Fixed-Term and Permanent Employment Contracts: Theory and Evidence

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Abstract: This paper constructs a theory of the coexistence of fixed-term and permanent employment contracts in an environment with ex ante identical workers and employers. Workers under fixed-term contracts can be dismissed at no cost while permanent employees enjoy labor protection. In a labor market characterized by search and matching frictions, firms find it optimal to discriminate by offering some workers a fixed-term contract while offering other workers a permanent contract. Match-specific quality between a worker and a firm determines the type of contract offered. We analytically characterize the firms' hiring and firing rules. Using matched employer-employee data from Canada, we estimate the wage equations from the model. The effects of firing costs on wage inequality vary dramatically depending on whether search externalities are taken into account.

JEL classification: H29, J23, J38, E24

Key words: employment protection, unemployment, dual labor markets, wage inequality

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Fixed-Term and Permanent Employment Contracts: Theory and Evidence

1 Introduction

The existence of two-tiered labor markets in which workers are segmented by the degree of job protection they enjoy is typical in many OECD countries. Some workers, which one could label temporary (or fixed-term) workers, enjoy little or no protection. They are paid relatively low wages, experience high turnover, and transit among jobs at relatively high rates. Meanwhile, other workers enjoy positions where at dismissal the employer faces a firing tax or a statutory severance payment. These workers' jobs are more stable; they are less prone to being fired, and are paid relatively higher wages. The menu and structure of available contracts is oftentimes given by an institutional background who seeks some policy objective . Workers and employers, however, can choose from that menu and agree on the type of relationship they want to enter.

This paper examines the conditions under which firms and workers decide to enter either a permanent or a temporary relationship. Intuitively firms should always opt for offering workers the contract in which dismissal is free, not to have their hands tied in case the worker under-performs. We construct a theory, however, in which match-quality between a firm and a worker determines the type of contract chosen. By match quality we mean the component of a worker's productivity that remains fixed as long as the firm and the worker do not separate and that is revealed at the time the firm and the worker meet. Firms offer workers with low match-quality a fixed-term contract, which can be terminated at no cost after one period and features a relatively low wage. If it is not terminated, the firm agrees to promote the worker and upgrade the contract into a permanent one, which features a higher wage and it is relatively protected by a firing tax. Firms find optimal to offer high-quality matches a permanent contract because temporary workers search on the job. Facing the risk of losing a good worker, the firm ties its hands promising to pay the tax in case of termination and remunerating the worker with a higher wage. Endogenous destruction of matches, both permanent and temporary, arises from changes in a time-varying component of a worker's productivity: if these changes are negative enough, they force firms to end relationships.

Our set-up is tractable enough to allow us to characterize three cut-off rules. First, we show that there exists a cut-off point in the distribution of match-specific shocks above which the firm offers a permanent contract, and below which the firm offers a temporary contract. There is also a cut-off point in the distribution of the time-varying component of productivity below which the relationship between a temporary worker and a firm ends and above which it continues. Finally, we show the existence of a cut-off point also in the distribution of the time-varying component of productivity below which the relationship between a permanent worker and a firm ends and above which it continues.

Naturally, workers stay longer in jobs for which they constitute a good-match. Permanent workers enjoy stability and higher pay. Temporary workers on the other hand experience high job-to-job transition rates in lower-paid jobs while they search for better opportunities. We emphasize that our theory delivers all of these results endogenously.

The paper does not examine the social or policy goals that lead some societies to establish firing costs or to regulate to some degree the relationships between workers and employers. Rather, we build a framework in which the menu of possible contracts is given by an institutional background that we do not model explicitly. We then use this framework to evaluate under what conditions employers and workers enter in to temporary or permanent relationships. Not addressing the reasons for why governments introduce firing costs does not preclude us from making positive statements about the effects of changing those firing policies. This is precisely the goal of the second part of the paper: to quantitatively evaluate how the existence of firing costs helps shape the wage distribution. To perform this quantitative evaluation, we apply the theory to the economy of Canada. We choose to study the Canadian economy for two reasons. First, it has a rich enough dataset that allows us to distinguish workers by type of contract. Second, it is an economy with a significant amount of temporary workers who represent 14% of the total workforce. We use the Workplace and Employee Survey (WES), a matched employer-employee dataset, to link wages of workers to average labor productivities of the firms that employ them. This relationship, together with aggregate measures of turnover for permanent and temporary workers also obtained from the WES, forms the basis for our structural estimation procedure. We employ a simulated method of moments - indirect inference approach to structurally estimate the parameters of the model. The method uses a Markov Chain Monte Carlo algorithm proposed by Chernozhukov and Hong (2003) that overcomes computational difficulties often encountered in simulation-based estimation.

Having estimated the vector of structural parameters, we use the model to assess the impact of firing costs on income inequality. We find that this impact greatly depends on whether one allows the level of labor market tightness to vary with the policy change or not. If the ins and outs of unemployment into employment, and vice-versa, do not change the job-finding and job-filling probabilities, inequality rises substantially. The reason for this rise is twofold. On the one hand, the relative wage of an average permanent worker rises relative to that of an average temporary worker. This is caused by the firm wanting to hire more productive permanent workers to lower the probability of having to fire them and pay the higher firing cost. On the other hand, the increase in firing costs causes the fraction of temporary workers to rise because they are relatively less expensive. However, if the degree of labor market tightness is allowed to adjust, the fraction of temporary workers falls because there are fewer upgrades of temporary contracts into permanent contracts (i.e. there are fewer promotions). The higher ratio of permanent to temporary wages still obtains, but the lower fraction of temporary workers lowers the variance of the wage distribution. The result is that increasing the firing costs has a very small effect on inequality.

To the best of our knowledge, the literature lacks a theory of the existence of two-tiered labor markets in which some some worker-firm pairs begin relationships on a temporary basis and other worker-firm pairs on a permanent basis.[1] Again, by temporary and permanent relationships we have something specific in mind; namely contracts with different degrees of labor protection. Our study is not the first one that analyzes this question within a theoretical or quantitative framework, so by theory we mean not assuming an ex-ante segmentation of a labor market into temporary workers or permanent workers. This segmentation can occur for a variety of reasons: related to technology (e.g. assuming that workers under different contracts are different factors in the production function); due to preferences - assuming that workers value being under a permanent contract differently than being under a temporary contract), or that they are subject to different market frictions. There are several examples which feature such an assumption: Wasmer (1999), Alonso-Borrego, Galdón-Sánchez, and Fernández-Villaverde (2006), or Bentolila and Saint-Paul (1992). Blanchard and Landier (2002) take a slightly different route, associating temporary contracts with entry-level positions: a worker begins a relationship with a firm in a job with a low level of productivity. After some time, the worker reveals her true - perpetual - productivity level.[2] If such level is high enough, the firm will retain the worker offering her a contract with job security.[3] Cahuc and Postel-Vinay (2002) construct a search and matching framework to analyze the impact on several aggregates

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[1] In the data, many workers that meet a firm for the first time are hired under a permanent contract.

[2] Faccini (2009) also motivates the existence of temporary contracts as a screening device. In his work, as in Blanchard and Landier, all relationships between workers and firms begin as temporary.

[3] A theory somewhat related to ours is due to Smith (2007). In a model with spatially segmented labor markets, it is costly for firms to re-visit a market to hire workers. This leads firms to hire for short periods of time if they expect the pool of workers to improve shortly and to hire for longer time periods if the quality of workers currently in a market is high. He equates a commitment by a firm to never revisit a market, as permanent duration employment. The route we take is to specify a set of contracts that resemble arrangements observed in many economies and ask when do employers and workers choose one arrangement over another.

of changing firing costs. Their concept of temporary and permanent workers is similar to the one used here. However, it is the government that determines randomly what contracts are permanent and which are temporary. In other words, the fraction of temporary worker is itself a policy parameter. That model is unable to answer why these two contracts can co-exist in a world with *ex-ante* identical agents. The fraction of temporary workers ought to be an endogenous outcome and this endogeneity should be a necessary ingredient in any model that analyzes policies in dual labor markets. In the development literature, Bosch and Esteban-Pretel (2009) construct a similar approach to analyzing informal versus formal labor markets.[4]

None of the studies mentioned in this summary of the literature is concerned with building a theory that explains why firms and workers begin both temporary and permanent relationships and analyzing policy changes once that framework has been built. We build such a theory, estimate its parameters and analyze its policy implications for wage inequality in the subsequent sections.

2 Economic Environment

We assume an labor market populated by a unit mass of *ex-ante* identical workers. These workers can be either employed or unemployed as a result of being fired and hired by firms. The mass of firms is potentially infinite. Unemployed workers search for jobs and firms search for workers. A technology to be specified below determines the number of pairwise meetings between employers and workers. We depart from standard search and matching models of labor markets (e.g. Mortensen and Pissarides (1994)) by assuming that two types of contracts are available. The first type - which we label a permanent contract -

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[4] There is a related branch of the literature that looks at the effect of increasing firing taxes on job creation, job destruction and productivity. An example is Hopenhayn and Rogerson (1993). They find large welfare losses of labor protection policies as they interfere with labor reallocation from high productivity firms to low productivity firms. Other examples would be Bentolila and Bertola (1990) or Álvarez and Veracierto (2000,2006).

has no predetermined length, but we maintain, however, the typical assumption of wage renegotiation at the beginning of each period. Separating from this kind of contract is costly. If a firm and a worker under a *permanent* contract separate, firms pay a firing tax f that is rebated to all workers as a lump-sum transfer τ. The second type of contract - a *temporary* contract - has a predetermined length of one period. Once that period is over, the employer can fire the worker at no cost. If the firm and the worker decide to continue the relationship, the temporary contract is upgraded to a permanent one. This upgrade - which one could label a *promotion* costs the firm a small fee c.

The production technology is the same for the two types of contracts. If a firm hires worker i, the match yields $z_i + y_{i,t}$ units of output in period t. The random variable z represents match-quality: a time-invariant - while the match lasts - component of a worker's productivity which is revealed at the time of the meeting. In our theory, the degree of match-quality determines the type of contract agreed upon by the firm and the worker. This match-specific shock is drawn from a distribution $G(z)$. The time-varying component $y_{i,t}$ is drawn every period from a distribution $F(y)$ and it is responsible for endogenous separations. From our notation, it should be clear to the reader that both shocks are independent across agents and time. The supports of the distributions of both types of shocks are given by $[y_{min}, y_{max}]$ and $[z_{min}, z_{max}]$ and we will assume throughout that $y_{min} < y_{max} - c - f$

A matching technology determines the number of pairwise meetings between workers and employers. This technology displays constant returns to scale and implies a job-finding probability $\alpha^w(\theta)$ and a vacancy-filling probability $\alpha^f(\theta)$. which are both functions of market tightness θ. The job-finding and job-filling rates satisfy the following conditions: $\alpha^{w\prime}(\theta) > 0$, $\alpha^{f\prime}(\theta) < 0$ and $\alpha^w(\theta) = \theta\alpha^f(\theta)$. The market tightness is defined as the ratio of the number of vacancies to number of workers searching for jobs. Every time a firm decides to post a vacancy, it must pay a cost k per vacancy posted. If a firm and a worker meet, z is revealed and observed by both parties. The realization of y, however, occurs after the worker and the firm have agreed on a match and begun their relationship.

Let us first fix some additional notation:

- Q : Value of a vacancy.
- U : Value of being unemployed.
- V^P : Value of being employed under a permanent contract.
- V^R : Value of being employed following promotion from a temporary position to a permanent one.
- V^T : Value of being employed under a temporary contract.
- J^P : Value of a filled job under a permanent contract.
- J^R : Value of a filled job that in the previous period was temporary and has been converted to permanent.
- J^T : Value of a filled job under a temporary contract.

It will be convenient to define by,

$$A \equiv \left\{ z \in [z_{min}, z_{max}] \,|E_y J^P(y,z) \geq E_y J^T(y,z) \right\}$$

the set of realizations of z for which the firm prefers to offer a permanent contract. For convenience, let $\mathbb{I}_A$ denote an indicator function defined as,

$$\mathbb{I}_A = \begin{cases} 1 & z \in A, \\ 0 & z \notin A. \end{cases}$$

We now turn to define some recursive relationships that must hold between asset values of vacant jobs, filled jobs, and employment and unemployment states. Let us begin by describing the law of motion for the asset value of a vacancy:

$$\begin{aligned} Q &= -k + \beta\alpha^f(\theta)\int_{z_{min}}^{z_{max}} \max\left(E_y J^P(y,z), E_y J^T(y,z)\right) dG(z) \\ &\quad +\beta\left(1-\alpha^f(\theta)\right) Q, \end{aligned} \tag{1}$$

This equation simply states that the value of a vacant position is the expected payoff from that vacancy net of posting costs k. Both workers and firms discount expected payoffs with a factor β. The firm forecasts that with probability $\alpha^f(\theta)$, the vacant position gets matched to a worker, turning the vacancy into either a permanent job, or a temporary job, depending on the realization of the match-specific shock.[5] With probability $1-\alpha^f(\theta)$ the vacant position meets no worker and the continuation value for the firm is having that position vacant. The following equation states the value of being unemployed as the sum of the flow from unemployment benefits b and the lump-sum transfer τ plus the discounted value of either being matched to an un-filled job - which happens with probability $\alpha^w(\theta)$ - or remaining unemployed.

$$\begin{aligned} U &= b + \tau + \beta\alpha^w(\theta)\int_{z_{min}}^{z_{max}} \left[\mathbb{I}_A E_y V^P(y,z) + (1-\mathbb{I}_A) E_y V^T(y,z)\right] dG(z) \\ &\quad +\beta\left(1-\alpha^w(\theta)\right) U, \end{aligned} \tag{2}$$

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[5] In principle the vacancy could remain unfilled if the value of the match-specific shock is low. Specifically, for a firm and a worker to match, the drawn value of z must be greater than $b - E(y)$, where b is the unemployment benefit. This should not be obvious to the reader at this point, but it will be once we reach equation (15). To avoid notational clutter we eliminate the possibility of meetings left un-matched when we describe the model economy and we impose $z > b - E(y)$ in our estimation procedure. Consequently, all meetings turn into matches. Modifying our setup to allow for the possibility of certain meetings left unmatched is straightforward.

We now turn to describing the value of being employed which will depend on the type of contract agreed upon between the worker and the firm. In other words, the value of being employed under a permanent contract differs from being employed under a temporary contract. We begin by describing the evolution of V^P, the value being employed under a permanent contract, given by:

$$V^P\left(y,z\right) = w^P\left(y,z\right) + \tau + \beta \int_{y_{min}}^{y_{max}} \max\left(V^P\left(x,z\right), U\right) dF\left(x\right), \tag{3}$$

The flow value of being employed under a permanent contract is a wage $w^P(y,z)$; the discounted continuation value is the maximum of quitting and becoming unemployed or remaining in the relationship. As the match-specific shock is time-invariant, only changes in the time-varying productivity drive separations and changes in the wage. However, note that the firing decision occurs before production can even take place: the realization of y that determines the wage is not the realization of y that determines the continuation of the relationship.

The worker employed under a temporary contract earns $w^T(y,z)$. At the end of the period, she searches for alternative employment. The job finding probability the worker faces is the same as that faced by the unemployed. Should the temporary worker not find a job, she faces the promotion decision after her new productivity level is revealed. She becomes unemployed if her realization of y falls below a threshold to be defined later. Formally,

$$\begin{aligned} V^T\left(y,z\right) &= w^T\left(y,z\right) + \tau + \beta\left(1-\alpha^w\left(\theta\right)\right) \int_{y_{min}}^{y_{max}} \max\left(V^R\left(x,z\right), U\right) dF\left(x\right) \\ &\quad + \beta\alpha^w\left(\theta\right) \int_{z_{min}}^{z_{max}} \left[\mathbb{I}_A E_y V^P\left(y,x\right) + \left(1-\mathbb{I}_A\right) E_y V^T\left(y,x\right)\right] dG\left(x\right). \end{aligned} \tag{4}$$

After earning $w^T(y,z)$ for one period, conditional on her time-varying productivity not being too low, the worker has a chance of being “promoted”. This promotion costs the firm c and earns the worker a larger salary $w^R(y,z)$. This salary is not at the level of $w^P(y,z)$, as the firm has to face the cost c, but it is higher than $w^T(y,z)$. The worker earns this higher salary for one period, and as long as she does not separate from the firm, she will earn $w^P(y,z)$ in subsequent periods. Consequently, the value of a just-promoted worker evolves as,

$$V^R(y,z) = w^R(y,z) + \tau + \beta \int_{y_{min}}^{y_{max}} \max\left(V^P(x,z), U\right) dF(x), \tag{5}$$

Regarding capital values of filled positions, the flow profit for a firm is given by the total productivity of the worker, $y+z$, net of the wage paid. This wage is contingent on the type of contract the worker is under. In the case of a just-promoted worker, the firm must pay a cost c to change the contract from temporary to permanent. The asset values of filled jobs under permanent, promoted, and temporary contracts are given by,

$$J^P(y,z) = y + z - w^P(y,z) + \beta \int_{y_{min}}^{y_{max}} \max\left(J^P(x,z), Q - f\right) dF(x), \tag{6}$$

$$J^R(y,z) = y + z - w^R(y,z) - c + \beta \int_{y_{min}}^{y_{max}} \max\left(J^P(x,z), Q - f\right) dF(x), \tag{7}$$

$$\begin{aligned} J^T(y,z) = y + z - w^T(y,z) + \beta\left(1 - \alpha^w(\theta)\right) \int_{y_{min}}^{y_{max}} \max\left(J^R(x,z), Q\right) dF(x) \\ + \beta\alpha^w(\theta) Q, \end{aligned} \tag{8}$$

Using the definition of $\mathbb{I}_A$, the value of a vacancy, equation (1) can be re-written as:

$$\begin{aligned} Q = & -k + \beta\alpha^f(\theta) \int_{z_{min}}^{z_{max}} \left[\mathbb{I}_A E_y J^P(y,z) + \left(1 - \mathbb{I}_A\right) E_y J^T(y,z)\right] dG(z) \\ & + \beta\left(1 - \alpha^f(\theta)\right) Q. \end{aligned} \tag{9}$$

So far we have been silent about wage determination. Following much of the search and matching literature we assume that upon meeting, firms and workers Nash-bargain over the total surplus of the match. Clearly, the sizes of the surpluses will vary depending on whether the worker and the firm agree on a temporary contract or a permanent contract. We assume that workers and firms compute the sizes of the different surpluses and choose the largest one as long as it is positive. Since we have three different value functions for workers and firms, we have three different surpluses depending on the choices faced by employers and workers.

Denoting by ϕ the bargaining power of workers, the corresponding total surpluses for each type of contract are given by:

$$\begin{aligned} S^P(y,z) &= J^P(y,z) - (Q - f) + V^P(y,z) - U, \\ S^R(y,z) &= J^R(y,z) - Q + V^R(y,z) - U, \\ S^T(y,z) &= J^T(y,z) - Q + V^T(y,z) - U. \end{aligned}$$

As a result of the bargaining assumption, surpluses satisfy the following splitting rules:

$$\begin{aligned} S^P(y,z) &= \frac{J^P(y,z) - Q + f}{1-\phi} = \frac{V^P(y,z) - U}{\phi}, \\ S^R(y,z) &= \frac{J^R(y,z) - Q}{1-\phi} = \frac{V^R(y,z) - U}{\phi}, \\ S^T(y,z) &= \frac{J^T(y,z) - Q}{1-\phi} = \frac{V^T(y,z) - U}{\phi}. \end{aligned} \tag{10}$$

Free entry of firms takes place until any rents associated with vacancy creation are exhausted, which in turn implies an equilibrium value of a vacancy Q equal to zero. Replacing Q with its equilibrium value of zero in equation (1) results in the free-entry condition:

$$k = \beta\alpha^{f}(\theta)\int_{z_{min}}^{z_{max}} \max\left(E_{y}J^{P}(y,z), E_{y}J^{T}(y,z)\right) dG(z)$$

The interpretation of this equation is that firms expect a return equal to the right-hand-side of the expression, to justify paying k.Combining equation (9) with the free entry condition and using the surplus sharing rule in (10), we can derive the following relationship:

$$\int_{z_{min}}^{z_{max}} \left[\mathbb{I}_{A}E_{y}S^{P}(y,z) + (1-\mathbb{I}_{A})E_{y}S^{T}(y,z)\right] dG(z) = \frac{k+\beta\alpha^{f}(\theta)\mu_{G}(A)f}{(1-\phi)\beta\alpha^{f}(\theta)}, \tag{11}$$

where $\mu_{G}(A)$ is the probability measure of A. Equation (11) says that the expected surplus - before firms and workers meet - is equal to the sum of two components. The first component, given by $\frac{k}{(1-\phi)\beta\alpha^{f}(\theta)}$, is the expected value of a filled job divided by $(1-\phi)$. This is another way of rewriting the surplus in a model with no firing costs and obtains in other models of search and matching in labor markets. The introduction of firing costs implies the total surplus needs to include the second component, $\frac{k+\beta\alpha^{f}(\theta)\mu_{G}(A)f}{(1-\phi)\beta\alpha^{f}(\theta)}$. This is the "compensation" to the firm for hiring a permanent worker - which occurs with probability $\alpha^{f}(\theta)\mu_{G}(A)$ and having to pay the firing cost f. Using this relationship together with equation (10) to substitute into equation (2), one can rewrite an expression for the value of being unemployed as,

$$U = \frac{1}{1-\beta}\left\{b+\tau+\frac{\phi\alpha^{w}(\theta)\left(k+\beta\alpha^{f}(\theta)\mu_{G}(A)f\right)}{(1-\phi)\alpha^{f}(\theta)}\right\}. \tag{12}$$

The value of unemployment can be decomposed into two components: a flow value represented by $b+\tau$ and an option value represented by the large fraction on the right-hand-side. Closer inspection facilitates the interpretation of that option value. Note that the expected surplus given by equation (11) equals this option value divided by $\phi\alpha^w(\theta)$. The worker, by being unemployed and searching, has the chance of finding a job, which happens with probability $\alpha^w(\theta)$, and obtaining a share ϕ of the expected surplus of that match.

Substituting equation (12) into equations (3)-(8) and using (10), yields the following convenient form of rewriting the surpluses under different contracts.

$$\begin{aligned}
S^P(y,z) &= y + z + \beta\int_{y_{min}}^{y_{max}} \max\left(S^P(x,z),0\right) dF(x) + (1-\beta) f \\
&\qquad -b - \frac{\phi\alpha^w(\theta)\left(k + \beta\alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)}, && (13)\\
S^R(y,z) &= y + z + \beta\int_{y_{min}}^{y_{max}} \max\left(S^P(x,z),0\right) dF(x) - c - \beta f \\
&\qquad -b - \frac{\phi\alpha^w(\theta)\left(k + \beta\alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)}, && (14)\\
S^T(y,z) &= y + z + \beta\left(1-\alpha^w(\theta)\right)\int_{y_{min}}^{y_{max}} \max\left(S^R(x,z),0\right) dF(x) - b. && (15)
\end{aligned}$$

In all three cases the continuation values for the surpluses are bounded below by zero. They cannot be negative because were the drawn value of y to imply a negative surplus, workers and firms would separate before production took place. Proposition 1 shows the existence of these values of y - conditional on the type of contract and the match-specific quality of the match - such that the relationship between a worker and a firm ends. Before stating that proposition we assume the following:

Assumption 1 *Suppose θ is bounded and belongs to $[\theta_{min}, \theta_{max}]$, i.e., $0 \leq \alpha^w(\theta_{min}) < \alpha^w(\theta_{max}) \leq 1$ and $0 \leq \alpha^f(\theta_{max}) < \alpha^f(\theta_{min}) \leq 1$. The following inequalities hold for exogenous parameters:*

$$y_{max} + z_{min} > b + \frac{\phi}{1-\phi} \left(\theta_{max} k + \beta \alpha^w \left(\theta_{max}\right) f\right) - \left(1-\beta\right) f, \tag{16}$$

$$b + \frac{\phi}{1-\phi} \theta_{min} k - \left(1-\beta\right) f > y_{min} + z_{max} + \beta \int_{y_{min}}^{y_{max}} \left(1 - F\left(x\right)\right) dx \tag{17}$$

Assumption 2 *In addition,*

$$y_{max} + z_{min} - c - f > b + \frac{\phi}{1-\phi} \left(\theta_{max} k + \beta \alpha^w \left(\theta_{max}\right) f\right) - \left(1-\beta\right) f. \tag{18}$$

Proposition 1 *Under Assumption 1, for any* z*, there exists an unique cut-off value* $y^P\left(z\right) \in \left(y_{min}, y_{max}\right)$ *and such that* $S^P\left(y^P\left(z\right), z\right) = 0$*. If Assumption 2 also holds then the unique cut-off value* $y^R\left(z\right) \in \left(y_{min}, y_{max}\right)$ *exists where* $S^R\left(y^R\left(z\right), z\right) = 0$*. The cut-off values solve the following equations:* [6]

$$y^P + z + \beta \int_{y^P}^{y_{max}} \left(1 - F\left(x\right)\right) dx = \quad b - \left(1-\beta\right) f + \frac{\phi \alpha^w\left(\theta\right)\left(k + \beta \alpha^f\left(\theta\right) \mu_G\left(A\right) f\right)}{\left(1-\phi\right) \alpha^f\left(\theta\right)}, \tag{19}$$

$$y^P + c + f = y^R. \tag{20}$$

Proposition 2 establishes the existence and uniqueness of a cut-off point $\bar{z}$ above which a firm and a worker begin a permanent relationship.

Proposition 2 *There exists a unique cut-off value* $\bar{z} \in \left[z_{min}, z_{max}\right]$ *such that when* $z > \bar{z}$ *the firm only offers a permanent contract, while* $z < \bar{z}$*, only temporary contract is offered*

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[6] Proofs for all propositions stated in the main body of the paper are relegated to an Appendix.

if the following conditions hold:

$$\beta\left[\int_{y^P(z_{min})}^{y_{max}}(1-F(x))\,dx-(1-\alpha^w(\theta_{max}))\int_{y^R(z_{min})}^{y_{max}}(1-F(x))\,dx\right] < \left[\frac{1}{1-\phi}-(1-\beta)\right]f+\frac{\phi}{(1-\phi)}\theta_{min}k \tag{21}$$

and,

$$\left[\frac{1}{1-\phi}-(1-\beta)\right]f+\frac{\phi}{(1-\phi)}\left(\theta_{max}k+\beta\alpha^w(\theta_{max})f\right) < \beta\left[\int_{y^P(z_{max})}^{y_{max}}(1-F(x))\,dx-(1-\alpha^w(\theta_{min}))\int_{y^R(z_{max})}^{y_{max}}(1-F(x))\,dx\right] \tag{22}$$

To obtain expressions for wages paid under different contracts we can substitute the value functions of workers and firms into the surplus sharing rule (10), which gives:

$$w^P(y,z)=\phi(y+z)+(1-\phi)b+\phi\left[(1-\beta)f+\frac{\alpha^w(\theta)}{\alpha^f(\theta)}\left(k+\beta\alpha^f(\theta)\mu_G(A)f\right)\right], \tag{23}$$

$$w^R(y,z)=w^P(y,z)-\phi(c+f), \tag{24}$$

$$w^T(y,z)=\phi(y+z)+(1-\phi)b. \tag{25}$$

Finally, we need to explicitly state how the stock of employment evolves over time. Let u_t denote the measure of unemployment, and n_t^P and n_t^T be the measure of permanent workers and temporary workers. Let's begin by deriving the law of motion of the stock of permanent workers, which is given by the sum of three groups of workers. First, unemployed workers and temporary workers can search and match with other firms and become permanent workers. This happens with probability $\alpha^w(\theta_t)\mu_G(A)$. Second, after the realization of the aggregate shock, the permanent worker remains at the current job. The aggregate quantity of this case is $\int_{z_{min}}^{z_{max}}\mu_F\left(\left[y^P(z),y_{max}\right]\right)dG(z)\,n_t^P$.

Third, some of temporary workers who cannot find other jobs get promoted to permanent workers which adds to the aggregate employment pool for permanent workers by $(1-\alpha^w(\theta_t))\int_{z_{min}}^{\bar{z}} \mu_F\left(\left[y^R(z), y_{max}\right]\right) dG(z)\, n_t^T$. Notice that $\mu_G(A) = 1 - G(\bar{z})$ and $\mu_F([y, y_{max}]) = 1 - F(y)$. The law of motion for permanent workers is then:

$$\begin{aligned} n_{t+1}^P &= \left(u_t + n_t^T\right)\alpha^w(\theta_t)\left(1 - G(\bar{z})\right) + \int_{z_{min}}^{z_{max}} \left[1 - F\left(y^P(z)\right)\right] dG(z)\, n_t^P \\ &\quad + \left(1 - \alpha^w(\theta_t)\right)\int_{z_{min}}^{\bar{z}} \left[1 - F\left(y^R(z)\right)\right] dG(z)\, n_t^T. \end{aligned} \tag{26}$$

Unemployed workers and temporary workers who are unable to find high-quality matches, join the temporary worker pool the following period. Therefore the temporary workers evolve according to:

$$n_{t+1}^T = \left(u_t + n_t^T\right)\alpha^w(\theta_t)\, G(\bar{z}). \tag{27}$$

Since the aggregate population is normalized to unity, the mass of unemployed workers is given by:

$$u_t = 1 - n_t^T - n_t^P.$$

The standard definition of market tightness is slightly modified to account for the on-the-job search activity of temporary workers:

$$\theta_t = \frac{v_t}{u_t + n_t^T}.$$

3 Partial Equilibrium Analysis

To understand the intuition behind some of the results we show in the quantitative section, we perform here some comparative statics in "partial" equilibrium, by which we mean keeping θ constant. The goal is to understand how changes in selected variables impact the hiring and firing decisions.

Proposition 3 *The hiring rule has the following properties:*

1. $d\bar{z}/df > 0$,

2. $\begin{cases} d\bar{z}/d\alpha^w < 0 & \text{when } \phi < \bar{\phi} \\ d\bar{z}/d\alpha^w > 0 & \text{when } \phi > \bar{\phi} \end{cases}$,

3. $d\bar{z}/dc < 0$.

The intuition behind proposition 3 can be illustrated in Figures 1 to 3 . Figure 1 shows the effects of an increase in the firing cost f. This increase has two effects on the (net) value of a filled job.[7] The direct effect causes a drop in the value of a permanent job because the firm has to pay more to separate from the worker. As a result the permanent contract curve shifts downward. An increase in f also increases the job destruction rate of temporary workers by raising the threshold value $\bar{y}^R$, lowering the value of a temporary job. In equilibrium, the first effect dominates resulting in fewer permanent contracts.

Increasing the job finding probability has an ambiguous effect on the hiring decision because it depends on the worker's bargaining power. If it is easier for unemployed workers to find a job, the value of being unemployed increases because the unemployment spell is shortened. This lowers the match surplus since the worker's outside option rises. Therefore, the value of filled jobs falls and (both permanent and temporary) contract curves will shift outward. We call this the *unemployment effect.* However, there are two additional effects on temporary jobs. Since the temporary worker can search on-the-job, the higher job finding probability increases the chance than a temporary worker remains employed. Therefore, the match surplus will go up due to the rise in the value of temporary employment. We call this effect the *job continuation* effect. For workers under temporary contract, these two effects exactly cancel out. On the other hand, the higher

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[7] The size of the surplus determines the type of contract chosen or whether matches continue or are destroyed. By Nash bargaining the value of a filled job is proportional to the total surplus, so it is sufficient to compare the changes in the values of filled jobs to determine the effects on the total surpluses.

job finding probability causes more separations of temporary contracts. This so-called *job turnover* effect will reduce the value of a temporary job which moves the temporary contract curve outward. If a worker has more bargaining power, then the unemployment effect dominates the job turnover effect. This case is depicted in Figure 1. However if the worker's bargaining power is small, the job turnover effect dominates the unemployment effect which leads to fewer temporary workers. The latter case is shown in Figure 2.

Finally, the effect of an increase in promotion costs is depicted in Figure 3. As promotion costs affect only the value of a temporary contract, an increase in c reduces the incentive for promoting a temporary worker. As a result, the value of a temporary job decreases and the temporary contract curve shifts downward.

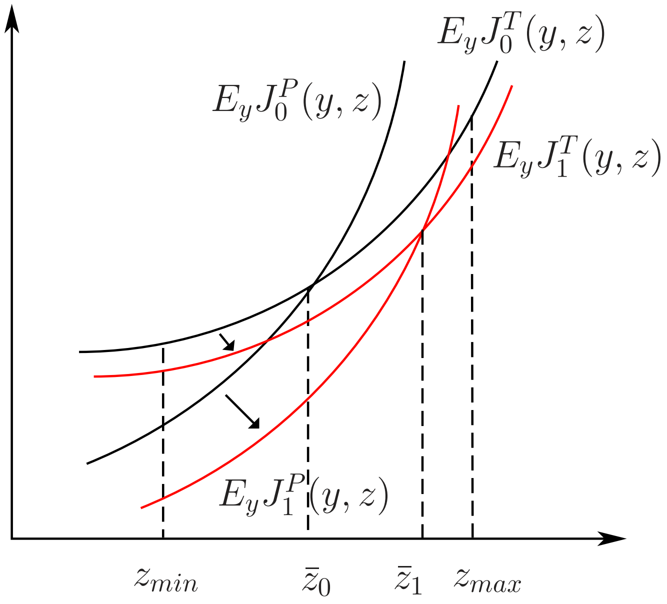

Figure 1: Effect of Firing Costs on Temporary Contracts

Proposition 4 *If the firm has most of the bargaining power, the job destruction rule has the following properties:*

1. $dy^P/df < 0$ *and* $dy^R/df > 0$,

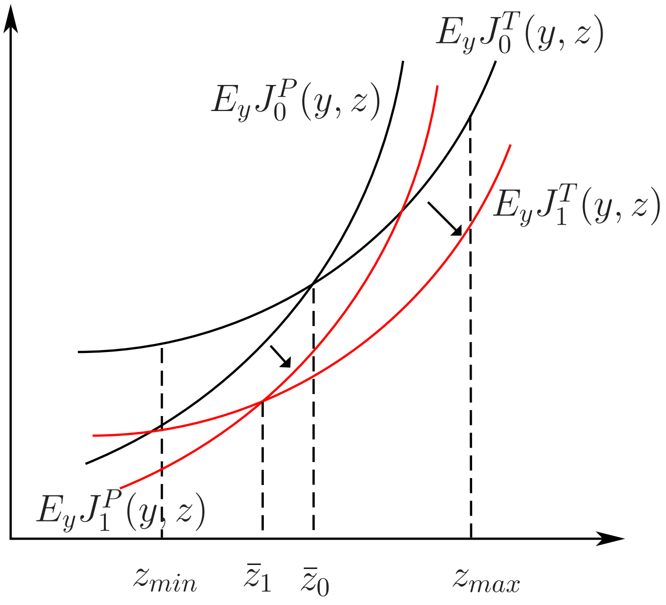

Figure 2: Effect of Job-Finding Probability on Temporary Contracts

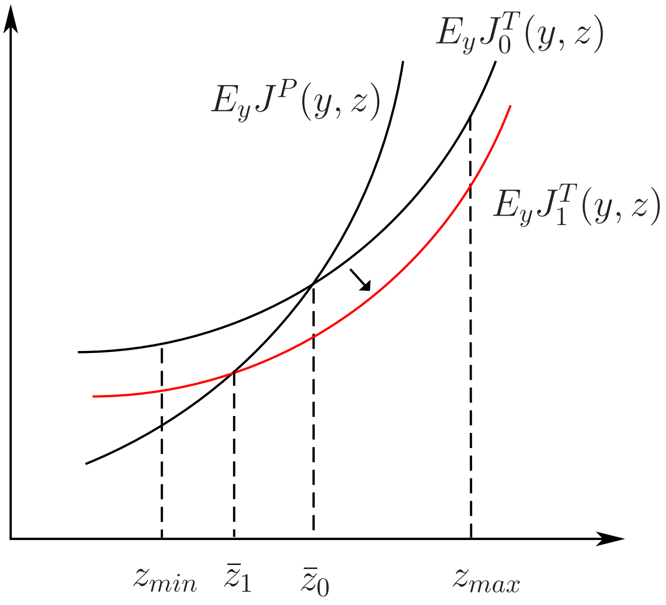

Figure 3: Effects of Promotion Costs on Temporary Contracts

2. $dy^P/d\alpha^w > 0$ *and* $dy^R/d\alpha^w > 0$.

3. y^P *is weakly increasing in* c *and* $dy^R/dc > 0$.

The first part of Proposition 4 states that the firing cost has opposite effects on the separation of permanent jobs and temporary jobs. An increase in the firing cost induces the firm to be less willing to pay the cost to fire a permanent worker. However, it makes the firm more willing to separate from a temporary worker now, in order to avoid paying the higher firing cost in the future. The second part of the proposition results mainly from changing the hiring threshold. The last part is straightforward: an increase in promotion costs discourages the firm to retain the temporary worker.

Finally, we can take the hiring and firing decisions as given and ask how changes in the firing cost and promotion cost affect the job creation (vacancy posting) decision. The following proposition summarizes the results.

Proposition 5 *Given the hiring and permanent job destruction rules, i.e.* $\bar{z}$ *and* $y^P(z)$ *are fixed,* $d\theta/df < 0$ *if* β *is not too small and* $d\theta/dc < 0$*.*

The explanation of this proposition is that an increase in firing costs and promotion costs discourages the firm to post more vacancies by reducing the expected profits of jobs.

4 Data

We used the Workplace and Employee Survey, a Canadian matched employer-employee dataset collected by Statistics Canada. It is an annual, longitudinal survey at the establishment level, targeting establishments in Canada that have paid employees in March, with the exceptions of those operating in the crop and animal production; fishing, hunting and trapping; households', religious organizations, and the government sectors. In 1999, it consisted of a sample of 6,322 establishments drawn from the Business Register maintained by Statistics Canada and the sample has been followed ever since. Every odd year the sample has been augmented with newborn establishments that have become part of the Business Register. The data are rich enough to allow us to distinguish employees by the type of contract they hold. However, only a sample of employees is surveyed from each establishment. [8] The average number of employees in the sample is roughly 20,000 each year. Workers are followed for two years and provide responses on hours worked, earnings, job history, education, and demographic information. Firms provide information about hiring conditions of different workers, payroll and other compensation, vacancies, and separation of workers.

Given the theory laid out above, it is important that the definition of temporary worker in the data matches as close as possible the concept of a temporary worker in the model. In principle, it is unclear that all establishments share the idea of what a temporary worker is when they respond to the survey: it could be a seasonal worker, a fixed-term consultant hired for a project or a worker working under a contract with a set termination date. As a result, Statistics Canada implemented some methodological changes to be consistent in its definition of a temporary worker. This affected the incidence of temporary employment in the survey forcing us to use data only from 2001 onwards. The definition of temporary workers we use, it is of those receiving a T-4 slip from an employer but who have a set termination date. For instance, workers from temporary employment agents or other independent contractors are not included in our definition. With the use of this definition the fraction of temporary workers among all workers is 14%.

Table 1 displays some descriptive statistics on workers' compensation by type of contract held. All quantities are in Canadian dollars and we use three different measures of compensation: total earnings reported by the employee, hourly wages with reported extra-earnings, and hourly wages without the reported extra earnings. According to the three measures, permanent workers earn more but they do work more as well. As a result, while total earnings of permanent workers are roughly double of those earned by temporary workers, when converted to hourly measures, that ratio drops to 1.14-1.15.

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[8] All establishments with less than four employees are surveyed. In larger establishments, a sample of workers is surveyed, with a maximum of 24 employees per given establishment.

Table 1: Worker's Compensation by Type of Contract

	Mean	Standard Deviation
Permanent		
Real Earnings	\$21,847	\$33,525
Real Hourly Wage (No Extra)	\$21.43	\$11.75
Real Hourly Wage	\$22.57	\$14.40
Temporary		
Real Earnings	\$9,737	\$26,469
Real Hourly Wage (No Extra)	\$18.87	\$15.22
Real Hourly Wage	\$19.54	\$18.85

The cross-sectional distribution of wages per hour has a larger variance in the case of temporary workers than of permanent workers. The standard deviation of permanent workers' hourly wages is about half of mean hourly wages. This ratio rises to 81% for temporary workers.

In Canada, job turnover is higher for temporary workers than for permanent workers, as extensively documented by Cao and Leung (2010). We reproduce some of their turnover statistics on Table 2. As it is typical, we measure turnover by comparing job creation and job destruction rates. If we denote by $EMP_{t,i}$ the total level of employment at time t at establishment i, the creation and destruction rates between periods t and $t+1$ are calculated as:

$$Creation = \sum_i \frac{Emp_{t+1,i} - Emp_{t,i}}{0.5(Emp_{t+1} + Emp_t)} \tag{28}$$

if $Emp_{t+1,i} - Emp_{t,i} > 0$ and 0 otherwise. And,

$$Destruction = \sum_i \frac{|Emp_{t+1,i} - Emp_{t,i}|}{0.5(Emp_{t+1} + Emp_t)} \tag{29}$$

if $Emp_{t+1,i} - Emp_{t,i} < 0$ and 0 otherwise.

Given the emphasis of our work on a labor market segmented by temporary and permanent workers, we use the previous expressions to provide measures of job destruction and creation by the type of contract held. However, we measure creation and destruction of temporary (or permanent) workers relative to the average *total* employment level. In other words, we measure the change in the stock of workers by contract type relative to the stock of total employment. These rates are given on the first two lines of Table 2. The job destruction rates are 6.2% for permanent workers and 6.4% for temporary workers. The creation rates are 8.4% and 5.4%. As the fraction of temporary workers is only 14% of the workforce, these rates point to a much higher degree of turnover for temporary workers.

The reader might have noticed that the sum of the destruction rates for temporary and permanent workers is not equal to the destruction rate for all workers. The same can be said for the creation rate. The reason is that establishments can change the number of temporary and permanent workers without altering the stock of all workers. If we restrict the sample to those establishments that increase or decrease the stock of both permanent and temporary workers, the rates for all workers are the sum of the rates of the two types of workers. These measures are reported in Table 2 under the "Alternative Definition" cell. Turnover decreases under this alternative definition, with creation and destruction rates for all workers that are 2% lower than using the conventional definition. The total job creation rate is 8.3% and the job destruction rate is 7.1%.

Table 2: Job Creation and Job Destruction (%)

Conventional Definition	All Workers	Permanent	Temporary
Job Creation	10.2	8.1	5.3
Job Destruction	9.2	6.4	11.5

Alternative Definition	All Workers	Permanent	Temporary
Job Creation	8.3	5.2	3.1
Job Destruction	7.1	4.1	3.0

5 Model Estimation

Our goal is to use our theory to understand patterns of inequality as they relate to employment contracts. More specifically, we want to assess how changes in firing policies affect inequality in wages and this goal demands our theory to be parameterized in a reasonable manner. This section describes the mapping between theory and data, goes over some technicalities of this mapping and shows its results.

Obtaining a solution for the model requires specifying parametric distributions for $G(z)$ and $F(y)$.[9] We assume that y is drawn from a log-normal distribution and z from a uniform distribution. In the model the overall scale of the economy in indeterminate and shifts in the mean of y plus z will have no impact. Consequently, we normalize the mean of y plus z to one, reducing the dimension of the parameter vector of interest. One needs a functional form for the matching technology as well. Denote by B the level of matches given vacancies v and searching workers $n^S = n^T + u$. We assume that matches are formed according to,

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[9] The reader can find much technical detail about our solution and estimation algorithms in a Technical Appendix

$$B(v, N^S) = \frac{vN^S}{(v^\xi + N^{S\xi})^{\frac{1}{\xi}}}.$$

This choice of technology for the matching process implies the following job-finding and job-filling rates, where, again we define $\theta = v/N^S$:

$$\alpha^w(\theta) = \frac{\theta}{(1 + \theta^\xi)^{\frac{1}{\xi}}},$$

and

$$\alpha^f(\theta) = \frac{1}{(1 + \theta^\xi)^{\frac{1}{\xi}}}.$$

Having specified parametric forms for G, F, and the matching technology we are now ready to describe our procedure in detail. Let $\gamma = (f, b, \phi, \xi, k, \mu_y, \mu_z, \sigma_y)$ be the vector of structural parameters that we need to estimate where μ_x and σ_x denote the mean and the standard deviation for a random variable x.[10] and The literature estimating search models is large and much of it has followed full-information estimation methodologies, maximizing a likelihood function of histories of workers.[11] These workers face exogenous arrival rates of job offers (both on and off-the-job) and choose to accept or reject such offers. Parameters maximize the likelihood of observing workers' histories conditional on the model's decision rules. In this paper, we depart from this literature by choosing a partial information approach to estimating our model. Our reason is twofold. First, our search model is an equilibrium one; the arrival rates of job offers are the result of aggregate behavior from the part of consumers and firms. Second, the lack of a panel dimension of the WES does not allow us to perform a maximum likelihood estimation. For these

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[10] Parameters c, β, and σ_z, should be included in the vector γ. We fix β to be 0.96 and c to be 1% of the firing cost f. The standard deviation of z, by our assumption of a uniform distribution, is given by knowing the μ_z and the normalization that $E(y) + E(z) = 1$.

[11] The list is far from being exhaustive but it includes Cahuc *et al.* (2006), Finn and Mabli (2009), Bontemps *et al.* (1999), Eckstein and Wolpin (1990). The reader is referred to Eckstein and Van den Berg (2007) for a survey of the literature that includes many more examples.

reasons, we take a partial-information route and estimate the model by combining indirect inference and simulated method of moments.

The first step involves choosing a set of empirical moments; set with a dimension at least as large as the parameter vector of interest. We estimate the parameters by minimizing a quadratic function of the deviations of those empirical moments from their model-simulated counterparts. Formally,

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}} M(\gamma, \mathbf{Y}_T)' W(\gamma, \mathbf{Y}_T) M(\gamma, \mathbf{Y}_T) \tag{30}$$

where $\hat{\gamma}$ denotes the point estimate for γ, W is a weighting matrix, and M is a column vector whose k-th element denotes a deviation of an empirical moment and a model-simulated moment. The vector $\mathbf{Y}_T$ describes time series data - of length T - from which we compute the empirical moments. The above expression should be familiar to readers, as it is a standard statistical criterion function in the method-of-moments or GMM literatures. Traditional estimation techniques rely on minimizing the criterion function (30) and using the Hessian matrix evaluated at the minimized value to compute standard errors. In many instances equation (30) is non-smooth, locally flat, and have several local minima. For these reasons, we use the quasi-Bayesian Laplace Type Estimator (LTE) proposed by Chernozhukov and Hong (2003). They show that under some technical assumptions, a transformation of (30) is a proper density function (in their language, a *quasi-posterior* density function) As a result, they show how moments of interest can be computed using using Markov Chain Monte Carlo (MCMC) techniques by sampling from that quasi-posterior density. We describe our estimation technique in more detail in the technical appendix, but MCMC essentially amounts to constructing a Markov chain that converges to the density function implied by a transformation of (30). Draws from that Markov Chain are draws from the quasi-posterior, and as a result, moments of the parameter vector such as means, standard deviations, or othe quantities of interest are readily available.

An important aspect of the estimation procedure is the choice of the weighting matrix W. We post-pone a description of how we weight the different moments and we now turn to describe the moments themselves.

Indirect inference involves positing an auxiliary - reduced-form - model which links actual data and model-simulated data. Given our focus on wage inequality, the auxiliary model we chose is a wage regression that links wages, productivity, and the type of contract held. Before being more specific about this regression let us first discuss an identification assumption needed to estimate it. An important element in our model's solution are wages by type of contract which are given by equations (23)-(25). Irrespective of the type of contract wages are always a function of a worker's productivity $y + z$. In the data, such productivity is unobserved; one observes an establishment's total productivity or the productivity for the entire sample. To overcome this difficulty we assume that the time-varying component of productivity y is firm or establishment-specific. Consequently, differences among workers' wages within a firm will be the result of working under a different contract or of having a different match-specific quality. We then posit that a wage of worker i of firm j at time t is given by:

$$w_{ijt} = \beta + \beta_{ALP} ALP_{jt} + \beta_{Type} \chi_{ijt} + \epsilon_{ijt} \tag{31}$$

where ALP_{jt} is an establishment's average labor productivity - output divided by total hours - and χ_{ijt} is an indicator variable describing a worker's temporary status. This is the equation we estimate from the data.[12] A panel of values for ALP_{jt} is easy to obtain, as we have observations on the number of workers and the amount of output per establishment. Note that variations over time in ALP_{jt} arise from changes in the time-varying productivity shock but also from the matches and separations that occur within a establishment over time. If as a result of turnover within a establishment, the mix of

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[12] We take logarithms for wages and ALP_{jt} as our model is stationary and displays no productivity growth.

workers changes- there are more temporary workers in some year, for instance- the average worker productivity will change, even without a change in y_{jt}. Let us now describe what is the analogous equation to (31) we estimate in our model-simulated data. Our theory is silent about firms or establishments; there are only matches of which one can reasonably speak. Note, however, that ALP_{jt} is the sum of the time-varying component y_{jt} plus an expectation of the match-specific productivity z at time t - assuming a large number of workers per establishment. Hence, we simulate a large number of values of y, z, and wages by contract type and regress the logarithm of wages on a constant, the logarithm of the sum of y and the mean simulated z and the contract type. Disturbances in this regression will be interpreted as deviations of the match-specific quality for a given match relative to its mean match-specific value (plus some small degree of simulation error).

Our sample of the WES dataset covers the years 2001 to 2006. We estimate equation (31) for each year which yields a series of estimates $(\beta, \beta_{ALP}, \beta_{Type}, \sigma_\epsilon)$. Returning to our criterion function (30), the first two moments we choose to match are the time-series average of the coefficients β_{ALP} and β_{Type}. For the remaining moments, we choose to include in our vector of moments time-series averages of job-creation and job-destruction (for permanent workers, the job-finding probability, the fraction of temporary workers, and the ratio of wages of permanent workers to those of temporary workers.[13]

Table 3 shows the means and standard deviations of the time series of the moments chosen in our estimation. The deviations of the empirical sample averages from their model counterparts comprise the vector M. Following much of the GMM literature, we weight elements of M according to the inverse of the covariance matrix of the deviations of the time series shown in Table 3 from their model equivalents.

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[13] We thank M. Zhang for sharing her data on the Canadian job-finding rate used in Zhang (2008).

Table 3: Statistical Properties: Empirical Moments

Series	Mean	Standard Deviation
JD	0.092	0.025
JC	0.102	0.018
JD_P	0.064	0.020
JD_T	0.062	0.015
JC_P	0.081	0.021
JC_T	0.053	0.009
$n_t/(1-u)$	0.140	0.040
α^w	0.919	0.004
f/w^P	0.182	0.002
w^p/w^t	1.140	0.034
u	0.071	0.004
βAPL	0.159	0.013
β_{Type}	0.193	0.019

6 Results

Table 4 shows the estimated parameter values along with their standard errors. The point estimates are the quasi-posterior means and the standard errors are the quasi-posterior standard deviations.[14] We estimate a bargaining power of workers ϕ equal to 0.17. This is smaller than values assigned in many calibrated studies but larger than empirical estimates such as Cahuc *et al.* (2006), who find values very close to zero. The estimation yields distributions for y and z whose means are far apart. Given our parametric assumptions $E(y) = e^{\mu_y + 0.5\sigma_y^2} = 1.47$ while the mean of the match-specific shock z is -0.47. These estimated parameters imply moments that we report on the first column of Table 5. The second column of the table reports the equivalent empirical moments. The fit is satisfactory: only one moment is significantly off its empirical value - the fraction of temporary workers. The reader should bear in mind that the fraction of temporary workers from the workers' survey is smaller than the number reported in

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[14] These results are based on 4000 draws of the Markov chain.

Table 4: Posterior Moments

	Mean	Std. Dev
f	0.177	0.003
k	0.030	0.003
b	0.841	0.020
ϕ	0.168	0.007
z_{min}	-0.588	0.029
σ_y	0.154	0.028
ξ	0.894	0.019
μ_y	0.375	0.015

the establishment survey. In the latter the number is the 14% we use as the actual empirical value, but in the workers' survey the number is much smaller; about 5%. The unemployment rate is perhaps another moment in which the model does not give a good fit. It is somewhat difficult to get it to be below 10%.

How do firing costs affect the wage distribution? The model delivers two endogenous objects that are functions of the firing costs and can affect the shape of the wage distribution: first, a different level of wages for each of the two different contracts, and second a fraction of workers under temporary contracts. We perform the experiment of tripling the level of firing costs from the estimated value of $f = 0.177$. Table 6 reports the result from this experiment. The first and the last column of that table show the same numbers as Table 5. The middle column shows the results for the economy with triple the level of firing costs. Increasing f has a modest effect in all moments except obviously the share of wages that the firm has to pays as a firing tax.

As intuition would suggest, creation and destruction of permanent matches drop. The function $Y^P(z)$ shifts downward (i.e. falls for every value of z) and the function $Y^R(z)$ shifts upward. As a result there are fewer promotions of temporary workers and fewer dismissals of permanent workers. The majority of workers work under permanent contracts, which causes the aggregate turnover measures to drop as well. The total destruction rate falls from 10.6% to 8.3% and aggregate creation rate falls from 11% to 8.7%. In

Table 5: Moments: Models vs. Data

	Model	Data
JD	0.106	0.092
JC	0.110	0.102
JD_P	0.105	0.064
JD_T	0.041	0.062
JC_P	0.105	0.081
JC_T	0.068	0.053
$n^T/(1-u)$	0.064	0.140
$\alpha^w(\theta)$	0.920	0.919
f/w^P	0.182	0.182
w^P/w^T	1.123	1.140
u	0.106	0.071
β_{APL}	0.174	0.159
β_{Type}	0.132	0.193

relative terms creation falls less, increasing the stock of employed workers and decreasing the unemployment rate from 10.6% to 8.9%. In light of these results it seems puzzling that turnover measures for temporary workers fall as well. The reason is the measure of creation and destruction we use. The mass of new created temporary jobs rises because $\bar{z}$ drops. But the creation rate is defined as the temporary jobs created divided by total employment. Total employment rises more than new temporary jobs generating the decline in the creation rate. Similar reasoning explains the fall in the job destruction rate. Recall that $Y^R(z)$ shifts upward, causing the hazard rate of losing one's temporary job when one faces the promotion decision, to rise. This increase is what intuition would suggest. Since the destruction rate of temporary contracts takes into account the total stock of employment, it can fall even with $Y^R(z)$ rising.

What is the contribution of the "search externality" in explaining these results? To eliminate the search externality we fix the labor market tightness θ to the value obtained under the estimated parameters.[15] We then triple the firing costs. Fixing θ clearly fixes the job-finding and job-filling probabilities; these do not take into account the flows

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[15] Fixing θ is what we labeled "partial equilibrium" in section 3.

Table 6: Increasing Firing Costs

	f = 0.177	f = 3(0.177)	Data
JD	0.106	0.083	0.092
JC	0.110	0.087	0.102
JD_P	0.105	0.081	0.064
JD_T	0.041	0.036	0.062
JC_P	0.105	0.081	0.081
JC_T	0.068	0.051	0.053
$n^T/(1-u)$	0.064	0.056	0.140
$\alpha^w(\theta)$	0.920	0.891	0.919
f/w^P	0.182	0.544	0.182
w^P/w^T	1.123	1.126	1.140
u	0.106	0.089	0.071
β_{APL}	0.174	0.174	0.159
β_{Type}	0.132	0.135	0.193

in and out of unemployment, the permanent workers pool or the temporary workers pool. Table 7 displays four columns of numbers. The first, second, and fourth columns correspond to three columns shown in Table 6. The third column reports the results for the model-implied moments of tripling the firing costs and keeping θ fixed. There are several large differences relative to the case in which θ can adjust. First, turnover measures increase (when compared to the low f case). In particular, destruction and creation of permanent jobs rises. The higher destruction of permanent jobs can be explained by Proposition 4. It states that if the firm has most of the bargaining power ($\phi \rightarrow 0$) the function $Y^P(z)$ falls when f rises. In our case, although ϕ is not large compared to values typically used in calibrated models, it is sufficiently far from zero to cause a *rise* in $Y^P(z)$. This rise is responsible for the increase in the destruction rate of permanent workers. The intuition comes from equation (13), which shows the value of the surplus under a permanent contract. The last term in that expression is the option value of unemployment. Note that when $\phi \rightarrow 1$, the ratio $\frac{\phi}{1-\phi} \rightarrow \infty$, increasing the option value of unemployment. When we fix θ the only elements that change in that option value are f and $\mu_G(A)$. Everything else remains fixed. If the ratio $\frac{\phi}{1-\phi}$ is large, an increase in f makes that option value have an even larger negative contribution to the surplus, implying that a small drop in y is enough to make the surplus negative. As a result the function $Y^P(z)$ rises, increasing along the destruction rate for permanent workers. This same option value appears in wages of the permanently employed which explains the rise in the relative wage. The larger destruction rate of permanent workers implies a larger share of temporary workers.

What are the implications of all this for the shape of the wage distribution? Figure 4 shows the wage distribution for the three cases discussed. The green line represents the density function of wages (using standard kernel-smoothing methods) when the parameters are set to their quasi-posterior means. If we increase the level of firing costs and do not take into account the search externalities, the result is the red line: higher mean wages, because of the rise in the wages of the permanent workers and a larger fraction of temporary workers (the hump in the distribution between the values of 0.8 and 0.9). The standard deviation of wages rises more than 10% and the wage of the average permanent worker rises relative to the ratio of the average temporary worker. When θ is permitted to adjust, the effects on inequality vanish. There is still a larger option value of unemployment which slightly increases the relative wages of permanent versus temporary workers, but as the fraction of temporary workers falls, inequality remains essentially the same.

7 Concluding Remarks

This study provides a theory of the co-existence of labor contracts with different firing conditions. Consistent with empirical evidence that points to employers choosing among contracts with different degrees of labor protection, firms here choose to offer *ex-ante* identical workers different contracts, and as a result, different wages. The reason is match-quality that varies among worker-firm pairs and that is revealed at the moment firms and workers meet. Firms offer permanent contracts to "good" matches, as they risk losing

Table 7: Inequality: Effect of Search Externalities

	f = 0.177	f = 3(0.177) (θ adjusts)	f = 3(0.177) (θ fixed)	Data
JD	0.106	0.083	0.125	0.092
JC	0.110	0.087	0.130	0.102
JD_P	0.105	0.081	0.12	0.064
JD_T	0.041	0.036	0.068	0.062
JC_P	0.105	0.081	0.120	0.081
JC_T	0.068	0.051	0.062	0.053
$n^T/(1-u)$	0.064	0.056	0.130	0.140
$\alpha^w(\theta)$	0.920	0.891	0.920	0.919
f/w^P	0.182	0.544	0.533	0.182
w^P/w^T	1.123	1.126	1.150	1.140
u	0.106	0.089	0.124	0.071
β_{APL}	0.174	0.174	0.173	0.159
β_{Type}	0.132	0.135	0.154	0.193

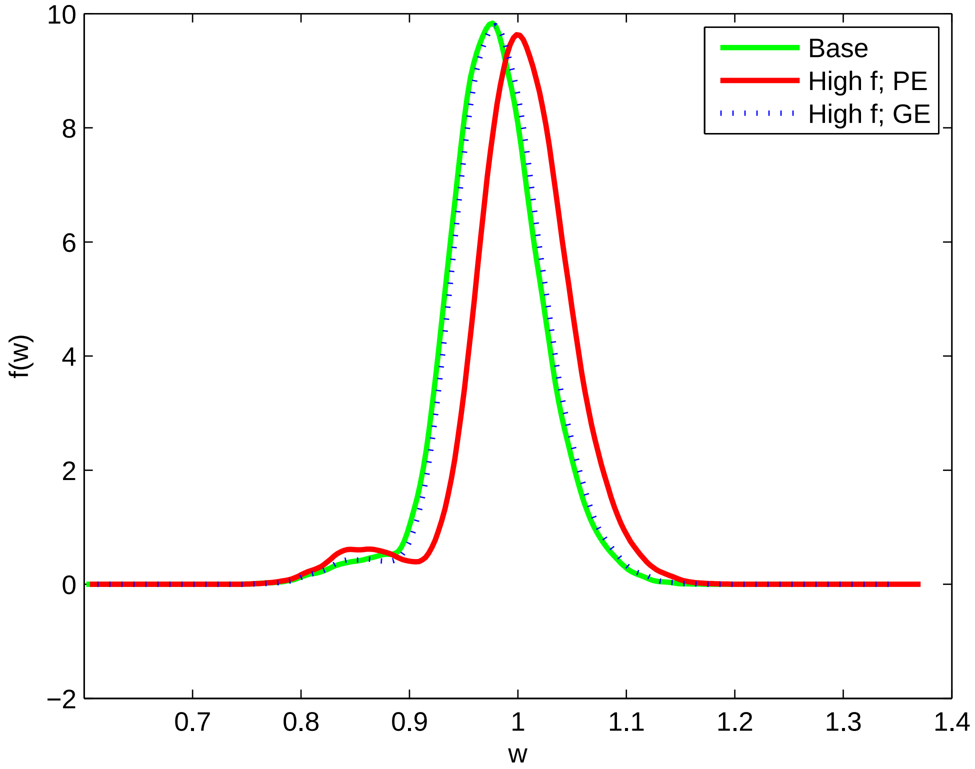

Figure 4: Effects of Promotion Costs on Temporary Contracts

the worker should they offer them a temporary contract. This risk results from the different on-the-job search behavior by the two types of workers: temporary workers search while permanent workers do not. Not-so-good matches are given a temporary contract under which they work for a lower wage but they are allowed to search for alternative opportunities. After one period, temporary workers have to be dismissed or promoted to permanent status.

The existence of search and matching frictions implies that workers might work temporarily in jobs with an inferior match quality, before transferring to better - and more stable - matches. Our assumption of including a time-varying component in the total productivity of a worker allows our environment to generate endogenous destruction rates that differ by type of contract. Our environment is simple enough to deliver several analytical results regarding cut-off rules for the type of relationship firms and workers begin and when and how they separate. Yet, it is rich in its implications.

One of these implications is that we can examine wage inequality from a different perspective. To what extent do firing costs help shape the wage distribution? We show that the answer to this question depends (greatly) on whether we take into account or not search externalities. If flows into and out of unemployment change the probabilities of finding a job or filling a vacancy that workers or firms face, the results are likely to be small. If those flows do not change the job-finding and filling rates, the impact on inequality of rising firing costs can be sizable.

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A Appendix: Proof of Propositions

Proof of Proposition 1. Equation (13) can be written as

$$\begin{aligned} S^P(y,z) &= y + z + \beta \int_{y^P}^{y_{max}} S^P(x,z)\,dF(x) + (1-\beta) f \\ &\quad - b - \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)}. \end{aligned} \tag{32}$$

From the fact that $\partial S^P/\partial y = 1$ and $\partial^2 S^P/\partial y \partial z = 0$, it implies that $S^P(y,z) = y + \varphi(z)$. The integral on the right-hand side of (32) is then

$$\begin{aligned} \int_{y^P}^{y_{max}} S^P(x,z)\,dF(x) &= \int_{y^P}^{y_{max}} x + \varphi(z)\,dF(x), \\ &= (x + \varphi(z)) F(x)\big|_{y^P}^{y_{max}} - \int_{y^P}^{y_{max}} F(x)\,dx. \end{aligned}$$

For any $z \in Z$, $S^P\left(y^P, z\right) = 0$ implies $y^P = -\varphi(z)$. Substitute $\varphi(z)$ with $-y^P$, the expression of the integral is

$$\int_{y^P}^{y_{max}} S^P(x,z)\,dF(x) = \int_{y^P}^{y_{max}} [1 - F(x)]\,dx. \tag{33}$$

To pin down y^P, we need to solve the equation $S^P(y^p, z) = 0$, thus

$$\begin{aligned} y^P + z + \beta \int_{y^P}^{y_{max}} [1 - F(x)]\,dx &= b + \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)} - (1-\beta) f. \\ &= b + \frac{\phi}{(1-\phi)}\left(\theta k + \beta \alpha^w(\theta)\mu_G(A) f\right) - (1-\beta) f. \end{aligned} \tag{34}$$

Denote left-hand side by $\Phi_z(y)$ and right-hand side by $\Phi(\theta)$. Notice that $\Phi(\theta)$ is increasing in θ and $\mu_G(A) \in [0,1]$, thus for any $\theta \in [\theta_{min}, \theta_{max}]$

$$b + \frac{\phi}{1-\phi}\theta_{min} k - (1-\beta) f < \Phi(\theta) < b + \frac{\phi}{1-\phi}\left(\theta_{max} k + \beta \alpha^w(\theta_{max}) f\right) - (1-\beta) f.$$

$\Phi_z(y)$ is increasing in y and z. If inequalities (16) and (17) holds then for given θ and z, we must have

$$\Phi_z(y_{min}) \leq \Phi_{z_{max}}(y_{min}) < \Phi(\theta) < \Phi_{z_{min}}(y_{max}) \leq \Phi_z(y_{max}).$$

We can conclude there is a unique solution $y^P(z) \in (y_{min}, y_{max})$ for equation (34) by the intermediate value theorem. That is, $y^P(z)$ exists for any $z \in [z_{min}, z_{max}]$.

Similarly, equation (14) can be rewritten as

$$S^R(y,z) = y + z + \beta \int_{y^P}^{y_{max}} [1 - F(x)]\, dx - c - \beta f - b - \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)}. \tag{35}$$

Following the same argument for the condition $S^P\left(y^P, z\right) = 0$, the above equation yields the cut-off value by solving:

$$y^R + z + \beta \int_{y^P}^{y_{max}} [1 - F(x)]\, dx = b + \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)\mu_G(A) f\right)}{(1-\phi)\alpha^f(\theta)} + c + \beta f. \tag{36}$$

Comparing equations (34) and (36), we get

$$y^R = y^P + c + f.$$

Then assumption 2 guarantees the existence of $y^P \in (y_{min}, y_{max} - c - f)$ which implies $y^R < y_{max}$ exists as well. ■

Proof of Proposition 2. Step 1. $E_y J^P(y,z)$ and $E_y J^T(y,z)$ are both strictly increasing in z. From the surplus sharing rule, it is sufficient to show that $S^P(y,z)$ and $S^T(y,z)$ are strictly increasing in z. Substitute equation (33) into (32), we obtain

$$S^P(y,z) = y+z+\beta\int_{y^P(z)}^{y_{max}}[1-F(x)]\,dF(x)+(1-\beta)f-b-\frac{\phi\alpha^w(\theta)\left(k+\beta\alpha^f(\theta)\mu_G(A)f\right)}{(1-\phi)\alpha^f(\theta)}. \tag{37}$$

Take the derivative of S^P with respect to z, we get

$$\frac{\partial S^P(y,z)}{\partial z} = 1-\beta\left(1-F\left(y^P(z)\right)\right)y^{P\prime}(z). \tag{38}$$

From equation (34), the implicit function theorem implies that

$$y^{P\prime}(z) = -\frac{1}{1-\beta\left(1-F\left(y^P\right)\right)} < 0. \tag{39}$$

Plug (39) into (38), we get $\partial S^P/\partial z > 0$. Similarly, the total surplus of a temporary contract can be rewritten as

$$S^T(y,z) = y+z+\beta(1-\alpha^w)\int_{y^P(z)+c+f}^{y_{max}}[1-F(x)]\,dx-b. \tag{40}$$

The derivative of S^T with respect to z is given by

$$\frac{\partial S^T(y,z)}{\partial z} = 1-\beta(1-\alpha^w)\left[1-F\left(y^P(z)\right)\right]y^{P\prime}(z) > 0.$$

Step 2. $E_yJ^P(y,z)$ and $E_yJ^T(y,z)$ are strictly convex. By the separability of y and z, it suffices to prove that S^P and $S^T$are convex in z. Twice differentiate S^P with respect to z, and get

$$\frac{\partial^2 S^P(y,z)}{\partial z^2} = \beta\left[F'\left(y^{P\prime}\right)^2-\left(1-F\left(y^P\right)\right)y^{P\prime\prime}\right].$$

Since $y^{P\prime\prime} = \beta F' y^{P\prime}/\left[1-\beta\left(1-F\left(y^P\right)\right)\right]^2 < 0$ and $F' > 0$, it must be the case that $\partial^2 S^P(y,z)/\partial z^2 > 0$. Similarly, $\partial^2 S^T(y,z)/\partial z^2 > 0$.

These two steps guarantee that if $E_y J^P(y,z) = E_y J^T(y,z)$ holds, the cut-off value z is unique. The last step is to verify the single crossing property. That is, if

$$\begin{aligned} E_y J^P(y, z_{min}) &< E_y J^T(y, z_{min}), \\ E_y J^P(y, z_{max}) &> E_y J^T(y, z_{max}) \end{aligned}$$

hold, then the cut-off value $\bar{z}$ exists. Denote

$$\begin{aligned} \Delta_\theta(z) &= \frac{E_y J^P(y,z) - E_y J^T(y,z)}{1-\phi} \\ &= \beta \int_{y^P}^{y_{max}} [1 - F(x)]\, dx - \beta(1 - \alpha^w(\theta)) \int_{y^R}^{y_{max}} [1 - F(x)]\, dx \\ &\quad - \left[\frac{1}{1-\phi} - (1-\beta)\right] f - \frac{\phi}{(1-\phi)} (\theta k + \beta \alpha^w(\theta) \mu_G(A) f). \end{aligned}$$

Let

$$\Gamma(z,\theta) \equiv \beta \int_{y^P}^{y_{max}} [1 - F(x)]\, dx - \beta(1 - \alpha^w(\theta)) \int_{y^R}^{y_{max}} [1 - F(x)]\, dx,$$

and

$$\Lambda(\theta) \equiv \left[\frac{1}{1-\phi} - (1-\beta)\right] f + \frac{\phi}{(1-\phi)} (\theta k + \beta \alpha^w(\theta) \mu_G(A) f).$$

The inequality (21) implies that for any $\theta \in [\theta_{min}, \theta_{max}]$,

$$\Gamma(z_{min}, \theta) \le \Gamma(z_{min}, \theta_{max}) < \Lambda(\theta_{min}) \le \Lambda(\theta).$$

While the inequality (22) implies that for any θ,

$$\Lambda(\theta) \le \Lambda(\theta_{max}) < \Gamma(z_{max}, \theta_{min}) \le \Gamma(z_{max}, \theta).$$

Thus, $\Delta_\theta(z_{min}) < 0 < \Delta_\theta(z_{max})$ for all $\theta \in [\theta_{min}, \theta_{max}]$. Figure 5 shows the single crossing

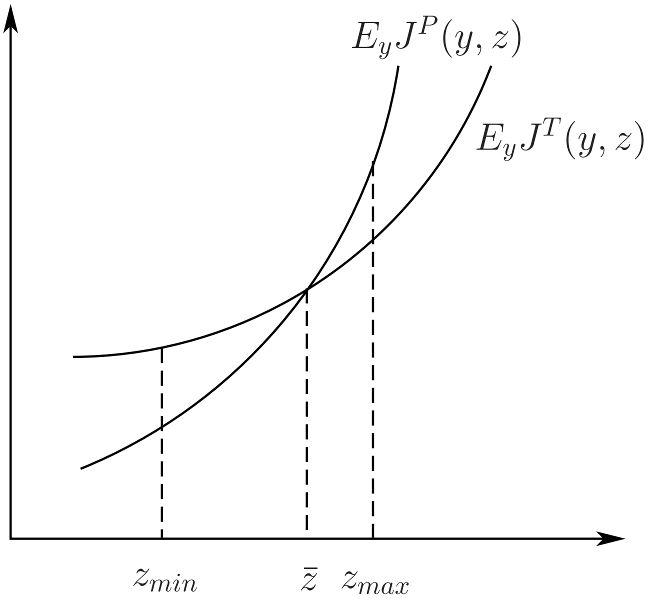

Figure 5: Permanent Contract vs. Temporary Contract

property.

∎

Proof of Proposition 3. The equilibrium condition for $\bar{z}$ is $E_y J^P(y,\bar{z}) = E_y J^T(y,\bar{z})$. By using the sharing rule (10) and equations (13) and (15), it implies that

$$\beta \int_{\bar{y}^P}^{y_{max}} (1-F(x))\,dx - \left[\frac{1}{1-\phi} - (1-\beta)\right] f - \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)(1-G(\bar{z}))f\right)}{(1-\phi)\alpha^f(\theta)} - \beta(1-\alpha^w)\int_{\bar{y}^R}^{y_{max}} (1-F(x))\,dx = 0 \quad (41)$$

where $\bar{y}^P \equiv y^P(\bar{z})$ and $\bar{y}^R \equiv y^R(\bar{z}) = \bar{y}^P + c + f$. From equation (19), we have

$$\bar{y}^P + \bar{z} + \beta \int_{\bar{y}^P}^{y_{max}} (1-F(x))\,dx + (1-\beta) f - b - \frac{\phi \alpha^w(\theta)\left(k + \beta \alpha^f(\theta)(1-G(\bar{z}))f\right)}{(1-\phi)\alpha^f(\theta)} = 0. \quad (42)$$

Denote the left hand sides of equations (41) and (42) by $\Pi\left(\bar{y}^P, \bar{z}\right)$. Totally differentiate Π, we get

$$D_{\left(\bar{y}^P,\bar{z}\right)}\Pi = \begin{bmatrix} 1-\beta\left(1-F\left(\bar{y}^P\right)\right) & 1+\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right) \\ -\beta\left[1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right] & \frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right) \end{bmatrix},$$

and

$$\begin{aligned} D_f\Pi &= \begin{bmatrix} 1-\beta-\frac{\phi}{1-\phi}\alpha^w\beta\left(1-G\left(\bar{z}\right)\right) \\ -\left[\frac{1}{1-\phi}-\left(1-\beta\right)\right]+\beta\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)-\frac{\phi}{1-\phi}\alpha^w\beta\left(1-G\left(\bar{z}\right)\right) \end{bmatrix}, \\ D_{\alpha^w}\Pi &= \begin{bmatrix} -\frac{\phi}{(1-\phi)\alpha^f}\left[k+\beta\alpha^f\left(1-G\left(\bar{z}\right)\right)f\right] \\ \int_{\bar{y}^R}^{y_{max}}\left(1-F\left(x\right)\right)dx-\frac{\phi}{(1-\phi)\alpha^f}\left[k+\beta\alpha^f\left(1-G\left(\bar{z}\right)\right)f\right] \end{bmatrix}, \\ D_c\Pi &= \begin{bmatrix} 0 \\ \beta\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right) \end{bmatrix}. \end{aligned}$$

The determinant of matrix $D_{\left(\bar{y}^P,\bar{z}\right)}\Pi$ is

$$\begin{aligned} \left|D_{\left(\bar{y}^P,\bar{z}\right)}\Pi\right| &= \beta\left[1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right] \\ &\quad +\left[1-\beta\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right]\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right) \\ &> 0, \end{aligned}$$

since $F\left(\bar{y}^P\right) < F\left(\bar{y}^R\right)$ and $G'\left(z\right) > 0$. Apply the implicit function theorem, we can calculate the following:

$$\frac{d\bar{z}}{df} = -\frac{a_1+a_2}{\left|D_{\left(\bar{y}^P,\bar{z}\right)}\Pi\right|},$$

where

$$a_1 = \beta\left[1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right]\left(1-\beta-\frac{\phi}{1-\phi}\alpha^w\beta\left(1-G\left(\bar{z}\right)\right)\right)$$

and

$$\begin{aligned} a_2 &= \left[1-\beta\left(1-F\left(\bar{y}^P\right)\right)\right]\left\{-\left[\frac{1}{1-\phi}-(1-\beta)\right]\right. \\ &\quad \left. +\beta\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)+\frac{\phi}{1-\phi}\alpha^w\beta\left(1-G\left(\bar{z}\right)\right)\right\}. \end{aligned}$$

Observe that the numerator $(a_1 + a_2)$ is decreasing in ϕ. When $\phi = 0$, the numerator becomes

$$\begin{aligned} a_1 + a_2 &= \beta\left(1-\beta\right)\left[1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right] \\ &\quad -\beta\left[1-\beta\left(1-F\left(\bar{y}^P\right)\right)\right]\left[1-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right]. \end{aligned}$$

Because $1-\beta < 1-\beta\left(1-F\left(\bar{y}^P\right)\right)$ and $1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right) < 1-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)$, we must have $a_1 + a_2 < 0$. Hence, for any ϕ, $d\bar{z}/df > 0$.

$$\frac{d\bar{z}}{d\alpha^w} = -\frac{a_3}{\left|D_{\left(\bar{y}^P,\bar{z}\right)}\Pi\right|},$$

where

$$\begin{aligned} a_3 &= \beta\left[1-F\left(\bar{y}^P\right)-\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)\right]\left\{-\frac{\phi}{\left(1-\phi\right)\alpha^f}\left[k+\beta\alpha^f\left(1-G\left(\bar{z}\right)\right)f\right]\right\} \\ &\quad +\left[1-\beta\left(1-F\left(\bar{y}^P\right)\right)\right]\left\{\int_{\bar{y}^R}^{y_{max}}\left(1-F\left(x\right)\right)dx\right. \\ &\quad \left. -\frac{\phi}{\left(1-\phi\right)\alpha^f}\left[k+\beta\alpha^f\left(1-G\left(\bar{z}\right)\right)f\right]\right\}. \end{aligned}$$

Again, a_3 is decreasing in ϕ. When $\phi \to 0$, $a_3 \to \left[1-\beta\left(1-F\left(\bar{y}^P\right)\right)\right]\int_{\bar{y}^R}^{y_{max}}\left(1-F\left(x\right)\right)dx >$

0, while $\phi \to 1$, $a_3 \to -\infty$. Therefore there exists $\bar{\phi}$ such that

$$\begin{cases} \frac{d\bar{z}}{d\alpha^w} < 0 & \text{when } \phi < \bar{\phi} \\ \frac{d\bar{z}}{d\alpha^w} > 0 & \text{when } \phi > \bar{\phi} \end{cases}$$

$$\begin{aligned} \frac{d\bar{z}}{dc} &= -\frac{\left[1-\beta\left(1-F\left(\bar{y}^P\right)\right)\right]\beta\left(1-\alpha^w\right)\left(1-F\left(\bar{y}^R\right)\right)}{\left|D_{(\bar{y}^P,\bar{z})}\Pi\right|} \\ &< 0. \end{aligned}$$

■

Proof of Proposition 4. Suppose $\phi \to 0$, equation (34) implies

$$\begin{aligned} \frac{dy^P}{df} &= -\frac{1-\beta-\frac{\phi}{1-\phi}\alpha^w\beta\left(1-G\left(\bar{z}\right)\right)+\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right)\frac{d\bar{z}}{df}}{1-\beta\left(1-F\left(\bar{y}^P\right)\right)} \\ &< 0, \\ \frac{dy^P}{d\alpha^w} &= -\frac{-\frac{\phi}{(1-\phi)\alpha^f}\left[k+\beta\alpha^f\left(1-G\left(\bar{z}\right)\right)f\right]+\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right)\frac{d\bar{z}}{d\alpha^w}}{1-\beta\left(1-F\left(\bar{y}^P\right)\right)} \\ &> 0, \\ \frac{dy^P}{dc} &= -\frac{\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right)\frac{d\bar{z}}{dc}}{1-\beta\left(1-F\left(\bar{y}^P\right)\right)} \to 0^+. \end{aligned}$$

Use these facts and combine equation (36), we derive

$$\begin{aligned} \frac{dy^R}{df} &= \frac{\beta F\left(y^P\right)-\frac{\phi}{1-\phi}\alpha^w\beta\left[\left(1-G\left(\bar{z}\right)\right)-fG'\left(\bar{z}\right)\frac{d\bar{z}}{df}\right]}{1-\beta\left(1-F\left(\bar{y}^P\right)\right)} > 0, \\ \frac{dy^R}{d\alpha^w} &= \frac{dy^P}{d\alpha^w} > 0, \\ \frac{dy^R}{df} &= 1+\frac{\frac{\phi}{1-\phi}\alpha^w\beta f G'\left(\bar{z}\right)\frac{d\bar{z}}{dc}}{1-\beta\left(1-F\left(\bar{y}^P\right)\right)} > 0. \end{aligned}$$

■

Proof of Proposition 5. The job creation rule is obtained by equation (11). Substitute equations (13) and (15), we get

$$\begin{aligned} & E(y+z)-b+(1-\beta)(1-G(\bar{z}))f \\ & \qquad -\frac{\beta+\phi\alpha^{w}(\theta)(1-G(\bar{z}))}{(1-\phi)\beta\alpha^{f}(\theta)}\left[k+\beta f(1-G(\bar{z}))\alpha^{f}(\theta)\right] \\ & \qquad +\beta(1-\alpha^{w}(\theta))\int_{z_{min}}^{\bar{z}}\int_{y^{P}(z)+c+f}^{y_{max}}(1-F(x))\,dxdz \\ & \qquad\qquad +\beta\int_{\bar{z}}^{z_{max}}\int_{y^{P}(z)}^{y_{max}}(1-F(x))\,dxdz=0 \end{aligned} \tag{43}$$

Denote the left hand side of equation (43) by h and differentiate it with respect to θ, f and c, one gets:

$$\begin{aligned} \frac{\partial h}{\partial \theta} &= -\beta\alpha^{w\prime}\int_{z_{min}}^{\bar{z}}\int_{y^{P}(z)+c+f}^{y_{max}}(1-F(x))\,dxdz \\ & \quad -\frac{k\left[\phi(1-G(\bar{z}))\alpha^{f}\alpha^{w\prime}-\left[\beta+\phi\alpha^{w}(\theta)(1-G(\bar{z}))\right]\alpha^{f\prime}\right]}{(1-\phi)\beta\left(\alpha^{f}\right)^{2}} \\ & \quad -\frac{\beta+\phi\alpha^{w}(1-G(\bar{z}))}{(1-\phi)}(1-G(\bar{z}))\alpha^{w\prime} \\ &< 0, \end{aligned}$$

due to $\alpha^{w\prime}(\theta)>0$ and $\alpha^{f\prime}(\theta)<0$,

$$\begin{aligned} \frac{\partial h}{\partial f} &= (1-\beta)(1-G(\bar{z}))-\frac{\beta+\phi\alpha^{w}(1-G(\bar{z}))}{(1-\phi)}(1-G(\bar{z})) \\ & \quad -\beta(1-\alpha^{w})\int_{z_{min}}^{\bar{z}}\left(1-F\left(y^{R}(z)\right)\right)dz. \end{aligned}$$

$\partial h/\partial f$ is negative provided that $\beta>1/2$. Hence

$$\frac{d\theta}{df}=-\frac{\partial h/\partial f}{\partial h/\partial \theta}<0.$$

Finally, since

$$\frac{\partial h}{\partial c} = -\beta\left(1-\alpha^{w}\right)\int_{z_{min}}^{\bar{z}}\left(1-F\left(y^{R}\left(z\right)\right)\right)dz < 0,$$

we can conclude that

$$\frac{d\theta}{dc} = -\frac{\partial h/\partial c}{\partial h/\partial \theta} < 0.$$

■

B Model Solution and Estimation

This section describes some technical aspects of the solution and estimation algorithms that produce the results shown in section 6. The model described can be defined as a function $\Xi : \Gamma \to \tilde{Y}$, where $\gamma \in \Gamma \subset \mathbb{R}^{n_\gamma}$, and $\tilde{y} \in \tilde{Y} \subset \mathbb{R}^{n_M}$. An element in the set $\tilde{Y}$ can be thought of an endogenous variable (e.g. the unemployment rate) that is an outcome of the model. The estimation procedure uses a statistical criterion function that minimizes the deviations of model-implied moments - weighted appropriately - from empirical moments.

Empirical moments are given by the means of time series that have a model-implied moment as a counterpart. Given a vector of time series of length T denoted by $\mathbf{Y}_T = \{Y_T^1, \ldots, Y_T^{n_M}\}$ define the vector $M_{n_M \times 1}$ as having typical element $m_j = (\tilde{y}(\gamma) - \bar{Y}_T^j)$ with $j = 1, \ldots, n_M$ and $\bar{Y}_T^j = (1/T)\sum_{t=1}^{T} y_t$. We construct the statistical criterion function,

$$H(\gamma, \mathbf{Y}_T) = M(\gamma, \mathbf{Y}_T)' W(\gamma, \mathbf{Y}_T) M(\gamma, \mathbf{Y}_T) \tag{44}$$

We sensibly choose the matrix $W(\gamma, \mathbf{Y}_T)$ to be the inverse of the covariance matrix of $\mathbf{Y}_T$. In our application $n_M = 13$ and $n_\gamma = 8$, since the parameter vector of interest is given by $\gamma = (f, b, \phi, \xi, k, \mu_y, \mu_z, \sigma_y)$. In principle one can obtain an estimate of γ by:

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}} H(\gamma, \mathbf{Y_T}).$$

Minimizing the function $H(\gamma, \mathbf{Y_T})$ by means of standard minimization routines e.g. any optimizer in the family of Newton-type methods, is seldom an easy task. Problems abound, and they include non-differentiabilities, flat areas, and local minima. To obtain estimates of γ we employ a Markov Chain Monte Carlo method (MCMC) that transforms the function $H(\gamma, \mathbf{Y}_T)$ into a proper density function. This transformation is given by:

$$p(\gamma, \mathbf{Y}_T) = \frac{e^{H(\gamma, \mathbf{Y}_T)}}{\int_\Gamma e^{H(\gamma, \mathbf{Y}_T)} d\gamma} \tag{45}$$

where $\pi(\gamma)$ is a prior distribution (or weight function) over the parameter space. This distribution can be uniform which implies a constant $\pi(\gamma)$ and we assume so in the estimation. Chernozhukov and Hong (2003) label $p(\gamma, \mathbf{Y}_T)$ a *quasi-posterior* density because it is not a posterior density function in a true Bayesian sense; there is no updating. It is, however, a proper density function with well-defined moments and as a result we can define, for instance, the quasi-posterior mean as:

$$\hat{\gamma} = \int_\Gamma \gamma p(\gamma, \mathbf{Y}_T) d\gamma \tag{46}$$

In practice, the way we compute the quasi-posterior mean is by a Monte Carlo procedure. Markov Chain Monte Carlo amounts to simulating a Markov Chain that converges to the quasi-posterior distribution. Beginning with an initial guess for the parameter vector γ^0, we iterate on the following algorithm:

1. Draw a candidate vector γ^i from a distribution $q(\gamma^i|\gamma^{i-1})$.
2. Compute $e^{H(\gamma^i,\mathbf{Y}_T)}$.
3. If $p_A = \frac{e^{H(\gamma^i,\mathbf{Y}_T)}}{e^{H(\gamma^{i-1},\mathbf{Y}_T)}} \geq 1$, accept γ^i.
4. Else, accept γ^i with probability p_A.
5. Set $i \leftarrow i+1$ and return to Step 1.

Repeating these 5 steps and generating a long sequence of draws for γ yields a sample of large size, hopefully drawn from the quasi posterior density $p(\gamma, \mathbf{Y}_T)$.[16] Any moment of interest (means, standard deviations, quantiles, etc...) can be readily computed. To evaluate the function $e^H(\gamma, \mathbf{Y}_T)$ one needs to solve for the model counterparts of the empirical series in $\mathbf{Y}_T$. For a given γ^i in the sequence of simulated draws, we obtain a model solution using the following steps:

1. We begin with guesses for θ, and $\bar{z}$.[17]
2. Find the surplus functions S^P, S^R and S^T by substituting and combining equations (13), (14), (15), (35), (37), and (40).
3. Update θ using equation (11). Using the functional form for the matching function specified above, θ is given by:

   $$\theta = \left(\left(\frac{\Phi}{k}\right)^{\xi} - 1\right)^{\frac{1}{\xi}}$$

   and

   $$\begin{aligned}\Phi &= \int_{z_{min}}^{z_{max}} \left[\mathbb{I}_A E_y S^P(y,z) +\right] dG(z)\,(1-\phi)\beta - \\ &\quad -\beta f \mu_G(A).\end{aligned}$$

4. Update $\bar{z}$ by solving the two-equation system defined by equations (41) and (42), which solve for $\bar{z}$ and $y^P(\bar{z})$.
5. Iterate on the previous two steps until the sequences of θ and $\bar{z}$ have converged.

---

[16] We used 5000 simulations and discarded the first 1000.

[17] We hope it is clear to the reader the implicit dependence of these variables on γ^i.

6. Having obtained $\bar{z}$ and θ we can update the employment measures - both temporary and permanent - using the steady-state versions of equations (26) and (27). These are given by:

$$
\begin{aligned}
n^P = & \frac{\left(u+n^T\right)\alpha^w(\theta)\left(1-G(\bar{z})\right)}{1-\int_{z_{min}}^{z_{max}}\left[1-F\left(y^P(z)\right)\right]dG(z)} \\
& + \frac{\left(1-\alpha^w(\theta)\right)\int_{z_{min}}^{\bar{z}}\left[1-F\left(y^R(z)\right)\right]dG(z)\,n^T}{1-\int_{z_{min}}^{z_{max}}\left[1-F\left(y^P(z)\right)\right]dG(z)}.
\end{aligned}
$$

and,

$$
n^T = \frac{u\alpha^w(\theta)G(\bar{z})}{1-\alpha^w(\theta)G(\bar{z})}.
$$

All integrals throughout are evaluated using quadrature.[18] With values for θ, $\bar{z}$, n^T, n^P (and clearly u as a byproduct), one can compute wages and simulate histories of workers to fit regression equation 31. In addition, it is easy to compute other moments. For example, the destruction rate of temporary workers is given by:

$$
JD_T = \frac{n^T\int_{zmin}^{\bar{z}}F(Y^R(z))dG(z)}{n^T+n^P}
$$

One can compute the remaining turnover measures in an analogous way.

---

[18] In particular, we use the Gauss-Kronrod integrator using the QDAG routine for Fortran provided in the IMSL package.