Measuring educational efficiency at student level with parametric stochastic distance functions: An application to Spanish PISA results

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Abstract

This study explicitly considers that education is a multi-input multi-output production process subject to inefficient behaviors that can be identified at student level. Therefore a distance function allows us to calculate different aspects of educational technology. The paper presents an empirical application of this model using Spanish data from the Programme for International Student Assessment implemented by the OECD. The results provide insights into how student background, peer-group and school characteristics interact with educational outputs. Findings also suggest that, once educational inputs are taken into account; there is no statistically significant difference in efficiency levels across schools regarding public-private ownership.

Keywords: Secondary schools, technical efficiency, stochastic frontier, distance function.

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Introduction

Since the early 1960s (Carroll, 1963) a wide range of studies (*e.g.* Coleman (1966); Jenks (1972); Summers and Wolfe (1977); Hanushek, (1986, 1996, 1997, 2003); Pritchett and Filmer, (1999)) have sought to define the relationship between school inputs, pupil's background and achievement at school. Despite all the research devoted to this issue, the well-known debate, "Does school matter? - Does money matter?", remains open. A great deal of evidence has established that a student's education takes place both at home and at school. However, the way that a student's own characteristics, home, peer-group and school interact with educational outputs continues to be largely unknown, and this is a serious drawback for policy-makers taking decisions about the allocation of the scarce public resources devoted to education.

We can roughly summarize as follows the main reasons put forward in the literature as to why empirical work does not find systematic relationships between school inputs and outputs. First, education is a highly complex process with variables, such as organization or non-monetary inputs, implied in production (see Vandenberghe (1999) for a review). Second, there is the inconsistency of the use of Cobb-Douglas specifications for the estimation of educational production functions (see, for example, Eide and Showalter (1998), Figlio (1999) or Baker (2001)). Third, most production function studies into the economics of education do not consider the theoretically potential role of the efficiency component (Farrell, 1957; Leibenstein, 1966). And, last but not least, in empirical research, student results are typically aggregated at school or district level, imposing a considerable limitation on disentangling the effect of a student's own background from peer-group and school inputs on student achievement[1].

In order to tackle the inefficiency issue in education, many studies use deterministic non-parametric Data Envelopment Analysis (DEA) in empirical evaluations. Pioneer studies applying DEA in education originate with Bessent and Bessent (1980), Charnes *et al*. (1981) and Bessent *et al.* (1982)[2]. Other studies have considered parametric methodologies, mainly using the Cobb-Douglas specifications, but also the translog functional form proposed by Christensen *et al*. (1971). These studies have included Jiménez (1986), Callan and Santerre (1990), Gyimah-Brempong and Gyapong (1992), Deller and Rudnicki (1993) and Grosskopf *et al*. (1997). The main advantage of the translog function is its highly flexible nature, which allows the study of interactions in the production process. Summers and Wolfe (1977) and Figlio (1999) used student level data in their econometric works; both concluded that student level is more appropriate than higher levels of aggregation. Their findings show that school inputs matter but that their impact on different types of student varies considerably.

In order to overcome simultaneously the difficulties underlined here before, in this paper we propose the use of frontier analysis techniques, more precisely, a parametric stochastic distance function. Under this specification, we explicitly consider that education is a process in which students use their own and school inputs in order to transform them into academic results, subject to inefficient behaviors that can be identified at student level. Moreover, parametric stochastic distance functions, flexible by definition, allow us to deal simultaneously with multiple outputs, *e.g.* mathematics and reading test scores, and multiple inputs, including school inputs, student background and peer-group characteristics in a stochastic framework. We adopt here a translog specification to estimate the parametric stochastic distance function at student level. This allows us to calculate several aspects of educational technology, mainly distance elasticities with respect to outputs and inputs, and output elasticities with respect to inputs. Furthermore, the technology set in education implies monotonicity in outputs because a student cannot continue belonging to the frontier by reducing the vector of outputs holding the vector of inputs fixed. For this reason we explore the imposition of monotonicity on outputs through the estimated educational distance function at each data point. This imposition requires the computing of an additional distance measurement or distance slack for those points breaking the monotonicity assumption.

In order to illustrate the potentialities of the approach proposed here, we provide an application to Spanish educational data from the Programme for International Student Assessment (PISA) implemented in 2000 by the Organization for Economic Co-operation and Development (OECD). Spanish student performance has been shown to be poor, both in PISA 2000 and in other international studies[3]. Furthermore, Spain is one of the European Countries (EU-25) with the highest percentage, 29.0%, among the 18-25 year old population either without a high school diploma, or who did not follow any other educational training[4]. These are puzzling results if we consider that 16 years old corresponds to the end of compulsory age schooling in Spain. For this reason, the PISA evaluations offer an exciting framework in which to analyze and to identify, as much as possible, the factors at work.

Two of these factors will be the focus of our attention in this study. On the one hand, we explore in detail distance function and output elasticities with respect to school inputs, student background and peer-group characteristics and show that student achievements in reading and mathematics react differently. On the other hand, we investigate differences in student performance across Spanish public and private schools and conclude that, once school inputs, student background and peer-group characteristics are taken into account, there is no statistically significant difference in school efficiency distribution regarding public-private ownership.

The paper is organized as follows: Section 2 provides an overview of educational production functions and presents the parametric stochastic distance function. Section 3 is devoted to describing data and provides results and a discussion of our empirical analysis. Finally, the paper ends with a summary and explores directions for further research.

1. Estimating an educational production function through distance functions

In most studies, a common conceptual framework for estimating the educational production function might take the following form (Levin, 1974; Hanushek, 1979):

$$A_{is} = f(B_{is}, S_s, P_{is}, I_{is}) \tag{1}$$

where A_{is} equals the achievement of student *i* at school *s*, B_{is} is the student's background, S_s are school inputs, P_{is} denotes the peer-group effect, and I_{is} are student innate abilities. Most often, equation (1) is estimated at school level. This analysis typically aggregates student inputs and achievements belonging to each school as an average by school, or even by school district when some non-controllable inputs are not observable at school level.

In this paper we propose to use parametric stochastic distance functions at student level in order to go further in the analysis of production functions in education. For this purpose equation (1) becomes:

$$D_{is} = g(A_{is}, B_{is}, S_s, P_{is}).I_i \tag{2}$$

where g represents the best practice technology used in the transformation of educational inputs in outputs, and D_{is} is the distance that separates each student from the technological boundary. Unobservable student innate abilities, I_i, are assumed to be randomly distributed in the population and to influence individual performance in a multiplicative way. This simple transformation places the empirical estimation of equation (2) naturally within the framework of parametric stochastic frontier analysis (SFA), which, under specific distributional assumptions, allows the disentangling of random effects from efficiency (distance to the frontier).

In the particular case of educational production analyzed here, distance functions D_{is} are expected to capture individual student performance measured with respect to the estimated frontier benchmark.

However, disentangling student and school sources of poor performance is an identification issue. Several factors could be responsible for observed differences in performance, among them the effort and motivation put into education by teachers on the one hand and by parents and students on the other. Other factors relate to the overall role of institutions, including main pedagogical choices, organizational structure and incentive schemes, among others. Within the context of this study, we are particularly interested in the comparison between public and private school scores.

The quality of public schools has recently been under scrutiny, with voices calling for the reform of the Spanish Law for Quality in Schooling (*Ley Orgánica de Calidad de la Enseñanza*). As in most other countries, there are three possible ways of financing a school in Spain: private schools, private schools financed by a public vouchers system (the so-called *escuela concertada*) and public schools. The argument calling for more private schools and for more public expenditure in education monitored by private hands is usually based on aggregate results, like those we obtained from PISA 2000, as presented in Table 1.

(Table 1)

A *naïve* interpretation of the descriptive results presented in Table 1 would bring us to conclude that there is a higher performance by private schools in terms of average scores, but also in terms of equity. The study presented here, based on the estimation of a parametric stochastic distance function, can provide the information to enable the confirmation or rejection of this assumption.

1.1. The parametric stochastic distance function approach

Defining a vector of inputs $x = (x_1, \ldots, x_K) \in \Re^{K+}$ and a vector of outputs $y = (y_1, \ldots, y_M) \in \Re^{M+}$, a feasible multi-input multi-output production technology can be defined using the output possibility set $P(x)$, which can be produced using the input vector x:

$P(x) = \{y: x \text{ can produce } y\}$, which is assumed to satisfy the set of axioms depicted in Färe and Primont (1995). This technology can also be defined as the output distance function proposed by Shephard (1970):

$$D_O(x,y) = inf\{\theta : \theta > 0, (x, y/\theta) \in P(x)\}$$

If $D_O(x,y) \leq 1$, then (x,y) belongs to the production set $P(x)$. In addition, $D_O(x,y) = 1$ if y is located on the outer boundary of the output possibility set.[5]

Figure 1 illustrates these concepts in a simple two-output setting. Let us assume that two decision-making units (DMU) in frontier analysis terminology, A and C, dispose of equal input endowments to perform outputs y_1 (mathematics) and y_2 (reading). Then C is efficient, $D_O(x, y_C) \equiv \theta_C = 1$, because it lies on the boundary of the output possibility set, whereas A, an interior point, is inefficient at a rate given by the radial distance function $D_O(x, y_A) \equiv \theta_A = OA/OB$ where $D_O(x,y) \equiv \theta \in [0;1]$.

(Figure 1)

In order to estimate the distance function in a parametric setting, a translog functional form is assumed. According to Coelli and Perelman (2000), this specification fulfills a set of desirable characteristics: flexible, easy to derive and allowing the imposition of homogeneity. The translog distance function specification herein adopted for the case of K inputs and M outputs is:

$$ln\, D_{Oi}(x,y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \, ln\, y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn}\, ln\, y_{mi}\, ln\, y_{ni} + \sum_{k=1}^{K} \beta_k \, ln\, x_{ki} \qquad (3)$$

$$+\frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl}\ ln\ x_{ki}\ ln\ x_{li}+\sum_{k=1}^{K}\sum_{m=1}^{M}\delta_{km}\ ln\ x_{ki}\ ln\ y_{mi}\ ,\quad i=1,2,\ldots,N,$$

where i denotes the i^{th} unit in the sample. In order to obtain the production frontier surface we set $D_O(x,y)=1$, which implies $ln\,D_O(x,y)=0$. The parameters of the above distance function must satisfy a number of restrictions, among them symmetry and homogeneity of degree + 1 in outputs. This latter restriction indicates that distances with respect to the boundary of the production set are measured by radial expansions, as illustrated in Figure 1.

According to Lovell *et al*. (1994), normalizing the output distance function by one of the outputs is equivalent to imposing homogeneity of a degree +1. Therefore, equation (3) can be represented as:

$$ln\left(D_{Oi}(x,y)/y_{Mi}\right)=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta),\qquad i=1,2,\ldots,N, \tag{4}$$

where $TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)=\alpha_0+\sum_{m=1}^{M-1}\alpha_m\ ln\left(y_{mi}/y_{Mi}\right)+\frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\alpha_{mn}\ ln\left(y_{mi}/y_{Mi}\right)ln\left(y_{ni}/y_{Mi}\right)$

$$+\sum_{k=1}^{K}\beta_k\ ln\,x_{ki}+\frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl}\ ln\,x_{ki}\ ln\,x_{li}+\frac{1}{2}\sum_{k=1}^{K}\sum_{m=1}^{M-1}\delta_{km}\ ln\,x_{ki}\ ln\left(y_{mi}/y_{Mi}\right)$$

Rearranging terms, the function above can be rewritten as follows:

$$-\ln\left(y_{Mi}\right)=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)-\ln D_{Oi}(x,y),\quad i=1,2,\ldots,N, \tag{5}$$

where $-ln\,D_{Oi}(x,y)$ corresponds to the radial distance function from the boundary. Hence we can set $u=-ln\,D_{Oi}(x,y)$ and add a term v_i capturing for noise to obtain the Battese and Coelli (1988) version of the traditional stochastic frontier model proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977):

$$-\ln\left(y_{Mi}\right)=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)+\varepsilon_i,\quad \varepsilon_i=v_i+u_i, \tag{6}$$

where $u=-ln\,D_{Oi}(x,y)$, the distance to the boundary set, is a negative random term assumed to be independently distributed as truncations at zero of the $N\left(0,\sigma_u^2\right)$ distribution, and the v_i term is assumed to be a two-sided random (stochastic) disturbance designated to account for statistical noise and is distributed as *iid* $N\left(0,\sigma_v^2\right)$. Both terms are independently distributed $\sigma_{uv}=0$.[6]

But these are the normal assumptions given to u_i and v_i error terms in frontier analysis literature dealing with the technical efficiency of firms in production. What is the interpretation we can give to these error terms in the particular case of student performance as discussed here? As indicated in Section 1, we think that they allow for a straightforward interpretation.

On the one hand, the stochastic term v_i is expected to capture unobserved student characteristics, mainly innate abilities, but also aptitude regarding the performance of tests and luck, as well as family-specific circumstances (*e.g*. parents' workplace status or family problems at home potentially affecting a student's results but not captured by the model). All of these characteristics are assumed to be distributed normally at random in the population.

On the other hand, the distance function term u_i is expected to capture students' and teachers' efforts and motivation as well as school performance and organization, not explained by input endowments, to be included in the distance function.

The emergence of inefficiencies in education can be outlined in the following way. Firstly, different methodologies exist to teach different subjects. However, not all pedagogical tools are equally productive for all students or all circumstances. Secondly, teachers are not homogenous or perfectly interchangeable for transforming educational inputs into academic results. Moreover, their efforts and motivation are likely to depend on financial incentives, as is the case in other production and service activities. Thirdly, factors such as effort, preferences, motivation or interest in learning and further education are not the same for all students, nor is the level of parent surveillance evenly distributed (controlling homework, skipping classes, assessment of education, conflicts and activities outside school). Last but not least, there is the role of educational institutions, which are nowadays considered the main explanatory factor for observed international differences in student achievement. This is particularly the case for institutions with a system that favors homogeneity in classroom composition, by means such as tracking students at an early age (Hanushek and Wossmann, 2005) or allowing private-public school competition (Nechyba, 2000).

Coming back to equation (6), note that, in practice, the parameters of the model are estimated together with two other parameters, σ^2 and γ, using a maximum likelihood analysis where, according to Battese and Corra (1977), $\sigma^2 = \sigma_v^2 + \sigma_u^2$, $\gamma = \sigma_u^2 / \left(\sigma_v^2 + \sigma_u^2\right)$.[7]

Using this information it is worthwhile analyzing the variance decomposition of the estimated endogenous variable $-\ln\left(\hat{y}_{Mi}\right)$ in equation (6) denoted by $S_{y_M}^2$ as follows:

$$S_{y_M}^2 = \hat{S}_{TL}^2 + \hat{S}_v^2 + \hat{S}_u^2 , \tag{7}$$

where $\hat{S}_{TL}^2$ denotes the percentage of variance of the left hand term in equation (7) explained by the estimated translog educational production model $\left[TL(x_i, y_i / y_{Mi}, \hat{\alpha}, \hat{\beta}, \hat{\delta})\right]$ in the mean inefficiency value, $\hat{S}_u^2$ captures the percentage of variance in the response corresponding to the inefficiency term $\left[-\ln \hat{D}\right]$ and finally $\hat{S}_v^2$ denotes the percentage of variance explained by the random error term. The latter term is computed as $\hat{S}_u^2 = S_{y_M}^2 - (\hat{S}_{TL}^2 + \hat{S}_v^2)$. To do this we calculate *R*-squared measures[8] defined as the proportion of the variation in the logs of the response chosen as numeraire explained by the three terms specified above.

Furthermore, assuming that differences across school performance are independent of differences among student performance only detected at the intra-school level, $\hat{S}_u^2$ can be decomposed through an analysis of variance as follows:

$$\hat{S}_u^2 = \hat{S}_{uB}^2 + \hat{S}_{uW}^2 \tag{8}$$

where $\hat{S}_{uB}^2$ and $\hat{S}_{uW}^2$ indicate the between-schools and within-schools inefficiency variance, respectively. Through this explanation of the variance of the endogenous variable we mainly seek to estimate the role of inefficiency in educational production.

1.2. Computation of elasticities

Once parameters of model (3) are estimated, it is interesting to calculate meaningful elasticities. In education, we are concerned with exploring three results: i) the elasticity of the distance function with respect to inputs and outputs; ii) the relative facility in substitution between outputs and the elasticity of each output with respect to each input; and iii) the elasticity between outputs themselves. Note that, in the general case under study here, the units of observation *i* are the students, the outputs y_{mi} are the student performances in *M* subjects and the x_{ki} are individual input variables corresponding to family background, peer-group characteristics and school factors (note that the *i* subscripts are suppressed in this section in order to simplify the presentation).

First, the distance function elasticity with respect to each input (and each output) provides information about how increases in one input (output) translate into more or less inefficiency for each pupil. These values can be obtained using the following expressions:

$$r_{D,x_k} = \frac{\partial D}{\partial x_k} = \frac{\partial \ln D(x,y)}{\partial \ln x_k}\frac{D(x,y)}{x_k}\;;\; r_{D,y_m} = \frac{\partial D}{\partial y_m} = \frac{\partial \ln D(x,y)}{\partial \ln y_m}\frac{D(x,y)}{y_m} \tag{9}$$

Positive values of r_{D,x_k} (r_{D,y_m}) indicate that greater input (output) implies higher distance values or, in other words, more efficiency. Negative values indicate, by contrast, less efficiency. These expressions, as with the others presented in this section, are calculated at the individual level.

It is also meaningful to measure how one output is marginally influenced by changes in inputs. Partial derivatives between output *m* and input *k* are obtained in the following way:

$$s_{y_m,x_k} \equiv \frac{\partial y_m}{\partial x_k} = \frac{r_{D,x_k}}{r_{D,y_m}} \tag{10}$$

In addition, we can compute how one particular output varies from another output. This ratio reflects the slope of the production frontier between the two outputs at the observed achievement mix:

$$s_{y_m,y_n} \equiv \frac{\partial y_n}{\partial y_m} = -\frac{r_{D,y_m}}{r_{D,y_n}} \tag{11}$$

According to Grosskopf *et al*. (1995), the ratio of the marginal rate of transformation (or relative opportunity cost) to relative output mix is a normalized marginal rate of transformation and can be defined in terms of distance functions as:

$$NMRT_{y_1y_2} = \frac{r_{D,y_1} / r_{D,y_2}}{y_2 / y_1} \tag{12}$$

where values of $NMRT_{y_1y_2}$ greater than unity reflect increasing difficulty in substitution between y_1 and y_2, whereas values of less than one reflect output substitutability.

2. An application to Spanish secondary schools

2.1. Data

In our empirical analysis, we use data from the Programme for International Student Assessment (PISA), implemented in 2000 by the OECD. The aim of PISA is to measure how well 15-year-old students are prepared to face up to the challenges of modern society. PISA tests students in the subjects of reading, mathematics and science. Because the home, school, and national contexts can play an important role in how students learn, PISA also collects extensive information about such background factors[9]. The entire database comprises 32 countries, but this study is limited to the Spanish case. The emphasis of PISA 2000 was on the assessment of reading literacy. This meant that the number of students evaluated in reading and other subjects (mathematics or science) was around 55%. However, only two-ninths of students were assessed in both mathematics and science. This fact implies a trade-off between the number of outputs and the number of students retained in the analysis. The best compromise in order to illustrate the model proposed here was to select the reading and mathematics tests combination. Given that the target 15-year-old population tends to be enrolled in two grades, we selected for this study upper 10th grade students. To sum up, the analysis is based on a homogenous population composed of 2449 students attending 10th grade at 185 different schools, who, in the year 2000, completed the mathematics and reading PISA tests.

It is worth noting that PISA is methodologically highly complex and it exceeds the aims of this paper to present a complete explanation of the procedures followed in the sampling design. Nevertheless, for a complete review, OECD (2001, 2002) may be consulted. Table 2 displays descriptive information on the output and input measures used in the analysis.

As mentioned above, we consider two outputs: the students' scores obtained in the international mathematics and reading tests. As reported in Table 2, average reading scores were higher and at the same time less widely distributed than mathematics scores.

(Table 2)

Two school inputs were selected: on the one hand, the *computer/student* ratio (corresponding to the total number of computers in the school divided by the total enrollment) and, on the other hand, the *teacher/student* ratio (corresponding to the total teaching staff divided by the total school enrollment).[10] We think that both inputs are plausible indicators for the level of physical and human capital inside each school. As most students in Spain spend their entire secondary education in the same school, we argue that specific school ratios are better input indicators than those obtained at the (10th grade) classroom level. As expected, the *computer/student* ratio varies dramatically across schools, from 0.9 to 31.0 *per* 100 students, but, less expectedly, the *teacher/student* ratio varies dramatically as well, from 3.62 to 17.67 teachers *per* 100 students.

We consider five student background inputs. All of these variables are represented by indices that summarize the answers given by students to a series of related questions. *Mother* and *father's level of education* corresponds to the International Standard Classification of Education (ISCED, OECD, 1999). The original categories were redefined as four major possibilities: 1 = did not go to school; 2 = primary school completed; 3 = secondary school completed; and 4 = tertiary education completed. The *cultural activities* index was derived from how often students had participated in the following activities during the preceding year: visiting a museum or art gallery, attending the opera, ballet, a classical symphony or a concert, or watching live theatre. The *cultural possessions* index was derived from student reports on the availability of the following items in their home: classical literature, poetry books and works of art.[11] *Time spent on homework* was also derived from student reports on the amount of time they devoted to homework *per* week in reading, mathematics and science. Together with this, and taking advantage of using student level data, we introduce a variable to control for potential *peer-group* effects. The variable considered here is the *average mother's level of education* of the peers measured at class level.

Given the nature and the treatment applied to the construction of these variables, their variation across the sample is limited. Even so, one can see in Table 2 that the highest variation corresponds to cultural activities.

2.2. Model estimation

A parametric output distance function was estimated assuming a stochastic translog technology, as indicated in Section 2. Homogeneity of degree +1 was imposed by selecting one of the outputs, the students' scores in mathematics y_1 as the dependent variable, and the ratio y_2/y_1 as the explanatory variable, instead of y_2, as described by equations (4) to (6). However, for presentation purposes, in Table 3 the parameters corresponding to y_1 are reported, as calculated by application of the homogeneity condition as in equation (3).

Two different specifications were estimated in order to test the non-separability hypothesis among outputs and inputs. For this purpose, following Coelli *et al*. (1998), we conducted a generalized likelihood ratio test (*LR*), which allows contrasting whether or not input-output cross effect parameters are statistically significant. The null hypothesis was retained on the basis of this test; therefore the results presented in Table 3 are those corresponding to the separable output distance function.[12]

As is usual for the estimation of translog functions, the original variables, $y_m(m = 1,2)$ and $x_k(k = 1,...,8)$, were transformed in deviations to mean values. Therefore, first-order parameters in Table 3 must be interpreted as distance function partial elasticities at mean values. For instance, those corresponding to the reading and mathematics scores are positive and indicate that student performance or efficiency increase (distance functions increase) when, *ceteris paribus*, their reading and mathematics scores increase. The opposite effect is observed for the scores in all first-order coefficients on inputs that are negative. This indicates that, at least at mean values and regardless of second-order effects, student performance decreases (distance functions decreases) when inputs increase. All these first-order coefficients are significant, with the sole exception of both school inputs: *computer/student* and *teacher/student* ratios.

Some general conclusions can, however, be drawn from these results without taking into account second-order coefficients affecting school inputs. Several of them are statistically significant, *e.g.* β_{22}, β_{12} and β_{23}, which correspond to the *teacher/student* ratio in its quadratic form and in interaction with the *computer/student* ratio and the *mother's level of education* index, respectively.

(Table 3)

In our case, a simpler Cobb-Douglas production function estimation would certainly be unable to discover cross effects between school inputs themselves or when combined with student background and peer-group inputs, and the conclusion would be "school does not matter". Therefore, one of the major advantages of parametric output distance function analysis at student level is that it can provide additional insights into the educational production process, overcoming at the same time model misspecification problems.

But before turning to a detailed analysis of elasticities, let us focus our attention on the statistics presented at the bottom of Table 3. The expected mean efficiency, computed as $E\left[\exp\left(-u_i|\varepsilon\right)\right]$, is equal to 0.8821, which indicates average student efficiency measured with respect to the stochastic frontier model. Using the information and the notation presented in equations (7) and (8), after proceeding to some further calculations, we obtained the variance decomposition of the estimated response presented in Table 4.

(Table 4)

This decomposition is also illustrative of the model potentialities to be used in the policy orientation debate. It shows us that student achievements can mainly be explained by the model (59.2%), that is, by school, by student endowments introduced as input factors and, finally, by the ratio of actual outputs. The random term that we attributed to non-observable factors, such as individuals' innate abilities, family circumstances or simply luck, account for only 7.8% of the total variance. Finally, estimated inefficiencies play an important role (33.0%), as expected, but these are mostly attributed to students (27.2 %), rather than to schools (5.8%). Remember, however, that this last decomposition is obtained under the strong assumption that schools are only responsible for differences in mean efficiency across the institution, and not at all for inefficiencies within them at student level.

2.3. Elasticity estimations

In this section, we present the results obtained by applying the elasticity equations (9) to (12) presented in Section 2.2. Given the flexible nature of the translog distance function, elasticities vary at each point and must be calculated for all observations in order to obtain a more convenient appraisal of the way they vary across the sample population. For presentation purposes only, inter-quartile elasticity values are reported here.

First of all, Table 5 presents distance function elasticities with respect to outputs and inputs. A positive (negative) sign indicates how inefficiency increases (decreases) when the analyzed variable augments in at one point. As a general comment on these results, it appears that elasticity values are in all cases rather stable. From distance elasticities with respect to outputs we conclude that, *ceteris paribus*, increasing reading and mathematics achievement increases efficiency. This positive effect is greater for improvements in mathematics. On the input side, three background variables show very low negative elasticities and negative effects on efficiency, on *mother's level of education*, and on *cultural activities* and *possessions*. However, the mother's level of education variable is compensated for by the high positive effect of the peer-group (*average mother's level of education*) variable. Moreover, out of the *computer/student* ratio that influences negatively student efficiency, input elasticities present high and positive values. And one of these is directly under the control of the educational institution: the *teacher/student* ratio, or, in other words, the size of school classrooms. For the median student, allocating one additional teacher *per* 100 students in each school would increase educational efficiency by around 4.3%.

(Table 5)

The way in which individual inputs affect outputs, reading and mathematics tests scores, is presented in Table 6, following equations 10 and 11. The negative derivatives between outputs show that, once a student is placed in the production frontier, the effort to augment mathematics results implies, as would be expected, a loss in reading results. The effect is not as great in the opposite direction, with a slight reduction in mathematics results when increasing reading results. The output/input derivatives signs and sizes, reported in Table 6, vary according to the input being considered, but the results are consistent with those obtained in Table 5. For example, increases in the *teacher/student* ratio, *father's level of education*, *time spent on homework* or *average mother's level of education*, clearly affect student performance in the expected direction. Table 6 shows the direct impact of input increases in terms of PISA scores for the Spanish students. It seems that the effect of educational inputs on outputs is greater for mathematics than for reading, at least in the Spanish case. Here, in terms of international scores, students performed better in reading than in mathematics. If decision were to be taken on the allocation of scarce public resources in a choice between teachers and computers, these results could be useful. Increasing the *teacher/student* ratio would probably be our recommendation based on a favorable trade-off for improvement in efficiency and test scores.

The positive effect of the peer-group over the two output variables brought into consideration the issue of school vouchers. It seems that, whatever a student's socioeconomic background, a more favorable peer-group tends to increase results, especially in mathematics. It is not the aim of this paper to go into a deep discussion of these results but, summing up, it seems that this framework could be a valid tool for policy makers concerned with education in order to advance further in the improvement of levels of achievement in a particular country or region.

(Table 6)

The ratio of the marginal rate of transformation to the relative output mix $NMRT_{y_1 y_2}$ was estimated as depicted in equation (12). The computed mean value is less than one (mean = 0.73, with standard deviation = 2.98), and must be interpreted as outputs; *reading* and *mathematics scores* are around the mean value relatively substitutes. The high variance points out that a significant number of students show values greater than one, suggesting that, for a set of students, the decision on how much effort to devote to each subject could damage results in the other output. This occurs, for instance, if the student dedicates too much time to reading and disregards mathematics. Changes in individual motivations and preferences, or in teacher and parental requirements, can have the effect that the proportion of time devoted to educational instruction becomes more balanced according to each particular situation.

2.4. Imposing curvature on the output distance function

The estimation of an output distance function can violate regularity conditions, monotonicity and convexity, for some of the evaluated units. For this reason, once we have calculated partial elasticities with respect to the distance function and outputs, it is worth evaluating the results from the point of view of educational economics. In our case, the rupture of some of these conditions is perfectly interpretable in this educational context. For example, the negative sign of output elasticities with respect to some of the inputs can be interpreted as local input congestion. For instance, it is possible that additional time spent on the computer reduces the amount of time devoted to reading comprehension or to solving mathematics exercises.

However, one assumption that must be maintained in education is monotonicity in outputs. In educational production theory it is inconsistent that a student could reduce both outputs remaining on the production frontier. The lack of theoretical sense of this result leads us to evaluate the estimations obtained at each observation. Following O'Donnell and Coelli (2005), monotonicity conditions involve constraints on distance function partial derivatives in equation (1) with respect to outputs:

$$r_m = \frac{\partial \ln D}{\partial \ln y_m} = \alpha_m + \sum_{n=1}^{M} \alpha_{mn} \ln y_n + \sum_{k=1}^{K} \delta_{km} \ln x_k \ .$$

In order for D to be non-decreasing in *y*, it is required that $r_m \geq 0$. For illustration purposes, in Figure 2 we show the empirical deterministic output distance function for the mean student.

(Figure 2)

As we can see in Figure 2, there are a number of pupils (153 cases) where monotonicity in outputs does not hold and the slope of the distance function becomes positive. This is probably due to the fact that, in real life, with very few exceptions, there are no pupils with outstanding results in reading (mathematics) and extremely bad results in mathematics (reading). If we fail to take this fact into account, we can underestimate inefficiency levels for those students projected at the stretches of the production frontier, which are breaking the monotonicity assumption in outputs.

In order to overcome this difficulty, one solution is to impose monotonicity conditions on outputs through the partial derivatives of distance function with respect to outputs adding up an inefficiency slack. In the case of our two outputs, and assuming separability, we have:

$$r_1 = \frac{\partial \ln D}{\partial \ln y_1} = \alpha_1 + \alpha_{11} \ln y_1 + \alpha_{12} \ln y_2 \geq 0 \,,$$

$$r_2 = \frac{\partial \ln D}{\partial \ln y_2} = \alpha_2 + \alpha_{22} \ln y_2 + \alpha_{21} \ln y_1 \geq 0 \,.$$

The slope of the distance function between the two outputs can be denoted as:

$$MRT_{y_2 y_1} = -\frac{\partial \ln y_2}{\partial \ln y_1} = -\frac{r_1}{r_2} = \frac{\alpha_1 + \alpha_{11} \ln y_1 + \alpha_{12} \ln y_2}{\alpha_2 + \alpha_{22} \ln y_2 + \alpha_{21} \ln y_1}$$

This expression fulfills monotonicity in outputs when $MRT_{y_2 y_1} \leq 0$. For imposing monotonicity in the empirical estimation we force up the slope of the deterministic production frontier to make it negative. To do this we evaluate the monotonicity conditions for each student. The process is shown graphically in Figure 3.

(Figure 3)

Theoretically, when a point exhibits a positive slope in the deterministic production frontier, then we can re-estimate the distance for the strict output distance function, which characterizes the fulfilling of monotonicity along the entire frontier. This implies the adding up of a distance slack $-\ln D_{Oi}^{Slack}(x, y)$ for the points projected against a section of the deterministic output distance function with a positive slope (A and B in Figure 2). We proceed in the following way. The first step is to equal the partial derivatives of pupils that do not fulfill monotonicity to be zero.

$$r_m = \frac{\partial \ln D}{\partial \ln y_m} = \alpha_m + \sum_{n=1}^{M} \alpha_{mn} \ln y_n + \sum_{k=1}^{K} \delta_{km} \ln x_k = 0$$

The second step is to calculate the output point in the strict frontier with the condition that the output ratio remains fixed. According to Figure 3, this step implies the calculus of A'' (B'') with respect to point A (B). To do this, we impose r_1 or r_2 to be zero, depending on the position of the student (A or B) on the production set.

$$\ln \tilde{y}_m = \frac{-\alpha_m + \sum_{n=1}^{M} \alpha_{mn} \ln \hat{y}_n + \sum_{k=1}^{K} \delta_{km} \ln x_k}{\alpha_{11}}$$

where $\ln \hat{y}_n = \ln \hat{y}_m - \ln\left(\frac{y_m}{y_n}\right)$. Once $\ln \tilde{y}_m$ is calculated, we can easily derive the other output in the strict deterministic production frontier through the exogenous ratio of outputs $\ln \tilde{y}_n = \ln \tilde{y}_m - \ln\left(\frac{y_m}{y_n}\right)$. Finally, we correct distance values of these students adding up the distance slack $-\ln D_{Oi}^{Slack}(x, y)$ that separates the estimated production frontier from the strict production frontier, which is graphically measured by the distance between OA' (OB') and OA'' (OB''). Following these steps, we calculate distance slacks in terms of additional inefficiency for some of the pupils in our sample. Results are shown in Table 7.

(Table 7)

Descriptive statistics show that the number of students that violate the monotonicity assumption is rather low (around 6% of the total). We corrected the biased estimated distance for 153 students. Most of them (134) presented extremely bad results in mathematics (points around A in Figure 3), while only 19 were in the opposite situation, with very low results in reading (points around B). The importance of inefficiency slacks when present is far from negligible, with a mean value of around 13.5%. However, mean slack in mathematics is around 10% of additional inefficiency, while slack in reading is greater, with a mean inefficiency of 37.7%. The corrected inefficiency values will be used in the analysis provided in the next section.

2.5. Does school ownership matter?

Finally, we focus our attention once again on school ownership. Table 8 reports school level average efficiency results by school type, which must be compared with descriptive statistics presented in Table 1. What can we learn from this comparison? Overall, that once school inputs, student background and peer-group, are taken into account, the observed differences across schools, as distinguished by ownership, vanish.

(Table 8)

This does not mean that the information was wrong, but that, if students attending private schools obtained better results, this was as a direct consequence of more favorable conditions: better family background, peer-groups and school inputs. Furthermore, a well-known selection process is at work in Spain, as well as in other countries, that offers the choice between public and private schools. As a consequence, public schools accept a higher percentage of students with less favorable backgrounds, *e.g.* foreigner populations with language difficulties and special needs. As the estimated parametric stochastic distance function model takes into account these and other student characteristics, public schools take as a benchmark this less favorable context and, as expected, their efficiency scores are better than when directly compared to private school scores in Table 1.

3. Concluding remarks

A comprehensive review of the literature of educational economics shows that the process of transforming educational inputs into test results is highly complex and little understood. Despite this generalized result, most studies continue to apply the traditional Cobb-Douglas analysis at school level. In this paper, we have proposed the use of frontier analysis techniques, more precisely, a flexible parametric stochastic distance function, in order to overcome the main criticisms directed at these studies. Moreover, we explicitly consider that education is a process in which students use their own and school inputs in order to transform them into academic results, subject to inefficient behaviors that can be identified at student level. We applied this methodology to the Spanish case using 15-year-old student tests scores and background data available from the PISA Project implemented by the OECD in 2000. The main results of this study can be summarized as follows:

1. Individual student achievements are explained, at a rate of 59%, by the education production model itself. The random term that we attributed to non-observable factors, such as an individual's innate abilities or family circumstances, accounts for only 8% of the total variance. Finally, 33% was the share attributed to the inefficiency component. According to this result, we think that the potential role of inefficiency in education should not be omitted from educational production models.

2. Several factors could be responsible for the observed differences in performance, among them the effort and motivation put into education by both teachers and students. Within the context of this study, we were particularly interested in a comparison between public and private school scores. The results showed that, in the case of Spain, the observed differences in favor of private schools were mainly accounted for by differences in school inputs, student background and peer-group characteristics considered as production factors in the education process.

3. The analysis conducted here reveals that school inputs matter, but that their effects are better captured with a translog specification, a non-linear second-order approximation, which takes into account the multi-output multi-input nature of education production. Only a detailed analysis of elasticities between distance functions, outputs and inputs revealed to us the complex interactions between them. One result of this analysis shows that the elasticities of reading and mathematics scores with respect to class composition had a positive straightforward effect on students' mathematics and reading achievement. This result leaves open for further research the convenience of a school voucher system and the quantification of "cream-skimming" in private schools financed by public funds. Another noteworthy finding is that the debate about which is the best way to spend public resources in school will need additional investigation. However, in general we cannot conclude that more money devoted to school resources is always effective whatever its allocation.

4. The rupture of the assumption of monotonicity in outputs cannot be admissible from the point of view of educational economics theory. In order to avoid this inconsistency, we propose in this paper a method for correcting the estimated distance through the calculation of distance slacks for the points that break this theoretical requirement.

To sum up, we think that the conceptual framework presented in this paper, based on the estimation of parametric stochastic output distance function, provides an appealing methodology for enhancing our understanding of the education process as it is subject to inefficiency. Furthermore, the measurement of educational technology and efficiency at student level sheds light on how to distribute school vouchers. From this point of view, we think that public schools have an important role to play in the allocation of school inputs according to student background.

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[1] Hanushek *et al*. (1996) showed how aggregation can dramatically influence upwards the statistical significance of inputs in the educational process.

[2] For an overview, Worthington (2001) provides an empirical survey of frontier efficiency techniques in education.

[3] For example, in the Third International Mathematics and Science Study (TIMSS) carried out in 1995, Spain performed in science and mathematics as rankings 27 and 31, respectively, in a survey of 41 countries. For an extensive review of all results of the TIMSS, see Gonzalez and Smith (1997).

[4] For illustration purposes, note that, for Portugal, the percentage is 45.5%, for Poland 7.6% and for Hungary 12.2% (MEC, 2004).

[5] Input distance functions can be defined in a similar way assuming input orientation and given output endowments.

[6] In order to estimate this model we used the computer program Frontier V4.1 developed by Coelli (1994), assuming that *u* was distributed as a semi-normal.

[7] For more details on these transformations, see Dios-Palomares (2002).

[8] Coelli and Perelman (1999) define the *R*-squared measure in terms of the logs of the radial distance. In the case of education, we think that is preferable to decompose the variance of the educational output into three components: i) the production model; ii) the distance or inefficiency score; and iii) the random noise.

[9] Unfortunately there is no information about IQ scores so we assume that innate ability is distributed normally among students regardless of other factors.

[10] Full-time and part-time teachers are accounted for by 1.0 and 0.5, respectively.

[11] Examples of each of these items were given to the students.

[12] In this case, the null hypothesis is rejected if the *LR* test exceeds χ_8^2 (α). For α=0.05 the critical value is 15.5, and we obtained *LR* = 10.74. The assumption of separability implies that the curvature of the production frontier is not affected by changes in scale.

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Table 1
Mathematics and reading scores by school type
Spain, 2000

School type	N	Mean	Standard deviation	Minimum	Maximum
Mathematics scores					
Private, government independent	16	539.09	34.23	468.22	589.44
Private, government dependent	56	510.11	39.80	401.97	577.87
Government	113	495.27	37.42	338.60	573.86
All	185	503.55	39.83	338.60	589.44
Reading scores					
Private, government independent	16	557.15	29.03	499.69	616.65
Private, government dependent	56	529.68	37.25	439.78	596.38
Government	113	513.27	38.01	388.43	582.82
All	185	522.03	39.18	388.43	616.65

Note: Mean differences are statistically significant, at 95% level, with F-test=10.5 and 11.5 for mathematics and reading, respectively. We cannot reject the hypothesis that variances are distributed homogenously, at 95% level, with Levene's test=0.169 and 0.587 for mathematics and reading, respectively.

Table 2
Descriptive statistics: outputs and inputs at student level in Spain

Outputs and inputs	Variable	Mean	Standard deviation	Minimum	Maximum
Outputs					
Mathematics score	y_1	505.3	82.9	202.1	815.9
Reading score	y_2	524.0	74.3	241.4	741.9
Inputs					
School					
Computers / 100 students	x_1	6.36	4.10	0.90	31.00
Teachers / 100 students	x_2	7.59	2.36	3.62	17.67
Background					
Mother's level of education	x_3	2.79	0.78	1.00	4.00
Father's level of education	x_4	2.89	0.82	1.00	4.00
Cultural activities	x_5	2.54	1.17	1.00	5.00
Cultural possessions	x_6	3.08	0.99	1.00	4.00
Time spent on homework	x_7	3.37	0.81	1.00	4.00
Peer-Group					
Average mother's level of education	x_8	2.88	0.43	1.90	4.00

Table 3
Parametric output distance function estimations

Variables and parameters			t-ratio	Variables and parameters			t-ratio
Intercept	α_0	-0.1429	19.52	*Inputs (Cont.)*			
Outputs				$(\ln x_1)(\ln x_5)$	β_{15}	0.0188	1.98
$\ln y_1$ (*mathematics score*)	α_1	0.3757		$(\ln x_1)(\ln x_6)$	β_{16}	-0.0152	1.28
$\ln y_2$ (*reading score*)	α_2	0.6243	41.45	$(\ln x_1)(\ln x_7)$	β_{17}	-0.0166	1.01
$(\ln y_1)^2$	α_{11}	1.5089		$(\ln x_1)(\ln x_8)$	β_{18}	-0.0857	2.26
$(\ln y_2)^2$	α_{22}	1.5089	17.38	$(\ln x_2)(\ln x_3)$	β_{23}	-0.0601	1.69
$(\ln y_1)(\ln y_2)$	α_{12}	-1.5089		$(\ln x_2)(\ln x_4)$	β_{24}	0.0616	1.69
Inputs				$(\ln x_2)(\ln x_5)$	β_{25}	-0.0073	0.42
$\ln x_1$ (*computers/students*)	β_1	-0.0002	0.05	$(\ln x_2)(\ln x_6)$	β_{26}	-0.0159	0.75
$\ln x_2$ (*teachers/students*)	β_2	-0.0046	0.54	$(\ln x_2)(\ln x_7)$	β_{27}	0.0017	0.06
$\ln x_3$ (*mother's level of education*)	β_3	-0.0357	3.35	$(\ln x_2)(\ln x_8)$	β_{28}	0.1638	2.42
$\ln x_4$ (*father's level of education*)	β_4	-0.0214	1.90	$(\ln x_3)(\ln x_4)$	β_{34}	-0.0570	1.96
$\ln x_5$ (*cultural activities*)	β_5	-0.0414	7.79	$(\ln x_3)(\ln x_5)$	β_{35}	0.0005	0.03
$\ln x_6$ (*cultural possessions*)	β_6	-0.0288	2.94	$(\ln x_3)(\ln x_6)$	β_{36}	0.0185	0.75
$\ln x_7$ (*homework*)	β_7	-0.0209	1.77	$(\ln x_3)(\ln x_7)$	β_{37}	-0.0063	0.22
$\ln x_8$ (*peer-group*)	β_8	-0.1497	7.81	$(\ln x_3)(\ln x_8)$	β_{38}	0.0240	0.30
$(\ln x_1)^2$	β_{11}	0.0124	1.17	$(\ln x_4)(\ln x_5)$	β_{45}	-0.0074	0.40
$(\ln x_2)^2$	β_{22}	0.1620	3.11	$(\ln x_4)(\ln x_6)$	β_{46}	-0.0162	0.70
$(\ln x_3)^2$	β_{33}	0.0930	2.01	$(\ln x_4)(\ln x_7)$	β_{47}	0.0121	0.43
$(\ln x_4)^2$	β_{44}	0.0250	0.59	$(\ln x_4)(\ln x_8)$	β_{48}	0.0879	1.15
$(\ln x_5)^2$	β_{55}	-0.0576	2.72	$(\ln x_5)(\ln x_6)$	β_{56}	0.0066	0.54
$(\ln x_6)^2$	β_{66}	-0.0189	0.70	$(\ln x_5)(\ln x_7)$	β_{57}	0.0288	1.82
$(\ln x_7)^2$	β_{77}	0.0015	0.04	$(\ln x_5)(\ln x_8)$	β_{58}	-0.0293	0.79
$(\ln x_8)^2$	β_{88}	0.0204	0.09	$(\ln x_6)(\ln x_7)$	β_{67}	0.0322	1.86
$(\ln x_1)(\ln x_2)$	β_{12}	-0.0656	3.70	$(\ln x_6)(\ln x_8)$	β_{68}	-0.0322	0.68
$(\ln x_1)(\ln x_3)$	β_{13}	-0.0079	0.43	$(\ln x_7)(\ln x_8)$	β_{78}	-0.0323	2.86
$(\ln x_1)(\ln x_4)$	β_{14}	0.0106	0.58				
Other ML parameters	γ	0.8067	30.84	*Expected mean efficiency*		0.8821	
	σ^2	0.0286	19.17				

Note: Underlined parameters are calculated by applying imposed homogeneity conditions.

Table 4
Estimated variance decomposition

Variance components		Shares	
Model	$\hat{S}^2_{TL}$		59.2 %
Random term	$\hat{S}^2_{v}$		7.8 %
School inefficiency	$\hat{S}^2_{uB}$	5.83 %	
Student inefficiency	$\hat{S}^2_{uW}$	27.17 %	
Total inefficiency	$\hat{S}^2_{u}$		33.0 %
Total Variance	$\hat{S}^2_{y_M}$		100.0 %

Table 5
Distance function elasticities

Outputs and inputs	Inter-quartile values		
	25%	50%	75%
With respect to outputs (r_{D,y_m})			
Mathematics score (y_1)	0.0009	0.0011	0.0013
Reading score (y_2)	0.0003	0.0006	0.0008
With respect to inputs (r_{D,x_k})			
School			
Computers / 100 students (x_1)	-0.0511	-0.0339	-0.0256
Teachers / 100 students (x_2)	0.0356	0.0429	0.0523
Background			
Mother's level of education (x_3)	-0.0484	-0.0303	-0.0195
Father's level of education (x_4)	0.0414	0.0519	0.0700
Cultural activities (x_5)	-0.0276	-0.0236	-0.0198
Cultural possessions (x_6)	-0.0311	-0.0233	-0.0190
Time spent on homework (x_7)	0.0439	0.0525	0.0653
Peer-Group			
Average mother's level of education (x_8)	0.0802	0.0968	0.1149

Table 6
Output/output and output/input derivatives

Inputs	Mathematics Inter-quartile values			Reading Inter-quartile values		
	25%	50%	75%	25%	50%	75%
Output with respect to output (s_{y_m,y_n})						
Mathematics score (y_1)	-	-	-	-3.03	-1.73	-1.02
Reading score (y_2)	-0.80	-0.50	-0.24	-	-	-
Output with respect to input (s_{y_m,x_k})						
School						
Computers / 100 students (x_1)	-106.38	-59.51	-33.36	-50.31	-31.90	-20.38
Teachers / 100 students (x_2)	47.53	72.01	115.25	29.03	38.93	52.93
Background						
Mother's level of education (x_3)	-89.33	-49.18	-26.73	-43.16	-27.38	-17.06
Father's level of education (x_4)	56.49	87.22	145.08	35.49	47.18	64.56
Cultural activities (x_5)	-59.50	-37.98	-25.11	-27.31	-20.99	-15.75
Cultural possessions (x_6)	-66.32	-40.53	-26.62	-30.41	-21.38	-16.05
Time spent on homework (x_7)	57.23	88.63	141.11	35.76	47.63	65.79
Peer-Group						
Average mother's level of education (x_8)	102.29	157.80	243.16	65.30	86.56	111.22

Table 7
Descriptive statistics for estimated distance slacks in mathematics and reading

Distance Slack	N	Mean	Standard deviation	Minimum	Maximum
Slacks in mathematics ($r_1 = 0$)	134	0.100	0.100	0.001	0.449
Slacks in reading ($r_2 = 0$)	19	0.377	0.076	0.295	0.539
Total	153	0.135	0.133	0.001	0.539

Table 8
Efficiency and school ownership

School type	N	Mean	Standard deviation	Minimum	Maximum
Private, government independent	16	0.8865	0.0304	0.81	0.93
Private, government dependent	56	0.8795	0.0268	0.82	0.93
Government	113	0.8775	0.0362	0.71	0.94
All	185	0.8854	0.0331	0.71	0.94

Note: Mean differences are not statistically significant, at 95% level, with *F*-test=0.522. Variances are distributed homogenously, at 95% level, with Levene's test=0.837.

Figure 1
Output possibility set *P(x)*

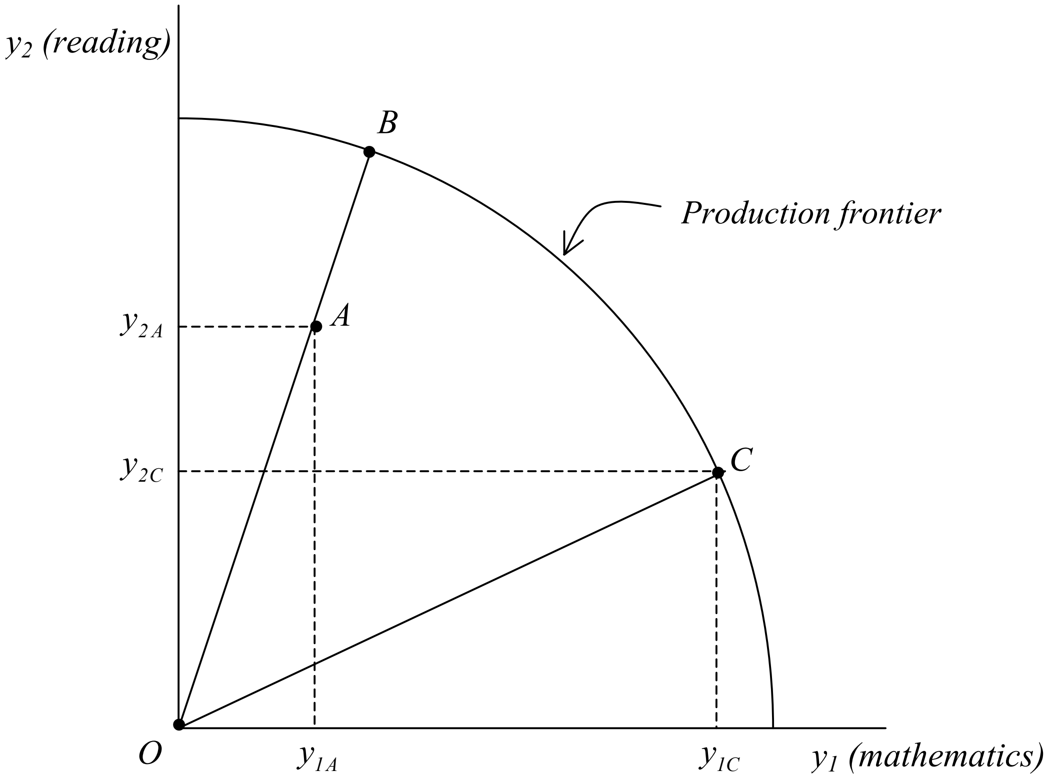

Figure 2
The deterministic output distance function at the mean student

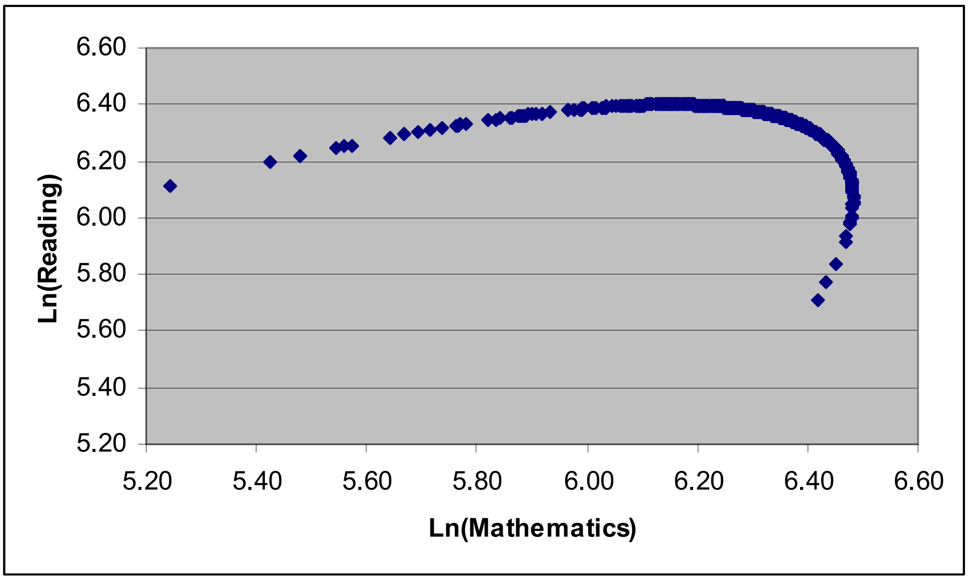

Figure 3
The deterministic output distance function: a measurement of distance slack