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The Effect of Market Structure on Pest Resistance Buildup

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Abstract:

In this paper we present a continuous time optimal control analysis of resistance buildup in agricultural chemicals when the supplier is a monopolist. We compare the monopolistic, competitive and socially optimal solutions after establishing the stability of each equilibrium. The central finding is that with a monopolistic supplier, resistance buildup is lower than socially optimal. This directly opposes the standard result in the resistance literature. This implies that policies to manage resistance should vary with the age of the chemical, with no intervention during the lifetime of a patent and an optimal tax once the patent expires.

Introduction

According to the Food and Agriculture Organization, the number of pest species with resistance to pesticides has increased from almost none 50 years ago to over 700 today (FAO, 2003). As the number of resistant species increases, so do losses incurred by farmers. Research by the International Resistance Action Committee (IRAC) finds that resistance raises the total U.S. insecticide bill by $40 million, while in Australia, sheep parasites resistant to drugs threaten to raise farmers' expenses by $800 million per year (ABC, 2002). In a 2001 *Science* article Stephen Palumbi estimated that the direct costs of resistance agriculture include $1.2 billion for respraying fields and $2 billion to $7 billion annually in crop loss costs (Schmid, 2001).

Given its importance to farmers, it is not surprising that resistance has inspired a large economic literature. Although much has been written regarding the role of farmers and regulators in managing resistance, the literature still harbors some conceptual holes, one of which we hope to fill with this paper. Regev and Hueth's (1974) seminal work was followed by many studies (see surveys by Carlson and Wetzstein (1993); and Regev, Shalit, and Gutierrez(1983)) that either theoretically or empirically describe different facets of resistance problems. The advent of transgenic crops has revitalized interest in resistance issues, with a particular focus on the introduction of Bt crops. Among these papers are Secchi and Babcock (2003), Morel et al (2003) and Wesseler (2003). With the exception of Wesseler, which is a static model, current research views resistance in the context of renewable or nonrenewable resources. The main result of this vein of research is that common pool problems and/or myopic behavior lead to the overuse of pesticides. This useful insight has been followed, logically, by a call for collective action or government intervention to reduce pesticide application and buildup of resistance. However, the existing literature's emphasis on the resource framework has ignored other features essential to understanding the dynamics of resistance in a more complex reality.

In this paper we turn the spotlight on one of these features: market structure. In particular, we look at the problem of pesticides produced by a monopolist. Given that chemical producers receive patents that effectively give them monopoly power for at least the first seventeen years of a product's life, this is a reasonably accurate depiction of the pesticide market. To our knowledge, there are two previous papers that introduce this characteristic, Alix and Zilberman (2003) and Noonan (2003). In contrast to the discrete analysis used by the former authors and the static framework introduced by the latter, we use a continuous time optimal control approach. We compare the result of the monopolist's problem with that of the social planner's and come up with a result that contradicts the previously cited literature. In the world of a monopolistic producer, resistance may be lower than is socially optimal.

Modeling Resistance

In the sections following this one, we model two distinct decision-making processes, that of a welfare-maximizing social planner and that of a profit-maximizing monopolist. We also mention the relevant results from the competitive solution. We begin with a discussion of the dynamics of pest resistance. In the interest of not obscuring our main result, we use a pared-down model of resistance relative to that of Hueth, Shalit and Gutierrez (1983).

We formalize our specification as follows. The change in the resistance stock over time is defined by $\dot{R} = g(X_t, R_t)$, where X_t is the amount of pesticide applied at time t and R_t is the percentage of the pest population resistant to chemical applications. The function $g(\cdot)$ describes the relationship of these two inputs to growth. Note that $0 \le R_t \le 1$. The above assumptions are integrated into the resistance buildup function as follows:

1. Pesticide applications increase resistance: $\frac{\partial g(X_t, R_t)}{\partial X_t} > 0$. From here onwards, we will denote partial derivatives with subscripts, so assumption (1) can be written: $g(X_t, R_t)_X > 0$. This assumption is a "worst case scenario", where there are no natural mitigating effects on resistance growth.
2. As the resistant population increases, the growth rate of resistance first increases and then decreases: $g(X_t, R_t)_R \underset{<}{\overset{>}{-}} 0 \; for \; R_t \underset{>}{\overset{<}{-}} \overline{R}$ and $| g(X_t, R_t)_R | < 1$. This gives us a typical, s-shaped population growth model.
3. The marginal effect of spraying on resistance is always increasing: $g(X_t, R_t)_{XR} > 0$.
4. There are decreasing returns to growth from chemical applications: $g(X_t, R_t)_{XX} < 0$ and
5. From resistance $g(X_t, R_t)_{RR} < 0$.

These relationships constitute the constraint set faced by each agent.

Society's Problem

In our society, we assume that production is competitive with constant returns to scale technology, and that there is a social planner who maximizes the farmers' surplus from pesticide consumption subject to the growth of resistance. We follow the assumption of Hueth and Regev that farmers do not take resistance into account when making their production decision either because they do not know about it, or they know that their individual effect on it will be approximately zero. This is a reasonable assumption in cases where farmers are small relative to the resistance problem, such as in developing countries in general and in the industrial world when resistance problems are not yet well understood or the geographical extent of the resistance problem is regional rather than local.

To formalize, let $W(P_t, X_t, R_t)$ be farmers' inverse demand for pesticides. P_t is the price of the output produced with the pesticide, so it follows that W, the price a farmer will pay for pesticides, will increase with P. The function behaves as follows:

1. As the amount of pesticide used increases, its VMP decreases: $W_X < 0$.
2. As farmers see increasing resistance, they will initially demand more pesticide, up to a particular level of resistance where they feel that the product is no longer effective, and then they will demand less: $W_R < 0$.
3. The marginal impact of resistance on the change in price is negative: $W_{XR} < 0$.
4. There are decreasing marginal effects of pesticide use and resistance growth on price: $W_{RR} < 0$, $W_{XX} < 0$.

To avoid messy notation, we will express farmer surplus as a benefit function that decreases in resistance and increases in pesticide use. Benefits are simply the area under the demand curve, which is equal to

(2) $$B(P_t, X_t, R_t) = \int_0^{X_t} W(P_t, v_t, R_t)dv$$

where $B_R < 0$, $B_X > 0$ *and* $B_{RR} > 0$. Note that $B_X = W$. There is also a cost of producing pesticides, which we assume increases at an increasing rate, therefore $C_X > 0, C_{XX} > 0$. We can now write society's optimization problem as one where the planner wishes to maximize the net present value of the stream of benefits of pesticide use subject to the biological constraint of resistance growth:

$$\max \int_0^{\infty} \left[e^{-rt} B(P_t, X_t, R_t) - C(X_t) \right] dt$$

$$\text{subject to} \quad \dot{R} = g(X_t, R_t)$$

The current value Hamiltonian can be expressed as follows:

(3) $$H = B(P_t, X_t, R_t) - C(X_t) + \lambda_t \left[g(X_t, R_t) \right]$$

This is a standard continuous optimal control problem. From here onwards we suppress the time subscripts. The necessary conditions for this problem are:

(4) $$H_X = W - C_X + \lambda g_X = 0$$

(5) $$\dot{\lambda} - r\lambda = -H_R = -B_R - \lambda g_R$$

(6) $$\dot{R} = g(X, R)$$

(7) $$\lim_{t \to \infty} e^{-rt}\lambda = 0$$

First, note that (4) yields a negative value for the costate, $\frac{C_X - W}{g_X}$, which is consistent with an interpretation of the multiplier as the cost of resistance. This result is comparable to Forster's (1977) model of pollution control and occurs because the benefits of pesticide use to farmers are greater than the marginal cost at the social optimum. As might be expected, rewriting this expression equates the marginal benefits of pesticide use to society to the marginal cost of production plus the marginal cost of resistance: $W = C_X - \lambda g_X$.

We now examine the properties of the equilibrium, focusing first on the equation of motion of society's shadow price of resistance:

The dynamics of the shadow price follows:

(8) $$\frac{\dot{\lambda}}{\lambda} = -\frac{B_R}{\lambda} + r - g_R$$

This result follows from the first order conditions above. The first part of this term represents the benefit from waiting one period to exploit the stock of resistance, an effect is always negative. The interest rate, *r*, is the benefit of exploiting resistance in the current period. This tradeoff between current and future benefits is quite standard (see Carlson, Zilberman and Miranowski, 1993). The final term is slightly trickier, as it changes with the level of the resistant population. When the population is small but growing quickly, there is additional benefit to withholding pesticide use until the next period. In this case, if $r > g_R + \frac{B_R}{\lambda}$, then the movement in the shadow price over time is positive. When this inequality is reversed, the movement is negative, an unusual result that is most likely to occur if the shadow price is very small and if consumers are very sensitive to increases in resistance – i.e., if the elasticity of demand with respect to resistance is very high. When the resistance level is high, increases in it decrease the growth rate of the population, and therefore there is a lower benefit to waiting another period to exploit the stock. It follows that in this case, the movement of the shadow price is positive when $r - g_R > \frac{B_R}{\lambda}$. The final piece of the solution is given by the dynamics of pesticide use.

The dynamics of pesticide use follows:

(9) $$\dot{X} = \frac{-g_X\dot{\lambda} - \left[W_R + \lambda g_{XR}\right]\dot{R}}{W_X - C_{XX} + \lambda g_{XX} + \lambda_X g_X}$$

This expression is derived from totally differentiating the first order condition (4), where we assume that prices are constant over time. It tells us that over time, change in pesticide use depends upon the buildup of resistance, both through its marginal effects and its direct effect in the state variable. The denominator of this equation is the second order condition for utility maximization. Assuming that resistance is increasing, when $g_X\dot{\lambda} < -[W_R + \lambda g_{XR}]\dot{R}$ is true, then pesticide use decreases over time. (9) is unambiguously negative in the unusual case where the shadow price decreases over time.

A note on stability

The equilibria for high and low levels of resistance are distinct, with the former being a stable saddlepath and the latter an unstable focus. The interested reader can refer to the proof of this proposition in the appendix. The general nature of our specification does not enable us to solve explicitly for the equilibrium levels of pesticide use and resistance, which we will do with specific functional forms at the end of the paper.

The Myopic Solution

In the case where the market structure is competitive and producers myopic, the producers solve their individual maximization problems without taking account of the resource constraint.

$$\max_{X_t} \int_{t=0}^{T} \left(e^{-rt} W(P, X, R) X - C(X) \right) dt \tag{10}$$

Since individual behavior does not affect price, we write the current value Hamiltonian as:

$$H = WX - C(X) \tag{11}$$

The first order condition is:

$$W = C_X \tag{12}$$

This price is unambiguously lower than the socially optimal price we derived from (4). In the myopic solution is the shadow value is equal to zero, so the equilibrium for high resistance is where $\dot{R} = 0$ crosses the x-axis. Since the isocline is positively sloped, this equilibrium is at a greater level of resistance than the socially optimal one.

This result is the standard one in most of the literature, and implies that there will be overuse of pesticides. It also implies that the appropriate policy response is to implement a tax of size λg_X in order to arrive at the socially optimal level of pesticide use. We now modify the market structure to reflect what is common in the chemical industry, at least when products are young: the producer is a monopolist. This situation will give us an entirely different policy prescription for the management of pesticide resistance.

***The Monopolist's Problem*:**

In the monopolist's maximization problem we have the same resistance constraint as before. The inverse demand curve for pesticides is the same as above, as are the assumptions on the farm sector. When pesticides are new, the monopolist derives his power from a patent. Chemicals over twenty years old are assumed to be produced by competitive industries. We define the patent length as T, and examine the problem where the monopolist maximizes expected profits subject to (2).

The same conditions hold as for the previous problem, with the monopolist maximizing profits as opposed to net benefits.

The monopolist's production choice problem can be described by

(13) $$\max_{X_t} \int_{t=0}^{T} \left(e^{-rt} W(P, X, R)X - C(X) \right) dt$$

subject to (1).

We assume that the problem begins with some resistance level $R(0) = R_0 \geq 0$, and that there are no constraints on the terminal level of resistance. It follows that the appropriate condition on the multiplier is $\mu(T) = 0$. This makes sense, since it means that at the end of the time period, the monopolist no longer cares about resistance. With this information in hand, we can proceed to the current value Hamiltonian:

(14) $$H = W(P, X, R)X - C(X) + \mu g(X, R)$$

where μ is the shadow price of resistance at time t. The optimality conditions are:

(15) $$H_X = W + W_X X - C_X + \mu g_X = 0$$

(16) $$\dot{\mu} - r\mu = -H_R = -W_R X - \mu g_R$$

(17) $$\dot{R} = g(X, R)$$

Condition (15) then tells the monopolist that he must produce where the marginal revenue of production is just equal to the marginal cost plus resistance cost.

$$W = C_X - \mu g_X - W_X X$$

Rewriting the equation in the form of elasticities gives us the monopolist's markup rule:

$$W = \frac{C_X - \mu g_X}{1 + \frac{1}{\eta_D}}$$

The dynamics of the shadow price for the monopolist are described by:

(18) $$\frac{\dot{\mu}}{\mu} = -\frac{W_R X}{\mu} + r - g_R$$

This follows from the first order conditions. The main difference between this case and the benchmark above is a reduction in cost represented by the change in demand with respect to resistance, $W_R X$, in lieu of the change in marginal benefits with respect to resistance. Similarly, with small, fast growing resistant populations, the movement of the shadow price is positive when the incentive to produce pesticides today, r , exceeds the incentives to wait, $\frac{W_R X}{\mu} + g_R$. When resistant populations are large, this expression is positive when $r > \frac{W_R X}{\mu} + g_R$ and negative otherwise. The monopolist has greater incentive to withhold pesticide from the market than in the low resistance case, since his putting it on the market today may decrease demand too much in the next period.

The dynamics of pesticide use with a monopolistic producer follows:

$$\text{(19)} \qquad \dot{X} = \frac{-g_X \dot{\mu} - [W_R + W_{XR} X + \mu g_{XR}]\dot{R}}{[2W_X + XW_{XX} - C_{XX} + \mu g_{XX} + \mu_X g_X]}$$

This follows from manipulation of the first order conditions. As with the social planner's problem, here we consider two cases. The denominator is negative given the assumptions made above on demand. The main difference between the dynamics here and in the benchmark case is that the change in social welfare is replaced by marginal revenue in the monopolist's case. $\dot{X} < 0$ when $g_X \dot{\mu} < W_R \dot{R} + W_{XR} X + \mu g_{XR}$.

Stability

A full analysis of stability of the monopolistic solution is in the appendix, and is quite similar to that of the social planner.

Comparisons

Although the point has already been made indirectly, this section describes in detail the conditions under which the monopolist will produce less pesticide than socially optimal. In order to compare the equilibrium situations, we must maintain the assumption of the monopolist's infinite patent. At the starting point, we know that the initial level of pesticide production will be lowest in the monopoly case, followed by the socially optimal equilibrium with the highest production in the myopic solution. If we first consider the social optimum, the price paid for pesticides at any given moment will be:

(20) $$W^* = C_X - \frac{B_R}{g_R - r} g_X$$

The monopolist will charge some price higher than his cost of production plus the resistance cost:

(21) $$W^M = C_X - XW_X - \frac{W_R X}{g_R - r} g_X$$

Society sets its price equal to marginal cost plus an expression that reflects the cost of resistance to consumer surplus, while the monopolist sets his price equal to marginal revenue plus a resistance cost that includes the marginal effect of resistance on demand. The socially optimal price is unambiguously more than marginal cost. The monopolist's price will be lower than socially optimal only when this marginal effect of resistance on demand is larger than the marginal effect of resistance on consumer surplus (the benefit function). This leads us to the following conclusion:

For high levels of resistance, the competitive solution results in too much pesticide use relative to the social optimum and the monopoly solution may or may not.

This result is essentially a dynamic version of the static result made by Lusky and Lusky (1975). Given that we are rarely in a situation where we have the choice of the "socially optimal solution", the real question is: which outcome is closer: the competitive or the monopoly solution? As is usually the case, we can only produce a set of conditions which depend upon the shape of the demand curve. In the myopic solution, the market under-prices pesticides by $\frac{g_X B_R}{g_R - r}$, while monopolist overprices pesticides by $-XW_X + \frac{g_X XW_R}{g_R - r} - \frac{g_X B_R}{g_R - r}$. If this is the case, then the monopolistic price is closer to the socially optimal price when: $-\frac{g_R - r}{g_X} W_X + W_R < \frac{2}{X} B_R$. The right hand side of this equation is always negative. At high levels of resistance, however, it is possible that the monopolist brings us closer to the social optimum, as both of the terms on the left-hand side will be negative. The intuition is that the negative externality caused by overuse of pesticides is larger when resistance levels are high. Therefore, it makes sense that the competitive price be farther away from the optimum.

Examples

Given that the conditions above are less than intuitive, we present a simple example using common demand forms in order to clarify the comparison between the monopolistic and the optimal long run equilibrium. We focus on the high resistance level equilibrium where we are guaranteed stability. The logistic growth function is one of the standard population growth specifications and satisfies the assumptions of our model (Haefner, 1996). Growth of the resistant population is given by:

$$g(X,R) = \phi RN\left(1-\frac{RN}{KX}\right)$$

As above, X is the amount of pesticide used and R the proportion of the total population which is resistant. The additional parameters ϕ, K, and N are the growth rate, carrying capacity, and actual population, respectively. It is easy to see that this choice satisfies our initial assumptions regarding pest population growth at high levels of resistance. In both of our examples we assume a cost that is quadratic in X.

Cobb-Douglas

The easiest form to solve involves assuming that farmers produce using a simple Cobb-Douglas technology. If $y = \left(\frac{X}{R}\right)^{\beta}$, the demand for pesticides is $W(X,R) = \frac{\beta X^{\beta-1}}{R^{B}}$, and the cost function is quadratic. This satisfies our assumptions for high levels of pest resistance. We first consider the optimal solution for this problem. Just as above, the problem is to maximize discounted net benefits subject to the growth constraint. Some messy algebra (available in the appendix) leads us to the solution for the socially optimal level of pesticide use:

$$(22) \qquad X^* = \sqrt{\frac{r\beta N^{\beta}}{K^{\beta}(2r + 2\phi N)}}$$

The optimal choice for the monopolist is:

$$(23) \qquad X^{M} = \sqrt{\frac{r\beta^{2} N^{B}}{K^{\beta}(2r + 2\phi N)}}$$

The competitive solution is:

$$(24) \qquad X^{C} = \left(\frac{2R^{\beta}}{\beta}\right)^{\frac{1}{\beta - 2}}$$

Since β is between zero and one, we see that the monopolist always produces less pesticide than is socially optimal and the competitive level of production is always greater than the other two. Since resistance increases in pesticide use, it follows that there is less resistance with a monopolistic producer and Cobb-Douglas farmers than in the socially optimal solution. What is the intuition behind this expression? In both cases, the equilibrium level of production is higher when the interest rate is higher, and it is also higher when the number of pests is larger. The only difference between the two expressions is β, which is inversely related to the elasticity of demand, $\frac{1}{\beta - 1}$. As β gets smaller, the difference between the socially optimal level of pesticide use and the monopolistic level increases. This is the case where demand is becoming relatively more inelastic, which means that the monopolist has more control over demand.

Conclusions

We have examined a dynamic model of pest resistance that takes into account the market structure in which pesticides are produced. In particular, we find that even in a situation where farmers do not take account of resistance, there may be underproduction of pesticides. The conclusions from this exercise are both methodological and policy-oriented. First, this finding suggests that we cannot continue to analyze pesticide and resource management issues in an industrial organizations vacuum. Given that chemical and biotech products produced by monopolies may later evolve into competitive management, we should keep these considerations in the forefront of our analysis.

On the policy side, we find that resistance by itself is not a reason to argue for pesticide restrictions, but that these policies should depend on both the structure and the timing of the market. That is to say, in markets where companies are able to renew and enforce their patent rights, pesticides will be undersupplied. There may be under-use and slow buildup of the resistance in the early stages of the product life when it is controlled by a monopolist. However, when the patent expires and the supply of the chemical becomes more competitive, then there may be tendency to overuse the chemicals. In principle, intervention to reduce the application may be more appropriate as products mature.

Moreover, in very developed markets, where farmers may well be dynamic optimizers, given their very large plots and easy access to information, it may even be true that the equilibrium is even "farther to the left" of the one described in the monopolistic situation. This is due to the fact that if both farmers and chemical companies take resistance into account, each without realizing that the other is doing so, chemical use may even fall below the monopolistic equilibrium. It remains true, however, that in countries where patents are not effectively enforced, chemicals with expired patents are commonly applied, and farmers have small plots of land, resistance is likely to move towards the myopic outcome.

Throughout our analysis, we maintain the assumption that demand for pesticides varies with the level of resistance growth. Our findings, however, are robust to changing this specification to a more traditional demand function. We additionally assume linear resistance growth in our general model, and though we find that a more complicated form gives us the same result in the examples section, concede that numerical analysis could be usefully applied here. Finally, we recognize that our model ignores several factors that could prove important in describing the environment in which pesticide policy decisions are made. One such factor that our model excludes is the impact of subsidization of agricultural commodities. This clearly has a positive effect on resistance buildup and would make an interesting extension to the analysis presented here. In addition, the extension of this model to an oligopoly setting could provide interesting insights. Further realism might be added to resistance research by considering a model that explicitly takes into account the effect of substitutes for the pesticide in question. Finally, a significant extension would be to consider the effect of market structure on the environmental externalities caused by pesticide use.

Appendix

Proof of the stability of socially optimal equilibrium:

To derive the properties of the isocline of $\dot{R} = 0$, we partially differentiate (6) with respect to R and λ and find $\lambda_R = -\frac{g_R}{g_X X_\lambda}$, where $X_\lambda = \frac{-g_X}{W - C_{XX} + \lambda_X g_X + \lambda g_{XX}} > 0$.

Given the assumptions of the model, $\lambda_R < 0$ for low levels of resistance ($\overline{R}$) and >0 for high levels of resistance. The stationary locus for the shadow price comes from (5). Specifically,

$$\lambda_R \big|_{\dot{\lambda}=0} = -\frac{-B_{RR} - \lambda g_{RR}}{r - g_R}$$

This expression is negative for low levels of resistance when $r > g_R$ and $-B_{RR} > \lambda g_{RR}$. It is also negative for high levels when the second condition holds. The coefficient matrix of the high resistance equilibrium is : $\begin{bmatrix} g_R & g_X X_\lambda \\ -B_{RR} - \lambda g_{RR} & r - g_R \end{bmatrix}$, with the corresponding signs: $\begin{bmatrix} - & + \\ + & + \end{bmatrix}$, so the determinant is negative, implying that the equilibrium is a saddlepoint. Note also that at the low level equilibrium, when the marginal growth of the resistance stock with respect to resistance switches sign, the trace of the matrix is just r and the sign of the determinant is positive if $r > g_R$. This implies that the focus is unstable (see Lim and Ferguson, 1998).

Proof of stability of monopolistic equilibrium

We follow a similar logic to derive the slopes of the isoclines for the monopolist's problem. To find the slope of $\dot{R} = 0$, we partially differentiate (17) with respect to R and μ. This results in

$\mu_R = -\frac{\dot{R}_R}{\dot{R}_\mu} = -\frac{g_R}{g_X X_\mu}$, where $X_\mu = \frac{-g_X}{2W_X - C_{XX} - W_{XX}X + \mu g_{XX} + \mu_X g_X} > 0$, which is positive.

Therefore we find that $\mu_R < 0$ for low levels of resistance and >0 for high levels of resistance. To find the stationary locus for $\dot{\mu} = 0$ we use equation (16). This gives us the following expression:

$$\mu_R\big|_{\dot{\mu}=0} = -\frac{-W_{RR} - \mu g_{RR}}{r - g_R}$$

The sign of this expression is negative for low levels of resistance when $r > g_R$ and $-W_{RR} > \mu g_{RR}$, and at high levels it is negative. The stability of the low level equilibrium is ambiguous as the trace is positive and the determinant of indeterminant sign. We focus instead on the high resistance equilibrium, whose coefficient matrix is: $\begin{bmatrix} g_R & g_X X_\mu \\ -W_{RR}X - \mu g_{RR} & r - g_R \end{bmatrix}$ which is negative as in the case above.

Derivation of the socially optimal level of pesticide use for Cobb-Douglas technology

The current value Hamiltonian for the social planner's problem is:

$$H = \left(\frac{X}{R}\right)^\beta - X^2 + \lambda\left(\phi RN\left(1 - \frac{RN}{KX}\right)\right)$$

The Pontryagin first order conditions are:

$$(28)\quad \frac{\beta X^{\beta-1}}{R^\beta} - 2X + \lambda\left(\frac{\phi R^2 N^2}{KX^2}\right) = 0$$

$$(29)\quad r\lambda + \frac{\beta X^\beta}{R^{\beta+1}} - \lambda\left(\phi N - \frac{2\phi RN^2}{KX}\right) = 0$$

$$(30)\quad \phi RN\left(1 - \frac{RN}{KX}\right) = 0$$

Here we simply have three equations and three unknowns. (30) gives us: $R = \frac{KX}{N}$. Substituting this into (28) we find: $\lambda = \frac{1}{\phi K}\left[2X - \frac{\beta N^{\beta}}{XK^{\beta}}\right]$. These two conditions, combined with (29), give us the socially optimal level of pesticide use: $X^* = \sqrt{\frac{r\beta N^{\beta}}{K^{\beta}(2r + 2\phi N)}}$.

The only difference between this solution and the monopolistic one is that the monopolist solves:

$$H = \frac{\beta X^{\beta}}{R^{\beta}} - X^2 + \lambda\left(\phi RN\left(1 - \frac{RN}{KX}\right)\right).$$

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