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# A note on convergence of Peck-Shell and Green-Lin mechanisms in the Diamond-Dybvig model

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FGV/EPGE

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## Abstract

We study the effects of population size in the Peck-Shell analysis of bank runs. We find that a contract featuring equal-treatment for almost all depositors of the same type approximates the optimum. Because the approximation also satisfies Green-Lin incentive constraints, when the planner discloses positions in the queue, welfare in these alternative specifications are sandwiched. Disclosure, however, is not needed since our approximating contract is not subject to runs.

keywords: bank fragility, role of population size, role of aggregate uncertainty

JEL codes: E4, E5.

## 1 Introduction

In the seminal model by Diamond and Dybvig (1983), an atomless population faces private liquidity needs. They remark that aggregate uncertainty poses a major problem for financial stability since the timing of expenditures becomes unpredictable. In this case, typical suspensions of payments should be avoided because they cannot remove bank panics and, at the same time, support the optimal provision of liquidity.

These conclusions led to the Diamond-Dybvig follow-up analysis of deposit insurance, received with reservations for ignoring constraints implied by the sequential nature of information flows. Reinstating the theory in a sequential-service environment became a goal for which finite traders and independent liquidity shocks are convenient assumptions. But then Green and Lin (2003) proved that bank runs cannot become equilibria using a specification with slackening incentive constraints.<sup>1</sup> The leading alternative in the field is now Peck and

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<sup>1</sup>See Andolfato et al. (2007) for an extension which assumes disclosure of all announcements made by early traders.

Shell (2003). They bring back the possibility of bank panics in examples with active constraints, and under the assumption that depositors are *not* informed about their relative position in the sequence of bank service.<sup>2</sup>

After the Peck-Shell model, it is natural to expect a renewed interest in arrangements giving rise to strong implementation (unique outcomes), possibly resembling the Green-Lin setting. Nosal and Wallace (2009) have already noticed that incentive constraints are relaxed when the planner withholds information from depositors. As a result, ‘Green-Lin disclosure’ (of positions) can only eliminate ‘Peck-Shell runs’ at an average-utility loss. In this note, we offer another perspective on this issue by showing that the optimal contracts of Peck-Shell and Green-Lin specifications essentially converge to the same mechanism as the population size increases. We also find a high speed of convergence in a numerical example. These results directly imply that the welfare cost of disclosure is zero. But there are other conclusions that remind us of intuitive ideas in Diamond and Dybvig (1983).

We find also that financial stability does not require disclosure of information. This important feature is revealed by approximations which we call *step* mechanisms. They are constructed for economies with finite population as follows. The first step is to target contracts that are desirable when the number of impatient depositors is about average. The planner proceeds making transfers sequentially, until indicators of the state of the world—the depositors’ withdrawals—trigger an one-shot correction forcing feasibility. In this second step, the new regime has impatient depositors receiving a lower level of consumption. We find three facts: (i) step mechanisms are also implementable with disclosure; (ii) they approach the Peck-Shell optimum as the population increases; (iii) they feature no bank runs since patient depositors are fully insured.

In summary, although Peck and Shell (2003) do reintroduce runs as equilibrium phenomena, their model does not generate sufficient aggregate uncertainty in order to make the problem quantitatively relevant. Hence, the Diamond-Dybvig emphasis on aggregate uncertainty is still an important issue. Progress requires avoiding approximations like ours in order to restore a trade-off between welfare and multiplicity in banking arrangements.<sup>3</sup>

## 2 The environment

A typical economy in our analysis is hit by a shock  $\omega$  with support  $\Omega \equiv \{0, 1\}^N$  according to the probability  $P(\omega) = p^{N-|\omega|}(1-p)^{|\omega|}$ , where  $|\omega| = \sum_{i=1}^N \omega_i$ . There are  $N$  ex ante identical depositors that live for two dates and derive utility from pairs  $(c_1, c_2)$  of consumption provided by a bank—the benevolent social planner, who controls the aggregate endowment  $Y$ —according to positions and

<sup>2</sup>See Ennis and Keister (2009) for examples of runs in a Green-Lin setting with correlated shocks.

<sup>3</sup>It also seems necessary to rule out a larger message space, such as the one used by Cavalcanti and Monteiro (2011) to achieve strong implementation even with correlated shocks.

announcements about preferences that are private information. Each individual draws a unique position  $i$  in  $\{1, \dots, N\}$  with probability  $\frac{1}{N}$  and, as a result, the realization  $\omega_i$ , without knowing the other coordinates of  $\omega$ . As a benchmark, we assume that the individual is not informed of his position  $i$ . We shall keep the parameter  $p$  and the per capita endowment  $e = \frac{Y}{N}$  constant when we consider changes in the population size  $N$ . Person  $i$  is called *impatient* if  $\omega_i = 0$  and called *patient* otherwise. The utility in the former case is  $Au(c_1)$  and in the latter is  $u(c_1 + c_2)$ , where  $A \geq 1$  and  $u$  is continuous, strictly increasing, concave, twice differentiable and satisfies the Inada condition  $u'(0) = +\infty$ . Thus only patient individuals can substitute consumption across dates. The resources not consumed in date 1 are reinvested at gross rate-of-return  $R > 1$ . These assumptions include the preferences in Green and Lin (2003) and Peck and Shell (2003) as particular cases.

Feasible transfers must be incentive-compatible and satisfy a sequential-service constraint. The sequential-service constraint prevents date-1 consumption transferred to a person in position  $i$  to depend on information provided by someone at position  $n$  for  $n > i$ .

A compact description of candidates for optimal allocations follows from additional notation. Let us denote by  $\omega^i$  the vector  $(\omega_1, \omega_2, \dots, \omega_i)$ , and by  $(\omega_{-i}, z)$  the profile that results from substituting the  $i$ -th coordinate of  $\omega$  by  $z$ . Given that  $R > 1$  we can restrict attention to transfers that assigns  $x_i(\omega^i)$  units of date-1 consumption to someone at position  $i$  if that person is impatient ( $\omega_i = 0$ ), and  $y_i(\omega)$  units of date-2 consumption if that person is patient ( $\omega_i = 1$ ), where  $1 \leq i \leq N$ . The sequential-service requirement has thus shaped the domains of  $x_i$  and  $y_i$ . We notice next that  $(x_i, y_i)_{i=1}^N$  is feasible if

$$\sum_{i=1}^N ((1 - \omega_i) x_i(\omega_i) + \omega_i R^{-1} y_i(\omega)) \leq Y, \quad (1)$$

and incentive-compatible if

$$E \left[ \frac{1}{N} \sum_{i=1}^N u(y_i(\omega_{-i}, 1)) \right] \geq E \left[ \frac{1}{N} \sum_{i=1}^N u(x_i(\omega^{i-1}, 0)) \right], \quad (2)$$

that is, when patient individuals that are not informed of their positions agree with revelation.

We also say that  $(x_i, y_i)_{i=1}^N$  is robust to *disclosure* if, in addition,

$$E[u(y_i(\omega_{-i}, 1))] \geq E[u(x_i(\omega^{i-1}, 0))], \quad 1 \leq i \leq N, \quad (3)$$

that is, when a patient individual agrees with revelation after being informed of his position.

The planner's problem is that of maximizing the representative-agent utility, before types and positions are assigned,

$$E \left[ \frac{1}{N} \sum_{i=1}^N ((1 - \omega_i) Au(x_i(\omega_i)) + \omega_i u(y_i(\omega))) \right], \quad (4)$$

subject to (1) and (2).

### 3 A ‘continuum’ with active constraints

In order to build some intuition on optimality, let us consider the following *continuum* maximization problem. The goal is to choose  $(c_1, c_2) \in \mathbb{R}_+^2$  to solve

$$\begin{aligned} & \max pAu(c_1) + (1-p)u(c_2) \\ & \text{s.t. } pc_1 + (1-p)R^{-1}c_2 \leq e \text{ and } c_2 \geq c_1. \end{aligned}$$

**Lemma 1** *If  $A \geq R$  the solution to this problem is*

$$c_1(p) = c_2(p) = \frac{Re}{p(R-1)+1} \in (e, Re).$$

**Proof.** Since  $u$  is strictly increasing at the optimum we have  $pc_1 + (1-p)R^{-1}c_2 = e$ . Thus  $c_1 = c_1(c_2) = \frac{e}{p} - \frac{(1-p)c_2}{R}$ . If  $f(c_2) = pu(c_1(c_2)) + (1-p)u(c_2)$  and  $c_1 \leq c_2 \leq \frac{Re}{1-p}$  then

$$f'(\cdot) = -\frac{(1-p)}{Rp}pAu'(c_1(\cdot)) + (1-p)u'(\cdot) = (1-p)\left(u'(\cdot) - \frac{A}{R}u'(c_1(\cdot))\right).$$

Since  $f'\left(\frac{Re}{1-p}\right) = (1-p)\left(u'\left(\frac{Re}{1-p}\right) - \frac{A}{R}u'(0)\right) < 0$  necessarily  $c_2 < \frac{Re}{1-p}$  at the optimum. However if  $f'(c_2) = 0$  then  $Ru'(c_2) = Au'(c_1) \geq Ru'(c_1)$  implies that  $c_1 \geq c_2$  or that the optimum cannot be interior. Therefore  $c_2 = c_1$ . ■

**Remark 2** *The proof of the lemma also demonstrates that any solution satisfies  $c_2(p) > c_1(p)$  if  $A < R$ .*

Given the previous remark, we restrict attention to the case  $A \geq R$  (also assumed by Peck and Shell, 2003), in order to derive a simple comparison between the solution of the planner’s problem for finite economies and that for the continuum problem. It is a straightforward extension to consider the case  $A < R$ .

**Assumption**  $A \geq R$ .

**Proposition 3** *Suppose  $Y = Ne$ . Let  $\alpha(N)$  be the optimal welfare (4) in a finite economy and  $\beta(e)$  the maximum of the ‘continuum’ problem. Then  $\alpha(N) \leq \beta(e)$ .*

**Proof.** We shall start with a candidate solution  $(x_i, y_i)_{i=1}^N$  for the planner’s problem, use it to define a feasible candidate for the continuum problem, and then rank the corresponding objectives. Define the numbers

$$\begin{aligned} \bar{y}_i &= E[y_i(\omega_{-i}, 1)]; \\ \bar{\bar{y}} &= \frac{1}{N} \sum_{i=1}^N \bar{y}_i; \\ \bar{x}_i &= E[x_i(\omega^{i-1}, 0)], 1 \leq i \leq N. \end{aligned}$$

And define implicitly

$$\begin{aligned} u(\tilde{x}_i) &= E[u(x_i(\omega^{i-1}, 0))], \\ u(\tilde{x}) &= \frac{1}{N} \sum_{i=1}^N u(\tilde{x}_i). \end{aligned}$$

From (2) and Jensen's inequality we get

$$\begin{aligned} u(\bar{y}) &\geq \frac{1}{N} \sum_{i=1}^N u(\bar{y}_i) \geq E\left[\frac{1}{N} \sum_{i=1}^N u(y_i(\omega_{-i}, 1))\right] \geq \\ &E\left[\frac{1}{N} \sum_{i=1}^N u(x_i(\omega^{i-1}, 0))\right] \geq \frac{1}{N} \sum_{i=1}^N u(\tilde{x}_i) = u(\tilde{x}) \end{aligned}$$

and therefore

$$\bar{y} \geq \tilde{x}. \quad (5)$$

If we take expectations in the feasibility constraint (1) we obtain

$$\sum_{i=1}^N \left( p\tilde{x}_i + (1-p) \frac{\bar{y}_i}{R} \right) \leq Ne. \quad (6)$$

From this we also obtain

$$p\tilde{x} + (1-p) \frac{\bar{y}}{R} \leq e$$

since  $\tilde{x} \leq \frac{1}{N} \sum_{i=1}^N \tilde{x}_i$ . We are now ready to evaluate the objective (4):

$$\begin{aligned} &E\left[\frac{1}{N} \sum_{i=1}^N ((1-\omega_i) Au(x_i(\omega^i)) + \omega_i u(y_i(\omega)))\right] = \\ &\frac{1}{N} \sum_{i=1}^N (pAE[u(x_i(\omega^{i-1}, 0))] + (1-p) E[u(y_i(\omega_{-i}, 1))]) \\ &\leq \frac{1}{N} \sum_{i=1}^N (pAu(\tilde{x}_i) + (1-p) u(\bar{y}_i)) \leq pAu(\tilde{x}) + (1-p) u(\bar{y}) \\ &= pAu(\tilde{x}) + (1-p) u(\bar{y}) \leq \beta(e), \end{aligned}$$

which completes the proof. ■

## 4 A 'lower bound' without runs

Any mechanism  $(x_i, y_i)_{i=1}^N$  for a finite economy defines a game of announcements, and a bank *run* is a Bayesian-Nash equilibrium of this game featuring misrepresentation of types. In this section we construct mechanisms that approximate the optimum and that are *immune* to runs. Because only the patient

individuals consider misrepresentation, in order to be immune to runs it suffices to have transfers to the patient that are invariant to  $\omega$ . For  $0 \leq q \leq 1$  we define

$$x(q) = y(q) = \frac{Re}{q(R-1) + 1},$$

and construct the *step* transfer-function according to

$$y_i(\omega) = y(q), \text{ if } \omega_i = 1;$$

$$x_i(\omega) = \begin{cases} x(q), & \text{if } \omega_i = 0 \text{ and } \sum_{j=1}^i (1 - \omega_j) \leq Nq; \\ R^{-1}x(q), & \text{if } \omega_i = 0 \text{ and } \sum_{j=1}^i (1 - \omega_j) > Nq. \end{cases}$$

Thus an impatient agent receives  $x(q)$  if less than  $Nq$  impatient agents are positioned before him. And he receives  $R^{-1}x(q)$  if there are at least  $Nq$  impatient agents before him.

We first notice that the constructed transfer function is feasible. We shall use  $a \wedge b$  to represent  $\min\{a, b\}$  and  $a^+$  to mean  $\max\{a, 0\}$  in what follows. For each  $\omega \in \Omega$  let  $I_\omega = N - |\omega|$  be the number of impatient agents. Now,

$$I_\omega \wedge (Nq) x(q) + (I_\omega - Nq)^+ \frac{x(q)}{R} + (N - I_\omega) \frac{y(q)}{R}$$

is equal, for  $I_\omega \leq Nq$ , to

$$I_\omega x(q) + (N - I_\omega) \frac{x(q)}{R} \leq Nq x(q) + (N - Nq) \frac{x(q)}{R} = Ne$$

and otherwise to

$$Nq x(q) + (N - Nq) \frac{x(q)}{R} = Ne.$$

Hence, by construction, the step function is a feasible and incentive-compatible (since  $y_i \geq x_i$ ) transfer scheme which we call the *step mechanism*.

The following simple lemma is useful for measuring convergence.

**Lemma 4** *There is a constant  $K$  such that if  $q > p$  is sufficiently near  $p$ ,*

$$|u(x(p)) - u(x(q))| \leq K |q - p|. \quad (7)$$

**Proof.** Since

$$|x(q) - x(p)| = \frac{Re(R-1)|q-p|}{(q(R-1)+1)(p(R-1)+1)},$$

a Lipschitz constant  $M$  for  $u$  in a neighborhood of  $x(p)$  yields

$$|u(x(q)) - u(x(p))| \leq M |x(q) - x(p)| \leq M \frac{Re(R-1)}{(p(R-1)+1)^2} |q-p|$$

and the desired constant  $K$ . ■



We now assess the optimality of a step mechanism. Its average utility,  $W$ , can be written as  $\frac{1}{N}$  times

$$E \left[ I_\omega \wedge (Nq) Au(x(q)) + (I_\omega - Nq)^+ Au \left( \frac{x(q)}{R} \right) + (N - I_\omega) u(x(q)) \right].$$

We notice that  $E[(N - I_\omega)] = N(1 - p)$  and

$$E[I_\omega \wedge (Nq)] = E \left[ I_\omega - (I_\omega - Nq)^+ \right] = Np - E \left[ (I_\omega - Nq)^+ \right],$$

since  $E[I(\cdot)] = Np$ . Given these observations,  $W$  can be easily compared with the maximum of the continuum problem according to the expression

$$W = \bar{A}u(x(q)) - \frac{1}{N}E \left[ (I_\omega - Nq)^+ \right] A \left[ u(x(q)) - u \left( \frac{x(q)}{R} \right) \right],$$

where  $\bar{A} \equiv pA + (1 - p)$ . We shall see that  $\frac{1}{N}E \left[ (I_\omega - Nq)^+ \right]$  can be bounded using inequalities, in the spirit of Tchebycheff's, regarding sums of bounded, independent random variables. A tight result is provided by Hoeffding (1963).<sup>4</sup> Because

$$\int (I_\omega - Nq)^+ dP \leq \int_{I_\omega > Nq} (N - Nq) dP = N(1 - q)P(I_\omega > Nq),$$

it follows that

$$\frac{1}{N}E \left[ (I_\omega - Nq)^+ \right] \leq (1 - q) \Pr(I_\omega > Nq)$$

and hence that

$$\frac{1}{N}E \left[ (I_\omega - Nq)^+ \right] \leq (1 - q) \Pr \left( \frac{I_\omega}{N} - p \geq q - p \right) \leq (1 - q) e^{-2(q-p)^2 N}.$$

Now, having bounded this component, it follows that

$$\beta(e) - W \leq \bar{A}k_1 + (1 - q) e^{-2(q-p)^2 N} Ak_2$$

where

$$k_1 = u(x(p)) - u(x(q))$$

and

$$k_2 = u(x(q)) - u \left( \frac{x(q)}{R} \right).$$

The next step is to use the Lipschitz constant (independent of  $N$ ), derived in the lemma, to put

$$k_1 \leq K |q - p|$$

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<sup>4</sup>If  $\bar{X} = (X_1 + \dots + X_n)/n$  and  $\mu = E[\bar{X}]$ , where  $0 \leq X_i \leq 1$  for  $i = 1, \dots, n$  are independent random variables, then Theorem 1 of Hoeffding (1963, p.15) establishes that  $\Pr\{\bar{X} - \mu \geq t\} \leq e^{-2nt^2}$ .

and

$$\beta(e) - W \leq \bar{A}K\delta + (1 - q) e^{-2\delta^2 N} Ak_2, \quad (8)$$

where  $\delta = |q - p|$ . Now, if  $\varepsilon > 0$  is given, we claim that a choice of a suitable  $\delta$  leads to an error-term in (8) less than  $\varepsilon$ . In order to have each of the two terms in the right-hand side of (8) less than  $\frac{\varepsilon}{2}$ , notice that setting  $\delta = \frac{\varepsilon}{2\bar{A}K}$  takes care of the first, and then a choice of  $N$  sufficient large (see remark below) takes care of the second, implying  $|\beta(e) - W| < \varepsilon$ .

**Remark 5** *Since  $p < q$  and  $A < \bar{A}$ , the right-hand side of (8) is bounded by  $AK\delta + (1 - p)e^{-2\delta^2 N} Ak_2$ . The cutoff value of  $N$  that makes this bound equal to  $\varepsilon$  for  $\delta = \frac{\varepsilon}{2\bar{A}K}$  can be found analytically. An approach that produces a tighter  $N$  is to choose  $\delta_N$  minimizing the bound in  $\delta$ , which gives*

$$\frac{K}{4N(1 - p)k_2} = \delta e^{-2\delta_N^2 N},$$

and then choose  $N$  such that the objective is made equal to  $\varepsilon$ .

We have thus proved the following.

**Theorem 6** *By choosing the population size, the step-mechanism welfare can be made arbitrarily close to the optimal one.*

Another obvious result is recorded in the following.

**Remark 7** *Since patient consumption is invariant to positions, the step mechanism is robust to disclosure.*

Because we have shown that the optimal welfare is bounded above by the continuum maximum, it follows that  $W$  sandwiches the welfare of optimal mechanisms with and without disclosure as the population increases.

## 5 Numerical examples

In this section we further explore the example of direct mechanisms in the Appendix B of Peck and Shell (2003). It follows by assuming homothetic preferences represented by  $u(c) = (1 - \gamma)^{-1} c^{1-\gamma}$ .<sup>5</sup> In order to find results for large  $N$  we use an algorithm (see Bertolai and Cavalcanti, 2011, available upon request), inspired by penalty functions, that iterates on candidate Lagrangian multipliers. Since the Peck-Shell model has a single (truth-telling) constraint, the problem is well behaved for large range of values of  $N$ . When the planner is forced to disclose positions, as in Green and Lin (2003), there is one truth-telling constraint for each position and we can compare Lagrangian multipliers in both settings for  $N \leq 15$ .

Figure 1 summarizes the main results.

<sup>5</sup>The parameters in the example are  $A = 10$ ,  $R = 1.05$ ,  $\gamma = 2$ ,  $p = 0.5$ , and  $e = 3$ .

Figure 1: Convergence under the parameterization in Peck and Shell (2003)

The graph on the left of Figure 1 displays welfare, for each  $N$ , in terms of its difference to average utility under autarky. The lowest curve is that for step mechanisms. It is computed using  $Q(j) \equiv \binom{N}{j} p^j (1-p)^{N-j}$  to represent the probability of  $I_\omega = j$ , which allow us to write average utility for the homothetic  $u$  as

$$W = u(x(p))[\bar{A} + A(1 - R^{\delta-1}) \sum_j Q(j) (\frac{j}{N} - p)^+].$$

The top line in the graph is the maximum for the continuum. In the scale established by these two bounds, the curves corresponding to the benchmark (PS) and the disclosure (GL) cases are almost identical.

The graph on the right of Figure 1 gives an explanation for this result. It plots the Lagrangian multiplier for the benchmark as a dot for each  $N$ . It also plots the set of multipliers for the disclosure case, linked by straight lines in a piecewise fashion, also for each  $N$ . The multipliers are also normalized by the value of the multiplier of the continuous problem.<sup>6</sup> It is evident that the set of GL multipliers converge rapidly to their PS counterparts, and that around  $N = 15$  there is no significant difference to the multiplier in the limit (represented by unity).

We also compute how much per capita endowment should be reduced in the continuum economy in order to deliver the same welfare as in the benchmark economy. The endowment reduction that does the job for  $N = 2$  is .102%. That for  $N = 15$  is .0061%, and for  $N = 100$  is .0008%. This documents the fact that the Peck-Shell specification with independent shocks does not produce significant aggregate uncertainty for values of  $N$  beyond 15.

By studying the best-response correspondence for each position in the game defined by disclosure (GL), we do not find banks runs for  $N \leq 15$ . Because the payoffs with and without disclosure change continuously and converge rapidly to the continuous case, we do not expect existence of runs for larger values of  $N$  under disclosure. Hence, provided that disclosure can be adopted freely, there is virtually no welfare cost of strong implementation. In terms of per capita endowment, the reduction that takes welfare from benchmark to disclosure for  $N = 2$  is .022%, but for  $N = 15$  is a tiny .00003%.

For completeness, we report the reduction in the benchmark endowment that is necessary for delivering the welfare of step mechanisms for some population levels in Table 1. In a region of population sizes, doubling the population halves the error, so that convergence is roughly linear in this metric.

<sup>6</sup> After straightforward algebra, the multiplier for the continuum problem can be expressed as  $\frac{(1-p)p(A-R)}{1+p(R-1)} u'(x(p))$ . Dividing this by  $u'(x(p))$  yields the multiplier when the truth-telling constraint is written in utility levels.

Table 1: Benchmark-endowment reduction for step mechanisms

Population ( $N$ )	2	15	100	200	300	400
$e$ reduction	1.022%	0.468%	0.279%	0.127%	0.104%	0.090%

## 6 Final Remarks

This note identifies two simple concepts for the Peck-Shell model with independent shocks. The first is a bound on welfare. Given convexity in preferences, we show that equal treatment is both desirable and feasible when the population is large. The shape of incentive constraints, with only patient individuals consuming in both dates, facilitates the argument, but it should hold more generally. The second construct is a simple mechanism that fully insures patient depositors, allocating ex-post distortions to impatient ones at the end of the queue. It is robust to disclosure and its welfare converges to the bound as the population increases. These ideas put together produce a sandwich result for both Peck-Shell and Green-Lin mechanisms.

Using the same functional forms and parameters of Peck-Shell examples, we find that the approximation error, expressed in terms of per capita endowment, is about .1% for a population near 300 individuals. Hence, the problem of eliminating runs is easily addressed by our simple mechanism at such population sizes.

For smaller populations, one can rely on a faster convergence between Peck-Shell and Green-Lin mechanisms. When the population size is 15 and the other parameters are maintained, allocations in both specifications are similar because the Lagrangian multipliers are essentially the same. No runs are however found for the setup with disclosure. The conclusion is that strong implementation does not have a bite when the population has size 15 either.

Future research thus seems necessary to find specifications in which aggregate uncertainty plays a more significant role in limiting the provision of insurance.

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