

# Regional Reserve Pooling Arrangements\*

Suman S. Basu  
International Monetary Fund

Ran Bi  
International Monetary Fund

Prakash Kannan  
International Monetary Fund

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## Abstract

Recently, several emerging market countries in East Asia and Latin America have initiated intra-regional reserve pooling mechanisms. This is puzzling from a traditional risk-diversification perspective, because country-level shocks are more correlated within rather than across regions. This paper provides a novel rationale for intra-regional pooling: if non-contingent reserve assets can be used to support production during a crisis, then a country's reserve accumulation decision affects not only its own production and consumption, but also its trading partners' consumption through terms of trade effects. These terms of trade adjustments can be fully internalized only by a reserve pool among trading partners. If trade linkages are stronger within rather than across regions, then intra-regional reserve pooling may dominate inter-regional pooling, even if shocks are more correlated within regions.

**Keywords:** Reserve pooling, Precautionary savings

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# 1 Introduction

Over the last decade, central banks in several emerging market economies have amassed substantial holdings of foreign exchange reserves. This trend has been particularly marked in Asian economies such as China, South Korea and Singapore. More recently, regional groupings such as the Chiang Mai Initiative in Asia, and the Latin American Reserve Fund (FLAR) in northern Latin America, have initiated discussions on establishing mechanisms to pool reserves amongst themselves.<sup>1</sup> On the face of it, the regional dimension of these arrangements seems puzzling from the perspective of the traditional theory on risk diversification. If shocks experienced by countries within a particular region are more correlated than shocks across regions, then inter-regional reserve pooling has better risk-sharing properties than intra-regional reserve pooling.

This paper provides a novel rationale for regional pooling arrangements, in a precautionary savings setup with noncontingent reserve assets. Consider a production economy whose output is consumed by both itself and its trading partners. The economy is subject to shocks to its inputs for production (e.g., capital). When negative shocks occur, it can use its reserves as inputs and boost production, which increases both its own consumption and its trading partners' consumption through terms of trade effects. Therefore, a country's reserve accumulation decision has ramifications for its trading partners' welfare. Such terms of trade effects are not taken into account when a country self insures, and are only partially internalized in pooling arrangements with non-trading partners. However, these effects are fully internalized by pooling with trading partners. As a result, if cross-country trade linkages are stronger within a region than across regions, reserve pooling within specific regions can be superior to inter-regional arrangements, even if the correlation of shocks is higher within rather than across regions.

To make our contributions as clear as possible, this paper is split into two parts. In the first part, we focus on the intratemporal risk-sharing dimension of a reserve pooling arrangement. We construct a one-period model of four countries with noncontingent reserve assets and endowment shocks. Each region contains two countries that trade with each other, but not with the countries in the other region. Every country uses its endowment to produce one variety of a tradable good, which it can trade for another variety produced by its single trading partner. In the one-period model, "self-insurance" simply means that the country uses its total savings and endowment as inputs into production. Even in this case, as in Cole and Obstfeld (1991), goods trade itself achieves some risk-sharing through fluctuations in the terms of trade. Alternatively, the country may insure against shocks by joining a reserve pool with another country, either inside or outside its own region.

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<sup>1</sup>The participants in the Chiang Mai Initiative are the ten member states of the Association of South East Asian Nations (ASEAN) – Brunei Darussalam, Cambodia, Indonesia, Laos, Malaysia, Myanmar, the Philippines, Singapore, Thailand and Vietnam – together with China, Japan and South Korea. The current membership of FLAR includes Bolivia, Colombia, Costa Rica, Ecuador, Peru and Venezuela. See Park and Wang (2005) and Eichengreen (2006) for more details on the Chiang Mai Initiative and FLAR respectively.

In the one-period model, a reserve pool can improve risk-sharing by transferring inputs across countries in advance of the production stage. Unlike Cole and Obstfeld (1991), where insurance is achieved via transfers of consumption goods, our setup allows countries to use the transferred inputs to boost production. This improves upon the Cole-Obstfeld optimum because it allows the reserve pool to affect the relative output of different varieties of final goods. By fixing the relative output of the two traded goods within the region, intra-regional reserve pooling fully internalizes the terms of trade effects of endowment shocks. This benefit is particularly valuable if the elasticity of substitution between the two varieties of traded goods is low. Reserve pooling between two countries in different regions, on the other hand, does not completely internalize the terms of trade effects because both countries suffer terms of trade shocks from trading partners outside the pool. Inter-regional pooling is only desirable if the correlation of shocks across regions is much lower than that within regions, or if the elasticity of substitution between goods is high.

The second part of this paper turns to the more general infinite horizon problem of consumption smoothing via noncontingent reserve assets. We show that the dynamic programming problem of any reserve pool can be split into two separate subproblems. The solution to the intratemporal subproblem follows the same logic as the one-period case. The intertemporal subproblem can be analyzed using standard techniques from the consumption smoothing literature. Moving from self-insurance to a reserve pooling arrangement brings two major benefits. Firstly, the unification of budget constraints results in a single borrowing limit for the reserve pool as a whole and this improves welfare by expanding the set of feasible allocations for insurance purposes. Secondly, production and reserve accumulation decisions can be coordinated across countries. Relative to the self-insurance case, such coordination generates a compensated Pareto improvement.

Two aspects of the model deserve further discussion. The first concerns the assumption that reserve pooling entails a transfer of inputs and not of final goods. This method of modeling foreign exchange reserves captures the ease with which they can be transferred across countries, and is consistent with their potential use by central banks to support the domestic production sector (via liquidity support to banks and firms with foreign currency denominated liabilities, for example). The second relates to the nature of shocks that affect an economy. The model clearly distinguishes between two broad categories of disturbances: financial and business cycle shocks that can affect a country's endowment, and the transmission of such shocks to other countries via trade linkages. For the purposes of a reserve pooling arrangement, the first category of shocks is regarded as exogenous and the second category as an endogenous outcome of the policies pursued by the reserve pool. This observation should inform how we measure intra-regional and inter-regional shock correlations from the data. The higher the correlation of financial/business cycle shocks within rather than across regions, the less attractive is a regional pool. The stronger the terms of trade effects of shocks (via trade linkages) within a specific region, the more attractive is such a pool.

Optimal reserve pooling across countries has received some attention in the literature. Imbs

and Mauro (2007) find that the country groupings that yield the largest welfare gains from pooling risks are usually made up of countries located in different regions, which have limited trade links with each other. This is because countries within the same region have higher output correlations. Imbs and Mauro (2007) speculate that contract enforceability may be one reason why we see more regional pooling, counter to the prediction of standard theory. Our model shows that stronger trade linkages within rather than across regions provide additional support for regional reserve pooling, due to the internalization of externalities. Aizenman and Lee (2006) provide another reason why regional pooling arrangements may be beneficial, which does not depend on risk-sharing. In their setup, regional reserve pooling can serve as a commitment device that can help prevent the negative externalities that exist in an environment where countries are engaged in a game of competitive devaluations.

This paper is also related to the broad literature on international risk-sharing. In particular, it builds on the work of Cole and Obstfeld (1991), who examine the role of international goods trade in the transmission (and insurance) of shocks through terms of trade adjustments. Heathcote and Perri (2002) extend the Cole-Obstfeld framework by incorporating a production sector familiar in the real business cycles literature. They find that only a financial autarky model can match both the observed volatility of terms of trade shocks and cross-country output, consumption and investment correlations. Gourinchas and Jeanne (2006) find limited gains from financial integration in a neoclassical growth model. Our work adds to the existing literature by focusing specifically on the interaction between optimal reserve accumulation and regional linkages via goods trade.

This paper focuses on the precautionary motives for holding reserves, whereby reserve assets serve as consumption stabilizers in the face of country-specific shocks. An alternative explanation attributes the build-up of reserves to a form of mercantilism, where the development strategy is aimed at export promotion via active exchange rate management. Empirical studies have produced conflicting results in their attempts to disentangle these two explanations for reserve accumulation. Ben-Bassat and Gottlieb (1992) test a version of the precautionary savings model on Israeli data. Jeanne and Ranciere (2006) assess the ability of the precautionary framework to explain observed reserve holdings, and conclude that it cannot explain the recent accumulation of reserves in Asia. Aizenman and Lee (2005) attempt to disentangle the two motives and conclude that the results support precautionary motives. The practice of most countries to invest reserves in high-quality foreign government bonds has also motivated us to model reserves as noncontingent assets. A relatively recent literature on country insurance (Cabellero and Panageas 2005, 2008, and Cordella and Levy Yeyati, 2006) emphasizes the benefits of investing in contingent assets.

The remainder of the paper is organized as follows. Section 2 analyzes the one-period version of the model. Section 3 looks at the more general intertemporal reserve pooling problem. Finally, section 4 provides a brief discussion of the results and concludes. The appendix contains the proofs of propositions in the main text.

## 2 One-period Model

**Regional Structure** The world comprises four countries,  $j = A, B, C, D$ . The collection of countries is partitioned into two regions, as shown in figure 1. Countries  $A$  and  $B$  are located in region I; countries  $C$  and  $D$  are located in region II.

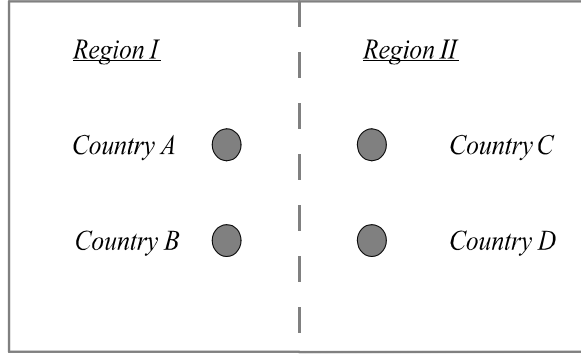


Figure 1: World with Two Regions and Four Countries

Each country is composed of a continuum measure one of identical agents. At the beginning of the period, each agent has  $x$  units of reserve assets. In addition to this, each agent in country  $j$  receives an identical stochastic endowment of  $\omega_j \in \{\omega^H, \omega^L\} = \Omega$ , where  $\omega^H > \omega^L$ . The unconditional probability of the high endowment is  $\frac{1}{2}$  for every country, but the probability is not independent. For countries  $j$  and  $j'$  in the same region:

$$\Pr(\omega_j = \omega^H | \omega_{j'} = \omega^H) = \Pr(\omega_j = \omega^L | \omega_{j'} = \omega^L) = q.$$

For countries in different regions, we impose the following structure for  $j, j' \in \{A, C\}$ :

$$\Pr(\omega_j = \omega^H | \omega_{j'} = \omega^H) = \Pr(\omega_j = \omega^L | \omega_{j'} = \omega^L) = r.$$

**Technology and Preferences** The endowment  $\omega_j$  and reserve assets  $x$  may be used as inputs to produce the country's unique tradable good  $y_j$ . The production function is linear in the input  $v_j$ :

$$y_j = v_j.$$

Final goods  $y_j$  can be traded (at zero transportation cost) within regions, but not across regions.

The utility function of the representative consumer in country  $j$  is defined over their consumption aggregator  $C_j$ :

$$U = \mathbb{E}[u(C_j)],$$

where

$$u'(C_j) > 0, u''(C_j) < 0 \text{ and } C_j = \left[ \frac{1}{2}c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2}c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

for  $j$  and  $j'$  in the same region.  $c_{jk}$  represents the quantity of good  $y_j$  consumed in country  $k$ . Country  $j$  derives utility from the consumption of all goods produced in the same region. The objective function of a reserve pooling arrangement between any two countries  $j$  and  $j'$ , whether within or across regions, is defined to be:

$$U^{RP} = \mathbb{E} \left[ \frac{1}{2}u(C_j) + \frac{1}{2}u(C_{j'}) \right].$$

**Market Incompleteness** Countries are not able to purchase insurance on private markets against the stochastic endowment shock.

## 2.1 First Best Benchmark

The first best optimum in this four country world solves the following social planner problem:

$$\tilde{U}^{FB} = \max_{\{c_{AA}, c_{AB}, c_{BB}, c_{BA}, c_{CC}, c_{CD}, c_{DD}, c_{DC}\}} \left[ \frac{1}{4}u(C_A) + \frac{1}{4}u(C_B) + \frac{1}{4}u(C_C) + \frac{1}{4}u(C_D) \right]$$

subject to

$$C_j = \left[ \frac{1}{2}c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2}c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$c_{AA} + c_{AB} + c_{BB} + c_{BA} + c_{CC} + c_{CD} + c_{DD} + c_{DC} \leq 4x + \sum_{j \in \{A, B, C, D\}} \omega_j,$$

where again,  $c_{jk}$  represents the quantity of good  $y_j$  consumed in country  $k$ . The choice of equal Pareto weights (of  $\frac{1}{4}$ ) reflects the fact that the ex ante expected value of each country's endowment is identical. From an ex ante perspective,  $U^{FB} = \mathbb{E}\tilde{U}^{FB}$ .

**Proposition 1** *At the first best:*

(i) *the quantity of each good produced is identical:  $y_j = x + \frac{1}{4} \sum_{j \in \{A, B, C, D\}} \omega_j$  for all  $j$ .*

(ii) *each country in a particular region consumes half the output of each good produced in that region:  $c_{jj} = c_{j'j} = \frac{1}{2}y_j$  for all  $j, j'$  in the same region.*

Results (i) and (ii) hold because the consumption aggregator  $C_j$  places equal weight on all goods  $y_j$  produced in the same region, and because of the equal Pareto weighting of all countries  $j$  in the social planner problem. Within each region, the social planner can transfer endowments one for one across countries' borders. The consumption aggregator  $C_j$  then implies that it is optimal for each country's consumption bundle to equally weight all the goods produced in its region. The

equal Pareto weighting instructs that consumption must also be equalized across regions. Together, these conditions yield the results above.

Henceforth, we use the terminology that the weighting  $(\frac{1}{2}, \frac{1}{2})$  is the socially optimal proportion of goods in consumption (given the transferability of endowments across countries without cost). Notice that at the social optimum, consumption levels are equalized across all countries in each state, but not across all states. This is because the world as a whole suffers from aggregate shocks.

## 2.2 “Self-insurance”

Now we re-introduce market incompleteness – namely, lack of insurance against the endowment shock. If the country decides to “self-insure”, it uses its savings to supplement its endowment and thereby boost its production. In the one-period model, every country  $j$  simply uses its entire endowment  $\omega_j$  and accumulated reserves  $x$  as inputs into domestic production. Such production exerts an external effect on its trading partners. However, inputs are not redistributed across countries in order to maximize the gains from trade; each individual country does not take into account the effect of its actions, via terms of trade shocks, on the welfare of its trading partner.

After production is completed, goods trade occurs within every region. Consumption levels and trade volumes are given by the solution to the representative consumer’s maximization problem subject to their budget constraint. The maximization problem for the representative consumer in country  $A$  can be written:

$$\tilde{U} = \max_{\{c_{AA}, c_{BA}\}} u(C_A)$$

subject to

$$C_A = \left[ \frac{1}{2} c_{AA}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{BA}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

$$p_A y_A = p_A c_{AA} + p_B c_{BA}.$$

The ex ante utility of country  $A$  is  $U = \mathbb{E}\tilde{U}$ . Goods market equilibrium requires market clearing within each region. For region I:

$$c_{AA} + c_{AB} \leq y_A$$

$$c_{BA} + c_{BB} \leq y_B.$$

Analogous conditions can be derived for other countries and regions.

**Proposition 2** *Under self-insurance:*

(i) *the quantity of each good produced is determined by the endowment and reserves in the specific country:  $y_j = x + \omega_j$  for all  $j$ .*

(ii) *consumption in countries  $j$  and  $j'$  in a particular region satisfies:*

$$\begin{aligned} c_{jj} &= \frac{y_j}{1 + p^{1-\theta}}, c_{j'j} = \frac{y_j}{p + p^\theta} \\ c_{jj'} &= \frac{y_{j'}p}{1 + p^{1-\theta}}, c_{j'j'} = \frac{y_{j'}}{1 + p^{\theta-1}}, \end{aligned}$$

where the price  $p$  is defined as follows:

$$p = \frac{p_{j'}}{p_j} = \left( \frac{y_j}{y_{j'}} \right)^{\frac{1}{\theta}}.$$

Shocks to country  $j$ 's endowment generates shocks to the quantity of good  $y_j$  produced. This affects not only country  $j$ , but also its trading partner in the same region, country  $j'$ , via terms of trade effects. The welfare effects of terms of trade shocks are not fully internalized by each individual country.

### 2.3 Reserve Pooling Within Regions

The reserve pool is not a social planner. Countries sign up to the pooling arrangement at the beginning of the period. The pool has the authority to require transfers of reserves and endowments across the members of the pool, prior to the production stage. Thereafter, countries are free to produce and trade their final goods output with other countries in the same region, without any intervention from the pool. The appropriate optimization problem of the reserve pool with countries  $A$  and  $B$  may be written:

$$\tilde{U}^{RPW} = \max_{\{y_A, y_B\}} \left[ \frac{1}{2}u(C_A) + \frac{1}{2}u(C_B) \right]$$

subject to

$$C_j = \left[ \frac{1}{2}c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2}c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$\begin{aligned} c_{AA} &= \frac{y_A}{1 + p^{1-\theta}}, c_{BA} = \frac{y_A}{p + p^\theta} \\ c_{AB} &= \frac{y_B p}{1 + p^{1-\theta}}, c_{BB} = \frac{y_B}{1 + p^{\theta-1}}, \end{aligned}$$

where  $p$  denotes the relative price of good  $B$  with respect to good  $A$ :

$$p = \frac{p_B}{p_A} = \left( \frac{y_A}{y_B} \right)^{\frac{1}{\theta}},$$



and subject to the resource constraint of the reserve pool:

$$y_A + y_B \leq 2x + \sum_{j \in \{A, B\}} \omega_j.$$

From an ex ante perspective,  $U^{RPW} = \mathbb{E}\tilde{U}^{RPW}$ .

**Proposition 3** *Reserve pooling within a particular region has the following properties:*

(i) *the quantity of each good produced in that region is identical:  $y_j = x + \frac{1}{2}\sum_j \omega_j$  for all  $j$  in the region.*

(ii) *each country consumes half the output of each good produced in that region:  $c_{jj} = c_{jj'} = \frac{1}{2}y_j$  for all  $j, j'$  in the same region.*

For pooling within a particular region, the reserve pooling arrangement is able to achieve the same allocation as a social planner who is constrained to operate only inside that region. Result (i) states that each country in the reserve pool produces the same level of output. This occurs because the consumption aggregator  $C_j$  places equal weight on all goods  $y_j$  produced in the same region. For more general constant elasticity of substitution (CES) consumption aggregators, the constrained social planner allocation is still achieved, but the optimal relative production of goods depends on the optimal proportion of the goods in consumption when the terms of trade is fixed at unity. Result (ii) holds because of the equal weighting of all countries in the objective function of the social planner. This is justified because the ex ante value of each country's endowment is identical.

How does this finding relate to Cole and Obstfeld's (1991) result on risk-sharing with goods trade? In their framework, each country is endowed with final goods rather than inputs. Taking the quantities of final goods as exogenous, they show that trade in goods may achieve most of the gains possible from insurance via transfers of final consumption goods. In the version of their model with goods trade only, risk sharing is achieved exactly as in our specification with "self-insurance". A country with a negative shock in their endowment of final goods experiences a positive terms of trade shock in the goods market. This means a negative terms of trade shock for the country's trading partner, which experiences a fall in relative demand for its good. In the limiting case as  $\theta \rightarrow 1$  (the Cobb-Douglas utility case), goods trade achieves full risk diversification in their model.

In Cole and Obstfeld's framework, perfect asset market integration entails insurance via transfers of consumption goods. The quantities of final goods are taken as exogenous, and the ratio of final goods may be far from  $(\frac{1}{2}, \frac{1}{2})$ . In our framework, on the other hand, reserve pooling arrangements are allowed to transfer inputs from one country to the other in advance of the production stage, and these inputs may be used to boost the production of final goods. Therefore, it is possible to both insure consumption and always keep the ratio of final goods equal to the socially optimal

proportion of the goods in consumption in our model,  $(\frac{1}{2}, \frac{1}{2})$ . This strictly improves welfare relative to the Cole-Obstfeld setup. The quantities of final goods are no longer exogenously given. Notice that this result holds for both the CES and Cobb-Douglas consumption aggregator functions.

## 2.4 Reserve Pooling Across Regions

In this subsection, we consider instead a reserve pool with member countries  $A$  and  $C$ . Both of these countries have the same utility under self-insurance, and are members of identical but separate regions. The optimization problem of the reserve pool is as follows:

$$\tilde{U}^{RPA} = \max_{\{y_A, y_C\}} \left[ \frac{1}{2}u(C_A) + \frac{1}{2}u(C_C) \right]$$

subject to

$$C_j = \left[ \frac{1}{2}c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2}c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$c_{AA} = \frac{y_A}{1 + p_I^{1-\theta}}, c_{BA} = \frac{y_A}{p_I + p_I^\theta} \text{ with } p_I = \frac{p_B}{p_A} = \left( \frac{y_A}{y_B} \right)^{\frac{1}{\theta}}$$

$$c_{CC} = \frac{y_C}{1 + p_{II}^{1-\theta}}, c_{DC} = \frac{y_C}{p_{II} + p_{II}^\theta} \text{ with } p_{II} = \frac{p_D}{p_C} = \left( \frac{y_C}{y_D} \right)^{\frac{1}{\theta}}$$

$$y_A + y_C \leq 2x + \sum_{j \in \{A, C\}} \omega_j \text{ and } y_B = x + \omega_B, y_D = x + \omega_D.$$

The ex ante version of the maximand is  $U^{RPA} = \mathbb{E}\tilde{U}^{RPA}$ .

This is a complicated problem because in addition to their own endowment shocks, both countries in the reserve pool are buffeted by terms of trade shocks originating from outside the pool. Nevertheless, some negative results follow immediately.

**Proposition 4** *Reserve pooling across regions has the following properties:*

- (i) *the quantity of each good produced is in general not identical, within or across regions.*
- (ii) *each country does not consume a constant fraction of the goods produced in its region.*

Result (i) constitutes a welfare loss relative to the first best, because the relative production of goods deviates from the socially optimal proportion of goods in consumption. Result (ii) is a symptom of imperfect risk-sharing across countries within each region. Both of these results show that the reserve pooling arrangement cannot fully take account of the terms of trade effects of endowment shocks originating outside the pool. The endowment shocks in countries  $B$  and  $D$  do affect the utility of countries in the reserve pool, even though they themselves are not members. However, unlike the intra-regional reserve pool, an inter-regional reserve pool can insure country  $A$  against region-wide shocks.

## 2.5 Main Result and Comparative Statics

We are now ready to present the main result of this paper, for the general case with concave utility.

**Main Result 1** *Even if  $q > r$ , there exist combinations of parameters  $(\sigma, \theta)$  such that intra-regional reserve pooling is superior to inter-regional reserve pooling for country A.*

Suppose that one member country of a reserve pooling arrangement suffers a negative endowment shock. If the members of the reserve pool do not trade with each other, the shock does not affect other member countries. However, risk sharing instructs that a transfer be made to the country with the negative shock, from the other member of the reserve pool. On the other hand, if the members of the reserve pool do trade with each other, then a negative endowment shock in one country reduces that country's demand for goods produced by the other member of the reserve pool. In the absence of a reserve pool, the latter country would suffer a decline in relative demand and a negative terms of trade adjustment. Risk sharing again requires a transfer from the member country without the endowment shock. In this case, the transfer is not a pure loss in consumption for the contributing country. By increasing production in the country affected by the negative endowment shock, the transfer increases the relative demand for the goods of the contributing country. Therefore the transfer insulates the recipient country from a decline in its endowment, and insulates the contributing country from an adverse terms of trade shock. Therefore, the benefits of risk sharing from an intra-regional arrangement may be superior to the benefits of an inter-regional arrangement, even if the correlation of shocks is higher within than across regions.

As in the risk neutral case, an intra-regional reserve pool improves risk sharing, and dampens terms of trade shocks, by aligning the relative production of each good with the socially optimal proportion of goods in consumption,  $(\frac{1}{2}, \frac{1}{2})$ , across states of nature. The concomitant welfare benefits are higher, the lower is the elasticity of substitution between goods in consumption,  $\theta$ .

Nevertheless, under risk aversion, inter-regional reserve pooling may be superior to intra-regional arrangements if the correlation of shocks within regions is sufficiently high relative to the correlation of shocks across regions (which translates into  $q$  being sufficiently high relative to  $r$ ), or if the countries are highly risk averse at low levels of consumption. The reason is that although the average level of the consumption aggregator is lower for an inter-regional arrangement than for an intra-regional one, pooling across regions can improve consumption in the worst states of nature. If all countries in one region suffer a negative shock, but all countries in the other region have high endowments, then an inter-regional arrangement is superior for risk-sharing purposes. This scenario is more likely to arise, the higher is  $q$  relative to  $r$ . The welfare benefits of improving consumption after a negative endowment shock depends on the country's risk aversion.

For the first two comparative statics exercises in this subsection, we use the utility function  $u(C_j) = \log(C_j)$ ; the endowment shocks and reserve assets are given by  $\omega^H = 10$ ,  $\omega^L = 2$ ,  $x = 5$ .

**Elasticity of Substitution  $\theta$**  The trade linkages-based motivation for intra-regional reserve pooling is illustrated in figure 2. To construct the figure, we keep constant the correlation of shocks across regions (as implied by  $r = 0.15$ ) and show whether intra-regional or inter-regional reserve pooling is superior for different values of  $q$  and  $\theta$ . In standard models of risk-sharing with a single final good, the elasticity of substitution  $\theta$  is implicitly set to infinity, and inter-regional reserve pooling dominates intra-regional reserve pooling if the parameter governing the correlation of shocks within regions,  $q$ , exceeds  $r$ . Consider the desirability of an inter-regional arrangement as the elasticity of substitution  $\theta$  declines. As this happens, endowment shocks in countries outside the inter-regional pool exert larger terms of trade shocks on their trading partners inside the pool, and it becomes more and more costly not to have member countries' trading partners inside the pooling arrangement. At some point then, it becomes optimal to switch to an intra-regional arrangement. Even for  $q > r$ , the figure shows that an intra-regional reserve pool may dominate an inter-regional arrangement.

**Correlation of shocks within and across regions ( $q$  and  $r$ )** Holding the elasticity of substitution  $\theta$  fixed, the standard risk-sharing argument predicts that an inter-regional pool is likely to be preferred if the correlation of shocks within regions is high relative to the correlation of shocks across regions (in other words, if  $q$  is high relative to  $r$ ). This is illustrated in figure 3, which is constructed for elasticity of substitution  $\theta = 1.5$ . Due to the trade linkages effects described above, there exists a set of points above the 45 degrees line where intra-regional reserve pooling is still superior for country  $A$ .

For the next comparative statics exercise, we use the more general constant relative risk aversion (CRRA) utility function  $u(C_j) = \frac{C_j^{1-\sigma}}{1-\sigma}$ .

**Relative risk aversion  $\sigma$**  Inter-regional reserve pooling reduces average consumption, but is superior to intra-regional pooling at insuring consumption after a region-wide shock. It is precisely after a region-wide shock that a self-insuring country would have the lowest consumption. Therefore, an inter-regional arrangement is more likely to dominate intra-regional reserve pooling in an ex ante sense if the country is more risk averse. Figure 4 holds the correlation of shocks within and across regions fixed ( $q = r = 0.15$ ), and for each value of the elasticity of substitution  $\theta$ , shows the degree of risk aversion such that country  $A$  is indifferent between an intra-regional and inter-regional pooling arrangement. The threshold risk aversion increases as the externality effect increases (namely, as  $\theta$  declines).

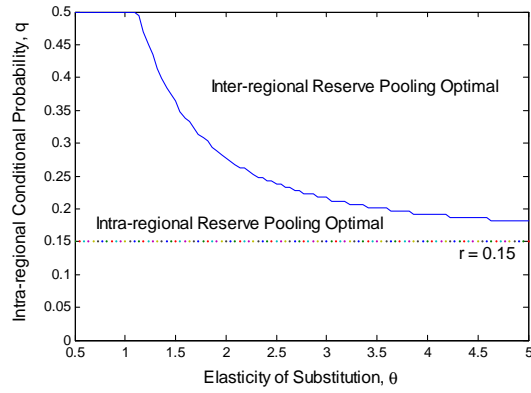


Figure 2: Elasticity of Substitution  $\theta$  and Optimality of Pooling Arrangements

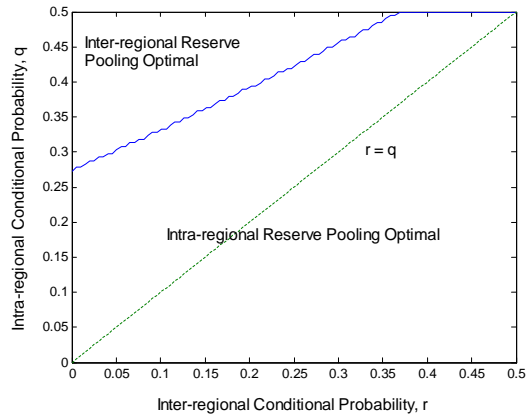


Figure 3: Intra-Regional and Inter-Regional Conditional Probabilities  $q$  and  $r$

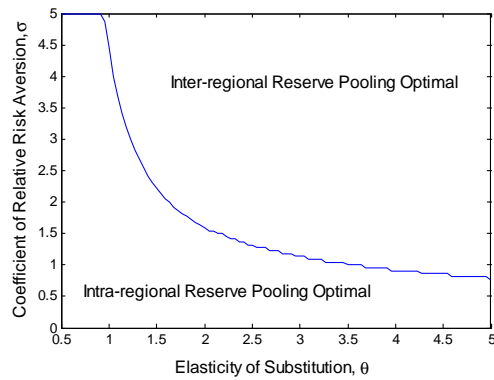


Figure 4: Risk Aversion  $\sigma$  And Optimality of Pooling Arrangements

**Comparison with Cole and Obstfeld (1991)** Finally, we compare the welfare of country  $A$  under three different specifications: (i) goods trade only, i.e. “self-insurance”; (ii) insurance via transfers of final consumption goods (as in Cole and Obstfeld 1991); and (iii) insurance via transfers of production inputs. For cases (ii) and (iii), we focus on the best outcomes achievable by a social planner constrained to operate in region I only. Figure 5 plots the welfare of country  $A$  for these three specifications, against the elasticity of substitution parameter  $\theta$ . Case (iii) yields the highest welfare. From Proposition 3,  $c_{AA} = c_{BA} = \tilde{c}_A$ . The welfare of country  $A$  is

$$U = \mathbb{E}[u(C_A)] = \mathbb{E}\left[u\left(\left[\frac{1}{2}c_{AA}^{\frac{\theta-1}{\theta}} + \frac{1}{2}c_{BA}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right)\right] = \mathbb{E}[u(\tilde{c}_A)].$$

which is independent of  $\theta$ . Case (ii), the Cole-Obstfeld optimum, is inferior in welfare terms because the relative quantities of the two final consumption goods are taken as exogenous rather than endogenously determined. In some states of nature, the ratio of final goods may be far from  $(\frac{1}{2}, \frac{1}{2})$ . The cost of such an outcome is higher, the more complementary the goods (i.e. the lower is the elasticity of substitution  $\theta$ ). As final goods become perfectly substitutable ( $\theta \rightarrow \infty$ ), country  $A$ 's welfare is just the weighted sum of the two goods; the relative proportions of the two goods do not matter. Therefore, welfare approaches the level in case (iii).

Case (i), with goods trade only, yields the worst welfare. It is identical in welfare terms to the Cole-Obstfeld optimum in the limiting case as  $\theta \rightarrow 1$  (Cobb-Douglas utility). As  $\theta$  increases, the welfare of country  $A$  under “self-insurance” is pulled in two different directions. Firstly, the terms of trade become less responsive to endowment shocks, which reduces the level of insurance, and thereby welfare. On the other hand, as  $\theta$  increases, the goods become perfectly substitutable. This improves welfare by reducing the welfare cost in those states of nature when the ratio of final goods in consumption is far from  $(\frac{1}{2}, \frac{1}{2})$ .

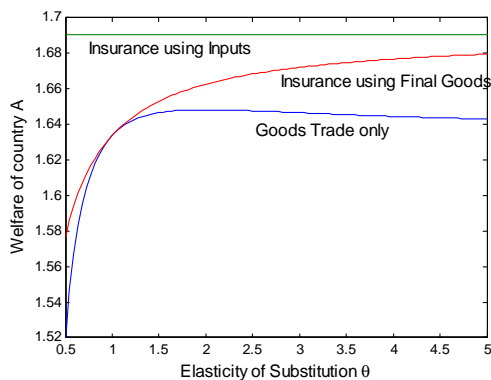


Figure 5: Comparison with Cole and Obstfeld (1991)

### 3 Infinite Horizon Model

This section of the paper is structured to mirror the previous one, but considers the more general infinite horizon version of our model. We show that the infinite horizon dynamic programming problem can be split into separate intratemporal and intertemporal subproblems. The intratemporal subproblem is identical to the one-period model of the previous section; the intertemporal subproblem has the same structure as standard problems in the consumption smoothing literature.

Again, we compare self-insurance to participation in a reserve pool for country  $A$ . The particular assumption for self-insurance is that countries  $A$  and  $B$  cannot coordinate their reserve usage via implicit promises of future continuation values, i.e. we assume that the game is memoryless. This is an extreme case. To the extent that such coordination may occur in practice, welfare under self-insurance would be strictly higher than the extreme case we consider, but still strictly lower than the reserve pooling case. Heathcote and Perri (2002) find that the empirical evidence on volatility in consumption, output and terms of trade match an extended version of the Cole-Obstfeld (1991) model under strict financial autarky. We believe this finding is consistent with the extreme form of self-insurance considered in this paper.

**Modifications to Technology and Preferences** The world unfolds in discrete time,  $t = 0, 1, 2, \dots$ . Within each period, the technology and correlation structure of shocks is identical to the one-period model. Endowment shocks are independent over time. Endowments can be transferred between periods (one for one) but final goods cannot. Countries do not enter the period with exogenous levels of the reserve asset; rather, the reserve level of a country in period  $t$  is equal to the endowment transferred from period  $t - 1$ . The notation is amended in the obvious manner (for example,  $x_{jt}$  represents the reserve level of country  $j$  at time  $t$ ).

The utility function of the representative consumer in country  $j$  is defined:

$$U = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(C_{jt}) \right],$$

where  $\beta \in (0, 1)$  is the discount factor. The objective function of the reserve pooling arrangement is amended in the same way.

**Market Incompleteness** The market incompleteness is unchanged relative to the one-period model: there are no private markets for insurance against countries' stochastic endowment shocks.

### 3.1 First Best Benchmark

The first best optimum solves the following social planner problem:

$$U^{FB} \left( x_t, \{\omega_{jt}\}_{j \in \{A,B,C,D\}} \right) = \max_{\{x_{t+1}\}_{t=0}^{\infty}, \{c_{AA_t}, c_{AB_t}, c_{BB_t}, c_{BA_t}, c_{CC_t}, c_{CD_t}, c_{DD_t}, c_{DC_t}\}_{t=0}^{\infty}} \mathbb{E} \left[ \sum_{j \in \{A,B,C,D\}} \frac{1}{4} \left( \sum_{t=0}^{\infty} \beta^t u(C_{jt}) \right) \right]$$

subject to

$$C_{jt} = \left[ \frac{1}{2} c_{jjt}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'jt}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$c_{AA_t} + c_{AB_t} + c_{BB_t} + c_{BA_t} + c_{CC_t} + c_{CD_t} + c_{DD_t} + c_{DC_t} \leq y_t,$$

$$x_{t+1} \leq x_t + \sum_{j \in \{A,B,C,D\}} \omega_{jt} - y_t \text{ with } x_{t+1} \geq 0,$$

where  $c_{jkt}$  represents the quantity of good  $y_j$  consumed in country  $k$  in period  $t$ . The final constraint reflects the world storage technology. The choice of equal Pareto weights (of  $\frac{1}{4}$ ) in the maximand again reflects the fact that the ex ante expected value of each country's endowment is identical.

This general optimization problem can be split into two related subproblems. The social planner's recursive Bellman equation captures the intertemporal subproblem and is used to derive the optimal savings decision. The intratemporal component of the social planner's problem refers to the consumption optimization decisions within each period  $t$ .

**Proposition 5** *At the first best:*

(i) *the optimal path of savings  $\{x_{t+1}\}_{t=0}^{\infty}$  solves the Bellman equation:*

$$V^{FB}(z_t) = \max_{x_{t+1} \geq 0} \left\{ W^{FB}(z_t - x_{t+1}) + \beta \mathbb{E} V^{FB}(z_{t+1}) \right\},$$

where  $z_t = x_t + \sum_{j \in \{A,B,C,D\}} \omega_{jt}$ .  $W^{FB}(\alpha)$  is defined from the intratemporal subproblem:

$$W^{FB}(\alpha) = \max_{\{c_{AA}, c_{AB}, c_{BB}, c_{BA}, c_{CC}, c_{CD}, c_{DD}, c_{DC}\}} \left\{ \frac{1}{4} u(C_A) + \frac{1}{4} u(C_B) + \frac{1}{4} u(C_C) + \frac{1}{4} u(C_D) \right\}$$

subject to

$$C_j = \left[ \frac{1}{2} c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$c_{AA} + c_{AB} + c_{BB} + c_{BA} + c_{CC} + c_{CD} + c_{DD} + c_{DC} \leq \alpha.$$

Therefore, within each period  $t$ :



- (ii) the quantity of each good produced is identical:  $y_{jt} = \frac{1}{4} \left( x_t + \sum_{j \in \{A,B,C,D\}} \omega_{jt} - x_{t+1} \right)$  for all  $j$ .
- (iii) each country in a particular region consumes half the output of each good produced in that region:  $c_{jjt} = c_{jj't} = \frac{1}{2} y_{jt}$  for all  $j, j'$  in the same region.

Notice that the intratemporal subproblem follows exactly the same structure as the one-period version of the model. Results (ii) and (iii) follow directly from the one-period model. For the first best case, the Bellman equation can be simplified.

**Corollary 1** *At the first best, the Bellman equation takes the following form:*

$$V^{FB}(z_t) = \max_{x_{t+1} \geq 0} \left\{ u \left( \frac{1}{8} [z_t - x_{t+1}] \right) + \beta \mathbb{E} V^{FB}(z_{t+1}) \right\}.$$

Therefore, the policy function  $x_{t+1}(z_t)$  does not depend on the elasticity of substitution parameter  $\theta$ . At the optimum, the social planner builds up some savings for intertemporal consumption smoothing. This is because the world suffers from aggregate shocks.

### 3.2 Self-insurance

In an infinite horizon model without private insurance contracts, country  $A$  can imperfectly smooth consumption over time by accumulating savings during benign periods and drawing down its assets after negative endowment shocks. Restricting the strategy space to memoryless strategies, country  $A$ 's optimization problem can be written:

$$U(x_{At}, \omega_{At}, x_{Bt}, \omega_{Bt}) = \max_{\{x_{At+1}, c_{AAAt}, c_{BAAt}\}_{t=0}^{\infty}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(C_{At}) \right]$$

subject to

$$C_{At} = \left[ \frac{1}{2} c_{AAAt}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{BAAt}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

$$p_{At} y_{At} = p_{At} c_{AAAt} + p_{Bt} c_{BAAt}$$

$$x_{At+1} \leq x_{At} + \omega_{At} - y_{At} \text{ with } x_{At+1} \geq 0.$$

The goods market clears in every period. For region I:

$$c_{AAAt} + c_{ABt} \leq y_{At}$$

$$c_{BAAt} + c_{BBt} \leq y_{Bt}.$$

**Proposition 6** *Under self-insurance:*

(i) *the optimal path of savings  $\{x_{At+1}\}_{t=0}^{\infty}$  solves the Bellman equation:*

$$V(z_{At}, z_{Bt}) = \max_{x_{At+1} \geq 0} \{W(z_{At} - x_{At+1}, z_{Bt} - x_{Bt+1}) + \beta \mathbb{E}V(z_{At+1}, z_{Bt+1})\}$$

subject to

$$x_{Bt+1} = x_{Bt+1}(z_{At}, z_{Bt}),$$

where  $z_{jt} = x_{jt} + \omega_{jt}$ .  $W(\alpha_A, \alpha_B)$  is defined from the intratemporal subproblem:

$$W(\alpha_A, \alpha_B) = \max_{\{c_{AA}, c_{BA}\}} u(C_A)$$

subject to

$$C_A = \left[ \frac{1}{2} c_{AA}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{BA}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

$$p_A y_A = p_A c_{AA} + p_B c_{BA},$$

where prices are derived from good market clearing:

$$c_{AA} + c_{AB} \leq \alpha_A$$

$$c_{BA} + c_{BB} \leq \alpha_B.$$

Therefore, within each period  $t$ :

(ii) *the quantity of each good produced in the region is not identical.  $y_{At}$  is determined by the endowment shock, inherited reserve level and savings decision of country A:  $y_{At} = x_{At} + \omega_{At} - x_{At+1}$ .*

(iii) *consumption in country A satisfies:*

$$c_{AA} = \frac{y_{At}}{1 + p_t^{1-\theta}}, c_{BA} = \frac{y_{At}}{p_t + p_t^\theta},$$

where the price  $p_t$  is defined as follows:

$$p_t = \frac{p_{Bt}}{p_{At}} = \left( \frac{y_{At}}{y_{Bt}} \right)^{\frac{1}{\theta}}.$$

The intratemporal component of the problem follows exactly the same structure as in the one-period model conditional upon the production levels  $y_{At}$  and  $y_{Bt}$ . Results (ii) and (iii), for the intratemporal subproblem, should be familiar from the one-period case. However, the savings decisions (and therefore, production levels) of countries  $A$  and  $B$  are the result of a more sophisticated infinite horizon maximization problem.

Let us consider the Bellman equation in more detail. Country  $A$  recognizes that the utility of its representative consumer depends not only on the domestic endowment shock and reserve level, but also on the endowment shock and reserve level in country  $B$  (through the effect of the latter variables on country  $B$ 's production decision, which in turn affects the relative price of final goods). Therefore, the optimal savings and production decision of country  $A$  is a function of the endowment shocks and reserve levels in all the countries in region I. Country  $A$  takes the optimal policies of country  $B$  as given. In equilibrium, of course, the optimal policy functions of the two ex ante identical countries are symmetric.

What does the optimal policy function  $x_{At+1}(z_{At}, z_{Bt})$  look like? Figure 6 illustrates the shape of the policy function for country  $A$ . We use the utility function  $u(C_j) = \log(C_j)$  and discount factor  $\beta = 0.6$ ; endowment shocks are given by  $\omega^H = 10$ ,  $\omega^L = 2$ . The elasticity of substitution between final goods produced in the same region is  $\theta = 1.5$ , and the parameter  $q$ , governing the correlation of shocks within regions, is set to 0.2. Country  $A$  decides to save more if it enters the period with higher levels of reserve assets, or if it receives a high endowment shock. However, it saves a fraction of any positive endowment shock, so production and consumption levels are also increasing in reserve assets and endowments. Moreover, trade linkages link the optimal savings and production decisions of country  $A$  to the reserve level and endowment of country  $B$ . The higher is the reserve level and/or endowment of country  $B$ , the higher is the production of good  $B$ , which means that country  $A$  benefits from a positive terms of trade shock. In response to this income effect, country  $A$  saves more and produces less, but the consumption aggregator of country  $A$  nevertheless increases.

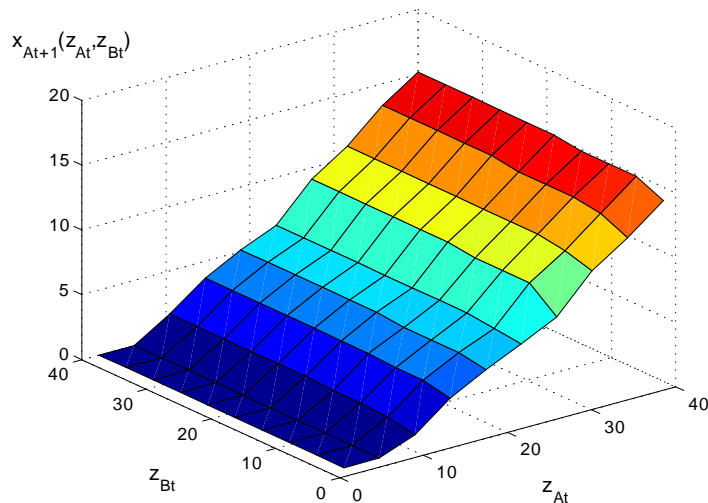


Figure 6: Optimal Policy Function  $x_{At+1}(z_{At}, z_{Bt})$  for Country  $A$

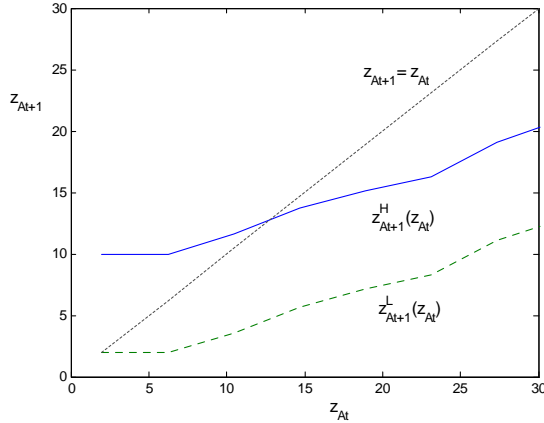


Figure 7: Evolution of Asset Levels for Country A

Figure 7 illustrates the evolution of asset levels over time, and the boundaries of the invariant distribution. To keep the exposition as simple as possible on a two dimensional diagram, the policy functions are plotted assuming that  $z_{At} = z_{Bt}$  throughout.  $z_{At+1}^H(z_{At})$  plots the sum of reserve assets and the endowment in period  $t + 1$  in the event of a high endowment shock in period  $t + 1$ , while  $z_{At+1}^L(z_{At})$  plots the sum after a low endowment shock. Subtracting the endowment levels from the chart, we see that reserve levels in the invariant distribution are contained within 0 and 3 units. Simulation exercises establish that the average level of reserves in the long run for this specification is  $\bar{x} = 1.4$ .

Finally, let us conclude by highlighting the externality due to trade linkages within regions. Due to the market incompleteness, country A uses its savings to self-insure against endowment shock, without taking into account the effect of its intervention on the welfare of country B. The concomitant externality can be identified from the Bellman equation. Country A selects its savings  $x_{At+1}$  to solve this equation, and this optimal choice does not in general maximize country B's utility. Therefore according to the Envelope condition, a marginal deviation in  $x_{At+1}$  has a second order effect on country A's welfare but may have a first order effect on country B's welfare. There exists a deviation constituting a compensated Pareto improvement. Such a Pareto improvement can be achieved by internalizing the externality, via creation of a reserve pooling arrangement between countries A and B. The externality is fully internalized by an intra-regional reserve pool, but not by an inter-regional reserve pool.

The second benefit from joining a reserve pooling arrangement comes from the unification of budget constraints, resulting in a single borrowing limit for the reserve pool as a whole. This improves welfare by expanding the set of feasible allocations, and is the traditional focus of the

literature on reserve pooling. The welfare improvement is higher, the lower the correlation of shocks between countries in the pool.

### 3.3 Reserve Pooling Within Regions

A reserve pooling arrangement between countries  $A$  and  $B$  maximizes the following expression:

$$U^{RPW}(x_t, \omega_{At}, \omega_{Bt}) = \max_{\{x_{t+1}, y_{At}, y_{Bt}\}_{t=0}^{\infty}} \mathbb{E} \left[ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(C_{At}) + \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(C_{Bt}) \right]$$

subject to

$$C_{jt} = \left[ \frac{1}{2} c_{jjt}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'jt}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$\begin{aligned} c_{AA_t} &= \frac{y_{At}}{1 + p_t^{1-\theta}}, c_{BA_t} = \frac{y_{At}}{p_t + p_t^\theta} \\ c_{AB_t} &= \frac{y_{Bt} p_t}{1 + p_t^{1-\theta}}, c_{BB_t} = \frac{y_{Bt}}{1 + p_t^{\theta-1}}, \end{aligned}$$

where  $p$  denotes the relative price of good  $B$  with respect to good  $A$ :

$$p_t = \frac{p_{Bt}}{p_{At}} = \left( \frac{y_{At}}{y_{Bt}} \right)^{\frac{1}{\theta}},$$

and subject to the resource constraint of the reserve pool:

$$x_{t+1} \leq x_t + \sum_{j \in \{A, B\}} \omega_{jt} - y_{At} - y_{Bt} \text{ with } x_{t+1} \geq 0.$$

**Proposition 7** *Reserve pooling within a particular region has the following properties:*

(i) *the optimal path of savings  $\{x_{t+1}\}_{t=0}^{\infty}$  solves the Bellman equation:*

$$V^{RPW}(z_t) = \max_{x_{t+1} \geq 0} \left\{ W^{RPW}(z_t - x_{t+1}) + \beta \mathbb{E} V^{RPW}(z_{t+1}) \right\},$$

where  $z_t = x_t + \sum_{j \in \{A, B\}} \omega_{jt}$ .  $W^{RPW}(\alpha)$  is defined from the intratemporal subproblem:

$$W^{RPW}(\alpha) = \max_{\{y_A, y_B\}} \left\{ \frac{1}{2} u(C_A) + \frac{1}{2} u(C_B) \right\}$$

subject to

$$C_j = \left[ \frac{1}{2} c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$\begin{aligned}
c_{AA} &= \frac{y_A}{1+p^{1-\theta}}, c_{BA} = \frac{y_A}{p+p^\theta} \\
c_{AB} &= \frac{y_{Bt}p_t}{1+p_t^{1-\theta}}, c_{BBt} = \frac{y_B}{1+p^{\theta-1}},
\end{aligned}$$

where  $p$  denotes the relative price of good  $B$  with respect to good  $A$ :

$$\begin{aligned}
p &= \frac{p_B}{p_A} = \left( \frac{y_A}{y_B} \right)^{\frac{1}{\theta}}, \\
y_A + y_B &\leq \alpha.
\end{aligned}$$

Therefore, within each period  $t$ :

(ii) the quantity of each good produced in that region is identical:  $y_{jt} = \frac{1}{2} \left( x_t + \sum_{j \in \{A,B\}} \omega_{jt} - x_{t+1} \right)$

for all  $j$  in the region.

(iii) each country consumes half the output of each good produced in that region:  $c_{jzt} = c_{j'zt} = \frac{1}{2}y_{jt}$  for all  $j, j'$  in the same region.

The intratemporal allocations are identical to the one-period model (conditional upon the total production level  $y_{At} + y_{Bt}$  in period  $t$ ). Furthermore, as in the one-period model, an intra-regional reserve pool is able to achieve the same allocation as a social planner who is constrained to operate only inside that region. In particular, the following corollary holds and the policy function of the reserve pool,  $x_{t+1}(z_t)$ , does not depend on the elasticity of substitution parameter  $\theta$ .

**Corollary 2** For intra-regional reserve pooling, the Bellman equation takes the following form:

$$V^{RPW}(z_t) = \max_{x_{t+1} \geq 0} \left\{ u \left( \frac{1}{4} [z_t - x_{t+1}] \right) + \beta \mathbb{E} V^{RPW}(z_{t+1}) \right\}.$$

### 3.4 Reserve Pooling Across Regions

Next we consider a reserve pooling arrangement between countries  $A$  and  $C$ , which are located in different regions. The optimization problem of the reserve pool can be written:

$$U^{RPA}(x_t, \omega_{At}, \omega_{Ct}) = \max_{\{x_{t+1}, y_{At}, y_{Ct}\}_{t=0}^{\infty}} \mathbb{E} \left[ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(C_{At}) + \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(C_{Ct}) \right]$$

subject to

$$C_{jt} = \left[ \frac{1}{2} c_{jzt}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'zt}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$\begin{aligned}
c_{AA_t} &= \frac{y_{At}}{1 + p_{It}^{1-\theta}}, c_{BA_t} = \frac{y_{At}}{p_{It} + p_{It}^\theta} \text{ with } p_{It} = \frac{p_{Bt}}{p_{At}} = \left(\frac{y_{At}}{y_{Bt}}\right)^{\frac{1}{\theta}} \\
c_{CC_t} &= \frac{y_{Ct}}{1 + p_{II_t}^{1-\theta}}, c_{DC_t} = \frac{y_{Ct}}{p_{II_t} + p_{II_t}^\theta} \text{ with } p_{II_t} = \frac{p_{Dt}}{p_{Ct}} = \left(\frac{y_{Ct}}{y_{Dt}}\right)^{\frac{1}{\theta}} \\
x_{t+1} &\leq x_t + \sum_{j \in \{A, C\}} \omega_{jt} - y_{At} - y_{Ct} \text{ with } x_{t+1} \geq 0 \\
x_{jt+1} &\leq x_{jt} + \omega_{jt} - y_{jt} \text{ with } x_{jt+1} \geq 0 \text{ for } j = B, D.
\end{aligned}$$

**Proposition 8** *Reserve pooling across regions has the following properties:*

(i) *the optimal path of savings  $\{x_{t+1}\}_{t=0}^\infty$  solves the Bellman equation:*

$$V^{RPA}(z_t, z_{Bt}, z_{Dt}) = \max_{x_{t+1} \geq 0} \left\{ W^{RPA}(z_t - x_{t+1}, z_{Bt} - x_{Bt+1}, z_{Dt} - x_{Dt+1}) + \beta \mathbb{E} V^{RPW}(z_{t+1}, z_{Bt+1}, z_{Dt+1}) \right\},$$

subject to

$$\begin{aligned}
x_{Bt+1} &= x_{Bt+1}(z_t, z_{Bt}, z_{Dt}) \\
x_{Dt+1} &= x_{Dt+1}(z_t, z_{Bt}, z_{Dt}),
\end{aligned}$$

where  $z_t = x_t + \sum_{j \in \{A, C\}} \omega_{jt}$ .  $W^{RPA}(\alpha, \alpha_B, \alpha_D)$  is defined from the intratemporal subproblem:

$$W^{RPA}(\alpha, \alpha_B, \alpha_D) = \max_{\{y_A, y_C\}} \left\{ \frac{1}{2} u(C_A) + \frac{1}{2} u(C_C) \right\}$$

subject to

$$C_j = \left[ \frac{1}{2} c_{jj}^{\frac{\theta-1}{\theta}} + \frac{1}{2} c_{j'j}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \text{ for } j, j' \text{ in the same region,}$$

$$\begin{aligned}
c_{AA} &= \frac{y_A}{1 + p_I^{1-\theta}}, c_{BA} = \frac{y_A}{p_I + p_I^\theta} \text{ with } p_I = \frac{p_B}{p_A} = \left(\frac{y_A}{y_B}\right)^{\frac{1}{\theta}} \\
c_{CC} &= \frac{y_C}{1 + p_{II}^{1-\theta}}, c_{DC} = \frac{y_C}{p_{II} + p_{II}^\theta} \text{ with } p_{II} = \frac{p_D}{p_C} = \left(\frac{y_C}{y_D}\right)^{\frac{1}{\theta}} \\
y_A + y_C &\leq \alpha \text{ and } y_B = \alpha_B, y_D = \alpha_D.
\end{aligned}$$

Therefore, within each period  $t$ :

- (ii) *the quantity of each good produced is in general not identical, within or across regions.*
- (iii) *each country does not consume a constant fraction of the goods produced in its region.*

There are three state variables in this problem. The optimal policy of the reserve pool is to share risks between countries  $A$  and  $C$ . This risk-sharing decision depends not only on the shocks

experienced by countries  $A$  and  $C$ , but also on the shocks afflicting their trading partners, countries  $B$  and  $D$  respectively (because such shocks are transmitted to countries  $A$  and  $C$  via terms of trade fluctuations). The policy functions  $x_{Bt+1} = x_{Bt+1}(z_t, z_{Bt}, z_{Dt})$  and  $x_{Dt+1} = x_{Dt+1}(z_t, z_{Bt}, z_{Dt})$  for countries  $B$  and  $D$  respectively are taken as given by country  $A$ ; they are themselves the solutions to corresponding Bellman equations for countries  $B$  and  $D$ .

### 3.5 Infinite Horizon Version of Main Result and Comparative Statics

We present the infinite horizon version of the main result in this paper.

**Main Result 2** *Even if  $q > r$ , intra-regional reserve pooling may be superior to inter-regional reserve pooling for country  $A$ .*

Figure 8 illustrates this result by plotting the value functions for country  $A$  for the three cases of self-insurance, intra-regional and inter-regional reserve pooling. Again, we use the utility function  $u(C_j) = \log(C_j)$  and discount factor  $\beta = 0.6$ ; endowment shocks are given by  $\omega^H = 10$ ,  $\omega^L = 2$ . We set the elasticity of substitution between final goods produced in the same region to  $\theta = 1.5$ . The conditional probability of country  $B$  receiving the same shock as country  $A$  is set to  $q = 0.2$ , while the same conditional probability for country  $C$  is set to  $r = 0.15$ . To be able to present the value functions on a two dimensional diagram, the value functions are plotted for the particular shock realization of  $z_{At} = z_{Bt} = z_{Ct} = z_{Dt}$  in the current period.

For the specification selected, the value function under inter-regional reserve pooling lies everywhere strictly between the value functions for self-insurance and intra-regional reserve pooling, even though the correlation of shocks is higher within than across regions.

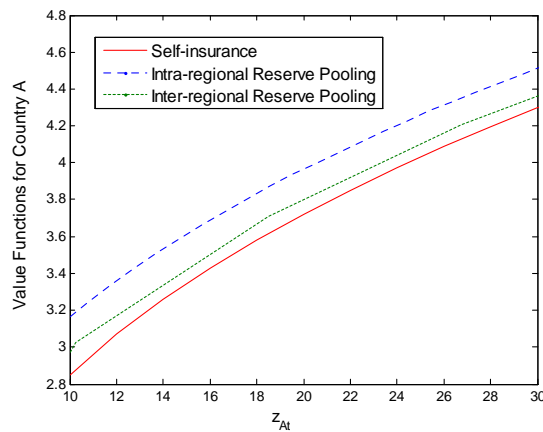


Figure 8: Value Functions for Country  $A$  under Self-insurance, Intra-regional Pooling and Inter-regional Pooling



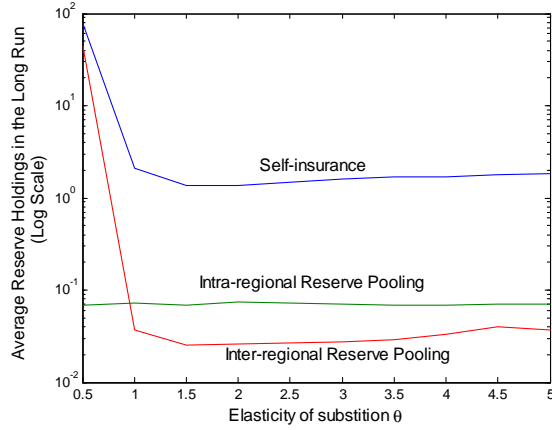


Figure 9: Long Run Average Reserve Holdings under Different Arrangements

Each arrangement for country  $A$  – whether self-insurance, intra-regional pooling or inter-regional pooling – entails the accumulation of reserves for at least some realizations of the endowment shock. In the long run, holdings of the reserve asset follow an invariant distribution. How do average reserve holdings for region I as a whole vary across the different arrangements? This is a comparative statics exercise which was not feasible in the one-period model. Figure 9 plots long run average reserve holdings (on a log scale) associated with the various arrangements, for different values of the elasticity of substitution between final goods  $\theta$ . To make the values comparable, the reserve holdings for the self-insurance case are the sum of the holdings of the two countries in region I. For the pooling arrangements, the reserve holdings shown are the accumulation of the entire pool.

Average reserve holdings in the long run do not change with  $\theta$  for an intra-regional reserve pooling arrangement (this is a direct implication of our prior result that the policy function of an intra-regional pool is invariant with respect to  $\theta$ ). Under self-insurance, long run average reserve holdings in region I are higher than in the intra-regional pooling case. Moreover, the average reserve holdings are non-monotonic in the elasticity of substitution parameter. The intuition for this result is as follows. If we set the elasticity of substitution to infinity, trade linkages within the region are non-existent. Reserves are accumulated by each country because they are the only tool available for insurance against its own endowment shocks. As the elasticity of substitution parameter declines, trade linkages strengthen and each country is partially insulated from a negative endowment shock due to a concomitant positive terms of trade movement. This diminishes the need for reserves to insure purely against the country’s own endowment shocks. However, for very low values of the elasticity of substitution, country  $A$  suffers severe negative terms of trade shocks when country  $B$  receives negative endowment shocks, and vice versa. Each country therefore has to increase reserve accumulation in order to insure itself not only against its own endowment shocks, but against those

of its trading partner.

As the elasticity of substitution parameter  $\theta$  tends to infinity, there are no terms of trade effects from endowment shocks, and the standard model of risk diversification applies. Since  $q > r$ , an inter-regional arrangement is both superior to, and requires lower average reserve holdings than, an intra-regional arrangement. As the elasticity of substitution parameter declines, inter-regional reserve pooling becomes dominated by intra-regional pooling because the externalities associated with uncoordinated reserve accumulation become larger, and these are not fully internalized via pooling across regions. However, the level of average reserves under inter-regional arrangements remains lower than the level under intra-regional pooling, because the intra-regional pool must accumulate reserves to insulate itself against region-wide shocks. For very low values of the elasticity of substitution, countries in the inter-regional pool suffer severe negative terms of trade shocks from countries outside the pool, and vice versa. Since these cannot be internalized in an inter-regional pool, the long run average reserve holdings increase sharply for low  $\theta$ .

## 4 Conclusion

The main result of this paper is that intra-regional reserve pooling arrangements may be superior to inter-regional arrangements, even if the correlation of shocks to countries is higher within rather than across regions. This result derives from the observation that trade linkages are higher within rather than across regions. In this context, self-insurance via noncontingent assets has regional ramifications, because countries use their savings to support domestic production without considering the effects of their actions on the welfare of their trading partners. Regional reserve pooling arrangements can internalize these terms of trade effects, and thereby improve welfare for all member countries. The stronger the terms of trade effects of shocks (via trade linkages) within a specific region, the more attractive is such a pool.

In the first part of the paper, we construct and solve a one-period model which specifically highlights the intratemporal risk-sharing dimension of a reserve pooling arrangement. Regional reserve pooling aligns the relative proportion of goods in production with the socially optimal proportion of goods in consumption, and is particularly valuable when the elasticity of substitution between goods is low. Reserve pooling between two countries in different regions does not fully take the intra-regional terms of trade effects into account; such an arrangement is only desirable if the correlation of shocks across regions is much lower than that within regions, or if the elasticity of substitution between goods is high. In the second part of the paper, we consider a more general infinite horizon model of reserve pooling. We prove that the infinite horizon problem can be split into intratemporal and intertemporal subproblems. The intratemporal subproblem follows exactly the same structure as the one-period model. The intertemporal subproblem can be analyzed using standard techniques in the consumption smoothing literature. On the theoretical front, we

characterize the externality associated with unilateral reserve accumulation. We also identify the compensated Pareto improvement that is generated when two trading partners coordinate their reserve accumulation and production decisions. Finally, we use the infinite horizon specification to examine how average reserve levels in the long run vary between self-insurance, intra-regional reserve pooling and an inter-regional arrangement.

For the purposes of our model, reserve pooling improves risk-sharing via the transfer of endowments across countries in advance of the production stage. We believe that this modeling choice is appropriate for two reasons. Firstly, it captures the ease with which foreign exchange reserves can be transferred across countries. Secondly, it reflects the potential use of reserves by central banks to support the domestic production sector via liquidity support (in particular, via foreign exchange provision for sectors with high levels of foreign currency denominated liabilities).

This paper restricts all reserve pooling arrangements to be pairwise arrangements. This enables a detailed comparison of the differences in risk-sharing properties between intra-regional and inter-regional arrangements of the same size. In future work, it would be useful to consider more general reserve pooling arrangements with a multitude of member countries. For the simple models considered in this paper, the optimal arrangement which implements the allocation of a world social planner is a global reserve pool. This is because we endow the reserve pool with the authority to decide the size of all transfers into and out of the pool; this is in turn sufficient to achieve full risk-sharing between all the members of the pool. We are currently extending our analysis to frameworks where the volume of transfers is decided in each period by member countries instead. This raises the possibility that a global pool may be able to sustain lower levels of contributions from member countries than a regional pool.

## 5 Appendix: Proofs of Propositions

Propositions 1 – 2 and 4 – 8 follow by inspection, as do Corollaries 1 – 2. Main Results 1 and 2 are statements that intra-regional pooling may be superior to inter-regional pooling in the infinite horizon case even if the correlation of shocks is higher within than across regions. They follow directly from the comparative statics exercises.

### Proof of Proposition 3.

First, consider the allocation achievable by a social planner who is constrained to operate only inside region I. Proposition 1 can be amended to prove that in this case, the optimal allocation has the following properties:

- (i) the quantity of each good produced is identical:  $y_j = x + \frac{1}{2} \sum_{j \in \{A, B\}} \omega_j$  for all  $j$ .
- (ii) each country in a particular region consumes half the output of each good produced in that region:  $c_{jj} = c_{jj'} = \frac{1}{2} y_j$  for all  $j, j'$  in the same region.

Next, let us return to the optimization problem of the reserve pool. The reserve pool is in fact able to induce the constrained social planner allocation described above, if and only if it sets  $y_A = y_B$ . ■

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