# The Impact of Firm Size and Market Size Asymmetries on National Mergers in a Three-Country Model*

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**Abstract**

This paper studies the impact of firm and market size asymmetries on merger decisions. To do that I consider a model where a small and a large country compete in a third (world) market. Each of the two countries has two firms (with potentially different costs) that supply the domestic market and export to the third market. Merger decisions in the two countries are modeled as a simultaneously move game. The paper finds that firms in the large country have more incentives to merge than firms in the small country. In contrast, the government of the large country has more incentives to block a merger than the government of the small country. Thus, the model predicts that conflicts of interest between governments and firms concerning national mergers are more likely in large countries than in small ones.

JEL Codes: F13, H77,L11, L41.
Keywords: Mergers; International Trade; Merger Policy; Size Asymmetry.

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# 1 Introduction

In many European countries there is a heated debate over whether governments and competition authorities should favor or oppose the creation of national champions.[1] An argument often put forth in favor of national champions is that bigger firms will be in a better position to compete in world markets.[2] It's true that the emergence of a national champion might improve a country's welfare if it has strong efficiency gains and shifts profits away from competitors in export markets. However, the emergence of a national champion might also reduce a country's welfare if the efficiency gains are smaller than the loss in consumer surplus. A national champion might also not be able to shift profits away from competitors in export markets due to losses in market share.

This paper contributes to this debate by setting up a three-country model in which firms in two countries serve their respective domestic markets and compete in a third (world) market. I use the model to study endogenous mergers and mergers that improve national welfare. By comparing the equilibrium outcomes of these two games I am able to clarify which factors contribute to the existence of conflicts of interest between firms and governments about the desirability of national champions.

In this model mergers have efficiency gains and are modeled as a simultaneous move game. Firms compete à la Cournot, markets are segmented, and there are no producers in the third market.[3] The novelty of my approach is that it allows for both firm size and market size asymmetries. Firms can have different costs of production and the three countries can have different market demands. These assumptions make the model more general than previous ones and allow me state new results that show how firm and market size asymmetries influence incentives for mergers to take place and merger equilibrium outcomes.

The paper starts by studying the incentives that firms have to merge when governments do not have an active role in merger decisions. To do that I provide conditions under which a merger of firms in one country is profitable conditional on a given market structure in the other country. My first result shows that the conditions for national firms to merge are less restrictive: *(i) when foreign firms are merged, (ii) when firm size asymmetries are high, (iii) when the export market is small, and (iv) in the country with the largest domestic market.*

The intuition behind the results is the following. When a merger takes place there are three effects that the firms involved in the merger need to take into consideration. First, there is an efficiency gain since the high cost firm transfers production to the low cost firm. Second, a merger leads to less competition both in the domestic market as well as in the export market. These two effects allow the merged firm to have a higher mark-up than the highest mark-up of the

[1] For example, the French government advocated a merger between the electricity and gas company SUEZ with the firm GAZ DE FRANCE.

[2] A recent example in Germany has been the aproval of the merger between the E.ON and RUHRGAS corporations where the German Minister of Economics argued that size was very important at the onset of the energy market liberalization in Europe.

[3] This set-up captures the idea that domestic markets are less competitive than export markets. See Brander and Spencer (1985).

individual firms. Thus, the market power of the firms involved in the merger increases in both markets. This effect creates an incentive for mergers to take place. However, in the export market the merger implies that the market share of the merged firm is lower than the sum of the pre-merger market shares of the firms involved in the merger. This third effect reduces the incentive for mergers to occur. We see that a merger increases profits in domestic markets but it does not necessarily increase profits in the export market.

The conditions for a merger of domestic firms are less restrictive when foreign firms are merged because the market-power gains are higher and the loss of market share is lower than when foreign firms are not merged. Incentives for a merger of national firms to occur are stronger when firm size asymmetries are high because the efficiency gains are larger. When firm size asymmetries are low the conditions for a merger of domestic firms are less restrictive when the export market is small because if the merger leads to losses in the export market these losses are smaller. Finally, a merger is more attractive to firms in the large country than to firms in a small country because if the merger leads to losses in the export market these are relatively smaller (compared to the domestic market gains) in the large country than in the small country.

Next the paper characterizes the equilibrium decisions of firms assuming that governments do not interfere in markets. In this game, firms in each country decide whether to merge or not to merge. The decisions of firms in both countries are made simultaneously. I find that for most firm size and market size configurations the equilibrium outcome is for mergers to take place in both countries. However, if firm size asymmetries are low and the size of the export market is sufficiently big there are no mergers. Additionally, I find that if the market of the large country is at least 1.26 times the market of the small country, there exists an asymmetric equilibrium with a merger in the large country but no merger in the small country.[4]

The paper proceeds by studying the incentives that governments have to merge national firms when firms have a passive role in merger decisions. To do that I provide conditions under which a merger of firms in one country increases national welfare conditional on a given market structure in the other country. I find that the conditions for a government to merge national firms are less restrictive: *(i) when foreign firms are merged, (ii) when firm size asymmetries are high, (iii) when the export market is big, and (iv) in the country with the smallest domestic market.*

The intuition behind the results is as follows. To find out whether a merger improves or worsens national welfare, a government must take into account the merger's impact on profits in the domestic and export market and on consumer surplus. If firm size asymmetries are high, then a merger has high efficiency gains. High efficiency gains raise profits in the domestic and in the export market (the increase in mark-up makes up for the loss of market share) and lead to lower consumer surplus losses. Thus, governments have more incentives to merge

[4] For a small set of firm size and market size configurations the merger game played by firms has multiple equilibria.

national firms when firm size asymmetries are high. If firm size asymmetries are moderate, there is a trade-off between welfare losses in the domestic market (profits in the domestic market increase less than the reduction in consumer surplus) and profit gains in the export market. In this case, governments have more incentives to merge national firms when the export market is big. Finally, a merger is more attractive to the government of the small country because when firm size asymmetries are moderate, the losses in the domestic market due to the merger are higher in the large country than in the small one.

The paper also characterizes the equilibrium decisions of governments when firms play a passive role in merger decisions. In this game, governments in each country decide simultaneously whether to merge or not merge national firms. I find that governments choose to merge national firms when (i) firm size asymmetries are high or (ii) firm size asymmetries are moderate and the export market is relatively bigger than the market of the small country. Governments choose not to merge national firms when (i) firm size asymmetries are low or (ii) firm size asymmetries are moderate and the export market is not very big. Additionally, I show if the market of the large country is at least 2.14 times the market of the small country, there exists an asymmetric equilibrium where the government of the small country merges firms but the government of the large country does not.[5]

I also use the model to state conditions under which firms and governments interests regarding the desirability of national mergers are aligned or in conflict. To do that I compare the set of equilibria of the merger game played by firms to that of the merger game played by governments. I find that if firm size asymmetries are high and the export market is big, then there are no conflicts of interest between national firms and governments: all favor the creation of a national champion. The interests of national firms and governments are also aligned if firm size asymmetries are small and the export market is big: all oppose the creation of a national champion. However, if firm size asymmetries are moderate or small and the export market is relatively small, then a conflict of interest arises: firms wish to merge but governments oppose the mergers.

Finally, I show that the conditions under which conflicts of interest occur are less restrictive in the large country than in the small country. This result is driven by the asymmetric equilibria of the merger game played by firms and by the asymmetric equilibria of the merger game played by governments. Comparing the set of equilibria of the two games I show that when one of these two types of asymmetric equilibria occur, firms in the large country prefer to merge but the government of the large country prefers to block the merger. Thus the model predicts that, everything else constant, competition authorities should be less actively involved in the regulation of export industries in small countries than in large ones.

The questions that this paper addresses have many links with the existing literature on merger and competition policy, specially with papers which ex-

[5] There is also a set of firm size and market size configurations where the merger game played by governments has multiple equilibria.

tend the analysis to open economies.[6] This literature has taken two different directions. One line of research focuses on nationally optimal merger policies and merger profitability when trade policy instruments are available to national governments–e.g., Richardson (1999), Horn and Levinsohn (2001), and Huck and Conrad (2004). The other line of research is based on the concept of "external effects" of a merger to outsiders. An important early contribution to this topic is Farrel and Shapiro (1990). This concept was extended to open economies by Barros and Cabral (1994). This literature has derived rather general conditions under which a merger benefits, or harms, the parties not participating in the merger. It does not, however, explicitly consider that a merger may lead to cost reductions and so it can not provide a complete characterization of post-merger equilibrium.

This paper takes a different approach by focusing on a three-country model where firms in each country share their domestic markets and jointly compete in a third market. This approach is similar to that of Haufler and Nielsen (2008) and Suedekum (2006). Haufler and Nielsen (2007) consider a similar set-up but assume that firms have identical costs of production and that the two competitor countries have the same market demand. However, they allow for market size asymmetries between the domestic markets and the export market. They find that the policies enacted by a national merger authority tend to be overly restrictive from a global efficiency perspective. In contrast, all international mergers that benefit the merging firms will be cleared by either a national or a regional regulator, and this laissez-faire approach is also globally efficient.

Suedekum (2008) analyzes mergers in three-country model and, like Haufler and Nielsen (2008), assumes that firms have identical costs of production and the two competitor countries have the same market demand. However, there are iceberg transport costs between the domestic markets and the third market. He uses this framework to study the profitability and social desirability of national versus international mergers. He finds that national mergers can have a negative impact in world surplus, and so national competition policy can be seen as too permissive. However, the promotion of national mergers can be in the interest of individual countries if rent extraction possibilities are strong enough. He also shows that cross-border mergers are more attractive than domestic mergers.

## 2 Set-up

Consider three countries: a small country, $s$, a large country, $L$, and a third country, $t$. Before any merger takes place there are 2 firms in the small country and 2 firms in the large country. There are no firms in the third country. Firms in the small and the large countries sell their product in the domestic markets and export it to the third country. Thus, there is no bilateral trade between the small country and the large country and firms compete in the third (or export) market.

[6] The traditional analysis of mergers and acquisitions in industrial organization–Salant et al. (1983) and Deneckere and Davidson (1985)–usually neglects the effects of country borders.

The inverse demand function in the small country is $P_s = a - Q_s$,with $a > 1$ while the inverse demand function in the large country is $P_L = a - \gamma Q_L$,with $0 < \gamma \leq 1$. The parameter $\gamma$ measures the level of market size asymmetry between the small and the large countries. If $\gamma = 1$ the inverse demand curves are the same and so the markets of the two countries have the same size. If $0 < \gamma < 1$ the market of the large country is bigger than the market of the small country, that is, for any given price, demand in the large country is greater than demand in the small country.[7] The inverse demand function in the export market is $P_t = a - \beta Q_t$, with $0 < \beta \leq 1$. The parameter beta represents the market size of the export market. Demand in the export market can be greater than (when $\beta < 1$) or equal to (when $\beta = 1$) demand in the small country market and can be smaller than (when $\gamma < \beta$), equal to (when $\beta = \gamma$), or greater than (when $\beta < \gamma$) demand in the large country market.

Firms in the small and large countries are fully owned by residents and produce a homogeneous good. Marginal costs are assumed to be nonnegative and constant. There are no fixed costs (this rules out gains from economies of scale in mergers).[8] Marginal costs of firms are given by $c_{v1} = c$, $c_{v2} = c + \Delta$, where $v = s, L$ and $\Delta \in [0, (a - c)/3]$. The parameter $\Delta$ is an index of cost asymmetry. If $\Delta = 0$ all firms have the same cost. I assume that $\Delta$ must be smaller than or equal to $(a - c)/3$ so that, in the absence of mergers, even the less efficient firm makes nonnegative profits in all markets. It is useful to define $\delta = \Delta/(a - c)$ and use it as a summary measure of asymmetry.

Following Barros (1998) I assume that when a merger between two firms occurs the less efficient firm ceases production.[9] Because marginal costs are constant, when two firms merge the merged entity will shut down the high-cost unit and use only the low-cost unit for production. Let $i + j$ stand for the merger between firms $i$ and $j$. Then, the merged entity's marginal cost is equal to $\min(c_{vi}, c_{vj})$. Therefore, a merger can be viewed as an acquisition of a high-cost firm by a low-cost firm.[10]

Firms compete in each market à la Cournot, that is, each firm chooses non-cooperatively and simultaneously the quantity that maximizes its individual profit.[11] Each firm sees the markets it serves as segmented, that is, it is respon-

[7]The reciprocal of $\gamma$ measures how many times the market of the large country is bigger than the market of the small country. For example, if $\gamma = 0.25$ the market of the large country is four times the market of the small country.

[8]Transportation costs between $s$ and $t$ and between $L$ and $t$ are assumed to be equal to zero. Transportation cost can be greater than zero (but lower than $a$) without changing qualitatively the results in the paper.

[9]Barros (1998) proposes a framework with three asymmetric firms and where mergers are endogenously determined. He tries in this way to explain the intuition behind the relationship between initial market concentration and size asymmetry of firms, showing that a negative relation should be expected.

[10]Barros (1998) approach is also used by Qiu and Zhou (2007). Perry and Porter (1985) and Farrell and Shapiro (1990) use other approaches to model the impact of a merger on an industry's cost structure.

[11]The assumption of Cournot competition is in line with much of the literature on mergers. Theoretical and empirical arguments in defence of the Cournot model are presented by Haufler and Nielsen (2005). The model proposed by Kreps and Scheinkman (1983) in which firms

sible for the choice of how much to produce in each market and it considers not just the output of other firms but their own choices about where to produce that output as unaffected by its actions. Thus, firms play separate Cournot games in each market as they take as given the output of each rival in each market and not the total output of each rival in all markets. This means that each market can be analyzed independently of the other markets.

The starting point of the analysis is a situation where no merger has yet taken place. So, at the start, the problem of firm $si$ is given by

$$\max_{q_{si};q_{si}^t} \left(a - \sum_{k=1}^{2} q_{sk} - c_{si}\right) q_{si} + \left(a - \beta\left(\sum_{k=1}^{2} q_{sk}^t + \sum_{k=1}^{2} q_{Lk}^t\right) - c_{si}\right) q_{si}^t.$$

The first-order conditions to this problem are:

$$\begin{aligned} a - 2q_{si} - q_{sj} - c_{si} &= 0. \\ a - 2\beta q_{si}^t - \beta\left(q_{sj}^t + \sum_{k=1}^{2} q_{Lk}^t\right) - c_{si} &= 0. \end{aligned}$$

Solving the first equation with respect to $q_{si}$ we obtain the best reply of firm $si$ in the domestic market:

$$q_{si} = \frac{a - c_{si}}{2} - \frac{1}{2} q_{sj}.$$

Solving the second equation with respect to $q_{si}^t$ we obtain the best reply of firm $si$ in the export market:

$$q_{si}^t = \frac{a - c_{si}}{2\beta} - \frac{1}{2}\left(q_{sj}^t + \sum_{k=1}^{2} q_{Lk}^t\right).$$

Similarly, at the start, the problem of firm $Li$ is:

$$\max_{q_{Li},q_{Li}^t} \left(a - \gamma\sum_{k=1}^{2} q_{Lk} - c_{Li}\right) q_{Li} + \left(a - \beta\left(\sum_{k=1}^{2} q_{Lk}^t + \sum_{k=1}^{2} q_{sk}^t\right) - c_{Li}\right) q_{Li}^t.$$

The first-order conditions to this problem are:

$$\begin{aligned} a - 2\gamma q_{Li} - \gamma q_{Lj} - c_{Li} &= 0 \\ a - 2\beta q_{Li}^t - \beta\left(q_{Lj}^t + \sum_{k=1}^{2} q_{sk}^t\right) - c_{Li} &= 0 \end{aligned}$$

Thus, the best reply of firm $Li$ in the domestic market is

$$q_{Li} = \frac{a - c_{Li}}{2\gamma} - \frac{1}{2} q_{Lj},$$

and the best reply of firm $Li$ in the export market is

$$q_{Li}^t = \frac{a - c_{Li}}{2\beta} - \frac{1}{2}\left(q_{Lj}^t + \sum_{k=1}^{2} q_{sk}^t\right).$$

choose capacities in the first period and compete in prices in the second period generates Cournot outcomes.

# 3 Profitability of Conditional Mergers

My first result provides conditions under which a merger in one of the countries is profitable for a given market structure in the other country.

**Proposition 1**:
*(i) If* $(\beta, \delta)$ *satisfy* $\frac{9}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_s^{L1,L2}(\delta) \leq \beta \leq 1$, *and firms in the large country are not merged, then a merger in the small country is profitable.*
*(ii) If* $(\beta, \delta)$ *satisfy* $\frac{1}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_s^{L1+L2}(\delta) \leq \beta \leq 1$, *and firms in the large country are merged, then a merger in the small country is profitable.*
*(iii) If* $(\beta, \delta)$ *satisfy* $\frac{9\gamma}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_L^{s1,s2}(\delta) \leq \beta \leq 1$, *and firms in the small country are not merged, then a merger in the large country is profitable.*
*(iv) If* $(\beta, \delta)$ *satisfy* $\frac{\gamma}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_L^{s1+s2}(\delta) \leq \beta \leq 1$, *and firms in the small country are merged, then a merger in the large country is profitable when.*

Corollary 1 summarizes the implications of Proposition 1.

**Corollary 1**: *When firms in two countries compete in an export market, the incentives for national firms to merge are higher: (a) when foreign firms are merged, (b) when firm size asymmetries are high, (c) when the export market is small, and (d) in the country with the largest domestic market.*

The best way to explain this result is with a picture.

Figure 1

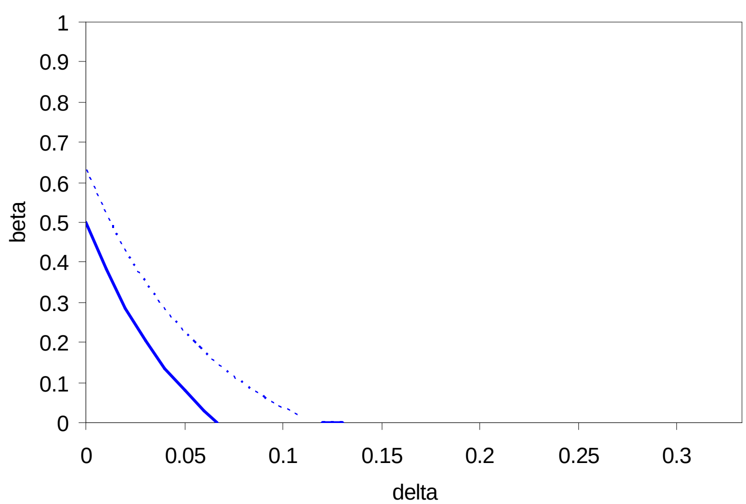



Figure 1 illustrates how the incentives for a merger to take place in the small country depend on $\beta$ and $\delta$. It displays on the horizontal axis the parameter delta, the index of firm size asymmetries and on the vertical axis the parameter beta, which represents the market size of the export market.

The thin dotted curve in Figure 1 represents the equation $\beta = f_s^{L1+L2}(\delta)$ and characterizes incentives for a merger in the small country when firms in the large country are not merged. To the right (left) of this curve firms in the small country choose (not) to merge. The thin dotted curve in Figure 1 tells us that if firms in the large country are not merged and firm size asymmetries are sufficiently high, $\delta \geq 0.11475$, then a merger of firms in the small country is profitable. However, if firms in the large country are not merged and firm size asymmetries are sufficiently low, $\delta \leq 0.11475$, then a merger of firms in the small country is only profitable if the export market is not too big, $f_s^{L1,L2}(\delta) \leq \beta$. This is the content of Proposition 1 part (i).

The thick solid curve in Figure 1 represents the equation $\beta = f_s^{L1,L2}(\delta)$ and characterizes incentives for a merger in the small country when firms in the large country are merged. To the right (left) of this curve firms in the small country choose (not) to merge. The thick solid curve in Figure 1 tells us that if firms in the large country are merged and firm size asymmetries are sufficiently high, $\delta \geq 0.0(6)$, then a merger of firms in the small country is profitable. However, if firms in the large country are merged and firm size asymmetries are sufficiently low, $\delta \leq 0.0(6)$, then a merger of firms in the small country is only profitable if the export market is not too big, $f_s^{L1+L2}(\delta) \leq \beta$. This is the message of Proposition 1 part (ii).

The intuition behind these two results is as follows. The merger raises profits in the domestic market since it implies moving from a duopoly to a monopoly. However, the impact of the merger on profits in the export market may be favorable or unfavorable depending on firm size asymmetries. If firm size asymmetries are high the merger also raises profits in the export market. This happens because high firm size asymmetries imply a large efficiency gain which raises mark-ups (the difference between price and marginal cost) enough to make up for the loss of market share. In contrast, if firm size asymmetries are low the merger reduces profits in the export market. In this case, the merger is only profitable if the profit gains in the domestic market are bigger than the profit losses in the export market. This happens when the export market is not too big.

Comparing parts (i) to (ii) of Proposition 1 we see that a merger of firms in the small country is profitable under less restrictive conditions when firms in the large country are merged than when firms in the large country are not merged. This happens because if firms in the large country are merged than a merger of firms in the small country always leads to losses in the export market. When firms in the large country are not merged and firms in the small country merge there is a move from four to three firms in the export market. By contrast, when firms in the large country are merged and firms in the small country merge there is a move from three to two firms in the export market. A move from three to two firms has associated a larger increase in mark-ups and a smaller loss of market share for small country firms than a move from four to three firms.

Parts (iii) and (iv) of Proposition 1 provide conditions under which a merger of firms in the large country is profitable when firms in the small country are not merged–part (iii)–and when they are merged–part (iv). The intuition is similar to that of parts (i) and (ii), respectively. The novelty here is that the conditions for a merger of firms to be profitable are less restrictive in the large country than in the small country. This happens because for low firm size asymmetries the gains in the domestic market due to the merger are higher in a large country than in a small one.

# 4 Merger Game Played by Firms

I will now characterize the equilibrium decisions of firms in the two countries assuming that governments do not interfere in markets. As the starting point of the analysis I assume that no merger has taken place in either country. Firms in the small country decide jointly whether they wish to merge or not. Firms in the large country also decide jointly if they wish to merge or not. The joint decisions of firms in each country are taken simultaneously. Thus, we have a simultaneous move game and we can use Nash equilibrium to make predictions about behavior.

Let the generic merger game played by firms be denoted by $F_{2,\gamma}$. Denote the strategies available to firms in the small country as $m$ (merger) and $n$ (no merger). Denote the strategies available to firms in the large country as $M$ (merger) and $N$ (no merger).The relevant payoffs of this game are summarized in Table I in the Appendix.

Proposition 2 characterizes the equilibria of the merger game played by firms when the market size of the large country is greater than or equal to that of the small country and less than or equal to 1.26 times the size of the small country's market. Before stating Proposition 2 define

$$p_s = \frac{(63\gamma - 100\beta) - (738\gamma + 800\beta)\delta + (1647\gamma + 2000\beta)\delta^2}{(13 + 162\delta - 603\delta^2)\gamma}, \quad (1)$$

and

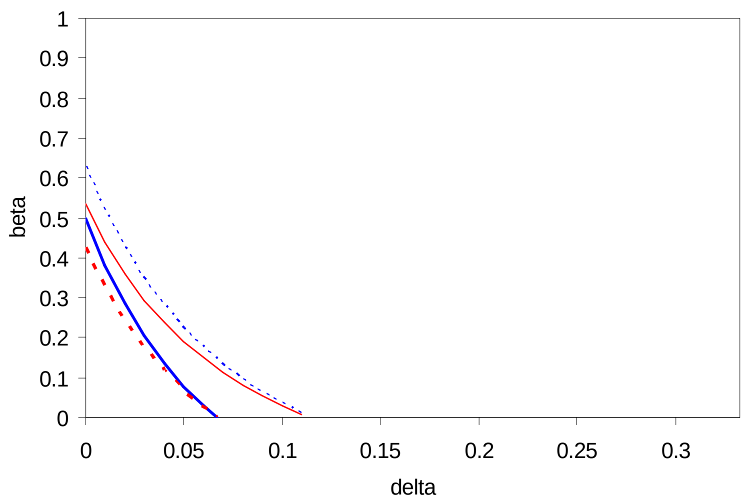
$$p_L = \frac{(63 - 100\beta) - (738 + 800\beta)\delta + (1647 + 2000\beta)\delta^2}{13 + 162\delta - 603\delta^2}. \quad (2)$$

**Proposition 2**: *Let* $0.79365 \leq \gamma \leq 1$.
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq f_s^{L1+L2}(\delta)$*, then* $NE(F_{2,\gamma}) = (n, N)$*;*
*(ii) If* $(\beta, \delta)$ *satisfy* $\max\{0, f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta)$*, then* $NE(F_{2,\gamma}) = \{(n, N), (m, M), (p_s, m; p_L, M)\}$ *where* $p_s$ *and* $p_L$ *are given by (1) and (2), respectively;*
*(iii) If* $(\beta, \delta)$ *satisfy* $\max\{0, f_L^{s1,s2}(\delta)\} < \beta \leq 1$*, then* $NE(F_{2,\gamma}) = (m, M)$.

Figure 2 illustrates how the set of equilibria of the merger game played by firms depends on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is low.[12]

[12] The figure is drawn for $\gamma = 0.85$, that is, when the market of the large country is 1.177 times the market of the small country. The qualitative predictions of the model are the same for any $\gamma \in [0.79365, 1]$.

Figure 2




As in Figure 1, the thick solid and the thin dotted curves in Figure 2 characterize incentives for a merger in the small country. The thick dotted curve in Figure 2 represents the equation $\beta = f_L^{s1+s2}(\delta)$ and characterizes incentives for a merger in the large country when firms in the small country are merged. The thin solid curve in Figure 2 represents the equation $\beta = f_L^{s1,s2}(\delta)$ and characterizes incentives for a merger in the large country when firms in the small country are not merged. To the right of each curve firms merge. To the left of each curve firms do not to merge.

The two solid curves in Figure 2 determine the areas that define the different equilibria of the merger game played by firms. The area in Figure 2 to the right of the thin solid curve represents the set of $\beta$ and $\delta$ where the game has a unique equilibrium where firms in the small country merge and firms in the large country merge: $(m, M)$. The area to the left of the thick solid curve represents parameter configurations where the game has a unique equilibrium where firms in the small country do not merge and firms in the large country do not merge: $(n, N)$. Finally, the area between the two solid curves represents parameter configurations where the game has multiple equilibria. In that area we have two PSNE, $(m, M)$ and $(n, N)$, and one MSNE where firms in the small country merge with probability $p_s$ and firms in the large country merge with probability $p_L$: $(p_s, m; p_L, M)$.[13]

Thus, Proposition 2 tells us that if $0.79365 \leq \gamma$ and firm size asymmetries are high, $\delta \geq 0.11475$, there will be mergers in both countries. This happens because when firm size asymmetries are high, mergers generate profit gains in the domestic and in the export markets. When $0.79365 \leq \gamma$ and firm size

[13] It follows from the definition of $p_s$ and $p_L$ that $p_s < p_L$ in the range of parameters where the MSNE is well-defined.

asymmetries are moderate, $0.0(6) \leq \delta \leq 0.11475$, mergers are not as attractive since they lead to gains in the domestic market but losses in the export market. In this case we have two possible situations. If the export market is sufficiently small, $f_L^{s1,s2}(\delta) \leq \beta$, then firms in the small country merge and firms in the large country merge: $(m, M)$. If the export market is small then the domestic profit gains are larger than the losses in the export market and firms merge. If the export market is big, $\beta \leq f_L^{s1,s2}(\delta)$, then we have multiple equilibria. If $0.79365 \leq \gamma$ and firm size asymmetries are low, $\delta \leq 0.0(6)$, mergers are the least attractive since they generate small profit gains in domestic market and large losses in the export market. In this case we have three outcomes. If the export market is sufficiently small, $f_L^{s1,s2}(\delta) \leq \beta$, firms in the small country merge and firms in the large country merge. If the size of the export market is intermediate, $f_s^{L1+L2}(\delta) \leq \beta \leq f_L^{s1,s2}(\delta)$, we have multiple equilibria. If the export market is sufficiently big, $\beta \leq f_s^{L1+L2}(\delta)$, there are no mergers in both countries.

Proposition 3 characterizes the equilibria of the merger game played by firms when the market size of the large country is more than 1.26 times the market size of the small country.

**Proposition 3**: *Let* $0 < \gamma < 0.79365$.
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$, *then* $NE(F_{2,\gamma}) = (n, N)$*;*
*(ii) If* $(\beta, \delta)$ *satisfy* $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$, *then* $NE(F_{2,\gamma}) = (n, M)$*;*
*(iii) If* $(\beta, \delta)$ *satisfy* $\max\{0, f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta)$, *then* $NE(F_{2,\gamma}) = \{(n, N), (n, N), (p_s, m; p_L, M)\}$, *where* $p_s$ *and* $p_L$ *are given by (1) and (2), respectively;*
*(iv) If* $(\beta, \delta)$ *satisfy* $\max\{0, f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\} < \beta \leq 1$, *then* $NE(F_{2,\gamma}) = (m, M)$.

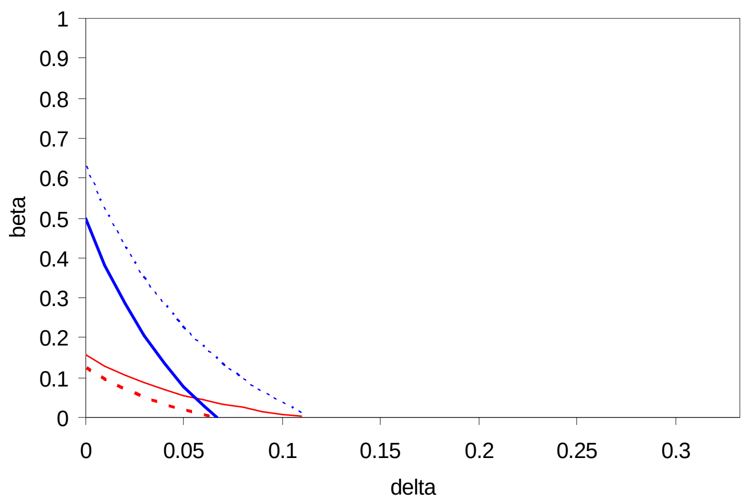
Figure 3 illustrates how the set of equilibria of the merger game played by firms depends on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is high.[14]

In Figure 3 the intersection of the area to the right of the thick solid curve with the area to the right of the thin solid curve represents parameter configurations for where national firms in each country merge: $(m, M)$. The intersection of the area to the left of the thick solid curve with the area to the left of the thin solid curve represents parameter configurations where national firms in each country do not merge: $(n, N)$.

The area to the right of the thin solid curve and to the left of the thick solid curve represents parameter configurations where firms in the small country do not merge and firms in the large country merge: $(n, M)$. Finally, the area to the right of the thick solid curve and to the left of the thin solid curve represents parameter configurations where the merger game played by firms has multiple equilibria: $(m, M)$, $(n, N)$, and $(p_s, m; p_L, M)$.

[14] The figure is drawn for $\gamma = 0.25$, that is, when the market of the large country is 4 times the market of the small country. The qualitative predictions of the model are the same for any $\gamma \in (0, 0.79365)$.

Figure 3




The thick solid and the thin dotted curves in Figure 3 are equal to the ones depicted in Figures 1 and 2 since incentives for mergers in the small country do not depend on the market size of the large country. However, an increase in the market size of the large country changes the incentives for mergers in the large country. Comparing Figures 2 and 3 we see that an increase in the market size of the large country moves the thin solid curve and the thick dotted curve closer to the delta axis. This means that for low firm size asymmetries, an increase in the market size of the large country makes mergers increasingly more attractive in the large country than in the small country. This happens because the bigger the market size of the large country, the greater are the domestic gains of a merger of firms in that country. So, when firm size asymmetries are low and the merger leads to losses in the export market, the bigger the size of the domestic market the more likely is that the domestic profit gains exceed the export market losses and the more attractive is becomes for firms to merge.

The fact that an increase in the market size of the large country makes a merger increasingly more attractive in the large country but not in the small country, implies that there now exists an equilibrium where firms in the large country merge but firms in the small country do not merge. This happens when firm size asymmetries are low, $\delta \leq 0.0(6)$, and the size of the export market is intermediate, $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$.

# 5 Welfare Impact of Conditional Mergers

This section provides conditions under which a merger in one country is welfare improving for a given market structure in the other country. National welfare is the sum of consumer surplus and profits in the domestic and export markets. Proposition 4 describes these conditions.

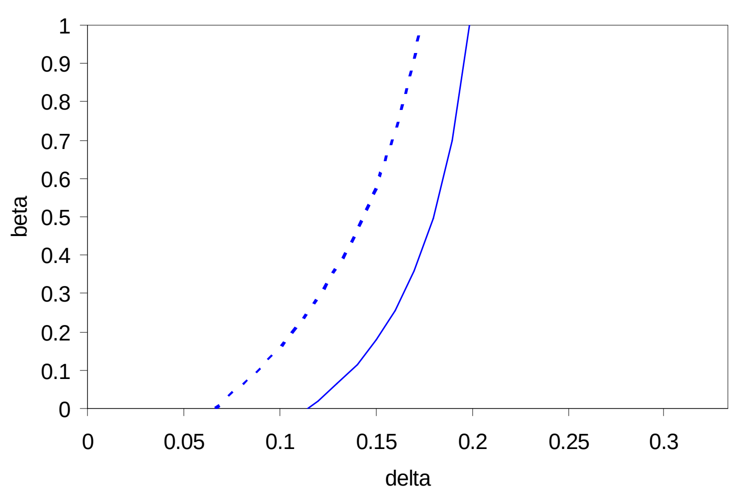
**Proposition 4**:
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq g_s^{L1,L2}(\delta) = \frac{9}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2}$, *and firms in the large country are not merged, then a merger in the small country improves that country's welfare .*
*(ii) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq g_s^{L1+L2}(\delta) = \frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2}$, *and firms in the large country are merged, then a merger in the small country improves that country's welfare.*
*(iii) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq g_L^{s1,s2}(\delta) = \frac{9\gamma}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2}$, *and firms in the small country are not merged, then a merger in the large country improves that country's welfare.*
*(iv) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq g_L^{s1+s2}(\delta) = \gamma\frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2}$, *and firms in the small country are merged, then a merger in the large country improves that country's welfare.*

Corollary 2 summarizes the implications of Proposition 4.

**Corollary 2**: *When firms in two countries compete in a third country, the incentives for governments to merge national firms are higher: (a) when foreign firms are merged, (b) when firm size asymmetries are high, (c) when the export market is big, and (d) in the country with the smallest domestic market.*

Figure 4 illustrates how the incentives for the government of the small country to merge national firms depend on $\beta$ and $\delta$.

Figure 4




The thin solid curve in Figure 4 represents the equation $\beta = g_s^{L1,L2}(\delta)$ and characterizes incentives for the government of the small country to merge national firms when firms in the large country are not merged. To the right (left) of this curve the government of the small country chooses (not) to merge national firms. The thin solid curve in Figure 4 tells us that if firms in the large country are not merged and firm size asymmetries are sufficiently high, $\delta \geq 0.1991$, then the government of the small country chooses to merge national firms. This is also the case when firm size asymmetries are moderate, $0.11475 \leq \delta \leq 0.1991$, and the export market is sufficiently big, $\beta \leq g_s^{L1,L2}(\delta)$. In contrast, if firms in the large country are not merged and either firm size asymmetries are sufficiently low, $\delta \leq 0.11475$, or firm size asymmetries are moderate and the export market is not sufficiently big, $g_s^{L1,L2}(\delta) \leq \beta$, then the government of the small country chooses not to merge national firms. This is the content of Proposition 4 part (i).

The thick dotted curve in Figure 4 represents the equation $\beta = g_s^{L1+L2}(\delta)$ and characterizes incentives for the government of the small country to merge national firms when firms in the large country are merged. To the right (left) of this curve the government of the small country chooses (not) to merge national firms. The thick dotted curve in Figure 4 tells us that if firms in the large country are merged and firm size asymmetries are sufficiently high, $\delta \geq 0.17371$, then the government of the small country chooses to merge national firms. This is also the case when firm size asymmetries are moderate, $0.0(6) \leq \delta \leq 0.17371$, and the export market is sufficiently big, $\beta \leq g_s^{L1+L2}(\delta)$. In contrast, if firms in the large country are merged and either firm size asymmetries are sufficiently low, $\delta \leq 0.0(6)$, or firm size asymmetries are moderate and the export market is not sufficiently big, $g_s^{L1+L2}(\delta) \leq \beta$, then the government of the small country chooses not to merge national firms. This is the message of Proposition 4 part (ii).

The intuition behind these two results is as follows. To know if a merger improves or worsens national welfare, a government must take into account the merger's impact on: (1) profits in the export market, (2) profits in the domestic market, and (3) consumer surplus. We know from Proposition 1 that if firm size asymmetries are sufficiently high (low) a merger increases (reduces) profits in the export market. Governments also need to take into account that a merger increases profits in the domestic market but reduces consumer surplus. Which effect dominates depends on firms size asymmetries. When firm size asymmetries are high the government should merge national firms because this increases profits in the export market and the increase in profits in the domestic market is larger than the decrease in consumer surplus. When for firm size asymmetries are low the government should not merge national firms since a merger would reduce profits in the export market and the increase in profits in the domestic market is not enough to make up for the reduction in consumer surplus. Finally, for moderate firm size asymmetries there is a trade-off between merger gains in the export market and merger welfare losses in the domestic market. This trade-off implies that for moderate firm size asymmetries, the government should merge national firms when the export market is big but not when the export market is small.

From parts (i) to (ii) of Proposition 4 we see that a merger of firms in the small country increases national welfare under less restrictive conditions when firms in the large country are merged than when firms in the large country are not merged. This happens because if firms in the large country are merged than a merger of firms in the small country always leads to losses in the export market. When firms in the large country are not merged and firms in the small country merge there is a move from four to three firms in the export market. In contrast, when firms in the large country are merged and firms in the small country merge there is a move from three to two firms in the export market. A move from three to two firms has associated a larger increase in mark-ups and a smaller loss of market share for small country firms than a move from four to three firms.

Parts (iii) and (iv) of Proposition 4 provide conditions under which a merger of firms in the large country improves national welfare when firms in the small country are not merged–part (iii)–and when they are merged–part (iv). The conditions show that a merger of firms improve national welfare in the small country under less restrictive conditions than in the large country since moderate firm size asymmetries, $0.0(6) \leq \delta \leq 0.2(27)$, the losses in the domestic market due to the merger are higher in a large country than in a small one.

# 6 Merger Game Played by Governments

I now assume that national governments determine the market structure in each country and that firms play no active role in merger decisions. Like before, I also assume that at the start no merger has taken place in either country. The government of each country decides whether to merge or not to merge national firms. Given the choice in the other government, each government takes the decision that maximizes its country's welfare.

Let the generic merger game played by governments be denoted by $G_{2,\gamma}$. The relevant payoffs of $G_{2,\gamma}$ are summarized in Table II in the Appendix. Proposition 5 characterizes the equilibria of the merger game played by governments when the market size asymmetry between the small and the large country is low. Before stating this result define

$$q_s = \frac{63\gamma + 250\beta - (738\gamma + 1600\beta)\delta + (1647\gamma + 2200\beta)\delta^2}{\left(13 + 162\delta - 603\delta^2\right)\gamma}, \tag{3}$$

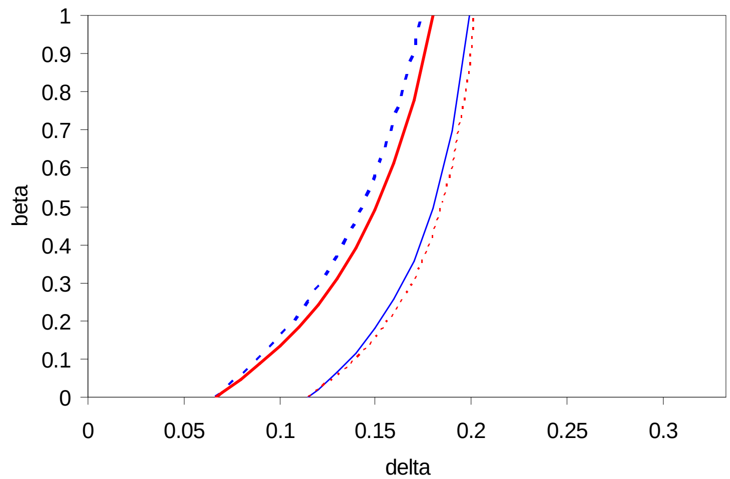
and

$$q_L = \frac{63 + 250\beta - (738 + 1600\beta)\delta + (1647 + 2200\beta)\delta^2}{13 + 162\delta - 603\delta^2}. \tag{4}$$

**Proposition 5**: *Let* $0.46621 \leq \gamma \leq 1$.
*(i) If* $(\beta, \delta)$ *satisfy* $\max\{0, g_L^{s1+s2}(\delta)\} < \beta \leq 1$, *then* $NE(G_{2,\gamma}) = (n, N)$.
*(ii) If* $(\beta, \delta)$ *satisfy* $\max\{0, g_s^{L1,L2}(\delta)\} < \beta \leq \min\{g_L^{s1+s2}(\delta), 1\}$, *then* $NE(G_{2,\gamma}) = \{(n, N), (m, M), (q_s, m; q_L, M)\}$ *where* $q_s$ *and* $q_L$ *are given by (3) and (4), respectively;*
*(iii) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$, *then* $NE(G_{2,\gamma}) = (m, M)$.

Figure 5 illustrates how the set of equilibria of the merger game played by governments depends on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is low.[15]

Figure 5




As in Figure 4, the thin solid and the thick dotted curves in Figure 5 characterize incentives for the government of the small country to merge national firms. The thin dotted curve in Figure 5 represents the equation $\beta = g_L^{s1,s2}(\delta)$ and characterizes incentives for the government of the large country to merge national firms when firms in the small country are not merged. The thick solid curve in Figure 5 represents the equation $\beta = g_L^{s1+s2}(\delta)$ and characterizes incentives for the government of the large country to merge national firms when firms in the small country are merged. To the right of each curve governments merge national firms. To the left of each curve governments do not to merge national firms.

The two solid curves in Figure 5 determine the areas that define the different equilibria of the game. The area to the right of the thin solid curve represents the set of parameters where the game has a unique PSNE in which governments merge national firms: $(m, M)$. The area to the left of the thick solid curve represents parameter configurations where the game has a unique PSNE in which governments do not merge national firms: $(n, N)$. Finally, the area between the two solid curves represents the set of parameters where the game has multiple equilibria: two PSNE, $(m, M)$ and $(n, N)$, and one MSNE where the government of the small country merges firms with probability $q_s$ and the government

[15] The figure is drawn for $\gamma = 0.85$, that is, when the market of the large country is 1.177 times the market of the small country. The qualitative predictions of the model are the same for any $\gamma \in [0.46621, 1]$.

of the large country with probability $q_L$: $(q_s, m; q_L, M)$.[16]

Proposition 5 tells us that if the market size of the large country is not sufficiently bigger than the market size of the small country, $0.46621 \leq \gamma$, and firm size asymmetries are sufficiently high, $\delta \geq 0.1991$, then governments merge national firms. This happens because when firm size asymmetries are sufficiently high, a merger generate welfare gains in both the export and the domestic markets. If the market size of the large country is not sufficiently bigger than the market size of the small country and firm size asymmetries are moderate, $0.0(6) \leq \delta \leq 0.1991$, then a merger leads to a welfare gain in the export market but a welfare loss in the domestic market. In this case we have multiple equilibria. Finally, if the market size of the large country is not sufficiently bigger than the market size of the small country and firm size asymmetries are sufficiently low, $\delta \leq 0.0(6)$, then governments do not merge national firms since a merger generates welfare losses in both the domestic and export markets.

Proposition 6 characterizes the equilibria of the merger game played by governments when the market size asymmetry between the small and the large country is high.

**Proposition 6**: *Let* $< \gamma < 0.46621$.
*(i) If* $(\beta, \delta)$ *satisfy* $\max\{0, g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta)\} < \beta \leq 1$, *NE*$(G_{2,\gamma}) = (n, N)$*;*
*(ii) If* $(\beta, \delta)$ *satisfy* $g_L^{s1+s2}(\delta) \leq \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$, *NE*$(G_{2,\gamma}) = (m, N)$*;*
*(iii) If* $(\beta, \delta)$ *satisfy* $\max\{0, g_s^{L1,L2}(\delta)\} < \beta \leq g_L^{s1+s2}(\delta)$, *then NE*$(G_{2,\gamma}) = \{(n, N), (n, N), (q_s, m; q_L, M)\}$ *where* $q_s$ *and* $q_L$ *are given by (3) and (4), respectively;*
*(iv) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta), 1\}$, *NE*$(G_{2,\gamma}) = (m, M)$.

Figure 6 illustrates how the set of equilibria of the merger game played by governments depends on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is high.[17]

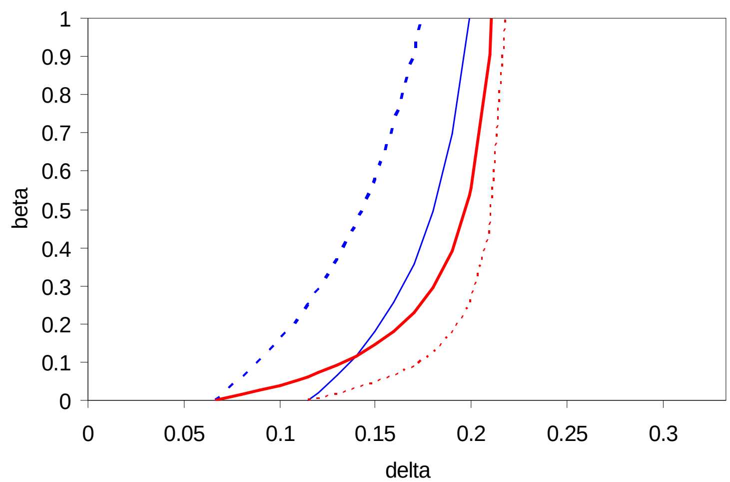
In Figure 6 the intersection of the area to the right of the thick solid curve with the area to the right of the thin solid curve represents parameter configurations for where governments choose to merge national firms: $(m, M)$. The intersection of the area to the left of the thick solid curve with the area to the left of the thin solid curve represents parameter configurations where governments choose not merge national firms: $(n, N)$.

The area to the right of the thin solid curve and to the left of the thick solid curve represents parameter configurations where the government of the small country chooses to merge national firms and the government of the large country chooses not merge national firms: $(m, N)$. Finally, the area to the right of the thick solid curve and to the left of the thin solid curve represents parameter configurations where the merger game played by governments has multiple equilibria: $(m, M)$, $(n, N)$, and $(q_s, m; q_L, M)$.

[16] It follows from the definition of $q_s$ and $q_L$ that $q_L < q_s$ in the range of parameters where the MSNE is well-defined.

[17] The figure is drawn for $\gamma = 0.25$, that is, when the market of the large country is 4 times the market of the small country. The qualitative predictions of the model are the same for any $\gamma \in (0, 0.46621)$.

Figure 6




Comparing Figures 5 and 6 we see that an increase in the market size of the large country moves the thick solid curve and the thin dotted curve closer to the delta axis. This means that for moderate firm size asymmetries, an increase in the market size of the large country makes mergers increasingly less attractive in the large country. This happens because for moderate firm size asymmetries a merger leads to welfare gains in the export market but welfare losses in the domestic market. The bigger the market size of the large country, the greater are the welfare losses of a merger in that country and the less attractive is for the government to merge national firms.

The fact that an increase in the market size of the large country makes a merger increasingly less attractive in the large country but not in the small country, implies that when the market size of the large country is sufficiently big there exist parameter configurations where the government of the small country chooses to merge firms but the government of the large country chooses not to merge firms.

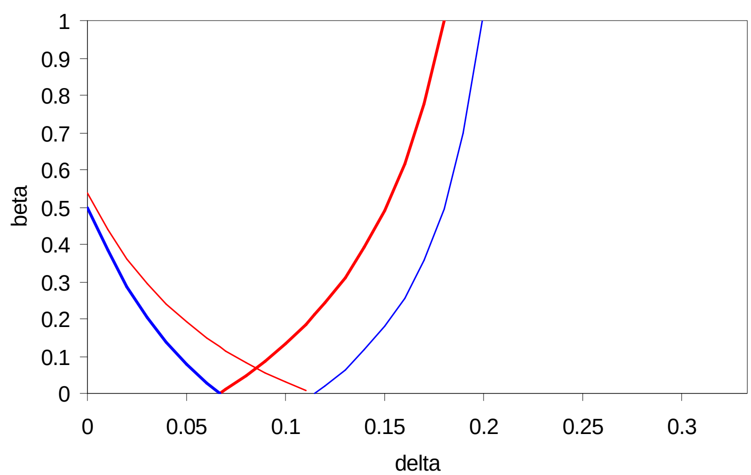
# 7 Conflicts of Interest

This section discusses the implications of the model regarding conflicts of interest between firms and governments about merger decisions. I show that the model predicts that if firms of a small and of a large country compete in a third country, then *the conditions under which conflicts of interest occur are less restrictive in the large country than in the small country.*

Figures 7, 8 and 9 display the inequalities that determine the set of equilibria of the two merger games. By comparing the equilibria of the two games we can find out what are the parameter configurations of the model that lead to conflicts of interest. Figure 7 illustrates how the set of equilibria of the two merger games depend on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is low.[18]

In Figure 7 the thin solid curve with a negative slope represents the equation $\beta = f_L^{s1,s2}(\delta)$ and characterizes incentives for firms in the large country to merge when firms in the small country are not merged. The thick solid curve with a negative slope represents the equation $\beta = f_s^{L1+L2}(\delta)$ and characterizes incentives for firms in the small country to merge when firms in the large country are merged. These two curves determine the set of equilibria of the merger games played by firms. The area to the right of the thin solid curve with a negative slope represents the $(m, M)$ equilibrium. The area to the left of the thick solid curve with a negative slop represents the $(n, N)$ equilibrium.

Figure 7




The thin solid curve with a positive slope represents the equation $\beta = g_s^{L1,L2}(\delta)$ and characterizes incentives for the government of the small country to merge national firms when firms in the large country are not merged. The thick solid curve with a positive slope represents the equation $\beta = g_L^{s1+s2}(\delta)$ and characterizes incentives for the government of the large country to merge national firms when firms in the small country are merged. These two curves determine the set of equilibria of the merger games played by governments. The are to the right of the thin solid curve with a positive slope represents the $(m, M)$ equilibrium. To are to the left of the thick solid curve with a positive slope represents the $(n, N)$ equilibrium.

Figure 7 tells us that the interests of national firms and governments are aligned when firm size asymmetries are high and the size of the export market is sufficiently big, $\beta \leq \min\{g_s^{L1,L2}(\delta), g_L^{s1+s2}(\delta), 1\}$ since the equilibrium

[18] The figure is drawn $\gamma = 0.85$, that is, when the market of the large country is 1.177 times the market of the small country. The qualitative predictions of the model regarding conflicts of interest are the same for any $\gamma \in [0.79365, 1]$ .

of the two games is $(m, M)$. The interests of national firms and governments are also aligned when firm size asymmetries are low and the size of the export market is sufficiently big, $\beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$ since the equilibrium of the two games is $(n, N)$. In contrast, when firm size asymmetries are moderate or low and the size of the export market is sufficiently small, $\max\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta), g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta)\} \leq \beta$ there is a conflict of interests since $(m, M)$ is the equilibrium of the merger game played by firms whereas $(n, N)$ is the merger game played by governments.

Figure 8 illustrates how the set of equilibria of the two merger games depend on $\beta$ and $\delta$ when the market size asymmetry between the small and the large country is moderate.[19]

Figure 8

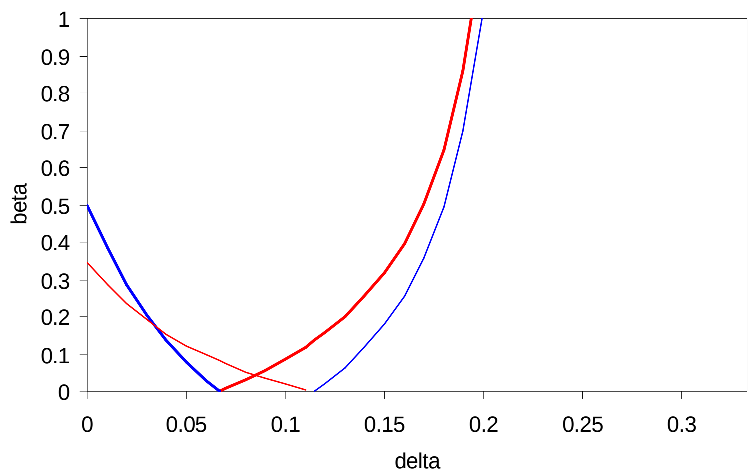



Figure 8 tells us that if the market size asymmetry between the small and the large country is moderate, then there is a set of parameter configurations where the merger game played by firms has an asymmetric PSNE in which firms in the large country choose to merge and firms in the small country choose not to merge.[20] For these parameter configurations, the equilibrium of the merger game played by governments is that governments choose not to merge national firms. Thus, we see that when the market size asymmetry between the small and the large country is moderate there are more parameter configurations where there is a conflict of interest between the firms and the government of the large country than when the market size asymmetry is low.

[19] The figure is drawn for $\gamma = 0.55$, that is, when the market of the large country is 1.(81) times the market of the small country. The qualitative predictions of the model regarding conflicts of interest are the same for any $\gamma \in [0.46621, 0.79365)$.

[20] However, when the market size asymmetry between the small and the large countries is moderate, there is no set of parameter configurations where the merger game played by governments has an asymmetric PSNE.

Figure 9 illustrates how the set of equilibria of the two merger games depend on $\beta$ and $\delta$ when $\gamma = 0.25$, that is, when the market of the large country is 4 times the market of the small country.[21]

Figure 9

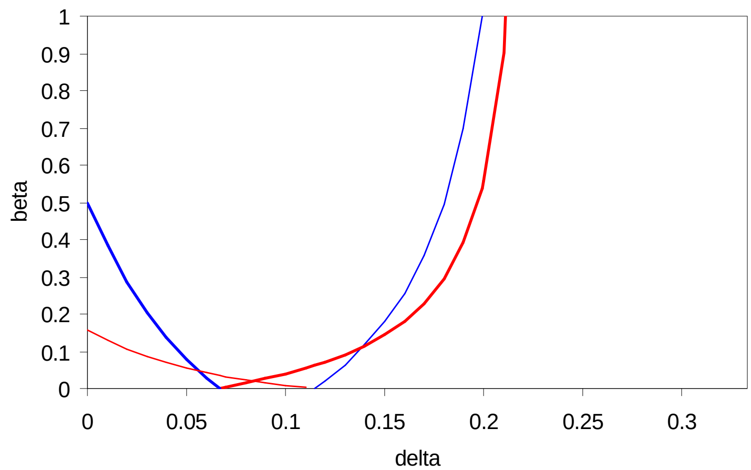



Figure 9 tells us that if the market size asymmetry between the small and the large country is high, then there is a set of parameter configurations where there is an asymmetric PSNE in the merger game played by firms and another set of parameter configurations where there is an asymmetric PSNE in the merger game played by governments. For the first set of parameter configurations we have the same situation as before: an absence of a conflict of interest between the firms and the government of the small country but the presence of a conflict of interest between the firms the government of the large country. For the second set of parameter configurations the equilibrium of the merger game played by firms is that firms merge in both the small and the large country.[22] However, the equilibrium of the merger game played by governments is that the government of the small country chooses to merge national firms but the government of the large country chooses not to merge national firms. Thus, we see that when the market size asymmetry between the small and the large country is high there are more parameter configurations where there is a conflict of interest between the firms and the government of the large country than when the market size asymmetry is moderate.

[21] The qualitative predictions of the model regarding conflicts of interest are the same for any $\gamma \in (0, 0.46621)$.

[22] However, when the market size asymmetry between the small and the large countries is moderate, there is no set of parameter configurations where the merger game played by governments has an asymmetric PSNE.

# 8 Extensions

There are many possible directions in which one could extend this model. For example, one could relax the assumption that there is no bilateral trade between the small and the large country. In this case competition in domestic markets would resemble competition in the third country market and so mergers would be less attractive to firms than in the model without bilateral trade between the small and the large countries. In contrast, mergers would be more attractive to governments since the gains in the export market are the same but the losses in the domestic are smaller. Thus, bilateral trade between the small and the large country reduces conflicts of interest between firms and governments.

Another possible extension is to assume that the firms of the small and large countries face competition from firms of the third country market. This would make mergers less attractive to governments since the losses in the domestic market are the same but the gains in the export market are smaller. The impact of competition from firms in the third country market on incentives for national firms to merger in the small and large countries depends on the size of efficiency gains induced by the mergers. Mergers would be less attractive to firms if firm size asymmetries are high and more attractive if firm size asymmetries are low. Thus, the presence of additional competitors in the export market should increase the likelihood of conflicts of interest between firms and governments.

One could also relax the assumption that there are only two national firms in each country. Doing that makes the analysis of the merger game considerably more complicated. For example, if there are three national firms in each country we would need to consider all possible merger combinations. We would need to state not only individually rational constraints for mergers to be viable but also stability conditions under which the firms outside the mergers would not make a better offer to one of the participants in the merger.[23]

Another possible modification of the model would be to model explicitly a game between national firms and competition authorities where firms propose mergers and competition authorities accept or reject mergers proposed by firms. This extension introduces a dynamic aspect to merger analysis in open economies that has not yet been sufficiently explored.[24] This model would be a middle ground between the merger game played by firms and the one played by governments.

Yet another extension of the model would be to break the assumption that the firm size asymmetries in the small and the large country are the same. For example, one could assume that firms in the large country are uniformly more (or less) efficient than firms in the small country. This extension complicates the analysis since it is no longer possible to find closed form solutions for market size thresholds that define the set of equilibria of the model. However, it is possible to parameterize the model numerically to study this possibility.

[23] See Barros (1998) and Horn and Persson (2001) for closed economy models of mergers in markets with two or more firms.

[24] See Motta and Vasconcelos (2005) for an example of this type of model in a closed economy.

# 9 Conclusion

This paper studies incentives for national mergers in a model where firms of two countries compete in a third market. The main novelty of the paper is that it characterizes incentives for national firms to merge and for governments to promote national mergers when firms can have different cost of production and countries can have different market demands.

The paper finds that firms in the large country have more incentives to merge than firms in the small country. In contrast, the government of the large country has more incentives to block a merger than the government of the small country. Thus, the model predicts that conflicts of interest between governments and firms concerning national mergers are more likely in large countries than in small ones.

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## 11 Appendix

**Proof of Proposition 1**: I start the proof by deriving the conditions under which a merger in $s$ is profitable conditional on a given market structure in $L$. If $s$ firms are not merged they sell $q_{s1} = (a-c+\Delta)/3$ and $q_{s2} = (a-c-2\Delta)/3$ in the $s$ market. In this case, profits of $s$ firms in the $s$ market are given by $\pi_{s1}^{s} = (a-c+\Delta)^2/9$ and $\pi_{s2}^{s} = (a-c-2\Delta)^2/9$. If $L$ firms are not merged they sell $q_{L1} = (a-c+\Delta)/3\gamma$ and $q_{L2} = (a-c-2\Delta)/3\gamma$ in the $L$ market. Profits of $L$ firms in the $L$ market are $\pi_{L1}^{L} = (a-c+\Delta)^2/9\gamma$ and $\pi_{L2}^{L} = (a-c-2\Delta)^2/9\gamma$. The market equilibrium in $t$ is given by:

$$
\begin{aligned}
q_{s1}^{t} &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s2}^{t} + q_{L1}^{t} + q_{L2}^{t}\right) \\
q_{s2}^{t} &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1}^{t} + q_{L1}^{t} + q_{L2}^{t}\right) \\
q_{L1}^{t} &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s1}^{t} + q_{s2}^{t} + q_{L2}^{t}\right) \\
q_{L2}^{t} &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1}^{t} + q_{s2}^{t} + q_{L1}^{t}\right)
\end{aligned}
$$

Solving this system we obtain $q_{s1}^{t} = q_{L1}^{t} = (a-c+2\Delta)/5\beta$ and $q_{s2}^{t} = q_{L2}^{t} = (a-c-3\Delta)/5\beta$. The profits of $s$ and $L$ firms in $t$ are given by $\pi_{s1}^{t} = \pi_{L1}^{t} = (a-c+2\Delta)^2/25\beta$ and $\pi_{s2}^{t} = \pi_{L2}^{t} = (a-c-3\Delta)^2/25\beta$.
If $s$ firms merge the $s$ market becomes a monopoly and the equilibrium quantity is $q_{s_1+s_2} = (a-c)/2$. The monopoly profits are $\pi_{s1+s2} = (a-c)^2/4$. If $s$ firms merge and $L$ firms are not merged, then the equilibrium in $t$ is given by

$$
\begin{aligned}
q_{s1+s2}^{t} &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{L1}^{t} + q_{L2}^{t}\right) \\
q_{L1}^{t} &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s1+s2}^{t} + q_{L2}^{t}\right) \\
q_{L2}^{t} &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1+s2}^{t} + q_{L1}^{t}\right)
\end{aligned}
$$

Solving this system we obtain $q_{s1+s2}^{t} = q_{L1}^{t} = (a-c+\Delta)/4\beta$ and $q_{L2}^{t} = (a-c-3\Delta)/4\beta$. The profits of the merged $s$ firm in $t$ are $\pi_{s1+s2}^{t} = (a-c+\Delta)^2/16\beta$. A merger of $s$ firms is profitable when $L$ firms are not merged if the total profits of the merged $s$ firm are greater than the sum of the profits of the $s$ firms before the merger, that is,

$$
\begin{aligned}
\frac{(a-c)^2}{4} + \frac{(a-c+\Delta)^2}{16\beta} \geq \frac{(a-c+\Delta)^2}{9} \\
\frac{(a-c-2\Delta)^2}{9} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-3\Delta)^2}{25\beta}.
\end{aligned}
$$

Solving this inequality with respect to $\beta$ we obtain $\beta \geq \frac{9}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_s^{L1,L2}(\delta)$ which proves part (i). If $s$ firms are not merged but $L$ firms are, the equilibrium in $t$ is given by:

$$\begin{aligned} q_{s1}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s2}^t + q_{L1+L2}^t\right) \\ q_{s2}^t &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1}^t + q_{L1+L2}^t\right) \\ q_{L1+L2}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s1}^t + q_{s2}^t\right) \end{aligned}$$

The solution to this system is $q_{L1+L2}^t = q_{s1}^t = (a-c+\Delta)/4\beta$ and $q_{s2}^t = (a-c-3\Delta)/4\beta$. The profits of $s1$ in $t$ are $\pi_{s1}^t = (a-c+\Delta)^2/16\beta$ and the profits of $s2$ are $\pi_{s2}^t = (a-c-3\Delta)^2/16\beta$. If $s$ firms merge and so do $L$ firms we have a duopoly in the $t$. In this case the equilibrium quantities in $t$ are $q_{s1+s2}^t = q_{L1+L2}^t = (a-c)/3\beta$ and profits of the merged $s$ firm by $\pi_{s1+s2}^t = (a-c)^2/9\beta$. Thus, a merger of $s$ firms is profitable when $L$ firms are merged if

$$\begin{aligned} \frac{(a-c)^2}{4} + \frac{(a-c)^2}{9\beta} \geq & \frac{(a-c+\Delta)^2}{9} \\ & + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9} + \frac{(a-c-3\Delta)^2}{16\beta}. \end{aligned}$$

Solving this inequality with respect to $\beta$ we obtain $\beta \geq \frac{1}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_s^{L1+L2}(\delta)$ which proves part (ii). Similarly, a merger of $L$ firms is profitable when $s$ firms are not merged if

$$\begin{aligned} \frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta} \geq & \frac{(a-c+\Delta)^2}{9\gamma} \\ & + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{25\beta}. \end{aligned}$$

Solving this inequality with respect to $\beta$ we obtain $\beta \geq \frac{9\gamma}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_L^{s1,s2}(\delta)$ which proves part (iii). A merger of $L$ firms is profitable when $s$ firms are merged if

$$\begin{aligned} \frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta} \geq & \frac{(a-c+\Delta)^2}{9\gamma} \\ & + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{16\beta}. \end{aligned}$$

Solving this inequality with respect to $\beta$ we obtain $\beta \geq \frac{\gamma}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_L^{s1+s2}(\delta)$ which proves part (iv). *Q.E.D.*

Table I

| $s\backslash L$ | $M$ | $N$ |
|---|---|---|
| $m$ | $\frac{\frac{(a-c)^2}{4}}{+\frac{(a-c)^2}{9\beta}}$, $\frac{\frac{(a-c)^2}{4\gamma}}{+\frac{(a-c)^2}{9\beta}}$ | $\frac{\frac{(a-c)^2}{4}}{+\frac{(a-c+\Delta)^2}{16\beta}}$, $\frac{\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}}{+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}}$ |
| $n$ | $\frac{\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}}{+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}}$, $\frac{\frac{(a-c)^2}{4\gamma}}{+\frac{(a-c+\Delta)^2}{16\beta}}$ | $\frac{\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}}{+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}}$, $\frac{\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}}{+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}}$ |

Table I displays the strategies and payoffs of firms in the merger game played by firms. The upper left part of each cell in Table I displays the profits of the merged firm in the small country (when firms in that country choose to merge) or the sum of profits of the two firms in the small country (when firms in that country choose not to merge) for each of the two possible market configurations in the large country. The lower right part of each cell displays the profits of the merged firm in the large country (when firms in that country choose to merge) or the sum of profits of the two firms in the large country (when firms that country choose not to merge) for each of the two possible market configurations in the small country. I will now state a lemma that will be helpful for determining the equilibria of the merger game played by firms (Propositions 2 and 3).

**Lemma 1**: *Let* $\delta^*(\gamma) = \frac{50-63\gamma}{750-549\gamma}$.
*(i) If* $(\gamma, \delta)$ *satisfy* $0 < \gamma < 0.79365$ *and* $0 \leq \delta \leq \delta^*(\gamma)$, *then* $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) < f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$.
*(ii) If* $\delta = \delta^*(\gamma)$, *then* $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) = f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$.
*(iii) If* $(\gamma, \delta)$ *satisfy* $0 < \gamma \leq 0.79365$ *and* $\delta^*(\gamma) < \delta \leq 0.0(6)$, *then* $f_L^{s1+s2}(\delta) < f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta) < f_s^{L1,L2}(\delta)$.
*(iv) If* $0.79365 < \gamma < 1$, *then* $f_L^{s1+s2}(\delta) < f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta) < f_s^{L1,L2}(\delta)$.
*(v) If* $\gamma = 1$, *then* $f_L^{s1+s2}(\delta) = f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta) = f_s^{L1,L2}(\delta)$.

**Proof of Lemma 1**: The expression for $\delta^*(\gamma)$ is obtained by solving $f_s^{L1+L2}(\delta) = f_L^{s1,s2}(\delta)$ with respect to $\delta$. Now, $0 < \gamma < 0.79365$ implies $0 \leq \delta^*(\gamma) \leq 0.0(6)$. So, if $0 < \gamma < 0.79365$ and $0 \leq \delta \leq \delta^*(\gamma)$, then $f_L^{s1,s2}(\delta) < f_s^{L1+L2}(\delta)$. However, if $0 < \gamma \leq 0.79365$ and $\delta^*(\gamma) < \delta \leq 0.0(6)$, then $f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta)$. The definitions of $f_L^{s1+s2}(\delta)$ and $f_s^{L1+L2}(\delta)$ imply that $f_L^{s1+s2}(\delta) < f_s^{L1+L2}(\delta)$ for $\gamma \in (0, 1)$ and $f_L^{s1+s2}(\delta) = f_s^{L1+L2}(\delta)$ when $\gamma = 1$. Similarly, the definitions of $f_L^{s1,s2}(\delta)$ and $f_s^{L1,L2}(\delta)$ imply that $f_L^{s1,s2}(\delta) < f_s^{L1,L2}(\delta)$ for $\gamma \in (0, 1)$ and $f_L^{s1+s2}(\delta) = f_s^{L1+L2}(\delta)$ when $\gamma = 1$. It is straightforward to show that these results imply (i) though (v). *Q.E.D.*

**Proof of Proposition 2**:
(i) If $0.79365 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$, then Lemma 1 part (iv) implies $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Proposition 1 parts (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that $m$ is a dominated strategy for firms in $s$. Thus, firms in $s$ choose $n$. If $\beta \leq f_s^{L1+L2}(\delta)$, then Lemma 1 part (iv) also implies $\beta < f_L^{s1,s2}(\delta)$. Proposition 1 part (iii) together with $\beta < f_L^{s1,s2}(\delta)$ imply that the best response of firms in $L$ to $n$ is $N$. So, firms in $L$ will play $N$. Thus, for $0.79365 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$, we have $NE(F_{2,\gamma}) = (n, N)$.
(ii) If $0.79365 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta)$, then Lemma 1 part (iv) implies $f_L^{s1+s2}(\delta) < f_s^{L1+L2}(\delta) < \beta \leq f_L^{s1,s2}(\delta) < f_s^{L1,L2}(\delta)$. If $f_s^{L1+L2}(\delta) < \beta < f_s^{L1,L2}(\delta)$, then Proposition 1 part (i) implies that the best response of firms in $s$ to $N$ is $n$ and Proposition 1 part (ii) implies that the best response of firms in $s$ to $M$ is $m$. If $f_L^{s1+s2}(\delta) < \beta \leq f_L^{s1,s2}(\delta)$, then Proposition 1 part (iii) implies that the best response of firms in $L$ to $n$ is $N$ and Proposition 1 part (iv) implies that the best response of firms in $L$ to $m$ is $M$. Thus, $(n, N)$ and $(m, M)$ are PSNE of $F_{2,\gamma}$ when $0.79365 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta)$. It is a well know result that the number of Nash equilibria of this type of game must be odd. Since there is no other PSNE we must have a MSNE. By definition, in a MSNE firms in $s$ randomize between $m$ and $n$ to make firms in $L$ indifferent between $M$ and $N$. Thus, in the MSNE we must have that

$$
\begin{aligned}
&p_s \left( \frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta} \right) + (1-p_s) \left( \frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta} \right) \\
&\quad = p_s \left( \frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{16\beta} \right) \\
&+(1-p_s) \left( \frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{25\beta} \right),
\end{aligned}
$$

where $p_s$ is the probability that firms in $s$ choose $m$. Solving this equation for $p_s$ we obtain (1). Firms in $L$ randomize between $M$ and $N$ to make firms in $s$ indifferent between $m$ and $n$. Let $p_L$ denote the probability that firms in $L$ choose $M$. Setting $\gamma = 1$ in (1) we obtain (2). Thus, for $0.79365 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta)$, we have $NE(F_{2,\gamma}) = \{(n, N), (m, M), (p_s, m; p_L, M)\}$.
(iii) If $0.79365 \leq \gamma \leq 1$ and $f_L^{s1,s2}(\delta) < \beta$, then Lemma 1 part (iv) implies $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) < \beta$. Proposition 1 parts (iii) and (iv) together with $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) < \beta$ imply that $N$ is a dominated strategy for firms in $L$. Thus, firms in $L$ choose $M$. If $f_L^{s1,s2}(\delta) < \beta$, then Lemma 1 part (iv) also implies $f_s^{L1+L2}(\delta) < \beta$. Proposition 1 part (ii) together with $f_s^{L1+L2}(\delta) < \beta$ imply that the best response of firms in $s$ to $M$ is $m$. So, firms in $s$ choose $m$. Thus, for $0.79365 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$, we have $NE(F_{2,\gamma}) = (m, M)$. *Q.E.D.*

**Proof of Proposition 3**:
(i) If $0 < \gamma < 0.79365$ and $\beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$, then Lemma 1 parts (i) or (iii) imply $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Proposition 1 parts (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that $m$ is a dominated strategy for firms in $s$. Thus, firms in $s$ choose $n$. If $\beta \leq f_L^{s1,s2}(\delta)$, then Proposition 1 part (iii) implies that the best response of firms in $L$ to $n$ is $N$. So, firms in $L$ will play $N$. Thus, for $0 < \gamma < 0.79365$ and $\beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$, we have $NE(F_{2,\gamma}) = (n, N)$.
(ii) If $0 < \gamma < 0.79365$ and $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$, then Lemma 1 part (i) implies $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Proposition 1 parts (iii) and (iv) together with $f_L^{s1+s2}(\delta) < f_L^{s1,s2}(\delta) \leq \beta$ imply that $N$ is a dominated strategy for firms in $L$. Thus, firms in $L$ choose $M$. Proposition 1 parts (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that $m$ is a dominated strategy for firms in $s$. So, firms in $s$ choose $n$. Thus, for $0 < \gamma < 0.79365$ and $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$ we have $NE(F_{2,\gamma}) = (n, M)$.
(iii) Similar to part (ii) of Proposition 2.
(iv) Similar to part (iii) of Proposition 2. *Q.E.D.*

**Proof of Proposition 4**: I start the proof by stating conditions under which a domestic merger improves national welfare for a given market structure in $L$. Consumer surplus at $s$ is given by $CS_s = (a-p_s)Q_s/2 = Q_s^2/2$,where $Q_s$ is total output produced by $s$ firms. If $s$ firms do not merge, then $Q_s = (2a-2c-\Delta)/3$ and $CS_{s1,s2}^s = (2a-2c-\Delta)^2/18$. If $s$ firms merge, then $Q_s = (a-c)/2$ and $CS_{s1+s2}^s = (a-c)^2/8$.
Thus, a merger of firms in $s$ will improve national welfare when $L$ firms are not merged if

$$\frac{(a-c)^2}{8} + \frac{(a-c)^2}{4} + \frac{(a-c+\Delta)^2}{16\beta} \geq \frac{(2a-2c-\Delta)^2}{18}$$
$$+\frac{(a-c+\Delta)^2}{9} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9} + \frac{(a-c-3\Delta)^2}{25\beta}.$$

Solving this inequality with respect to $\beta$ we obtain $\beta \leq \frac{9}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2} = g_s^{L1,L2}(\delta)$ which proves part (i). A merger of firms in $s$ will improve national welfare when $L$ firms are merged if

$$\frac{(a-c)^2}{8} + \frac{(a-c)^2}{4} + \frac{(a-c)^2}{9\beta} \geq \frac{(2a-2c-\Delta)^2}{18}$$
$$+\frac{(a-c+\Delta)^2}{9} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9} + \frac{(a-c-3\Delta)^2}{16\beta}.$$

Solving this inequality with respect to $\beta$ we obtain $\beta \leq \frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2} = g_s^{L1+L2}(\delta)$ which proves part (ii). I will now state conditions under which a $L$ merger improves $L$ welfare conditional on a given market structure in $s$. Consumer

surplus in $L$ is given by $CS_L = (a-p_L)Q_L/2 = \gamma Q_L^2/2$,where $Q_L$ is total output produced by $L$ firms. If $L$ firms do not merge, then $Q_L = (2a-2c-\Delta)/3\gamma$ and $CS^L_{L1,L2} = (2a-2c-\Delta)^2/18\gamma$. If $L$ firms merge, then $Q_L = (a-c)/2\gamma$ and $CS^L_{L1+L2} = (a-c)^2/8\gamma$.
So, a merger of firms in $L$ will improve national welfare when $s$ firms are not merged if

$$\frac{(a-c)^2}{8\gamma} + \frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta} \geq \frac{(2a-2c-\Delta)^2}{18\gamma} + \frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{25\beta}.$$

Solving this inequality with respect to $\beta$ we obtain $\beta \leq \frac{9\gamma}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2} = g_L^{s1,s2}(\delta)$ which proves part (iii). A merger of firms in $L$ will improve national welfare when $s$ firms are merged if

$$\frac{(a-c)^2}{8\gamma} + \frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta} \geq \frac{(2a-2c-\Delta)^2}{18\gamma} + \frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{16\beta}.$$

Solving this inequality with respect to $\beta$ we obtain $\beta \leq \gamma\frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2} = g_L^{s1+s2}(\delta)$ which proves part (iv). *Q.E.D.*

Table II

| $s\backslash L$ | $M$ | $N$ |
|---|---|---|
| $m$ | $\frac{(a-c)^2}{8}+\frac{(a-c)^2}{4}+\frac{(a-c)^2}{9\beta}$, $\frac{(a-c)^2}{8\gamma}+\frac{(a-c)^2}{4\gamma}+\frac{(a-c)^2}{9\beta}$ | $\frac{(a-c)^2}{8}+\frac{(a-c)^2}{4}+\frac{(a-c+\Delta)^2}{16\beta}$, $\frac{(2a-2c-\Delta)^2}{18\gamma}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$ |
| $n$ | $\frac{(2a-2c-\Delta)^2}{18}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$, $\frac{(a-c)^2}{8\gamma}+\frac{(a-c)^2}{4\gamma}+\frac{(a-c+\Delta)^2}{16\beta}$ | $\frac{(2a-2c-\Delta)^2}{18}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$, $\frac{(2a-2c-\Delta)^2}{18\gamma}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$ |

Table II displays the strategies and payoffs of governments in the merger game played by governments. The upper left part of each cell in Table II displays the sum of consumer surplus and profits of the merged firm in the small country (when firms in that country are merged) or with profits of the two firms in the small country (when firms in that country are not merged) for each market configuration in the large country. The lower right part of each cell displays the sum of consumer surplus and profits of the merged firm in the large country (when firms in that country are merged) or with profits of the two firms in the large country (when firms in that country are not merged) for each market configuration in the small country. I will now state a lemma that will be helpful for determining the equilibria of the merger game played by governments (Propositions 5 and 6).

**Lemma 2** *Let* $\hat{\delta}(\gamma) = \frac{63-50\gamma}{549-750\gamma}$.
*(i) If* $(\gamma, \delta)$ *satisfy* $0 < \gamma \leq 0.46621$ *and* $0.11475 < \delta \leq \hat{\delta}(\gamma)$, *then* $g_L^{s1,s2}(\delta) < g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta) < g_s^{L1+L2}(\delta)$.
*(ii) If* $\delta = \hat{\delta}(\gamma)$ *then* $g_L^{s1,s2}(\delta) < g_s^{L1,L2}(\delta) = g_L^{s1+s2}(\delta) < g_s^{L1+L2}(\delta)$.
*(iii) If* $(\gamma, \delta)$ *satisfy* $0 < \gamma \leq 0.46621$ *and* $\hat{\delta}(\gamma) \leq \delta \leq 0.1991$, *then* $g_L^{s1,s2}(\delta) < g_L^{s1+s2}(\delta) < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$.
*(iv) If* $0.46621 < \gamma < 1$, *then* $g_L^{s1,s2}(\delta) < g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta) < g_s^{L1+L2}(\delta)$.
*(v) If* $\gamma = 1$, *then* $g_L^{s1,s2}(\delta) = g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta) = g_s^{L1+L2}(\delta)$.

**Proof of Lemma 2**: The expression for $\hat{\delta}(\gamma)$ is obtained by solving $g_s^{L1,L2}(\delta) = g_L^{s1+s2}(\delta)$ with respect to $\delta$. Now, $0 < \gamma \leq 0.46621$ implies $0.11475 < \hat{\delta}(\gamma) \leq 0.1991$. So, if $0 < \gamma \leq 0.46621$ and $\hat{\delta}(\gamma) \leq \delta \leq 0.1991$, then $g_L^{s1+s2}(\delta) < g_s^{L1,L2}(\delta)$. However, if $0 < \gamma \leq 0.46621$ and $0.11475 < \delta \leq \hat{\delta}(\gamma)$, then $g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta)$. The definitions of $g_L^{s1+s2}(\delta)$ and $g_s^{L1+L2}(\delta)$ imply that $g_L^{s1+s2}(\delta) < g_s^{L1+L2}(\delta)$ for $\gamma \in (0, 1)$ and $g_L^{s1+s2}(\delta) = g_s^{L1+L2}(\delta)$ when $\gamma = 1$. Similarly, the definitions of $g_L^{s1,s2}(\delta)$ and $g_s^{L1,L2}(\delta)$ imply that $g_L^{s1,s2}(\delta) < g_s^{L1,L2}(\delta)$ for $\gamma \in (0, 1)$ and $g_L^{s1+s2}(\delta) = g_s^{L1+L2}(\delta)$ when $\gamma = 1$. It is straightforward to show that these results imply (i) though (v). *Q.E.D.*

**Proof of Proposition 5**:
(i) If $0.46621 \leq \gamma \leq 1$ and $g_L^{s1+s2}(\delta) < \beta$, then Lemma 2 part (iv) implies $g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta) < \beta$. Proposition 4 parts (i) and (ii) together with $g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta) < \beta$ imply that $m$ is a dominated strategy for the government of $s$. Thus, the government of $s$ chooses $n$. If $g_L^{s1+s2}(\delta) < \beta$, then Lemma 2 part (iv) also implies $g_L^{s1,s2}(\delta) < \beta$. Proposition 4 part (iii) together with $g_L^{s1,s2}(\delta) < \beta$ imply that the best response of the government of $L$ to $n$ is $N$. So, the government of $L$ plays $N$. Thus, for $0.46621 \leq \gamma \leq 1$ and $g_L^{s1+s2}(\delta) < \beta$ we have $NE(G_{2,\gamma}) = (n, N)$.
(ii) If $0.46621 \leq \gamma \leq 1$ and $g_s^{L1,L2}(\delta) < \beta \leq g_L^{s1+s2}(\delta)$, then Lemma 2 part (iv) implies $g_L^{s1,s2}(\delta) < g_s^{L1,L2}(\delta) < \beta \leq g_L^{s1+s2}(\delta) < g_s^{L1+L2}(\delta)$. If $g_s^{L1,L2}(\delta) < \beta < g_s^{L1+L2}(\delta)$, then Proposition 4 part (i) implies that the best response of the government of $s$ to $N$ is $n$ and Proposition 4 part (ii) implies that the

best response of the government of $s$ to $M$ is $m$. If $g_L^{s1,s2}(\delta) < \beta \leq g_L^{s1+s2}(\delta)$, then Proposition 4 part (iii) implies that the best response of the government of $L$ to $n$ is $N$ and Proposition 4 part (iv) implies that the best response of the government of $L$ to $m$ is $M$. Thus, $(n, N)$ and $(m, M)$ are PSNE of $G_{2,\gamma}$ when $0.46621 \leq \gamma \leq 1$ and $g_s^{L1,L2}(\delta) < \beta \leq g_L^{s1+s2}(\delta)$. It is a well know result that the number of Nash equilibria of this type of game must be odd. Since there is no other PSNE we must have a MSNE. By definition, in a MSNE the government of $s$ randomizes between $m$ and $n$ to make the government of $L$ indifferent between $M$ and $N$. Thus, in the MSNE we must have that

$$\begin{aligned}
q_s &\left(\frac{(a-c)^2}{8\gamma} + \frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta}\right) \\
&\qquad + (1-q_s)\left(\frac{(a-c)^2}{8\gamma} + \frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta}\right) \\
&= q_s\left(\frac{(2a-2c-\Delta)^2}{18\gamma} + \frac{(a-c+\Delta)^2 + (a-c-2\Delta)^2}{9\gamma}\right) \\
&\quad + q_s\frac{(a-c+\Delta)^2 + (a-c-3\Delta)^2}{16\beta} + (1-q_s)\frac{(2a-2c-\Delta)^2}{18\gamma} \\
+(1-q_s)&\left(\frac{(a-c+\Delta)^2 + (a-c-2\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2 + (a-c-3\Delta)^2}{25\beta}\right),
\end{aligned}$$

where $q_s$ is the probability that the government of $s$ chooses $m$. Solving this equation for $q_s$ we obtain (3). The government of $L$ randomizes between $M$ and $N$ to make the government of $s$ indifferent between $m$ and $n$. Let $q_L$ denote the probability that the government of $L$ chooses $M$. Setting $\gamma = 1$ in (3) we obtain (4). Thus, for $0.46621 \leq \gamma \leq 1$ and $g_s^{L1,L2}(\delta) < \beta \leq g_L^{s1+s2}(\delta)$, we have $NE(G_{2,\gamma}) = \{(n, N), (m, M), (q_s, m; q_L, M)\}$.
(iii) If $0.46621 \leq \gamma \leq 1$ and $\beta \leq g_s^{L1,L2}(\delta)$, then Lemma 2 part (iv) implies $\beta < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$. Proposition 4 parts (iii) and (iv) together with $\beta < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$ imply that $n$ is a dominated strategy for the government of $s$. Thus, the government of $s$ chooses $m$. If $\beta \leq g_s^{L1,L2}(\delta)$, then Lemma 2 part (iv) also implies $\beta < g_L^{s1+s2}(\delta)$. Proposition 4 part (iv) together with$\beta < g_L^{s1+s2}(\delta)$ imply that the best response of the government of $L$ to $m$ is $M$. So, the government of $L$ plays $M$. Thus, for $0.46621 \leq \gamma \leq 1$ and $\beta \leq g_s^{L1,L2}(\delta)$ we have $NE(G_{2,\gamma}) = (m, M)$. *Q.E.D.*

**Proof of Proposition 6**:
(i) If $0 < \gamma < 0.46621$ and $\max\{g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta)\} < \beta$, then Lemma 2 part (i) or (iii) imply $g_L^{s1,s2}(\delta) < g_L^{s1+s2}(\delta) < \beta$. Proposition 4 parts (iii) and (iv) together with $g_L^{s1,s2}(\delta) < g_L^{s1+s2}(\delta) < \beta$ imply that $M$ is a dominated strategy for the government of $L$. Thus, the government of $L$ chooses $N$. If $g_s^{L1,L2}(\delta) < \beta$, then Proposition 4 part (ii) implies that the best response of the government of $s$ to $N$ is $n$. So, the government of $s$ plays $n$. Thus, for $0.46621 \leq \gamma \leq 1$ and

$\max\{g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta)\} < \beta$ we have $NE(G_{2,\gamma}) = (n, N)$.
(ii) If $0 < \gamma < 0.46621$ and $g_L^{s1+s2}(\delta) \leq \beta \leq g_s^{L1,L2}(\delta)$, then Lemma 2 part (iii) implies $g_L^{s1,s2}(\delta) < g_L^{s1+s2}(\delta) \leq \beta \leq g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$. Proposition 4 parts (iii) and (iv) together with $g_L^{s1,s2}(\delta) < g_L^{s1+s2}(\delta) \leq \beta$ imply that $M$ is a dominated strategy for the government of $L$. Thus, the government of $L$ chooses $N$. Proposition 4 parts (i) and (ii) together with $\beta \leq g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$ imply that $n$ is a dominated strategy for the government of $s$. So, the government of $s$ plays $m$. Thus, for $0 < \gamma < 0.79365$ and $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$ we have $NE(G_{2,\gamma}) = (m, N)$.
(iii) Similar to part (ii) of Proposition 5.
(iv) Similar to part (iii) of Proposition 5. *Q.E.D.*

**Proposition 7**: *Let* $0.79365 \leq \gamma \leq 1$.
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq f_s^{L1+L2}(\delta)$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$;
*(ii) If* $(\beta, \delta)$ *satisfy* $\max\{f_L^{s1,s2}(\delta), g_L^{s1+s2}(\delta)\} \leq \beta \leq 1$, *then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$;
*(iii) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

**Proof of Proposition 7**: The proof follows from Propositions 2 and 5. *Q.E.D.*

**Proposition 8**: *Let* $0.46621 \leq \gamma < 0.79365$.
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$;
*(ii) If* $(\beta, \delta)$ *satisfy* $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$, *then* $NE(F_{2,\gamma}) = (n, M) \neq (n, N) = NE(G_{2,\gamma})$;
*(iii) If* $(\beta, \delta)$ *satisfy* $\max\ \{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta), g_L^{s1+s2}(\delta)\} \leq \beta \leq 1$, *then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$;
*(iv) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

**Proof of Proposition 8**: The proof follows from Propositions 3 and 5. *Q.E.D.*

**Proposition 9**: *Let* $0 < \gamma < 0.46621$.
*(i) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta)\}$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$;
*(ii) If* $(\beta, \delta)$ *satisfy* $f_L^{s1,s2}(\delta) \leq \beta \leq f_s^{L1+L2}(\delta)$, *then* $NE(F_{2,\gamma}) = (n, M) \neq (n, N) = NE(G_{2,\gamma})$;
*(iii) If* $(\beta, \delta)$ *satisfy* $\max\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta), g_L^{s1+s2}(\delta), g_s^{L1,L2}(\delta)\} \leq \beta \leq 1$, *then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$;
*(iv) If* $(\beta, \delta)$ *satisfy* $g_L^{s1+s2}(\delta) < \beta \leq g_s^{L1,L2}(\delta)$, *then* $NE(F_{2,\gamma}) = (m, M) \neq (m, N) = NE(G_{2,\gamma})$;
*(v) If* $(\beta, \delta)$ *satisfy* $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), g_L^{s1+s2}(\delta), 1\}$, *then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

**Proof of Proposition 9**: The proof follows from Propositions 3 and 6. *Q.E.D.*