Forecasting Value-at-Risk Using Nonlinear Regression Quantiles and the Intra-day Range

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Abstract

Value-at-Risk (VaR) is commonly used for financial risk measurement. It has recently become even more important, especially during the 2008-09 global financial crisis. We propose some novel nonlinear threshold conditional autoregressive VaR (CAViaR) models that incorporate intra-day price ranges. Model estimation and inference are performed using the Bayesian approach via the link with the Skewed-Laplace distribution. We examine how a range of risk models perform during the 2008-09 financial crisis, and evaluate how the crisis affects the performance of risk models via forecasting VaR. Empirical analysis is conducted on five Asia-Pacific Economic Cooperation stock market indices as well as two exchange rate series. We examine violation rates, back-testing criteria, market risk charges and quantile loss function values to measure and assess the forecasting performance of a variety of risk models. The proposed threshold CAViaR model, incorporating range information, is shown to forecast VaR more efficiently than other models, across the series considered, which should be useful for financial practitioners.

Keywords: Value-at-Risk; CAViaR model; Skewed-Laplace distribution; intra-day range; backtesting, Markov chain Monte Carlo.

1 Introduction

It is well known that the bursting of the global housing bubble, especially in the USA, caused a significant reduction in real estate-based securities and a subsequent increase in the default rate of mortgages, especially sub-prime mortgages. The resulting effect on global financial markets, initially via mortgage-based assets like collateralized-debt-obligations through to bankruptcies of financial institutions and collapsing stock markets, all contributed to the global financial crisis (GFC) of 2008-09. The crisis has (once again) called into question financial risk management practices and whether risk measures can be forecast accurately enough for that purpose. This paper adds to this debate by proposing some novel univariate, semi-parametric range-based conditional autoregressive VaR (CAViaR) models and evaluating them for forecasting tail risk, specifically Value-at-Risk (VaR), in a "horse-race" with some existing, competing models, during the GFC period, for some individual market returns, a portfolio of these market returns, plus two exchange rate return series. The motivation is to generate more accurate and efficient forecasts of VaR for univariate asset and market returns, single fixed-weight portfolio returns and exchange rate return series, to help achieve better risk measurement and risk management practice. We attempt this by incorporating intra-day high-low price range data, known to be more efficient, at least regarding volatility estimation, than simple daily returns data since at least Parkinson (1980), into the CAViaR model. We then examine whether this adds to the efficiency and accuracy of VaR forecasts during the GFC; the evidence presented suggests this is indeed the case.

Quantitative risk measure forecasting has become very important, at least since the market crash in 1987, and even moreso after the recent global financial disaster, which began with a liquidity crisis in the U.S. banking system: the "credit-crunch"; caused by the over-valuation of assets, and included the Lehman Brothers bankruptcy, AIG crisis, and the sub-prime mortgage debacle. Financial markets and products continue to become increasingly complex, and risk management and regulations need to keep pace with this rapid process. The Basel II Accord is designed to monitor and encourage sensible risk taking, using appropriate models to calculate VaR and daily capital charges. VaR is now a standard tool in risk management and became highly important following the 1995 amendment to the Basel Accord, whereby banks and other Authorized Deposit-taking Institutions (ADIs) were permitted to use internal models to forecast daily VaR. VaR was pioneered by J.P. Morgan Corporation, via their RiskMetrics system, in 1993 and is more formally defined by Jorion (1996), as an estimate of the probability and size of the worst potential or expected loss over a given time horizon with a specified probability. Mathematically:

$$P_r\left(\Delta V(l) \leq -\text{VaR}|\mathcal{F}_{t-1}\right) = \alpha,$$

where $\Delta V(l)$ is the change in the asset value over time period l, α is the probability level, and $\mathcal{F}_{t-1}$ denotes the information set at time $t-1$. Models and methods for VaR forecasting are an on-going challenge for financial practitioners and statisticians.

In this paper, we propose a new semi-parametric family of quantile risk CAViaR models; discuss the selection of optimal risk models; examine how risk management strategies performed during the 2008-09 GFC; evaluate how the crisis affected risk management practices, forecasts of VaR and daily capital charges; and discuss diagnostic checking of VaR methods. Further, we adapt the Bayesian estimation methods in Yu and Moyeed (2001), exploiting the link between quantile estimation and the Skewed-Laplace distribution, first discussed by Koenker and Machado (1999), to the range of models in the CAViaR family in a systematic way, conducting a comparison with the frequentist estimation of Engle and Manganelli (2004), in regards to the forecasts produced from these models.

The paper is structured as follows. In Section 2, methods for VaR are reviewed and the new CAViaR specifications, incorporating range information, are presented. Section 3 discusses estimation of VaR models and criteria for measuring VaR performance. Empirical analysis is conducted in Section 4 on five Asia-Pacific Economic Cooperation (APEC) stock market indices, including Standard and Poors 500 Index, Nikkei 225, TAIEX, HSI and KOSPI, to forecast VaR from August 2008 to April 2010. Finally, some concluding remarks are given in Section 5.

2 Value-at-Risk - Models and Methods

For a given value α, $0 \leq \alpha \leq 1$, the αth quantile of the variable y is defined as $q_\alpha(y) = \inf\{y|F(y) \geq \alpha\}$, where F is the CDF of y. There are several VaR estimation methods in the literature, classified into three over-arching categories:

Non-parametric: no, or very few or non-restrictive, assumptions are made on the distribution of returns, e.g. historical simulation, which uses past sample return quantiles.

Parametric: usually constructed assuming a specific choice for the unconditional and/or conditional return distribution and also the model dynamics. Kuester et al. (2006) conduct a review of some competing models, many with generalized autoregressive conditional heteroscedastic (GARCH) volatility equations (proposed by Engle, 1982, and Bollerslev, 1986) with specific noise distributions such as Gaussian, Student-t, skewed Student-t (see Hansen, 1994). McAleer and da Veiga (2008a) propose a parsimonious portfolio spill-over GARCH (PS-GARCH) model which accommodates aggregate spill-overs, and avoids the so-called curse of dimensionality. Chen et al. (2011) also consider a range of parametric models to forecast VaR, including standard, threshold nonlinear and Markov switching GARCH specifications (see e.g. Guidolin and Timmerman, 2006; or Haas, Mittnik and Paolella, 2006), plus standard and nonlinear stochastic volatility models, together with four probability distributions for the error component, namely Gaussian, Student-t, skewed Student-t, and generalized error distributions.

Semi-parametric: often makes assumptions about the model dynamics but not the error distribution, e.g. Engle and Manganelli (2004) propose direct dynamic quantile regression (see Koenker and Bassett, 1978) to calculate VaR, denoted CAViaR, which directly models the dynamics of each quantile. Gerlach, Chen and Chan (2011) propose a family of nonlinear CAViaR models, extended from those in Engle and Manganelli (2004).

In this paper we use methods from all three classes above. Historical simulation is employed in the non-parametric category, where we use two sample percentiles: a short-term (ST, the last 25 days) and a long-term (LT, last 100 days).

For parametric methods, RiskMetrics and GARCH models are used. More precisely, the IGARCH(1,1) of RiskMetrics with Gaussian errors, and the GARCH(1,1) model with Gaussian and Student-t errors, are considered in the empirical analysis. Much of the literature on VaR forecasting focuses on these models as benchmarks. The models are specified as follows:

Model A: GARCH model

$$\begin{aligned} y_t &= \mu_t + a_t, \ \mu_t = \phi_0 + \phi_1 y_{t-1}, \\ a_t &= \varepsilon_t \sqrt{h_t}, \text{ where } \varepsilon_t \overset{i.i.d}{\sim} D(0,1), \\ h_t &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}. \end{aligned}$$

Model B: RiskMetrics model

$$\begin{aligned} y_t &= a_t \\ a_t &= \varepsilon_t \sqrt{h_t}, \text{ where } \varepsilon_t \overset{i.i.d}{\sim} N(0,1) \\ h_t &= (1-\lambda) a_{t-1}^2 + \lambda h_{t-1}. \end{aligned}$$

Under each model, the one-step-ahead VaR at α% quantile level is computed, as $VaR_t = \mu_t + D_\alpha^{-1}\sqrt{h_t}$, where D^{-1} is the inverse CDF for the distribution D. The parameters of the GARCH models are estimated by Bayesian Markov chain Monte Carlo (MCMC), as in Chen, Chiang and So (2003) and discussed in the next section.

For semi-parametric models we consider the CAViaR models discussed in the next section. Giacomini and Komunjer (2005) find that CAViaR is most efficient at the 1% quantile level, but the GARCH model with normal distributed errors is better than CAViaR at the 5% quantile level. Gerlach et al (2011) find similar results across a range of financial market indices. McAleer et al. (2010a) consider mixing alternative risk models, and discuss the choice between conservative and aggressive risk management, as well as evaluating the effects of the Basel II

Accord for risk management. McAleer et al. (2010b) provide a method for choosing one risk model at the beginning of the period, and then modify the forecast depending on the recent history of violations.

2.1 Quantile regression and CAViaR

Koenker and Bassett (1978) suggest that, based on a sample of i.i.d. realizations $\{y_t\}$ of y, the quantile $b = q_\alpha(y)$ can be estimated by solving the following minimization problem:

$$\min_{b \in \Re} \left[\sum_t (y_t - b)\left(\alpha - I\{y_t < b\}\right) \right].$$

Engle and Manganelli (2004) propose some time series models for the quantile, i.e. b becomes b_t and the i.i.d. assumption is relaxed, called CAViaR, and apply this criterion to estimate the unknown parameters in the models for b_t. Let y_t be an asset, market, portfolio or exchange rate return at time t, and $\boldsymbol{\beta}_\alpha$ the vector of $q + r$ unknown parameters, $(\beta_1, \ldots, \beta_q, \beta_{q+1}, \ldots, \beta_{q+r})'$, for the α-quantile model. For notational convenience, we let $f_t(\boldsymbol{\beta}) = f_t(y_t, \boldsymbol{\beta}_\alpha)$ denote the time t conditional α level quantile. A general specification of VaR at time t is:

$$f_t(\boldsymbol{\beta}) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\boldsymbol{\beta}) + g(\beta_{q+1}, \ldots, \beta_{q+r}, \mathcal{F}_{t-1}),$$

where $g()$ is a function of a finite number of lagged returns and model parameters, thus linking the *alpha* quantile $f_t(\boldsymbol{\beta})$ to past returns, which are a subset of all past information, denoted as $\mathcal{F}_{t-1}$. $\beta_i f_{t-i}(\boldsymbol{\beta})$ is the autoregressive term which ensures smooth quantile changes over time. Three general CAViaR specifications in Engle and Manganelli (2004) are:

(1) Symmetric Absolute Value (SAV):

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 |y_{t-1}|. \quad (1)$$

(2) Asymmetric Slope (AS):

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + (\beta_3 I_{(y_{t-1}>0)} + \beta_4 I_{(y_{t-1}<0)})|y_{t-1}|. \quad (2)$$

(3) Indirect GARCH(1,1) (IG):

$$f_t(\boldsymbol{\beta}) = (\beta_1 + \beta_2 f_{t-1}^2(\boldsymbol{\beta}) + \beta_3 y_{t-1}^2)^{1/2}. \quad (3)$$

Yu, Li, and Jin (2011) extend CAViaR using two approaches, namely the threshold and mixture type indirect-GARCH CAViaR models. Gerlach et al (2011) propose a nonlinear CAViaR model to capture more flexible asymmetric and nonlinear responses via more general threshold nonlinear forms. We adopt the threshold CAViaR (TCAV) model of Gerlach et al (2011), and threshold-type indirect-VaR model of Yu, Li, and Jin (2011) as follows:

(4) Threshold CAViaR (TCAV)

$$f_t(\boldsymbol{\beta}) = \begin{cases} \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 |y_{t-1}|, & z_{t-1} \leq \gamma \\ \beta_4 + \beta_5 f_{t-1}(\boldsymbol{\beta}) + \beta_6 |y_{t-1}|, & z_{t-1} > \gamma, \end{cases} \tag{4}$$

where $\boldsymbol{z}$ is an observed threshold variable, which can be exogenous or self-exciting (i.e. $z_t = y_t$), and γ is the threshold value, typically set as $\gamma = 0$. We extend the model slightly by estimating this parameter in this paper, while Gerlach et al (2011) fix $\gamma = 0$. Further:

(5) Threshold Indirect GARCH(1,1) (TIG):

$$f_t(\boldsymbol{\beta}) = \begin{cases} (\beta_1 + \beta_2 f_{t-1}^2(\boldsymbol{\beta}) + \beta_3 y_{t-1}^2)^{1/2}, & \text{if } y_{t-1} < \gamma, \\ (\beta_4 + \beta_5 f_{t-1}^2(\boldsymbol{\beta}) + \beta_6 y_{t-1}^2)^{1/2}, & \text{if } y_{t-1} \geq \gamma. \end{cases} \tag{5}$$

2.2 Proposed Range-based CAViaR models

There are several advantages to using the intra-day high-low price range directly for volatility measurement and forecasting, relative to the use of absolute or squared return data, or intra-day returns. Many papers have shown the intra-day range to be an efficient measure of daily volatility (e.g. see Parkinson, 1980). Mandelbrot (1971) proposes the range to evaluate the existence of long-term dependence on asset prices; Garman and Klass (1980) show that high-low price-range data contain more information regarding volatility than opening to closing prices. Beckers (1983) applies the range estimator to incorporate past information for different variance measures. Gallant et al. (1999) and Alizadeh, Brandt and Diebold (2002) incorporate the range into the stochastic volatility model. Brandt and Jones (2006) proposed a range-based EGARCH model, using a link between the range and intra-day volatility, showing that the their model had favourable out-of-sample volatility forecasting performance. Chou (2005) proposes the Conditional Autoregressive Range (CARR) model for the high and low range of asset prices. Chen, Gerlach and Lin (2008) allow the intra-day high and low price range to depend nonlinearly on past information, or an exogenous variable such as US market information, finding increased accuracy for volatility estimation over the CARR and GARCH models. Here, we propose a family of CAViaR models that incorporates intra-day price range information.

In the same spirit as Chou (2005) and Chen et al (2008), we extend the CAViaR models in (2), (4), (5) and incorporate the intra-day high-low price range into the following models:

(6) Range Value (RV):

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 R_{t-1}. \tag{6}$$

(7) Threshold Range Value (TRV):

$$f_t(\boldsymbol{\beta}) = \begin{cases} \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 R_{t-1}, & \text{if } R_{t-1} \leq \gamma, \\ \beta_4 + \beta_5 f_{t-1}(\boldsymbol{\beta}) + \beta_6 R_{t-1}, & \text{if } R_{t-1} > \gamma. \end{cases} \tag{7}$$

The first model has the same form as the SAV model in (1), but replaces the absolute return with the intra-day price range R_{t-1}. The TRV has the same form as the TCAV in (4), again replacing return data with range data. The following model makes the same adjustments to the TIG model in (5):

(8) Threshold Range Indirect GARCH(1,1) (TRIG):

$$f_t(\boldsymbol{\beta}) = \begin{cases} (\beta_1 + \beta_2 f_{t-1}^2(\boldsymbol{\beta}) + \beta_3 R_{t-1}^2)^{1/2}, & \text{if } R_{t-1} \leq \gamma, \\ (\beta_4 + \beta_5 f_{t-1}^2(\boldsymbol{\beta}) + \beta_6 R_{t-1}^2)^{1/2}, & \text{if } R_{t-1} > \gamma, \end{cases} \tag{8}$$

Here R_t is the intra-day range at time t, and γ is the threshold value. The RV model responds symmetrically to past range, while the TRV and TRIG allow for different responses to high and low ranges.

3 Estimation and Forecast Evaluation

Using the Koenker and Bassett (1978) regression quantile framework, the unknown parameters of CAViaR models can be estimated by optimising a criterion function. The αth regression quantile is defined as the solution, $\boldsymbol{\beta}_\alpha$, of the criterion function:

$$\min \sum (y_t - f_t(\boldsymbol{\beta})) \left\{\alpha - I_{(-\infty,0)}(y_t - f_t(\boldsymbol{\beta}))\right\}, \tag{9}$$

where $f_t(\boldsymbol{\beta})$ is the model for the αth regression quantile. Based on a sample of data $y_1, \ldots, y_n$, the function (9) can be numerically minimised to find $\hat{\boldsymbol{\beta}}_\alpha$, as was done by Engle and Manganelli (2004) for CAViaR models (1)-(3). Chen et al (2011) also use this method for estimation in the

TCAV in (4), then show that for simulated data, the Bayesian estimate using MCMC is more efficient for that model. We discuss this approach now.

It has recently been shown that the quantile regression criterion function is related to the likelihood function for the skewed-Laplace distribution. This result allows (maximum) likelihood estimation, and has motivated Bayesian solutions for this problem, as proposed in Yu and Moyeed (2001), Tsionas (2003), Yu and Zhang (2005) and Geraci and Bottai (2007), and subsequently extended in Chen et al (2011). These designs all involve MCMC computational methods due to the non-standard form of the posterior resulting from the skewed-Laplace likelihood.

3.1 Frequentist estimation

First, we define the data vectors as $\boldsymbol{y} = (y_1, \ldots, y_n)'$ for the asset returns and $\boldsymbol{R} = (R_1, \ldots, R_n)'$ for the intra-day range data. If we assume the returns follow a skewed-Laplace, i.e. $y_t \overset{i.i.d}{\sim} SL(f_t(\boldsymbol{\beta}), \tau, \alpha)$, then, the following density function results:

$$f(y_t; f_t(\boldsymbol{\beta}), \tau, \alpha) = \frac{\alpha(1-\alpha)}{\tau} \exp\left[-\rho_\alpha(\frac{y_t - f_t(\boldsymbol{\beta})}{\tau})\right],$$

where $\rho_\alpha(u) = u(\alpha - I(u < 0))$, $f_t(\boldsymbol{\beta})$ is the mode and $\tau > 0$ is a scale parameter. Under this assumption, the likelihood function for any CAViaR model, including (1) -(8), is then:

$$L_\alpha(\boldsymbol{\beta}, \tau, \gamma; \boldsymbol{y}, \boldsymbol{R}) \propto \tau^{-n} \exp\left\{-\tau^{-1}\left[\sum_{t=1}^{n}(y_t - f_t(\boldsymbol{\beta}))\left(\alpha - I_{(-\infty,0)}(y_t - f_t(\boldsymbol{\beta}))\right)\right]\right\}. \tag{10}$$

As such, the $\hat{\boldsymbol{\beta}}_\alpha$ that minimises (9) also maximises (10). This estimate can then simply be plugged into the formula for $f_{n+1}(\boldsymbol{\beta})$ to forecast VaR.

3.2 Bayesian estimation and forecasting

Bayesian inference requires specifying a prior distribution for the unknown parameters, combined with the likelihood function. Assuming the parameters, $(\boldsymbol{\beta}, \tau, \gamma)$, are a priori independent, we choose $\pi(\tau) \propto \tau^{-1}$, the standard Jeffreys' prior, and $\pi(\boldsymbol{\beta}) \propto 1$, as in Gerlach et al (2011). When considering two regimes, a flat prior on the threshold limit γ is $Unif(u, l)$, where the (u, l) are chosen as suitable quantiles of the threshold variable to allow reasonable sample size in each regime for inference.

MCMC methods sample from the joint posterior distribution of the unknown model parameters for estimation, inference and forecasting. Here groups of parameters are defined for the following sampling scheme:

$$p(\boldsymbol{\beta}|\boldsymbol{y}, \boldsymbol{R},\ \gamma), p(\tau|\boldsymbol{y}, \boldsymbol{R}, \boldsymbol{\beta}, \gamma), \text{and } p(\gamma|\boldsymbol{y}, \boldsymbol{R}, \boldsymbol{\beta}, \tau)$$

which is iteratively sampled from to form a dependent sample from the joint posterior distribution. The density $p(\tau|\boldsymbol{y}, \boldsymbol{R}, \boldsymbol{\beta}, \gamma)$ is an inverse gamma distribution, allowing τ to be integrated out of the full posterior to obtain the marginal posterior distribution $p(\boldsymbol{\beta}|\boldsymbol{y}, \boldsymbol{R}, \gamma)$. As all parameters have non-standard posterior densities, we use the Metropolis-Hastings (MH) algorithms (Metropolis et al., 1953, and Hastings, 1970).

In order to speed convergence and allow optimal mixing properties, we use the combined Random Walk and Independent Kernel MH algorithms. The Random Walk Metropolis algorithm is used for the first M iterations, the so-called burn-in period, while the Independent Kernel MH algorithm is employed from iteration $M+1$ onwards, employing the sample mean and covariance matrix of the burn-in iterates for each parameter grouping. This procedure is discussed in detail in Chen and So (2006).

Bayesian forecasts of VaR can be constructed via the MCMC sampling scheme. For each MCMC iterate of parameter values $\boldsymbol{\beta}^{(j)}, \gamma^{(j)}$, $j = 1, \ldots, M$, a 1-day α-level VaR estimator is obtained by plugging in $\boldsymbol{\beta}^{(j)}, \gamma^{(j)}$ to the formula for $f_{n+1}(\boldsymbol{\beta})$, obtaining $f_{n+1}(\boldsymbol{\beta})^{(j)}$. Under the TRV model, this is:

$$f_{n+1}(\boldsymbol{\beta})^{(j)} = \left(\beta_1^{(j)} + \beta_2^{(j)} f_n(\boldsymbol{\beta}) + \beta_3^{(j)} R_n\right) I(R_n \le \gamma^{(j)}) + \left(\beta_4^{(j)} + \beta_5^{(j)} f_n(\boldsymbol{\beta}) + \beta_6^{(j)} R_n\right)(1 - I(R_n \le \gamma^{(j)})).$$

These iterated values $f_{n+1}(\boldsymbol{\beta})^{(j)}$ are then simply averaged over the iterates $j = M+1, \ldots, N$, to obtain a posterior mean estimate $\widehat{VaR}_{n+1}$ that is a forecast of VaR, where the parameters have been integrated out in the MCMC sampling scheme.

3.3 Parametric GARCH estimation

The parametric GARCH models, labelled Model A above, are here estimated by MCMC methods, following the method in Chen, Chiang and So (2003). First, the standard prior choices are made, so that the the usual stationarity and positivity conditions are enforced, i.e.:

$$\alpha_0 > 0\,;\; 0 < \alpha_1 + \beta_1 < 1\,;\; \alpha_1, \beta_1 \ge 0\,;\; |\phi_1| < 1 \tag{11}$$

and the degrees of freedom for the Student-t errors is restricted to be above 4, ensuring its' first four moments are finite. These are achieved by placing a flat prior over the parameters constrained to the region B, which is equivalent to (11), and defining $\eta^* = 1/\eta$ and using a flat prior $\eta^* \sim \text{Unif}(0, 0.25)$. Thus, the prior becomes:

$$p(\phi_1, \phi_1, \alpha_0, \alpha_1, \beta_1) = I(B) \times I(\eta > 4).$$

The likelihood is defined by the choice of error distribution, combined with the GARCH volatility equation.

Multiplying the likelihood and the prior gives the posterior density function (up to a proportionality constant). The standard Gaussian random walk Metropolis method is employed for the first M MCMC iterations (M is the size of the burn-in sample) for each of the parameter groups: (i) (ϕ_1, ϕ_1);(ii) $(\alpha_0, \alpha_1, \beta_1)$; and (iii) η^*; in turn. After the burn-in period, the sample mean and sample variance-covariance of the iterates for $(\alpha_0, \alpha_1, \beta_1)$ are collected. These are then used as the proposal mean and covariance matrix in an independent kernel Metropolis-Hastings method, with a Gaussian proposal distribution. The overall method is thus adaptive, because it learns from the burn-in period. This has the added advantage of capturing the posterior correlations among the $\boldsymbol{\alpha}$ in the burn-in period for use in the sample period proposal, which should also increase efficiency. In particular, since the burn-in sample's mean (now the proposal mean) is likely not too close to the boundaries in (11), the sampler should be more efficient in that region for these parameters. For more details of this method, see Gerlach and Chen (2008) or Chen, Chiang and So (2003).

We note that this method will only work if the MCMC sample has converged and sufficiently covered the posterior inside the burn-in period. Convergence is thus monitored heavily using trace and ACF plots, while the tuning algorithm will also help to ensure sufficient coverage of the posterior by moderating the acceptance rate of the Metropolis method. MCMC results and convergence are extensively examined by starting the scheme from many different and varied starting values.

3.4 Forecast Evaluation

In this section we discuss assessing the accuracy of VaR estimates and forecasts. The Basel II Accord requires financial institutions to use back-testing, so that at least one year of actual returns are compared with VaR forecasts. There are some common criteria for comparing the forecasting performance of VaR models, that is, the violations $(I(y_t < -VaR_t))$ and the violation rate (VRate). For an in-sample period of size n, and forecast sample of size m, VRate is defined as:

$$\text{VRate} = \frac{1}{m} \sum_{t=n+1}^{n+m} I\left(y_t < -VaR_t\right),$$

which is simply the proportion of violating returns. Naturally, the VRate should be close to the risk level, α, for accurate risk models.

Three formal back-testing methods for assessing forecasting performance are the unconditional coverage (uc) test of Kupiec (1995), the conditional coverage (cc) test of Christoffersen

(1998), and the dynamic quantile (DQ) test of Engle and Manganelli (2004). Under the null hypothesis $\alpha = \alpha_0$, Kupiec (1995) employs a likelihood ratio to test whether VaR estimates, on average, provide correct coverage of the lower α percent tails of the forecast distributions. Christoffersen (1998) develops an independence test, employing a two-state Markov process, and combines this with the uc test to develop a joint likelihood ratio conditional coverage test, that examines whether VaR estimates display correct conditional coverage at each point in time. The conditional coverage test thus examines simultaneously whether the violations appear independently and the unconditional coverage is α. The DQ test is also a joint test of the independence of violations and correct coverage. It employs a regression-based model of the violation-related variable 'hits', defined as $I\left(y_t < -VaR_t\right) - \alpha$, which will on average be α if unconditional coverage is correct. A regression-type test is then employed to examine whether the 'hits' are related to lagged 'hits', lagged VaR forecasts, or other relevant regressors, over time; a model producing accurate and independent violations and 'hits' will not be. The DQ test is well known to be more powerful than the CC test, see e.g. Berkowitz, Christofferson and Pelletier (2010).

The tests and criteria above do not consider whether the magnitude of the VaR forecasts is appropriate; only that the violations occur independently and in the right proportion. Naturally, however, it is also important to assess the accuracy of the magnitudes of the forecasts. For example, a simple method that sets VaR to be -100% on a randomly chosen number of days, each with probability α, and with probability $1-\alpha$ sets VaR to be 500% (say), will automatically pass all the statistical tests mentioned above, since violations occur with probability α and are independent over time. But this method is very poor at setting appropriate risk or capital allocation limits. We thus consider three more measures, all assessing the accuracy of the magnitude of VaR forecasts. In our opinion, these measures should be employed once it is established that a VaR forecast method passes the test above.

The Basel II Accord stipulates that market risk charges (MRC) (also called Daily Capital Charge) should also be used to assess appropriate risk models, where lower MRC is desirable. The optimization problem facing ADIs, with the number of violations and forecasts of risk as endogenous choice variables, is as follows:

$$\text{Daily Capital Charge}_t = \sup\left\{\text{VaR}_{t-1}, (3+k)\overline{\text{VaR}_{60}}\right\},$$

where VaR_{t-1} is the VaR of the previous trading day, $\overline{\text{VaR}_{60}}$ is the average VaR over the last 60 trading days, and k is the penalty term from the Basel Accord Penalty Zone. The daily capital charge is set to be the supremum of the last trading day VaR and the average VaR over the past 60 trading days multiplied by a violation penalty weight factor $(3+k)$. Models with lower daily capital charge values are preferred for risk management. The daily capital charge attempts to give a conservative estimate of capital required to cover market risk that tries to correct for under-estimation of risk levels by applying a penalty factor to the average of previous VaR estimates. The penalty is higher the more risk has been under-estimated in the past. MRC is the average of the daily capital charges during the forecast period. McAleer and da Veiga (2008a, Table IV) displays the penalty zones at the 1% level, and the number of violations is given for 250 trading days: "Another feature of regulatory back-tests that is not easy to understand is why they require only 250 days in the back-test. With such a small sample the power of the test to reject a false hypothesis is very low indeed. So, all in all, it is highly likely that an inaccurate VaR model will pass the regulatory backtest."(Market Risk Analysis p. 336). We increase the forecast sample size here and extend the traffic light approach to obtain the penalty weight factor for such larger samples, which are given in Table 1; as discussed in detail in Section 4.

As we are also interested in the magnitude of violating returns, McAleer and da Veiga (2008a) propose the absolute deviation (AD) of violating returns, as follows:

$$AD_t = |y_t - (-VaR_t)|\, I(y_t < -VaR_t).$$

This measure is related to the size of the loss for violating returns. We evaluate forecast performance based on the mean and maximum of AD, where smaller values are preferred, over the forecast sample.

Note that if models are consistently under-estimating risk and thus have too many violations, it is likely they will have smaller values for MRC, AD mean and AD maximum. Models that consistently over-estimate risk levels for violations, as the simple diabolical method mentioned above (which sets VaR randomly to -100% thus forcing a violation), will have very large MRC, AD mean and AD maximum values. As such, models with small AD and MRC values are preferred only if they are generating independent violations at the correct rate α.

Finally, the accuracy of quantile forecasts can be directly assessed using the quantile criterion loss function, given in equation (9). Here, $f_t(\boldsymbol{\beta})$ is replaced by the forecast VaRs for each method. The true VaR series should give the minimum of (9) and thus the most accurate model under consideration, during the forecast period, should return the minimum value of this loss function.

4 Empirical Applications

In order to demonstrate and compare the forecasting performance of the proposed models, we first consider daily financial returns from five Asia-Pacific Economic Cooperation (APEC) stock markets: Standard and Poor's 500 Composite Index (U.S.), Nikkei 225 Index (Japan), TAIEX Index (Taiwan), HANG SENG Index (Hong Kong) and KOSPI Index (Korea). An equally-weighted daily return portfolio is formed from the five individual market returns, on days when all markets traded. For this portfolio, the range data from Standard and Poor's 500 Composite Index is used in the equations (6)-(8). This choice is based on the global economic scale of the US market and its strong impact on economic growth of other countries. Moreover, we consider stock market returns and their intra-day ranges. McAleer and da Veiga (2008b) compare the performance of single-index and portfolio models in forecasting VaR. All market data are obtained from Datastream International for the period January 1, 2002 to April 30, 2010. A further example concerns two exchange rate series, the Euro vs US and Japan vs US exchange rates. The range of the exchange rates were obtained from Thomson Reuters Tick History database.

The percentage returns series are calculated by taking differences of the logarithms or the daily price indices, $r_{t,j} = (\ln(P_t^j) - \ln(P_{t-1}^j)) \times 100$, where P_t^j is the closing price index on day t of asset j. We consider a single equal-weighted portfolio of assets, with returns: $y_t = \sum_{i=1}^{5} r_{t,i} \times 20\%$. In addition, the differences of the logarithm of the daily series of intra-day high and low prices are taken as the range data, R_t, and are defined as $R_t = (\ln(R_{t,max}) - \ln(R_{t,min})) \times 100$. We use the S&P 500 range for the portfolio as the explanatory variable, and the threshold variable, in the risk models using range data, and use domestic daily range for this purpose in each individual stock market.

We examine how risk management strategies perform during the 2008-09 GFC, evaluate how the financial crisis affects risk management practices, and forecast VaR and daily capital charges, that is, diversification of more than a single investment for the purpose of risk control and management.

4.1 VaR forecasting for the portfolio

The full sample is divided into an in-sample period (from January 1, 2002 to July 31, 2008), and a forecast period of 400 trading days from August 1, 2008 to April 30, 2010), which covers the 2008-09 GFC.

Table 1: Modified Traffic light approach (Basel Committee, 1996) based on 400 trading days; true coverage is 99%

Zone	Number of Violation	Cumulative probability	Increase in scaling factor
Green	0	0.01795	0
	1	0.09048	0
	2	0.23663	0
	3	0.43249	0
	4	0.62884	0
	5	0.78592	0
	6	0.89037	0
	7	0.94976	0
Yellow	8	0.97923	0.39820
	9	0.99220	0.48142
	10	0.99732	0.56080
	11	0.99915	0.63705
	12	0.99975	0.71069
Red	13 or more	0.99993	1

Figure 1 shows the time series plots of the portfolio returns and the S&P500 range data: both highlight sharp increases in volatility in September, 2008 and for the subsequent early months in 2009, plus a very low volatility period approximately from 2005 to 2007; clearly the range data reflects these periods well. Table 2 presents summary statistics for each market and the portfolio returns for the full sample (contains in-sample and out-of-sample) and for the forecast (out-of-sample) sample. As expected, the forecast period displays consistently higher standard deviations and average intra-day ranges than the full sample, across all markets. Also expected, all six return series have heavy-tailed distributions and most are mildly negatively skewed. The p-values of the Jarque-Bera test for departure from normality are all very small: normality is rejected in all markets/series.

Table 2: Summary statistics: Stock index returns and ranges for five stock markets and equal weights portfolio from January 1, 2002 to April 30, 2010.

	TAIEX		Nikkei225		HSI		KOSPI		S&P500		Portfolio	
Statistics	return	range	return	range	return	range	return	range	return	range	return	range
Observations	2055	2055	2042	2042	2056	2056	2062	2062	2096	2096	1790	1790
Mean	0.017	1.494	0.001	1.506	0.030	1.467	0.043	1.770	0.001	1.499	0.005	1.504
Median	0.064	1.277	0.044	1.287	0.055	1.188	0.150	1.531	0.074	1.167	1.164	1.167
Std.	1.478	0.852	1.618	0.988	1.662	1.072	1.639	1.097	1.391	1.185	0.056	1.174
Minimum	-6.912	0.146	-12.111	0.299	-13.582	0.285	-11.172	0.408	-9.470	0.239	-7.336	0.239
Maximum	6.525	7.403	13.235	11.743	13.407	17.647	11.284	15.841	10.957	10.904	8.297	10.904
Q1	-0.664	0.885	-0.781	0.905	-0.657	0.846	-0.726	1.092	-0.587	0.786	-0.539	0.797
Q3	0.793	1.851	0.875	1.844	0.785	1.761	0.931	2.108	0.606	1.811	0.617	1.818
Skewness	-0.257	1.653	-0.377	3.527	0.100	4.378	-0.453	3.715	-0.144	3.162	-0.297	3.009
Normality test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Excess kurtosis	2.316	4.264	7.433	23.035	9.345	39.355	4.581	28.018	9.155	15.320	6.531	14.031
Hold-out set												
Mean	0.023	1.802	-0.050	1.914	-0.003	2.252	0.032	2.109	-0.013	2.332	-0.035	2.306
Std.	1.825	1.035	2.360	1.558	2.578	1.631	2.066	1.690	2.241	1.849	1.708	1.760
Minimum	-5.933	0.333	-12.111	0.314	-13.582	0.462	-11.172	0.408	-9.470	0.375	-7.336	0.375

The specific VaR models and methods considered are now listed:

1. Non-parametric: short-term (ST, last 25 days) and long-term (LT, last 100 days) sample percentiles.

2. Parametric methods: GARCH(1,1) with normal and Student-t errors; RiskMetrics with normal errors and $\lambda = 0.94$.

3. Semi-parametric methods: The family of CAViaR models in (1)-(8) is considered with estimation either by frequentist (denoted by "E&M", to indicate that the same or similar estimation as in Engle and Manganelli, 2004 was used) and/or Bayesian methods, both as detailed in Section 3.

We use Fortran codes to obtain the MCMC iterates, and use the 'fminsearch' routine in the Matlab software to minimise (9) numerically. The Matlab code is adapted and updated for the RV-E&M model from freely available code kindly provided by Simone Manganelli (downloadable from http://www.simonemanganelli.org/Simone/Research.html). The Econometrics toolbox in the software Matlab is employed to estimate both GARCH model via maximum likelihood.

For the Bayesian estimation, priors are as stated in Section 3, e.g. a uniform prior is used for the threshold value γ, that is $\gamma \sim Unif(l, u)$, where l and u are the 1st and 3rd quantiles of the range data. MCMC sampling is performed with 20,000 iterations in total, including the first 10,000 burn-in iterations. The last 10,000 iterations are used for inference.

We only report Bayesian estimates of the parameters for the RV, TRV and TRIG specifications in Table 3 which include posterior means, standard deviations (Std.), and the 95% credible interval (95% CI) for each parameter. All estimated parameters are significant, except for β_3 of the TRV model at the 1% level and β_1 of the TRV model at the 5% level. The former belongs to the low range which does not respond strongly to VaR, and the latter is the intercept of the low range. Convergence of the MCMC iterates is examined via trace plots and autocorrelation function plots as diagnostic checks, not presented due to space limitations; these show that the Markov chain appears to have reached a stationary distribution in each case and indicate low autocorrelation and fairly efficient sampling and hence suggest good convergence and mixing properties of the MCMC sampling scheme. A rolling window approach is used, where a fixed in-sample size, of approximately 2000 days, is employed for estimation to forecast each day in the forecast period. Thus each method is completely re-estimated for each day in the forecast sample. This gives each method a chance to adapt to changing risk dynamics and levels.

The traffic light approach suggested by the Basel Committee (1996) deems a VaR model acceptable (green zone) if the number of violations of 1 % VaR remains below the binomial(p=0.01) 95% quantile. A model is disputable (yellow zone) up to the 99.99% quantile and is deemed seriously flawed (red zone) whenever more violations occur. Translated to our sample size (n=400) in Table 1, a model passes regulatory performance assessment if, at most, 7 violations occur, is disputable when between 8 and 12 violations occur and is seriously flawed for above 12 violations.

The results reported in Table 4 show numbers of violations, zone colour, VRate, VRate/α, AD, Penalty Charge, MRC and the quantile criterion function, at the 1% and 5% confidence levels for the portfolio return series, for each of the 17 VaR models/methods considered; these are informal comparison metrics. When comparing VRate/α, a value of 0.9 is considered better than VRate/α=1.1, as the loss estimates are conservative in the former and anti-conservative in the latter, case. The methods are formally tested by UC, CC and DQ, with results in Table 7 (bottom right corner). At the 1% confidence level the ST, LT, RM, both GARCH, both SAV, the AS-E&M and the TIG-Bayesian methods are rejected by at least one test, at 5% significance. All the models using range information can not be rejected; while all the models surviving the tests are CAViaR-type models. In table 4, the top three ranked models, of those surviving the tests, for each metric are boxed. For these, the RV-Bayesian, TRV-Bayesian and TRIG-Bayesian are the top three by VRate, while these plus RV-E&M are all in the green zone and have attracted no penalty. Looking across the metrics, the TRIG-Bayesian and RV-E&M both rank in the top 3 models for four criteria: AD mean, Penalty, Daily Capital Charge and loss function; while RV-Bayesian, TRV-Bayesian and TRIG-Bayesian rank in the top 3 models for three criteria. Informally, these are the most accurate VaR forecast methods for the portfolio 1% risk level.

At the 5% confidence level, again the ST, LT, RM, both GARCH and both SAV models are rejected by at least test, as are all the CAViaR models estimated via the E&M method. As, such only the mnodels estimated via Bayesian methods survive all the tests here. For these models, the TIG, TRIG, RV and TRV-Bayesian models are closest to nominal in terms of VRate/α. Across the metrics, the RV-Bayesian and TRV-Bayesian also rank best on Mean AD and the criterion loss function. Note that the Basel Accord only has 1% level penalties for MRC, not 5%, thus these metrics are not reported in this case.

Table 5 shows the VaR forecasting results separately for each of the five individual markets making up the portfolio. For the Nikkei225, at the 1% risk level, only three methods survive the tests, shown in Table 7: the IG-Bayesian, TIG-Bayesian and RV-Bayesian. All models under-predict risk for the Nikkei, since all VRates are larger than 0.01. The IG-Bayesian and

Table 3: Bayesian estimation of parameters for the RV, TRV and TRIG specifications

		1%			5%		
Model	Parameter	Mean	Std.	95% CI	Mean	Std.	95% CI
RV	β_1	0.233	0.042	(0.163,0.333)	0.251	0.052	(0.157,0.360)
	β_2	0.608	0.035	(0.521,0.672)	0.339	0.055	(0.235,0.445)
	β_3	0.548	0.044	(0.457,0.647)	0.582	0.047	(0.490,0.671)
TRV	β_1	0.221	0.036	(0.136,0.284)	0.125	0.069	(-0.003,0.270)
	β_2	0.850	0.029	(0.788,0.905)	0.546	0.104	(0.360,0.775)
	β_3	-0.030	0.057	(-0.132,0.092)	0.440	0.125	(0.168,0.659)
	β_4	1.541	0.196	(1.096,1.897)	0.497	0.182	(0.139,0.846)
	β_5	0.237	0.073	(0.070,0.375)	0.162	0.090	(0.015,0.360)
	β_6	0.400	0.051	(0.319,0.517)	0.616	0.087	(0.426,0.778)
	γ	1.447	0.004	(1.435,1.452)	1.424	0.047	(1.339,1.543)
TRIG	β_1	0.335	0.053	(0.237,0.444)	0.170	0.082	(0.030,0.351)
	β_2	0.802	0.025	(0.749,0.847)	0.514	0.108	(0.287,0.737)
	β_3	0.044	0.036	(0.001,0.140)	0.544	0.167	(0.191,0.838)
	β_4	4.604	0.518	(3.526,5.553)	1.008	0.405	(0.282,1.811)
	β_5	0.359	0.079	(0.180,0.497)	0.162	0.094	(0.013,0.371)
	β_6	0.405	0.066	(0.312,0.569)	0.614	0.091	(0.406,0.784)
	γ	1.447	0.004	(1.434,1.452)	1.429	0.039	(1.365,1.548)

RV-Bayesian do best (of all models) on VRate, but IG-Bayesian and does best, of these three models, on four of the six criteria (inclding 'Zone'). For the HSI index returns, all models under-predict risk levels, but only the ad hoc ST and LT methods fail the tests. Among the surviving models, the AS-Bayesian, TCAV-Bayesian and SAV-Bayesian consistently rank in the top three models across the criteria, for HSI. For the KOSPI only the SAV, AS, IG, RV and TRV, all estimated via the Bayesian method, survive the statistical tests. Of these, the SAV, TIG and RV-Bayesian all consistently rank in the top three models over the metrics. For the TAIEX, only the SAV, IG and RV models, using both E&M and BAyesian estimation, survive all the tests and only the RV-Bayesian is a conservative risk model (with VRate < 0.01). Of these, the IG-Bayesian and IG-E&M rank consistently well across the criteria. Finally, for the S&P500, no model survives the tests at the 1% VaR level.

At the 5% level, a similar story ensues, see Table 6. All models are rejected for the Nikkei225, though not at the 1% significance level for TCAV-Bayesian and TIG-Bayesian, which also do comparatively well across the criteria in Table 5. For the HSI, the ST, LT, GARCH-t, TIG-Bayesian and TRIG-Bayesian are all rejected. For the other models, the AS-Bayesian, TCAV-Bayesian and TRV-Bayesian ranked best across the criteria. For the KOSPI and the TAIEX, the IG-Bayesian and RV-Bayesian ranked most consistently among surviving models; while the

Table 4: VaR prediction performance: using 17 model specifications and the 400 forecasts for the portfolio returns

α=1%	Violations	Zone	VRate/α	AD		Penalty	Daily Capital	Quantile criterion
				Mean	Max		Charge	function
ST	23	Red	5.75	0.724	1.976	1.000	12.507	28.360
LT	11	Yellow	2.75	1.066	2.234	0.631	15.069	27.312
RiskMetrics	13	Red	3.25	0.534	1.819	0.762	14.884	20.837
GARCH-n	11	Yellow	2.75	0.637	1.829	0.631	13.014	20.430
GARCH-t	8	Yellow	2.00	0.606	1.665	0.400	13.113	19.329
SAV-Bayesian	8	Yellow	2.00	0.628	1.384	0.400	14.307	20.810
SAV-E&M	13	Red	3.25	0.823	2.074	0.762	13.278	23.462
AS-Bayesian	7	Green	1.75	1.396	3.670	0.150	10.900	23.636
AS-E&M	13	Red	3.25	1.053	3.648	0.762	12.428	25.786
IG-Bayesian	7	Green	1.75	0.498	1.676	0.150	12.283	18.901
IG-E&M	7	Green	1.75	0.479	1.672	0.150	12.229	18.684
TCAV-Bayesian	7	Green	1.75	1.277	4.078	0.150	11.104	23.022
TIG-Bayesian	8	Yellow	2.00	0.739	1.482	0.400	12.723	20.177
RV-Bayesian	3	Green	0.75	0.727	1.899	0.000	12.124	17.388
RV-E&M	6	Green	1.50	0.527	1.991	0.000	11.049	17.092
TRV-Bayesian	3	Green	0.75	0.824	1.825	0.000	11.719	17.152
TRIG-Bayesian	4	Green	1.00	0.993	1.794	0.000	11.545	18.459
α=5%	Violations	VRate	VRate/α	AD				Quantile criterion
				Mean	Max			function
ST	38	9.50	1.90	0.809	2.574			28.360
LT	21	5.25	1.05	1.577	4.241			27.312
RiskMetrics	26	6.50	1.30	0.761	2.958			20.837
GARCH-n	29	7.25	1.45	0.795	2.708			20.430
GARCH-t	30	7.50	1.50	0.814	2.892			19.329
SAV-Bayesian	26	6.50	1.30	0.766	3.013			20.810
SAV-E&M	30	7.50	1.50	0.837	2.981			23.462
AS-Bayesian	25	6.25	1.25	0.986	2.300			23.636
AS-E&M	28	7.00	1.40	0.955	2.526			25.786
IG-Bayesian	24	6.00	1.20	0.742	2.306			18.901
IG-E&M	25	6.25	1.25	0.748	2.459			18.684
TCAV-Bayesian	23	5.75	1.15	0.975	2.292			23.022
TIG-Bayesian	20	5.00	1.00	0.821	2.321			20.177
RV-Bayesian	22	5.50	1.10	0.633	2.608			17.388
RV-E&M	23	5.75	1.15	0.682	2.669			17.092
TRV-Bayesian	22	5.50	1.10	0.724	2.677			17.152
TRIG-Bayesian	21	5.25	1.05	0.743	2.687			18.459

TIG-Bayesian ranked best for the S&P500 among surviving models.

Tables 5-6 about here

From above, we note that the CAViaR family of models are consistently dominating the models surviving the formal back-tests, while traditional methods like RM and GARCH are consistently rejected. Further, CAViaR models estimated by the Bayesian method consistently rank higher across all or most series than models estimated by the traditional method in E&M.

Finally, the models with IG and RV form consistently survive the tests and rank highly in most series.

We now consider two more series, both exchange rates, being the Euro vs US and Japan vs US exchange rates. Due to the availability of intra-day data for these series, the dates for sample and forecast periods are: December 21, 2004 to July 3, 2009 and July 6, 2009 to February 8, 2011. While it is well known that exchange rate returns do not, in general, exhibit significant volatility asymmetry, we choose to keep the same set of 17 models/methods, including the asymmetric ones, for consistency. Table 8 contains the p-values for the tests for the 1% and 5% VaR forecasts for these two series over the 17 methods. Tables 9-10 show the violation rates and other accuracy metrics for these two series. At the 1% confidence level for the Euro/US series, only the two adhoc ST, LT methods fail the tests. Of the surviving models, the AS-Bayesian, TCAV-E&M and the TRIG-Bayesian consistently rank well across the metrics. For JP/US rates at 1% risk level, the ST, SAV-E&M, TCAV-E&M and RV-E&M models fails the tests. Of the surviving models, the TIG, TRIG, both Bayesian and the GARCH-t model consistently rank in the top 3 over the metrics.

At the 5% risk level, the ST, LT, RM, both GARCH models, both SAV and both AS models are rejected, for the Euro/US series. Of the remaining models, the TIG and RV, both Bayesian, consistently rank highly. For the JP/US series, only the AS models, plus RV-Bayesian and TRV-Bayesian survive the tests. Of these, AS-Bayesian and RV-Bayesian rank highest across the metrics.

Tables 9-10 about here

To summarise, for the exchange rate series, similar conclusions can be formed: the CAViaR family of models are consistently the only models surviving the formal back-tests; CAViaR models estimated by the Bayesian method rank highest across most series and; the models with IG and RV form consistently survive the tests and rank highly in most series.

Table 11 shows counts of the rejections over the three tests (UC, CC, and DQ) and across the markets and exchange rate series (counts the number of series where each, and at least one, test rejected that model) for each model. The DQ statistic is clearly the most powerful test and rejects the most models in the most markets. For the 1% VaR forecasts, the non-parametric ST and LT methods are rejected in all markets by almost all tests: clearly these are the poorest methods for this data period. The RM, GARCH with Gaussian and Student-t errors and the SAV-E&M and AS-E&M methods are rejected in five of the eight series. On the other hand, the RV-Bayesian and IG-Bayesian models are each only rejected in one series, the S&P500, and only by the powerful DQ test, while all other methods are rejected at least three times. The detection of violations in the portfolio return series using the TRIG-Bayesian model at the 1% level is shown in Figure 2. There are four violations within the forecasting period, three of which occur during the period September-December 2008. The closeness of these violations, in time, is sufficient for the DQ test to reject this model. Other model befell a similar fate of not reacting, in time or magnitude, enough to the onset of the GFC extreme period. On the contrary, the RV-Bayesian and IG-Bayesian models, we clearly able to do so effectively across almost all of these eight series.

For the 5% VAR forecasts, table 11 shows that again the ST, LT, GARCH-t and AS-E&M methods are the worst performed, rejected in at least 6 series. Again, the IG-Bayesian and the RV-Bayesian models perform the best, being rejected in only two out of eight series.

Table 12 shows summary statistics of the various criteria in Tables 4 and 5 for 1% VaR forecasting. While these are informal criteria, this table highlights the performance of each model across the six series and may show consistent out-performance, or otherwise. For each criteria, the mean and median across the eight series are shown. For the VRate/α the square root of the average squared distance from 1 is also shown, labeled 'RMSD'. The best three models are boxed for each criterion summary, while the worst model appears in bold. For 1% VaR forecasting, clearly the non-parametric models are anti-conservative, under-estimating risk levels and performing the worst among these 17 methods across the six series. The CAViaR models are clearly the best performing as a group, since they get all the boxes except one, with two stand-outs across the criteria: the IG-Bayesian model, which is in the top three ranked models for eight of the eleven criterion summaries; and the RV-Bayesian model, in the top three ranked models for six of criterion summaries.

Table 13 shows these summaries for 5% VaR forecasting. Now it is the TRV-Bayesian, with seven top three rankings across the nine criteria, and the RV-Bayesian model, with five top three rankings, that consistently do best across the criterion summaries over the eight markets combined.

Tables 11-13 about here

4.2 Summary and Discussion

Overall, the non-parametric methods have performed consistently the worst at VaR forecasting, over a range of criteria, in each of the eight series and combined across these series. The ST method's performance is perhaps explainable by the fact that 25 days is not sufficient to estimate quantiles at the 1% and 5% risk level, however the LT method's poor performance is harder to explain. Regardless, clearly these methods are not suitable for VaR forecasting at 1% and 5% levels. Further, the fully parametric RiskMetrics and GARCH models only did marginally better than the non-parametric ones, being rejected by at least one test in most or almost all series and generally and consistently under-estimating risk levels and poorly capturing risk dynamics across the data analysed.

The CAViaR models as a group did uniformly better than these four models on all metrics in almost all eight series and also in almost all metrics combined across the series, for both 1% and 5% VaR forecasting. Further, when focusing on models estimated by Bayesian or classical methods (as in E&M), the Bayesian models also performed almost uniformly better in all markets and metrics than the same model estimated via E&M. Gerlach et al (2011) found in simulations that the Bayesian estimates of the TCAV model parameters and forecasts of VaR were more efficient than those estimated/forecasted using classical estimation. The results here suggest that this is a more general result across the CAViaR family of models: Bayesian estimation and forecasting of CAViaR models is more efficient and accurate than classical estimation of CAViaR models, at least for those models and the data considered here. Finally, three models stood out as performing the best across the back-tests and the range of forecast accuracy criteria applied: the IG-Bayesian model and the RV-Bayesian model for 1% VaR forecasting; and the TRV-Bayesian and RV-Bayesian models for 5% VaR forecasting. These models consistently out-performed all others across the eight series on most of the metrics considered, at each risk level.

The Basel Committee (1996) classified the reasons for model back-testing failures into the following categories:

1. Basic integrity of the model: The system is unable to capture the risk of the positions or there is a problem in calculating volatilities and correlations.

2. Model's accuracy could be improved: Risk of some instruments not measured with sufficient precision.

3. Bad luck, or markets moved in a fashion that could not be anticipated by the model. For

instance, volatilities or correlations turned out to be significantly different than what was predicted.

4. Intra-day trading: There is a change in positions after the VaR estimates were computed.

The US market result, where all models are rejected at VaR 1% forecasting, may be explained by point 3; which points to possible future research on developing a model that can capture the risk dynamics in the US market, especially during extreme or crisis conditions.

5 Concluding Remarks

A novel family of risk models, namely nonlinear threshold CAViaR models using intra-day price range, are proposed and the selection of optimal risk models during the 2008-09 global financial crisis is assessed and discussed. Risk management strategies and performance are assessed during this period and the performance of VaR models during the gobal financial crisis is evaluated and compared. Further, both Bayesian and frequentist (quasi-Newton) methods of estimation and forecasting are assessed and compared with real financial return data. Bayesian MCMC methods are adapted to the new family of CAViaR models, employing the link between the quantile criterion function and the Skewed-Laplace density. Five APEC stock market indices are considered, individually and via and equally weighted portfolio, and VaR is forecast for these series over roughly a two year period. Two exchange rate series are added to the analysis. The empirical evidence reveals several phenomena:

1. Risk levels and dynamics during the financial crisis seem to be predictable at a one day horizon, at least by some models (but not by others), in most markets and the portfolio considered.

2. By comparing the same model using different estimation methods, the forecasting performance of the Bayesian method is more accurate than the frequentist quasi-Newton method, in almost all cases.

3. The forecasting performance of VaR models with range information outperform models without range information, in most cases.

4. Semi-parametric models, i.e. CAViaR, consistently ranked best via statistical testing and informal assessment criteria. Next came the parametric models, though they were consistently rejected by back-tests. The non-parametric methods consistently ranked worst and were rejected in most cases.

5. The two most favoured models for 1% VaR forecasting were the simple CAViaR models IG-Bayesian and RV-Bayesian. These were favoured by the statistical back-tests, being acceptable in almost all series, as well as by most of the informal criteria across the series.

6. The two most favoured models for 5% VaR forecasting, were the two range-based CAViaR models: TRV-Bayesian and RV-Bayesian.

7. When incorporating range information and employing the Bayesian approach, for dynamic quantile VaR forecasting, CAViaR models are competitive at worst, and far more accurate at best, when compared with a range of popular and well known VaR methods.

Each of these findings should be useful to financial practitioners and institutions. Many additional questions emerge for future research. Is the crisis period as predictable using similar time series models for longer horizons (for example 10-day forecasts)? Due to space limitations, we only focus on 1-day forecasting. Extensions to include different state variables in the information set, more than two regimes, and a smooth transition function are potential directions for further research.

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Table 5: VaR prediction performance: over 450 forecasts at the 1% level for each market

α=1%		Violations	Zone	VRate/α	AD Mean	AD Max	Daily Capital Charge	Quantile Criterion function
Nikkei225	ST	20	Red	4.44	0.952	4.945	16.252	35.920
	LT	11	Yellow	2.44	1.659	5.497	19.008	40.879
	RiskMetrics	8	Yellow	1.78	1.336	4.492	15.060	31.756
	GARCH-t	5	Green	1.11	1.609	4.039	14.556	29.227
	GARCH-n	6	Green	1.33	1.471	4.218	15.407	29.657
	SAV-Bayesian	8	Yellow	1.78	0.819	3.034	15.776	28.659
	SAV-E&M	14	Red	3.11	0.737	3.873	18.139	29.605
	AS-Bayesian	9	Yellow	2.00	0.812	1.627	17.352	28.867
	AS-E&M	14	Red	3.11	0.988	2.077	17.761	32.905
	IG-Bayesian	7	Green	1.56	0.832	3.263	15.074	27.009
	IG-E&M	9	Yellow	2.00	0.688	3.267	16.605	26.860
	TCAV-Bayesian	10	Yellow	2.22	0.906	1.815	17.288	30.144
	TIG-Bayesian	8	Yellow	1.78	1.094	2.277	14.562	29.212
	RV-Bayesian	7	Green	1.56	0.952	3.191	14.864	27.455
	RV-E&M	8	Yellow	1.78	0.911	2.895	14.218	27.305
	TRV-Bayesian	9	Yellow	2.00	0.663	2.907	17.317	27.279
	TRIG-Bayesian	9	Yellow	2.00	0.940	3.688	16.685	29.158
HSI	ST	21	Red	4.67	1.008	4.952	17.194	39.719
	LT	10	Yellow	2.22	1.699	5.062	18.255	39.872
	RiskMetrics	5	Green	1.11	1.345	2.400	16.995	30.880
	GARCH-t	4	Green	0.89	1.179	1.611	16.449	30.401
	GARCH-n	7	Green	1.56	0.997	2.311	17.952	30.263
	SAV-Bayesian	5	Green	1.11	0.426	1.008	17.187	26.822
	SAV-E&M	4	Green	0.89	0.752	1.770	17.303	27.743
	AS-Bayesian	3	Green	0.67	0.500	0.903	15.986	24.628
	AS-E&M	2	Green	0.44	0.688	1.117	16.587	25.235
	IG-Bayesian	5	Green	1.11	0.517	1.341	16.848	26.750
	IG-E&M	6	Green	1.33	0.726	1.760	16.035	27.337
	TCAV-Bayesian	6	Green	1.33	0.467	1.216	15.454	25.140
	TIG-Bayesian	6	Green	1.33	0.635	1.343	15.337	25.916
	RV-Bayesian	3	Green	0.67	1.099	1.104	17.984	28.764
	RV-E&M	5	Green	1.11	1.248	2.331	15.990	29.134
	TRV-Bayesian	5	Green	1.11	0.688	1.509	17.570	28.365
	TRIG-Bayesian	3	Green	0.67	1.088	1.232	17.820	28.507
KOSPI	ST	22	Red	4.89	1.001	3.692	15.966	39.185
	LT	9	Yellow	2.00	1.929	3.922	17.816	39.794
	RiskMetrics	12	Yellow	2.67	1.151	3.011	16.081	33.103
	GARCH-t	10	Yellow	2.22	1.044	2.274	15.402	32.545
	GARCH-n	11	Yellow	2.44	1.234	2.659	16.236	30.831
	SAV-Bayesian	8	Yellow	1.78	1.261	1.998	14.933	31.693
	SAV-E&M	10	Yellow	2.22	0.986	1.979	16.029	30.051
	AS-Bayesian	8	Yellow	1.78	1.316	2.972	14.326	31.417
	AS-E&M	13	Yellow	2.89	1.152	3.434	15.096	33.246
	IG-Bayesian	8	Yellow	1.78	1.080	2.211	15.062	30.435
	IG-E&M	10	Yellow	2.22	0.947	2.120	16.543	30.217
	TCAV-Bayesian	10	Yellow	2.22	1.111	3.182	16.485	31.879
	TIG-Bayesian	10	Yellow	2.22	1.162	2.889	16.704	32.655
	RV-Bayesian	8	Yellow	1.78	1.196	3.255	15.244	31.346
	RV-E&M	10	Yellow	2.22	1.158	3.876	15.849	31.328
	TRV-Bayesian	8	Yellow	1.78	1.345	2.994	15.541	32.951
	TRIG-Bayesian	6	Green	1.33	1.672	2.597	16.276	33.138

Table 5 (Continued)

α=1%		Violations	Zone	VRate/α	AD		Daily Capital Charge	Quantile Criterion function
					Mean	Max		
TAIEX	ST	21	Red	4.67	0.801	1.841	13.911	32.464
	LT	11	Yellow	2.44	0.735	1.315	14.931	26.866
	RiskMetrics	11	Yellow	2.44	0.669	1.417	14.499	25.582
	GARCH-t	8	Yellow	1.78	0.481	0.960	14.285	25.145
	GARCH-n	11	Yellow	2.44	0.649	1.208	13.439	23.812
	SAV-Bayesian	6	Green	1.33	0.471	0.994	13.415	22.923
	SAV-E&M	7	Green	1.56	0.442	0.999	13.174	22.844
	AS-Bayesian	9	Yellow	2.00	0.558	2.042	14.649	24.434
	AS-E&M	13	Yellow	2.89	0.728	2.521	14.243	27.034
	IG-Bayesian	6	Green	1.33	0.448	0.838	13.728	23.242
	IG-E&M	6	Green	1.33	0.468	0.908	13.563	23.114
	TCAV-Bayesian	11	Yellow	2.44	0.371	0.942	15.369	23.597
	TIG-Bayesian	9	Yellow	2.00	0.613	1.733	15.043	25.371
	RV-Bayesian	4	Green	0.89	0.737	1.255	13.690	23.308
	RV-E&M	5	Green	1.11	0.643	1.217	13.487	23.291
	TRV-Bayesian	6	Green	1.33	0.931	1.944	13.731	26.026
	TRIG-Bayesian	6	Green	1.33	0.783	2.248	13.695	25.123
S&P500	ST	24	Red	5.33	0.958	4.390	15.903	39.708
	LT	11	Yellow	2.44	1.205	4.390	18.777	35.553
	RiskMetrics	13	Yellow	2.89	0.648	3.749	17.522	28.498
	GARCH-t	9	Yellow	2.00	0.713	3.437	18.200	28.410
	GARCH-n	14	Red	3.11	0.656	3.850	16.726	27.212
	SAV-Bayesian	10	Yellow	2.22	0.854	4.008	16.379	28.492
	SAV-E&M	13	Yellow	2.89	1.178	4.962	16.597	34.410
	AS-Bayesian	11	Yellow	2.44	0.790	3.926	16.450	28.306
	AS-E&M	15	Red	3.33	0.986	4.828	16.658	32.543
	IG-Bayesian	9	Yellow	2.00	0.860	3.482	16.369	28.084
	IG-E&M	11	Yellow	2.44	0.878	4.227	17.221	30.176
	TCAV-Bayesian	12	Yellow	2.67	0.963	3.746	17.123	31.511
	TIG-Bayesian	11	Yellow	2.44	0.701	3.702	17.029	28.016
	RV-Bayesian	9	Yellow	2.00	0.849	3.976	15.385	26.847
	RV-E&M	17	Red	3.77	0.835	4.680	16.061	31.357
	TRV-Bayesian	10	Yellow	2.22	1.028	4.286	15.409	29.033
	TRIG-Bayesian	11	Yellow	2.44	1.010	5.015	15.657	29.835

Table 6: VaR prediction performance: over 450 forecasts at the 5% level for each market

α=5%		Violations	VRate	VRate/α	AD		Quantile Criterion function
					Mean	Max	
Nikkei225	ST	39	8.67	1.73	0.926	5.532	35.920
	LT	20	4.44	0.89	2.280	8.979	40.879
	RiskMetrics	34	7.56	1.51	1.002	6.062	31.756
	GARCH-t	36	8.00	1.60	1.022	5.868	29.227
	GARCH-n	34	7.56	1.51	1.054	5.961	29.657
	SAV-Bayesian	29	6.45	1.29	0.958	5.564	28.659
	SAV-E&M	34	7.56	1.51	1.040	6.101	29.605
	AS-Bayesian	32	7.11	1.42	0.914	4.379	28.867
	AS-E&M	33	7.33	1.47	1.038	4.810	32.905
	IG-Bayesian	32	7.11	1.42	0.851	5.748	27.009
	IG-E&M	32	7.11	1.42	0.958	5.942	26.860
	TCAV-Bayesian	28	6.22	1.24	1.016	3.991	30.144
	TIG-Bayesian	28	6.22	1.24	0.986	4.081	29.212
	RV-Bayesian	33	7.33	1.47	0.926	5.242	27.455
	RV-E&M	36	8.00	1.60	1.113	5.676	27.305
	TRV-Bayesian	30	6.67	1.33	0.825	3.986	27.279
	TRIG-Bayesian	34	7.56	1.51	0.899	5.470	29.158
HSI	ST	40	8.89	1.78	0.972	5.275	39.719
	LT	25	5.56	1.11	1.536	8.475	39.872
	RiskMetrics	22	4.89	0.98	1.162	5.675	30.880
	GARCH-t	27	6.00	1.20	1.058	5.893	30.401
	GARCH-n	25	5.56	1.11	1.041	5.613	30.263
	SAV-Bayesian	24	5.33	1.04	0.810	4.240	26.822
	SAV-E&M	22	4.89	0.98	1.014	5.187	27.743
	AS-Bayesian	28	6.22	1.24	0.660	2.751	24.628
	AS-E&M	22	4.89	0.98	0.814	4.147	25.235
	IG-Bayesian	25	5.55	1.11	0.854	4.573	26.750
	IG-E&M	24	5.33	1.07	0.945	5.277	27.337
	TCAV-Bayesian	27	6.00	1.20	0.676	2.779	25.140
	TIG-Bayesian	29	6.44	1.29	0.795	3.556	25.916
	RV-Bayesian	22	4.89	0.98	0.987	4.356	28.764
	RV-E&M	22	4.89	0.98	1.011	5.232	29.134
	TRV-Bayesian	25	5.56	1.11	0.735	2.912	28.365
	TRIG-Bayesian	28	6.22	1.24	0.824	4.480	28.507
KOSPI	ST	38	8.44	1.69	1.213	4.483	39.185
	LT	22	4.88	0.98	1.942	4.302	39.794
	RiskMetrics	30	6.67	1.33	1.181	5.033	33.103
	GARCH-t	30	6.67	1.33	1.225	4.921	32.545
	GARCH-n	28	6.22	1.24	1.311	4.784	30.831
	SAV-Bayesian	28	6.22	1.24	1.252	4.604	31.693
	SAV-E&M	30	6.67	1.33	1.176	4.671	30.051
	AS-Bayesian	28	6.22	1.24	1.243	4.695	31.417
	AS-E&M	28	6.22	1.24	1.204	4.592	33.246
	IG-Bayesian	27	6.00	1.20	1.168	4.329	30.435
	IG-E&M	26	5.78	1.16	1.190	4.325	30.217
	TCAV-Bayesian	28	6.22	1.24	1.197	4.702	31.879
	TIG-Bayesian	26	5.78	1.16	1.217	4.586	32.655
	RV-Bayesian	23	5.11	1.02	1.169	5.321	31.346
	RV-E&M	23	5.11	1.02	1.235	5.478	31.328
	TRV-Bayesian	23	5.11	1.02	1.319	5.446	32.951
	TRIG-Bayesian	24	5.33	1.07	1.358	5.287	33.138

Table 6 (Continued)

α=5%		Violations	VRate	VRate/α	AD		Quantile Criterion function
					Mean	Max	
TAIEX	ST	44	9.78	1.96	0.802	2.223	32.464
	LT	25	5.56	1.11	1.053	2.761	26.866
	RiskMetrics	29	6.44	1.29	0.819	2.289	25.582
	GARCH-t	29	6.44	1.29	0.884	2.274	25.145
	GARCH-n	28	6.22	1.24	0.877	2.192	23.812
	SAV-Bayesian	26	5.78	1.56	0.847	2.199	22.923
	SAV-E&M	28	6.22	1.24	0.833	2.191	22.844
	AS-Bayesian	29	6.44	1.29	0.858	2.944	24.434
	AS-E&M	33	7.33	1.47	0.876	3.424	27.034
	IG-Bayesian	25	5.56	1.11	0.883	2.016	23.242
	IG-E&M	27	6.00	1.20	0.881	2.272	23.114
	TCAV-Bayesian	30	6.67	1.33	0.724	2.967	23.597
	TIG-Bayesian	31	6.89	1.38	0.811	1.966	25.371
	RV-Bayesian	23	5.11	1.02	0.850	2.111	23.308
	RV-E&M	26	5.78	1.16	0.884	2.351	23.291
	TRV-Bayesian	24	5.33	1.07	0.928	2.246	26.026
	TRIG-Bayesian	26	5.78	1.56	0.817	2.538	25.123
S&P500	ST	43	9.56	1.91	1.047	4.577	39.708
	LT	29	6.44	1.29	1.397	6.176	35.553
	RiskMetrics	29	6.44	1.29	1.139	5.352	28.498
	GARCH-t	31	6.89	1.38	1.169	5.423	28.410
	GARCH-n	31	6.89	1.38	1.175	5.418	27.212
	SAV-Bayesian	31	6.89	1.38	1.110	5.541	28.492
	SAV-E&M	30	6.67	1.33	1.231	5.885	34.410
	AS-Bayesian	31	6.89	1.38	1.241	6.060	28.306
	AS-E&M	40	8.89	1.78	1.296	6.374	32.543
	IG-Bayesian	29	6.44	1.29	1.153	5.370	28.084
	IG-E&M	31	6.89	1.38	1.156	5.546	30.176
	TCAV-Bayesian	30	6.67	1.33	1.060	5.137	31.511
	TIG-Bayesian	30	6.67	1.33	1.061	5.034	28.016
	RV-Bayesian	33	7.33	1.47	0.997	5.602	26.847
	RV-E&M	41	9.11	1.82	1.141	5.879	31.357
	TRV-Bayesian	34	7.56	1.51	0.943	6.005	29.033
	TRIG-Bayesian	33	7.33	1.51	0.970	5.786	29.835

Table 7: P-values of the unconditional and conditional coverage tests, and Dynamic Quantile tests

	Nikkei225						HSI					
	1%			5%			1%			5%		
	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ
ST	0.000	0.000	0.000	0.001	0.003	0.000	0.000	0.000	0.000	0.001	0.002	0.000
LT	0.009	0.025	0.000	0.582	0.146	0.000	0.025	0.004	0.000	0.595	0.126	0.000
RiskMetrics	0.135	0.281	0.000	0.020	0.061	0.005	0.816	0.919	0.740	0.914	0.697	0.156
GARCH-n	0.499	0.731	0.000	0.020	0.032	0.004	0.273	0.489	0.331	0.595	0.756	0.194
GARCH-t	0.816	0.919	0.000	0.007	0.021	0.001	0.809	0.938	0.562	0.345	0.604	0.044
SAV-Bayesian	0.135	0.281	0.001	0.177	0.302	0.004	0.816	0.919	0.607	0.748	0.779	0.632
SAV-E&M	0.000	0.001	0.000	0.020	0.032	0.001	0.809	0.938	0.615	0.914	0.697	0.181
AS-Bayesian	0.061	0.142	0.043	0.053	0.090	0.006	0.449	0.739	0.564	0.251	0.414	0.073
AS-E&M	0.000	0.001	0.000	0.033	0.055	0.004	0.183	0.411	0.366	0.914	0.697	0.184
IG-Bayesian	0.273	0.489	0.243	0.053	0.090	0.006	0.816	0.919	0.653	0.595	0.756	0.214
IG-E&M	0.061	0.142	0.043	0.053	0.013	0.002	0.499	0.731	0.464	0.748	0.779	0.224
TCAV-Bayesian	0.025	0.063	0.007	0.251	0.079	0.047	0.499	0.731	0.595	0.345	0.542	0.085
TIG-Bayesian	0.135	0.281	0.132	0.251	0.079	0.037	0.499	0.731	0.283	0.177	0.395	0.026
RV-Bayesian	0.273	0.489	0.239	0.033	0.096	0.006	0.449	0.739	0.753	0.914	0.992	0.180
RV-E&M	0.135	0.281	0.000	0.007	0.021	0.000	0.816	0.919	0.432	0.914	0.697	0.176
TRV-Bayesian	0.061	0.142	0.025	0.122	0.210	0.009	0.816	0.919	0.535	0.595	0.756	0.027
TRIG-Bayesian	0.061	0.142	0.036	0.020	0.061	0.008	0.449	0.739	0.610	0.251	0.144	0.010

	KOSPI						TAIEX					
	1%			5%			1%			5%		
	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ
ST	0.000	0.000	0.000	0.002	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LT	0.061	0.064	0.000	0.914	0.697	0.001	0.009	0.018	0.000	0.595	0.806	0.101
RiskMetrics	0.003	0.009	0.000	0.122	0.297	0.158	0.009	0.025	0.000	0.177	0.284	0.056
GARCH-n	0.009	0.025	0.002	0.251	0.414	0.104	0.009	0.025	0.000	0.251	0.331	0.071
GARCH-t	0.025	0.063	0.007	0.122	0.210	0.094	0.135	0.281	0.000	0.177	0.284	0.061
SAV-Bayesian	0.135	0.281	0.111	0.251	0.414	0.183	0.499	0.731	0.438	0.460	0.694	0.196
SAV-E&M	0.025	0.063	0.007	0.122	0.210	0.104	0.273	0.489	0.275	0.251	0.331	0.070
AS-Bayesian	0.135	0.281	0.125	0.251	0.414	0.002	0.061	0.142	0.026	0.177	0.302	0.018
AS-E&M	0.001	0.003	0.000	0.251	0.414	0.001	0.001	0.003	0.000	0.033	0.055	0.000
IG-Bayesian	0.135	0.281	0.127	0.345	0.542	0.145	0.499	0.731	0.251	0.595	0.756	0.232
IG-E&M	0.025	0.063	0.007	0.460	0.678	0.198	0.499	0.731	0.278	0.345	0.604	0.168
TCAV-Bayesian	0.025	0.063	0.010	0.251	0.414	0.001	0.009	0.025	0.001	0.122	0.210	0.017
TIG-Bayesian	0.025	0.063	0.007	0.460	0.152	0.019	0.061	0.142	0.013	0.081	0.140	0.006
RV-Bayesian	0.135	0.281	0.088	0.914	0.287	0.220	0.809	0.938	0.177	0.914	0.978	0.516
RV-E&M	0.025	0.063	0.007	0.914	0.978	0.441	0.816	0.919	0.193	0.460	0.694	0.514
TRV-Bayesian	0.135	0.281	0.027	0.914	0.978	0.472	0.499	0.731	0.026	0.748	0.779	0.198
TRIG-Bayesian	0.499	0.731	0.300	0.748	0.912	0.287	0.499	0.731	0.022	0.460	0.694	0.369

	S&P500						Portfolio					
	1%			5%			1%			5%		
	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ
ST	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
LT	0.009	0.025	0.000	0.177	0.271	0.000	0.004	0.009	0.000	0.820	0.267	0.000
RiskMetrics	0.001	0.003	0.000	0.177	0.312	0.035	0.000	0.001	0.000	0.187	0.400	0.040
GARCH-n	0.000	0.001	0.000	0.081	0.146	0.028	0.004	0.009	0.000	0.052	0.148	0.015
GARCH-t	0.061	0.142	0.000	0.081	0.146	0.028	0.077	0.176	0.022	0.032	0.096	0.007
SAV-Bayesian	0.025	0.063	0.000	0.081	0.146	0.006	0.077	0.070	0.000	0.187	0.400	0.092
SAV-E&M	0.001	0.003	0.000	0.122	0.218	0.009	0.000	0.001	0.000	0.032	0.059	0.005
AS-Bayesian	0.009	0.025	0.000	0.081	0.146	0.018	0.173	0.346	0.084	0.269	0.502	0.098
AS-E&M	0.000	0.000	0.000	0.001	0.003	0.000	0.000	0.001	0.000	0.083	0.164	0.011
IG-Bayesian	0.061	0.142	0.000	0.177	0.302	0.068	0.173	0.346	0.105	0.373	0.596	0.071
IG-E&M	0.009	0.025	0.000	0.081	0.146	0.012	0.173	0.346	0.079	0.269	0.502	0.044
TCAV-Bayesian	0.003	0.009	0.000	0.122	0.210	0.108	0.173	0.346	0.111	0.501	0.667	0.223
TIG-Bayesian	0.009	0.025	0.000	0.122	0.210	0.052	0.077	0.176	0.049	1.000	0.645	0.232
RV-Bayesian	0.061	0.142	0.000	0.033	0.055	0.024	0.599	0.853	0.642	0.651	0.878	0.295
RV-E&M	0.000	0.000	0.000	0.000	0.001	0.000	0.349	0.586	0.425	0.501	0.318	0.015
TRV-Bayesian	0.025	0.063	0.000	0.020	0.061	0.004	0.599	0.853	0.772	0.651	0.878	0.293
TRIG-Bayesian	0.009	0.025	0.000	0.033	0.055	0.023	1.000	0.960	0.561	0.820	0.302	0.177

Table 8: P-values of the unconditional and conditional coverage tests, and Dynamic Quantile test for each exchange rate series

		1%			5%		
		LR_{uc}	LR_{cc}	DQ	LR_{uc}	LR_{cc}	DQ
Euro vs US	ST	0.000	0.000	0.000	0.083	0.002	0.000
	LT	1.000	0.960	0.024	0.651	0.301	0.004
	RiskMetrics	0.629	0.833	0.735	0.651	0.301	0.033
	GARCH-n	1.000	0.960	0.698	0.651	0.301	0.043
	GARCH-t	0.266	0.536	0.591	0.373	0.314	0.046
	SAV-Bayesian	1.000	0.960	0.575	0.817	0.168	0.011
	SAV-E&M	0.629	0.833	0.674	0.817	0.168	0.011
	AS-Bayesian	1.000	0.960	0.425	1.000	0.220	0.009
	AS-E&M	1.000	0.960	0.432	1.000	0.220	0.011
	IG-Bayesian	0.349	0.586	0.427	0.373	0.596	0.054
	TCAV-Bayesian	1.000	0.960	0.763	0.641	0.385	0.111
	TCAV-E&M	1.000	0.960	0.816	0.342	0.586	0.200
	TIG-Bayesian	0.349	0.586	0.193	0.651	0.704	0.133
	RV-Bayesian	0.173	0.346	0.191	0.651	0.704	0.247
	RV-E&M	0.266	0.536	0.588	0.231	0.425	0.199
	TRV-Bayesian	0.349	0.586	0.430	0.817	0.557	0.112
	TRIG-Bayesian	0.349	0.586	0.408	0.820	0.696	0.103
JP vs US	ST	0.004	0.011	0.000	0.373	0.596	0.000
	LT	0.266	0.536	0.584	1.000	0.348	0.195
	RiskMetrics	0.349	0.586	0.362	0.501	0.667	0.014
	GARCH-n	1.000	0.960	0.724	0.817	0.557	0.020
	GARCH-t	0.599	0.853	0.559	0.817	0.557	0.028
	SAV-Bayesian	0.599	0.853	0.589	0.817	0.557	0.008
	SAV-E&M	0.629	0.833	0.001	0.817	0.557	0.009
	AS-Bayesian	0.599	0.853	0.510	0.641	0.385	0.052
	AS-E&M	1.000	0.960	0.191	0.480	0.369	0.056
	IG-Bayesian	0.599	0.853	0.573	0.817	0.557	0.018
	TCAV-Bayesian	0.599	0.853	0.428	0.817	0.971	0.010
	TCAV-E&M	1.000	0.960	0.009	0.501	0.753	0.010
	TIG-Bayesian	0.599	0.853	0.602	0.817	0.377	0.025
	RV-Bayesian	0.599	0.853	0.575	0.817	0.971	0.125
	RV-E&M	0.629	0.833	0.000	0.641	0.447	0.013
	TRV-Bayesian	0.599	0.853	0.594	0.641	0.882	0.062
	TRIG-Bayesian	0.599	0.853	0.591	1.000	0.645	0.036

Table 9: VaR prediction performance: using 17 model specifications and the 400 forecasts for the exchange returns

α=1%		Violations	Zone	VRate/α	AD		Penalty	Daily Capital	Quantile criterion
					Mean	Max		Charge	function
Euro vs US	ST	19	Red	4.75	0.297	0.803	1.000	5.508	11.074
	LT	4	Green	1.00	0.390	0.879	0.000	5.108	8.358
	RiskMetrics	5	Green	1.25	0.196	0.392	0.000	4.756	7.242
	GARCH-n	4	Green	1.00	0.246	0.453	0.000	4.816	7.314
	GARCH-t	2	Green	0.50	0.374	0.398	0.000	5.054	7.436
	SAV-Bayesian	4	Green	1.00	0.220	0.367	0.000	4.769	7.157
	SAV-E&M	5	Green	1.25	0.199	0.405	0.000	4.766	7.266
	AS-Bayesian	4	Green	1.00	0.166	0.247	0.000	4.827	7.002
	AS-E&M	4	Green	1.00	0.175	0.258	0.000	4.819	7.024
	IG-Bayesian	6	Green	1.50	0.243	0.447	0.000	4.560	7.474
	TCAV-Bayesian	4	Green	1.00	0.319	0.466	0.000	4.828	7.611
	TCAV-E&M	4	Green	1.00	0.167	0.227	0.000	4.916	7.105
	TIG-Bayesian	6	Green	1.50	0.242	0.483	0.000	4.662	7.583
	RV-Bayesian	7	Green	1.75	0.159	0.520	0.000	4.438	7.014
	RV-E&M	2	Green	0.50	0.327	0.381	0.000	5.157	7.452
	TRV-Bayesian	6	Green	1.50	0.177	0.447	0.000	4.517	7.082
	TRIG-Bayesian	6	Green	1.50	0.166	0.443	0.000	4.502	7.000
JP vs US	ST	11	Yellow	2.75	0.442	1.003	0.637	5.067	10.364
	LT	2	Green	0.50	0.478	0.608	0.000	5.399	8.163
	RiskMetrics	6	Green	1.50	0.472	1.132	0.000	4.696	8.882
	GARCH-n	4	Green	1.00	0.497	0.683	0.000	4.780	8.222
	GARCH-t	3	Green	0.75	0.427	0.529	0.000	5.305	8.206
	SAV-Bayesian	3	Green	0.75	0.343	0.495	0.000	5.567	8.311
	SAV-E&M	5	Green	1.25	0.579	1.193	0.000	5.101	9.643
	AS-Bayesian	3	Green	0.75	0.339	0.436	0.000	5.939	8.730
	AS-E&M	4	Green	1.00	0.331	0.437	0.000	5.852	8.933
	IG-Bayesian	3	Green	0.75	0.371	0.448	0.000	5.561	8.369
	TCAV-Bayesian	3	Green	0.75	0.470	0.596	0.000	5.860	9.047
	TCAV-E&M	4	Green	1.00	0.796	1.160	0.000	5.649	10.568
	TIG-Bayesian	3	Green	0.75	0.178	0.325	0.000	5.812	8.098
	RV-Bayesian	3	Green	0.75	0.187	0.429	0.000	6.021	8.448
	RV-E&M	5	Green	1.25	0.606	1.216	0.000	5.240	9.985
	TRV-Bayesian	3	Green	0.75	0.248	0.436	0.000	5.809	8.336
	TRIG-Bayesian	3	Green	0.75	0.251	0.340	0.000	5.749	8.263

Table 10: VaR prediction performance: using 17 model specifications and the 400 forecasts for the exchange returns

α=5%		Violations	VRate/α	AD		Quantile criterion function
				Mean	Max	
Euro vs US	ST	28	1.400	0.308	0.917	30.873
	LT	22	1.100	0.307	1.214	29.164
	RiskMetrics	22	1.100	0.278	0.964	28.226
	GARCH-n	22	1.100	0.269	1.016	28.271
	GARCH-t	24	1.200	0.272	1.035	28.455
	SAV-Bayesian	19	0.950	0.284	0.771	28.154
	SAV-E&M	19	0.950	0.284	0.783	28.118
	AS-Bayesian	20	1.000	0.272	0.759	28.254
	AS-E&M	20	1.000	0.273	0.775	28.241
	IG-Bayesian	24	1.200	0.265	0.888	28.302
	TCAV-Bayesian	18	0.90	0.309	0.822	28.483
	TCAV-E&M	16	0.800	0.317	0.791	28.363
	TIG-Bayesian	22	1.100	0.267	0.818	27.910
	RV-Bayesian	22	1.10	0.264	0.888	28.017
	RV-E&M	15	0.750	0.266	0.772	28.596
	TRV-Bayesian	19	0.950	0.313	0.864	28.396
	TRIG-Bayesian	21	1.050	0.284	0.938	28.454
JP vs US	ST	24	1.200	0.336	1.063	30.350
	LT	20	1.000	0.287	1.219	27.933
	RiskMetrics	23	1.150	0.338	1.473	28.957
	GARCH-n	19	0.950	0.343	1.114	28.362
	GARCH-t	19	0.950	0.361	1.155	28.187
	SAV-Bayesian	19	0.950	0.340	1.078	28.413
	SAV-E&M	19	0.950	0.335	1.088	28.378
	AS-Bayesian	18	0.900	0.329	1.200	28.855
	AS-E&M	17	0.850	0.345	1.235	28.818
	IG-Bayesian	19	0.950	0.332	1.123	28.068
	TCAV-Bayesian	19	0.950	0.330	1.217	29.462
	TCAV-E&M	23	1.150	0.288	1.145	28.908
	TIG-Bayesian	19	0.95	0.295	1.169	28.403
	RV-Bayesian	19	0.95	0.332	1.138	28.412
	RV-E&M	18	0.900	0.319	1.021	28.447
	TRV-Bayesian	18	0.900	0.341	1.307	29.142
	TRIG-Bayesian	20	1.000	0.337	1.255	29.324

Table 11: Counts of model rejections for the three quantile tests across each market, the portfolio and two exchange rate series

Specification	1%				5%			
	LR_{uc}	LR_{cc}	DQ	total	LR_{uc}	LR_{cc}	DQ	total
ST	8	8	8	8	6	7	8	8
LT	5	5	7	7	0	0	6	6
RiskMetrics	4	4	5	5	1	0	5	5
GARCH-n	4	4	5	5	1	1	5	5
GARCH-t	1	0	5	5	2	1	6	6
SAV-Bayesian	1	0	3	3	0	0	4	4
SAV-E&M	4	3	5	5	2	1	5	5
AS-Bayesian	1	1	3	3	0	0	5	5
AS-E&M	5	5	5	5	3	1	6	6
IG-Bayesian	0	0	1	1	0	0	2	2
IG-E&M	2	1	4	4	0	1	4	4
TCAV-Bayesian	4	2	4	4	0	0	4	4
TIG-Bayesian	2	1	4	4	0	0	5	5
RV-Bayesian	0	0	1	1	2	0	2	2
RV-E&M	2	1	4	4	2	2	4	4
TRV-Bayesian	1	0	4	4	1	0	3	3
TRIG-Bayesian	1	1	3	3	2	0	4	4

Table 12: Summary statistics for the 1% VaR forecast criteria in Tables 4 and 5

	VRate/α			AD mean		AD max		MRC		Quantile criterion	
Model	Mean	Median	RMSD	Mean	Median	Mean	Median	Mean	Median	Mean	Median
ST	**4.656**	**4.710**	**3.286**	0.773	0.878	2.951	2.834	12.788	14.907	**29.599**	**34.192**
LT	1.974	2.330	1.160	**1.146**	**1.136**	**2.989**	**3.078**	**14.295**	**16.442**	28.350	31.432
RiskMetrics	2.111	2.110	1.256	0.794	0.660	2.302	2.110	13.062	14.972	23.347	27.040
GARCH-n	1.954	2.000	1.170	0.799	0.653	2.152	2.071	12.688	14.421	22.712	26.778
GARCH-t	1.406	1.445	0.754	0.805	0.660	1.865	1.638	12.904	14.423	22.093	25.512
SAV-Bayesian	1.496	1.555	0.677	0.628	0.550	1.661	1.196	12.792	14.620	21.858	24.872
SAV-E&M	2.053	1.890	1.308	0.712	0.745	2.157	1.875	13.048	14.654	23.128	25.602
AS-Bayesian	1.549	1.765	0.810	0.735	0.675	1.978	1.835	12.554	14.488	22.128	24.531
AS-E&M	2.239	2.890	1.612	0.763	0.858	2.290	2.299	12.931	14.669	24.088	26.410
IG-Bayesian	1.473	1.530	0.574	0.607	0.508	1.714	1.509	12.436	14.395	21.283	24.996
IG-E&M	1.603	1.540	0.793	0.622	0.584	1.877	1.717	12.861	14.799	21.631	24.987
TCAV-Bayesian	1.829	1.985	0.973	0.758	0.852	2.046	1.516	12.924	15.412	22.871	24.369
TIG-Bayesian	1.753	1.890	0.848	0.671	0.669	1.780	1.609	12.734	14.802	22.128	25.644
RV-Bayesian	1.269	1.225	0.601	0.739	0.794	1.954	1.578	12.469	14.277	21.321	25.078
RV-E&M	1.656	1.375	1.148	0.782	0.740	2.324	2.162	12.131	13.852	22.118	25.298
TRV-Bayesian	1.430	1.415	0.664	0.738	0.757	2.044	1.885	12.702	14.570	22.028	26.653
TRIG-Bayesian	1.378	1.330	0.689	0.863	0.967	2.170	2.021	12.741	14.676	22.435	26.815

Table 13: Summary statistics for the 5% VaR forecast criteria in Tables 4 and 5

	VRate/α			AD mean		AD max		Quantile criterion	
Model	Mean	Median	RMSD	Mean	Median	Mean	Median	Mean	Median
ST	**1.696**	**1.755**	**0.791**	0.802	0.868	3.331	3.529	83.535	101.520
LT	1.066	1.075	0.138	**1.298**	**1.467**	**4.976**	**5.209**	**89.503**	**104.950**
RiskMetrics	1.244	1.29	0.307	0.843	0.900	3.726	3.996	80.308	96.915
GARCH-n	1.248	1.24	0.327	0.873	0.974	3.590	3.747	80.341	97.492
GARCH-t	1.306	1.31	0.384	0.868	0.968	3.694	3.907	80.487	97.563
SAV-Bayesian	1.168	1.2	0.241	0.827	0.913	3.377	3.627	80.803	97.653
SAV-E&M	1.224	1.285	0.336	0.887	0.997	3.611	3.827	80.669	97.541
AS-Bayesian	1.215	1.245	0.291	0.814	0.907	3.140	2.865	79.225	97.340
AS-E&M	1.274	1.32	0.429	0.859	0.923	3.486	3.786	80.108	97.864
IG-Bayesian	1.185	1.2	0.241	0.816	0.889	3.316	3.318	80.335	97.826
IG-E&M	1.166	1.18	0.258	0.864	0.987	3.483	3.393	80.024	97.234
TCAV-Bayesian	1.180	1.22	0.256	0.801	0.868	2.976	2.874	79.121	97.403
TIG-Bayesian	1.181	1.2	0.249	0.796	0.862	3.046	3.181	79.260	97.353
RV-Bayesian	1.139	1.06	0.258	0.767	0.896	3.408	3.482	77.814	94.308
RV-E&M	1.173	1.085	0.406	0.846	0.945	3.635	3.951	78.904	94.397
TRV-Bayesian	1.124	1.085	0.242	0.769	0.810	3.180	2.795	78.927	95.640
TRIG-Bayesian	1.194	1.115	0.286	0.760	0.812	3.555	3.584	78.876	95.402

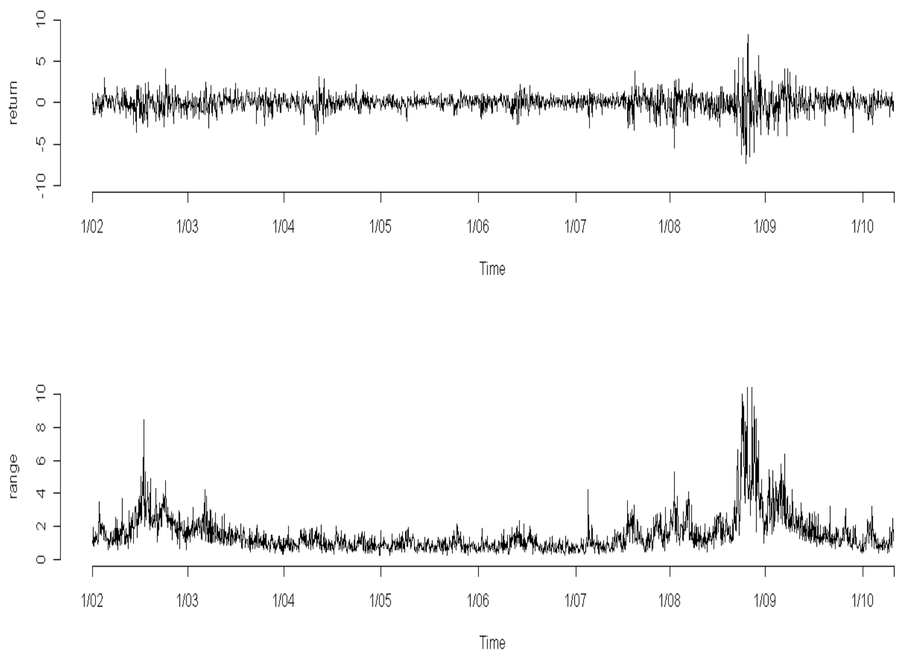

Figure 1: Time series plots for (a) portfolio's returns, (b) S&P500 range.

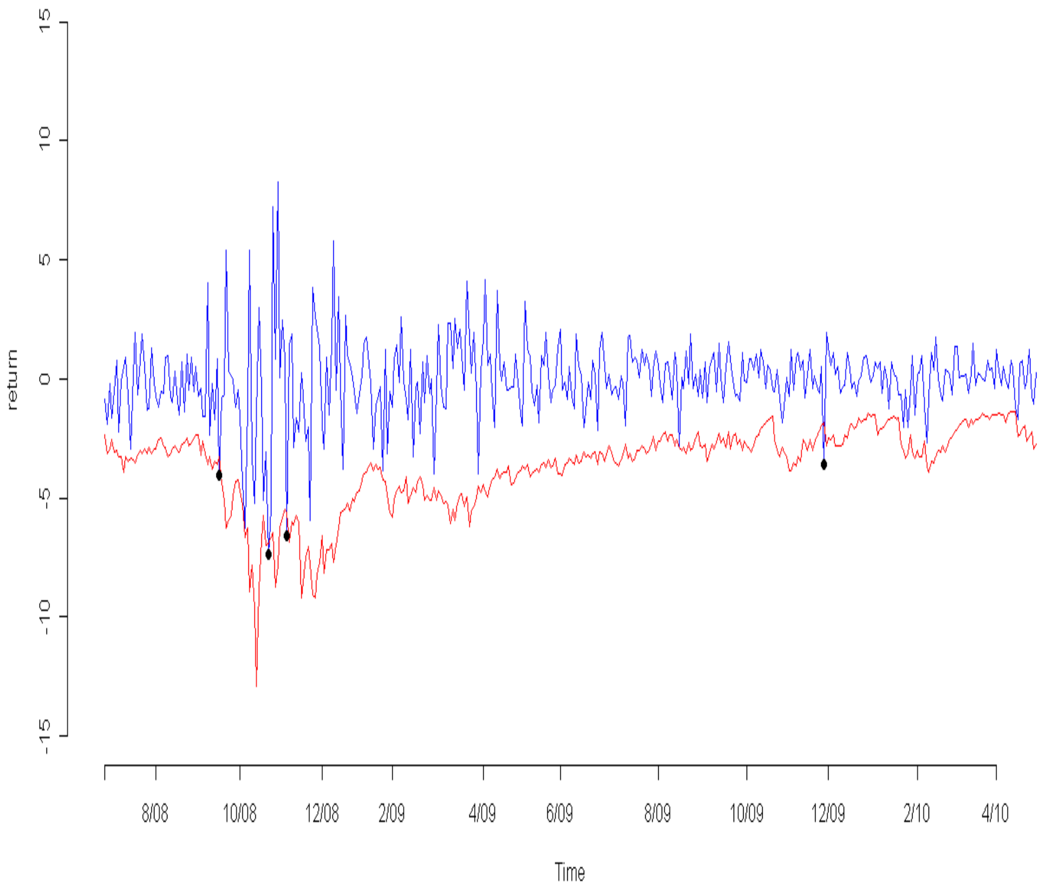

Figure 2: VaR forecasts of the TRIG model at the 1% level.