

Shock Absorbing Prices: A look at Cattle and Feed

Carlos Arnade¹

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¹ Carlos Arnade is an economist in the Speciality Crops Branch of the Economic Research Service. The bulk of this analysis was undertaken when serving in the Animal Products Branch of the Economic Research Service. Copy right 2006 by Carlos Arnade. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes, provided copyright notice appears on all such copies..

Abstract

This paper explores the time series properties of cattle and feed prices to determine the effect shocks may have on price evolution. Two different unit roots tests are applied to the data and compared and the issue of fractional integration is discussed. A Geweke Porter-Hudak test finds that at least of three of the four price series are fractionally integrated. VAR models are estimated using level data and fractionally differenced data and impulse responses compared across various degrees of fractional differencing.

Introduction

Domestic prices of agricultural goods are increasingly exposed to shocks as international agricultural markets have become more integrated. If commodity prices are nonstationary these disruptions can have a permanent impact on prices. Less widely appreciated is the potential for shocks to influence commodity prices for extended periods of time and yet, not have a permanent effect on prices.

Typically economists have classified data series into two categories: those which are stationary, where the impact of shock dies out in a predictable way over time and those that are nonstationary, where the impact of a shock can have an permanent effect on the evolution of the data series. Over the past 20 years a significant amount of effort has been devoted to determining whether macroeconomic data, stock prices, or commodity prices are fractionally integrated (Cromwell, Labys, and Kouassi (2000), Geweke and Porter-Hudak (1983), Lo (1991), Hassler (1991). Empirical methods have been developed (Geweke and Porter-Hudak (1983), Lo (1991)) to determine if time series data is fractionally integrated and, generally, these methods have been applied to macro-economic data, stock prices, and prices of nonagricultural commodities.

One effect of this research has been the creation of a third category for classifying time series data--a category where a data series is fractionally integrated and has a long but not permanent memory of past events. When a price series is fractionally integrated, a shock may influence prices for long period of time and

often in unpredictable ways. Yet unlike a data series which is integrated to an integer order (i.e. $I(1)$) eventually the impact of the shock dies out. By revealing that some stock and commodity price series are fractionally integrated and follow a stationary long memory process these studies raise the importance of investigating whether agricultural prices display such behavior (Cromwell, Labys, and Kouassi (2000), Lo (1991), Jin and Frechette (2004), Hassler (1991). This is particularly important given the increased potential for disruptive shocks to influence agricultural prices, as markets become more global in scope.

Livestock and feed markets provide an example of markets that are particularly subject to disruptive shocks. In the spring of 2000 bovine spongiform encephalopathy, BSE, (also known as mad cow disease) was discovered beyond England spreading to several countries around the globe. In May of 2003 a case of BSE was detected in Canada leading many countries to restrict Canadian imports of beef. Both countries have subsequently discovered additional isolated cases of the disease. Outside the beef sector the Russian currency crisis in August 1998 and rapid imposition of new restrictions on imports of U.S. chicken meat led to a sharp fall in the wholesale price of certain cuts of U.S. chicken.

This paper examines the potential for shocks to have long run effects on weekly prices of four commodities: corn, soymeal, fed cattle, and feeder calves. Several methods are applied to explore the time series properties of these prices. In particular prices are examined to determine the order of integration, both integer (Kwiatkowski, et al. (1992) Nelson and Plosser (1982) and fractional, (Geweke

and Porter-Hudak (1983)) and relate this finding to the impact that shocks may have on each of these price series. Following this, a series of Vector Autoregressive (VAR) Models, each using various levels of fractionally differenced data, are estimated and used to evaluate the effects of data differencing. Impulse response functions are calculated from each of these differenced models and compared. It is found that using traditional unit root tests to determine the order of differencing prior to estimating a VAR could lead to over differencing the data and produce oscillating impulse response functions. In contrast, when 3 out of 4 price series are differenced using a more fractional order of integration, impulse response functions are more stable and reveal (preserve) a longer memory of shocks.

Testing for the Order of Integration

Typically, there is a lack of distinction between a data series that has a long memory and a data series that either has a permanent memory or a short memory of an event. A series that has unit roots (integrated to order 1 or $I(1)$) has infinite memory so that shocks have a permanent impact on the series. On the other hand, a series that is $I(0)$ is stationary and the impact of a shock or innovation will decay geometrically over time. Two major methods used to test the integer order of integration of any particular data series are the widely used Dickey Fuller (DF) test and the less popular KPSS (Kwiatkowski, Phillips, Schmidt, and Shin) test. Ideally both tests should agree. Disagreement between these two tests is a sign that the data could be fractionally integrated (Jin and Frechette (2004)).

Jin and Frechette (2004) discuss the four outcomes, summarized in table 1, of using both the KPSS and DF tests to determine if a data series is nonstationary. Given these arguments, then the common practice of using only one method to test for unit roots should be questioned. If a test outcome leads to first differencing a time series that is fractionally integrated then it can be doubly damaging. Critical information may be thrown away and differencing may not produce a stationary series (or even create an explosive oscillating series). In contrast, models estimated with fractional differenced data would contain more information than had the data been first differenced, yet adhere to the stationarity requirement, which is critical to hypothesis testing.

Fractional Integration

While the properties of data which are integrated to integer orders has been widely established and applied, the literature which has emerged around the issue of fractional integration is less widely known or used. Cromwell, Labys, and Kouassi (2000), Jin and Frechette (2004), for example, describe the behavior of a time series for different levels of fractional integration, which, depending on the level of fractional integration, can display properties of similar associated with *both* I(0) and I(1) series. To introduce the concept, write the difference operator as: $(1-B)^d$ where B is the backshift operator ($BX=X_{t-1}$ and $B^2X=X_{t-2}$)

Setting $d=1$ differences the series and, if the series is $I(1)$, produces a stationary series. What the fractional integration literature emphasizes is that a series can be integrated of order $I(d)$ where d need not be of integer value. Time series data with different fractional values of " d " can behave in very different ways (Cromwell, Labys, and Kouassi, 2000). For example, for a time series where $0 < d < .5$, the series remains stationary, has a short memory, can be inverted (into a moving average form). For a series with $d < .5$ the autocorrelation function will smoothly decline. In contrast if $.5 < d < 1$ the series has a long but not infinite memory (autocovariances may be high at unusually long lags) has an undefined variance and is nonstationary. A level of $d=.5$ represents a critical boundary. Here the series can still be inverted (into moving average form) but the series lies on the boundary of stationarity and nonstationarity. Cromwell notes that a data series that is integrated of the order $d=.5$, may follow complex cyclical paths that appear to be random. In other words a data series where $d=.5$ displays what can be viewed as chaotic behavior.

Hosking (1981) made the concept of fractional differencing operational by utilizing a binomial expansion of the difference operator. Jin and Frechette (2004) provide an example of Hosking's binomial expansion which can be written as:

$$\begin{aligned}
1) (1-B)^d &= \sum_{k=1}^{\infty} \binom{d}{k} (-B^k) \\
&= 1-dB - (1/2)*d(1-d)B^2 - (1/6)d*(1-d)(2-d)B^3 \\
&\dots\dots\dots - (1/k!)*d*(1-d)*(2-d)\dots\dots((k-1)-d)B^k \dots
\end{aligned}$$

Perhaps the best way to view the above operator is treat it as a data filter.

Ideally if the data is filtered using the correct level of d , the filtered series should be stationary or $I(0)$.

Testing Weekly Price Data

Weekly USDA wholesale prices of corn, soymeal, fed cattle, and feeder cattle were tested, in turn, for each price, in 3 different ways to determine the order of fractional integration. Weekly prices of the four commodities were calculated from USDA series of daily wholesale prices from January 1998 to August 2004. Central Illinois price were used to represent corn and soymeal. Fed cattle prices were represented by Nebraska choice Steers 1100 to 1300 lb category, while feeder calves were represented by Oklahoma City Feeder steers 750-800 lb category. Prices reported with such a high frequency data should be ideal for analysis. However there are enough holidays or other occasions when the market is closed to insure that there is a significant number of missing daily observations. By averaging up to weekly observations it was possible to reduce the number of missing observations to a reasonable level.

In creating weekly averages careful notice was taken in locating daily observations to the first and last week of the year. This was a particular problem for cattle prices which are often not reported over parts of the Christmas season. However at the weekly level some cattle price observations were not available. This was particularly true for feeder steer prices which had about 10 missing observations (out of 347). All but two missing observations were filled in by taking an average of the price from the previous week and the following week. The relationship between feeder calf and fed cattle prices were used to fill in the missing fed cattle observations for the two sequential weeks that the prices were missing.

Method: 1-Standard Unit Root (UR) Tests

As noted a correctly filtered series should be stationary. Suppose a data series were filtered through the fractional difference operator for *varying* levels of d and both Augmented Dickey Fuller (ADF) Unit Root and KPSS tests applied to the different representations of " d " filtered data. Once filtered by the correct level of d both tests should indicate that the filtered series is $I(0)$. This exercise would not be precise because neither the KPSS and ADF test are meant, (nor might be refined enough) to determine a fractional order of integration. For example, either test could incorrectly lead to the inference that a stationary series (where $0 < d < .5$) is $I(0)$. However, such an exercise could provide insights concerning permissible levels of fractional differencing.

Testing Level Price Data

Following the example of Jin and Frechette (2004) (who test a time series of variances) *both* the Augmented Dickey Fuller (ADF) test and the KPSS tests were applied to a time series data on the prices of each of our four commodities: corn, soybean meal, feeder calves, and fed cattle. Table 2 evaluates the four time series in light of each of the four outcomes. The top two rows of table 2 summarize these tests when applied to the data in its original form. Using critical values joint DF and KPSS tests established by Keblowski and Welfe (2004) at a .05 level of confidence, each of the four data series falls into outcome 4; stationarity cannot be rejected by the KPSS tests nor can unit roots be rejected by the Augmented Dickey Fuller Test.

On the other hand, using a .1 level of confidence, and Keblowski's and Welfe's critical values, feeder calf prices fall under outcome 1 and can be considered non-stationary. For corn prices the ADF tests reject nonstationarity and the KPSS test rejects stationarity. Using a .1 level of confidence, soybean meal prices fall into category 4 of table 1. That is, the null hypothesis cannot be rejected in either the KPSS test (which has a stationary null hypothesis) nor the ADF test (which has a nonstationary null hypothesis). When testing fed cattle prices the ADF tests does not reject unit roots while the KPSS tests cannot reject stationarity. For feeder calves nonstationarity is rejected by the KPSS test and the ADF test cannot reject nonstationary. These differences in outcome,

between tests as well as differences in outcome at different confidence levels, warrant further investigation of the time series properties of each series.¹

Fractionally Filtered Data

Here fractional filtering methods are combined with joint Dickey Fuller and KPSS test to choose the lowest level of fractional differencing which insures stationary data. Corn, soybeans, fed cattle, and feeder calf prices were filtered using varying values of d , varying in increments of .1 ranging from .1 to 1.5. Both the Dickey Fuller test and the KPSS test were applied to each of the filtered series. When filtered with the correct difference operator both tests should indicate the data is stationary (outcome 2). This could be viewed as a way of *ruling out* possible levels of fractional integration.² That is, permissible levels of d should be those values that when filtered, produce a series where the Dickey Fuller test rejects non-stationarity *and* the KPSS test cannot reject stationary.

The lower rows of table 2 report both KPSS and DF tests results for various levels of fractionally filtered data. Using Keblowki's and Welfe's (2004) critical values for testing data jointly with the KPSS and DF tests, a fractional difference filter of .3 produced a stationary corn prices series. Soybeans prices required first differencing, fed cattle a difference filter of .5 and feeder calves a difference filter of .6 to produce a stationary series. This suggests that corn prices follow a short memory mean reverting process, soybean prices have

permanent memory of shocks, fed cattle prices follow a long memory but stationary process, and feeder cattle prices follow a long memory process which is nonstationary. A .5 filter insures that the price of fed cattle is stationary.

Method 2: The GPH test.

The Geweke and Porter-Hudak (GPH) test (Geweke and Porter-Hudak. (1983), Hassler (1991)) is the most commonly applied method to determine the order of fractional integration of a data series. This test uses estimates of the periodogram (Box and Jenkins (1976), pp.36, Hamilton (1992), pp. 158) which can be calculated for any data series. Specifically GPH show that if logged estimates of the periodogram, at every frequency are regressed on $4 * \sin^2(\lambda_j / 2)$ where $\lambda \in [0, \Pi]$ the negative of the estimated regression coefficient provides a reliable estimate of the level of fractional integration of the series.

After regressing the variable on harmonic variables representing various frequencies, the periodogram for the data series can be constructed from the estimated coefficients of the different sine and cosine variables (Hamilton (1992) pp. 160).³ In the GPH test the periodogram then is regressed on the squared sine variable. The negative of the coefficient from this regression provides the estimate of the order of fractional integration. Higher frequencies of the periodogram should be removed from this regression. GPH suggest

including, in the regression, frequencies from the lowest up until T^τ where T equals the number of observations in the data and recommended setting $\tau=.5$.⁴

Table 3 reports both GPH based estimates of the level of fractional integration and the estimated 95% confidence intervals at various levels of τ . Using the lower frequencies up to the recommended level of τ to perform the GPH regression, the estimated fractional integration of each price series is corn: 0.81 soymeal: 1.25, fed cattle: 0.59 feeder cattle: 0.53. For three prices, (soymeal, fed cattle, and feeder cattle) this is not far from that obtained by applying combined KPSS and DF tests to d filtered data (corn: .3, soymeal: 1, fed cattle: .5, feeder cattle: .7) However corn prices are nonstationary (.81) by the GPH test but stationary (.3) by the combined filtering tests.

Table 2 also reports estimated levels of fractional integration when higher frequencies are included. Setting $\tau=.7, .9$ reveal that the data is close to having unit roots. Including all frequencies (a practice not recommend by GPH) produces the estimate order of fractional integration of corn: .37, soymeal: .36, fed cattle: .17, feeder cattle: .37.

These GPH tests (based on the preferred level of $\tau=.5$) reveal that three of the price series are nonstationary and that the prices of feeder calves (at a level of integration of .53) while stationary, lie close to the boundary between being stationary and being nonstationary. In fact, estimates for feeder calves d are not

far from those associated with chaotic behavior ($d=.5$). The top row in table 2 indicates that using standard DF tests (when applied to unfiltered data) suggests that at least 3 of the price series should be first differenced to achieve stationarity. Yet, table 3 indicates may it be possible to produce a stationary series with a lower order of differencing

Method 3: Vector Autoregression & Comparing Impulse Responses

This section follows a more direct approach toward determining how prices may respond to shocks. The price response to shocks is simulated using models estimated with various levels of fractionally differenced data. Various VAR models were estimated, with data which were fractionally differenced to various degrees and impulse response functions calculated and compared. The goal is to find the smallest degree of differencing which produces stable (and reasonable) response to price shocks. Impulses were also generated from a model estimated in levels.

Specification

Rather than estimate a full blown VAR, a recursive triangular structure was imposed on the exogenous variable matrix (Sims, 1980). Correlation coefficients, *conditional* correlation coefficients, and Fisher Z's statistics, were calculated as an aid in determining the specification of the recursive VAR (see Bessler, 2004).⁵ This showed that corn prices were not caused *directly by* lags of soybean, feeder calf, or fed cattle prices and vice versa.⁶ A corn price

equation was estimated as a stand alone equation. Unconditional correlations did not reveal such clear results for the other three variables but were consistent with a recursive (triangular) VAR structure consisting of a top equation with fed cattle prices regressed on lags of fed cattle prices, a middle equation consisting of feeder cattle prices regressed on its own lags and fed cattle prices, and a bottom soybean price equation regressed on its own lags and fed and feeder calf prices. Therefore the following system was specified:

$$\begin{aligned} \Delta P_{fd} &= \sum_{k=1}^9 \eta_{1k} \Delta P_{fd,t-k} + \mu_1 \\ 2) \quad \Delta P_{fdr} &= \sum_{k=1}^9 \eta_{2k} \Delta P_{fd,t-k} + \sum_{k=1}^9 \alpha_{2k} \Delta P_{fdr,t-k} + \mu_2 \\ \Delta P_{sy} &= \sum_{k=1}^9 \eta_{3k} \Delta P_{fd,t-k} + \sum_{k=1}^9 \alpha_{3k} \Delta P_{fdr,t-k} + \sum_{k=1}^9 \lambda_{3k} \Delta P_{fsy,t-k} + \mu_3 \end{aligned}$$

The three equation model (soybeans, feeder calves, fed cattle) models were specified with 9 lags.⁷ The corn price equation estimated with 12 lags. The models were estimated by SUR while the corn model was estimated by OLS.

Impulse Response Function

Impulse response functions measure the response of each of the endogenous variables across time to the impact of a shock (or innovation) to any variable represented in a VAR model.⁸ Impulse responses represent an autoregressive (AR) process converted in moving average form. However, a general practice is

to preserve the AR structure, shock the system one variable at a time, and iteratively map out the response. For a multi-equation model, this procedure rests on the assumption that the VCV matrix of errors is diagonal. Following standard practice, the data for the 3 equation model was estimated and a Cholesky decomposition of the estimated VCV matrix calculated. Both the endogenous variable vector and exogenous variable vector were multiplied by the inverse of the cholesky *matrix* --at *each observation*--a GLS transformation of the data, sufficient to insure that the VCV matrix is diagonal (Bessler 2004).

Shocks to the 3 Equation VAR

Impulses generated from a shock to fed cattle prices were not reasonable for the model estimated in levels. While the fed cattle price response to the fed cattle price shock peaked in 6 weeks and was 11% higher than the original shock the feeder calf price response peaked in 24 weeks and was more than 12 and a half times the size of the shock. And the soy price response peaked in 43 week and was 173 times the size of the shock. The response of soymeal prices to feeder calf prices shock was also unreasonable, peaking 21 weeks later as a size over 9 times the size of the original shock. Estimating the 3 equation VAR model with nonstationary level data clearly produced unreasonable impulse responses to a shock.

On the other hand the impulse response for the stand alone corn price model was reasonable, producing a peak response 15% lower than the size of price shock, 3

weeks later. Note these impulse responses are consistent with the joint KPSS and ADF tests which finds the corn prices are stationary only after being fractionally differenced at $d=.3$.

Having generated impulse responses from a VAR model estimated using data in levels the procedure of fractionally differencing data, estimating a VAR model, transforming the data to insure a diagonal VCV matrix, re-estimating the VAR model, and generating impulse responses was applied several times over. Data was, in turn, fractionally differenced by $d=.1$, $d=.2$, $d=.3$, $d=.5$, $d=.8$, and by d set equal to the first set of GPH estimates (when $\tau=.5$) of .8 for corn, 1.25 for soymeal, .5 for feeder calves, and .6 for fed cattle.

Table 5 presents impulse responses for models estimated with fractionally differenced data with d set equal to the GPH estimated levels and for $d=.5$. Table 6 presents key information about impulse responses for the 3 variable VAR model estimated with various degrees of fractionally differenced data. When data is differenced according to that suggested by the standard GPH test ($\tau=.5$) the peak fed cattle price response to a fed cattle price shock is after 4 weeks and is about a quarter the size of the original shock. The peak feeder cattle price (about 1/3 the size of the shock) and soy price response (a negative response about 1/3 greater in absolute value than the original shock) is within a week. While all responses are stable, an oscillating soy price response suggests that setting $d=1.25$ is too high when differencing soymeal prices.

A more reasonable response can be viewed when all data are fractionally differenced to equal degrees. For example, when data are fractionally differenced at $d=.5$ and the model estimated and shocked, the peak fed cattle price response is 6 weeks later and about a third larger than the size of the original shock. The peak feeder cattle price response is about nine weeks later and is negative and is slightly more than half the size of the shock in absolute value. The soymeal price response about double the size of the original shock and occurs 12 weeks later. Table 6 also reveals that setting $d=.5$ produces a reasonable and stable response when feeder cattle prices or soymeal prices are shocked. When a model is estimated using data that is differenced with $d=.8$ shocks produce stable impulse responses which decline rather quickly. However the impulse response of soymeal prices tends to oscillate which is indicative of over differencing of the data.

Stable impulse responses to price shocks also can be generated when the degree of fractional differencing is reduced to .2 and even .1 prior to estimation of model parameters. While this preserves more information, table 6 shows that impulse response to price shocks simply are not reasonable when generated from such models. For example, soymeal prices rise (or fall) by more than 10 times the size of the shock and considerably large impulse responses extend out almost two years. In general, from this exercise it appears that, prior to estimating the 3 equation VAR model, differencing of a degree less than .5 may

not be reasonable. However, differencing of a degree greater than .5 seems to throw away information which need not be thrown away.⁹

A Corn Price Equation

The corn equation was estimated alone with 12 lags and impulses generated for a corn price shock. Similar to the exercise performed for the 3 equation VAR model, corn prices were repeatedly differenced using various levels of d , the model repeatedly estimated using various levels of differenced data, and impulse responses to a price shock repeatedly generated. In contrast to the 3 equation VAR model, in the single equation corn price model, every level of differencing, including setting $d=0$, produced a reasonable impulse response to a corn price shock. However shocking the model estimated with data difference by $d=.8$ produced a damped oscillating response suggestive of over differencing. These results from the corn price model should be viewed in light of the various tests for the level of integration. Among the four tested prices corn prices produced most inconsistent results across test methods. Using a .1 level of confidence an ADF test alone suggests corn prices are stationary. Joint GPH and ADF test suggest that corn prices should be fractionally differenced with $d=.3$ while the GPH test suggest that corn prices should be fractional differencing with $d=.8$. Two test results indicate suggest a stationary series

($d=0, .3$). In any case it is not surprising that low levels of differencing produced stable impulses from a corn price shock.

Conclusion

As international agricultural markets expand, the prices of agricultural products are increasingly exposed to shocks. Livestock markets, in particular, have been forced to absorb sudden new information related to outbreaks of disease. It is likely that these shocks to livestock markets will continue to occur in the future. Therefore it is important to obtain a better understanding of how both livestock prices and feed prices respond to such shocks, in the short run, in the intermediate run, and in the long run.

This paper analyzed the properties of the prices of corn, soymeal, fed cattle, and feeder calf prices with the intent of obtaining a better understanding of how a shock may influence these prices in the ensuing periods. Techniques which are typically used for analyzing macro-economic variables or stock prices were applied to the prices of these agricultural commodities. Of particular interest was the ability of each these 4 agricultural prices to retain the memory of a shock which can remain for an extended period of time but remain stationary. Both Dickey Fuller and KPSS tests were used to test if these prices were stationary (or non-stationary). Contradictions between these tests, an indicator

that a series may be fractionally integrated, led to further exploration of the properties of each the series.

Two approaches were used to determine if any of these four prices were fractionally integrated. First, each price series was repeatedly filtered through various fractional difference operators and both KPSS and ADF tests applied to filtered data. Second, the Geweke and Porter-Hudak test (GPH) was applied to each of these price series to estimate the degree of fractional integration of each price series. These combined tests demonstrated that prices for 2 commodities (fed cattle, feeder cattle) are fractionally integrated with the degree of integration, close to the boundary between stationary and nonstationary behavior. For corn prices one test indicated the data were stationary while the other did not.

A Vector Autoregression model was estimated for these prices and subsequent impulse responses calculated. Impulses responses for soymeal, (feeder calf) prices shock to feed (feeder calf) were usually large and extended for long periods indicating that data required differencing. VAR models were estimated with fractionally differenced data, using various levels of fractional differencing, prices were shocked, and impulse responses generated. Impulse responses from VAR models estimated with fractional differenced data showed a tendency to dissipate. However for models estimated with low levels of fractional differenced data (.1, .2) impulse responses were unreasonably large and

extended for any unreasonable period of time. Fractionally differencing to slightly higher level (.5) produced reasonable impulse responses. In some instances, fractionally differencing to even a higher level, (.8) produced oscillating impulses indicating over differencing. These results indicated the ideal level of differences lies between 0 and 1, a level that for 3 of the 4 prices is consistent with the results obtained in the fractional integration tests.

Since the degree of fractional differencing used in this paper preserves more information than first differencing, yet insures that data is stationary, impulse responses using fractionally differenced data should be a better indicator of the response of prices to a shock. Therefore, it is not surprising that the impulses generated from the parameters of a fractionally differenced VAR seem to be most reasonable.

Footnotes

1) Accounting for serial correlation (equation 10 of Kwiatkowski et al., 1992) produced different test results than reported in table 2. However the alternative version of the test was highly sensitive to arbitrary choice of lags and the chosen parameters of the weighting function. This variance in outcome, even *within* the KPSS test only serves to illustrate the need to pursue testing other means of testing the order of integration.

2) While this may not be the most precise method for determining the level of fractional integration (d), it is consistent with traditional methods of evaluating a data series.

3) An alternative method for calculating the periodogram using autocovariances (Hamilton) can produce occasionally negative periodogram estimates and often requires weighting or smoothing schemes (Hassler, 1991) to insure that the periodogram is everywhere positive.

4) Many computer programs set the default level of $\tau=.5$.

5) Fisher Z statistics were based on data in levels, while each of the VAR models were estimated using some form of differenced data

6) Corn prices appear to contemporaneously correlated with feeder calf price. In a VAR model, consisting of lags on corn prices, lag feeder calf prices would be redundant.

7) Box-Pierce Q tests for white noise residuals indicated that a nine lag length was sufficient for each equation in the 3 equation VAR model but the corn price equation required 12 lags.

8) With the recursive structure above, not every shock will affect every variable.

9) This is particularly interesting since a series which fractionally integrated to a degree where $d=.5$, lies on the boundary between stationary and nonstationarity.

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Appendix

Fractional cointegration

Suppose that the degree of fractional integration of two time series is the same or similar. This leads to the possibility that the two series may be cointegrated or even fractionally cointegrated (Dittman, 2001). Two series that are fractionally cointegrated may depart for an *extended* period from their cointegrating relationship but will eventually return to that relationship. A simple method to determine if two series are fractionally cointegrated, similar in spirit to standard bivariate cointegration tests, is to regress one series on the other and apply fractional integration tests to the errors. Candidate series should have similar level of fractional integration. If the degree of fractional integration of the errors is different, the series can be said to be fractionally cointegrated. The Table A1 reveals that when using a level of $\tau=.5$ to apply the GPH test, that fed cattle and feeder cattle appear to be fractionally cointegrated. This implies that both series therefore may take quite long trips away from, but eventually return to, their cointegrating relationship.

Table 1: Four Outcomes of Joint Dickey Fuller and KPSS test.

	DF Null=NS	KPSS Null=Stat	
Case 1	Not reject	Reject	I(1)
Case 2	Reject	Not Reject	I(0)
Case 3	Reject	Reject	Prob Fractional
Case 4	Not Reject	Not Reject	Not Informative

1/NS=nonstationary, S=stationary or I(0)

Table 2: Joint Unit Root Tests, KPSS and ADF

<u>KPSS TEST^{1,2} : Stationary Null</u>					<u>ADF TEST: Nonstationary Null</u>							
	<u>Corn</u>	<u>Soy</u>	<u>Fed Cattle</u>	<u>Feeder Cattle</u>	<u>Crn</u>	<u>Soy</u>	<u>Fed Cattle</u>	<u>Fdr Cat</u>	<u>Corn</u>	<u>Soy</u>	<u>Fed Cattle</u>	<u>Feeder Cattle</u>
Level Data	0.353	0.256	0.338	0.380	C4 C3	C4 C4	C4 C4	C4 C4	-3.08	-2.15	-2.26	-1.07
Fractional Differenced Data												
<u>Order of fractional differencing</u>					<u>Table 1 Categories</u>							
D=.1	0.348	0.255	0.335	0.374	C4	C4	C4	C4	-3.05	-2.14	-2.30	-1.10
D=.2	0.340	0.250	0.330	0.368	C4	C4	C4	C4	-3.09	-2.20	-2.44	-1.18
D=.3	0.326	0.241	0.321	0.362	C2	C4	C4	C4	-3.22*	-2.36	-2.57	-1.38
D=.4	0.306	0.227	0.306	0.358	C2	C4	C4	C4	-3.45*	-2.40	-2.82	-1.70
D=.5	0.281	0.208	0.286	0.357	C2	C4	C2	C4	-3.78*	-2.51	-3.16*	-2.17
D=.6	0.255	0.188	0.264	0.363	C2	C4	C2	C4	-4.18*	-2.55	-3.57*	-2.77
D=.7	0.232	0.171	0.246	0.374	C2	C4	C2	C2	-4.57*	-2.47	-3.97*	-3.45*
D=.8	0.213	0.159	0.233	0.389	C2	C4	C2	C2	-4.87*	-2.23	-4.28*	-4.14*
D=.9	0.197	0.151	0.225	0.402	C2	C4	C2	C2	-5.99*	-1.85	-4.44*	-4.74*
D=1	0.185	0.146	0.221	0.411	C2	C2	C2	C2	-6.07*	-5.15*	-5.82*	-5.16*
D=1.1	0.176	0.142	0.218	0.416	C2	C2	C2	C2	-6.06*	-5.17*	-5.58*	-5.40*
D=1.2	0.168	0.139	0.216	0.419	C2	C2	C2	C2	-4.89*	-5.20*	-5.41*	-5.54*
D=1.3	0.162	0.136	0.214	0.420*	C2	C2	C2	C2	-5.54*	-5.34*	-5.14*	-5.69*
D=1.4	0.157	0.134	0.213	0.420*	C2	C2	C2	C2	-5.84*	-5.66*	-5.14*	-5.90*
D=1.5	0.153	0.133	0.212	0.420*	C2	C2	C2	C2	-6.25*	-6.16*	-5.32*	-6.22*

1/ KPSS (Kwiatkowski, Phillips, Schmidt & Shin Test with a stationary null hypothesis ADF (Augmented Dickey Fuller) test with a nonstationary null hypothesis.

2/ Equation (6, pg 161) of KPSS. Augmented KPSS tests were also applied but were not robust across lag lengths and weight choices (see Kwiatkowski et al. (1992) eq. 10, pg 164). Critical values are for *joint* DF and KPSS tests at the .05 were used (Keblowki and Welfe, 2004). Note these critical values are different than those used for either for the DF test alone and the KPSS test alone.

3/ C categories (table 1) refer to relative performance of both tests. For example, C4: joint tests contradict and the order of integration cannot be determined. C2: joint tests imply that the data is stationary. In categorizing level data two sets of C's, are listed. The top representing 05 confidence levels and the 2nd line of C's representing categories at a .1 (lower) level .

Table 3: Estimated Level of Fractional Integration of Price Data

Gwenke, Porter, Hudak Test^{1,2}

$\tau = .5$ ³

	<u>LB</u> ⁴	<u>Estimate</u>	<u>UB</u>
P-Corn	0.38	0.81	1.26
P-Soymeal	0.844	1.25	1.65
P-Fed Cattle	0.30	0.59	0.88
Feeder Cattle	0.22	0.53	0.84

$\tau = .7$

	<u>LB</u>	<u>Estimate</u>	<u>UB</u>
P-Corn	0.83	1.03	1.23
P-Soymeal	0.816	1.00	1.185
P-Fed Cattle	1.02	1.23	1.43
Feeder Cattle	0.80	0.98	1.16

$\tau = .9$

	<u>LB</u>	<u>Estimate</u>	<u>UB</u>
P-Corn	0.82	0.95	1.08
P-Soymeal	0.86	1.0	1.13
P-Fed Cattle	0.8	1.01	1.14
Feeder Cattle	0.94	1.08	1.22

All Fr³

	<u>LB</u>	<u>Estimate</u>	<u>UB</u>
P-Corn	0.254	0.37	0.49
P-Soymeal	0.22	.36	0.49
P-Fed Cattle	0.01	0.16	0.30
Feeder Cattle	0.23	0.37	0.50

Alternative Method (Table 2)

P-Corn	0.30
P-Soymeal	1.00
P-Fed Cattle	0.50
Feeder Cattle	0.70

1)GPH: Geweke Porter-Hudak test for fractional integration. The periodogram of data is regressed on estimates of 4 sin squared variable (see text) and the estimated fractional integration equals the negative of the coefficient on this variable.

2) Periodogram estimates of each series were calculated from coefficients on each frequency (see Hamilton, pp. 160).

3) α refers to the number of frequencies used in the GHP test. The number is t^α . where t is the number of observations. GHP found setting $\alpha = .5$ produced reliable estimates of d and recommended sticking to low frequencies when d estimates were sensitive to α . (Geweke Porter-Hudak,(1983), pp 226-231).

4) LB represents the lower bound of the 90% confidence interval, UB represents the Upper Bound.

5) All Fr: all frequencies used. This last estimate illustrates sensitivity of GPH test to frequencies included.

Table 4: Impulse Responses of VAR Model Estimated with Fractional Differenced Data, $d=0.5$

Price Response: Weeks Out ¹	<u>Fed Cattle Shock</u>			<u>Feeder Calf Shock</u>		<u>Soymeal Shock</u>
	Shock=1 <u>Fed</u>	<u>Feeder</u>	<u>Soymeal</u>	Shock=1 <u>Feeder</u>	<u>Soymeal</u>	<u>Soymeal</u>
1	0.146	-0.169	1.474	0.288	-1.433	0.306
2	0.234	-0.378	1.698	0.380	-0.826	0.085
3	0.262	-0.460	1.382	0.368	-0.927	0.161
4	0.325	-0.419	0.366	0.289	0.246	-0.141
5	0.305	-0.497	1.779	0.237	-0.057	-0.104
6	0.358	-0.521	1.459	0.294	-0.078	0.221
7	0.249	-0.484	1.496	0.178	-0.487	0.204
8	0.116	-0.591	1.006	0.240	-0.382	0.290
9	0.165	-0.632	1.381	0.200	-0.679	0.208
10	0.200	-0.592	1.622	0.180	-1.069	0.252
11	0.187	-0.575	1.351	0.171	-0.629	0.036
12	0.156	-0.593	1.924	0.163	-0.548	-0.005
13	0.132	-0.595	1.871	0.155	-0.299	-0.001
14	0.109	-0.575	1.834	0.131	-0.408	0.034
15	0.088	-0.584	1.443	0.136	-0.205	0.086
16	0.100	-0.569	1.672	0.118	-0.432	0.123
17	0.103	-0.541	1.689	0.113	-0.580	0.182
18	0.086	-0.526	1.619	0.105	-0.615	0.122
19	0.069	-0.515	1.753	0.097	-0.619	0.092
20	0.061	-0.500	1.826	0.091	-0.525	0.035
21	0.057	-0.479	1.836	0.084	-0.486	0.020
22	0.053	-0.463	1.691	0.080	-0.310	0.006
23	0.051	-0.443	1.762	0.072	-0.330	0.030
24	0.047	-0.422	1.693	0.068	-0.325	0.067
25	0.039	-0.405	1.633	0.063	-0.395	0.080
26	0.034	-0.389	1.598	0.059	-0.426	0.088
27	0.031	-0.371	1.631	0.055	-0.455	0.068
28	0.030	-0.352	1.631	0.051	-0.449	0.051
29	0.028	-0.335	1.595	0.048	-0.379	0.021
30	0.025	-0.319	1.607	0.044	-0.336	0.013
31	0.022	-0.303	1.559	0.041	-0.280	0.015
32	0.019	-0.287	1.506	0.038	-0.276	0.027
33	0.017	-0.273	1.440	0.036	-0.271	0.041
34	0.016	-0.258	1.418	0.033	-0.301	0.048
35	0.015	-0.244	1.383	0.031	-0.317	0.049
36	0.013	-0.231	1.357	0.029	-0.317	0.037
37	0.012	-0.218	1.342	0.027	-0.300	0.026
38	0.011	-0.206	1.316	0.025	-0.267	0.015
39	0.010	-0.195	1.281	0.023	-0.239	0.012
40	0.009	-0.184	1.232	0.022	-0.211	0.013
41	0.008	-0.173	1.194	0.020	-0.207	0.019
42	0.007	-0.163	1.148	0.019	-0.207	0.026
43	0.007	-0.154	1.111	0.017	-0.216	0.028

Table 4 (cont.)

Price Response:	<u>Fed Cattle Shock</u>			<u>Feeder Calf Shock</u>		<u>Soymeal Shock</u>
	<u>Shock=1</u>			<u>Shock=1</u>		<u>Shock=1</u>
	<u>Fed</u>	<u>Feeder</u>	<u>Soymeal</u>	<u>Feeder</u>	<u>Soymeal</u>	<u>Soymeal</u>
Wk						
44	0.006	-0.145	1.079	0.016	-0.218	0.026
45	0.005	-0.136	1.053	0.015	-0.214	0.021
46	0.005	-0.128	1.025	0.014	-0.201	0.015
47	0.004	-0.120	0.993	0.013	-0.181	0.010
48	0.004	-0.113	0.962	0.012	-0.165	0.009
49	0.004	-0.106	0.924	0.011	-0.151	0.010
50	0.003	-0.100	0.888	0.010	-0.147	0.013
51	0.003	-0.094	0.852	0.010	-0.146	0.016
52	0.003	-0.088	0.823	0.009	-0.148	0.016
53	0.002	-0.082	0.794	0.008	-0.147	0.015
54	0.002	-0.077	0.769	0.008	-0.141	0.012
55	0.002	-0.073	0.744	0.007	-0.132	0.009
56	0.002	-0.068	0.717	0.007	-0.121	0.007
57	0.002	-0.064	0.689	0.006	-0.111	0.006
58	0.001	-0.060	0.661	0.006	-0.104	0.007
59	0.001	-0.056	0.633	0.005	-0.101	0.008
60	0.001	-0.052	0.607	0.005	-0.100	0.009
61	0.001	-0.049	0.583	0.005	-0.099	0.009
62	0.001	-0.046	0.561	0.004	-0.096	0.008
63	0.001	-0.043	0.540	0.004	-0.092	0.007
64	0.001	-0.040	0.520	0.004	-0.086	0.005
65	0.001	-0.038	0.499	0.004	-0.079	0.004
66	0.001	-0.035	0.478	0.003	-0.074	0.004
67	0.001	-0.033	0.457	0.003	-0.070	0.005
68	0.001	-0.031	0.437	0.003	-0.067	0.005
69	0.000	-0.029	0.418	0.003	-0.066	0.006
70	0.000	-0.027	0.401	0.002	-0.065	0.005
71	0.000	-0.025	0.385	0.002	-0.062	0.005
72	0.000	-0.024	0.369	0.002	-0.059	0.004
73	0.000	-0.022	0.354	0.002	-0.055	0.003
74	0.000	-0.021	0.339	0.002	-0.051	0.003
75	0.000	-0.019	0.324	0.002	-0.048	0.003
76	0.000	-0.018	0.309	0.002	-0.046	0.003
77	0.000	-0.017	0.295	0.002	-0.044	0.003
78	0.000	-0.016	0.282	0.001	-0.043	0.003

1/ Refers to the response of prices n weeks after the initial shock of 1.

Table 5: Impulse Response Peaks for Various d Differenced Models

<u>PFed=1 Shock</u>							
Candidate Or Not ⁴	<u>Price:</u> Difference Degree $d=GPH^3$	<u>Fed</u> <u>Cattle</u>		<u>Feeder</u> <u>Cattle</u>		<u>Soymeal</u>	
		<u>Wks</u>	<u>Sz</u>	<u>Wks</u>	<u>Sz</u>	<u>Wks</u>	<u>Sz</u>
C		4	0.25	1	0.338	4	-1.38
N	$d=.1$	6	0.91	22	-7.28	48	-178.80
N	$d=.2$	5	0.66	20	-4.06	37	30.75
N	$d=.3$	6	0.58	17	-2.19	37	8.41
C	$d=.5$	6	0.36	9	-0.63	12	1.92
C	$d=.8$	1	-0.14	2	-0.18	1	1.26
N	Level model	6	1.11	24	-12.74	43	173

<u>Pfeedr =1 Shock</u>				<u>Psymeal=1 Shock</u>			
	$d=GPH$	<u>Feeder</u> <u>Cattle</u>		<u>Soymeal</u>		<u>Soymeal</u>	
		<u>Wks</u>	<u>Sz</u>	<u>Wks</u>	<u>Sz</u>	<u>Wks</u>	<u>Sz</u>
C		2	-0.23	4	1.44	4	-0.30
N	$d=.1$	6	0.91	7	-7.23	48	-178.67
N	$d=.2$	3	0.7	10	-3.35	39	30.60
C	$d=.3$	3	0.57	1	-1.56	1	0.49
C	$d=.5$	2	0.38	1	-1.43	1	0.30
C	$d=.8$	2	0.21	1	-1.37	4	0.25
N	Level Model	6	1.09	21	-9.42	1	0.88

1/ GPH levels of d : fed cattle=.6 feeder calves=.5 and soymeal =1.25

2/ wks= number of wks when impulse response peaks. Sz, size of peak response, relative to original shock of 1. For example, when a model is estimated with fractionally differenced data of $d=.5$, the maximum response of soymeal price is 1.92 to a fed cattle price shock, is 92% higher and occurs 12 weeks after the shock.

3/Soy price response oscillates the value follow several weeks of soy price rises

4/ n= though stable, clearly does not provide reasonable answer, c=candidate differencing level

Table 6: Peak Impulses Corn Price Model

Corn=1 Shock

<u>Price:</u>			<u>Corn</u>
<u>Candidate</u>		<u>Wks</u>	<u>Sz</u>
<u>Or Not</u>			
C	D=GPH	11	-0.18
C	D=.1	3	0.72
C	D=.2	3	0.6
C	D=.3	3	0.507
C	D=.5	3	0.36
C	D=.8	11	-0.18
	D=0	3	0.85

1/ Similar interpretation as table 5

Table A: Estimated Fractional Integration: Selected Errors

	Frc coin:	<u>LB</u>	<u>Estimate</u>	<u>UB</u>
$\tau = .5$				
cr/fed	ND	0.048	0.532	1.016
cr/fdr	ND	0.111	0.534	0.957
Fd/fdr	Yes	-0.047	0.187	0.421
$\tau = .7$				
cr/sy	Yes	-0.234	0.374	0.608
cr/fd	Yes	-0.194	0.440	0.634
cr/fr	Yes	-0.207	0.510	0.717
sy/fd	Yes	-0.216	0.380	0.596
sy/fr	Yes	-0.224	0.426	0.649
Fd/fr	Yes	-0.184	0.480	0.664
$\tau = .9$				
cr/sy	Yes	0.156	0.296	0.436
cr/fr	Yes	0.336	0.471	0.606
cr/fd	Yes	0.239	0.384	0.529
sy/fd	Yes	0.342	0.498	0.654
sy/fr	Yes	0.218	0.376	0.534
fd/fr	Yes	0.043	0.189	0.335
All FR				
cr/sy	Yes	0.064	0.146	0.228
cr/fr	Yes	0.083	0.182	0.281
cr/fd	NO	0.206	0.300	0.394
sy/fd	ND	0.167	0.277	0.387
sy/fr	Yes	0.029	0.139	0.249
fd/fr	Yes	-0.229	-0.124	-0.019

1/Errors of a regression between selected price. Errors chosen based on possibilities for cointegration as indicated in table 2. If level of fractional integration is the same between prices, but is different for these errors then the series are fractionally cointegrated to a degree equal to the difference in the degree of integration of the series and the errors.

2/ NA: there was a significant difference in fractional integration of original series and tests do not apply. Yes: appears to be fractionally cointegrated. NO: Despite similar levels of fractional integration, the two price series do not appear to have a fractionally cointegrated relationship.

3/ Higher frequencies or the periodogram are not typically applied in the GPH test. Elimination based on GPH's τ rule as discussed in text. All FR refers to a GPH test applied when using all frequencies of the periodogram.