

POLICY RESEARCH WORKING PAPER

5693

Disaster Risk Financing and Contingent Credit

A Dynamic Analysis

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June 2011



Abstract

This paper aims to assist policy makers interested in establishing or strengthening financial strategies to increase the financial response capacity of developing country governments in the aftermath of natural disasters, while protecting their long-term fiscal balance. Contingent credit is shown to increase the ability of governments to self-insure by relaxing their short-term liquidity constraints. In many situations, contingent credit is most effectively used to facilitate risk retention for middle layers, with reserves used for bottom layers and risk transfer (for example, reinsurance) for top layers.

Discussions with governments on the optimal use of contingent credit instruments as part of a sovereign catastrophe risk financing strategy can be guided by the output of a dynamic financial analysis model specifically developed to allow for the provision of contingent credit, in addition to reserves and/or reinsurance. This model is illustrated with three country case studies: agricultural production risks in India; tropical cyclone risk in Fiji; and earthquake risk in Costa Rica.

This paper is a product of the Non-Banking Financial Institutions Unit of the Global Capital Market Development Department, Finance and Private Sector Development Vice Presidency. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The authors may be contacted at omahul@worldbank.org. The authors thank the participants of the World Bank seminar on March 7, 2011, and the Research Workshop at the Center for the Study of African Economies on May 11, 2011, University of Oxford, for their comments and feedback.

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DISASTER RISK FINANCING AND CONTINGENT CREDIT: A DYNAMIC ANALYSIS

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JEL codes: G22, G32, H60, O16.

Keywords: Disaster risk financing, contingent credit, developing countries, natural disasters, dynamic financial analysis, insurance.

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1. Introduction

How should a government vulnerable to natural disasters arrange its public finances?

One simple approach would simply be to 'wait and see', with the intention of arranging a loan after the occurrence of a disaster to finance reconstruction. While this is likely to be a reasonable strategy for much reconstruction, despite the potentially higher post-disaster cost of borrowing (Ozcan 2005), it is not appropriate for the financing of immediate post-disaster liquidity needs, such as the reconstruction of key public infrastructure such as bridges or hospital or the financial compensation of affected households, since arranging a reasonably priced loan in the aftermath of a disaster takes time (Ghesquiere and Mahul, 2010).

A second approach would be to pre-fund a budget allocation or reserve fund, with government either borrowing or accumulating tax revenues to hold in liquid assets to act as a quick disbursement facility. However, while some states have access to substantial sovereign wealth funds which can be disbursed quickly in the aftermath of a disaster (Kern 2007), for many others there are significant political economy constraints on the size of such a fund in the short or medium term, limiting their ability to fully pre-fund post-disaster liquidity needs.

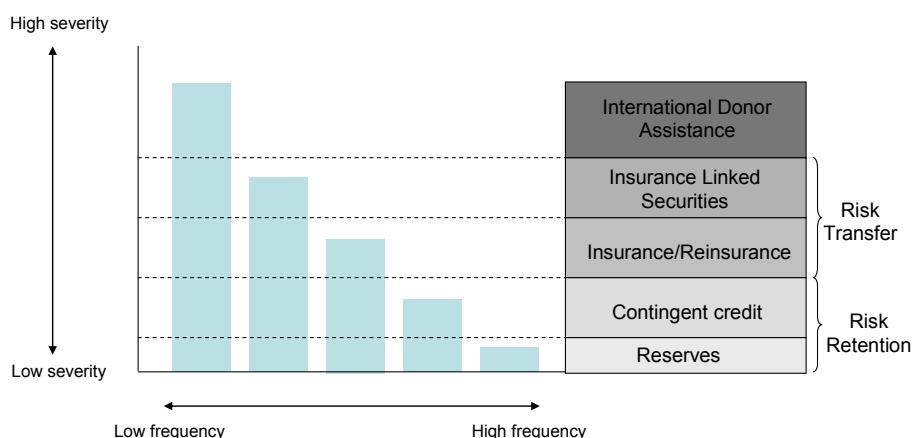
Third, a government could purchase reinsurance or other risk transfer instruments such as index linked securities, paying an annual premium in return for quick cash in the event of a predefined disaster. Whilst such risk transfer may be appropriate for extreme events, it is likely to be expensive for more frequent events. Moreover, reinsurance operates using a *hard trigger* (e.g., the magnitude of an earthquake or the intensity of an hurricane), with claim payments a clearly defined function of realized events, and may not provide protection against all possible catastrophes. This is particularly true for the recently popular indexed insurance, which offers only a hedge against exposure, and exposes policyholders to basis risk, the risk that insurance claim payments do not accurately reflect the incurred loss.

Finally, a government could arrange a line of credit in advance, to be drawn down in the event of a disaster. As for a commitment loan, the credit contract between lender and government would specify how the interest rate would be determined, the maturity of the loan, and how the loan could be put to use (Greenbaum and Thakor 2007). However, the contract could also specify that a loan could only be disbursed in the event of a disaster. This could have either a hard trigger, as for reinsurance, or a *soft trigger*, such as the declaration of state of emergency by the government. As with the first and second option, government would still retain risk, and like the second option this could be used to finance post-disaster liquidity needs, spreading the effect of any disaster over multiple years. However, ex-post liquidity financing may be subject to fewer political economy constraints than ex-ante reserving.

Contingent credit with a soft trigger is similar to relationship banking since, although a government could trigger drawdown at any time, this would be costly and would harm their reputation with lenders (Boot 2000); by improperly declaring an emergency, government would essentially be swapping 'relationship capital' for financial capital (Boot et al. 1993).

The paper argues that the optimal strategy for financing post-disaster liquidity needs for a government subject to restrictions on their budget allocation or reserve fund is likely to include both risk retention through reserving and contingent credit and risk transfer through reinsurance and other such instruments. The underlying theoretical model suggests a risk layering approach with reserves financing small losses and contingent credit and reinsurance providing additional financial capacity for moderate and catastrophic losses (see Figure 1).

Figure 1. Risk layering and disaster risk financing strategy



Source: Ghesquiere and Mahul (2010)

In practice governments may find the decision about what precise disaster risk financing strategy to implement to be somewhat challenging. In addition to theoretical models, this paper presents output of a dynamic financial model, developed to run in MS Excel, which allows decision makers to compare alternative risk financing strategies. The model has been designed as a tool to assist countries in developing cost-effective disaster risk financing strategy based on a combination of reserves, contingent credit and risk transfer instruments (e.g. reinsurance). The tool does not calculate an optimal strategy, but rather calculates projections that can help financial specialists to engage with governments on disaster risk financing.

While the principles in this paper apply more generally, particular attention is given to the World Bank's Development Policy Loan (DPL) with Catastrophe Risk Deferred Drawdown Option, DPL with CAT DDO. This product offers IBRD-eligible countries immediate liquidity up to USD\$500 million or 0.25 percent of GDP (whichever is less) if they suffer a natural disaster, providing bridge financing while other sources of post-disaster funding are being mobilized and allowing for budget support to governments hit by a natural disaster. Funds are disbursed when a country suffers a natural disaster and declares a state of emergency; since a state may declare a state of emergency for a number of reasons, some difficult to foresee, this *soft trigger* may capture more events than reinsurance. Eligible borrowers must have an adequate macroeconomic framework in place at inception and renewal, and a disaster risk management program that is monitored by the World Bank.

The first DPL with CAT DDO was approved by the World Bank's Board of Executive Directors in September 2008. The US\$65 million contingent loan to the Government of Costa Rica aims to enhance its capacity to "implement a Disaster Risk Management Program for natural disasters." This program is described in the loan document and agreed upon before signing. Following the 6.2 magnitude earthquake that hit Costa Rica on January 8, 2009, the Government of Costa drew down approximately US\$15 million. DPLs with CAT DDO have since been negotiated with Colombia (\$150m), Peru (\$100m), El Salvador (\$50m) and Guatemala (\$85m) and are currently under preparation in various other countries.

As of May 2010, the DPL with CAT DDO has the same lending base rate as regular IBRD loans. The front-end-fee, payable upon effectiveness, is 0.5% and there is no commitment fee. The draw down period is for three years, renewable up to four times (with a renewal fee of 0.25%). Repayment terms may be determined either upon commitment, or upon drawdown within prevailing maturity policy limits. Repayment schedule would commence from date of drawdown. The CAT DDO is an example of contingent credit that governments in developing countries can use to strengthen their

disaster risk financing strategy, as part of their overall disaster risk management program. Contingent credit can complement other risk retention tools (such as reserves) and risk transfer instruments (including catastrophe insurance).

The rest of this paper is structured as follows. Section 2 presents theoretical rationales for optimal contingent credit purchase. It derives a simple formula for the contingent credit multiple, which can be compared with reinsurance multiples to assess value for money from a risk neutral government. It then offers an analysis of the contingent credit purchase decision using a variation of the two period canonical lifetime consumption model for a risk-averse government. Section 3 discusses the results of the contingent credit DFA model, as applied to three illustrative risks: agricultural production risk in India; tropical cyclone risk in Fiji; and earthquake risk in Costa Rica. Section 4 concludes.

2. Theoretical rationales for contingent credit

This section provides two alternative theoretical frameworks for thinking about the optimal use of savings, direct and contingent credit and reinsurance. First we extend the concept of an insurance multiple, the ratio of the annual premium to the annual expected loss, to the context of direct and contingent debt. This allows us to analyze the risk financing decision for a risk neutral government exposed to a potential liquidity crunch. This approach is illustrated with a worked example. Second, we present a stylized lifetime consumption model to characterize optimal financial planning for a risk-averse government. Both models provide a theoretical motivation for the strategies modeled in Section 3.

2.1. Calculating the effective multiple for contingent credit

For a given layer of risk, the reinsurance (or insurance) multiple is typically defined as:

$$\text{Reinsurance Multiple} = \frac{\text{Annual reinsurance premium}}{\text{Annual expected loss from layer}} \quad (1)$$

The reinsurance multiple for a given layer is simple to calculate for a given reinsurance premium and a given loss distribution. Reinsurance multiples are typically greater than unity, to reflect the costs to the reinsurer of administration and accessing risk capital. Multiples are often used by purchasers and sellers of reinsurance as an indication of the value for money of the product, with a lower multiple indicating better value for money for the reinsured. For given operating costs, the multiple is expected to increase with high risk layers as the cost of capital increases with respect to the annual expected loss.

Calculating an equivalent multiple for contingent credit is more complex for two reasons. First, although reinsurance is typically an annual contract, a contingent credit contract may last for many years. Multiples for contingent credit must therefore incorporate some subjective assumptions about how future cash flows will be discounted. Second, contingent credit is typically purchased with a multi-year drawdown period, and is used for different layers in different years, depending on the start year savings available for retention. Calculating the multiple for contingent credit on a multi-year basis would therefore require detailed assumptions about the entire risk financing strategy. Given this, it is suggested that the multiple for contingent credit is calculated on an annual basis for a specific layer, even if the contingent credit contract is multi-year and expected to cover different layers in different years.

The basic formula suggested for calculating the multiple for contingent credit is as follows:

$$\begin{aligned} &\text{Contingent Credit Multiple} \\ &= \frac{\text{Annualised fee} + \mathbb{E}PV[\text{Contingent debt repayment cashflow}]}{\text{Annual expected loss from layer}} \end{aligned} \quad (2)$$

where $\mathbb{E}PV$ denotes expected present value, with discounting using a subjective discount factor appropriate for the government, and the contingent credit repayment cash flow is only for debt drawn down during the year. The formula is of the same basic structure as that for the reinsurance multiple; both multiples are defined by the expected present value of the cost of cover for a given layer, divided by the annual expected loss from that layer.

Equation (2) can be simplified as follows. Let $i > 0$ denote the subjective interest rate used by government for discounting future cashflows and n and ρ denote the term and contractual rate of interest applicable to any drawn down contingent loan respectively. For the layer of interest let D_{CC} denote the maximum loss in the layer and L_{CC} be a random variable denoting the incurred loss in the layer. The average loss in the layer is therefore given by $\mathbb{E}[L_{CC}]$ and the loss-on-line is $\mathbb{E}[L_{CC}]/D_{CC}$. The annualized fee in equation (2) may therefore be expressed as $\alpha \times D_{CC}$ where α is the annualized fee rate. α may be calculated as a simple average of total upfront and renewal fee rates over the maximum drawdown period.³ Similarly denoting the expected present value of future loan repayments, per unit of drawn down loan, as $G_n(i, \rho)$, then $\mathbb{E}PV[\text{Contingent debt repayment cashflow}] = G_n(i, \rho) \times \mathbb{E}[L_{CC}]$. Substituting these equations into equation (2) and rearranging gives the following formula for the contingent credit multiple:

$$\text{Contingent Credit Multiple} = \frac{\alpha}{\text{loss-on-line}} + G_n(i, \rho) \quad (3)$$

where, as defined in the previous paragraph, α is the annualized fee rate and $G_n(i, \rho)$ is the expected present value of future loan repayments per unit of drawn down loan, with term n , subjective interest rate i and contractual rate of interest for loan repayment ρ .

The first term in formula (3) arises from the upfront and renewal fee and the second arises from the interest and capital cost of servicing any drawn down debt. Perhaps surprisingly, the final term does not depend on the distribution of losses in the layer, but depends only on the subjective discount rate to be used and the structure of the contingent credit contract. This is because it is assumed that the expected present value of paying back contingent credit is linear in drawn down loan, resulting in a $\mathbb{E}[L_{CC}]$ term in the second term of the numerator in equation (2) which cancels with the $\mathbb{E}[L_{CC}]$ term of the denominator. The contingent credit multiple is therefore linearly separable in the loss distribution and the terms for repayment of any drawn down loan.

$G_n(i, \rho)$ may be written explicitly in terms of subjective and contractual interest rates i and ρ for a given schedule of repayment for any drawn down debt. For example, if any loan was to be repaid by n level annual repayments, with the first payment due one year after the loan was originally drawn down, then the annual repayments would be $(1 - \frac{1}{(1+\rho)^n})/\rho$. The expected present value of these level repayments using government's subjective interest rate of i would then be:

$$G_n^{LEVEL}(i, \rho) = \frac{\rho(1 + \rho)^n}{i(1 + i)^n} \times \frac{(1 + i)^n - 1}{(1 + \rho)^n - 1} \quad (4)$$

For full repayment at the end of one year, equation (4) simplifies to

³ α could alternatively be calculated as a weighted average of fee rates, where the weights applicable to renewal fees reflected the expected loan available for drawdown at time of renewal. However, such a calculation would depend on the entire strategy chosen by the government and would not necessarily be transparent. In most circumstances it should be possible to derive an upper and lower bound for the annualized fee rate to use in equation (3), without reference to the precise strategy or loss distribution. The contingent credit multiple is only sensitive to α for high layers with a very low loss-on-line, and so the approximation suggested in the main text, or the bounded approach suggested in this footnote should be adequate for the purposes of assessing the cost-effectiveness of contingent credit (see Figure 3).

$$G_1^{LEVEL}(i, \rho) = \frac{1 + \rho}{1 + i}, \quad (5)$$

and for level annual repayments in perpetuity, equation (4) becomes

$$G_\infty^{LEVEL}(i, \rho) = \frac{\rho}{i}. \quad (6)$$

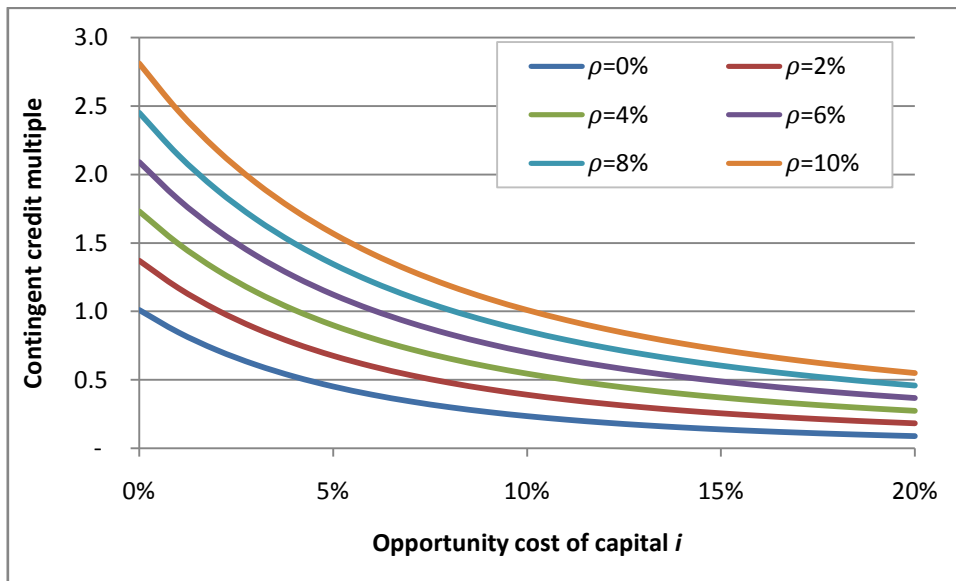
Under the standard structure for the DPL with CAT DDO, only interest is paid for the first m years, with level annual capital repayments for the final $m - n$ years resulting in an expected present value of repayments of:

$$G_n^{CATDDO}(i, \rho) = \frac{\rho}{i} \left(1 - \frac{1}{(1+i)^n} \right) + \sum_{k=m+1}^n \left[\frac{1}{(1+i)^k} \times \frac{1 - \rho(k-m-1)}{(n-m)} \right] \quad (7)$$

Figures 2 and 3 display illustrative CAT DDO multiples for a range of parameter values, using equations (3) and (7). Consistent with Appendix 2 it is assumed that the term of any drawn down loan (n) is 30 years, the grace period (m) is 5 years, and the annualized fee rate (α) is $\frac{0.5\% + 4 \times 0.25\%}{15} = 0.1\%$. Figure 2 takes the loss-on-line to be fixed at 10% and Figure 3 takes the rate of interest applicable to any draw down loan (ρ) to be fixed at 4.4%.

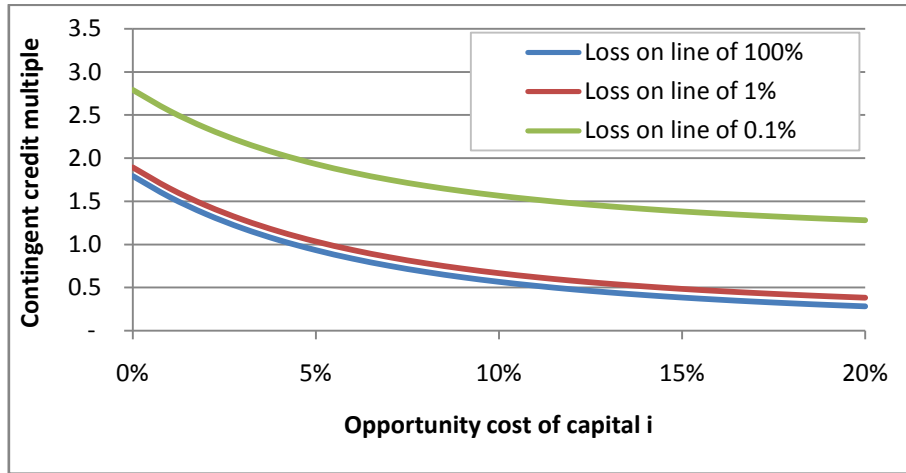
As can be seen from Figure 2, the contingent credit multiple is increasing in the contractual rate of interest ρ and decreasing in the opportunity cost of capital i . The second term of equation (3) is equal to unity when $i = \rho$, and so for the assumed parameter values the contingent credit multiple equals 1.01 when $i = \rho$.

Figure 2. Contingent credit multiples, varying with contractual rate of interest payable on credit (loss-on-line = 10%, $\alpha = 0.1\%$, $n = 30$, $m = 5$)



As can be seen from Figure 3 the contingent credit multiple is not materially sensitive to the loss-on-line or annualized fee rate when the former is more than ten times the latter; multiples are almost unchanged when the loss-on-line is decreased from 100% to 1%, but change substantially when further reduced to 0.1%. This arises from the first term of equation (3) equating to less than 0.1 when $(\text{loss-on-line}/\alpha) > 10$. One implication of this is that for low and medium layers for which the loss-on-line divided by the annualized fee rate is likely to be greater than ten, it is not necessary to calculate either the loss-on-line or α precisely to obtain a fairly good estimate for the contingent credit multiple.

Figure 3. Contingent credit multiples, varying with loss on line ($\rho = 4.4\%$, $\alpha = 0.1\%$, $n = 30$, $m = 5$)



Formula (3) can also be used for calculating multiples for direct credit, in which case the upfront fee α would typically be zero and:

$$\text{Direct Credit Multiple} = G_n(i, \rho) \quad (8)$$

where $G(i, \rho)$ is defined as above as the discounted present value of a loan repayments of unit debt, calculated at subjective interest rate i .

2.2. Using multiples to compare risk financing strategies

The choice of risk financing strategy by a risk neutral policy maker can be simplified through use of the multiples for direct credit, contingent credit and reinsurance.

For example, consider the following situation. A risk neutral government must finance a loss which takes the values L and zero with probabilities p and $1 - p$ respectively, and makes decisions with a subjective rate of interest of $i > 0$. The government must finance the loss through liquid assets but is otherwise unrestricted in its use of savings, direct credit, contingent credit and reinsurance.

The government may borrow at low interest rate ρ_L up to a country exposure limit of $D > L$, and at a high interest rate ρ_H above D , where $\rho_H > \rho_L > 0$. Contingent credit within the country exposure limit may not contribute 100% towards the limit, but rather the direct and contingent credit within the limit, D_D and D_{CC} respectively, must satisfy $D_D + \beta \times D_{CC} < D$, where $p < \beta \leq 100\%$. β might be set by the loan issuer to be less than 100% to reflect the low probability that a loan will be disbursed.

Any debt must be repaid in full one year after draw down and the upfront fee for contingent credit is not material to the decision, and therefore assumed to be zero in the following calculations. The multiples for direct or contingent credit up to and above the country exposure limit of D are therefore given by $G_1^{LEVEL}(i, \rho_L) = \frac{1+\rho_L}{1+i}$ and $G_1^{LEVEL}(i, \rho_H) = \frac{1+\rho_H}{1+i}$ respectively. The reinsurance multiple is denoted M_R .

If subjective interest rate i is greater than ρ_H or less than ρ_L a risk neutral government's decision is trivial. If $i > \rho_H$ then the government will optimally borrow up to the full country exposure limit of D at low rate ρ_L and the maximum possible amount above the country exposure limit at high rate ρ_H , with loss L financed through reinsurance or contingent credit above the country exposure limit. If $i < \rho_L$ the government would borrow the minimum amount possible, self-insuring through savings or contingent credit, or reinsuring. The case of $\rho_L < i < \rho_H$, where borrowing within the country exposure limit is cheap but borrowing above it is expensive, is explored in more detail below.

Three intuitive strategies are described in column [2] of Table 1. Under the first strategy the loss is fully reinsured, and under the second and third strategies the loss is financed through contingent credit above and within the country exposure limit of D , respectively. A risk neutral government choosing between these three strategies would select the strategy with the smallest expected cost (column [5] of Table 1).

Table 1. Potential risk financing strategies

Strategy	Description of strategy	Realized cost if no loss incurred	Realized cost if incur loss of L	Expected cost of strategy $= (1 - p) \times [3] + p \times [4]$
[1]	[2]	[3]	[4]	[5]
1	Direct credit of D , invested in illiquid asset. Full reinsurance of potential loss.	$pLM_R - D \left(1 - \frac{1+\rho_L}{1+i}\right)$	$pLM_R - D \left(1 - \frac{1+\rho_L}{1+i}\right)$	$pLM_R - D \frac{i - \rho_L}{1+i}$
2	Direct credit of D , invested in illiquid asset. Borrow L at ρ_H if a loss occurs.	$-D \left(1 - \frac{1+\rho_L}{1+i}\right)$	$\frac{L(1+\rho_H)}{1+i} - D \left(1 - \frac{1+\rho_L}{1+i}\right)$	$\frac{pL(1 + \rho_H) - D(i - \rho_L)}{1+i}$
3	Direct credit of $D - \beta L$. Contingent credit of L at ρ_L if a loss occurs.	$-(D - \beta L) \times \left(1 - \frac{1+\rho_L}{1+i}\right)$	$\frac{L(1+\rho_L)}{1+i} - (D - \beta L) \times \left(1 - \frac{1+\rho_L}{1+i}\right)$	$\frac{pL(1 + \rho_L) - (D - \beta L)(i - \rho_L)}{1+i}$

A risk neutral government would prefer contingent credit within the country exposure limit to contingent credit above the limit if the expected cost for strategy 3 is lower than that for strategy 2, that is if:

$$\beta(i - \rho_L) < p(\rho_H - \rho_L) \quad (9)$$

The left hand side of equation (9) reflects the opportunity cost of reducing the amount of direct credit within the country exposure limit to allow for an increase in contingent credit within the limit, and the right hand side is the expected increase in interest rate that would be payable if loss L was financed through contingent credit above the country exposure limit, instead of within the country exposure limit. Reserving part of the country exposure limit for contingent credit is more likely to be preferred when the contribution of contingent credit towards the country exposure limit is low compared to the probability of drawdown (low β/p), where borrowing within the country exposure limit is relatively unattractive (low $i - \rho_L$) and where borrowing above the country limit exposure is much more expensive than borrowing within the limit ($\rho_H - \rho_L$ is high).

A risk neutral government would prefer to finance the loss through contingent credit instead of reinsurance if the expected cost of either strategy 2 or 3 is lower than that for strategy 1, that is if:

$$M_R > \min \left(\frac{1 + \rho_H}{1 + i}, \frac{1 - \rho_L \left(\frac{\beta}{p} - 1 \right) + \frac{\beta i}{p}}{1 + i} \right) \quad (10)$$

This is more likely to bind when reinsurance is expensive (high reinsurance multiple M_R), the subjective discount rate is high (high i), borrowing above the country exposure limit is attractive (low ρ_H), borrowing within the country exposure limit is unattractive (high ρ_L), or the contribution of contingent credit towards the country exposure limit is low compared to the probability of drawdown (low β/p). The latter two dependencies arise from the following observation. When $i > \rho_L$, borrowing within the country exposure limit is valuable, and the reservation of part of the country exposure limit for contingent credit is costly in that with probability $1 - p$ only $D - L$ of cheap credit will be drawn down, instead of the maximum amount D . The opportunity cost of foregoing cheap credit with probability $1 - p$ is lowest when β/p is low and the value of cheap credit is low (ρ_L is high). Governments for whom credit within the country exposure limit is particularly valuable (ρ_L is low) are therefore unlikely to purchase contingent credit within the country exposure limit, although they may purchase contingent credit above the country exposure limit.

For the three strategies examined above, the potential loss of L is financed by reinsurance or contingent credit. The loss could instead be financed by savings or direct credit. However, such financing strategies would be suboptimal if, for example, the rate of return available on liquid assets was less than ρ_L , and the rate of return available on illiquid assets was greater than ρ_L . In such a case, savings or direct loans would be optimally invested in illiquid assets, which would not provide cash in the event of a loss L having been incurred.

Calculations like those conducted above can be performed to compare strategies for a risk neutral government. However, such calculations only allow comparison of the expected cost of alternative strategies, and do not make any allowance for any benefits from the transfer of risk to globally diversified pools.

In the next section we introduce a lifetime consumption model which allows us to characterize optimal decision making when the government is risk averse, and acts as a prelude to the analysis of Section 3.

2.3.A lifetime consumption model of contingent credit and risk financing

A government lives for two periods, is exposed to one uncertain shock x in the first period and makes decisions that maximize expected utility, where time separable utility is given by:⁴

$$U(C) = u(C_1) + \beta u(C_2) \quad (11)$$

Constant discount factor $\beta < 1$ embodies the degree of impatience, per period utility function u is strictly concave and net consumption C_t may be interpreted as required government expenditure net of any incurred shock. The effect of a shock is therefore to increase the marginal utility of government expenditure. Shock x is assumed to be atomless with full support over $[0, \bar{x}]$, and probability density function denoted by $f(x)$.

⁴ We ignore the possibility of shocks after year 1 for mathematical convenience, and therefore ignore any incentive for precautionary saving. An N or infinite period model would have qualitatively similar optimal strategies (see Gollier, 2002).

Immediately before the shock occurs, the policy maker must choose an indemnification schedule for insurance $\{I(x)|I(x) \geq 0\}_{x \in [0, \bar{x}]}$, and an amount of contingent credit to purchase $D \geq 0$. The markets for insurance and contingent credit are assumed to be perfectly competitive and so the agent is a price taker. The insurance premium P is determined with a constant multiple of $\theta > 1$, that is $P = \theta \mathbb{E}(\bar{x}) = \theta \int_0^{\bar{x}} f(x)I(x)dx$. Contingent credit is subject to a front end fee of $\alpha > 0$ and any drawn down loan is subject to interest rate of ρ , where ρ is strictly greater than the rate of return on savings r .

At time-1 the agent experiences a shock of x , receives net (certain) income of Y_1 , pays front end fee of αD , and receives net income from insurance of $I(x) - P$. The agent must also decide how much of the loan to draw down, denoted $L(x)$ where $0 \leq L(x) \leq D$, and how much to save for the next period, denoted $S(x) \geq 0$.⁵ Total net consumption at time-1 is therefore:

$$C_1(x) = Y_1 - \alpha D - P + I(x) + L(x) - S(x) - x \quad (12)$$

At time-2 the agent receives income Y_2 , must fully repay any loan and will consume any remaining assets, resulting in consumption of:

$$C_2(x) = Y_2 - (1 + \rho)L(x) + (1 + r)S(x) \quad (13)$$

The agent's problem is to choose constant D and state contingent functions $I, L, S: X \rightarrow [0, \infty)$ to maximize expected utility, subject to the restriction that $L(x) \leq D$ for all x . Formally, the problem may be written as the following program:

$$\max_{P, D, I(x), L(x), S(x)} \int_0^{\bar{x}} f(x) [u(Y_1 - x - \alpha D - P + I(x) + L(x) - S(x)) + \beta u(Y_2 - (1 + \rho)L(x) + (1 + r)S(x))] dx \quad (14)$$

such that:

$$P - \int_0^{\bar{x}} f(x)\theta I(x)dx = 0 \quad (14a)$$

$$D \geq 0; \quad I(x), S(x), L(x), D - L(x) \geq 0 \quad \forall x \quad (14b)$$

Objective function (14) is the agent's expected utility, equation (14a) is the insurance premium equation and constraints (14b) restrict the class of available products to those of insurance and contingent credit, and ensure that the agent is subject to a liquidity constraint on time-1 consumption.

We may now characterize the optimal solution with the following proposition.

Proposition 1. In any solution to program (14) with $D^* > 0$ there exist $\{x_i\}_{i=1,2,3,4}$, with x_i nondecreasing in i such that the optimal savings, contingent credit drawdown and insurance purchase decisions are as follows:

$$0 \leq x < x_1: \quad S^*(x) > 0, -1 < S^{*'}(x) < 0 \text{ and } I^* = L^*(x) = 0;$$

$$x_1 \leq x \leq x_2: \quad S^*(x) = I^* = L^*(x) = 0;$$

$$x_2 < x < x_3: \quad S^*(x) = I^*(x) = 0, L^*(x) > 0 \text{ and } 0 < L^{*'}(x) < 1;$$

$$x_3 \leq x \leq x_4: \quad S^*(x) = I^*(x) = 0, L^*(x) = D^*;$$

⁵ The solution would have the same form if the restriction was $S(x) \geq \underline{S}$ for some $\underline{S} \neq 0$. For $\underline{S} < 0$, the interpretation would be that the agent could borrow up to $-\underline{S}$ at rate of interest r , or save any positive amount at rate of interest r .

$$x_4 \leq x \leq \bar{x}: \quad S^*(x) = 0, I^*(x) = x - x_4 \text{ and } L^*(x) = D^*$$

Proof. See Appendix 1.

Any optimal strategy therefore involves the layering of risk, with risk retention through savings in the bottom layer, risk retention through contingent credit in a middle layer and risk transfer through insurance in the top layer. Figure 4 and Figure 5 offer an illustration of an optimal schedule, as characterized in Proposition 1.

The result that the optimal insurance schedule offers full marginal indemnification for losses above the deductible of x_4 is driven by the assumption that insurance is priced with a fixed multiple of θ , and corresponds to Arrow's (1963) result. Losses below x_4 are retained by the government, and spread between time-1 and time-2 consumption through reduced savings and increased loan drawdown as appropriate.

Figure 4. Illustrative optimal plans for savings, debt and insurance under an assumption of a constant insurance multiple

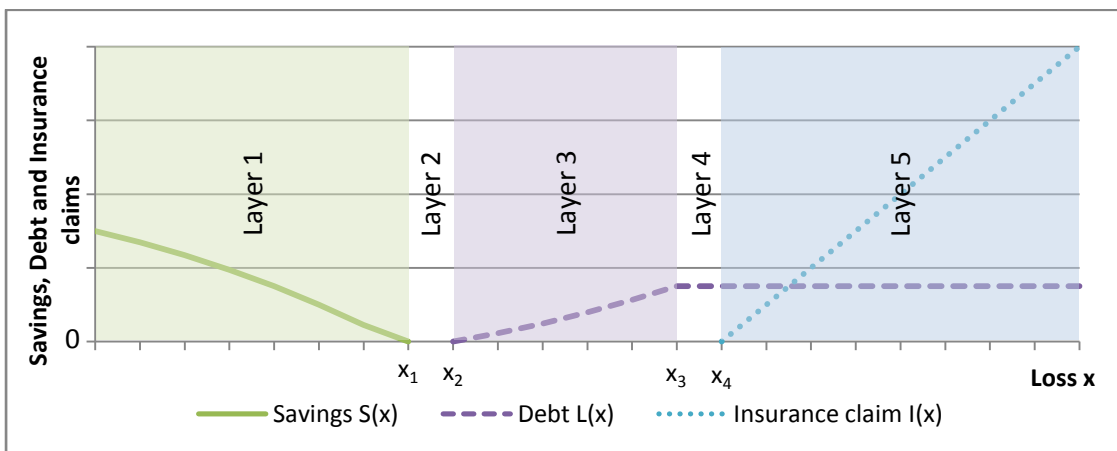
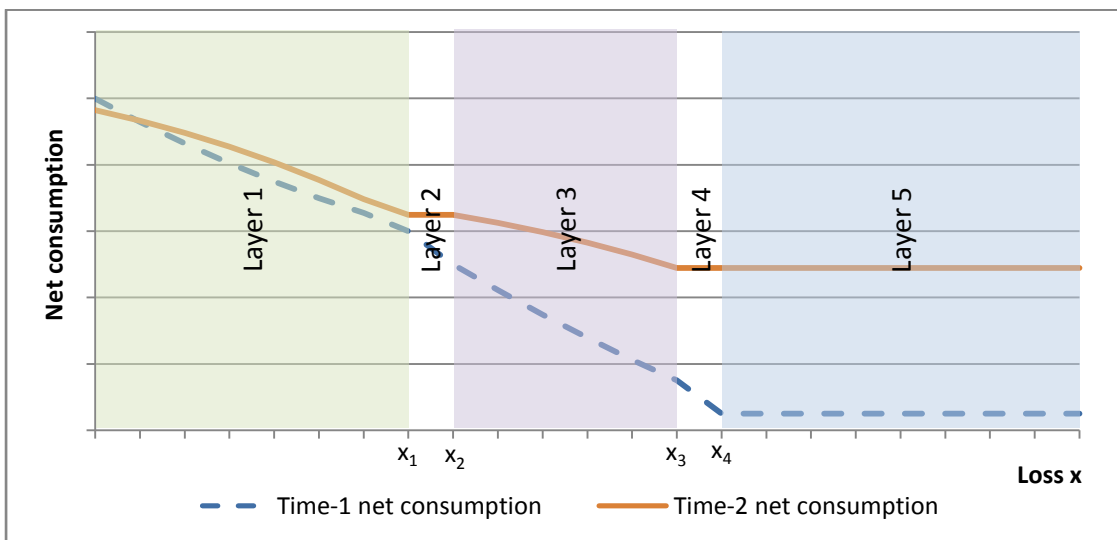


Figure 5. Illustrative optimal consumption plans under an assumption of a constant insurance multiple



For some parameter values some of these layers may have zero size, that is, $x_i = x_{i+1}$, $x_1 = 0$ or $x_4 = \bar{x}$. For example, if insurance is too expensive (θ is large) it may be optimal to not purchase any insurance.⁶ However, as the following corollary shows, so the potential effect of a shock on government is large enough, the front end fee for contingent credit is small enough, and insurance is expensive enough, it will always be optimal for a government to purchase a positive amount of contingent credit.

Corollary 1. If there exists some K such that $\lim_{c \rightarrow K^+} u'(c) = \infty$, then for small enough α , and large enough \bar{x} and θ , it will always be optimal to purchase a positive amount of contingent credit, $D^* > 0$.

Proof. See Appendix 1.

In the above model, in which insurance is priced with a fixed multiple, contingent credit is only ever optimally purchased for consumption smoothing in middle layers, with savings used for lower layers and insurance for higher layers. The result that savings are employed for consumption smoothing in lower layers than for contingent credit is fairly robust, relying only on the assumption that the effective rate of interest applicable to the loan is higher than the effective rate of interest applicable to savings ($\rho > r$). Were the government to borrow positive amounts in a state in which savings were positive, the government could reduce $L(x)$ and $S(x)$ by $\min(L(x), S(x))$ and consume an additional $(\rho - r) \times \min(L(x), S(x)) > 0$ at time-1, with unchanged consumption in time-2. Therefore, in our model contingent credit will only be drawn down when the government faces a liquidity crunch, that is where $S(x) = 0$.

However, it may be optimal to purchase contingent credit for top layers and insurance for middle layers if the insurance multiple is sufficiently increasing in the loss.

3. Country Case Studies

The theoretical models of Section 2 provide a justification for risk layering strategies combining savings, contingent credit and risk transfer. In this section we will build upon the intuition from these models by presenting results generated from a stochastic Dynamic Financial Analysis (DFA) model for three illustrative risks: agriculture in India; tropical cyclones in Fiji; and earthquakes in Costa Rica. We will therefore be able to give an indication of the likely order of magnitude of benefit from using contingent credit as part of a Disaster Risk Financing strategy. We begin with a discussion of the DFA model used for the simulations.

3.1. The DFA model

Real life financing decisions for a government are typically more complex than those modeled theoretically in Section 2: governments exist for more than two periods and are subject to multiple risks. Whilst governments will often have a good understanding of the strategies that could be employed, financial modeling is typically required to understand the mapping from strategies to outcomes. The purpose of a dynamic financial analysis (DFA) model is to aid decision making, by illustrating the medium and long term differences between strategies.

Just as a good economic theory model does not include all features of reality, a DFA model should be parsimonious, and should not necessarily be designed to provide a precise best estimate stochastic projection of the future. The purpose of the contingent credit DFA (CC-DFA) model is to guide discussions with government on DRF and it should enable alternative strategies to be compared,

⁶ Conversely, if $\theta = 1$, full insurance is optimal.

with differing outcomes having a clear and robust explanation. It is therefore acceptable to refrain from modeling any features of reality that affect all potential strategies in the same way, without having a material impact on the differences between strategies.

In the CC-DFA model, a risk financing strategy is a decision about the size of the contingent credit facility to purchase at the outset and rules about reinsurance purchase and the drawdown of any contingent credit. The only source of uncertainty in the model is that of the losses incurred in each year, which are assumed to be drawn from a specified loss distribution. The model abstracts from the nature of the losses, and can therefore be applied to a range of disaster risks, such as earthquakes, hurricanes or droughts.

The CC-DFA model outlined in this paper does not allow for other potential elements of a risk financing strategy, such as a dynamic fiscal allocation rule, or other sources of variation, such as investment returns, future portfolio size or composition, or reinsurance prices. These features could be easily added but, whilst adding further realism to the model, are second order from the point of view of contingent credit purchase, and may make interpretation of the model outputs more difficult.

At its core, a DFA model takes a risk financing strategy and a set of assumptions about future experience as inputs and deterministically projects the outcome. The core equations at the heart of the CC-DFA model are as follows:

$$\begin{aligned}
 Reserves_{t+1} = & [(Reserves_t + AnnualNetAllocation_t - ReinsCost_t - CCFee_t) \times (1 + WithinYearInt) \\
 & + OutstandingCCLoan_{t+1} - OutstandingCCLoan_t \times (1 \\
 & + CInt) - Loss_t + ReinsIncome_t] \\
 & \times (1 + BetweenYearInt)
 \end{aligned} \tag{15}$$

$$NetReserves_t = Reserves_t - OutstandingCCLoan_t \tag{16}$$

Equation (15) can be explained as follows. It is assumed that the government has a dedicated reserve fund which starts year t with liquid reserves of $Reserves_t$ and outstanding drawn down contingent credit of $OutstandingCCLoan_t$. The fund is assumed to immediately receive an annual budget allocation to a dedicated reserve fund, resulting in net increase in liquid reserves of $AnnualNetAllocation_t$. The fund then pays any reinsurance ($ReinsCost_t$), and in some years will pay a contingent credit upfront or renewal fee ($CCFee_t$). Reinsurance and contingent credit purchase is determined by the risk financing strategy, with reinsurance premiums calculated based on the assumed multiples. The financial conditions of the World Bank's DPL with CAT DDO are used: the upfront fee of 0.50% of the contingent loan facility is payable at the start of year 1 and renewal fee of 0.25% of debt that has not yet been drawn down is payable at the start of years 4, 7 and 10. The fund earns interest of $WithinYearRoR$ on its reserves, and then at the end of the policy year may receive claim income from reinsurers ($ReinsClaim_t$) and must meet additional expenditure of $Loss_t$. The government must meet the interest cost of any loan outstanding at the start of the year ($OutstandingCCLoan_t \times CInt$) and may receive additional cash from drawing down additional debt of $OutstandingCCLoan_{t+1} - OutstandingCCLoan_t$. Finally, the insurer earns interest on its reserves between the end of the policy year and the start of the next of $BetweenYearRoR$.

Net reserves at time t , $NetReserves_t$, are equal to the total liquid reserves of the fund minus any outstanding drawn down contingent loan $OutstandingCCLoan_t$.

Given this core functionality, a large number of Monte Carlo simulations can be run using a specified risk financing strategy but with assumptions about future experience drawn from a specified probability distribution. A decision maker can then compare the simulated distribution of outcomes for different risk financing strategies, and choose the strategy with the preferred distribution of outcomes.

For decision makers to be able to make use of the power of the CC-DFA model, the presentation of results from the model is critical. Since the purpose of a DFA model is to aid decision making, outputs from the model should directly relate to the preferences of decision makers. A government might have preferences over: (i) The probability of a liquidity crunch occurring within ten years, that is where $Reserves_t < 0$ for some $t \leq 10$; (ii) The distribution of net reserves at the end of ten years; and (iii) The amount of outstanding drawn down contingent credit at the end of ten years.

For the distribution of net savings at the end of 10 years to hold meaning, one cannot simply ignore the consequences of a liquidity crunch occurring. In the following we will quote the probability of a liquidity crunch occurring, and when quoting net savings will assume that in the event of a liquidity crunch the government can borrow further, albeit at a high interest rate, to meet its obligations.

3.2. Description of the country case studies

The CC-DFA model is illustrated through the discussion of three worked examples: multi-peril crop yield risk in India; tropical cyclones risk in Fiji; and earthquake risk in Costa Rica. An overview of the assumptions and strategies used for projections may be found in Appendix 2. We now describe the strategies and some of the assumptions in detail.

There are a number of real life differences between these three examples but for the purpose of choosing an optimal amount of contingent debt to purchase, the key distinction is the distribution of losses: losses from Indian agricultural are frequent, but moderate; those from Fijian tropical cyclones are infrequent but severe; and those from Costa Rican earthquakes are rare but very severe (see Figure 6).

Figure 6. Mean return period of losses as proportion of Annual Expected Loss

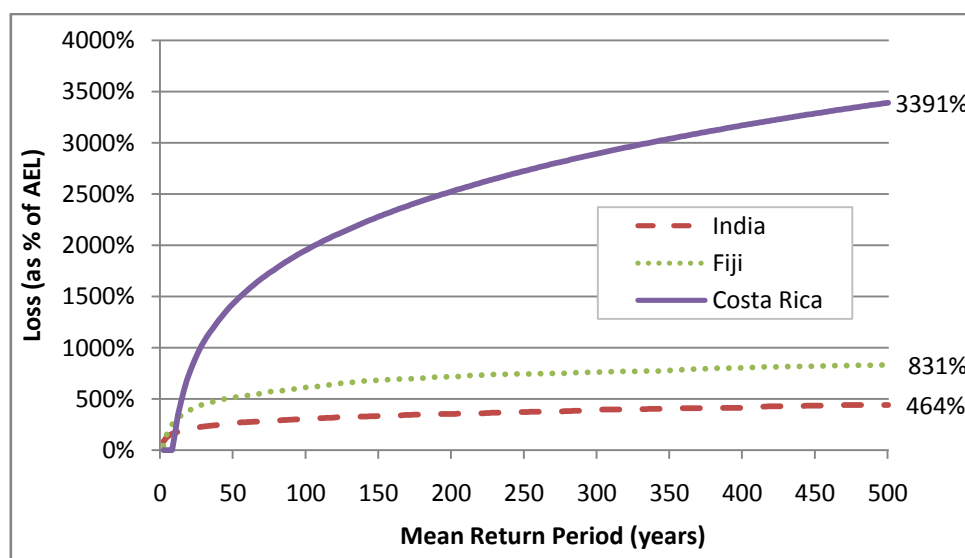


Figure 6 shows how much the risk profiles of these three illustrative examples differ. The median loss is estimated at 86 percent for agriculture in India, 42 percent for tropical cyclones in Fiji, and 0 percent for earthquakes in Costa Rica, all expressed in percentage of the respective annual expected loss (AEL) to facilitate comparison.

For each example, the CC-DFA model has been calibrated with a collection of realistic financial assumptions about the cost of reinsurance, the initial reserves, the annual budget allocation to the reserve fund, the terms and conditions of the contingent credit (front end fee, renewal fee, interest rate, grace period, maturity, etc) and the maximum amount of contingent credit, etc. Appendix 2 offers a summary of these assumptions.

For each example we generate 5,000 ten year loss histories using the respective assumption about the distribution of losses. We then apply two different strategies to the same set of loss histories: the first strategy involves no contingent credit purchase and the second involves positive contingent credit purchase. By using the same loss histories for both strategies we are able to more precisely compare the effect of the strategies on the outcomes.

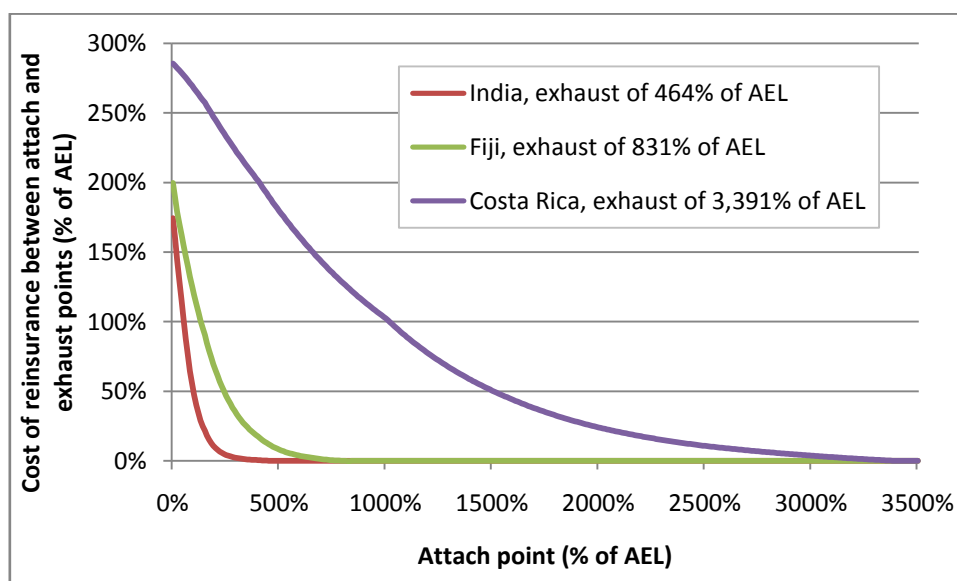
The strategies assumed in each example are described below:

Contingent credit strategy: In the simulations with contingent credit, it is assumed that contingent credit is only ever drawn down if reserves and any reinsurance income is not sufficient to meet the incurred loss. In the event of such a loss, the minimum amount of contingent credit is drawn down, to ensure that end year liquid *Reserves* are zero. Any drawn down loan is paid back according to the schedule in Appendix 2.

Reinsurance purchase strategy: The reinsurance purchase rules applied are as follows. For all three examples we assume that in each year the government’s dedicated reserve fund purchases full stop loss reinsurance between an attachment point and an exhaust point, that is to say the reserve fund’s liability is capped at the attachment point, unless losses exceed the exhaust point. The exhaust point is always assumed to be equal to the 1-in-500 year loss, but the attachment point depends on the start year reserves and contingent debt not yet drawn down. If start year reserves plus contingent credit not yet drawn down as a percentage of AEL are denoted by Z , the attachment point as a percentage of AEL is given by $\max(Z - 80\%, 80\%)$. Therefore, we assume that the attachment point is always at least 80% of AEL, and increases if reserves plus contingent credit is greater than 160% of AEL.

Annual budget allocation: Although we assume the same structure for reinsurance multiples in all three examples (see Appendix 2), the difference in loss distributions leads to a difference in reinsurance costs. For example, as shown in Figure 7 the cost of full reinsurance for losses between the AEL and 1-in-500 year exhaust point varies from 47% of AEL (US\$265m) for Indian agriculture to 267% (US\$20m) of AEL for Costa Rican earthquakes. This is because we assume that reinsurance pricing multiple is higher for catastrophic shocks than for more moderate shocks. For the dedicated reserve funds to be sustainable, we assume that initial reserves and annual budget allocation expressed as a multiple of the AEL are largest for Costa Rica, next largest for Fiji, and smallest for India.

Figure 7. Cost of stop loss reinsurance between attachment points and 1-in-500 year exhaust point



In practice it is likely that annual budget allocation from the government would be a dynamic strategy; if the dedicated reserve fund's reserves were particularly high the budget allocation would likely be low and if the dedicated reserve fund's reserves were particularly low the budget allocation would likely be high. However, we assume a static budget allocation rule, where the annual allocation is constant. Our key results would follow through under a dynamic budget allocation strategy, but where good experience resulted in low average budget allocation from government as opposed to high end of ten year net reserves for the reserve fund.

Borrowing in the event of a liquidity crunch: If risk retention ever leads to a liquidity crunch, we record the liquidity crunch event and in future years assume that all additional losses are funded by borrowing at the high rate of 8% per annum (see Appendix 2).

Agriculture in India: For agriculture in India we assume that the dedicated reserve fund begins with reserves equal to 100% of the AEL (US\$560m) and the annual net allocation from farmer insurance premiums and government subsidies is 130% of the AEL (US\$730m). The reinsurance exhaust point is set to equal the 1-in-500 year loss of 464% of the AEL (US\$2.6bn) and the minimum attach point is set at 80% of the AEL (US\$450m). In one simulation we assume that the fund purchases contingent credit of 25% of AEL (US\$140m) and in the other we assume that no contingent credit is purchased.

Tropical cyclones in Fiji: For tropical cyclones in Fiji we assume that the dedicated reserve fund begins with reserves equal to 150% of the AEL (US\$130m) and the annual net allocation from farmer insurance premiums and government subsidies is 190% of the AEL (US\$160m). The reinsurance exhaust point is set to equal the 1-in-500 year loss of 831% of the AEL (US\$720m) and the minimum attachment point is set at 80% of the AEL (US\$70m). In one simulation we assume that the fund purchases contingent credit of 35% of AEL (US\$30m) and in the other we assume that no contingent credit is purchased.

Earthquakes in Costa Rica: For earthquakes in Costa Rica we assume that the dedicated reserve fund begins with reserves equal to 400% of the AEL (US\$30m) and the annual net allocation from farmer insurance premiums and government subsidies is 300% of the AEL (US\$24m). The reinsurance exhaust point is set to equal the 1-in-500 year loss of 3,391% of the AEL (US\$260m) and the minimum attachment point is set at 80% of the AEL (US\$6m). In one simulation we assume that the fund purchases contingent credit of 1,100% of AEL (US\$85m) and in the other we assume that no contingent credit is purchased.

3.3. Scenario analysis

Before presenting the results of the stochastic simulations, we will discuss three scenarios for the case of agriculture in India that will help us interpret the results of the simulations (see Appendix 3).

Figure 13 illustrates the modeled strategies for a scenario where realized losses are low, with average loss over the ten years of 85% of AEL. In both strategies the fund's reserves increase significantly to the end of the tenth year, with the strategy with contingent credit leading to reserves that are higher by 212% of AEL.

Figure 14 illustrates the modeled strategies for a scenario where large losses are incurred in the final five years, but losses are low on average in the first five years. In this scenario at the end of the tenth year the fund's reserves are 71% of AEL for the strategy with no contingent credit and 155% of AEL for the strategy with contingent credit, an increase of 84% of AEL.

Figure 15 finally illustrates a scenario in which a very large loss is incurred in the first year and a fairly large loss in the second year. In the strategy with contingent credit, the reinsurance attachment point in the first year is higher than that in the strategy without contingent credit, leading to a larger retained loss. The fund draws down a loan of 2% of AEL in the first year, and a further 15% in the second year, and reserves take longer to recover. At the end of the tenth year the fund's reserves

are 7% of AEL for the strategy with contingent credit and 42% of AEL for the strategy without, a decrease of 35% of AEL.

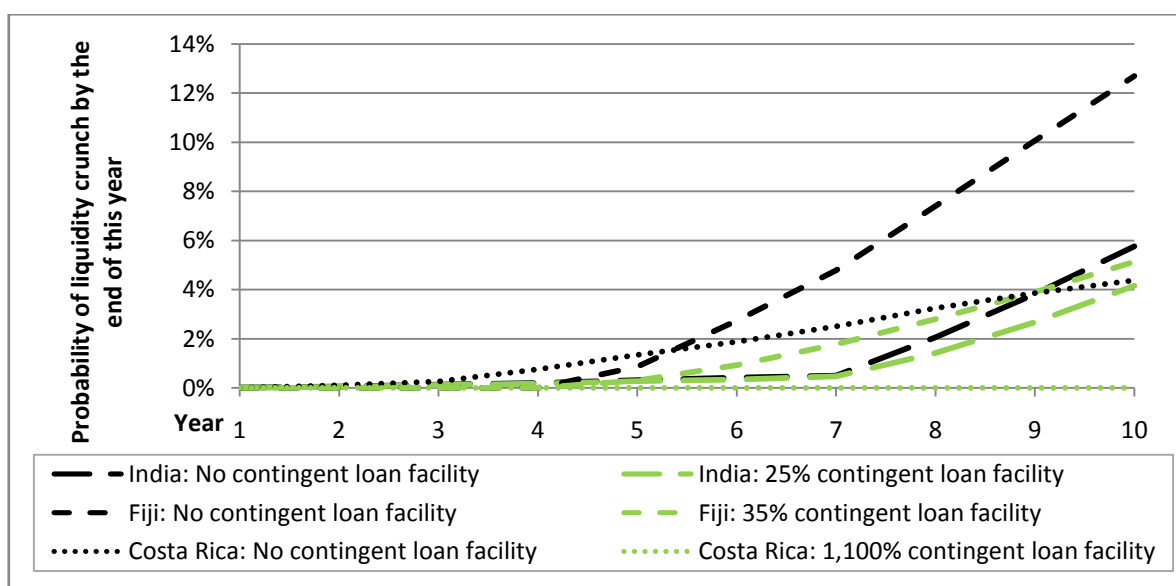
These strategies demonstrate the main advantages and disadvantages of contingent credit. Contingent credit allows the reserve fund to bear more risk and therefore save on reinsurance costs. However, by bearing more risk the fund exposes itself to the possibility that a series of years with high losses deplete reserves and force a drawdown of contingent debt, before reserves have built up sufficiently.

3.4. Stochastic projections: Probabilities of liquidity crunch and contingent debt drawdown

While it is important to analyze individual scenarios to understand the potential effects of contingent credit purchase, these three scenarios are not in any way representative of the likely outcomes. To understand the difference in outcomes between a strategy with contingent credit purchase and one without, we run the CC-DFA model with 5,000 simulations, with losses drawn from the assumed probability distribution.

As can be seen from Figure 8, the strategies incorporating contingent debt do not seem to lead to increases in the probability of a liquidity crunch, despite the increase in retained risk. Indeed, by saving on reinsurance premiums in early years, and therefore facilitating a larger increase in the average growth in reserves in early years, strategies with contingent credit reduce the 10 year probability of a liquidity crunch occurring from 5.8% to 4.4% in the case of India, from 12.7% to 5.1% in the case of Fiji and from 4.4% to 0.0% in the case of Costa Rica.

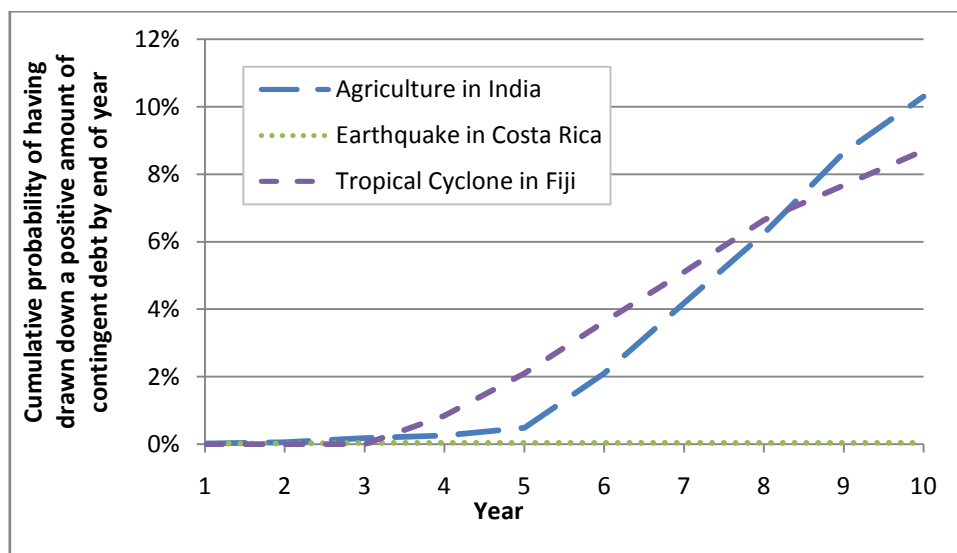
Figure 8. Probability of uninsured loss exceeding reserves and contingent credit



The cumulative probability that part of the contingent credit has been drawn down increases as time goes on (see Figure 9). The probability of drawdown in early years is particularly low since we assume that the facilities begin with substantial reserves and receive annual net allocations from government that allow significant purchase of reinsurance. In early years losses have to exceed the reinsurance exhaust point to trigger a drawdown of contingent credit. In later years, the probability of drawdown is much higher since drawdown can be triggered by a series of years with reasonably high losses, a much more likely event a very high loss occurring in any one year. The probability that

contingent debt is at least partially drawn down by the end of year ten in the strategies involving contingent credit are 10.3% in the case of India, 8.7% in the case of Fiji and only 0.0% in the case of Costa Rica.

Figure 9. Cumulative probability that contingent credit drawdown is positive



3.5. Stochastic projections: Net reserves

Finally, we may compare the development of net reserves, by which we mean savings minus any outstanding debt, over time. Recall that by assuming a static budget allocation strategy, we implicitly assume that any favorable loss experience results in an increase in reserves of the dedicated reserve fund. In practice, any increase in reserves could instead be passed back to government through a reduced future allocation to the dedicated reserve fund. For the contingent debt purchase decision it is of only second order concern as to how any increase in reserves is distributed. We will assume a static budget allocation strategy, but the end of ten year reserves need not be interpreted literally as an increase in dedicated reserve fund; any increase in reserves could just as well be passed back to government.

Figure 10 offers fan charts of the six projections: one with and one without contingent credit for each of the three examples. As can be seen, the path of net reserves is similar to that of a random walk, and so the variance of net reserves increases over time. The annual budget allocations for each of the three examples have been chosen so that the median net reserve also increases over time. Table 2 and Figure 11 present the distribution of net reserves at the end of ten years both with and without contingent credit purchase. The purchase of contingent credit, and associated increase in risk retention, leads to an average increase in net reserves of 93% of AEL (US\$520m) for agriculture in India, 312% of AEL (US\$270m) for tropical cyclones in Fiji, and 1,313% of AEL (US\$100m) for earthquakes in Costa Rica. These average increases in net reserves are driven by the savings in reinsurance costs, with the higher relative increase for catastrophic risks, for which the average reinsurance multiple is high.

Not only does contingent credit purchase increase the expected increase in reserves, in all three examples it leads to a distribution of net reserves which very nearly first order stochastic dominates the distribution without contingent credit purchase; the red lines in Figure 11 for the strategies with contingent credit are almost always below and to the right of the blue lines. Therefore, for these three examples contingent credit purchase is likely to be optimal for all decision makers; contingent

credit purchase increases the potential upside without materially affecting the downside. Figure 16 in Appendix 3 presents the probability density functions of outcomes, to complement the cumulative density functions of Figure 11.

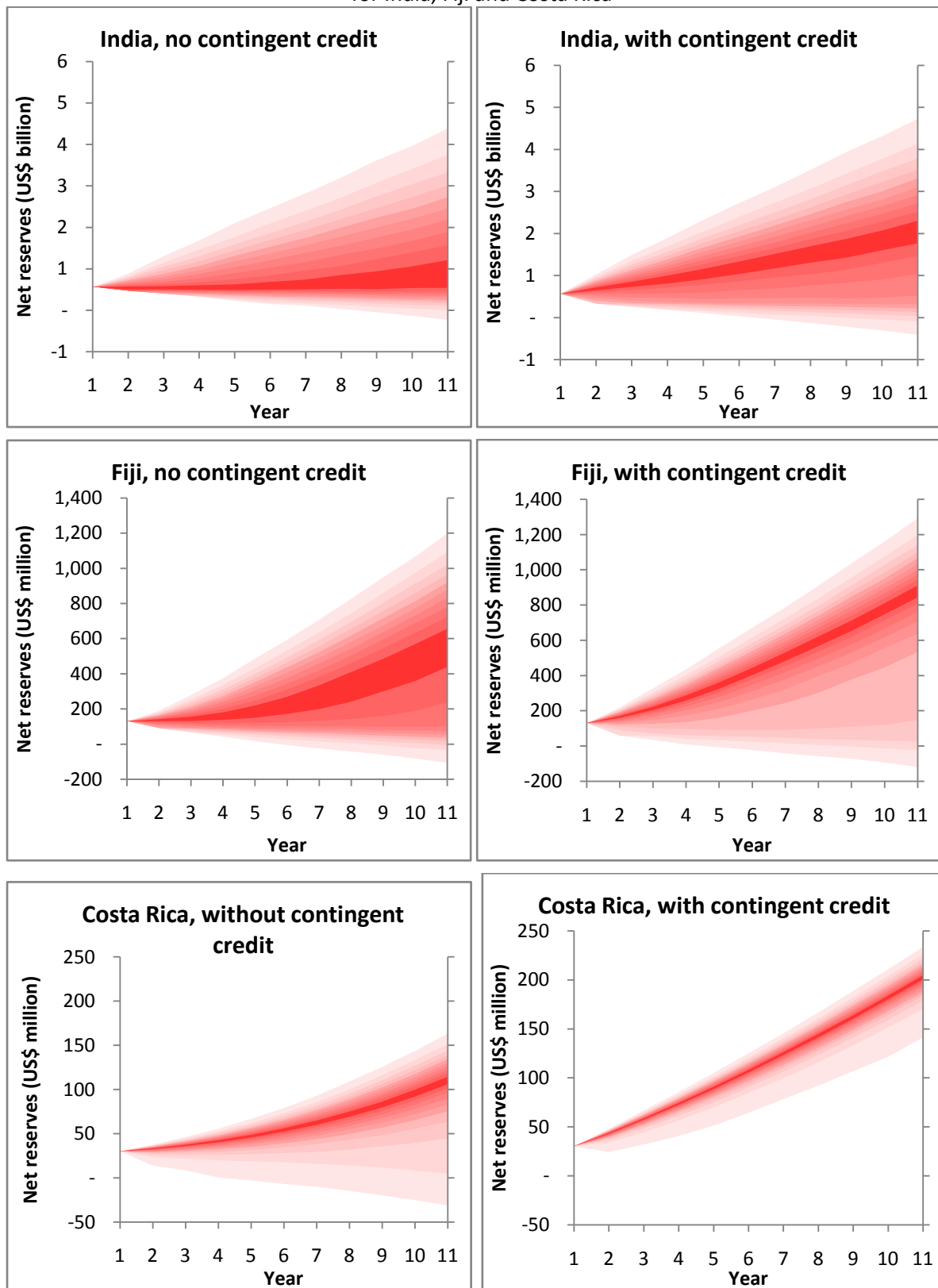
Table 2. Net reserves of dedicated reserve fund at end of ten years

Risk	Strategy	1st	10th	Median	90th	99th	Average
Agriculture in India	No contingent credit	-33%	19%	153%	594%	784%	245%
	Contingent credit of 25% AEL	-56%	6%	364%	676%	845%	338%
	Benefit from contingent credit	-23%	-13%	+212%	+82%	+61%	+93%
Tropical Cyclones in Fiji	No contingent credit	-97%	-7%	665%	1179%	1383%	573%
	Contingent credit of 35% AEL	-105%	31%	1016%	1319%	1493%	885%
	Benefit from contingent credit	-8%	+37%	+351%	+140%	+110%	+312%
Earthquakes in Costa Rica	No contingent credit	-337%	594%	1465%	1916%	2167%	1347%
	Contingent credit of 1,100% AEL	1983%	2369%	2686%	2933%	3096%	2660%
	Benefit from contingent credit	+2,321%	+1,776%	+1,221%	+1,017%	+929%	+1,313%

Notes: 1. Net reserves equal savings minus any outstanding debt

2. All figures expressed as percentage of respective Annual Expected Losses

Figure 10. Ten year projections of net reserves under strategies with and without contingent credit for India, Fiji and Costa Rica



Note: The fan charts show the relative likelihood of possible outcomes. In each chart the central, dark band is the central projection: there is judged to be a 10% chance that net reserves will be within that central band at any date. The next deepest shade, on both sides of the central band, takes the distribution out to 20%; and so on, in steps of 10 percentage points until the outermost band which includes all but the best and worst 1% outcomes.

Figure 11. Modeled cumulative density functions of end of 10 year net reserves for strategies with and without contingent credit

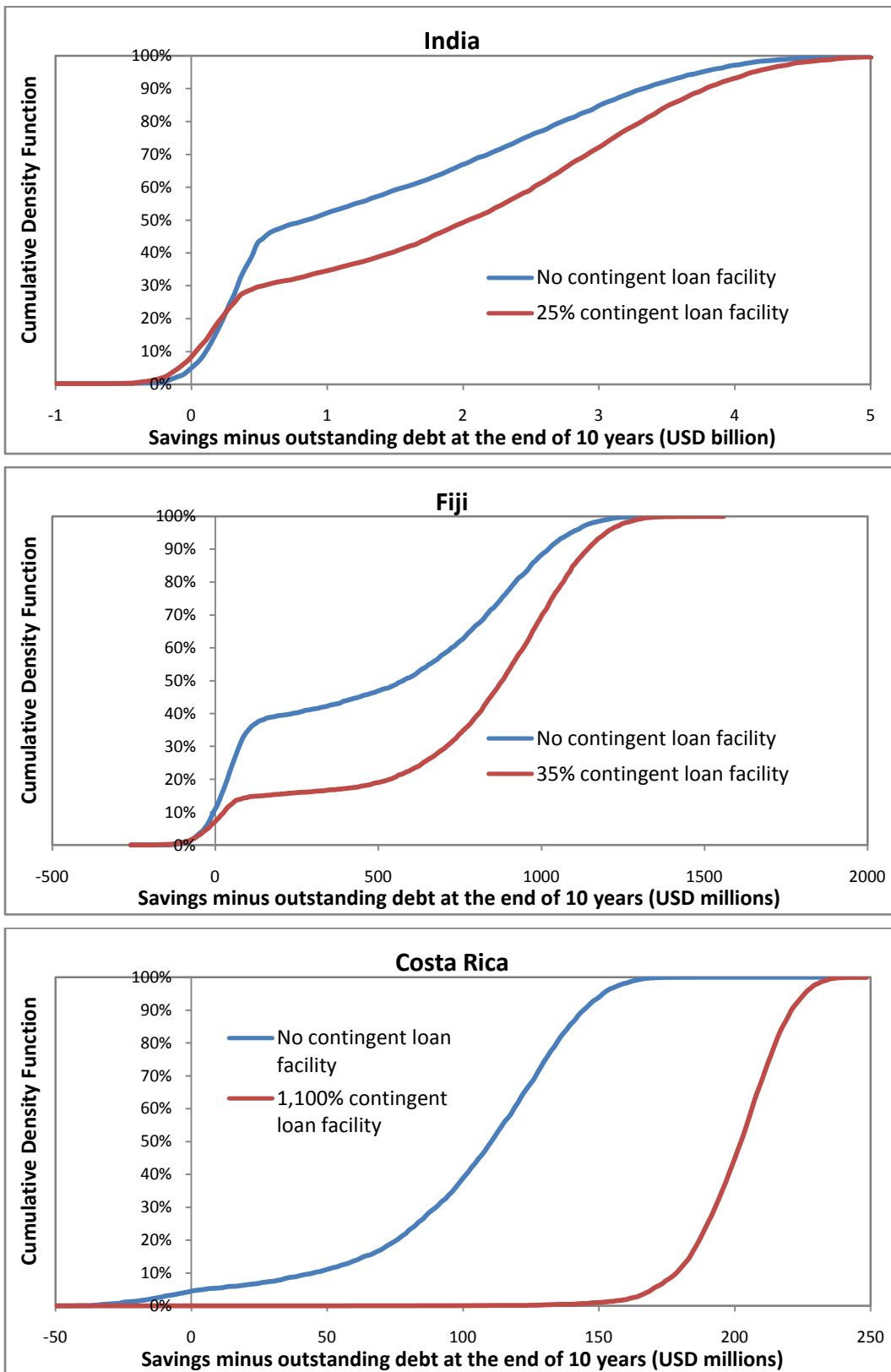
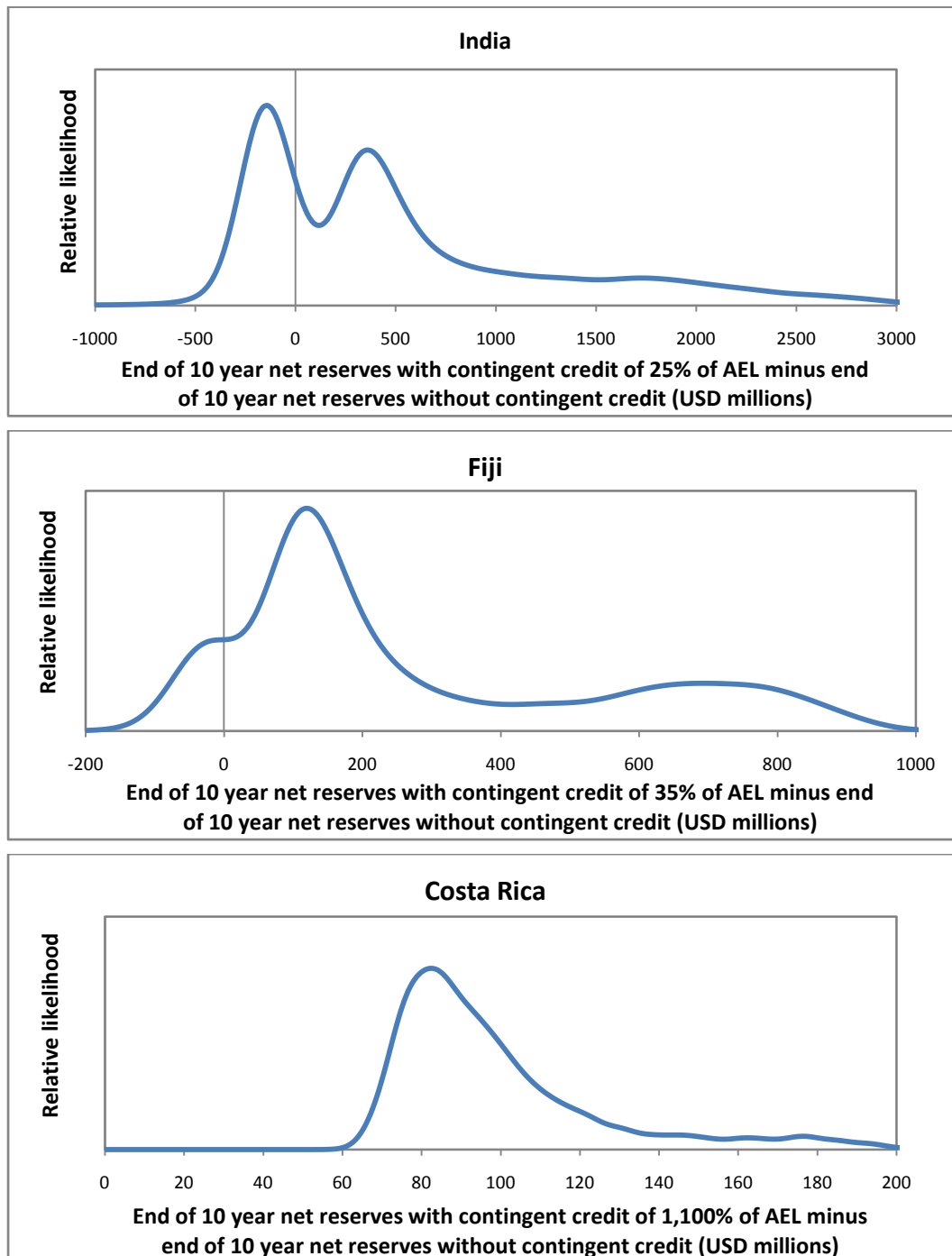


Figure 12 offers an alternative breakdown of the effect of contingent credit purchase on the distribution of net reserves at the end of ten years for each of the three examples. For 66% of the loss histories for agriculture in India the strategy with contingent credit leads to larger end of ten year net reserves than the strategy with no contingent credit. For the remaining 34% of loss histories, contingent credit purchase, and the corresponding increase in risk retention, leads to a decrease in end of ten year net reserves. The corresponding percentages for tropical cyclones in Fiji and earthquakes in Costa Rica are 87%/13% and 100%/0% respectively.

Figure 12. The within-simulation effect of using contingent credit on end of 10 year net reserves



In all three examples, the benefit from contingent credit purchase would be lower if the reserve funds were to start with much larger initial reserves. The benefit of contingent credit is to allow risk retention in layers for which reinsurance is poor value. If the reserve fund can retain an appropriate amount of risk without contingent credit then contingent credit purchase is unnecessary. In practice, a government may be restricted in the level of initial reserves that may be allocated to a dedicated reserve fund. In such a case, contingent credit purchase is likely to be optimal, to allow the reserves of the fund to increase quickly in initial years.

4. Conclusion

This paper is part of the World Bank policy work to assist policy makers interested in establishing or strengthening financial strategies to increase the financial response capacity of governments of developing countries in the aftermath of natural disasters while protecting their long-term fiscal balance. It provides a technical background for the use of contingent credit in sovereign risk financing and offers a dynamic financial tool to guide policy makers in the design of optimal sovereign financial strategies based on a combination of reserves, contingent credit and reinsurance.

The theoretical models first provide an economic rationale for catastrophe risk layering: retention/self-insurance finances the bottom risk layer; contingent credit is usually efficient to finance middle-risk layer; and risk transfer instruments like reinsurance finances the top risk layer. It also provides a simple model to compare the cost of reinsurance and the cost of contingent credit for a given risk layer by deriving the multiple of each financial product.

The dynamic financial tool is illustrated in three case studies: agricultural production risk in India, tropical cyclone risk in Fiji and earthquake risk in Costa Rica. The model offers visualization tools to assist in the dialogue of catastrophe risk financing and contingent credit with the governments of developing countries. It calculates in particular the potential savings generated by a contingent credit facility but also shows that, by allowing a government to retain risk, purchase of contingent credit can make the country worse off in case of very bad years.

The financial protection of the state against natural disasters has gained increasing attention among the developing countries, the donor community and the international financial institutions, and is a major pillar of the proactive disaster risk management framework. It is incorrect to reduce disaster risk financing to catastrophe insurance; not only is protection against 1-in-5 year events important, just as protection against 1-in-100 year events, but in practice it may be difficult to sustain a program that protects against 1-in-100 year events if adequate protection is not available for 1-in-5 year events.

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Appendix 1

Proof of Proposition 1

Clearly, in any optimal schedule $L(x) \times S(x) = 0$ or the agent could reduce $L(x)$ and $S(x)$ by $\min(L(x), S(x))$ and consume an additional $(\rho - r) \times \min(L(x), S(x)) > 0$ at time-1, with unchanged consumption in time-2.

Under the parameter restrictions specified above this program is convex and so we may characterize the solution through the first-order conditions.⁷ Denoting λ as the Lagrange multiplier for the insurer's zero profit constraint and μ_1 to μ_5 as the Lagrange multipliers for the sign restrictions of (14b), the first-order conditions include the following:

$$\frac{\partial \mathcal{L}}{\partial I(x)} = f(x)[u'(C_1(x)) - \lambda\theta] + \mu_2(x) = 0 \quad (17)$$

$$\frac{\partial \mathcal{L}}{\partial L(x)} = f(x)[u'(C_1(x)) - (1 + \rho)\beta u'(C_2(x))] + \mu_4(x) - \mu_5(x) = 0 \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial S(x)} = f(x)[-u'(C_1(x)) + (1 + r)\beta u'(C_2(x))] + \mu_3(x) = 0 \quad (19)$$

$$\mu_1, \mu_2(x), \mu_3(x), \mu_4(x), \mu_5(x) \geq 0 \text{ for all } x \quad (20)$$

$$\mu_1 C, \mu_2(x)I(x), \mu_3(x)S(x), \mu_4(x)L(x), \mu_5(x)(D - L(x)) = 0 \text{ for all } x \quad (21)$$

First we prove the characteristics of layer 5. Due to the concavity of u , (17) implies that if $I(x) > 0$ then $C_1(x)$ is some constant floor \underline{C}_1 which solves $u'(\underline{C}_1) = \lambda\theta$. Moreover, if $C_1(x) > \underline{C}_1$ then $I(x) = 0$.

So we have shown that $I(x)$ acts to offer a lower floor on consumption. We must now show that $L(x) = D$ whenever $I(x) > 0$. Consider an x with $0 < L(x) < D$. The constraint $C_1(x) \geq \underline{C}_1$ applied to equation (18) puts an upper bound on the maximum possible $L(x)$ as the \bar{L} which solves $(1 + \rho)\beta u'(Y_2 - (1 + \rho)\bar{L}) = \lambda\theta$. This upper bound \bar{L} must be at least as large as D or D could be reduced, reducing the cost of contingent credit but not reducing the benefit. Therefore:

$$(1 + \rho)\beta u'(Y_2 - (1 + \rho)D) \leq \lambda\theta \quad (22)$$

and for $I(x) > 0$ it cannot be that $L(x) < D$ or equations (18) and (20) could not both hold. If $L(x) = D > 0$ then $S(x) = 0$ from above. Putting all this together we can define x_4 as the solution to $u'(Y_1 - x_4 - \alpha D - P + D) = \lambda\theta$. This proves the characteristics of layer 5.

Next we prove the characteristics of layers 1 to 4. Define x_1 as the solution to $u'(Y_1 - x_1 - \alpha D - P) = (1 + r)\beta u'(Y_2)$, x_2 as the solution to $u'(Y_1 - x_2 - \alpha D - P) = (1 + \rho)\beta u'(Y_2)$ and x_3 as the solution to $u'(Y_1 - x_3 - \alpha D - P + D) = (1 + \rho)\beta u'(Y_2 - \rho D)$. Now $x_1 < x_2 < x_3$ due to the strict concavity of u , $\rho > r$ and our restriction of attention to solutions with $D > 0$. $S(x) = 0$ for $x \geq x_1$ from (19) and $L(x) = 0$ for $x \leq x_1$ from (20). All we need prove now is that $0 \leq L^*(x) < 1$ for $x \in [x_2, x_3]$. This follows from equation (18) and the restriction $L^*(x) \leq D$. ■

Proof of Corollary 1

⁷ The optimal insurance schedule can be characterized in terms of stochastic dominance arguments alone (Gollier and Schlesinger, 1996). Conditional on the optimal insurance schedule the savings and contingent credit purchase decision problem is clearly convex.

Suppose that $\alpha = 0$ and $\theta = \infty$. $I^*(x) = 0$ for all x . D can be set to anything without increasing the cost and so $\mu_5(x) = 0$ for all x . We therefore need only show that there is some state x in which $L^*(x) > 0$. There will be such a state if $u'(Y_1 - \bar{x} - P) > (1 + \rho)\beta u'(Y_2)$, and this will hold if $Y_1 - \bar{x} - P \leq K$, that is if \bar{x} is large enough. The result follows from the twice differentiability of the objective function.

Appendix 2

Table 3: Assumptions used in stochastic projections

Worked Example	1. Agricultural insurance in India	2. Tropical cyclone insurance in Fiji	3. Earthquake insurance in Costa Rica
Loss Distribution (USD million and % of AEL):			
Annual Expected Loss	560 (100%)	87 (100%)	8 (100%)
Median	480 (86%)	40 (42%)	0 (0%)
1-in-10 years	960 (172%)	250 (288%)	10 (159%)
1-in-20 years	1,180 (210%)	340 (392%)	60 (760%)
1-in-100 years	1,720 (307%)	530 (614%)	150 (1,953%)
1-in-200 years	1,980 (354%)	620 (715%)	190 (2,525%)
1-in-500 years	2,600 (464%)	720 (831%)	260 (3391%)
Initial reserves (USD million and % of AEL)	560 (100%)	130 (150%)	30 (400%)
Annual budget allocation (USD million and % of AEL)	730 (130%)	160 (190%)	24 (300%)
Contingent credit amount: USD million and % of AEL	140 (25%)	30 (35%)	85 (1,100%)
% of GDP	0.01%	0.85%	0.30%
Contingent credit conditions:	0.50% of loan facility, payable at start of year 1		
Front-end fee	0.25% of debt that has not yet been drawn down, at start of years 4, 7 and 10		
Renewal fee	5 years		
Grace period	30 years		
Term of drawn down loan	1/25 of drawn down loan		
Annual capital repayment after grace period			
Investment return	0% between premiums paid in year t and claims paid in year t 3.0% between claims paid in year t and premiums paid in year t+1		
Contingent credit lending rate	4.4%		
Liquidity crunch lending rate	8.0%		
Reinsurance minimum attachment point	80% of AEL		
Reinsurance exhaust point	1-in-500 years		
Reinsurance multiple:			
0% to 50%	1.5		
50% to 100%	1.8		
100% to 150%	2.0		
150% to 400%	2.5		
400% to 1000%	3.0		
1000% +	4.0		

Note: Reinsurance multiple is cost of purchasing reinsurance for layer divided by expected loss from layer.

Appendix 3

Figure 13. Scenario 1: Low average claims throughout ten years

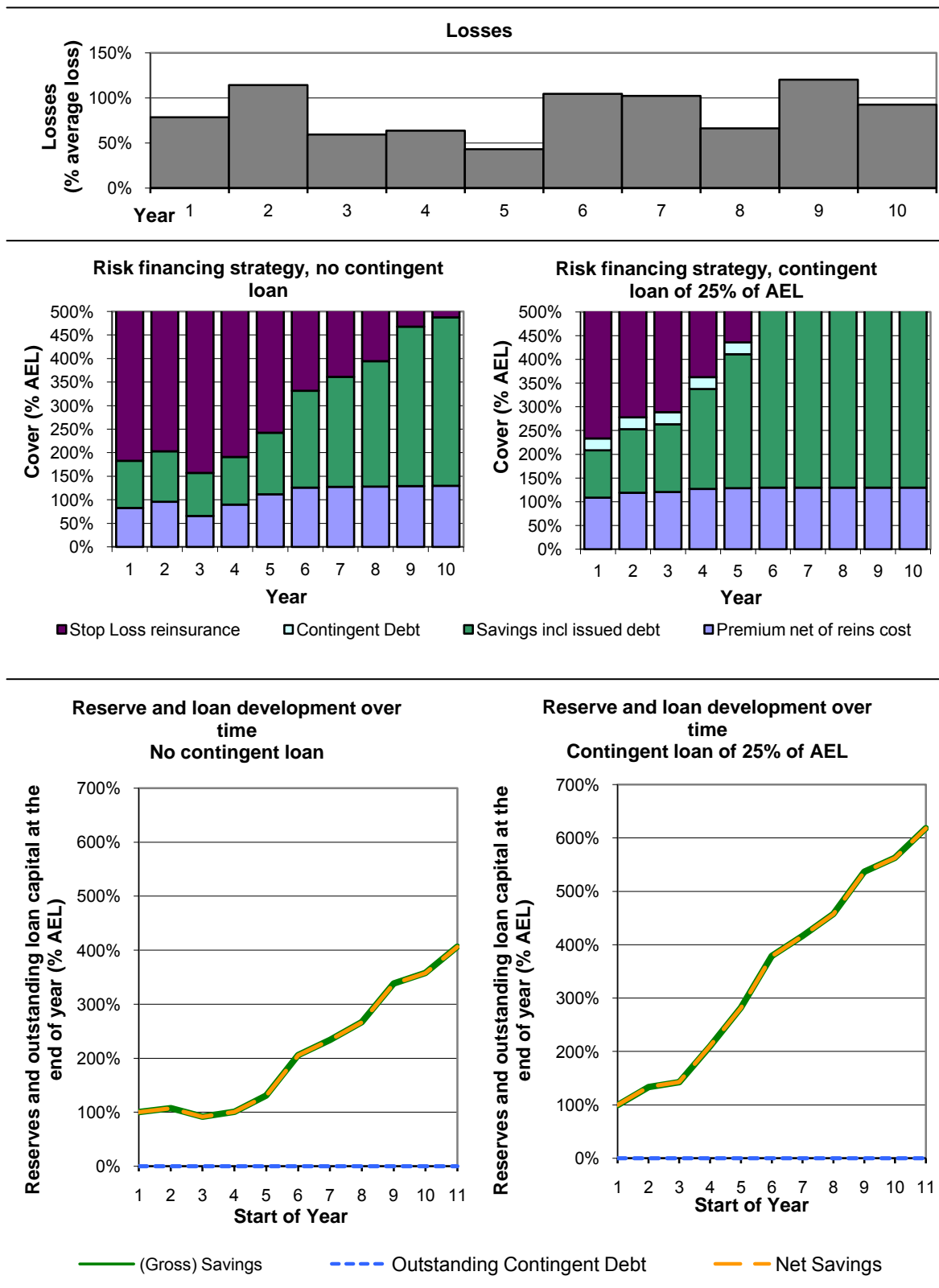


Figure 14. Scenario 2: High loss once savings have been built up

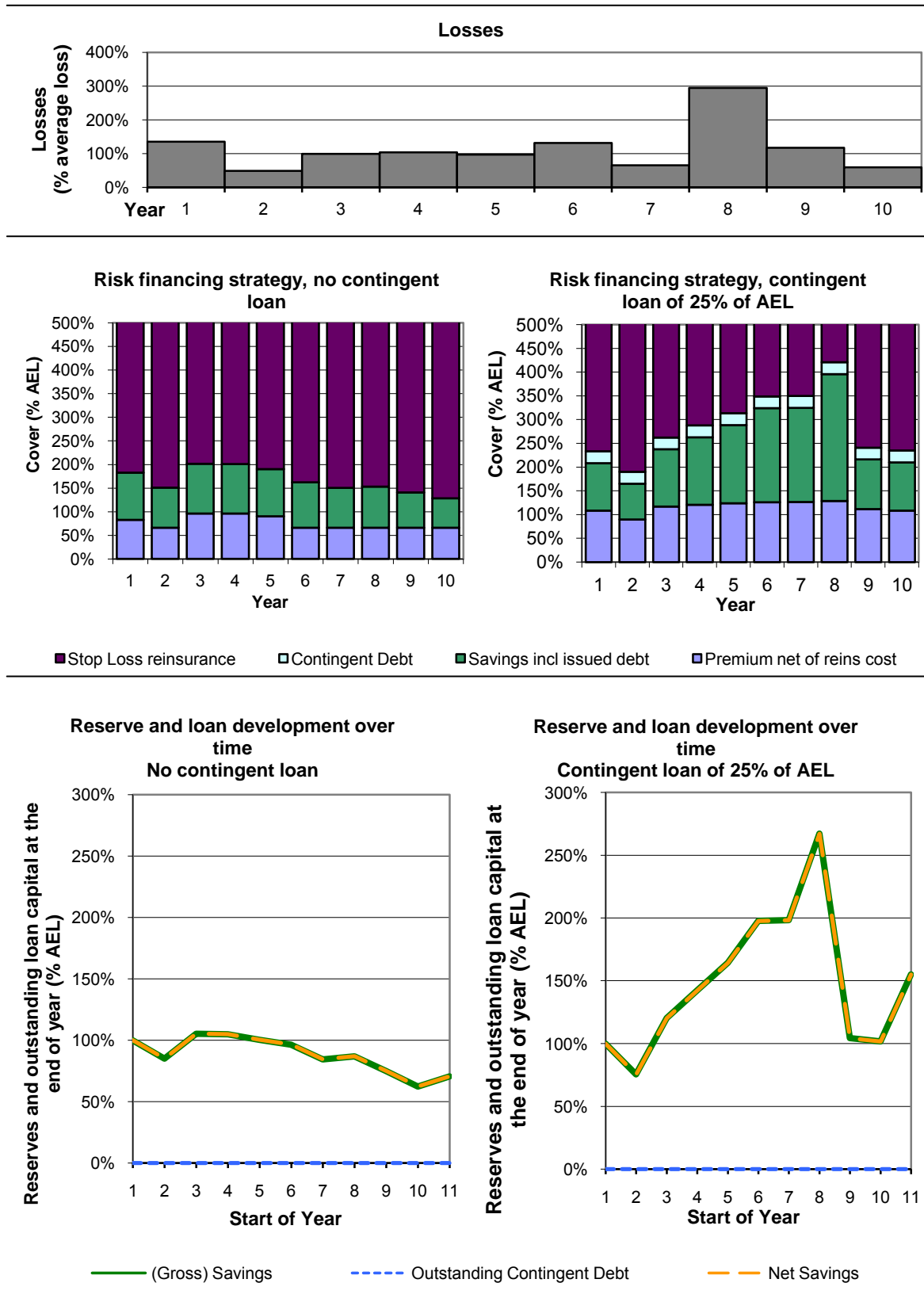


Figure 15. Scenario 3: High loss before savings have been built up

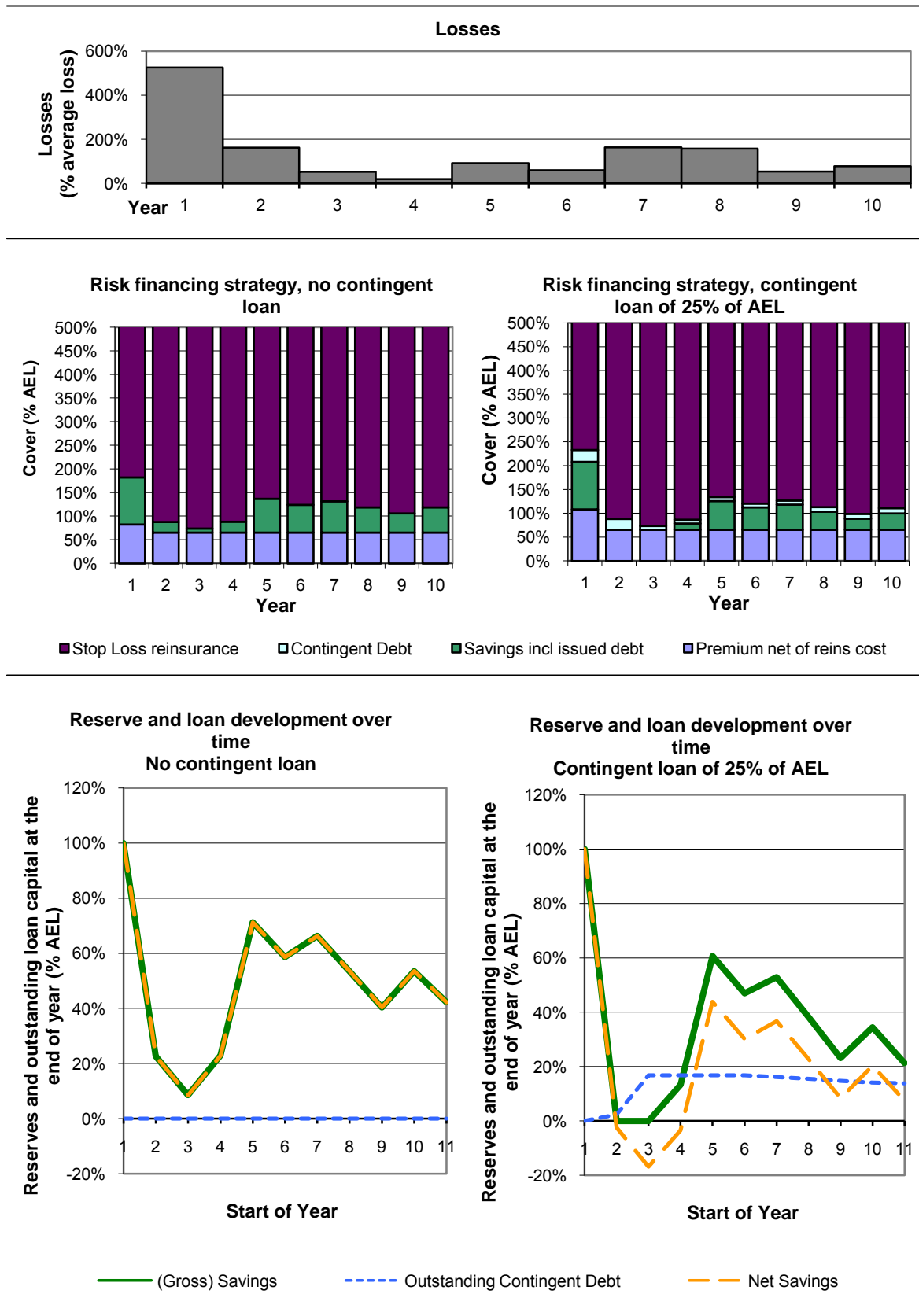


Figure 16. Modeled probability density functions of end of 10 year net reserves for strategies with and without contingent credit

