CHAPTER 3

EMPIRICAL MODELLING OF TRADE AND THE ENVIRONMENT

JOHN BEGHIN, SÉBASTIEN DESSUS, DAVID ROLAND-HOLST, AND DOMINIQUE VAN DER MENSBRUGGHE

1. INTRODUCTION

This chapter provides a complete and technical description of the computable general equilibrium (CGE) model, which underlies our country case studies. The model attempts to capture some of the key features relating to environmental emissions. These features include (*a*) linking emissions to the consumption of polluting inputs (as opposed to output); (*b*) including emissions generated by final demand consumption; (*c*) integrating substitutability between polluting and non-polluting inputs (including capital and labour); (*d*) capturing important dynamic effects, such as capital accumulation, population growth, productivity and technological improvements, and vintage capital (through a putty/semi-putty specification); and (*e*) including emission taxes to limit the level of pollution.

In addition to these important elements for studying environmental linkages, the model includes other structural features, which may be of interest to policy-makers. For example, detailed labour markets and household specifications are conducive to an analysis of the incidence of economic policies. While the model is rich in structure, it also lacks some elements for a more complete analysis of environmental linkages. In its current form, the model is useful only for calculating the economic costs of limiting emissions, without the concomitant, but certainly important, evaluation of the benefits. Chapter 5 makes a partial attempt to address the measurement of benefits related to the health impact of pollution applied in the case study of Chile (Chapter 6). The second major lacuna is the lack of abatement technology which is a relevant decision variable for producers. The results of the analysis therefore may tend to overstate the costs of controlling emissions if more cost-effective alternatives exist in the form of abatement equipment or “cleaner” capital. The third deficiency is that the study focuses on “industry-based” pollution and ignores other significant environmental issues such as deforestation, soil degradation and erosion, solid waste and its disposal, and other potentially serious problems.

Next, the chapter provides a brief overview of some of the key features of the model. A complete description of each block of the model follows. Then we provide a list of the differences among the data and model specifications implemented across countries. This is followed by a few concluding remarks.

2. OVERVIEW OF THE MODEL

2.1. Production

All sectors are assumed to operate under constant returns to scale and cost optimisation. Production technology is modelled by a nesting of constant-elasticity-of-substitution (CES) functions. See Figure A.1 in Appendix A for a schematic diagram of the nesting. The implementation of the model allows for all permissible special cases of the CES function, notably Leontief and Cobb-Douglas.

In each period, the supply of primary factors—capital, land, and labour—is usually predetermined.[1] The model includes adjustment rigidities. An important feature is the distinction between old and new capital goods. In addition, capital is assumed to be partially mobile, reflecting differences in the marketability of capital goods across sectors.[2] Once the optimal combination of inputs is determined, sectoral output prices are calculated assuming competitive supply (zero-profit) conditions in all markets.

2.2. Consumption and Closure Rule

All income generated by economic activity is assumed to be distributed to consumers. Each representative consumer allocates optimally his or her disposable income among the different commodities and saving. The consumption/saving decision is completely static: saving is treated as a “good,” and its amount is determined simultaneously with the demand for the other commodities, the price of saving being set arbitrarily equal to the average price of consumer goods.[3]

The government collects income taxes and indirect taxes on intermediate inputs, outputs, and consumer expenditures. The default closure of the model assumes that the government deficit/saving is exogenously specified.[4] The indirect tax schedule will shift to accommodate any changes in the balance between government revenues and government expenditures. The current account surplus (deficit) is fixed in nominal terms. The counterpart of this imbalance is a net outflow (inflow) of capital, which is subtracted from (added to) the domestic flow of saving. In each period, the model equates gross investment to net saving (equal to the sum of saving by households, the net budget position of the government, and foreign capital inflows). This particular closure rule implies that saving drives investment.

2.3. Foreign Trade

Goods are assumed to be differentiated by region of origin. In other words, goods classified in the same sector are different according to whether they are produced domestically or imported. This assumption is often called the Armington assumption. The degree of substitutability and the import penetration shares are allowed to vary across commodities and across agents. The model assumes a single Armington agent. This strong assumption implies that the propensity to import and the degree of substitutability between domestic and imported goods are uniform across economic agents. This assumption reduces tremendously the dimensionality of the model. In many cases, this assumption is imposed by the data. A symmetric assumption is made on the export side where domestic producers are assumed to differentiate between the domestic market and the export market. This is implemented using a constant-elasticity-of-transformation (CET) production possibility frontier.

2.4. Dynamic Features and Calibration

The current version of the prototype has a simple recursive dynamic structure, as agents are assumed to be myopic and to base their decisions on static expectations about prices and quantities. Dynamics in the model originate in three sources: (*a*) accumulation of productive capital and labour growth; (*b*) the putty/semi-putty specification of technology; and (*c*) shifts in production technology.

2.4.1. Capital accumulation

In the aggregate, the basic capital accumulation function equates the current capital stock to the depreciated stock inherited from the previous period plus gross investment. However, at the sectoral level, the specific accumulation functions may differ because the demand for (old and new) capital can be less than the depreciated stock of old capital. In this case, the sector contracts over time by releasing old capital goods. Consequently, in each period, the new capital vintage available to expanding industries is equal to the sum of disinvested capital in contracting industries plus total saving generated by the economy, consistent with the closure rule of the model.

2.4.2. The putty/semi-putty specification

The substitution possibilities among production factors are assumed to be higher with the new than with the old capital vintages—technology has a putty/semi-putty specification. Hence, when a shock to relative prices occurs (e.g., the imposition of an emissions tax), the demands for production factors adjust gradually to the long-run optimum because the substitution effects are delayed over time. The adjustment path depends on the values of the short-run elasticities of substitution and the replacement rate of capital. As the latter determines the pace at which new vintages are installed, the larger is the volume of new investment, the greater is the possibility to achieve the long-run total amount of substitution among production factors.

2.4.3. Dynamic calibration

The model is calibrated on exogenous growth rates of population, labour force, and gross domestic product (GDP). In the so-called business-as-usual (BAU) scenario, the dynamics are calibrated in each region by imposing the assumption of a balanced growth path. This implies that the ratio between labour and capital (in efficiency units) is held constant over time.[5] When alternative scenarios around the baseline are simulated, the technical efficiency parameter is held constant, and the growth of capital is endogenously determined by the saving/investment relation.

The following indices are used extensively in subsequent equations. Note that the time index generally is dropped from the equations.

- *i* Represents production sectors; *j* is an alias for *i*.
- *nf* Represents the non-fuel commodities.
- *e* Represents fuel commodities.
- *l* Represents the labour types.
- *v* Represents the capital vintages.
- *h* Represents the households.
- *g* Represents the government expenditure categories.
- *f* Represents the final demand expenditure categories (including *g* as a subset).
- *r* Represents trading partners.
- *p* Represents different types of effluents.
- *t* Represents the time index.
- *d* Represents demand.
- *k* Represents capital.
- *m* Represents trade.

3. MODEL DESCRIPTION

3.1. Production

Production is based on a nested structure of CES functions. Each sector produces a gross output,[6] *XP*, which, given the assumption of constant returns to scale, is undetermined by the producer. It will be determined by equilibrium conditions. The producer therefore minimises costs subject to a production function, which is of the CES type. At the first level, the producer chooses a mix of a value-added aggregate, *VA*,[7] and an intermediate demand aggregate, *ND*.[8] In mathematical terms, this leads to the following formulation:

$$\min PVA_i VA_i + PN_i ND_i$$

$$\text{s.t.} \quad XP_i = \left[a_{va,i} VA_i^{r_i^p} + a_{nd,i} ND_i^{r_i^p} \right]^{1/r_i^p},$$

where *PVA* is the aggregate price of value added, *PN* is the price of the intermediate aggregate, a_{va} and a_{nd} are the CES share parameters, and r is the CES exponent.[9] The exponent is related to the CES elasticity (s), via the following relationship:

$$r_i^p = \frac{s_i^p - 1}{s_i^p} \Leftrightarrow s_i^p = \frac{1}{1 - r_i^p} \quad \text{and} \quad s_i^p \geq 0 .$$

Note that in the model, the share parameters incorporate the substitution elasticity using the following relationships:

$$a_{va,i} = \left(a_{va,i} \right)^{s_i^p} \quad \textit{and} \quad a_{nd,i} = \left(a_{nd,i} \right)^{s_i^p} .$$

Solving the minimisation problem above yields equations (1) and (3), which make up part of the top-level production equations [(1)-(6)]. Because of the assumption of vintage capital, we allow the substitution elasticities to differ according to the vintage of the capital. Depending on the available data, and due to the importance of energy in terms of pollution, we separate energy demand from the rest of intermediate demand and incorporate the demand for energy directly in the value-added nest. Hence, the equations below are not specified in terms of a value-added bundle, but are specified as a value-added plus energy bundle.

Equation (1) determines the volume of aggregate intermediate non-energy demand by vintage, *ND*. Equation (2) determines the total demand for intermediate non-energy aggregate inputs (summed over vintages). Equation (3) determines the level of the composite bundle of value-added demand and energy, *KEL*. *PKEL* is the price of the *KEL* bundle. The CES dual price of *ND* and *KEL*, *PX,* is defined by equation (4). Equation (5) determines the aggregate unit cost, *PX*, exclusive of an output subsidy/tax.[10] Finally, we allow the possibility of an output subsidy (j) or tax (t), generating a wedge between the producer price and the output price, *PP*, yielding equation (6). The production tax is multiplied by an adjustment factor (d), which normally is fixed at unit value. However, it is possible to endogenise the average level of the production tax to achieve a pre-determined fiscal target.

$$ND^v_j = a^v_{nd,j} \left[\frac{PX^v_j}{PN_j} \right]^{s^{p,v}_j} XP^v_j \tag{1}$$

$$ND_j = \sum_v ND^v_j \tag{2}$$

$$KEL^v_j = a^v_{kel,j} \left[\frac{PXv^v_j}{PKEL^v_j} \right]^{s^{p,v}_j} XP^v_j \tag{3}$$

$$PX^v_j = \left[a^v_{nd,j} PN_j^{1-s^{p,v}_j} + a^v_{va,j} PKEL^{v\,1-s^{p,v}_j}_j \right]^{1/(1-s^{p,v}_j)} \tag{4}$$

$$PX_j XP_j = \sum_v PXv^v_j XP^v_j \tag{5}$$

$$PP_j XP_j = PX_j \left(1 + d^p t^p_j - j^p_j \right) XP_j \tag{6}$$

The next level of the CES nest concerns aggregate intermediate demand, *ND*, on the one side, and the *KEL* bundle on the other side. The split of *ND* into intermediate demand is assumed to follow the Leontief specification; in other words, it has a substitution elasticity of zero. (We also assume that the share coefficients are independent of the vintage.) Equations (7)-(11) represent the second-level CES production equations. The demand for non-fuel intermediate goods (XAp_{nf}) is determined by equation (7). The intermediate demand coefficients are given by $a_{nf,j}$. The price of aggregate intermediate demand (*PN*) is given by adding up the unit price of intermediate demand. This is specified in equation (8). Demand for each good is specified as a demand for the Armington composite (described in more detail below), an aggregation of a domestic good and an import good, which are imperfect substitutes. Therefore, while there is no substitution of one intermediate good for another, there will be substitution between domestic demand and import demand, depending on the relative prices. The price of the Armington good is given by *PA*.

At the same level, the *KEL* bundle is split between labour and a capital-energy bundle, *KE*. We assume here, as well, that the substitution possibilities between labour and the *KE* bundle depend on the vintage of the capital. The optimisation problem is similar to that above, i.e., cost minimisation subject to a CES aggregation function. If *AW* is the aggregate sectoral wage rate, and *PKE* is the price of the *KE* bundle, aggregate labour demand *AL* and demand for the *KE* bundle are given by equations (9) and (10). Parameters $a_{l,i}$ and $a_{ke,i}$ are the CES share parameters, and s^v is the CES elasticity of substitution. The price of the *KEL* bundle, *PKEL*, is determined by equation (11), which is the CES dual price.

$$XAp_{nf,j} = \sum_v a_{nf,j} ND_j^v \tag{7}$$

$$PN_j = \sum_{nf} a_{nf,j} PA_{nf} \tag{8}$$

$$AL_j^d = \sum_v a^v_{l,j} \left[\frac{PKEL_j^v}{AW_j} \right]^{s_j^{v,v}} KEL_j^v \tag{9}$$

$$KE_j^v = a^v_{ke,j} \left[\frac{PKEL_j^v}{PKE_j^v} \right]^{s_j^{v,v}} KEL_j^v \tag{10}$$

$$PKEL_j^v = \left[a^v_{l,j} \left(AW_j \right)^{1-s_j^{v,v}} + a^v_{ke,j} \left(PKE_j^v \right)^{1-s_j^{v,v}} \right]^{1/(1-s_j^{v,v})} \tag{11}$$

The combined labour bundle is split into labour demand by type of labour, each with a specific wage rate, *W*.[11] (Though labour markets are assumed to clear for each skill category, we allow for differential wage rates across sectors, reflecting the potential for different institutional arrangements.) Equation (12) determines labour demand by skill type (L^d) in each sector, using a CES aggregation function. We allow for changes in labour efficiency (l), which can be specified by both skill type and by sector. The producer decision can also be influenced by a wage tax, which is represented by the variable t^l. Φ represents the productivity coefficient. The dual price, or the average sectoral wage, *AW*, is defined by equation (13).

$$L^d_{l,j} = \frac{a^l_{lj}}{l_{lj}} \left[\frac{l_{lj} AW_j}{\Phi_{lj} W_l \left(1 + t^l_l\right)} \right]^{s^l_j} AL^d_j \tag{12}$$

$$AW_j = \left[\sum_l a^l_{lj} \left(\frac{\Phi_{lj} W_l \left(1 + t^l_l\right)}{l_{lj}} \right)^{1-s^l_j} \right]^{1/(1-s^l_j)} \tag{13}$$

The next set of equations [(14)-(16)] describe the disaggregation of the capital-energy bundle, *KE*, into its energy and capital-land components. Equation (14) determines the demand for aggregate energy (E*v*). *PE* is the price of the energy component. Equation (15) determines the demand for the capital-land bundle by vintage, *KT*, where *PKT* is the price of the capital-land bundle. Equation (16) defines the dual price of the *KE* bundle.

$$E^v_j = a^v_{e,j} \left[\frac{PKE^v_j}{PE^v_j} \right]^{s^{k,v}_j} KE^v_j \tag{14}$$

$$KT^{d,v}_j = a^v_{kt,j} \left[\frac{PKE^v_j}{PKT^v_j} \right]^{s^{k,v}_j} KE^v_j \tag{15}$$

$$PKE^v_j = \left[a^v_{e,j} \left(PE^v_j\right)^{1-s^{k,v}_j} + a^v_{k,j} \left(PKT^v_j\right)^{1-s^{k,v}_j} \right]^{1/(1-s^{k,v}_j)} \tag{16}$$

The next level in the nest determines the demand for the capital and land factors [equations (17)-(21)]. Equation (17) defines land demand by sector and vintage, where *PT* is the price of land. Similarly, equation (18) determines demand for capital by sector and vintage, where *R* is the rental rate of capital. Note that the rental rate is both sector and vintage specific. Both equations incorporate technology shifters (which will be explained in the section on dynamics).[12] Equations (20) and (21) determine, respectively, aggregate sectoral land demand (T^d) and capital demand (K^d).

$$T^{d,v}_j = \frac{a^v_{t,j}}{l^v_{t,j}} \left[\frac{l^v_{t,j} PKT^v_j}{PT_j} \right]^{s^{s,v}_j} KT^v_j \tag{17}$$

$$K_j^{d,v} = \frac{a_{k,j}^{v}}{l_{k,j}^{v}} \left[\frac{l_{k,j}^{v} PKT_j^{v}}{R_j^{v}} \right]^{s_j^{s,v}} KT_j^{v} \tag{18}$$

$$PKT_j^{v} = \left[a_{t,j}^{v} \left(\frac{PT_j}{l_{t,j}^{v}} \right)^{1-s_j^{s,v}} + a_{k,j}^{v} \left(\frac{R_j^{v}}{l_{k,j}^{v}} \right)^{1-s_j^{s,v}} \right]^{1/\left(1-s_j^{s,v}\right)} \tag{19}$$

$$T_j^d = \sum_v T_j^{d,v} \tag{20}$$

$$K_j^d = \sum_v K_j^{d,v} \tag{21}$$

The energy bundle determined by equation (14) is further disaggregated by energy type. The number of fuel types will depend on the available data. We let the index *e* range over the number of fuel types (eventually the dimension of *e* could even be 1). Equations (22) and (23) represent the decomposition of the energy bundle. Equation (22) determines the demand for the different types of fuels (XAp_e). The *?* factor allows for energy-efficiency improvement over time, which can be sector specific, as well as vintage specific. Equation (23) determines the CES dual price, *PE*, of the energy bundle.

$$XAp_{e,j} = \sum_v \frac{a_{e,j,v}^{f}}{l_{jv}^{e}} \left[\frac{l_{jv}^{e} PE_j^{v}}{PA_e} \right]^{s_{jv}^{f}} E_j^{v} \tag{22}$$

$$PE_j^{v} = \left[\sum_e a_{e,j,v}^{f} \left(\frac{PA_e}{l_{jv}^{e}} \right)^{1-s_{jv}^{f}} \right]^{1/(1-s_{jv}^{f})} \tag{23}$$

3.2. Income Distribution

Production generates income, both wage and non-wage, which is distributed in some form to three main institutions: households, government, and financial institutions (both domestic and foreign). Equations (24)-(27) represent the corporate earnings equations. Equation (24) determines gross operating surplus, *KY*. It is the sum across all vintages and all sectors of capital remuneration, and it incorporates factor payments from abroad (*FP*). *ER* is the exchange rate. Equation (25) defines company income, *CY*, as equal to a share of gross operating surplus (the rest being distributed to households and to foreigners). Equation (26) determines corporate taxes, Tax^c. The base tax rate is given by the parameter $?^c$. However, corporate taxes can be endogenised (in order to meet a fiscal target, for example), in which case the adjustment parameter, d^c, becomes endogenous. Equation (27) defines retained earnings, i.e., corporate saving (Sav_c). Corporate saving is equal to a residual share (f^c) of after-tax company income, net of transfers to the rest of the world, TR_c^r.[13] The remaining amount of net company income is distributed to households characterized by share f_h^c:

$$KY = \sum_v \sum_i R_i^v Kv_i^{d,v} + ER \sum_r FP_r^k \tag{24}$$

$$CY = c^k KY \tag{25}$$

$$Tax^c = d^c k^c CY \tag{26}$$

$$Sav_c^p = \left(1 - \sum_h f_h^c\right)\left(1 - d^c k^c\right) CY - ER \sum_r TR_c^r \tag{27}$$

Household income derives from two main sources: capital and labour income. Additionally, households receive transfers from the government and from abroad. The next set of equations [(28)-(31)] make up the household income equations. Equation (28) defines total labour income, *YL*, as the product of total labour demand and the wage rate. There are two adjustments. One comes from wages earned abroad (*FW*); the other concerns wages remitted to foreign labour. In the latter case, a fixed share of total domestic labour income is assumed to be distributed to foreign labour, while in the former case, foreign wage income is assumed to be constant (in dollar terms).

Labour income is distributed to the households. Equation (29) defines total household income, *YH*. It is the sum of labour income, distributed capital income and net company income, income from land, and transfers from the government, TR_g^h, and from abroad, TR_r^h. Capital, company, and land income are distributed to households using fixed shares (f). The adjustment factor d^{HTr} on government transfers can be used as a fiscal instrument in order to achieve a specified target, similar to the adjustment factors on other taxes in the model. *P* is the price index. Household direct tax, Tax_h^H, is given by equation (30), where $?^h$ is the tax rate. The adjustment factor d^{HTr} can be endogenous if the government saving/deficit is exogenous. In this case, the household tax schedules shifts in or out to achieve the net government balance. Otherwise, the household tax schedule is exogenous, and the factor stays at its initial value of 1. Finally, equation (31) defines household disposable income, *YD*. Disposable income is equal to total household income less taxes and transfer payments to the rest of the world.

$$YL_l = c_l^l \sum_i \Phi_{lj} W_l L_{l,j}^d + ER \sum_r FW_r^l \tag{28}$$

$$\begin{aligned} YH_h \; = \; & \sum_l f_h^l YL_l + f_h^k c^h KY + f_h^c \left(1 - d^c k^c\right) CY + f_h^t \sum_j PT_j T_j^d \\ + \; & Pd^{HTr} \; TR_{g,h}^h + ER \sum_r TR_r^h \end{aligned} \tag{29}$$

$$Tax_h^H = d^h k_h^h YH_h \tag{30}$$

$$YD_h = YH_h - Tax_h^H - ER \sum_r TR_h^r \tag{31}$$

3.3. Household Consumption and Savings

Household disposable income is allocated to goods, services, labour, and savings using the Extended Linear Expenditure System (ELES) specification.[14] The consumption function of each household follows the same specification, but income elasticities are household specific. The consumer problem can be set up as follows:

$$\max \; U = \sum_{i=1}^{n} m_i \ln\left(C_i - q_i\right) + m_s \ln\left(\frac{S}{cpi}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} PC_i C_i + S = YD$$

$$\text{and} \quad \sum_{i=1}^{n} m_i + m_s = 1$$

where U is the utility function, C_i is consumption by commodity, S is household saving, PC is the vector of consumer prices, and YD is disposable income. *?* and *µ* are preference parameters, which will be given an interpretation below. Variable S can be thought of as demand for a future bundle of consumer goods. For reasons of simplification, we assume that the saving bundle is evaluated using the consumer price index, *cpi*. Lluch (1973) provides a more detailed theoretical analysis of how savings enters the utility maximisation problem.

Solving the above optimisation problem leads to the following demand functions:

$$C_i = q_i + \frac{m_i Y^*}{PC_i},$$

$$S = m_s Y^* = YD - \sum_{i=1}^{n} PC_i C_i \ ,$$

$$Y^* = YD - \sum_{j=1}^{n} PC_j q_j \ .$$

Consumption is the sum of two parts, ?, which is often called the subsistence minima or floor consumption, and a fraction of Y^*, which is often called the supernumerary income. Variable Y^* is equal to disposable income less total expenditures on the subsistence minima.

The following six equations represent the equations of the consumer demand system. Equation (32) defines the consumer price vector (for goods and services), *PC*, as the Armington price (*PA*), incorporating household-specific indirect taxes and subsidies. Equation (33) defines supernumerary income, that is, disposable income less total expenditures on the subsistence minima. (The subsistence minima are adjusted each period by the growth rate in population, *Pop*.) Consumer demand for goods and services (XA_c) is given by equation (34).[15] Household savings (*HSav)* is determined as a residual and is given in equation (35). Aggregate household saving (S_H) is determined by equation (36). Equation (37) defines the consumer price index.

$$PC_{ih} = PA_i(1 + t^h_{ih})(1 - j^h_{ih}) \qquad (32)$$

$$Y^*_h = YD_h - Pop_h \sum_i PC_{ih} q_{ih} \qquad (33)$$

$$XAc_{ih} = Pop_h\, q_{ih} + m_{ih} Y_h^* / PC_{ih} \quad (34)$$

$$HSav_h^p = YD_h - \sum_i PC_{ih} XAc_{ih} \quad (35)$$

$$S_H = \sum_h HSav_h^p \quad (36)$$

$$cpi_h = \frac{\sum_i PC_{ih} XAc_{ih}}{\sum_i PC_{ih,0} XAc_{ih}} \quad (37)$$

3.4. Other Final Demands

All other final demand accounts (except stock changes) are integrated into a single demand matrix component. In the most general version of the model, the final demand components are government current expenditures, government capital expenditures, private capital expenditures, trade and transport margins for domestic sales, and trade and transport margins for imports. All the final demand vectors are assumed to have fixed expenditure shares. The closure of the final demand accounts will be discussed below.

Equations (38)-(43) make up the final demand expenditure equations. Equation (38) determines the composition of final demand for each of the final demand components (*XAFD*). The demand for goods is determined as constant shares (*afd*) of the volume of total final demand, *TFD*. The index *f* covers government current and capital expenditures, private capital expenditures, and both domestic and import trade margin expenditures. Equation (39) determines the value of final demand expenditures, *TFDV*. Equation (40) determines the price of final demand expenditures (*PAFD*) inclusive of taxes and subsidies, *PFD*. Equation (41) determines the aggregate final demand price deflator for each type of final demand account, *PTFD*. Trade and transport margins, will be discussed in more detail in section 3.6. Equations (42) and (43) determine the revenue side of the margins. *PD* is the price of the domestic good. *XD* is the demand for the domestic good. *PM* is the aggregate import price and *XM* is the demand for aggregate imported good.

$$XAFD_i^f = afd_i^f\, TFD_f \quad (38)$$

$$TFDV_f = \sum_i PFD_i^f \, XAFD_i^f \tag{39}$$

$$PFD_i^f = PAFD_i^f \left(1 + t_i^f\right)\left(1 - j_i^f\right) \tag{40}$$

$$PTFD_f = \sum_i afd_i^f \, PFD_i^f \tag{41}$$

$$TFDV_{Margd} = \sum_i x_i^d \, PD_i \, XD_i \tag{42}$$

$$TFDV_{M\arg m} = \sum_i x_i^m \, PM_i \, XM_i \tag{43}$$

Government current expenditures include expenditures on goods and services. Government aggregate expenditures on goods and services are fixed in real terms. Equations (44)-(46) represent the current government expenditure equations. Total nominal government expenditures, *GExp*, are determined by equation (44). There are several exogenous elements that enter this equation, including transfers to households, TR_g^h. TR_g^r is government transfers abroad. *FDIT* denotes indirect taxes paid by final demand expenditures and *FDSubs* is the level of subsidies for other final demand expenditures. Note that there is a potential adjustment factor attached to the household transfer variable. Also note that all domestic transfers are typically held fixed and are multiplied by a price index (*p*) in order to ensure the homogeneity of the model. Equation (45) defines the government expenditure deflator, *PG*. Finally, equation (46) is simply an identity, which equates aggregate real government expenditures (*TG*) to the variable *TFD* (for the accounts indexed by *g*).

$$\begin{aligned} GExp \;&=\; \sum_g \sum_i PA_i \, XAFD_i^g + \sum_g \left(FDIT_g - FDSubs_g\right) \\ &+\; P\sum_h \left(d^{HTr} TR_{g,h}^h\right) + ER\sum_r TR_g^r \end{aligned} \tag{44}$$

$$PG_g \; TG_g = \sum_i PFD_i^g \, XAFD_i^g \tag{45}$$

$$TFD_g = TG_g \tag{46}$$

3.5. Government Revenues and Saving

Government derives most of its revenues from direct corporate and household taxes and indirect taxes. Subsidies are also provided, which enter as negative revenues. Equations (47)-(50) list all the different indirect taxes paid by production activities, household consumption, final demand expenditures, and exports, represented by *PITx*, *HITx*, *FDITx*, and *EITx*, respectively. Equation (51) describes the sum of all indirect taxes, *TIndTax*. In equation (50), *PEr* denotes the export price and *ESr* the export supply.

$$PITx = \sum_i \left(d^p t_i^p - j_i^p\right) PX_i XP_i \tag{47}$$

$$HITx = \sum_h \sum_i PA_i t_{ih}^h XAc_{ih} \tag{48}$$

$$FDITx_f = \sum_i PA_i t_i^f XAFD_i^f \tag{49}$$

$$EITx = \sum_r \sum_i PEr_{ir} t_{ir}^E ESr_{ir} \tag{50}$$

$$TIndTax = PITx + HITx + \sum_f FDITx_f + EITx \tag{51}$$

Equations (52)-(54) define the level of subsidies for household consumption, other final demand expenditures, and exports, represented by *HSubs*, *FDSubs*, and *ESubs*, respectively. Total subsidies (*TSubs*) are given by equation (55).

$$HSubs = \sum_h \sum_i PA_i (1 + t_{ih}^h) j_{ih}^h XAc_{ih} \tag{52}$$

$$FDSubs_f = \sum_i PA_i \ (1 + t_i^f) \ j_i^f XAFD_i^f \tag{53}$$

$$ESubs = \sum_r \sum_i PEr_{ir}\ (1 + t_{ir}^E)\ j_{ir}^E\ ESr_{ir} \tag{54}$$

$$TSubs = HSubs + \sum_f FDSubs_f + ESubs \tag{55}$$

The next set of equations [(56)-(60)] define fiscal closure for the government. Equation (56) describes total income from import tariffs, where *WPM* are world prices, t^m are tariffs, and *XMr* represents import volumes. All the relevant import variables are doubly indexed because they represent variables by sector and region of origin. The exchange rate (*ER*) is used to convert world prices (e.g., in dollars) into local currency. There is an additional adjustment factor d^{Tar}, which allows the aggregate tariff rate to vary endogenously. Equation (57) identifies miscellaneous government revenues *(MiscRev*) as all revenues less household direct taxes.

Equation (58) provides total current government nominal revenues, *GRev*. Equations (59) and (60) define respectively the nominal and real level of government saving, S_G *and* RS_g. *GExp* denotes government expenditure. Two government closure rules are implemented. Under the default rule, government saving is held fixed (typically at its base value), and one of the taxes (or government transfers to households) is allowed to adjust (uniformly) to achieve the government fiscal target. Under the second closure rule, all tax levels and transfers are fixed, and real government saving is endogenous. This latter rule can have a significant effect on the level of investment, as investment is savings driven.

$$YTrade = ER\, d^{Tar} \sum_r \sum_i WPM_{ri}\, t_{ri}^m\, XMr_{ri} \tag{56}$$

$$\begin{aligned} MiscRev \quad = \quad & Tax^c + TIndTax - TSubs + YTrade + ER \sum_r TR_r^g \\ + \quad & \sum_l \sum_i t_l^l W_l \Phi_{li} L_{l,j}^d + \sum_i \left[t_i^{vd} PD_i XD_i + t_i^{vm} PM_i XM \right] \end{aligned} \tag{57}$$

$$GRev = MiscRev + \sum_h Tax_h^H \tag{58}$$

$$S_G = GRev - GExp \tag{59}$$

$$RSg = S_G / P \tag{60}$$

3.6. Trade, Domestic Supply, and Demand

Similar to many trade CGE models, we have assumed that imported goods are not perfect substitutes for goods produced domestically.[16] The degree of substitution will depend on the level of disaggregation of the commodities. For example, wheat is more substitutable as a commodity than are grains, which in turn are more substitutable than a commodity called primary agricultural products. The Armington assumption reflects two stylised facts. First, trade data shows the existence of two-way trade, which is consistent with the Armington assumption. Second, and related to the first fact, the Armington assumption leads to a model where perfect specialisation, which is rarely observed, is avoided.

In this version of the model, we have adapted the CES functional specification for the Armington assumption. This has some undesirable properties, which have been explored in more detail elsewhere,[17] but alternative formulations have proven to be deficient as well. The adoption of the constant elasticity of transformation (CET) specification for exports alleviates to some extent the deficiencies of the Armington CES specification. We also assume that there is only one domestic Armington agent, which is sometimes known as the border-level Armington specification. It is parsimonious in both data requirements and computational resources.

To allow for the existence of multiple trading partners, the model adopts a two-level CES nesting to represent the Armington specification (see Figure A.2 in Appendix A).[18] At the top level, agents choose an optimal combination of the domestic good and an import aggregate, which is determined by a set of relative prices and the degree of substitutability. Let *XA* represent aggregate demand for an Armington composite, with the associated Armington price of *PA*. Each agent then minimises the cost of obtaining the Armington composite, subject to an aggregation function. This can be formulated as follows:

$$\begin{aligned} \min \quad & PD\ XD + PM\ XM \\ \text{s.t.} \quad & XA = \left[a_d XD^r + a_m XM^r \right]^{1/r}, \end{aligned}$$

where *XD* is demand for the domestic good, *PD* is the price of obtaining the domestic good, *XM* is demand for the aggregate imported good, *PM* is the aggregate import price, *a* are the CES share parameters, and ? is the CES exponent. Exponent ? is related to the CES substitution elasticity (s) via the following:

$$r = \frac{s - 1}{s} \Leftrightarrow s = \frac{1}{1 - r} \ .$$

At the second level of the nest, agents choose the optimal choice of imports across regions, again as a function of the relative import prices and the degree of substitution across regions. Note that the import prices are region specific, as are the tariff rates. The second-level nest also uses a CES aggregation function. The CES formulation implies that the substitution of imports between any two pairs of importing partners is identical. The following set of equations [(61)-(64)] lists the solution of the optimisation problem described above and represents the top-level Armington equations. Equation (61) determines domestic demand for the Armington aggregate across all agents of the economy, *XA*. Equations (62) and (63) determine, respectively, the optimal demand for the domestic component of the Armington aggregate, *XD*, and aggregate import demand, *XM*. Equation (64) defines the price of the Armington bundle, *PA*, which is the CES dual price. Both the domestic price of domestic goods and the price of the aggregate import bundle are adjusted to incorporate a value-added tax (t) and trade and transportation margins (*?*). Both the tax and margin are assumed to differ between domestic and import goods.

$$XA_i = \sum_j XAp_{ij} + \sum_h XAc_{ih} + \sum_f XAFD_f \tag{61}$$

$$XD_i = b_i^d \left(\frac{PA_i}{PD_i \left(1 + t_i^{vd} + x_i^d\right)} \right)^{s_i^m} XA_i \tag{62}$$

$$XM_i = b_i^m \left(\frac{PA_i}{PM_i \left(1 + t_i^{vm} + x_i^m\right)} \right)^{s_i^m} XA_i \tag{63}$$

$$PA_i = \begin{bmatrix} b_i^d \left(PD_i \left(1 + t_i^{vd} + x_i^d\right)\right)^{1-s_i^m} \\ + b_i^m \left(PM_i \left(1 + t_i^{vm} + x_i^m\right)\right)^{1-s_i^m} \end{bmatrix}^{1/(1-s_i^m)} \tag{64}$$

Equations (65)-(67) describe the decomposition of the aggregate import bundle, *XM*, into its components, i.e., imports by region of origin and represent the second-level Armington equations characterised by substitution elasticities s_i^w. Each demand component will be a function of the price of the exporting partner, as well as of partner-specific tariff rates. Equation (65) determines import volume by sector and region of origin, *XMr*, where *PMr* is the partner-specific import price, in domestic currency and inclusive of tariffs. Equation (66) defines the price of the aggregate import bundle, *PM*, which is the CES dual price. Finally, equation (67) defines the domestic import price, *PMr*, which is equal to the import price of the trading partner, converted into local currency, and inclusive of the partner-specific tariff rate.

$$XMr_{ri} = b_{ri}^{r}\left(\frac{PM_i}{PMr_{ri}}\right)^{s_i^w} XM_i \tag{65}$$

$$PM_i = \left[\sum_r b_{ri}^{r}\left(PMr_{ri}\right)^{1-s_i^w}\right]^{1/\left(1-s_i^w\right)} \tag{66}$$

$$PMr_{ri} = ERWPM_{ri}\left(1+t_{ri}^{m}\right) \tag{67}$$

Treatment of domestic production is symmetric to the treatment of domestic demand. Domestic producers are assumed to perceive the domestic market as different from the export market. The reason is similar: a high level of aggregation. Further, export markets might be more difficult to penetrate, perhaps forcing different quality standards than those applicable for the domestic market. This formulation assumes a production possibilities frontier where each producer maximises sales, subject to being on the frontier and influenced by relative prices. The optimisation problem is formulated somewhat differently because the object of the local producer is to maximise sales, not to minimise costs. We therefore have the following:

$$\begin{aligned} &\max \quad PD\ XD + PE\ ES \\ &\text{s.t.} \quad XP = \left[g_d XD^{l} + g_e ES^{l}\right]^{1/l} \end{aligned}$$

where *XD* is aggregate domestic sales of domestic production, *PD* is the domestic price, *ES* is foreign sales of domestic production (exports) with a producer export price of *PE*, *XP* is aggregate domestic production with a producer price of *PP*, ? are the CET share parameters, and ? is the CET exponent. The CET exponent is related to the CET substitution elasticity, ? , via the following:

$$l = \frac{\Lambda+1}{\Lambda} \Leftrightarrow \Lambda = \frac{1}{l-1}\ .$$

Analogous to the Armington specification, producer supply decisions are assumed to be undertaken in two steps (see Figure A.3 in Appendix A). First, producers choose the optimal combination of domestic supply and aggregate export supply. Then, an additional step is taken which optimises export supply across trading partners. The top-level producer supply decisions, in reduced form, are given by equations (68) and (69), where the share parameters are a^t and the CET substitution elasticity is s^t.[19]

Equation (70) is the CET dual price function, which determines sectoral domestic output. If the CET elasticity is infinite, producers perceive no differentiation across markets, in which case both domestic and export goods are sold at the uniform producer price, *PP*, and output is simply the sum of domestic supply and export supply. (The formulas reflect an adjustment for stock building. Domestic changes in stocks are priced at the aggregate producer price, *PP*, and imported stock changes are priced at the aggregate import price. Trade margins and the value-added tax are not applied to stock changes.) Sectoral stock building is modelled as a fixed share of a volume of stock building, *StB*. This formulation implies that stock building is simply subtracted from (added to) total current output, *XP*.

$$\begin{cases} XD_i = a^t_{d,i}\left(\dfrac{PD_i}{PP_i}\right)^{s^t_i}\left(XP_i - a^{d,st}_i StB\right) & \text{if} \quad s^t_i < \infty \\ PD_i = PP_i & \text{if} \quad s^t_i = \infty \end{cases} \tag{68}$$

$$\begin{cases} ES_i = a^t_{e,i}\left(\dfrac{PE_i}{PP_i}\right)^{s^t_i}\left(XP_i - a^{d,st}_i StB\right) & \text{if} \quad s^t_i < \infty \\ PE_i = PP_i & \text{if} \quad s^t_i = \infty \end{cases} \tag{69}$$

$$\begin{cases} PP_i = \left[a^t_{d,i} PD_i^{1+s^t_i} + a^t_{e,i} PE_i^{1+s^t_i}\right]^{1/(1+s^t_i)} & \text{if} \quad s^t_i < \infty \\ XP_i = XD_i + ES_i + a^{d,st}_i StB & \text{if} \quad s^t_i = \infty \end{cases} \tag{70}$$

The second-level CET nest determines the optimal supply of exports to individual trading partners, *ESr* characterised by transformation elasticities s^z_i. Equation

(71) defines export supply by region of destination. Equation (72) determines the aggregate export price, *PE*.

$$ESr_{ir} = a_{ir}^{r}\left(\frac{PEr_{ir}}{PE_i}\right)^{s_i^z} ES_i \qquad (71)$$

$$PE_i = \left[\sum_r a_{ir}^{r}\left(PEr_{ir}\right)^{1+s_i^z}\right]^{1/\left(1+s_i^z\right)} \qquad (72)$$

Equations (73)-(75) represent the equations that determine export demand by the regional trading partners and the export market equilibrium condition. Equation (73) defines export demand by trading partner, *ED*. If the exporting country has some market power, it will face a downward-sloping demand curve. This is implemented using a constant elasticity function, with the elasticity given by s^e. Export demand will also be influenced by the price of competing exports. This is reflected in the variable *WPINDEX*, which is exogenous because it is assumed the domestic economy does not influence export prices of its trading partners. (Changes in the *WPINDEX* could show the impacts of exogenous changes in the terms-of-trade.) Under the small-country assumption, the export demand elasticity is infinity, and the exporting country faces a flat demand curve; i.e., the export price is fixed (in dollar terms). Equation (74) converts the domestic export producer price (*WPE*) into the domestic export price inclusive of taxes and subsidies (however, it is still in local currency). Equation (75) defines the export market equilibrium, i.e., the equality between domestic export supply and foreign demand (*ED*).

$$\begin{cases} ED_{ir} = a_{ir}^{E}\left(\frac{ERWPINDEX_{ir}}{WPE_{ir}}\right)^{s_{ir}^{E}} & \text{if} \quad s_{ir}^{E} < \infty \\ WPE_{ir} = ERWPINDEX_{ir} & \text{if} \quad s_{ir}^{E} = \infty \end{cases} \qquad (73)$$

$$WPE_{ir} = (1+t_{ir}^{E})(1-j_{ir}^{E})PEr_{ir} \qquad (74)$$

$$ESr_{ir} = ED_{ir} \qquad (75)$$

3.7. Equilibrium Conditions

The first factor market equilibrium condition concerns labour [equations (76) and (77)]. Labour demand (L^d), by skill type, is generated by production decisions. In terms of supply, the model implements a simple labour supply curve, where labour supply is a function of the real wage. Equation (76) defines the labour supply curve (L^s). If the supply elasticity (W) is less than infinity, labour supply is a function of the equilibrium real wage rate. In the extreme case where the elasticity is zero, labour is fully employed and fixed. If the elasticity is infinite, the real wage is fixed and there is no constraint on labour supply. This may be an appropriate assumption in cases where the level of unemployment is relatively high.

Equation (77) determines equilibrium on the labour market. If the labour supply curve is not flat, it determines the equilibrium wage rate. If the labour supply curve is flat, it sets labour supply identically equal to aggregate labour demand. Labour by skill type is assumed to be perfectly mobile across sectors; therefore, equation (77) determines the uniform wage by skill type. Because the model allows for wages to vary across sectors, the uniform wage is actually the aggregate wage, which varies uniformly across sectors for each skill type. The relative wages across sectors are held fixed at their base levels.

$$\begin{cases} L_l^s = a_l \left(\dfrac{W_l}{P} \right)^{w_l} & \text{if} \quad w_l < \infty \\ W_l = P\,\overline{W_l} & \text{if} \quad w_l = \infty \end{cases} \tag{76}$$

$$\sum_i L_{l,i}^d = L_l^s \tag{77}$$

Land demand, similar to demand for labour and capital, is generated by the production sector. Land supply is modelled using the CET specification. If the elasticity is infinite, land is perfectly mobile across sectors. If the elasticity is zero, land is fixed and sector specific. Between these two extreme values, land is partially mobile and sectoral supply will reflect the relative rate of return of land across sectors.

Equations (78)-(80) reflect either situation (finite or infinite). In the case of a finite CET elasticity, equation (78) determines the aggregate price of land, *PLand*, which is the CET dual price. The variable *TLand* represents aggregate land supply, which is exogenous. Equation (79) determines sectoral supply of land, T^s, and equation (80) is the equilibrium condition, which determines the sector-specific land price, *PT*. In the case of infinite elasticity, equation (78) determines the aggregate (uniform) price of land through an equilibrium condition, which equates total land supply, *TLand*, to aggregate land demand (T^d). Equation (79) trivially sets the sectoral land price equal to the economy-wide land price, and equation (80) equates sectoral supply to sectoral demand.

$$\begin{cases} PLand = \left[\sum_i a_i^{ts} PT_i^{1+w^s} \right]^{1/(1+w^s)} & \text{if} \quad w^s < \infty \\ TLand = \sum_i T_i^d & \text{if} \quad w^s = \infty \end{cases} \tag{78}$$

$$\begin{cases} T_i^s = a_i^{ts} \left(\frac{PT_i}{PLand} \right)^{w^s} TLand & \text{if} \quad w^s < \infty \\ PT_i = PLand & \text{if} \quad w^s = \infty \end{cases} \tag{79}$$

$$T_i^s = T_i^d \tag{80}$$

3.8. Determination of Vintage Output and Capital Market Equilibrium

The model is set up to run in either comparative static mode or in recursive dynamic mode. Capital market equilibrium is different in the two cases, and each will be described separately. In comparative static mode, no distinction is made between old and new capital. Each sector determines demand for a single aggregate capital good. On the supply side, the model implements a CET supply allocation function (similar to land above). There is a single “capitalist” who owns all the capital in the economy and supplies it to the different sectors based on each sector’s rate of return. Capital mobility across sectors is determined by the “capitalist’s” CET substitution elasticity. The substitution elasticity is allowed to vary from zero to infinity. If the elasticity is zero, there is no capital mobility. This is an adequate description of a short-term scenario. In the polar case, the substitution elasticity is infinite and there is perfect capital mobility. An intermediate value would allow for partial capital mobility.

The next set of equations [(81)-(83)] determines the equilibrium conditions for the capital market in comparative static mode. Equation (81) determines the aggregate rental rate (*TR*). If there is partial capital mobility, the aggregate rental rate is the CET dual price of the sector-specific rates of return. If there is perfect capital mobility, the aggregate rental rate is determined by an equilibrium condition that equates aggregate capital demand (K^d) to total capital supply

(TK^s). Equation (82) determines either sectoral capital supply (K^s) or the sectoral rental rate (R). If capital is partially mobile, sectoral capital supply is determined by the CET first-order condition; i.e., sectoral capital supply is a function of each sector's relative rate of return. If capital is perfectly mobile, the equivalent condition identically sets the sectoral rate of return to the economy-wide rate of return. Finally, equation (83) determines the sectoral rate of return in the case of partial capital mobility. Under perfect capital mobility, it trivially equates capital supply to capital demand.

$$\begin{cases} TR = \left[\sum_i a_i^s R_i^{1+\Lambda} \right]^{1/(1+\Lambda)} & \text{if} \quad \Lambda < \infty \\ TK^s = \sum_i K_i^d & \text{if} \quad \Lambda = \infty \end{cases} \tag{81}$$

$$\begin{cases} K_i^s = a_i^s \left(\frac{R_i}{TR} \right)^{\Lambda} TK^s & \text{if} \quad \Lambda < \infty \\ R_i = TR & \text{if} \quad \Lambda = \infty \end{cases} \tag{82}$$

$$K_i^s = K_i^d \tag{83}$$

The second case is capital market equilibrium in the recursive-dynamic mode. Sectoral output is essentially determined by aggregate demand for domestic output; see equation (70). (In the simplest case, with no market differentiation, output is equal to the sum of domestic demand for domestic output, plus export demand, i.e., $XP = XD+ED$.) The producer decides the optimal way to divide production of total output across vintages. At first, the producer will use all of the capital installed at the beginning; this is the depreciated installed capital from the previous period. If demand exceeds what can be produced with the old capital, the producer will demand new capital. If demand is lower than the output that can be produced with the old capital, the producer will disinvest some of the installed capital.

Equations (84)-(86) determine vintage output. Equation (84) provides the capital/output ratio for old capital, c (note that $K^{d,Old}$ reflects the optimal capital demand for old capital by the producer). Once the capital/output ratio is determined, it is easy to determine the optimal output using old capital. Equation (85) determines this quantity, XP^{Old}, where an upper bound is given by total output.

If the producer owns too much old capital, i.e., the desired output exceeds total demand, the producer will disinvest the difference between the initial capital stock and the capital stock, which will produce the desired demand. Equation (86) determines output produced with new capital as a residual.

$$c_i^{Old} = \frac{K_i^{d,Old}}{XP_i^{Old}} \tag{84}$$

$$XP_i^{Old} = \min\left(K_{i,0}^s / c_i^{Old}, XP_i\right) \tag{85}$$

$$XP_i^{New} = XP_i - XP_i^{Old} \tag{86}$$

If a sector is in decline, i.e., it has too much installed capital given its demand, it will disinvest. The capital supply curve is a simple constant elasticity function of the relative rental rates. The higher is the rental rate on old capital, the higher is the supply of old capital. The formula is

$$K_i^{s,Old} = K_{i,o}^s \left[\frac{R_{i,t}^{Old} / R_{i,t}^{New}}{R_{i,t-1}^{Old} / R_{i,t-1}^{New}}\right]^{h_i^k}$$

where h^k is the disinvestment elasticity. Another way to think of this is to subtract the two capital numbers, i.e.,

$$K_{i,0}^s - K_i^{s,Old} = K_{i,o}^s \left[1 - \left(\frac{R_{i,t}^{Old} / R_{i,t}^{New}}{R_{i,t-1}^{Old} / R_{i,t-1}^{New}}\right)^{h_i^k}\right]$$

This represents the supply of disinvested capital, which increases as the relative rental rate of old capital decreases. At the limit, when the rental rates are equalised, there is no disinvested capital. At equilibrium, demand for old capital (in each declining sector) must equal supply of old capital. We can therefore invert the first equation to determine the rental rate on old capital, assuming that the sector is in decline and supply equals demand. Equations (87)-(90) represent the capital market equilibrium. Equation (87) determines the relative rental rate (*RR*) on old capital for sectors in decline, i.e., the ratio of the old rental rate to the new rental rate. It is bounded above by 1, because the rental rate on old capital in declining sectors is not allowed to exceed the rental rate on new capital.

Equation (88) determines the rental rate on mobile capital. Mobile capital is the sum of new capital, disinvested capital, and installed capital in expanding sectors. It is not necessary to subtract immobile capital from each side of the capital equilibrium condition, i.e., the rental rate on mobile capital can be determined from the aggregate capital equilibrium condition. Equation (89) is an identity that sets the rental rate on new capital (R^{New}) equal to the rental rate on mobile capital (*TR*). Equation (90) determines the rental rate of old capital (R^{Old}). If a sector is disinvesting, the rental rate on old capital is essentially determined by equation (87). If a sector is expanding, then *RR* is equal to 1, and therefore the rental rate on old capital in expanding sectors will be equal to the rental rate of new capital.

$$RR_{i,t} = \min\left(1, RR_{i,t-1}\left(\frac{K_i^{d,v}}{K_{i,0}^s}\right)^{1/h_i^k}\right) \tag{87}$$

$$\sum_i K_i^d = K^s \tag{88}$$

$$R_i^{New} = TR \tag{89}$$

$$R_i^{Old} = TR\ RR_i \tag{90}$$

3.9. Macro Closure

Government closure was discussed above. Current government savings are determined either endogenously, with fixed tax rates, or exogenously, with one of the tax adjustment factors endogenous. Equation (91) is the ubiquitous savings-equals-investment equation. In equation (91), $TFDV_{zp}$ is the value of private investment expenditures, whose value must equal total resources allocated to the private investment sector: retained corporate earning, Sav_c^p; total household savings, S_H; government savings, S_G; the sum across regions of foreign capital flows, S_{fr}; and net of stock building expenditures.

The last closure rule concerns the balance of payments. First, we make the small-country assumption for imports, i.e., local consumption of imports will not affect the border price of imports, *WPM*. Equation (92) is the overall balance-of-payments equation. The value of imports at world (border) prices must equal the value of exports at border prices (i.e., inclusive of export taxes and subsidies) plus net transfers and factor payments, and net capital inflows. The balance-of-payments constraint is dropped from the model due to Walras's Law.

The final equations of the model, equations (93)-(95), are used to calculate the domestic price index (*P*), which is used to inflate real domestic transfers. Note that real GDP (*RGDP*) is measured in efficiency units. The numéraire of the model is the exchange rate. *GDPVA* is nominal *GDP*.

$$\begin{aligned} TFDV_{zp} &= Sav_c^p + S_H + ER\sum_r S_{fr} + S_G \\ &-\sum_i \left(a_i^{d,st} PP_i + a_i^{m,st} PM_i \right) StB \end{aligned} \tag{91}$$

$$\begin{aligned} ER\sum_r\sum_i WPM_{ri} XMr_{ri} \quad = \quad & \sum_r\sum_i WPE_{ir} ESr_{ir} + ER\sum_r S_{fr} \\ - \quad & \left(1 - c_l^l\right)\sum_i \Phi_{li} W_l L_{l,i}^d + ER\sum_l\sum_r FW_r^l \\ - \quad & c_c^r KY + ER\sum_r FP_r^c - ER\sum_r TR_c^r \\ - \quad & ER\sum_r\sum_r \left(TR_h^r - TR_r^h\right) - ER\sum_r \left(TR_g^r - TR_r^g\right) \end{aligned} \tag{92}$$

$$GDPVA = \sum_l W_l \sum_i \Phi_{li} L_{li}^d + \sum_i\sum_v PT_i T_i^{d,v} + \sum_i\sum_v R_i^v K_i^{d,v} \tag{93}$$

$$\begin{aligned} RGDP &= \sum_l W_{l,0} \sum_i \Phi_{li}\, l_{li}^l L_{li}^d \\ &+\sum_i\sum_v PT_{i,0}\, l_{t,i}^v T_i^{d,v} + \sum_i\sum_v R_{i,0}^v\, l_{k,i}^v K_i^{d,v} \end{aligned} \tag{94}$$

$$P = \frac{GDPVA}{RGDP} \tag{95}$$

3.10. Dynamics

We first address predetermined variables; then, we describe capital stock accumulation. We follow this with our assumptions regarding factor productivity and also discuss capital vintage recalibration. Equations (96)-(100) present the variables that are predetermined, i.e., they do not depend on any contemporaneous endogenous variables. Equation (96) determines the labour supply shift factor (a_l), which is equal to the previous period's labour supply shift factor multiplied by an exogenously specified labour supply growth rate (?^{l}). (All dynamic equations reflect the fact that the time steps may not be of equal size. The growth rates are always given as per cent-per annum increases.)

Equation (97) provides a similar equation for population. The population and labour growth rates are allowed to differ. Government (real) expenditures (*TG*) and the transfers between government and households (TR_g^h) grow at the growth rate of GDP ($?^y$). This latter growth rate is exogenously specified (for the BAU scenario). Equations (98) and (99) provide the relevant formulas. Users can input their own exogenous assumptions about these variables. Equation (100) determines the amount of installed capital at the beginning of the period. If a sector is expanding, this will equal the amount of old capital in the sector at the end of the period. If a sector is declining, the amount of old capital at the end of the period will be less than the initial installed capital. The depreciation rate *d* is exogenous.

$$a_{l,t} = \left(1+g^{\,l}\right)^n a_{l,t-n} \tag{96}$$

$$Pop_t = \left(1+g^{\,p}\right)^n Pop_{t-n} \tag{97}$$

$$TG_t = \left(1+g^{\,y}\right)^n TG_{t-n} \tag{98}$$

$$TR^h_{g,t} = \left(1+g^{\,y}\right)^n TR^h_{g,t-n} \tag{99}$$

$$K^s_{i,0,t} = \left(1-d\right)^n K^d_{i,t-n} \tag{100}$$

The motion equation for the aggregate capital stock is given by the following one-step formula:

$$K_t = (1-d)K_{t-1} + I_{t-1},$$

where *K* is the aggregate capital stock, *d* is the annual rate of depreciation, and I_{t-1} is the level of real investment in the previous period. Using mathematical induction, we can deduce the multiperiod transition equation as follows:

$$\begin{aligned} K_t &= (1-d)\left[(1-d)K_{t-2} + I_{t-2}\right] + I_{t-1} \\ &\vdots \\ K_t &= (1-d)^n K_{t-n} + \sum_{j=1}^{n}(1-d)^{j-1} I_{t-j} \end{aligned} .$$

If the step size is greater than one, the model does not calculate the intermediate values for the path of real investment. The investment path is estimated using a simple linear growth model, i.e.,

$$I_j = (1+g^i)I_{j-1}$$

where

$$g^i = \left(\frac{I_t}{I_{t-n}}\right)^{1/n} - 1 .$$

Note that the formula for the investment growth (g) depends on the contemporaneous level of real investment. This explains why the current capital stock is not predetermined. If real investment increases (e.g., because foreign transfers increase), this will have some effect on the current capital stock by way of its influence on the estimated growth rate of real investment. Inserting the formula for the estimated real investment stream in the capital stock equation, we derive

$$K_t = (1-d)^n K_{t-n} + \sum_{j=1}^{n}(1-d)^{j-1}(1+g^i)^{n-j} I_{t-n} \ .$$

A little bit of algebra yields equation (101) for the aggregate capital stock. Equation (102) defines the annualised growth rate of real investment, which is used to calculate the aggregate capital stock. Equation (103) determines the level of normalised capital. There are two indices of capital stock. The first index is the normalised level of capital stock. This index is called normalised because it is the level of capital stock in each sector that yields a rental rate of 1. The second index is the actual level of the capital stock, given in base-year prices. The latter variable is used only in two equations. It is used to determine the depreciation allowance and to update the level of the capital stock in equation (101) (because it is in the same units as the level of real investment).[20]

$$K_t = (1-d)^n K_{t-n} + \frac{(1+g^i)^n - (1-d)^n}{g^i + d} I_{t-n} \qquad (101)$$

$$g^{i} = \left(\frac{I_t}{I_{t-n}}\right)^{1/n} - 1 \quad (102)$$

$$K_t^s = \frac{K_{t-n}^s}{K_{t-n}} K_t \quad (103)$$

Productivity enters the value-added bundle—labour, land, and capital—as separate efficiency parameters for the three factors, differentiated by sector and by vintage. In the current version of the model, and for lack of better information, the labour efficiency factor (and the energy efficiency factor) is exogenous. In defining the reference simulation, the growth path of real GDP is prespecified, and a single economy-wide efficiency factor for land and capital is determined endogenously. In subsequent simulations, i.e., with dynamic policy shocks, the capital and land efficiency factors are exogenous, and the growth rate of real GDP is endogenous.

Equations (104)-(107) represent capital-land efficiency. Equation (104) defines the growth rate of real GDP. In defining the reference simulation, both lagged real GDP and the growth rate ?^{y} are exogenous; therefore, the equation is used to determine the common efficiency factor for land and capital. In subsequent simulations, equation (104) determines ?^{y}, i.e., the growth rate of real GDP. Equations (105) and (106) determine respectively the efficiency factors (*?*) for capital and land. Both are set to the economy-wide efficiency parameter determined by equation (104); however, the model allows for a partition of sectors, where *i*' indexes a subset of all the sectors. It is assumed that the sectors not indexed by *i'* have no efficiency improvement in land-capital. Equation (107) determines the common capital-labour efficiency growth factor, which is stored in a file for subsequent simulations. There are alternative methods for specifying and implementing the reference scenario.

$$RGDP_t = \left(1 + g^{y}\right)^n RGDP_{t-n} \quad (104)$$

$$l^{v}_{k,i'} = l_{kt} \quad (105)$$

$$l^{v}_{t,i'} = l_{kt} \quad (106)$$

$$l_{kt,t} = \left(1 + g^{kt}_{t}\right)^n l_{kt,t-n} \quad (107)$$

At the beginning of each new period, the parameters of the production structure need to be modified to reflect the changing composition of capital. As a new period begins, what was new capital gets added to old capital, i.e., the new old capital has a different composition from the previous old capital. A simple rule is used to recalibrate the production structure: the parameters are calibrated such that they can reproduce the previous period's output using the aggregate capital of the previous period but with the old elasticities. (The parameters of the new production structure are not modified.) The relevant formulas are not reproduced here but can be found in the GAMS code.

3.11. Emissions

Emissions data at a country and detailed level rarely have been collated. An extensive data set exists for the United States, which includes thirteen types of emissions; see Table 1.[21] The emissions data for the United States has been collated for a set of over 400 industrial sectors. Generally, the emissions data has been directly associated with the volume of output. This has several consequences. First, the only way to reduce emissions with a given (abatement) technology is to reduce output. This is often an unpleasant message for developing country policymakers. The second consequence is that the data set ignores important sources of pollution outside the production side of the economy, namely, household consumption. In an attempt to ameliorate this situation, the pollution data of the United States has been regressed on a small subset of inputs in the U.S. input/output table. Using econometric estimates, we have shown that the level of emissions can be explained by a very small subset of inputs.[22] This allows producers to substitute away from polluting inputs, and to use the same pollution coefficients for final demand consumption.

Because the emission factors are originally calculated from a U.S. database, they are appropriately scaled so as to be consistent with the definition of outputs and inputs of the designated country. The following example shows how this is done in practice. Assume, in a specific sector, that output in 1990 has the value $1 billion, and that the estimated amount of lead emitted from that sector is 13,550 pounds. If we normalise the output price to 1 in 1990, the emission factor has units of 1.355×10^{-5} pounds per (1990) U.S.$, or 13.55 pounds per million (1990) U.S.$. If output, in the same sector, is 300 billion pesos (in Mexico in 1988), the dollar equivalent is $131.5 million (1988 U.S.$). Abstracting from inflation, this leads to lead emissions of 1,782 pounds. The emission factor for lead in Mexico (in this sector) would then be 5.94 pounds per billion 1988 pesos.

Equation (108) defines the total level of emissions for each pollutant E_p. The bulk of the pollution is assigned to the direct consumption of goods, which is the second term in the expression. The level of pollution associated with the consumption of each good is constant (across a row of the social accounting matrices [SAMs]); i.e., there is no difference in the amount of pollution emitted

Table 1. Emission Types

Air Pollutants		
1.	Suspended particulates	PART
2.	Sulphur dioxide	SO_2
3.	Nitrogen dioxide	NO_2
4.	Volatile organic compounds	VOC
5.	Carbon monoxide	CO
6.	Toxic air index	TOXAIR
7.	Biological air index	BIOAIR
Water Pollutants		
8.	Biochemical oxygen demand	BOD
9.	Total suspended solids	TSS
10.	Toxic water index	TOXWAT
11.	Biological water index	BIOWAT
Land Pollutants		
12.	Toxic land index	TOXSOL
13.	Biological land index	BIOSOL

per unit of consumption, whether it is generated in production or in final demand consumption. The first term in equation (108) represents what we call process pollution. It is the residual amount of pollution in production that is not explained by the consumption of inputs. In the estimation procedure, a process dummy proved to be significant in certain sectors. Parameter p_i^p are the estimates of emissions per unit of input *i*. If emissions taxes (t_{Poll}) are exogenous, they are specified in physical units, i.e., dollars per pound (or metric ton). Equation (109) converts this into a nominal amount.

$$E_p = \sum_i u_i^p XP_i + \sum_i p_i^p \left(\sum_j XAp_{ij} + \sum_h XAc_{ih} + \sum_f XAFD_f^i \right) \qquad (108)$$

$$t_{Poll} = P \, \bar{t}_{Poll} \qquad (109)$$

Equations (5'), (64'), (62'), (63') and (58') reproduce the corresponding equations in the text if a pollution tax is imposed. The tax can be generated in one of two ways. It can be specified either exogenously (in which case it is multiplied by a price index to preserve the homogeneity of the model) or endogenously, by determining a constraint on the level of emissions. In the latter case, equation (108) is used to define the pollution-level constraint. The tax that is generated by the constraint is the shadow price of equation (108), and equation (109) is not active.

The tax is implemented as an excise tax; i.e., it is implemented as a tax per unit of emission in the local currency. For example, in the United States it would be the equivalent of $x per metric ton of emission. It is converted to a price wedge on the consumption of the commodity (as opposed to a tax on the emission), using the commodity-specific emission coefficient. For example, in equation (5'), the tax adds an additional price wedge between the unit cost of production, exclusive of the pollution tax, and the final unit cost of production. Let production equal 100 (million dollars for example), and let the amount of pollution be equal to 1 metric ton of emission per $10 million of output. Then the total emission in this case is 10 metric tons. If the tax is equal to $25 per metric ton of emission, the total tax bill for this sector is $250. In the formula below, *?* is equal to 0.1 (metric tons per million dollars of output), *XP* is equal to 100 (million dollars), and t_p is equal to $25. The consumption-based pollution tax is added to the Armington price; see equation (64'). However, the Armington decomposition occurs using basic prices. Therefore, the taxes are removed from the Armington price in the decomposition formulae; see equations (62') and (63'). Equation (58') determines the modification to the government revenue equation.

$$PX_j XP_j = \sum_v PX_j^v XP_j^v + \sum_p u_j^p XP_j t_{Poll} \tag{5'}$$

$$PA_i = \left[b_i^d PD_i^{1-s_i^m} + b_i^m PM_i^{1-s_i^m} \right]^{1/(1-s_i^m)} + \sum_p p_i^p t_{Poll} \tag{64'}$$

$$XD_i = b_i^d \left(\frac{PA_i - \sum_p p_i^p t_{Poll}}{PD_i} \right)^{s_i^m} XA_i \tag{62'}$$

$$XM_i = b_i^m \left(\frac{PA_i - \sum_p p_i^p t_{Poll}}{PM_i} \right)^{s_i^m} XA_i \tag{63'}$$

$$GRev = MiscRev + Tax^h + \sum_p t_{Poll} E_p \tag{58'}$$

The change in emission can be decomposed into three effects: composition effect, technical effect, and scale effect. The decomposition is derived from the following formulas:

$$E_i = \left(\frac{XP_i}{XP}\right)\left(\frac{E_i}{XP_i}\right)(XP)$$

$$\sum_i dE_i \cong \underbrace{\sum_i d\left(\frac{XP_i}{XP}\right)\left(\frac{E_i}{XP_i}\right)(XP)}_{Composition} + \underbrace{\sum_i \left(\frac{XP_i}{XP}\right) d\left(\frac{E_i}{XP_i}\right)(XP)}_{Technical} + \underbrace{\sum_i \left(\frac{XP_i}{XP}\right)\left(\frac{E_i}{XP_i}\right) d(XP)}_{Scale}$$

where E represents sectoral emissions, XP_i is sectoral output, and XP is aggregate output. The first term in the second expression is the composition effect, the second term is the technical effect, and the final term is the scale effect.

3.12. Country-Specific Details

This section describes the characteristics of each of the countries' SAMs used to calibrate the model. Table 2 reports the number of sectors (and the corresponding number of products) available in each of the countries' SAM, the number of households and labour types, the number of partner regions, and the number of capital and land types. When two numbers are reported in the same cell, the first denotes the number in the original SAM, and the second denotes the number in the model. For instance, the original Chilean SAM contains 74 sectors of production, but the model is run with 72 sectors, after aggregation. Table 2 also reports the year for which the SAM is constructed, its currency and unit, the unit used in the model, and the exchange rate of the country.

The level of structural detail in each SAM is country specific. Table 3 provides a description of the available accounts for each one of the individual countries (all the other flows in the SAM are present for all countries).

Table 2. SAM Dimensions

	Chile	*China*	*Costa Rica*	*Indonesia*	*Mexico*	*Morocco*	*Vietnam*
Sectors	74/72	64/64	40/38	22/22	93/93	48/48	50/50
Households	5/5	10/10	10/10	10/10	20/20	5/5	1/1
Labour	20/20	16/16	16/16	16/16	8/8	3/3	1/1
Partner region	26/5	1/1	1/1	1/1	1/1	3/3	1/1
Capital	1/1	1/1	1/1	1/1	3/1	1/1	1/1
Land	--	--	--	7/7	--	--	--
Base year of the SAM	1992 and 1995	1987	1991	1990	1989	1995	1989
Currency	peso	yuan	colone	rupiah	peso	dirham	dong
SAM unit	10^6	10^4	10^6	10^9	10^6	10	10^9
Model unit	10^6	10^8	10^9	10^{12}	10^9	10	10^{15}
Exchange rate local currency unit (LCU/$)[a]	3.6259	3.7221	122.43	1842.8	2.4615	8.24234	4300

[a] Source: IMF, series *rf*.

Table 3. Flow of Funds Availability by Country

	Chile	*China*	*Costa Rica*	*Indonesia*	*Mexico*	*Morocco*	*Vietnam*
Enterprise direct taxes	x	x	x	x	x	x	.
Income distributed to households	x	x	.	x	x	x	x
Enterprise saving	.	x	x	x	x	x	x
Export taxes	.	.	x	.	.	.	.
Payments from foreign labour	x	.	.	.	.	x	.
Foreign labour income	x	.	.	.	.	x	.
Capital income to government	.	.	.	x	.	.	.
Government transfers to households	.	x	.	.	.	.	.
Household direct taxes	x	.	x	x	x	x	x
Labour tax	x	.	.	.	.	.	.
Trade and transport margins	x	.	.	x	.	.	x
Capital income distribution matrix	.	.	x	x	.	.	.
Fixed capital consumption	x	.	.	.	.	.	.
Stock change	x	x	x	.	.	.	x
Intra-enterprise transfers	.	.	.	x	.	.	.
Intra-government transfers	.	.	.	x	.	.	.
Intra-household transfers	.	.	.	x	.	.	x
Corporate transfers to ROW	.	.	x	x	x	x	x
Foreign capital income	x	.	x	x	.	.	x
Government transfers to ROW	x	.	x	x	.	.	x

Table 3. Continued

	Chile	*China*	*Costa Rica*	*Indonesia*	*Mexico*	*Morocco*	*Vietnam*
Household transfers to ROW	x	.	x	.	.	.	x
Capital income from ROW	x	.	x	x	x	x	x
Transfers from ROW to enterprises	.	.	.	x	.	.	x
Transfers from ROW to government	x	.	x	x	.	.	x.
Transfers from ROW to households	x	x	x	x	x	x	x

Note: An "x" indicates availability.

4. CONCLUDING REMARKS

This chapter presented the detailed specification of the prototype CGE model used for assessing the links between economic activity and the environment. Quantifying the response of both economic and environmental variables to policy changes, such as trade or environmental measures, is a necessary condition for the design of coherent reforms. Three main aspects of the CGE model presented in this chapter account for its specificity with respect to previous analyses.

First, it embodies a high level of disaggregation for pollutants, products, sectors, and types of households. This model has been used to simulate the impacts of abatement policies targeted to specific air emissions, measuring at the same time the effect on related water and soil pollutants. Trade policy reform, and the induced resource reallocation, does not have a uniform outcome across sectors. The expansion or contraction of specific activities has differentiated environmental consequences. The product disaggregation of the model highlights certain environmental outcomes of trade policy. Moreover, income distribution issues arising from environmental and trade policies and the question of the redistribution of environmental tax receipts are briefly discussed and can be further investigated due to the detailed classification of households.

Second, this model explicitly includes dynamic features, allowing the introduction of exogenous factors, such as productivity shifts and demographic changes, that affect the growth and pollution trajectory. The modelling of a vintage structure for capital also captures import dynamic effects, such as the relation between capital accumulation and the adjustment capacity of the economy to environmental regulation. It is possible to assess to what extent new investment favours the substitution from polluting factors to non-polluting factors. Therefore, negative outcomes of growth in terms of pollution arising from scale effects can be compared to positive ones, to determine the aggregate impact.

Third, most economy-wide studies on growth and environmental linkages rely on effluent intensities associated with output and do not take into account substitution between non-polluting and polluting factors. Abating pollution is then achieved principally by reducing output in pollution-intensive sectors, with a significant cost in terms of growth. By contrast, in our model, pollution emissions are linked to polluting input use rather than to output. Technical adjustment by substituting non-polluting factors for polluting factors therefore may be assessed. Moreover, the model includes emissions generated by final consumption and thus describes the abatement of emissions from both the production and the final consumption sides.

This model has been used to assess the environmental and economic linkages in a diverse group of countries. While it represents progress in the tools used to design optimal policy interventions, there is still a wide scope for improving the methodology. First is the need to assess not only the economic costs of abatement but also the economic and non-economic benefits. Further research is necessary in the valuation of a clean environment for households, and in the identification of the potentially important feedbacks between environmental damage and the economy (e.g., soil degradation and harm to human capital). Finally, a proper assessment of abatement technology, embodied in new capital, would provide a more complete set of policy options for policymakers.

NOTES

[1] Capital supply is to some extent influenced by the current period's level of investment.

[2] For simplicity, it is assumed that old capital goods supplied in second-hand markets and new capital goods are homogeneous. This formulation makes it possible to introduce downward rigidities in the adjustment of capital without increasing excessively the number of equilibrium prices to be determined by the model (see Fullerton 1983).

[3] The demand system is a version of the Extended Linear Expenditure System (ELES), which was first developed by Lluch (1973). The formulation of the ELES in this model is based on atemporal maximisation; see Howe 1975. In this formulation, the marginal propensity to save out of supernumerary income is constant and independent of the rate of reproduction of capital.

[4] In the reference simulation, the real government fiscal balance converges (linearly) towards zero by the final period of the simulation.

[5] This involves computing in each period a measure of Harrod-neutral technical progress in the capital-labour bundle as a residual. This is a standard calibration procedure in dynamic CGE modelling; see Ballard et al. 1985.

[6] Gross output is divided into two parts, one part produced with *old* capital and the residual amount produced with *new* capital.

[7] The value-added bundle also contains demand for energy; see below.

[8] Some models of this type assume a top-level Leontief, i.e., a substitution elasticity of zero, in which case there is no substitution possibility between intermediate demand and value added. The GAMS implementation of the model can handle all of the special cases of the CES, i.e., Leontief and Cobb-Douglas.

[9] The CES is described in greater detail in Appendix B.

[10] The unit cost equation will be affected by production-specific emission taxes. Emission taxes are discussed in section 3.11.

[11] The current model specification includes only a single-level nest for disaggregating the aggregate labour bundle. In other words, the substitution across any pair of labour skills is uniform.

[12] Only the Indonesian model includes land as a specific factor of production. All the other country models incorporate the land specification if the data were to be developed from the existing social accounting matrices (SAMs).

[13] In the reference simulation, both the private corporate saving rate and the household saving rate are adjusted (upwards), under the assumption that domestic saving, as a share of GDP, will increase in the future. The adjustments are based on rules of thumb but could be made explicit in the model.

[14] For references, see Lluch 1973 or Deaton and Muellbauer 1980.

[15] As noted earlier, the m parameters are adjusted in the reference simulation in order to increase the level of domestic saving.

[16] This is known as the Armington assumption; see Armington 1969.

[17] See, for example, Robinson, Soule and Weyerbrock 1992.

[18] The current Vietnamese SAM has a single rest-of-the-world account, i.e., an aggregate trading partner. The dual nesting is therefore redundant. However, both the data processing facility and the model retain the multiple-trading-partner specification in order to maintain flexibility for future data developments.

[19] Note the difference between the Armington CES and the CET. First, the relation between the exponent and the substitution elasticity is different. Second, the ratio of the prices and the share parameter in the reduced forms are inverted. This is logical as the goal of the producer is to maximise revenues. For example, an increase in the price of exports, relative to the composite aggregate price, will lead to an increase in export supply.

[20] The following numerical example may clarify the issue. Assume that the value of the capital stock is 100. Assume, as well, that capital remuneration is 10. Capital remuneration is simply rK where r is the rental rate and K is the demand for capital. In this example, rK is equal to 10, which implies a rental rate of 0.1. The model assumes a normalisation rule such that the rental

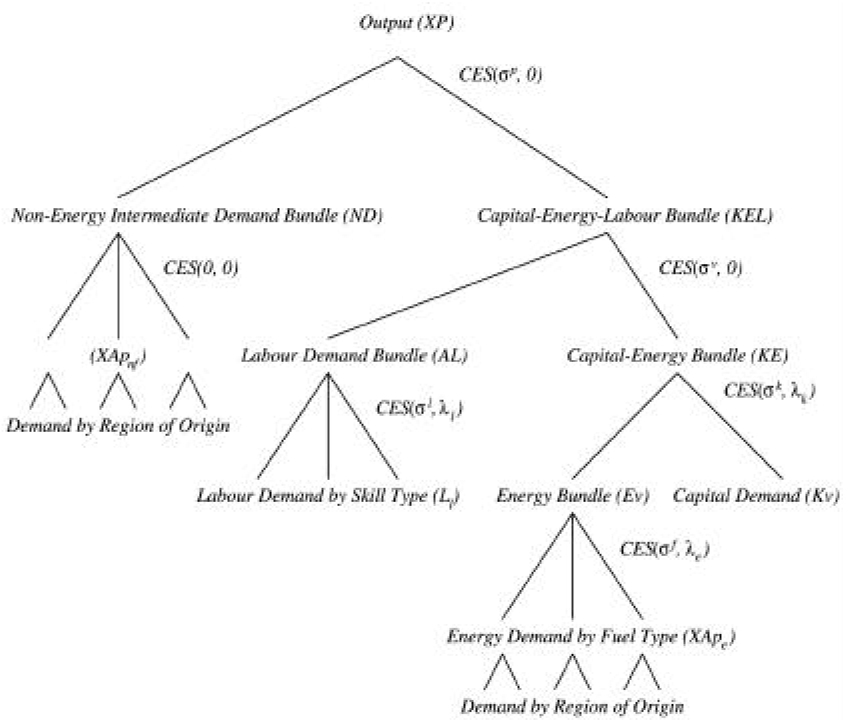
rate is 1, and it normalises the capital data to be consistent with the normalisation rule. In other words, the normalised capital demand is 10, and it is really an index of capital volume. The non-normalised level of capital is used only in the accumulation function and in determining the value of the depreciation allowance. All other capital stock equations use the normalised value of capital.

[21] See Martin et al. 1991.

[22] See Chapter 4.

APPENDIX A – FIGURES

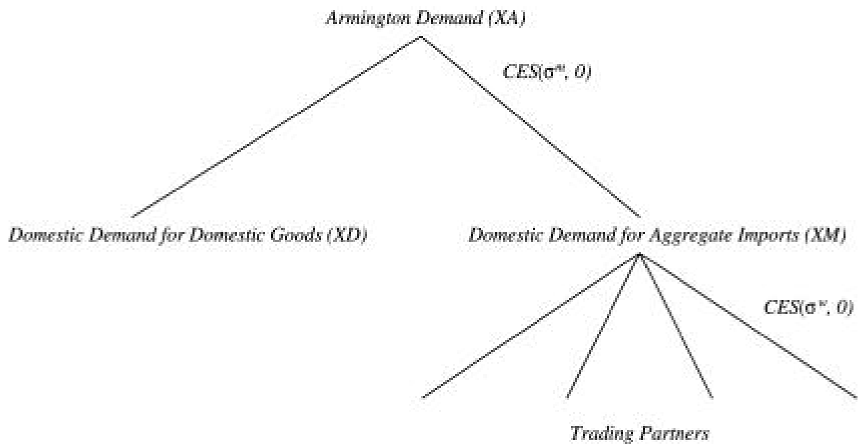
Figure A.1: Production Nesting



Notes:

1. Each nest represents a different CES bundle. The first argument in the CES function represents the substitution of elasticity. The elasticity may take the value zero. Because of the putty/semi-putty specification, the nesting is replicated for each type of capital, i.e., *old* and *new*. The values of the substitution elasticity will generally differ depending on the capital vintage, with typically lower elasticities for *old* capital. The second argument in the CES function is an efficiency factor. In the case of the *KE* bundle, it is only applied on the demand for capital. In the case of the decomposition of labour and energy, it is applied to all components.
2. Intermediate demand, both energy and non-energy, is further decomposed by region of origin according to the Armington specification. However, the Armington function is specified at the border and is not industry specific.
3. The decomposition of the intermediate demand bundle, the labour bundle, and the energy bundle will be specific to the level of aggregation of the model. The diagram only schematically represents the decomposition and is not meant to imply that there are three components in the CES aggregation.

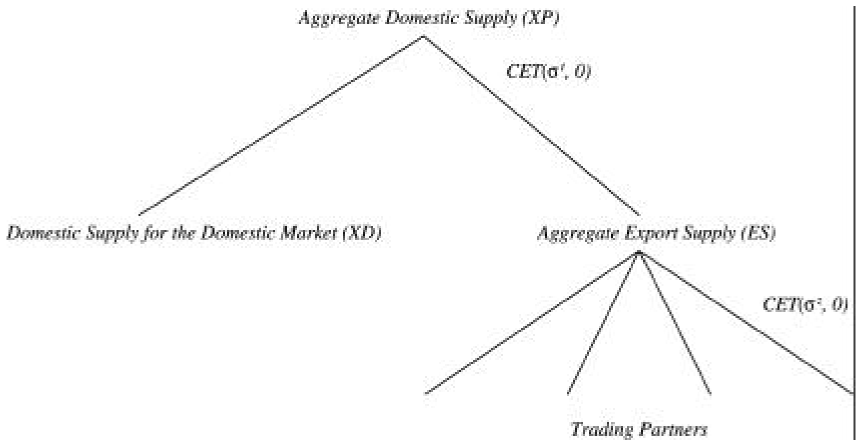
Figure A.2: Armington Nesting



Note:

1. The base SAM includes a single trading partner with Vietnam, though the specification of import demand uses the multiple nesting approach in order to provide flexibility for the future as trade data is developed further. Import demand is modelled as a nested CES structure. Agents first choose the optimal level of demand for the so-called Armington good (*XA*). In a second stage, agents decompose the Armington aggregate good into demand for the domestically produced commodity (*XD*), and an aggregate import bundle (*XM*). At the third and final stage, agents choose the optimal quantities of imports from each trading partner. Import prices and tariffs are specific to each of the trading partners.

Figure A.3: Output Supply (CET) Nesting



Note:

1. The market for domestic output is modelled as a nested CET structure (similar to the note above, the current version of the Vietnamese data only concerns a single trading partner). Producers first choose the optimal level of output (*XP*). (Note that in a perfectly competitive framework, output is determined by equilibrium conditions, and is not a producer decision.) In a second stage, producers choose the optimal mix of goods supplied to the domestic market (*XD*) and an aggregate export supply (*ES*). At the third and final stage, producers choose the optimal mix of exports to each of the individual trading partners. The export price of each trading partner is region specific. Under the small-country assumption, the export price is fixed (in foreign currency terms); otherwise, each trading partner has a downward-sloping demand curve, and the export price is determined endogenously through an equilibrium condition.

APPENDIX B – THE CES/CET FUNCTIONS

Because of the frequent use of the constant-elasticity-of-substitution (CES) function, this appendix will develop some of the properties of the CES, including some of its special cases. The CES function can be formulated as a cost-minimisation problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$

$$\text{s.t.} \quad V = \left[\sum_i a_i (l_i X_i)^r \right]^{1/r}$$

where V is the aggregate volume (of production, for example), X are the individual components ("inputs") of the production function, P are the corresponding prices, and a and ? are technological parameters. Parameters a are most often called the share parameters. Parameters ? are technology shifters. The parameter ? is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

A bit of algebra produces the following derived demand for the inputs, assuming V and the prices are fixed:

$$X_i = a_i l_i^{s-1} \left(\frac{P}{P_i} \right)^s V \qquad \text{(B.1)}$$

where we define the following relationships:

$$r = \frac{s-1}{s} \quad \Leftrightarrow \quad s = \frac{1}{1-r} \quad \text{and} \quad s \geq 0$$

$$a_i = a_i^s$$

and

$$P = \left[\sum_i a_i l_i^{s-1} P_i^{1-s} \right]^{1/(1-s)} . \qquad \text{(B.2)}$$

Variable P, called the CES dual price, is the aggregate price of the CES components. The parameter s is called the substitution elasticity. This term comes from the following relationship, which is easy to derive from equation (B.1):

$$\frac{¶\left(X_i / X_j\right)}{¶\left(P_i / P_j\right)} \frac{\left(P_i / P_j\right)}{\left(X_i / X_j\right)} = -s \ .$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note that we are assuming that the substitution elasticity is a positive number.) For example, if the price of input *i* increases by 10 per cent with respect to input *j*, the ratio of input *i* to input *j* will decrease by (around) s times 10 per cent.

The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero; in other words, there is no substitution between inputs, no matter what the input prices are. Equations (B.1) and (B.2) become

$$X_i = \frac{a_i V}{l_i} , \qquad \text{(B.1')}$$

$$P = \sum_i a_i \frac{P_i}{l_i} . \qquad \text{(B.2')}$$

The aggregate price is the weighted sum of the input prices. The Cobb-Douglas function is for the special case when s is equal to one. It should be clear from equation (B.2) that this case needs special handling. The following equations provide the relevant equations for the Cobb-Douglas:

$$X_i = a_i \frac{P}{P_i} V , \qquad \text{(B.1'')}$$

$$P = A^{-1} \prod_i \left(\frac{P_i}{a_i l_i} \right)^{a_i} , \qquad \text{(B.2'')}$$

where the production function is given by

$$V = A\prod_i \left(l_i X_i \right)^{a_i}$$

and

$$\sum_i a_i = 1 .$$

Note that in equation (B.1") the value share is constant and does not depend directly on technology change.

8.1. Calibration

Typically, the base data set and a given substitution elasticity are used to calibrate the CES share parameters. Equation (B.1) can be inverted to yield

$$a_i = \frac{X_i}{V}\left(\frac{P_i}{P}\right)^s ,$$

assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalised to 1, simplifying the above expression to a true value share. Let us take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalised to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by a, not the share parameters represented by *a*. This saves on computation time because the *a* parameters never appear explicitly in any equation, whereas *a* raised to the power of the substitution elasticity, i.e., s, occurs frequently.)

With less detail, the following describes the relevant formulas for the CET function, which is similar to the CES specification:

$$\max \quad \sum_i P_i X_i$$

$$\text{s.t.} \quad V = \left[\sum_i g_i X_i^l \right]^{1/l} ,$$

where V is the aggregate volume (e.g., aggregate supply), X are the relevant components (sector-specific supply), P are the corresponding prices, g are the

CET share parameters, and *?* is the CET exponent. The CET exponent is related to the CET substitution elasticity, *?*, via the following relation:

$$l = \frac{\Lambda + 1}{\Lambda} \Leftrightarrow \Lambda = \frac{1}{l - 1}.$$

Solution of this maximisation problem leads to the following first-order conditions:

$$X_i = g_i \left(\frac{P_i}{P} \right)^{\Lambda} V,$$

$$P = \left[\sum_i g_i P_i^{1+\Lambda} \right]^{1/(1+\Lambda)},$$

where the ? parameters are related to the primal share parameters, *g*, by the following formula:

$$g_i = g_i^{-\Lambda} \Leftrightarrow g_i = \left(\frac{1}{g_i} \right)^{1/\Lambda}.$$

REFERENCES

Armington, P. (1969), "A Theory of Demand for Products Distinguished by Place of Production," *International Monetary Fund (IMF) Staff Papers*, vol. 16. Washington, D.C.: International Monetary Fund, pp. 159-78.

Ballard, C.L., D. Fullerton, J.B. Shoven, and J. Whalley (1985), *A General Equilibrium Model for Tax Policy Evaluation*. Chicago: University of Chicago Press.

Deaton, A., and J. Muellbauer (1980), *Economics and Consumer Behaviour*. Cambridge, UK: Cambridge University Press.

Fullerton, D. (1983), "Transition Losses of Partially Mobile Industry-Specific Capital," *Quarterly Journal of Economics* 98February): 107-25.

Howe, H. (1975), "Development of the Extended Linear Expenditure System from Simple Savings Assumptions," *European Economic Review* 6: 305-10.

International Monetary Fund (IMF) (various issues), *International Financial Statistics*. Washington, D.C.

Lluch, C. (1973), "The Extended Linear Expenditure System," *European Economic Review* 4: 21-32.

Martin, P., D. Wheeler, M. Hettige, and R. Stengren (1991), "The Industrial Pollution Projection System: Concept, Initial Development, and Critical Assessment," mimeo., The World Bank, Washington, D.C.

Robinson, S., M.J. Soule, and S. Weyerbrock (1992), "Import Demand Functions, Trade Volumes, and Terms of Trade Effects in Multi-Country Trade Models," mimeo., Department of Agricultural and Resource Economics, University of California at Berkeley, January.