technology adoption focusing on the complementarities between technologies and skills. Workers' skills and firms' technological profiles are complementary because the level of available skills limits the profile of technologies firms can use, while the technological profile determines the rate of learning. Benhabib and Spiegel (1994), by focusing on the role of human capital in economic development, suggest that human capital facilitates the adoption of technology from abroad and creates domestic technology. This evidence reinforces the importance of matching among the skills and the technological profile. There are two well known approaches. The first, more conventional, approach is the "Skill-biased Technological Change" approach (see papers by Berman *et al*. (1994) in the US, Haskel and Heden (1999) in the UK, and Machin and Van Reenen (1998) extend the approach to continental Europe) where the investments in R&D, new products, new processes, new technologies and even the ICT (Information and Communication Technologies) increase the firms' demand for skilled workers, assuming firms and workers know how to implement the new technologies (see the "absorptive capacity" by Cohen and Levinthal (1990, 131) where "an organization's absorptive capacity will depend on the absorptive capacities of its individual members"). Skilled labour is a necessary complement to R&D activities in reinforcing the absorptive capacity of a given firm and new technologies become more effective. The second theory supports the endogeneity of the phenomenon (referring to approaches that have complementarities), *i.e.*, the endogenous skill-biased suggesting that skilled workers are responsible for inducing firms to invest in new technologies, see Kiley (1999), and Funk and Vogel (2004).

In this vein, this paper studies the strategic complementarities between R&D and Human Capital. But our approach is quite different from the above references, since we analyze the evolutionary dynamics with an evolutionary game theoretical approach. Evolutionary games introduce two new points of view in the classical approach given by the game theory of the economic problems. The first one is the introduction of the concept of Evolutionarily Stable Strategy (ESS) and the second one is the evolutionary dynamic. The first one was introduced by Maynard Smith in 1982 in his book Evolution and the Theory of Games. This concept, basically, is a refinement of the Nash Equilibrium (NE) concept, indicating the stability of a strategy as a best response. An evolutionarily stable strategy is a best response that continuous being a best response, even after perturbations of the original conditions, Taylor (1979). The second point of view that evolutionary game theory introduces, refers to the evolution of system modeled as a game, and here the understanding of evolution, in evolutionary game theory, is not the usual as in repeated games and/or differential games. The evolution in this theory is modeled by a dynamical system, whose equations are determined by the behavioral rules existing in a given time, and in a given system, biological, social or economic, that is being modeled as a game. So under this approach we can consider an economy as an evolutionary system, where the behavioral rules of the individuals determine the evolution of the economy. These rules are mathematically modeled by a dynamical system, and under additional hypothesis, the solutions of this system are evolutionarily stable strategies. Moreover, evolutionary game theory does not explain the evolution of the economy for individuals or agents that are completely rational or rational maximizers.

Instead, evolutionary game theory takes into account that in some cases (in most of them) individuals do not act in a rational way, at least in the perspective of the economy as a whole trying to reach a social maximum benefit. Even, if the individuals understand that they need to do the best, at least for themselves, their end behavior, some times, leads the economy into a path of low economical performance or even caught in poverty trap. We know that standard growth theory teaches us that poverty traps are stable-low level balanced growth paths to which economies gravitate due to adverse initial conditions or poor equilibrium selection. In other words, economies fail to generate sustained growth because they started poor, as Azariadis and Drazen (1990) mention due to low longevity or poor human capital, or else because they cannot invent better institutions that coordinate their investments successfully. Therefore, in this paper, we explain this pernicious form of coordination as one equilibrium of an evolutionary game characterized by complementarities between firms and workers. Hence, our research question states that:

"From the basic ideas of a standard growth model of human capital and R&D, what happens if we drop the assumption of rational maximizers agents (or rational expectations) and apply imitation rules (or imitative agents) instead?"

Hence, we can analyze the dynamical stability of the solution of a dynamical system that represents the change in the behavior of the individuals, and at the same time the stability of this behavior is a strategy followed in the conflict. We find that the rationality of the economic agents, can be one of the main factors to maintain the economy in a path going to a low level of equilibrium, but in other cases this rationality can be an engine to overcome a situation of low performance, in particular a poverty trap. The role that rationality plays, depend on the initial conditions.

Therefore, we analyze a two-population game as a matching process between the economic agents that are firms and workers. First, we assume that firms have two strategies, innovative or non-innovative, depending on whether they invest in R&D. Workers' strategies are skilled or unskilled, depending on whether they invest in human capital or not. Wages are conditional on worker's type but are exogenously fixed in the model. There are two pure Nash equilibria: innovative firms with high ability workers and non-innovative firms with low ability workers. We analyze the game in an evolutionary setting with random matching between firms and workers. We consider imitative behavioral rules as the mobilizing force behind cultural evolution, Matsuy (1996). In this vein, we assume that the probability that an agent (either a firm or a worker) will imitate a successful agent is based on two factors: first, the probability that an agent meets a more successful agent, and, second, the probability that the agent will be better off if they adopt the successful agent's strategy, for more details refer to Apesteguia *et al.* (2007) and Schlag (1998, 1999). Therefore, we study an imitation game where the nature of interactions among agents creates a potential for multiple equilibria. These equilibria are characterized by different levels of "activity" (high-skilled workers, innovative firms) in the economy. A low-level equilibrium arises when non-innovative firm and low-skilled workers predominate. We show that when firms invest in R&D to become innovative, they are successful only in the presence of a sufficiently high number of high-skilled workers, following Redding (1996). At the same time, workers are only encouraged to improve their skills when a large number of firms invest in R&D. Conversely, firms that do not invest in R&D do not look for high-skilled workers, and therefore make the accumulation of skills unprofitable for workers. Hence, the distribution of the types of firms and workers changes over time. These changes can be understood as cultural evolution, where cultural evolution refers to changes in beliefs or behavior over time.

We argue that, this research is relevant because it shows that imitation[1] can drive economies toward prosperity or poverty depending on initial conditions and on exogenous shocks. The model suggests that imitation determines a worker's choice about education (or human capital accumulation) and a firm's decision to invest in R&D. Moreover, the model shows that the market cannot solve the problem itself. But, by imputing agency to individuals, or by giving power to a government (central planner) we can overcome the problem and use either collective action or central planning to overcome an equilibrium and move to another. This work therefore reinforces other works on market failures, while showing that implementable solutions still exist. The crucial parameters that encourage firms and workers to adopt innovation and skills are education or training costs, skill premia and income taxes as policies to create incentives for investment in R&D. To model evolution we maintain the perspective of evolutionary game theory with its models characterized by the replicator dynamics. This perspective allows to relax the strong rationality assumptions characteristic of classical game theory and to capture explicitly the dynamics of the decision-making process.

The structure of the paper is as follows. Section 1 presents the one-shot game. Section 2 presents the evolutionary game, stating the main definitions about evolutionarily stable strategies, population flows and the imitation rule to get the system of replicator dynamics. Subsection 2.3 studies the stability of the steady state using the replicator dynamics. Section 3 proposes how the poverty trap can be avoided when an agent faces the current economic environment, playing against the field. Subsection 3.1 analyzes the intervention of policy makers through an implementation of payoff taxation and subsidies to overcome the poverty trap. The last section concludes.

1. The Game

It is considered a model of two populations: workers, W, and firms, F, based on Accinelli *et al*. (2009). This is a normal form game with two-players: a worker and a firm. The potential worker decides whether to improve her skills or not. Though it seems initially counterintuitive to think that a skilled worker can become "unskilled," a loss of skill could depend on the type of skills or knowledge under consideration and how the change is motivated. Hence, we refer to the decision of being on the "knowledge frontier." We

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[1] As a driving force of cultural evolution.

assume that a worker's tendency, to be skilled or not, follows an imitation rule, this means that every worker knows other workers' economic conditions, and having imitated a high-skilled individual, takes a training course with a 100 percent probability of being successful.[2] Then, the firm decides whether to invest in R&D or not. If a firm invests in R&D it becomes innovative, otherwise it is not. Such decisions may depend on the current state of the economy. Thus if the economy is composed mainly of workers and firms with low-profiles, then workers and firms decide not to invest in education and R&D departments.

Hence, consistent with the literature, we consider an economy composed of two populations: workers, *W* and firms, *F*. Let us assume that each population is divided into two types:

• Occasionally, the *W*-population divides into the *S*-type that invests in improving their individual skills, who will be the skilled workers, and the *NS*-type corresponds to unskilled workers.

• Occasionally, the *F*-population divides into the *I*-type that invests in R&D or are innovative firms, and the *NI*-type of non-innovative firms.

The contractual period between types of firms and workers is described by the following characteristics:

• **Asymmetric information**. At the beginning of the contractual period, workers do not know the type of the firm that hired them. On the contrary, workers must certify their skill levels, for example they must show a CV, and therefore firms know the worker's type. A worker enters a firm without understanding if the company invests in R&D or not, but if an innovative firm is successful, then the worker gets a premium.

• **Training costs and Investment costs**. The worker has to pay an associated cost to become a high-skilled worker. That is, the *S*-type worker must invest in education by going to a training school, at an associated cost, *CE*. To become an innovative firm implies that the firm carries out investment in R&D departments at an associated cost, *CI.*

• **Gross income**. Let $B_i(j)$ be the gross-benefit of the *i*-th firm hiring the *j*-th worker, $\forall \;\; i = \{I, NI\}, \;\; j = \{S, NS\}$. For all firms, a *S*-type worker gets a salary $\bar{S}$, and a *NS*-type worker gets a salary salary $0 < S < \bar{S}$.

[2] But, of course, entering a training course is not a guarantee to pass it and to find a skill-adequate job.

• **Skill premia**.[3] The "skill premia hypothesis" assumes that innovative firms give premia to their workers at the end of the contractual period. Assume that the innovative firms, *I*, give premia to their workers, while *NI*-firms do not share these benefits.[4] Thus, skilled workers, *S*, receive a premium $\bar{p}$ and unskilled workers, *NS*, receive a premium p, $(0 < p < \bar{p})$, when both types work for an innovative firm, *I*.

Moreover, there are strategic complementarities between types of firms and workers.[5] Thus,

• If a firm is innovative, the payoff to a skilled worker is greater than the payoff to an unskilled worker, *i.e.*, $\bar{s} + \bar{p} - CE > s + p$.

• If a firm is non-innovative, then the payoff to an unskilled worker is at least as great as the payoff to a skilled worker, *i.e.*, $s \geq \bar{s} - CE$. This means that if there are no rewards, then there are no incentives to be a skilled worker.

• If a worker is skilled, then the payoffs obtained by an innovative firm are greater than those obtained by a non-innovative firm, *i.e.*, $B_I(S) - \bar{p} > B_{NI}(S)$.

• If a worker is unskilled, then the benefits for a non-innovative firm are greater than those for an innovative firm, *i.e.*, $B_I(NS) - p < B_{NI}(NS)$.

Table 1 presents a summary of the two population normal form game.

Table1
Payoff Matrix of the Game

W \ F	I	NI
S	$\bar{s} + \bar{p} - CE,\ B_I(S) - (\bar{s} + \bar{p} + CI)$	$\bar{s} - CE,\ B_{NI}(S) - \bar{s}$
NS	$s + p,\ B_I(NS) - (s + p + CI)$	$s,\ B_{NI}(NS) - s$

[3]Acemoglu (2003) provides a seminal contribution about skill premia.
[4]Recall that workers do not know the type of contracting firm. So, at the beginning of the productive process each worker does not know if she is going to receive a premium or not; this information is revealed only at the end of the period, once she learns the contracting firm´s type.
[5]Two references on strategic complementarities among types of firms and workers are: 1) Acemoglu (1999); when there is a sufficient fraction of workers who are skilled, firms find profitable to create jobs specifically targeted at this group, and as a result, unskilled wages fall and skilled wages increase. 2) Acemoglu (1998); as the economy accumulates more skills, technical change responds to make new technologies more complementary to skilled labor.

The expected payoffs of the *S*-type and *NS*-type worker, given the probabilities of being hired by either the *I*-type or *NI*-type firms are *E(S)* and *E(NS)* defined as follows:

$$E(S) = prob(I)\left[\overline{s} + \overline{p} - CE\right] + prob(NI)(\overline{s}) - CE \tag{1}$$

$$E(NS) = prob(I)\left[s + p\right] + prob(NI)(s) \tag{2}$$

Where *prob(I)* represents the worker's probability of being hired by an innovative firm and *prob(NI)* the probability of being hired by a non-innovative firm. Hence, workers prefer to be a *S*-type if *E(S)> E(NS)*, and, conversely, they prefer to be a *NS*-type *if E(NS) > E(S).* Then, *E(S) > E(NS)* if and only if *prob(I)* is sufficiently large *i.e.*

$$prob(I) > \frac{CE - (\overline{s} - s)}{(\overline{p} - p)} \tag{3}$$

Workers are indifferent between being skilled or unskilled if and only if[6]

$$prob(I) = \frac{CE - (\overline{s} - s)}{(\overline{p} - p)} \tag{4}$$

Let us denote a firm's probability to employ a skilled worker by *prob*(S) and the probability to employ an unskilled worker by *prob (NS)*. Hence, a firm prefers to be innovative if and only if the expected payoff of being innovative is greater than the expected payoff of being non-innovative, *i.e.*, *E(I) > (E(NI)* or,

$$prob(S) > \frac{B_I(NS) - B_{NI}(NS) - p - CI}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) + \left(\overline{p} - p\right)} \tag{5}$$

Let us call $prob(I) = \overline{y}_I$ and $prob(S) = \overline{x}_s$. Therefore, the game has three Nash equilibria. Two in pure strategies: (*I,S*) and (*NI,NS*) and a mixed strategy Nash equilibrium given by

$$NE = \left(\overline{x}_s, (1-\overline{x}_s);\ \overline{y}_I, (1-\overline{y}_I)\right) \tag{6}$$

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[6]Note that, $0 < \frac{CE-(\bar{s}-s)}{(\bar{p}-p)} < 1$ holds due to the above assumptions of strategic complementarities.

Hence, a threshold level where both firms and workers prefer to be high-profile (innovative and skilled) is above $\left(\bar{x}_s, \bar{y}_I\right)$.

2. The Evolutionary Game

A key concept in evolutionary game theory is the Evolutionarily Stable Strategy (ESS), for more details see Maynard Smith and Price (1973), Maynard Smith (1974, 1982) and Taylor (1979). An evolutionary stable strategy must have the property of noninvadability, that is, if almost every member of the population adheres to the (ESS), no mutant[7] can successfully invade. This concept is relevant only for monomorphic populations, but it can be extended to polymorphic populations. It corresponds to a stable state in which only pure strategies, or behaviors, can be played, but in which different players with different strategies can co-exist; such that, every change in the distribution of strategies implies a reduction in the expected value of fitness or welfare for the population as a whole.

Originally, this concept was developed in the framework of a symmetric normal form game. In order to apply the (ESS) concept to asymmetric games, we can think of agents being randomly assigned, with equal probabilities, to be either player i or player j. The agents are informed of the role to which they have been assigned and are allowed to condition their choice upon this assignment.

Let us briefly state the notion of ESS that we follow in this paper. Hence, we are interested in the evolution and in the stability of both populations, firms F and workers W playing the game. We analyze if small changes in the individuals' behavior from one population imply changes in the behavior of the individuals from the other population.

Let $p \in \Delta^F$ be the profile distribution of firms' behavior from population F in a given period t_0, and that in the same period the profile distribution of workers' behavior in population W is $q \in \Delta^W$. Assume that in $t_1 > t_0$ a small mutation affects the workers' behavior from population W. Hence, the profile distribution from population W after the mutation, is given by the number of offspring:

[7] A mutant is an individual who adopts a novel strategy.

$$q_{\varepsilon} = ((1-\varepsilon)q + \varepsilon\overline{q} \tag{7}$$

for any $\bar{q} \in \Delta^W$: $\bar{q} \neq q$, the new distribution q_{ε} is called the fitness of the post-entry population. Analogously, the profile distribution from population F after a small mutation is:

$$p_{\varepsilon} = (1-\varepsilon)p + \varepsilon\overline{p} \tag{8}$$

for any $\bar{p} \in \Delta^F$: $\bar{q} \neq p$. Now, we can state the following definition:

Definition 1. Let $(p^*, q^*) \in \Delta^F \times \Delta^W$ be a profile of mixed strategies. We say that the profile (p^*, q^*) is an ESS for an asymmetric two-population normal form game, if for each $(\overline{p}, \overline{q}) \in \Delta^F \times \Delta^W$ there exists $\overline{\varepsilon}$ such that:

$$\begin{aligned} &1)\quad E^F(p^*/q_{\varepsilon}) > E^F(\overline{p}/q_{\varepsilon}) \;\; \forall \overline{p} \in \Delta^F, \overline{p} \neq p^* \\ &2)\quad E^W(q^*/p_{\varepsilon}) > E^W(\overline{q}/p_{\varepsilon}) \;\; \forall \overline{q} \in \Delta^W : \overline{q} \neq q^* \end{aligned} \tag{9}$$

for all $0 < \varepsilon \leq \overline{\varepsilon}$, then $p_{\varepsilon} = (1-\varepsilon)p^* + \varepsilon\overline{p}$ and $q_{\varepsilon} = (1-\varepsilon)q^* + \varepsilon\overline{q}$, hold for their respective post-entry populations.

Hence, from the above definition, it follows that if (p^*, q^*) is an ESS, then, the following assertions are verified:

1. The profile (p^*, q^*) is a NE.

2. If there exists some $p \in \Delta^F$, $p \neq p^*$ such that $E^F(p/q^*) = E^F(p^*/q^*)$, then $E^F(p^*/\overline{q}) > E^F(p/\overline{q})$.

3. If there exists some $q \in \Delta^W$, $q \neq q^*$ such that $E^W(q/p^*) = E^W(q^*/p^*)$, then $E^W(q^*/\overline{p}) > E^W(q/\overline{p})$.

So, players who adopt an ESS perform better than the mutants given the post-entry population.

In the following sections, we study the dynamic complementarities for both types of firms and workers. We consider the workers' population dynamic to characterize the dynamic equilibria and find the threshold value of skilled workers and innovative firms required to overcome the low level equilibrium.

2.1 Replicator Dynamics by Imitation

The second approach introduced by evolutionary game theory is the replicator dynamics. The replicator dynamics explicitly models the process by which the frequency of a strategy changes in the population, this allows us to study the model's evolutionary dynamics. In the replicator dynamics, individuals imitate strategies of randomly sampled members of the population with a probability proportional to the difference in payoffs between players, given that the difference is positive. Assuming that individuals always imitate better performing agents, otherwise the dynamics change.

Consider that during each period $t \in (0, \infty)$ an agent from each population is randomly matched with an agent from the other population to play a bilateral finite game. Note that $prob(I) = PI = y_I$ is the worker's probability of being hired by an innovative firm, and $prob(S) = x_S$ is the firm's probability of hiring a skilled worker.

Let N_i^τ be the total of *i*-strategist, $i \in \{(I, NI); (S, NS)\}$, from the population, $\tau \in \{F, W\}$, with both populations normalized to 1, that is, $y_I + y_{NI} = 1$ and $x_S + x_{NS} = 1$.

Hence, the fractions of *S*-type strategists and *I*-type strategists are,

$$x_S = \frac{N_S^W}{N_S^W + N_{NS}^W}$$

$$y_I = \frac{N_I^F}{N_I^F + N_{NI}^F}$$

In the following system, the vectors $y_F = (y_I, y_{NI})$ and $x_W = (x_S, x_{NS})$ are distributions of agents playing a certain pure or mixed strategy in the space $\{(I, NI); (S, NS)\}$ from the two populations *F* and *W*. The expected payoffs are defined as:

$$E(S) = y_I \overline{p} + \overline{s} - CE$$
$$E(NS) = y_I p + s$$
$$E(I) = x_S \left[B_I(S) - B_I(NS) - (\overline{s} + \overline{p}) + (s + p) \right] + B_I(NS) - \left(s + p - CI \right) \quad (10)$$
$$E(NI) = x_S \left[B_{NI}(S) - B_{NI}(NS) - \overline{s} + s \right] + B_{NI}(NS) - s$$

2.2 The Specific Imitation Rule

Our imitation rule implies that players stick to a pure strategy for some time interval, and occasionally review their strategies, sometimes resulting in a change of strategy, early contributions start with Björnerstedt and Weibull (1995). Behavioral rules driven by imitation have a long tradition in the literature of evolutionary game theory,[8] see Weibull (1995). One of the best known evolutionary models, the replicator dynamics, describes an evolutionary process driven purely by imitation of other players, which corresponds to cultural evolution. In our case the engine of the replicator dynamics is the self-improvement of the expected payoffs by imitation of the benefits of each economic agent. But we do not assume complete rational maximizers, instead agents imitate the most successful behavior of their neighbors. But, to imitate their neighbors they do not need to know the complete list of states of the world.

In any game with *N* players, there are N populations; one population for each type of player, from which we randomly draw agents who are programmed to play some pure strategies available to that type of player. Let these agents play the game. Over time, players may change their pure strategy. This is embodied by the so-called behavioral rules; such behavioral rules generate a system of differential equations that describes the evolution of the relative frequency of a pure strategy occurring in a population.

There is one differential equation for each pure strategy available to a population and every differential equation describes the evolution of a strategy's population share, that is the number x_i^τ for all $1 \le \tau \le N$ and $1 \le i \le n_\tau$.

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[8]There are two basic elements common to these models. The first is a specification of the time rate at which agents in the population review their strategy choice. This rate may depend on the current performance of the agent's pure strategy and of other aspects of the current population state. The second element is a specification of the choice probabilities of a reviewing agent. The probability that an *i*-strategist will switch to some pure strategy *j* may depend on the current performance of these strategies and other aspects of the current population state.

Definition 2. A behavioral rule is a map from current aggregate behavior to conditional switch rates. The map is given by two basic elements:

1. The time rate $r_i(x)$ at which agents review their strategy choice. This time rate depends on the performance of the agent's pure strategy and other aspects of the current population state.[9]

2. The probability $p_{ij}(x)$ that a reviewing $i-$strategist will switch to some pure strategy *j*. The vector of this probabilities is written as: $p_i(x) = \left(p_{i1}(x),..., p_{ik}(x)\right)$, and it is a distribution on the set of pure strategies.

The *outflow* from club i in population τ is $q_i^\tau r_i^\tau(x) p_{ij}^\tau(x)$ and the *inflow* is $q_j^\tau r_j^\tau(x) p_{ji}^\tau(x)$, where $q_i^\tau = q^\tau x_i^\tau$ is the number of i-strategists from population τ and q^τ represents the whole population τ, hence $q^\tau = q_i^\tau + q_j^\tau$. Therefore, given the population τ and assuming that the size of the population τ is constant, by the law of large numbers, we can model these processes as a deterministic flow. Rearranging terms, for each pair $i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$, from population $\tau \in \{F, W\}$, we get the following mathematical expression summarized in Definition (2):

$$\dot{x}_i^\tau = r_j^\tau(x) p_{ji}^\tau(x) x_j^\tau - r_i^\tau(x) p_{ij}^\tau(x) x_i^\tau \tag{11}$$

Consider that an agent's decision depends upon the expected payoff associated with their own behavior, given the composition of the other population,[10] $E^\tau(e_i, x^{-\tau})$ and depends on the characteristics of the populations represented by $x = (y^f, x^W)$. Therefore, we may assume that

$$r_i^\tau(x) = f_i^\tau\left(E^\tau\left(e_i, x^{-\tau}\right), x\right) \tag{12}$$

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[9]This is the "behavioural rule with inertia," see Björnerstedt and Weibull (1996), Weibull (1995) and Schlag (1998; 1999) that allows an agent to reconsider her action with probability $r \in (0,1)$ each round.

[10] Where τ represents the population to which the agent following the *i-th* behavior belongs, and $-\tau \in \{F, W\}, -\tau \neq \tau$)

where e_i indicates vectors of pure strategies, $i, j \in \{(I, NI);(S, NS)\}$, independently from population τ, and $x^{-\tau}$ is the population state of the other population, $-\tau \neq \tau$. Functions such as $f_i^{\tau}(E^{\tau}(e_i, x^{-\tau}), x)$ can be interpreted as the propensity of a member from the *i*-th type to switch membership as a function of the expected utility gains from such a switch. Agents with less successful strategies on average review their strategy at a higher rate than agents with more successful strategies.

Assume f_i^{τ} is population-specific, but the same across all its components independent of club membership, and assume, furthermore, that it is linear in utility levels, see Weibull (1995). Thus, the propensity to switch behavior will be decreasing in the level of utility, *i.e.*,

$$f_i^{\tau}(E^{\tau}(e_i, x^{-\tau})) = \alpha^{\tau} - \beta^{\tau} E^{\tau}(e_i, x^{-\tau}) \in 0,1 \tag{13}$$

where $\alpha^{\tau}, \beta^{\tau} \geq 0$ and $\frac{\alpha^{\tau}}{\beta^{\tau}} \geq E^{\tau}(e_i, x^{-\tau})$.

To simplify the notation, for each pair $i, j \in \{(I, NI);(S, NS)\}$, let us label:

- $r_i^{\tau}(x) = r_i$ and $r_j^{\tau}(x) = r_j$,
- $p_{ji}^{\tau}(x) = p_{ji}$ and $p_{ij}^{\tau}(x) = p_{ij}$,
- $E^{\tau}(e_i, x^{-\tau}) = E(i)$ and $E^{\tau}(e_j, x^{-\tau}) = E(j)$.

From Equations (12) and (13), and after some algebraic manipulation, system (11) becomes a replicator dynamic driven by imitation,[11] *i.e.*,

$$\dot{x}_i^{\tau} = x_i^{\tau}\left(1 - x_i^{\tau}\right)\left[\lambda\left(\alpha^{\tau} + \beta^{\tau}\right)\left(E(i) - E(j)\right)\right] \tag{14}$$

where $x_i^{\tau} \in [0,1]$ and $\dot{x}_i^{\tau} + \dot{x}_j^{\tau} = 0$ for each pair $i, j \in \{(I, NI);(S, NS)\}$, $j \neq i$ from population[12] $\tau \in \{F, W\}$, and $\lambda = \frac{1}{|E(i) + E(j)|}$, $\alpha^{\tau}, \beta^{\tau} \geq 0$, $\frac{\alpha^{\tau}}{\beta^{\tau}} \geq E(\cdot)$

[11]See appendix numbered list 1 for the specific cases of the dynamic flows of firms and workers.

Consider that an agent does not know the true values of the expected payoffs that other agents receive, but she can take a sample of such true values in order to estimate the average. Let $\tilde{E}(i)$ and $\tilde{E}(j)$ be the estimators for the true values *E(i)* and *E(j)*, $\forall i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$.

Hence, each $i-$strategist changes her current strategy if and only if $P[\tilde{E}(i) < \tilde{E}(j)]$. A reviewing worker who changes her current strategy must take into consideration both: i) a probability of imitating at least one strategy that performs better than her current strategy, $P[\tilde{E}(i) < \tilde{E}(j)]$, and ii) the probability of meeting an agent, x_j^τ, who uses such strategy. Consider that $P[\tilde{E}(j) - \tilde{E}(i) > 0]$ increases proportionally to the true value $E(j)$, we therefore obtain the following definition.

Definition 3. A reviewing i-strategist changes to j with a probability, p_{ij}, equals a positive average rule times the probability of encountering a $j-$ strategist in the whole population, *i.e.* $\forall i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$,

$$p_{ij} = P[\tilde{E}(j) - \tilde{E}(i) > 0]x_j = \begin{cases} \lambda E(j)x_j^\tau & if \quad E(j) > 0 \\ 0 & if \quad E(j) \leq 0 \end{cases} \tag{15}$$

where $\lambda = \dfrac{1}{|E(i) + E(j)|}$

Now, we introduce the behavioral rule from definition (3) and expression (15), the substitution of expected payoffs $E(\cdot)$ from system 10, and, after some algebraic manipulation from system (14), we get the worker's replicator dynamic (RD) and the firm's replicator dynamic (RD) driven by imitation:

$$\begin{aligned} \dot{x}_S &= -\dot{x}_{NS} = x_S(1 - x_S)A(\cdot) \\ \dot{y}_I &= -\dot{y}_{NI} = y_I(1 - y_I)B(\cdot) \end{aligned} \tag{16}$$

Where the functions $A(x_S, y_I)$ and $B(x_S, y_I)$ depend on shares of firms and workers and their payoffs, for the explicit expressions see the Appendix.

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[12]Then, the trajectory $x^\tau(t) = \{(x_i^\tau(t), x_j^\tau(t)), t_0 \leq t\}$ is bounded in the unit square $C = [0,1] \times [0,1]$.

2.3 Stability and Equilibria Analysis

The RD system, $(\dot{y}_I, \dot{x}_S)$, admits five stationary states or dynamic equilibria, *i.e.*, $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ and $(\hat{y}_I, \hat{x}_S)$, where:

$$\begin{aligned} \hat{y}_I &= \frac{CE - (\overline{s} - s)}{\overline{p} - p} \\ \hat{x}_S &= \frac{B_I(NS) - B_{NI}(NS) - p - CI}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) + \left(\overline{p} - p\right)} \end{aligned} \tag{17}$$

In fact, the interesting case occurs when $(\hat{y}_I, \hat{x}_S)$ is an equilibrium lying in the interior of the square $C = [0,1] \times 0,1]$, which occurs when $0 < \hat{y}_I < 1$ and $0 < \hat{x}_S < 1$.

The next proposition summarizes the evolutionary dynamics for the replicator dynamics of firms and workers:

Proposition 1. *The evolutionary dynamics of firms and workers driven by imitation is as follows:*

1. Equilibria $(1,0)$ and $(0,1)$ are nodal sources and unstable.

2. Equilibria $(0,0)$ and $(1,1)$ are asymptotically stable points or nodal sinks, and therefore ESS.

3. Equilibrium $G = (\hat{y}_I, \hat{x}_S)$ is a saddle point, and therefore a threshold since it separates the basins of attraction of the low-level and high-level equilibria.

Proof. *We can assess whether the five equilibria are asymptotically stable points by analyzing the Jacobean Matrix* $J(\dot{x}_S, \dot{y}_I)$*, such that, equilibria fitting* $\det(J) > 0$ *and* $tr(J) < 0$ *are asymptotically stable (see Appendix), and then by the Defition (1) they are ESS of the game. There are two asymptotically stable strategy profiles. In particular, it is enough to look at the phase diagram, Figure 1, where a graphic representation is given.*

The point $G = (\hat{y}_I, \hat{x}_S)$ *is hypothetically centered but it is not necessarily always centered since it depends on the model's parameters.* ▪

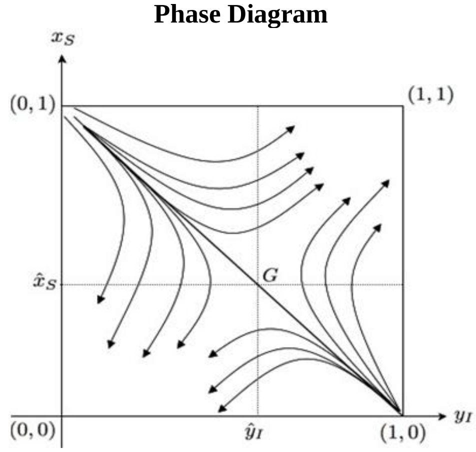
These equilibria can be interpreted as follows:

• Equilibrium $(0,0)$ is the low-level equilibrium in which firms and workers are low-profile (NI, NS).

• Equilibrium $(0,1)$ and $(1,0)$ is a mismatch of profiles (NI, S), (I, NS).

• Equilibrium $(1,1)$ is the high-level equilibrium in which firms and workers are high-profile (I, S).

Figure 1
Phase Diagram



Concluding, we can say, first, that players behave rationally, though they follow an imitation rule and this may lead the economy to a poverty trap; second that there is a threshold value of high-profile agents above which firms and workers will rationally adopt high profiles and high growth will be maintained.

3. How to Overcome a Poverty Trap

Since the results of the model depend on initial conditions, that is, the profile distributions, y_F and x_W, the model is historically dependent where an optimal strategy of the dynamic replicator system converges towards two distinct attractors, $(0,0)$ and $(1,1)$.

Let us denote by $y^F = (y_I^F, y_{NI}^F)$ and $x^W = (x_S^W, x_{NS}^W)$ the initial distributions of firms and workers. Consequently, the following case may arise:

1. This consideration can be analyzed as a "game against the field" in which there is no specific "opponent" to a given agent and payoffs depend on what everyone in the population is doing and the agent is facing the other population state. That is, from the side of workers, an individual worker is playing against the field represented by the profile distribution of firms and workers, this is, the population state.

Hence, let us consider the population of workers. If the fraction of innovative firms, y_I^F, is smaller than the threshold value $\hat{y}_I^F$, *i.e.*,

$$y_I^F < \hat{y}_I^F = \frac{CE - (\overline{s} - s)}{\overline{p} - p} \tag{18}$$

then all potential workers prefer not to become high-skilled, and thus the initial distribution of workers $x^W(t_0) = (x_S^W(t_0), x_{NS}^W(t_0)) \to (0,1)$ in pure strategies. Suppose now, that a small part of the non-innovative firms become innovative, $y'^F = \left(y'^F_I, y'^F_{NI}\right)$ where $y(t_0) < y'^F_I < y_I^F$, then workers prefer to be unskilled and the population of the high-skilled workers continues to decrease. In this case a worker will be unskilled. So, we need a big change in the distribution of the population of firms to obtain workers to become skilled. The threshold value is $\hat{y}_I^F$, if the new distribution verifies that $y_I^F > \hat{y}_I^F$, and therefore the rational imitator worker chooses to be skilled.

Now, we define an ESS against the field as follows:

Definition 4. Consider a two population normal form game where each population *F* and *W* has two possible behaviors. Suppose that the distribution of the population *F* is given by $y_F = (y_{1F}, y_{2F})$. We say that the the strategy $x_W^* = (x_{1W}^*, x_{2W}^*)$ is an ESS against the field if there exists an ε such that if

$$E^W(x_W^*, y_F') \geq E^W(x_W, y_F') \tag{19}$$

for all $x_W \in S_W$ *where* $| y_F - y'_F | \leq \varepsilon$. *Therefore, if the distribution of the initial population of firms is given by* $y^F = (y_I^F, y_{NI}^F)$ *where* $y_I^F < \hat{y}_I^F$ *then* $x^W = (0,1)$ *is an* ESS *against the field.*

To overcome a poverty trap requires that we change the rationally determined outcome. To do so the economy has to move beyond the threshold point of low-skilled workers and non-innovative firms. Focusing on workers, a central planner could implement a policy that decreases the costs to become high-skilled, or, given the incentive to become an innovative firm, to increase rewards to skilled workers.

Therefore, to reverse the inequality (18), and to overcome the poverty trap, consider the following:

• Either Costs of Education *CE* must decrease or the differences in skill premia $\Delta P = \overline{p} - p$ must increase positively. Consequently, with a perturbation σ_ε, workers will become skilled and firms will become innovative and both perform better. The economy simultaneously overcomes its historical poverty trap. We thus consider the perturbation as a shock that compels agents to adopt new strategies, *i.e.*, alternative strategies will invade the equilibrium as a consequence of the exogenous shock and, because the economy is in the basin of attraction of the higher-level equilibrium, the economy will converge to the higher-level equilibrium.

Decreasing *CE* means investing in human capital, which implies an increasing ΔP. In particular, from Equation (17) the threshold $\hat{x}_S = 0$ if, given the gross benefits of firms, wich are determined in the market, $B_i(j)$, $\forall \ i \in \{I, NI\} \ \ j \in \{S, NS\}$, then, the premia of unskilled workers is equal to $p = B_{NI}(NS) - B_I(S) - CI$, which means that an innovative firm must offer a premium that encourages the unskilled workers to supply their labor or become employed in the R&D sector. In turn, due to imitation and to strategic complementarities, such an unskilled worker must become a skilled worker. This means that workers with better profiles should get the largest possible premium, $\overline{p}$. In this way, the number of innovative firms (R&D) and the number of skilled workers (human capital) are large enough to overcome the poverty trap, *i.e.* $\left(x_S^W, y_I^F\right) > \left(\hat{x}_S^W, \hat{y}_I^F\right)$.

Any situation in the game against the field can be modified by a central planner or policy-maker who is able to change the profile distributions y_F and x_w. Alternatively, each player who wonders whether their current behavior is the optimal response for the long run outcome of the economy could hypothetically participate in collective action to change their profiles. Either way, the economy becomes more efficient. The market alone is incapable of overcoming this kind of poverty trap. Policy-makers could intervene, for instance, by providing some kind of financial incentive for R&D investment or by imposing a minimum period of schooling.

3.1. Replicator Dynamic with Fiscal Incentives

Focusing on firms, the central planner could implement a policy that would increase the number of innovative firms above the threshold. A policy option is to give fiscal incentives to those firms that implement skill premia, which can be provided by a well-implemented income tax.

Let us assume that the policy-maker imposes some income taxations at rate $\gamma \in [0,1]$ and $\delta \in [0,1]$ for firms and workers, respectively. Where means of non intervention are denoted as $\gamma = 0$, $\delta = 0$, while means of complete intervention are defined as $\gamma = 1$, $\delta = 1$.

Suppose that at the time when this policy starts, the economy has the following characteristics:

• Let N_i^τ be the total of *i*-strategists, $i \in \{(I, NI);(S, NS)\}$, from the population $\tau \in \{F, W\}$, and let $H = \sum_{i,\tau} N_i^\tau$ be the total number of players, the whole economy is normalized to 1.

The mass or number of firms and workers that adopt a strategy *i*, $\forall\ i \in \{(I, NI); (S, NS)\}$, is given by:

$$m_i = \frac{N_i^\tau}{H} \tag{20}$$

or:

$$m_I = \frac{N_I^F}{H}, m_{NI} = \frac{N_{NI}^F}{H}, m_S = \frac{N_S^W}{H}, m_{NS} = \frac{N_{NS}^W}{H} \tag{21}$$

Then, we denote by, $\Delta = \{m \in R_+^k : \sum_{i=1}^k m_i = 1\}$ the simplex of R_k. In our case $k = 4$.

If such taxes are imposed on each population $\tau \in \{F, W\}$, then the total revenue collected in the economy is:

$$T = \gamma[m_I E(I) + m_{NI} E(NI)] + \delta[m_S E(S) + m_{NS} E(NS)] \tag{22}$$

The implemented policy is characterized by taxation and subsidies. The subsidies are awarded to skilled workers and innovative firms. A proportion $\theta \in (0,1)$ of the total taxation collected is shared to skilled workers and the rest to innovative firms.

The new expected payoff, after transference, for strategist S and NS are respectively:

$$\begin{aligned} E_T(S) &= E(S)(1-\delta) + \theta T \\ E_T(NS) &= E(NS)(1-\delta) \end{aligned} \tag{23}$$

And for strategist I and NI are now respectively

$$\begin{aligned} E_T(I) &= E(I)(1-\gamma) + (1-\theta)T \\ E_T(NI) &= E(NI)(1-\gamma) \end{aligned} \tag{24}$$

So the dynamical system (14), $\forall \quad i, j \in \{(I, NI); (S, NS)\}$, is now substituted by the new dynamical system given by:

$$\dot{x}_i^\tau = x_i^\tau(1 - x_i^\tau)[\lambda(\alpha^\tau + \beta^\tau)(E_T(i) - E_T(j)] \tag{25}$$

Consider that the the initial conditions are: $z(t_0) = (x_S(t_0), y_I(t_0))$, then the solution of this system will be unique and symbolized by $\xi(t, t_0 z(t_0))$. The threshold value corresponding to this system, is given by the equations:

$$\begin{aligned} E_T(S) &= E(NS) - \frac{\theta T}{1-\delta} \\ E_T(I) &= E(NI) - \frac{(1-\theta)T}{(1-\gamma)} \end{aligned} \tag{26}$$

From these equations we obtain the threshold value in terms of the share of skilled workers and innovative firms, and these values are given by:

$$n_s = \hat{x}_s - \left(\frac{(1-\theta)\,T}{1-\delta}\right)\frac{1}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) - p + \bar{p}}$$
$$n_I = \hat{y}_I - \left(\frac{\theta T}{1-\gamma}\right)\frac{1}{\bar{p} - p} \tag{27}$$

Where $\hat{y}_I$ and $\hat{x}_S$ are the former threshold value defined in (17).

So, if the initial value $z(t_0) = (x_S(t_0), y_I(t_0))$ in a time $t = t_0$ of the economy, is below the threshold value $G = (\hat{x}_S, \hat{y}_I)$ then the policy-maker needs to implement taxes such that:

$$x_s(t_0) \geq \hat{x}_s - \left(\frac{(1-\theta)\,T}{1-\delta}\right)\frac{1}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) - p + \bar{p}}$$
$$y_I(t_0) \geq \hat{y}_I - \left(\frac{\theta T}{1-\gamma}\right)\frac{1}{\bar{p} - p} \tag{28}$$

and then, the initial conditions are outside of the basin of attraction of the low-level equilibrium, corresponding to the system of equations (25). So, the economy is increasing to a high-level equilibrium. At the moment where the solution of this dynamical system surpasses the threshold value *G* the policy-makers may leave the economy evolving by its own rules, i.e, the evolution of the economy will be again, determined by the system (14), but now, the initial conditions are in the basin of attraction of the high-level equilibrium. This means that the policy-makers should withdraw taxation and subsidies once the economy, follows a trajectory corresponding to a solution of (25), surpasses the level *G*. After this moment the rationality of the agents will drive the economy to a high-level equilibrium. Therefore, the intervention of the policy-makers in the economy, in a time $t=t_0$, could be justified for a given time period of z(t_0) below the threshold value *G*.

Summarizing, the policy-makers could intervene in the economy to implement a tax and subsidy policy such that it encourages workers to become skilled and firms to become innovative. The purpose of this policy is withdraw the economy from a trajectory converging to an inefficient equilibrium (the poverty trap). This intervention of the policy-maker ends in the moment where the solution of the dynamical system given by (25) $\xi(t, t_0, z(t_0))$ surpasses the threshold value *G* from this moment the economy will follow a trajectory, corresponding to a solution of the dynamical system (14), of high growth, converging to a high level equilibrium, the participation of the social planner in the economy from now on will be superfluous.

Concluding remarks

We presented an evolutionary game characterized by complementarities between firms and workers. The model shows that rates of return on R&D depend on average human capital, and rates of return on human capital depend on aggregate R&D spending. The outcome is a self-confirming equilibrium in evolutionarily stable strategies determined by the steady states of the replicator dynamic system. The poverty trap or low-level equilibrium is such that in poor economies with a large fraction of low-human-capital workers or low-R&D firms, imitation rules can act against sustained growth. To achieve economic take-off, a central planner could subsidize the costs of education and/or R&D until the economy builds a critical mass of human capital and R&D which allows the economy to move beyond the threshold to achieve virtuous growth.

The model we presented is practical and/or real, for instance the Bombardier Aeronautica Firm in Queretaro, Mexico, is a real approximation of our model, where, in an article entitled Mexico's Jet Set from the Latin Trade Magazine,[13] pointed out what we studied:

• Innovative firms: "The levels of technological development of manufacturing sector are the strictest worldwide."

• Imitation: "If we can make this industry grow in our country, other industries will follow, and wages will inevitably go up."

• Skill premia: "You need very disciplined, structured, high-paid employees with a lot of experience," said consultant John F. Walsh, president of the Annapolis-based Walsh Aviation. "There are not many places in the world that do that. It would be a very big deal."

• Decreasing the cost of education: "the state government founded Mexico's first aeronautical university next door to the airport."

Hence, from our model, to overcome a poverty trap is necessary to surpass a threshold number of innovative firms and skilled workers. Unskilled workers are given an incentive to become skilled when the premia to skilled work increases. Hence, firms, in turn, can obtain more benefits being $I-$strategists. If the economy does not surpass this threshold value, a policy-maker could implement an incentive-based policy to reach the high-level equilibrium of innovative firms and skilled workers. For instance, they could

[13] See http://latintrade.com/2009/12/mexico%E2%80%99s-jet-set

implement a policy to lower the cost of attaining skills. The market alone is incapable of overcoming this poverty trap, and policy makers should intervene. For instance, policy-makers could provide a tax scheme that encourages R&D investment, or they could impose a minimum level of high-quality education for the whole population. However, the policy-makers' intervention in the economy can finalize from the moment where the economy surpasses the threshold value.

Conversely, a poverty trap can arise because agents imitate the best-responding strategies when the state of the economy lacks both R&D and human capital. For low numbers of skilled labor, innovation will not be profitable, not just in terms of final output but also in terms of the generated rate of technical change. Only after an economy is sufficiently developed in terms of its supply and demand for skilled labor, it will undertake R&D. So, in this case, one of the main obstacles to overcoming a poverty trap is the agents' rationality by imitation. Using evolutionary game theory to tackle the problem of poverty traps, we conclude that a kind of bounded rationality characterized by imitation offers a clue to understand the dynamics of poverty traps. Boundedly rational agents act to maximize their own payoffs, pursuing, as best as they can, their own self-interest. Hence, we have shown the strategic foundations of high and low level equilibria when firms and workers imitate, and thus they adopt the best strategy given the state of the economy. A future research could go in the direction to offer a feasible answer about: how much and for how long a government should subsidize R&D activities and human capital accumulation?

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Appendix

1. In particular, if we consider the distribution of workers' profile, $x_W = (x_S, x_{NS})$, and firms, $y_F = (y_I, y_{NI})$, then we can model the flow of high-skilled workers and innovative firms over time: $\dot{x}_S$ and $\dot{y}_I$. Since we consider large populations, we invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each flow is being set equal to the expected rate of the corresponding Poisson arrival process. Hence, rearranging terms, we obtain the system of differential equations that characterizes the dynamic flow of workers,

$$\begin{aligned} \dot{x}_S &= r_{NS}\, p_{NSS}\, x_{NS} - r_S\, p_{SNS}\, x_S \\ \dot{x}_{NS} &= -\dot{x}_S \end{aligned} \tag{29}$$

and the differential equations that characterize the dynamic flow of firms,

$$\begin{aligned} \dot{y}_I &= r_{NI}\, p_{NII}\, y_{NI} - r_I\, p_{INI}\, y_I \\ \dot{y}_{NI} &= -\dot{y}_I \end{aligned} \tag{30}$$

2. The systems of worker's replicator dynamic (RD) and firm's replicator dynamic (RD) driven by imitation:

$$\dot{x}_S = -\dot{x}_{NS} = x_S\left(1 - x_S\right) A\left(\cdot\right) \tag{31}$$

$$\dot{y}_I = -\dot{y}_{NI} = y_I\left(1 - y_I\right) B\left(\cdot\right) \tag{32}$$

where the functions $A(x_S, y_I)$ and $B(x_S, y_I)$ are defined by:

$$A(x_S, y_I) = \alpha^W \frac{y_I(\overline{p} - p) + s - \overline{s} + CE}{y_I(p + \overline{p}) + s + \overline{s} - CE},$$

$$B(x_S, y_I) = \alpha^F \left[\frac{B_I(NS) - B_{NI}(NS) + x_S\left(\Delta B_I + \Delta B_{NI} - \Delta P\right) - p - CI}{B_I(NS) + B_{NI}(NS) + x_S\left(\Delta B_I - \Delta B_{NI} - \Delta P - 2s - 2\overline{s}\right) - p + 2s}\right]$$

where: $\Delta B_I = B_I(S) - B_I(NS)$, $\Delta B_{NI} = B_{NI}(NS) - B_{NI}(S)$ and $\Delta P = \overline{p} - p$

3. **Proof of Proposition 1**. The idea is to use the eigenvalues of the Jacobian matrix evaluated at a critical point to understand the behavior of the system near that critical point. The Jacobean Matrix is given by,

$$J\left(\dot{y}_I, \dot{x}_S\right) = \begin{pmatrix} \dfrac{\left(1-2x_S\right)\alpha^W\left(y_I \Delta P + \overline{s} - s - CE\right)}{y_I\left(\overline{p}+p\right)+\overline{s}+s-CE} & \dfrac{2\left(1-x_S\right)\alpha^W x_S\left(s\overline{p} - \left(\overline{s}+CE\right)p\right)}{\left(y_I\left(\overline{p}+p\right)+\overline{s}+s-CE\right)^2} \\ 2\left(1-y_I\right)\alpha^F y_I Q & \left(1-2y_I\right)\alpha^F Z \end{pmatrix}$$

where:

$$Q = \frac{\left(\Delta B_{NI} + \Delta P\right)\left(B_I(NS) + s - p - CI\right) + \Delta B_I\left(B_{NI}(NS) + s\right) - \left(\overline{s} + s\right)\left(p + B_{NI}(NS) - B_I(NS)\right)}{x_S\left(\Delta B_{NI} - \Delta B_I + \Delta P + 2\overline{s}\right) - \left(1 - x_S\right)2s + p - B_I(NS) - B_{NI}(NS)}$$

and

$$Z = \frac{CI + p - B_I(NS) + B_{NI}(NS) - x_S\left(\Delta B_I + \Delta B_{NI} + \Delta P\right)}{x_S\left(\Delta B_{NI} - \Delta B_I + \Delta P + 2\overline{s}\right) - \left(1 - \sigma x\right)2s + p - B_I(NS) - B_{NI}(NS)}$$

It can be verified that evaluating such Jacobean to each of the five equilibrium points, the equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are: $(0,0)$ and $(1,1)$ and then, they are ESS. Moreover, equilibrium $G = \left(\hat{y}_I, \hat{x}_S\right)$ fits $\det(J) < 0$, then it is a saddle point which separates the basin of attractions from the low-level to the high-level equilibrium.