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Market Integration and Structural Transformation in a Poor Rural Economy

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Abstract

By developing a simple theoretical model of the impact of market integration on sectoral output and employment in a poor rural setting, this paper demonstrates that trade can induce asymmetric growth. Under certain, plausible, assumptions, the non-farm sector will grow much faster than the agricultural sector when markets become integrated. Promoting market integration may thus be an effective way of encouraging diversification beyond agriculture and catalysing structural change in poor rural economies.

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1 Introduction

Economic development is an uneven process, characterized by severe inequalities in growth patterns across sectors and across space (Harris, 1985). Theories of structural change typically explain the shift from agriculture to manufacturing and services that accompanies the development process as being triggered by increases in agricultural productivity ("push"), manufacturing productivity ("pull") or a combination of both (for an overview of the literature, see Matsuyama, 2008). This paper contributes to the literature by demonstrating that, even in the absence of technological progress, asymmetric sectoral output growth and employment shifts can be induced by trade. Such shifts result in spatial disparities in economic activity that are qualitatively similar to those conventionally associated with rural transformation. By highlighting the possible role of preferences as part of the explanation for these disparities, the paper also aims to contribute to the New Economic Geography literature (see Krugman, 1998, for an overview of this literature).

To demonstrate these effects, we use a simple theoretical model to analyze the effects of market integration on food and non-food production and the sectoral composition of employment in a poor rural setting, where utility is highly sensitive to food consumption. Our motivation for focusing on poor rural areas is twofold. At the inception of the industrial revolution, most societies were rather poor and dominated by agriculture. Currently, diversification beyond agriculture is often considered a promising way out of poverty for poor rural economies. Yet, in many countries, market fragmentation constrains the growth of the rural non-farm sector (see e.g. Loening et al, 2008). If people cannot trade, they have no choice but to produce what they need to eat.[1] We analyze the benefits of

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[1]See e.g. de Janvry et al. (1991) for an analysis of peasant household behaviour when markets

market integration and show that, under certain plausible assumptions, as trade is facilitated, output and employment in the non-food sector grow more quickly than output and employment in the food sector as trade. Sectoral growth patterns are thus asymmetric. A related result is that the non-farm sector may benefit more from market integration than the farm sector.

2   The Model

We consider a setting in which there are two representative agents (communities) who are potential trading partners. We assume they produce, and consume, food and non-food products, and that they have Cobb-Douglas utility functions:

$$\ln U\left(C_F^i, C_{NF}^i\right) = a \ln C_F^i + (1-a) \ln C_{NF}^i,$$

where C_F^i, C_{NF}^i denote consumption of food and non-food products, respectively, for agent i, and a is a preference parameter bounded between 0 and 1. In rural areas of developing countries, where poverty is widespread and a substantial proportion of the population is malnourished, we think of a as being closer to 1 than 0, reflecting a relatively high sensitivity of utility to changes in food consumption.[2]   Each producer has a vector of product-specific productive skills denoted (A_F^i, A_{NF}^i), and production of food (F) and non-food (NF) products is given by

$$F_i = \alpha_i A_F^i,$$
$$NF_i = (1-\alpha_i) A_{NF}^i,$$

where α_i is the time (bounded between 0 and 1) agent i allocates to food production, which we shall refer to as the employment share of sector i. Thus there

are missing.

[2]Dasgupta and Ray (1986; 1987) discuss how physical wellbeing is highly sensitive to changes in nutritional intake at low nutrition levels, in their analysis of the effects of inequality on malnutrition. Private consumption expenditure on food accounts for the bulk of consumer spending in the least developed countries (Grigg, 1994).

are constant returns to scale in both sectors.

Under autarky, each agent has to produce the products to be consumed. It is straightforward to show that, in our model, the optimal amount of time allocated to producing food in autarky is $\alpha_i = a$. Thus, if the preference parameter a is high, each agent will allocate most of his or her time producing food, independent of the underlying skills vector (A_F^i, A_{NF}^i).

Now consider the effects of integrating the market, enabling the two producers to trade with each other. Suppose that community 1 has a comparative advantage in the production of food, so that

$$\frac{A_F^1}{A_F^2} \frac{A_{NF}^2}{A_{NF}^1} > 1.$$

This assumption is maintained throughout the analysis. We assume that the agents bargain over the total utility surplus generated by trade. The outside option in the bargaining game is the utility under autarky, denoted $\overline{U}_i$. In equilibrium, the Nash product

$$\Omega = \ln\left(U_1 - \overline{U}_1\right) + \ln\left(U_2 - \overline{U}_2\right),$$

is maximized with respect to inputs α_1, α_2. The solutions for α_1, α_2 are summarized in Proposition 1.

Proposition 1 *Optimal time allocation under trade is determined by preferences and productivity differentials as follows:*

1. If $A_F^1/A_F^2 < a/(1-a)$, then

$$\alpha_1 = 1,$$

$$\alpha_2 = a - (1-a)\left(A_F^1/A_F^2\right) \in (0,1)$$

2. If $A^1_{NF}/A^2_{NF} \leq a/(1-a) \leq A^1_F/A^2_F$, *then*

$$\alpha_1 = 1,$$
$$\alpha_2 = 0.$$

3. If $A^1_{NF}/A^2_{NF} > a/(1-a)$, *then*

$$\alpha_1 = a + a\left(A^1_{NF}/A^2_{NF}\right) \in (0,1)$$
$$\alpha_2 = 0.$$

Proof: See Appendix. Clearly, small changes in preferences can result in sharply discontinous patterns of spatial specialization, as preferences a are a key determinant of which solution will be optimal and thus of the sectoral composition of employment across communities.

3   Application: A poor rural economy

Subject to the assumptions made, the results summarized in Proposition 1 are general. In the context of a poor rural economy, it seems reasonable to assume that utility is quite sensitive to food consumption (a close to 1), and that heterogeneity in productivity levels (skills) across agents is modest (A^1_F/A^2_F close to 1). We thus focus on the solution scenario where $A^1_F/A^2_F < a/(1-a)$. It then follows from Proposition 1 that, under trade, community 1 will specialize in food production while community 2 will adopt a mixed production strategy. Now consider some implications for output and sectoral employment shares under this scenario.

Effects of shifting from autarky to trade on output.   We highlight two striking results. The first is that total volume of food produced is *the same* under

trade as under autarky:

$$F_{autarky} = a\left(A_F^1 + A_F^2\right),$$

$$F_{trade} = A_F^1 + \left[a - (1-a)\left(A_F^1/A_F^2\right)\right]A_F^2,$$

$$F_{trade} = A_F^1 + aA_F^2 - (1-a)A_F^1,$$

$$F_{trade} = a\left(A_F^1 + A_F^2\right).$$

Total consumption of food will thus not change as a result of market integration.

The second result is that total volume of non-food produced is *strictly higher* under trade than under autarky:

$$NF_{autarky} = (1-a)\left(A_{NF}^1 + A_{NF}^2\right)$$

$$NF_{trade} = \left[1 - a + (1-a)\left(A_F^1/A_F^2\right)\right]A_{NF}^2$$

$$NF_{trade} = (1-a)\left[\frac{A_F^1 + A_F^2}{A_F^2}\right]A_{NF}^2,$$

hence the percentage growth in non-food output is

$$\Delta_{NF} = \frac{NF_{trade} - NF_{autarky}}{NF_{autarky}}$$

$$\Delta_{NF} = \frac{\left[\frac{A_F^1}{A_F^2}\frac{A_{NF}^2}{A_{NF}^1} - 1\right]A_{NF}^1}{A_{NF}^1 + A_{NF}^2}$$

$$\Delta_{NF} = [R - 1]\frac{A_{NF}^1}{A_{NF}^1 + A_{NF}^2} > 0$$

where $R = \frac{A_F^1}{A_F^2}\frac{A_{NF}^2}{A_{NF}^1} > 1$ measures the comparative advantage of community 1 in food production. Thus, subject to $A_F^1/A_F^2 < a/(1-a)$, the more pronounced the comparative advantage in agriculture for community 1, and the higher the relative non-food productivity of community 1 to that of community 2, the higher the output gain from trade in the non-food sector. This effect arises because the increase in the relative efficiency with which non-food is produced resulting from trade exceeds the increase in the efficiency with which non-food goods are produced.

Shifting from autarky to trade thus results in asymmetric growth: food output does not change; non-food output increases.

Effects of shifting from autarky to trade on sectoral employment shares.

The effects on sectoral employment shares depend on absolute advantages. It follows from part 1 in Proposition 1 that if community 1 has an absolute advantage in food production, i.e. $A_F^1/A_F^2 > 1$, total employment in agriculture in the economy falls, and total non-farm employment increases. Conversely, if $A_F^1/A_F^2 < 1$ (agent 1 has an absolute disadvantage in food production), total employment in agriculture in the economy increases, and total employment in the non-farm sector falls. If $A_F^1/A_F^2 = 1$, there is no change in employment in the agricultural vs the non-farm sector.

Both the increase in non-farm output and the increase in the employment in the non-farm sector are consistent with historically documented patterns of rural transformation. Also note that the average productivity with which both farm and non-farm goods are produced rises.

3.1 A numerical example

We now illustrate these effects by means of a simple numerical example. Suppose $A_F^1 = 0.6$, $A_{NF}^1 = 0.4$, $A_F^2 = 0.4$, $A_{NF}^1 = 0.6$; $a = 0.8$. Community 1 thus has a comparative (and absolute) advantage in the production of food, and community 2 a comparative (and absolute) advantage in the production of the non-food product. In autarky, we have $\alpha_i = 0.8$ for $i = 1, 2$, hence total production is as follows:

$$F_{Autarky} = F_1 + F_2 = 0.8 \times (0.6 + 0.4) = 0.8,$$

$$NF_{Autarky} = NF_1 + NF_2 = 0.2 \times (0.4 + 0.6) = 0.2$$

Once the communities are allowed to trade, we obtain $\alpha_1 = 1$ and $\alpha_2 = a - (1-a)(A_F^1/A_F^2)$ in equilibrium (see proposition 1). In this particular example, we obtain $\alpha_2 = 0.5$. The total output volume in equilibrium is as follows:

$$F = F_1 + F_2 = 1 \times 0.6 + 0.5 \times 0.4 = 0.8,$$

$$NF = NF_1 + NF_2 = 0 \times 0.4 + 0.5 \times 0.6 = 0.3.$$

Thus, as a result of allowing the agents to trade with each other, the volume of non-food products grows by 50%, whereas the volume of food products does not change at all. Furthermore, since in this example $A_F^1/A_F^2 > 1$, more labor is allocated to non-food production and less is allocated to food production.

These effects are depicted in Figure 1. The solid straight lines indicate the production possibility curves for the two agents, under autarky, and the indifference curves are drawn for the Cobb-Douglas utility function with $a = 0.8$. Optimal production and consumption in autarky for producers 1 and 2 are indicated in the graph by the hollow small circle, and the hollow small diamond, respectively. Once the agents engage in trade, the solution to the bargaining problem is such that production occurs in the points indicated by the solid large circle for producer 1, and the solid large diamond for producer 2. The terms of trade are determined as part of the bargaining process, and are shown in the graph by the gray dashed lines. Producer 1 thus consumes in the point indicated by the solid small circle, and producer 2 consumes in the point indicated by the small solid diamond. Both agents increase their consumption of nonfarm goods, and so total production of nonfarm products increases. The total level of food production and consumption is unchanged compared to autarky.

3.2 Generalized preferences

The above results are obtained under the assumption that the utility function is Cobb-Douglas. In cases where the elasticity of substitution is different from 1, the results will be different. Given our context, the substitutability between food and non-food products is probably rather limited. In the extreme case where the elasticity of substitution is zero (Leontief utility function), so that the ratio of food to non-food products consumed in equilibrium is constant, output in the two sectors will always grow at the same rate when shifting from autarky to trade. Suppose the utility function exhibits constant elasticity of substitution (CES),

$$\ln U \left(C_F^i, C_{NF}^i \right) = \frac{1}{r} \ln \left[a^{\frac{1}{s}} C_F^i + (1-a)^{\frac{1}{s}} C_{NF}^i \right],$$

where $r = (s-1)/s$ and s is the elasticity of substitution. Suppose $s = 0.5$, so that there is some substitutability, but less than under Cobb-Douglas preferences. Using the same calibration as in the numerical example based on Cobb-Douglas preferences above, we then solve numerically for the effects of trade on output. We find that total food production grows by 5% (cf. 0% under Cobb-Douglas), and that non-food output grows by 30% (cf. 50% under Cobb-Douglas). Thus, there is still asymmetric growth - the non-farm sector grows more quickly - but, this is less pronounced than under Cobb-Douglas preferences. For completeness we also consider $s = 2$, implying high substitutability between the two products (of course, it seems unlikely that individuals in a poor rural economy are willing to substitute food for non-food consumption at such a rapid rate). In this case. the volume of food produced *falls* by 10%, whereas non-food production grows by 85%, when the economy shifts from autarky to trade.

4 Discussion

Our model is highly stylized. However, some of the mechanisms that we have abstracted from might enhance our results further. For example, Engel effects, (which are often emphasized in the literature on structural change, see e.g. Matsuyama, 1992, Laitner, 2000, Caselli and Coleman, 2001) are likely to reinforce our results; if one of the effects of market integration is to raise individuals' incomes and this in turn lowers the food preference parameter a (an Engel effect)[3], then this will enhance the pattern of asymmetric growth in non-farm production documented above. Alternatively, suppose the agents have Stone-Geary utility functions

$$\ln U \left(C_F^i, C_{NF}^i \right) = a \ln \left(C_F^i - \overline{S} \right) + (1 - a) \ln C_{NF}^i,$$

where $\overline{S}$ is the subsistence level of food consumption. It is easy to verify that this specification would only reinforce our finding that trade stimulates the output of non-farm products and non-farm employment disproportionately (given reasonable choices of $\overline{S}$). If in the numerical example above $\overline{S} = 0.35$, the time allocation of producer 2 under autarky will change towards more production of food. The effect of trade on total non-food production and non-farm employment will therefore be even greater than with conventional preferences.

Increasing returns to scale (Romer, 1987, Krugman, 1991), endogenous technological progress (Matsuyama, 1992), knowledge spillovers, pecuniary externatlities and other agglomeration economies are also likely to be important drivers of structural change (see e.g. WDR 2009 and the references therein for an overview of agglomeration economies). The contribution of our analysis is to show that, even

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[3]That Engel-effects cancontribute to asymmetric sectoral growth rates is well-known - see e.g. Caselli & Coleman (2001), Matsuyama (1992), and Laitner (2000).

in the absence of such auxiliary mechanisms which are probably very important in practice, trade could result in asymmetric sectoral output growth as well as employment shifts, and that small changes in preferences can result in sharply discontinuous patterns of spatial specialization. Since facilitating trade is likely to benefit the non-farm sector most, market integration appears an effective way of catalyzing economic development in economies dominated by agriculture, i.e. much of the developing world.

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Appendix

A1. Optimal time allocation under autarky. Individual i chooses the time share parameter α_i so as to maximize utility:

$$\ln U_i = \max_{\alpha_i} a \ln \left(\alpha_i A_F^i \right) + (1 - a) \ln \left((1 - \alpha_i) A_{NF}^i \right),$$

subject to $0 \leq \alpha_i \leq 1$. The Cobb-Douglas functional form implies that corner solutions can be ruled out. The first-order condition (f.o.c.) is :

$$\frac{a A_F^i}{\alpha_i A_F^i} = \frac{(1 - a) A_{NF}^i}{(1 - \alpha_i) A_{NF}^i},$$

which implies $a = \alpha_i$.

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A2. Optimal time allocation under trade. In equilibrium, the Nash product

$$\Omega = \ln\left(U_1 - \overline{U}_1\right) + \ln\left(U_2 - \overline{U}_2\right),$$

is maximized with respect to inputs α_1, α_2, subject to

$$0 \le \alpha_i \le 1, i = 1, 2. \tag{1}$$

The resulting utility sharing rule implies:

$$U_1 - \overline{U}_1 = U_2 - \overline{U}_2.$$

Since the threat points $\overline{U}_1, \overline{U}_2$ are fixed, the time allocation parameters α_1, α_2 will be chosen in order to maximise total utility:

$$V = \max_{\alpha_1, \alpha_2} \left[U\left(F_1\left(\alpha_1, \alpha_2\right), NF_1\left(\alpha_1, \alpha_2\right)\right) + U\left(F_2\left(\alpha_1, \alpha_2\right), NF_2\left(\alpha_1, \alpha_2\right)\right) \right],$$

where F_i, NF_i denote food and non-food consumption for producers $i = 1, 2$. Let F, NF denote total food and non-food production:

$$
\begin{aligned}
F &= F_1 + F_2 = \alpha_1 A_F^1 + \alpha_2 A_F^2 \\
NF &= NF_1 + NF_2 = (1 - \alpha_1) A_{NF}^1 + (1 - \alpha_2) A_{NF}^2.
\end{aligned}
$$

For any F, NF, optimal consumption is such that:

$$
\begin{aligned}
\frac{U\left(F_1, NF_1\right)}{F_1} &= \frac{U\left(F_2, NF_2\right)}{F_2} \\
\frac{U\left(F_1, NF_1\right)}{N_1} &= \frac{U\left(F_2, NF_2\right)}{N_2},
\end{aligned}
$$

hence $F_1/NF_1 = F_2/NF_2 = F/NF$. It follows that producer i will consume the same share of total food production as of total non-food production:

$$
\begin{aligned}
F_1 &= \theta F, \\
NF_1 &= \theta NF, \\
F_2 &= (1-\theta)\, F, \\
NF_2 &= (1-\theta)\, NF,
\end{aligned}
$$

where θ is determined by the bargaining process. The utility maximization problem therefore simplifies to

$$
\begin{aligned}
V &= \max_{\alpha_1,\alpha_2} \left[U\left(\theta F\left(\alpha_1,\alpha_2\right), \theta NF\left(\alpha_1,\alpha_2\right)\right) + U\left(\left(1-\theta\right) F\left(\alpha_1,\alpha_2\right), \left(1-\theta\right) NF\left(\alpha_1,\alpha_2\right)\right) \right], \\
V &= \max_{\alpha_1,\alpha_2} \left[\left(\theta + \left(1-\theta\right)\right) U\left(F\left(\alpha_1,\alpha_2\right), NF\left(\alpha_1,\alpha_2\right)\right) \right], \\
V &= \max_{\alpha_1,\alpha_2} \left[U\left(F\left(\alpha_1,\alpha_2\right), NF\left(\alpha_1,\alpha_2\right)\right) \right],
\end{aligned}
$$

subject to (1). Using the Cobb-Douglas functional form, and taking logarithms, we thus obtain

$$
\ln V = \max_{\alpha_1,\alpha_2} a \ln\left(\left[\alpha_1 A_F^1 + \alpha_2 A_F^2\right]\right) + (1-a) \ln\left(\left[\left(1-\alpha_1\right) A_{NF}^1 + \left(1-\alpha_2\right) A_{NF}^2\right]\right),
$$

subject to (1).

A2.1 Types of solutions. The following table is useful for characterizing the types of solutions to the maximization problem above

Table A1 - Types of solutions

	$\alpha_1 = 0$	$0 < \alpha_1 < 1$	$\alpha_1 = 1$
$\alpha_2 = 0$	A	g	h
$0 < \alpha_2 < 1$	B	F	i
$\alpha_2 = 1$	C	D	E

Assume that producer 1 has a comparative advantage in the production of food,

$$
\frac{A_F^1}{A_F^2} \frac{A_{NF}^2}{A_{NF}^1} > 1.
$$

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It can then be seen immediately that the solutions A,C,E can never be optimal: A & E result in zero consumption of one of the goods and so are inadmissible; and C is always inferior to h. In what follows we first prove that B,F,D can never be optimal, leaving us with three types of solutions: g,h,i. We then derive the conditions determining which of the types g,h,i will be the optimal solution, depending on skills and preferences.

A2.2 Optimal α_2, conditional on α_1   Suppose the solution for α_2 is interior, $0 < \alpha_2 < 1$. The f.o.c. with respect to α_2:

$$\frac{a}{F} A_F^2 = \frac{(1-a)}{NF} A_{NF}^2.$$

After some tedious algebra,we obtain the solution for α_2:

$$aA_F^2 \left[(1-\alpha_1) A_{NF}^1 + (1-\alpha_2) A_{NF}^2 \right] = (1-a) A_{NF}^2 \left[\alpha_1 A_F^1 + \alpha_2 A_F^2 \right]$$

$$aA_F^2 (1-\alpha_1) A_{NF}^1 + aA_F^2 A_{NF}^2 - \alpha_1 (1-a) A_{NF}^2 A_F^1 = \alpha_2 (1-a) A_{NF}^2 A_F^2 + \alpha_2 a A_{NF}^2 A_F^2$$

$$aA_F^2 (1-\alpha_1) A_{NF}^1 + aA_F^2 A_{NF}^2 - \alpha_1 (1-a) A_{NF}^2 A_F^1 = \alpha_2 A_{NF}^2 A_F^2,$$

thus

$$\alpha_2 = a + a (1-\alpha_1) \frac{A_{NF}^1}{A_{NF}^2} - \alpha_1 (1-a) \frac{A_F^1}{A_F^2}. \tag{2}$$

Provided the solution for α_2 is interior, total food consumption is equal to

$$
\begin{aligned}
F &= \alpha_1 A_F^1 + \alpha_2 A_F^2 \\
F &= \alpha_1 A_F^1 + \left(a + a (1-\alpha_1) \frac{A_{NF}^1}{A_{NF}^2} - \alpha_1 (1-a) \frac{A_F^1}{A_F^2} \right) A_F^2 \\
F &= \alpha_1 A_F^1 + aA_F^2 + a (1-\alpha_1) \frac{A_{NF}^1}{A_{NF}^2} A_F^2 - \alpha_1 (1-a) \frac{A_F^1}{A_F^2} A_F^2 \\
F &= \alpha_1 A_F^1 + aA_F^2 + a (1-\alpha_1) \frac{A_{NF}^1}{A_{NF}^2} A_F^2 - \alpha_1 (1-a) A_F^1 \\
F &= a\alpha_1 A_F^1 + aA_F^2 + a (1-\alpha_1) \frac{A_{NF}^1}{A_{NF}^2} A_F^2.
\end{aligned}
$$

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Total non-food consumption is equal to

$$NF \; = \; (1 - \alpha_1) A_{NF}^1 + \left(1 - \left(a + a(1 - \alpha_1)\frac{A_{NF}^1}{A_{NF}^2} - \alpha_1(1 - a)\frac{A_F^1}{A_F^2}\right)\right) A_{NF}^2$$

$$NF \; = \; (1 - \alpha_1) A_{NF}^1 + (1 - a) A_{NF}^2 - a(1 - \alpha_1) A_{NF}^1 + \alpha_1(1 - a)\frac{A_F^1}{A_F^2} A_{NF}^2$$

$$NF \; = \; (1 - \alpha_1)(1 - a) A_{NF}^1 + (1 - a) A_{NF}^2 + \alpha_1(1 - a)\frac{A_F^1}{A_F^2} A_{NF}^2.$$

Total utility is thus:

$$\ln V \; = \; a \ln \left(a\alpha_1 A_F^1 + a A_F^2 + a(1 - \alpha_1)\frac{A_{NF}^1}{A_{NF}^2} A_F^2\right)$$

$$+ (1 - a) \ln \left((1 - \alpha_1)(1 - a) A_{NF}^1 + (1 - a) A_{NF}^2 + \alpha_1(1 - a)\frac{A_F^1}{A_F^2} A_{NF}^2\right).$$

Differentiate with respect to α_1:

$$\frac{d \ln V}{d\alpha_1} \; = \; \frac{a^2}{F}\left(A_F^1 - \frac{A_{NF}^1}{A_{NF}^2} A_F^2\right) + \frac{(1 - a)^2}{NF}\left(-A_{NF}^1 + \frac{A_F^1}{A_F^2} A_{NF}^2\right)$$

$$\frac{d \ln V}{d\alpha_1} \; = \; \frac{a^2}{F}\left(1 - \frac{A_{NF}^1}{A_{NF}^2}\frac{A_F^2}{A_F^1}\right) A_F^1 + \frac{(1 - a)^2}{NF}\left(\frac{A_F^1}{A_F^2}\frac{A_{NF}^2}{A_{NF}^1} - 1\right) A_{NF}^1$$

$$\frac{d \ln V}{d\alpha_1} \; = \; \frac{a^2}{F}\left(\frac{R - 1}{R}\right) A_F^1 + \frac{(1 - a)^2}{NF}(R - 1) A_{NF}^1, \qquad (3)$$

where

$$R = \frac{A_F^1}{A_F^2}\frac{A_{NF}^2}{A_{NF}^1} > 1$$

is the ratio of the relative food-productivity of individual 1 to that of individual 2. All terms on the right-hand side of (3) are non-negative. It follows that utility is a monotonic function of α_1, which implies the solution for α_1 is a corner solution (specialization) whenever the solution for α_2 is interior. This implies F in Table A1 cannot be optimal. It also follows that R determines whether $\frac{d \ln V}{d\alpha_1}$ is positive or negative, i.e. whether α_1 will be equal to one or zero:

$$\alpha_1 \; = \; 1 \text{ if } R > 1$$

$$\alpha_1 \; = \; 0 \text{ if } R < 1.$$

Hence, if producer 1 has a comparative advantage in food production, B in Table A1 cannot be optimal. A corollary is that, if α_1 is interior, then α_2 must be a corner solution, and since producer 2 has a comparative advantage in non-food production, $\alpha_2 = 0$ must be the solution in this case; hence D cannot be optimal.

A2.3 Distinguishing between potential solutions   The optimal solution will be such that it falls into one of the cells g,h,i shown in Table A2:

Table A2 - Remaining solutions

	$0 < \alpha_1 < 1$	$\alpha_1 = 1$
$\alpha_2 = 0$	g	h
$0 < \alpha_2 < 1$	Not optimal	i

In regime i, we have

$$
\begin{aligned}
\alpha_1 &= 1 \\
\alpha_2 &= a - (1-a)\left(A_F^1/A_F^2\right) > 0,
\end{aligned}
$$

where the latter equation follows from (2). In this regime, then,

$$
\frac{A_F^1}{A_F^2} < \frac{a}{(1-a)}. \tag{4}
$$

In regime g, we have

$$
\begin{aligned}
\alpha_1 &= a + a\left(A_{NF}^2/A_{NF}^1\right) < 1 \\
\alpha_2 &= 0,
\end{aligned}
$$

thus

$$
\frac{A_{NF}^1}{A_{NF}^2} > \frac{a}{1-a}. \tag{5}
$$

Producer 1 has a comparative advantage in food production,

$$
\frac{A_F^1}{A_F^2}\frac{A_{NF}^2}{A_{NF}^1} > 1,
$$

hence:

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- If (4) holds then (5) does not hold.
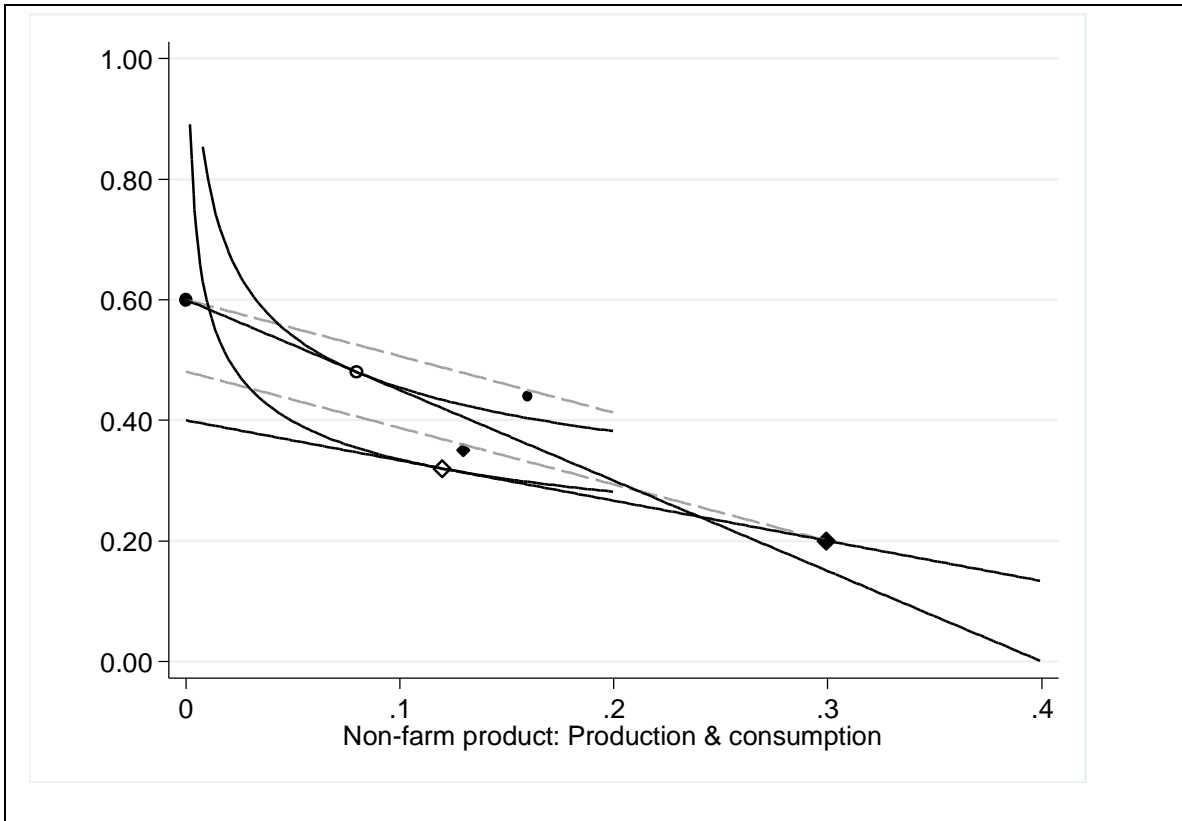
- If (5) holds then (4) does not hold.

Finally, suppose **neither** (4) nor (5) holds, so that

$$\frac{A_{NF}^1}{A_{NF}^2} \leq \frac{a}{1-a} \leq \frac{A_F^1}{A_F^2}.$$

In this case, neither g nor i can be optimal, leaving h (complete specialization) as the only remaining candidate solution.

This completes the proof of Proposition 1 in the text.

Figure 1: The asymmetric effect of trade on production and consumption of the non-farm product

Note: The solid straight lines indicate the production possibility curves for the two hypothetical individuals discussed in the text, under autarky. The indifference curves are drawn for the Cobb Douglas utility function discussed in the text, with $a = 0.8$. Optimal production and consumption in autarky for individual 1 and 2 is indicated by the hollow small circle, and the hollow small diamond, respectively. Once the individuals engage in trade, the solution to the bargaining problem is such that production occurs in the points indicated by the solid large circle for individual 1, and the solid large diamond for individual 2. Individual 1 consumes in the point indicated by the solid small circle, and individual 2 consumes in the point indicated by the small solid diamond. The terms of trade are indicated by the gray dashed lines. Both individuals increase their consumption of nonfarm goods.