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Rubens Penha Cysne

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The n -Dimensional Bailey-Divisia Measure as a General-Equilibrium Measure of the Welfare Costs of Inflation*

Rubens Penha Cysne[†]

July 14, 2011

Abstract

This paper shows that in economies with several monies the Bailey-Divisia multidimensional consumers' surplus formula may emerge as an exact general-equilibrium measure of the welfare costs of inflation, provided that preferences are quasilinear.

1 Introduction

An important problem a researcher may face when trying to assess the welfare costs of inflation in modern economies is the necessity to take into consideration the existence of interest-bearing deposits performing monetary functions, a fact that has been very well documented since the 70's¹. From this period on, most economies have faced a huge process of financial innovations leading to the existence of several quasi-monies.

Using Lucas's (2000) or Bailey's (1956) unidimensional² formulas to calculate the welfare costs of inflation in such cases can be very misleading.

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[†]Professor at the Graduate School of Economics of the Getulio Vargas Foundation (FGV/EPGE). Praia de Botafogo 190 s. 1100, Rio de Janeiro, RJ, CEP 22250-900. Brasil. Phone +5521 3799-5832 . Fax: +5521 2553-8821. E-mail: rubens@fgv.br.

¹See, e.g., Goldfeld (1973 and 1976), Garcia and Pak (1979), Judd and Scadding (1982), as well as, more recently, Teles and Zhou (2005).

²The dimension here refers to the number of different types of money considered in the underlying theoretical model. In the unidimensional case worked out by Bailey (1956) and by Lucas (2000) only non-interest-bearing money is considered.

Indeed, consumers may protect themselves from inflation by using interest-bearing assets as alternative means of payment. For this reason, the demand for narrow definitions of money (as M_1), when defined as a function of only one opportunity cost, turns out to be very unstable. All money or quasi-money demand functions in this case should be understood as defined over \mathbb{R}^n , $n > 1$, rather than over \mathbb{R} , and we should consider all of them when measuring welfare. This leads us, as conjectured by Lucas (2000, p. 270) and later offered by Cysne (2003), to n -dimensional welfare measures based on Divisia indices of monetary services.

Marty and Chaloupka (1988), Marty (1994, 1999) and Baltensperger and Jordan (1997) are examples of papers in the literature where Bailey's formula is used in a higher-than-one dimensional context (with currency and interest-bearing deposits). These contributions, though, lack a general-equilibrium setting in which money demands can be endogenously derived as a function of technology and tastes.

Recently, Cysne (2009) has shown that Bailey's usual 1-dimensional formula, rather than as an approximation, as in Lucas (2000), can be obtained as an exact general-equilibrium measure of the welfare costs of inflation, provided that preferences are quasi-linear. The importance of this result arises from the fact that, to this day, the use of Bailey's measure to calculate the welfare costs of inflation is widespread in the profession (see, e.g., Lucas (2000), Mulligan and Sala-i-Martin (2000), Attanasio et al. (2002) and, more recently, Ireland (2009)). Looking at Bailey's formula as a general-equilibrium one, even if under particular conditions, allows us to precisely know how money demand, inflation, interest rates and spreads may be generated and may be related to the welfare calculations.

A natural question to ask would be: can the conclusion obtained by Cysne (2009) be extended to an n -dimensional setting? This paper shows it can. Provided that preferences are quasi-linear, the n -dimensional Bailey-Divisia measure³ turns out to be the exact measure of the welfare costs of inflation which emerges from a generalization of Sidrauski (1967) general-equilibrium model to an n -dimensional context.

³Let B_D stand for the Bailey-Divisia measure and s and w for the general-equilibrium measures of the welfare costs of inflation which emerge, respectively, from the shopping-time and the Sidrauski models defined in Lucas (2000). It is shown in Cysne (2003) that $s < B_D$ and in Cysne and Turchick (in press) that

$$s < B_D < w$$

is true both in the unidimensional and in the multidimensional case. When inflation is low, B_D usually turns out to be a good approximation to both measures.

2 The Model

Our model is an extension of Sidrauski's (1967) to an economy with several monies. Let

$$\mathbf{m} = (m_1, \dots, m_n) \in [0, +\infty]^n$$

represent the vector of real quantities of each type of money, as a fraction of nominal *GDP*. Real output is supposed to be constant and equal to one. Each m_i yields a nominal interest rate of r_i , and

$$\mathbf{r} := (r_1, \dots, r_n) \in \mathbb{R}_+^n$$

The first monetary asset (m_1) is assumed to be real currency, in which case $r_1 = 0$.

Let b stand for the real value of bonds, h for real transfers from the government to the representative consumer and π for inflation. Bonds perform no monetary services and pay an interest rate equal to $r \geq r_i \forall i \in \{1, \dots, n\}$. We shall write the vector of opportunity costs (relatively to holding bonds) as:

$$\mathbf{u} := (u_1, \dots, u_n) := (r, r - r_2, \dots, r - r_n) \in \mathbb{R}_+^n$$

Given a concave utility function $U(c, \mathbf{m})$, the maximization problem of the representative consumer reads:

$$\max_{c>0, \mathbf{m} \geq 0} \int_0^{+\infty} e^{-\rho t} U(c, \mathbf{m}) dt \quad (\text{P}_S^n)$$

subject to

$$\begin{aligned} \dot{b} + \mathbf{1} \cdot \dot{\mathbf{m}} &= 1 + h - c + (r - \pi)b + (\mathbf{r} - (\pi \cdot \mathbf{1})) \cdot \mathbf{m} \\ b_0 &> 0 \text{ and } \mathbf{m}_0 > 0 \text{ given,} \end{aligned}$$

where we write $\mathbf{1}$ for the vector $(1, \dots, 1) \in \mathbb{R}^n$, and ‘ \cdot ’ for the canonical inner product of \mathbb{R}^n .

Given a rate of monetary expansion equal to θ , in the steady state we have $\pi = \theta$ and the usual Euler equations imply:

$$r = \theta + \rho \quad (1)$$

$$u_i : = r - r_i = \frac{U_{m_i}}{U_c}, \quad i \in \{1, \dots, n\}, \quad (2)$$

U_x standing for the partial derivative of U with respect to variable x . In equilibrium $c = 1$ and the n equations given by (2) determine the demand for the n monetary assets in the economy.

2.1 An n -Dimensional (Sidrauski) General-Equilibrium Measure of the Welfare Costs of Inflation

This subsection is based on Cysne and Turchick (in press), where technical details are duly worked out. Assume for a moment that our representative agent's utility had the general form⁴:

$$U(c, \mathbf{m}) = \frac{1}{1-\sigma} \left(c \varphi \left(\frac{G(\mathbf{m})}{c} \right) \right)^{1-\sigma} \quad (3)$$

where $\sigma > 0$, $\sigma \neq 1$. $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a function satisfying:

Assumption φ . There exists an $\bar{m} \in (0, +\infty]$ such that $\varphi|_{[0, \bar{m}]}$ is strictly increasing, concave and $\varphi|_{[\bar{m}, +\infty)}$ is constant.

The number \bar{m} in Assumption φ is unique and equal to infinity if φ is strictly increasing. The money aggregator function $G : [0, +\infty]^n \rightarrow [0, +\infty]$ is a twice-differentiable 1-degree homogeneous concave function such that $G_{m_i} > 0$, $\lim_{m_i \rightarrow 0} G_{m_i}(\mathbf{m}) = +\infty$, $\lim_{m_i \rightarrow +\infty} G_{m_i}(\mathbf{m}) = 0$, $G_{m_i m_i} < 0$ for all $i \in \{1, \dots, n\}$.

Equations (2) now imply:

$$u_i(\mathbf{m}) = \frac{\varphi'(G(\mathbf{m}))}{\varphi(G(\mathbf{m})) - G(\mathbf{m}) \varphi'(G(\mathbf{m}))} G_{m_i}(\mathbf{m}), \forall i \in \{1, \dots, n\}. \quad (4)$$

Let $C_{\bar{m}} := \{\mathbf{m} \in \mathbb{R}_{++}^n : G(\mathbf{m}) = \bar{m}\}$. The consumer is satiated with monetary balances when $G(\mathbf{m}) = \bar{m}$. We denote by $\bar{\mathbf{m}}$ those \mathbf{m} which lie in $C_{\bar{m}}$.

Throughout this paper we shall follow the same methodology set forth by Lucas (2000) and implicitly define the welfare costs of inflation $w(\mathbf{m})$ by:

$$U(1 + w(\mathbf{m}), \mathbf{m}) = U(1, \bar{\mathbf{m}}) \quad (5)$$

Take the partial derivatives of (5) and divide by $\varphi'(G((1 + w(\mathbf{m}))^{-1} \mathbf{m}))$ to obtain:

$$w_i(\mathbf{m}) \frac{\varphi \left(G \left(\frac{1}{1+w(\mathbf{m})} \mathbf{m} \right) \right)}{\varphi' \left(G \left(\frac{1}{1+w(\mathbf{m})} \mathbf{m} \right) \right)} + \left[G_{m_i}(\mathbf{m}) - w_i(\mathbf{m}) G \left(\frac{1}{1+w(\mathbf{m})} \mathbf{m} \right) \right] = 0.$$

⁴One can easily show, by the appropriate choice of φ , that (3) includes the CES form:

$$U(c, m) = \frac{1}{1-\sigma} \left(\rho c^\beta + (1-\rho)(\gamma m)^\beta \right)^{\frac{1-\sigma}{\beta}}$$

Using (4):

$$\frac{\varphi\left(G\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right)\right)}{\varphi'\left(G\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right)\right)} = \frac{G_{m_i}(\mathbf{m})}{u_i\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right)} + G\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right),$$

This leads to w being represented by the n differential equations:

$$w_i(\mathbf{m}) = -u_i\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right)$$

with initial condition $w(\bar{\mathbf{m}}) = 0$.

Alternatively, consider a C^1 path

$$\chi : [0, 1] \rightarrow [0, +\infty]^n \quad (6)$$

such that $\chi(0) = \bar{\mathbf{m}}$ and $\chi(1) = \mathbf{m}$. Then, $w(\mathbf{m})$ can be written as the line integral⁵:

$$w(\mathbf{m}) = -\int_{\chi} \mathbf{u}\left(\frac{1}{1+w(\mathbf{m})}\mathbf{m}\right) \cdot d\mathbf{m} \quad (7)$$

2.2 The Bailey-Divisia Measure as a General-Equilibrium Measure

Note that (7), the measure of the welfare costs of inflation which emerges from Sidrauski's general-equilibrium model under (3), is not equal to the Bailey-Divisia (B_D) measure, defined by:

$$B_D(\mathbf{m}) = -u_i(\mathbf{m}), \quad B_D(\bar{\mathbf{m}}) = 0 \quad (8)$$

or, alternatively, when expressed as a line integral, and considering the path χ defined above:

$$B_D(\mathbf{m}) = -\int_{\chi} \mathbf{u}(\mathbf{m}) \cdot d\mathbf{m} \quad (9)$$

⁵Path independence of (7) and of a second line integral to be defined later, (9), is proved in Cysne and Turchick (in press).

Our main purpose here is showing that $B_D = w$ when preferences, rather than defined by the general form (3), can be expressed by the quasi-linear form:

$$U(c, \mathbf{m}) = g(c + \varphi(G(\mathbf{m}))) \quad (10)$$

where c , φ and G have the same properties as before and $g : [0, +\infty] \rightarrow \mathbb{R}$ is assumed to be a twice-differentiable function with $g' > 0$ and $g'' \leq 0$. Any U in the class of functions represented by (10) is concave in (c, m) , since it is given by the composition of concave and increasing functions. This will lead to concavity of $e^{-gt}U$ with respect to $(b, \dot{b}, \mathbf{m}, \dot{\mathbf{m}})$, making the Euler equations sufficient for an optimum.

In this particular case, equations (2) give, $\forall i \in \{1, \dots, n\}$:

$$u_i = \varphi'(G(\mathbf{m}))G_{m_i}(\mathbf{m}), \quad i \in \{1, \dots, n\} \quad (11)$$

Here, (5) and (10) imply:

$$g(1 + w(\mathbf{m}) + \varphi(G(\mathbf{m}))) = g(1 + \varphi(G(\bar{\mathbf{m}}))) \quad (12)$$

Proposition 1 *Let an economy be described as above. Then*

$$B_D(\mathbf{m}) = w(\mathbf{m}) \quad (13)$$

Proof. The demonstration follows basically the same steps as in Cysne (2009). Since $g'(\cdot) > 0$, we can write, from (12):

$$w(\mathbf{m}) = \varphi(G(\bar{\mathbf{m}})) - \varphi(G(\mathbf{m})) \quad (14)$$

Taking the derivative with respect to m_i :

$$w_i(\mathbf{m}) = -\varphi'(G(\mathbf{m}))G_{m_i}(\mathbf{m}), \quad \forall i \in \{1, \dots, n\} \quad (15)$$

By using (11) in (15):

$$w_i(\mathbf{m}) = -u_i(\mathbf{m}), \quad \forall i \in \{1, \dots, n\} \quad (16)$$

Normalize $w(\mathbf{m})$ by making $w(\bar{\mathbf{m}}) = 0$, which is equivalent to writing $B_D(\chi(0)) = 0$. Now use the definition of B_D in (9) to obtain the main result:

$$w(\mathbf{m}) = - \int_{\chi} \mathbf{u}(\mathbf{m}) \cdot d\mathbf{m} = B_D(\mathbf{m}) \quad (17)$$

■

General-equilibrium considerations remind us that the vector \mathbf{m} in (17) is a function not only of the nominal interest rate r , but also of all remaining spreads u_i , $i \in \{2, \dots, n\}$.

Remark 1 Note, by using the parameterized path (6), that (17) can also be written as:

$$w(\mathbf{m}) := \int_0^1 \frac{d}{d\lambda} w(\boldsymbol{\chi}(\lambda)) d\lambda = \int_0^1 [-\mathbf{u}(\boldsymbol{\chi}(\lambda)) \cdot \nabla \boldsymbol{\chi}(\lambda)] d\lambda \quad (18)$$

In (18), λ stands for a real parameter taking values in $[0, 1]$, $\boldsymbol{\chi}(\lambda)$ for the vector of monetary aggregates and $\nabla \boldsymbol{\chi}(\lambda)$ for the vector of derivatives of $\boldsymbol{\chi}$ with respect to λ .

3 Applications

Example 1 (*Area Under the Inverse Demand for Monetary Base*) Here we assume supply and spreads u_i , $\forall i \in \{2, \dots, n\}$, to be competitively determined by a costless banking system operating under constant and non-remunerated reserve requirements. In this case $u_i := r - r_i = k_i r$, k_i standing for the reserve requirements on deposit i . Let z stand for the monetary base, $\mathbf{k} := (k_2, \dots, k_n)$, $\mathbf{m}_{(-1)} := (m_2, \dots, m_n) \in [0, +\infty]^{n-1}$. With $u_i = k_i r$ for all i (17) becomes:

$$B_D(\mathbf{m}) = w(\mathbf{m}) = - \int_{\boldsymbol{\chi}} r(dm_1 + \mathbf{k} \cdot d\mathbf{m}_{(-1)}) \quad (19)$$

But in such an economy the monetary base (equal to currency plus non-interest-bearing reserves deposited in the Central Bank) reads $z := m_1 + \mathbf{k} \cdot \mathbf{m}$. Since \mathbf{k} is a vector of constants, $dz = dm_1 + \mathbf{k} \cdot d\mathbf{m}_{(-1)}$, in which case (19) can be written as:

$$B_D(\mathbf{m}) = w(\mathbf{m}) = - \int_{\bar{m}_1 + \mathbf{k} \cdot \bar{\mathbf{m}}_{(-1)}}^{m_1 + \mathbf{k} \cdot \mathbf{m}_{(-1)}} r dz$$

The conclusion of this example is that under quasilinear preferences the Bailey-Divisia measure leads to an exact general-equilibrium welfare measure equal to the area under the inverse demand for monetary base (rather than the inverse demand for M_1 , as used, for instance, by Lucas (2000)).

Example 2 (*Constant Spreads*). Suppose that supply is determined by the government by making all banking spreads u_i other than that on currency constant. This is to say that, with respect to m_2, m_3, \dots, m_n , government makes interest payments increase *pari-passu* with the interest rate on bonds, the only nonmonetary asset in the economy. In this case, keeping in mind

that $u_1 = r$ and using Remark 1, $B_D(\mathbf{m})$ can be written as a function of the nominal interest rate in the following way:

$$B_D(\mathbf{m}) = - \int_0^r [xm'_1(x) + \sum_{i=2}^n \bar{u}_i m'_i(x)] dx$$

An important conclusion emerges: having all spreads constant does not imply that Bailey's unidimensional formula is the correct one to be used. The remaining term $\sum_{i=2}^n \bar{u}_i m'_i(x)$ reminds us that one should also take into consideration the implications of the changes of the nominal interest rate on the demand for all other monies. Cysne and Turchick (2010) concentrate on discussing this issue and calculating the bias originated by measurements which neglect this fact.

Example 3 Take, in (10), g as the identity function, $\varphi|_{[0, \bar{m})}$ (see Assumption φ) defined by $\varphi(v) = \frac{v^{1-\sigma}}{1-\sigma}$, where $0 < \sigma < 1$, or $\varphi(v) = \ln v$ when $\sigma = 1$ and $G(m_1, m_2) = m_1^\mu m_2^{1-\mu}$, with $0 < \mu < 1$. From (11) we have $u_i(\mathbf{m}) = G(\mathbf{m})^{-\sigma} G_{x_i}(\mathbf{m})$. So, from (9),

$$\begin{aligned} B_D(\mathbf{m}) &= - \int_{\mathbf{x}} G(\mathbf{m})^{-\sigma} (G_{m_1}(\mathbf{m}) dm_1 + G_{m_2}(\mathbf{m}) dm_2) \\ &= - \int_{G(\bar{\mathbf{m}})}^{G(\mathbf{m})} G^{-\sigma} dG = \frac{1}{1-\sigma} (G(\bar{\mathbf{m}})^{1-\sigma} - G(\mathbf{m})^{1-\sigma}) \quad (20) \end{aligned}$$

The definition of G is only used in the final step to generate:

$$B_D(\mathbf{m}) = \frac{1}{1-\sigma} [\bar{m}_1^{\mu(1-\sigma)} \bar{m}_2^{(1-\mu)(1-\sigma)} - m_1^{\mu(1-\sigma)} m_2^{(1-\mu)(1-\sigma)}]$$

When $\sigma = 1$:

$$B_D(\mathbf{m}) = \ln \frac{G(\bar{\mathbf{m}})}{G(\mathbf{m})} = \mu \ln \frac{\bar{m}_1}{m_1} + (1-\mu) \ln \frac{\bar{m}_2}{m_2} \quad (21)$$

In this case the welfare costs of inflation are given by a weighted average which measures how relatively distant the representative agent is from satiation with respect to each monetary asset. Note that in both cases $B_D(\mathbf{m})$ may be unbounded.

Finally, note that both (20) and (21) can also be directly obtained from Proposition 1 and (14).

4 Conclusions

Lucas (2000) has shown that Bailey's formula for the welfare costs of inflation can be regarded as a very good approximation to general-equilibrium measures originating from the shopping-time and the Sidrauski models. Cysne (2009) has deepened such a result by showing that, under quasilinear preferences, rather than as an approximation, Bailey's measure emerges as the exact measure of the welfare costs of inflation in Sidrauski's one-dimensional model.

However, both Bailey's and Lucas's one-dimensional formulas do not take into consideration an important fact common to most economies nowadays: the presence of several types of money other than currency or demand deposits.

The present work has tackled this issue and extended Cysne's (2009) result by showing that in economies with several monies a Divisia-index version of Bailey's original measure can also be regarded as an exact general-equilibrium measure of the welfare costs of inflation. Three applications of this result have been presented.

The intuition is the same as before, now applied to a higher dimension: in the absence of wealth effects, the consumers' surplus (here, the multidimensional consumers' surplus defined by the Bailey-Divisia measure), rather than to an approximation, leads to an exact measure of the deadweight loss stemming from taxation.

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