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Cristina Davino, Rosaria Romano
University of Macerata

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Cristina Davino, Rosaria Romano[1]
University of Macerata

Abstract

The paper proposes a new approach for analysing the stability of Composite Indicators. Starting from the consideration that different subjective choices occur in their construction, the paper emphasizes the importance of investigating the possible alternatives in order to have a clear and objective picture of the phenomenon under investigation. Methods dealing with Composite Indicator stability are known in literature as *Sensitivity Analysis*. In such a framework, the paper presents a new approach based on a combination of explorative and confirmative analysis aiming to investigate the impact of the different subjective choices on the Composite Indicator variability and the related individual differences among the statistical units as well.

JEL classification: C38, C52, C63.
Keywords: composite indicators, sensitivity analysis, analysis of variance, principal component analysis.

Corresponding author: Cristina Davino (cristina.davino@unimc.it).

Department Informations:
Piazza Oberdan 3, 62100 Macerata - Italy Phone: +39 0733 258 3960 Fax: +39 0733 258 3970 e-mail: csampaoli@unimc.it

[1] University of Macerata, Dipartimento di Studi sullo Sviluppo Economico, Piazza Oberdan 3, 62100 Macerata-Italy, cristina.davino@unimc.it, rosaria.romano@unimc.it

1 Introduction

The aim of the paper is to develop a new approach for the analysis of Composite Indicators (CIs) in the theoretical framework of explorative and confirmative analysis.

It is a matter of fact that the requirement to synthetize univariate indicators by means of a CI is becoming more and more common in all those contexts where the interesting phenomenon cannot be directly observed and measured due to the presence of several and different concurrent factors. Once a CI is constructed, a post-analysis of its stability is advisable before employing it in a decision process. The values of a CI and/or the ranking deriving from these values depend on the methodological choices faced in its construction. These choices are well known in literature as *uncertainty factors* (Nardo *et al.*, 2008) and they involve all the steps followed in the CI definition process: definition of the phenomenon to be measured (selection of factors, indicators and statistical units), pre-processing of the original indicators (missing data imputation, indicator transformations), construction of the CI (identification of the system of weights, selection of the aggregation method).

The paper is embedded in the Sensitivity Analysis (SA) (Saltelli *et al.*, 2008) framework where the aim is to identify the contribution of each uncertainty factor on the obtained CI.

The proposal of the present contribution is to present an innovative CI Sensitivity Analysis based on a combination of Mixed Model Analysis of Variance models (McCulloch *et al.*, 2001) and multivariate methods (Mardia *et al.*, 1979). This strategy has already been proposed by Naes (Naes *et al.*, 2010) in the context of consumers' preferences. Aim of the present work is to adapt such approach to the CI Sensitivity Analysis framework.

Besides the evaluation of the impact of the uncertainty factors on the CI variability, the proposed approach allows to highlight the individual differences among the observations as well. Classical sensitivity methods and the proposed approach will be compared by means of a case study based on the *Technology Achievement Index* (Desai *et al.*, 2002).

2 Constructing Composite Indicators

A Composite Indicator is obtained by synthetizing individual indicators (quantitative/qualitative measures observed on a set of units) into a single index.

The requirement to synthesize univariate indicators by means of CIs is becoming more and more urgent in all those contexts, e.g. social, sanitary and economic, where the object of analysis cannot be directly observed and measured due to the presence of several and different concurrent factors acting as determinants. Examples of CIs proposed over the years by international organizations in different fields of applications are listed below:

- *Human Development Index* [2]: a CI which takes into account the three main dimensions of the human development (life expectation, education, income).
- *Human Poverty Index* [3]: it is computed differently for specific group of countries (developing countries and selected OECD countries) and it measures the standard of living of a country according to the longevity, knowledge, standard of living and social exclusion.
- *Global Risk Index* [4]: it measures the financial risk connected to investments and it is based on the volatility index of 34 financial assets.
- *Economic Competitiveness Index* [5]: it measures the ability of a nation to guarantee favorable economical conditions for firm competitiveness.
- *Index of Healthy Conditions* [6]: it combines 6 healthy conditions in order to measure the healthy state of PAN American countries.

These and other additional examples of CIs give rise to an increasing interest on the topic among the politicians, the workers from the different socio-economic sectors, the researchers, the news agencies and the public opinion. It is a matter of fact that CIs are recognized as fundamental tools according to which important political decisions, often aiming to share financial resources, are made. They are also widely used to communicate the relative performance of countries.

[2] http://hdr.undp.org/docs/statistics/indices/technote1.pdf

[3] http://hdr.undp.org/docs/statistics/indices/technote1.pdf

[4] http://www.weforum.org/en/initiatives/globalrisk/index.htm

[5] http://www.cforic.org/pages/european-competitiveness.php

[6] http://www.paho.org/English/D/Annual-Report1996/ops96ar4.htm

Whatever the applicative context is, the construction of CIs involves stages where subjective decisions have to be taken. The first requirement pertains to the characterization of the dimensions underlying the concept to be measured. Once these have been identified, the quantitative and qualitative variables (indicators) able to measure each dimension must be specified. A pre-processing of the univariate indicators is then performed to deal, for example, with missing values and transformation of raw values. Finally, several aggregation methods and systems of weights can be adopted. A full checklist for building composite indicators is provided by OECD Handbook (Nardo *et al.*, 2008). All the required choices are defined *uncertainty* or *input factors* since they introduce variability in the model output, namely in the Composite Indicator.
A list of possible uncertainty factors in case of quantitative observed indicators, with their corresponding alternatives (levels), is presented in Table 1.

Table 1: *Uncertainty Factors and levels*

Factors	**Levels**
Normalization	raw, ranking, standardization, minmax, distance to a reference unit
Aggregation	linear, geometric, multi criteria (Munda, 2007)
Weighting	equal, factorial analysis, participatory approaches (budget allocation process - BAP (Moldan *et al.*, 1997), analytic hierarchy process - AHP (Saaty, 1987))

In order to introduce some basic notation, let $\mathbf{X}(N \times P)$ be a data matrix of P indicators observed on N units, $Y(N \times 1)$ the CI and Z_h $(h = 1, ..., H)$ the generic uncertainty factor assuming different levels (l_h). According to the choices occurring in the CI construction, the output Y will vary obtaining as many different outputs as the number of possible levels combinations $(l_1 \times l_2 \ldots \times l_H)$. For instance if the selected combination of levels is $Z_{normalization} = raw(r), Z_{aggregation} = geometric(g), Z_{weights} = equal(e)$, the resulting CI for the generic unit n $(n = 1, \ldots, N)$ will be:

$$Y_n^{rge} = \prod_{p=1}^{P} X_{np} \tag{1}$$

3 Uncertainty and sensitivity analysis

Studies on Uncertainty and Sensitivity analysis find their origin in the experimental sciences where the value of the variable in itself is the prominent uncertainty input factor. Thus, different methods have been proposed over the years for handling this type of uncertainty. More recently, these methods have been generalized to the context of the CIs (Nardo *et al.*, 2008): *Uncertainty Analysis* aims to quantify the uncertainty associated to the CI while *Sensitivity Analysis* aims to identify the contribution of each factor involved in the construction of the CI (weighting schemes, aggregation methods, etc...) on its variability.
Sensitivity methods can be classified into three categories (Saltelli *et al.*, 2000):

- Factor screening
- Local sensitivity analysis
- Global sensitivity analysis

The methods belonging to the first group are very useful in case of many input factors and they allow preliminary evaluations on the input factors aiming to identify the most important ones. Typical screening methods are represented by one-at-a-time (OAT) experiments (Daniel, 1958) and factorial experiments (Box *et al.*, 1978). All of them provide a ranking of the input factors according to their importance but they do not quantify the impact of each input factor on the composite indicator. This aim can be achieved by the local sensitivity analysis methods which measure the exact impact of the input factors on the model output. They are based on the computation of partial derivatives of the output functions with respect to the input factors. Even if local sensitivity methods are computationally efficient, they are strongly dependent from the specific point (nominal value of each input factor) investigated in the space of parameters.

Global sensitivity analysis methods are the most widespread approaches to sensitivity analysis. They allow overcoming the main drawbacks of the previously mentioned methods and they measure the uncertainty of the output deriving from the uncertainty of each input factor. The term global refers to the capability of such methods to simultaneously analyze all input factors and to inspect their entire distribution. Global sensitivity analysis methods are performed via simulation techniques such as Monte Carlo methods be-cause it is necessary to generate multiple evaluations of the model output according to randomly selected model inputs.

In order to perform a global sensitivity analysis it is firstly necessary to define a probability distribution function for each input factor. This phase is strongly dependent from expert's subjective choices and it influences both the next phases and the final results. Once a probability distribution function is identified for each input factor, a sampling procedure must be chosen to select a sample form those distributions. Several sampling procedures can be functional to such phase: random sampling, stratified sampling, quasi random sampling, etc. The generated samples are used to evaluate the model output in terms of sequences of output values or rankings of the output values or differences between the output values and a benchmark reference output. Finally, a global sensitivity analysis closes with the evaluation of the uncertainty and of the sensitivity of the model output to the input factors.

In particular uncertainty analysis aims to quantify the uncertainty of the model output through the analysis of simple statistics such as the expected value and the variance of the output or its density function.
Sensitivity analysis, instead, aims to identify the contribution of each input factor on the uncertainty of the model output. Several tools and methods have been proposed to perform a sensitivity analysis. The simplest one is represented by the inspection of the scatter plot displaying the output and input values generated by the Monte Carlo procedure. It is a matter of fact that a careful inspection of the scatter plot can reveal the presence or the absence of a relationship between the variables and, in case of a relationship, its form (linear or not). A further investigation can be performed through a regression analysis between inputs and outputs thus providing a regression coefficient measuring the effect of a unitary variation of the input on the model output. Regression analysis can be executed on data replaced by ranks in case of non linear relationships between variables. *Variance-based methods* represent a widespread class of methods to perform a global sensitivity analysis. They provide, for each input factor, a measure of the impact of the given factor on the model output.

3.1 Variance based methods

Variance based methods (VBM) provide quantitative measures evaluating the variability in the model output, for each sources of uncertainty. The underlying concept is that fixing one important source of variation to a given value ($Z_h^{l_h}$, where l_h is one of the levels of the h factor), the variance of the model output ($Var(Y|Z_h^{l_h})$, conditional variance) should be less than the total variance of the composite indicator (Var(Y), total or unconditional variance).

The origins of VBM date back to the proposal of Cukier (Cukier *et al.*, 1973) based on Fourier transformations (Fourier Amplitude Sensitivity Test). It was followed by the introduction of measures of importance such as the one proposed by Hora and Iman (1986).

In the 90s, total sensitivity indexes were introduced by Homma and Saltelli (1996) after the Sobol (1990) systematization of the theory about variance-based methods.

Whatever the variance-based method is used, all the steps for a global sensitivity analysis must be performed for each observed unit:

- Definition of a probability density function (f) for each h uncertainty factor ($h = 1, ..., H$):
$$Z_h \sim f_h(\theta_h) \tag{2}$$
- Selection of an R dimension random sample for each f_h, one independently from the other: $\mathbf{Z}_{(R\times H)}$. Each row of the $\mathbf{Z}$ matrix represents a sample corresponding to a given combination of levels of the H factors.
- For each row of the $\mathbf{Z}$ matrix, evaluation of the CI: $\mathbf{Y}_r$ ($r = 1, ..., R$).

The first order sensitivity index is computed evaluating for each source of uncertainty Z_h with l_h levels the following quantities:

- Compute the CI values corresponding to a given level l_h of the Z_h factor: $Y|Z_h^{l_h}$.
- Take the expected value on the conditioned Y values: $E(Y|Z_h^{l_h})$.
- Measure the variance of the expected values over the l_h levels of the Z_h factor: $V_h = Var_{Z_h}(E(Y|Z_h^{l_h}))$
- Compute the fractional contribution to the model output variance due to the uncertainty in Z_h:
$$S_h = \frac{V_h}{V} \tag{3}$$
where V=Var(Y).

First order sensitivity indexes refer to an additive model without interactions among factors and where $V = \sum_{h=1}^{H} V_h$ and $\sum_{h=1}^{H} S_h = 1$. In order to take into account interaction effects in case of non additive models, a total effect index is introduced (Homma and Saltelli, 1996). For example, in case of $H = 3$ uncertainty factors, the total effect index of the first uncertainty factor is: $S_{T_1} = S_1 + S_{12} + S_{13} + S_{123}$ where S_1 is the first order sensitivity index and all the other terms are sensitivity indexes based on the interactions among factors.
A technical drawback of the VBM is its computational cost since it requires many simulations of the CIs. Moreover, these methods provide information on the different uncertainty factors without highlighting the role of the corresponding levels. This lack of information also affects the analysis of the interactions since it is limited to verify how much a factor is sensitive to the interactions with the others but no information is provided on which are the affecting factors and levels.

4 The proposed approach: ANOVA-PCA based method

The main focus of this paper is to propose an alternative method for Sensitivity Analysis of CIs which investigates the impact of the different sources of uncertainty in the CI construction (factors, levels, units), taking also into account external information available for each statistical unit (e.g., continent, dimension, etc...). The use of external information is of crucial importance in this type of analysis since it provides additional information very useful for a suitable interpretation of the final results.
The proposed strategy consists of a simultaneous approach combining both explicative and explorative methods. Specifically, the approach consists of two main steps:

1. *evaluation of the significance of uncertainty factors and additional information by Analysis of Variance (ANOVA):*
   estimating the effect of each uncertainty factor on the CI variability by means of a *Mixed Model ANOVA* with *units* as random factor;
2. *exploration of interactions among factors and units by Principal Component Analysis (PCA):*
   (a) estimating a Mixed Model ANOVA without the *units* factor and taking the residuals;

(b) exploring individual differences among units by PCA on the obtained residuals.

4.1 Evaluation of the significance of uncertainty factors and additional information by ANOVA

Analysis of Variance is a very useful method when the objective is an assessment of the impact of some controllable factors (categorical variables) on a specific response (continuous variable) (Searle, 1997). The impact is significant if the variability *between* the groups defined by the factor levels (categories) is much larger than the variability *within* the groups. ANOVA model is equivalent to a linear model where the response variable becomes the dependent variable, and each of the factors is transformed into *dummy variables* according to the number of levels.

Given the data matrix $\mathbf{X}$ $(N \times P)$ of P indicators observed on N units as introduced in section (2), let's consider for simplicity only two uncertainty factors $Z^{'}$ and $Z^{''}$, respectively with I and J levels. The *units* factor U will consist of as many levels as the number of observed units and it is nested in the *external information* factor δ, with M levels. A factor is nested when subgroups of units match only one of the levels of the nesting factor and not each one of them, as usually happens in a crossed design. The model can then be written as:

$$y_{ijn} = \mu + z^{'}_{i} + z^{''}_{j} + u_n(\delta) + z^{'}z^{''}_{ij} + z^{'}u_{in} + z^{''}u_{jn} + e_{ijn} \tag{4}$$

where y_{ijn} is the n^{th} observation obtained using the i^{th} (i=1,...I) level of the $z^{'}$ factor and the j^{th} (j=1,...J) level of the $z^{''}$ factor. In model (4), the general mean is represented by μ, while $z^{'}_{i}$ and $z^{''}_{j}$ are the main effects of the two uncertainty factors and $z^{'}z^{''}_{ij}$ is their interaction effect. All these factors are considered fixed. The main effect of the factor represented by the units and nested in the external information factor δ is u_n, while $z^{'}u_{in}$ and $z^{''}u_{jn}$ are the interactions between units and the two uncertainty factors. Finally, the term e_{ijn} is the random error. As the set of units can be viewed as one specific 'sample' of the whole population of statistical units, the related factor is a random factor. An ANOVA model including both fixed and random factors is called Mixed Model ANOVA.

Model (4) corresponds to a simultaneous ANOVA for all statistical units and it is estimated by stacking in a pile the same matrix containing the different

combinations of factor levels and the corresponding CI obtained for each statistical unit.
Results from model (4) show which uncertainty factors strongly affect or not the stability of the CI and also the impact of these effects on each single unit. In order to better explore such differences and similarities among the units, a PCA exploiting all the advantages of the factorial methods is performed on the residuals of an ANOVA model without the units factor, as shown in the next section.

4.2 Exploration of interactions among factors and units by Principal Component Analysis

A study of the differences among the units in their CI values is already achieved by introducing the *units* effect as random factor in the joint ANOVA (4). In fact, the variance components for the random effects provide information on the relative size of the individual differences in the model, whilst the main effects and the interaction effects plots show graphically such differences. However, in case of many statistical units these plots are unreadable and more sophisticated exploratory methods are required. Principal Component Analysis is very appropriate at this aim since it allows to synthetize multivariate data in a few linear combinations to be plotted by means of factorial planes. Specifically, individual differences will be explored by PCA on the residuals obtained in a model with only fixed uncertainty factors and the random *unit* effect:

$$y_{ijn} = \mu + z_i^{'} + z_j^{''} + z^{'}z_{ij}^{''} + u_n(\delta_m) + e_{ijn} \tag{5}$$

Residuals from this second model contain information on individual differences among units with respect to the uncertainty factors plus the random error. The same individual differences are modeled differently in model (4) as interactions between the units and the uncertainty factors.
In order to run the PCA on the residuals from model (5), these have to be rearranged in a data matrix $(N \times (I \times J))$ with the units as rows and the CIs, corresponding to the different combinations among the uncertainty factors levels, as columns. Results from this PCA will highlight units with CI values, due to a specific combination of uncertainty factor, either higher or lower than the average unit. These units will be identified as those which are more sensitive to a specific uncertainty factors combination. The impact of

the external information is investigated by including it in the PCA as supplementary variable and projecting it on the factorial planes obtained by the residuals from model (5).

5 Case study

5.1 The Technology Achievement Index

The Technology Achievement Index (TAI) is a composite indicator developed by the United Nations for the Human Development Report (United Nations, 2001) aiming to assess the national capacities in technology of a certain number of countries.
It is composed by 8 individual indicators observed on 72 countries:

- **patents**: the number of patents granted per capita, to reflect the current level of invention activity (per million people);
- **royalties**: receipt of royalty and license fees from abroad per capita, to reflect the stock of successful past innovations still useful (US$ per 1000 people);
- **internet**: Internet host (per 1000 people);
- **exports**: exports of high-technology and medium-technology products (as % of all total goods exports);
- **telephones (log)**: expressing the measure in logarithms ensures that, as the level increases, it contributes less to the index (mainlines and cellular per 1000 people);
- **electricity (log)**: expressing the measure in logarithms ensures that, as the level increases, it contributes less to the index (kWh consumption per capita);
- **schooling**: mean years of schooling as proxy for basic education to develop cognitive skills and skills in science and mathematics (age 15 and older);
- **university**: enrolment in tertiary education in science, mathematics and engineering (ratio %).

For the purpose of the paper only the first 23 of the 72 original countries measured by the TAI are considered and listed in Table 2:

Table 2: Countries analyzed for the TAI

Finland (Fi)	United States (US)	Sweden (Sw)	Japan (Ja)
Korea (Ko)	Netherlands (Ne)	United Kingdom (UK)	Canada (Ca)
Australia (Aul)	Singapore (Si)	Germany (Ge)	Norway (No)
Ireland (Ir)	Belgium (Be)	New Zealand (NZ)	Austria (Au)
France (Fr)	Israel (Is)	Spain (Sp)	Italy (It)
Czech Republic (CzR)	Hungary (Hu)	Slovenia (Sl)	

Countries can be classified according to the region as belonging to *Europe* or not. In the following this information on the *region* will be used in modeling data as *external information*.
Descriptive statistics of the eight TAI indicators are presented in Table 3.

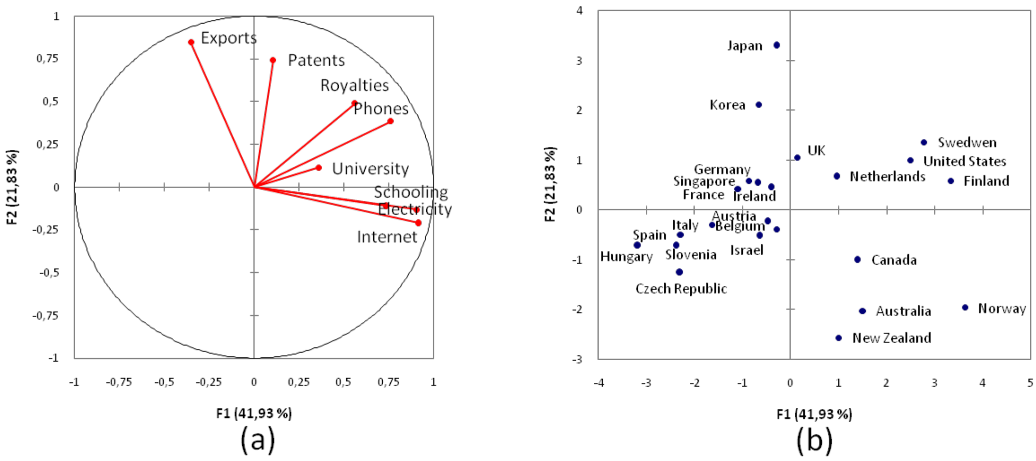
Table 3: Descriptive Statistics of the TAI indicators

indicator	**mean**	**variation coefficient**	**skewness (Pearson)**
Patents	182	1.3	2.5
Royalties	54	1	0.8
Internet	80	0.7	0.7
Exports	52	0.3	−0.9
Telephones (log)	3	0.0	−0.6
Electricity (log)	4	0.1	0.8
Schooling	10	0.2	−0.1
University	15	0.4	1.2

The first 3 indicators have the highest coefficient of variation; in addition *Patents* also presents a relevant positive skewness.

Results from the PCA on the standardized raw data highlight similarities and differences among the countries with respect to the simple indicators. Factorial planes in Figure 1 show how indicators span only three of the four quadrants. Specifically, all the indicators with the exception of *Patents*, *Royalties* and *Exports*, which characterize the second principal component, are highly correlated to the first principal component discriminating between coutries with high values for almost all individual TAI indicators and countries performing in the opposite way.

Figure 1: Variables (a) and units (b) first factorial plane from the PCA on the TAI indicators



The methodology used to calculate the TAI consists in a simple average of the observed indicators whose values are normalized to a scale from 0 to 1 according to the *minmax* normalization (see table 1):

$$X_p = \frac{X_p - min(X_p)}{max(X_p) - min(X_p)} \tag{6}$$

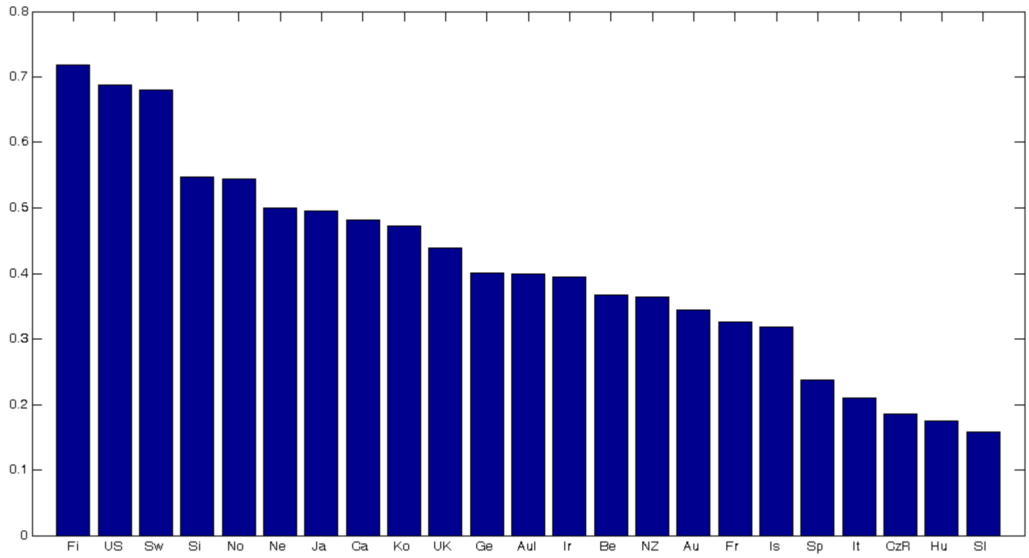
$$Y_n = \frac{\sum_{p=1}^{P} X_{np}}{P} \tag{7}$$

Figure 2 shows the TAI values calculated according to equations (6) and (7) for each country.

5.2 Uncertainty and sensitivity analysis for TAI

The methodology used for TAI calculation considers as uncertainty factors the normalization in (6), the aggregation in (7) and equal weights. As a matter of fact, this is only one of the possible strategies and a study of the sensitivity of the TAI distribution over the coutries is advisable. At this aim, the evaluation of TAI sensitivity is proposed according to the global sensitivity analysis procedure presented in section (3) and it is performed through the following steps:

Figure 2: TAI distribution over the countries



- Definition of three uncertainty input factors: normalization, aggregation, weights.
- Definition of factor levels: the minmax transformation (MinMax) and the distance to the average (NI) are considered as normalization methods together with the linear (LIN) and the geometric (GEOM) aggregation methods. Two systems of weights are taken into account: budget allocation process (BAP) and Analytic Hierarchy Process (AHP).
- Definition of the probability distribution function for each input factor: uniform distribution in $[0, 1]$ (Table 4).
- Selection of a sample from those distributions according to a sampling procedure: a random sample of 10.000 levels combinations of normalization, aggregation and weighting.
- Evaluation of the model outputs: 10.000 TAI values for each country corresponding to different combinations of uncertainty factors.
- Evaluation of the uncertainty and of the sensitivity of the model output to the input factors.

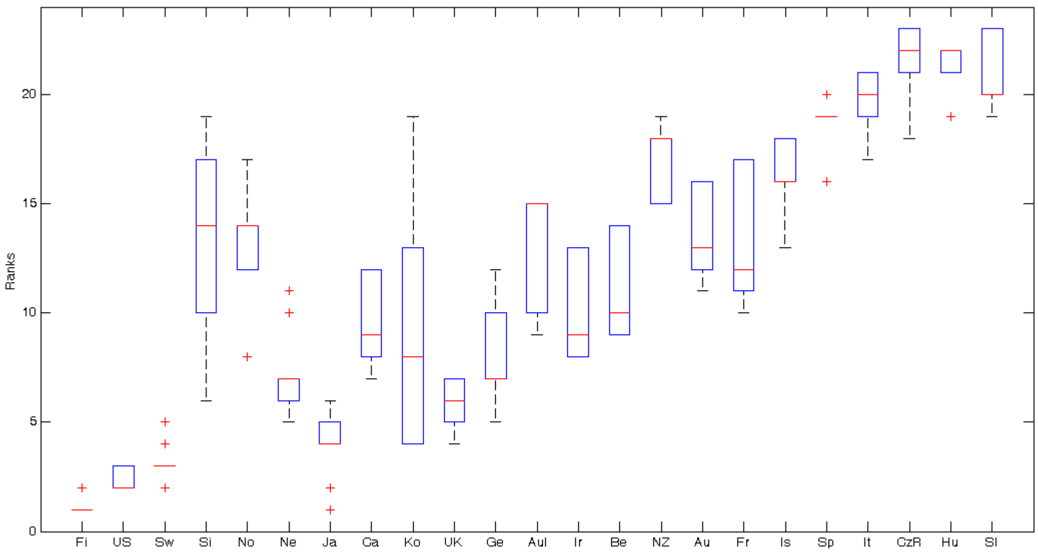
In order to investigate the variability of the simulated TAI values for each country and to have comparable measures, the values are tranformed in ranks

Table 4: Reference scheme of the Uncertainty Analysis

Factor	Definition	pdf	Levels
X_1	Normalization	Uniform $[0, 1]$	$[0, 0.5] = NI; (0.5, 1] = MinMax$
X_2	Aggregation	Uniform $[0, 1]$	$[0, 0.5] = LIN; (0.5, 1] = GEOM$
X_3	Weighting	Uniform $[0, 1]$	$[0, 0.5] = BAP; (0.5, 1] = AHP$

which are graphically represented by boxplots in Figure 3 (countries are ordered according to the original TAI values). Results in Figure 3 show

Figure 3: TAI Unceratinty Analysis results



how the variability is related to the position in the ranking: the higher/lower the ranks the lower the variability. Singapore and Korea present the most variable position with ranks going from the higher to the lower positions in the ranking.

Once explored the uncertainty in the CI values, SA is used to investigate which uncertainty facors are more responsible for such variability. At this aim the sensitivity measures S_h and S_{T_h} introduced in section (3.1) are computed for each country and represented in percentage in Figures 4 and 5.

Figure 4: TAI First Order Sensitivity Measures

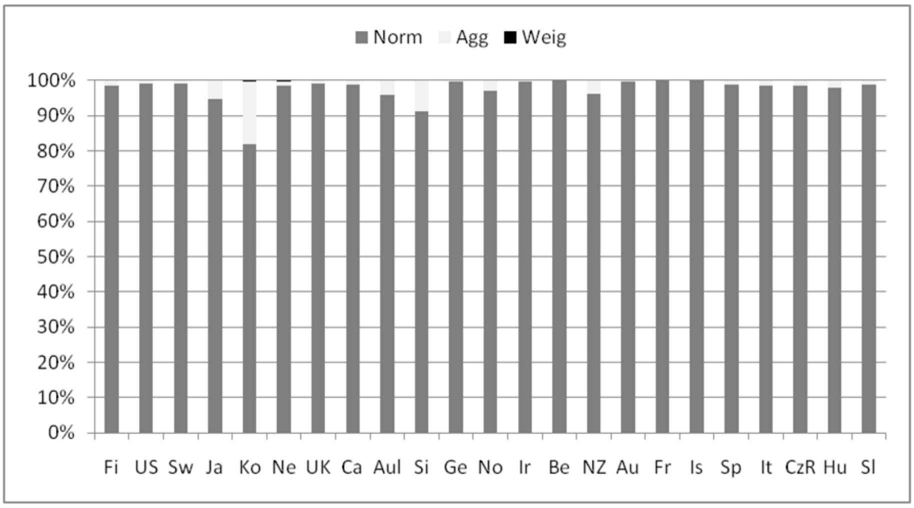
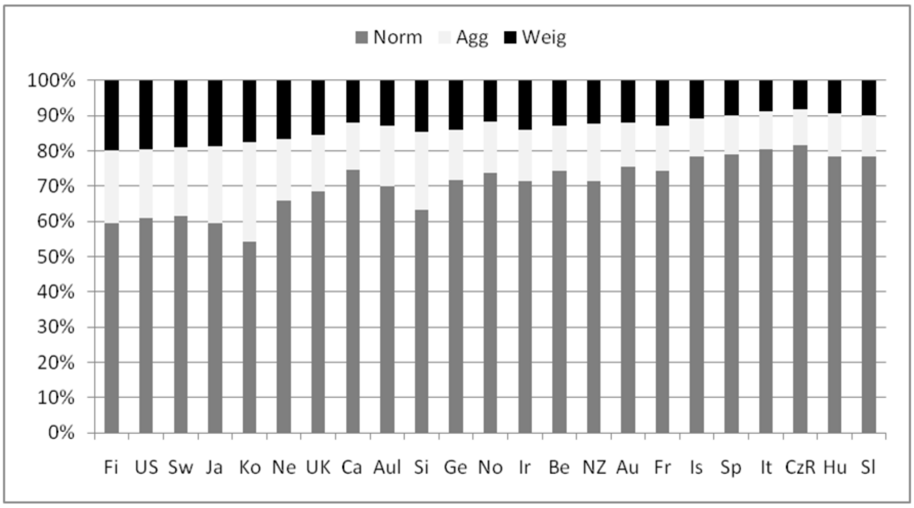


Figure 5: TAI Total Order Sensitivity Measures



First order sensitivity measures in Figure 4 highlight the importance of the *normalization* factor on the variation of the TAI values for all countries. The *aggregation* factor also gives a little contribution, especially for Korea and Singapore. The choice of the *weighting* scheme does not affect the TAI values variability.

However, the total order sensitivity measures in Figure 5 point out that the aggregation and the weights factors are involved in the interactions with the others factors.

5.3 The ANOVA-PCA based method for TAI

In order to evaluate the significance of the uncertainty factors, a full ANOVA model with all uncertainty factors, external information and individual factors is estimated and the results are presentd in Table 5.

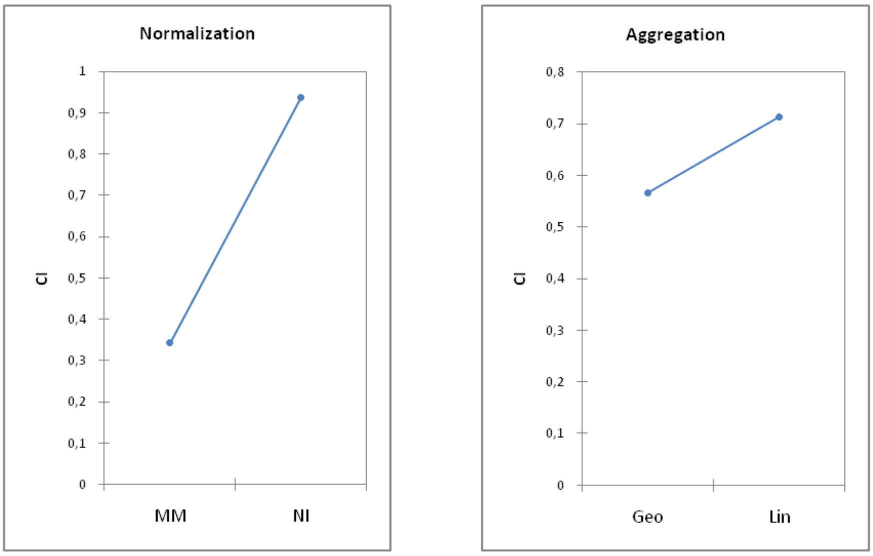
Table 5: Full ANOVA results

Source	Type	F	Pr > F
Normalization	Fixed	8115.9	< 0.0001
Aggregation	Fixed	598.9	< 0.0001
Weighting	Fixed	1.0	0.324
Country	Random	377.6	< 0.0001
Norm*Agg	Fixed	6.6	0.012
Norm*Weig	Fixed	1.7	0.200
Norm*Country	Random	29.8	< 0.0001
Agg*Weig	Fixed	0.1	0.712
Agg*Country	Random	23.1	< 0.0001
Weig*Country	Random	2.1	0.009

The *normalization* has the strongest effect on the CI variability, followed by a significant effect of *aggregation* while *weighthing* has no effect. Specifically, the TAI average value (Figure 6) increses in case of *NI* normalization and *linear* aggregation while behaves in the opposite way in case of the other two levels of the respective uncertainty factors. Table 5 also shows a significant interaction effect between *normalization* and *aggregation*. Note that these results are coeherent with results from the *variance based method*. The significant *country* factor and its interactions with all uncertainty factors point out individual differences among countries in their own TAI values and in their behaviour with respect to the different choices occurring in the TAI composite indicator construction.

The second step of the proposed approach consists in exploring residuals from the model using only the uncertainty factors, their interactions and

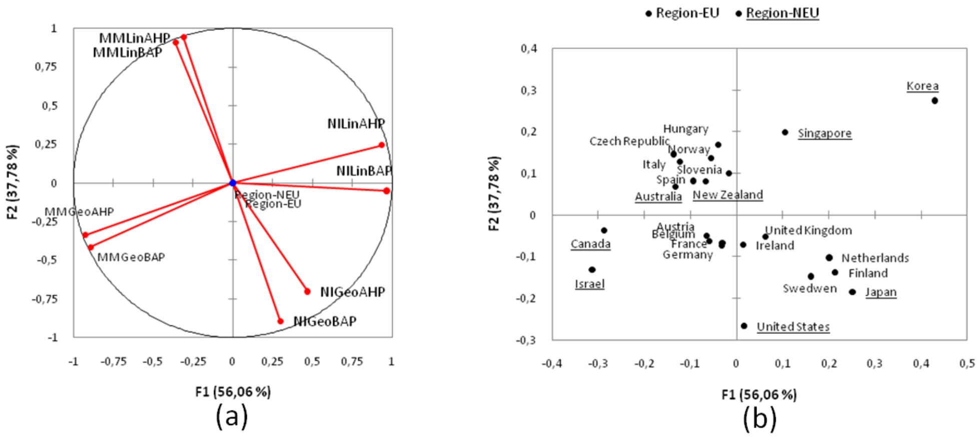
Figure 6: ANOVA means plots



the country factor. These residuals are computed and arranged into a matrix with the countries as rows and the different combinations of the three uncertainty factors on the columns. A PCA is then run on these data, including *region* as supplementary categorical variable and the related score and loading plots are shown in Figure 7.

The explained variance for the first two components is 94%. The score plot shows which countries are sensitive to the different uncertainty factor combinations represented in the related loading plot. For instance, *United States* increases its position in the ranking if the TAI is built by using the *NI* normalization combined with the *geometric* aggregation, while its position decreases if the *minmax* normalization and the *linear aggregation* are used. Note that there are no significant differences in the TAI values between *european* and *not european* countries since the two modalities fall in the middle of the score plot meaning that the two averages are very close to each other. Moreover a different system of weights does not cause variations on the TAI values whatever the normalization and aggregation method is used.

Figure 7: Loading (a) and score (b) first factorial plot from the PCA on the ANOVA residuals



6 Conclusion

The proposed approach aims to investigate the impact of the different sources of uncertainty in the CI construction taking into account external information too. In addition to classical Sensitivity Analysis uncertainty sources, the effect of units (e.g. countries) is also evaluated. All such uncertainty factors are simultaneosly analysed through a multidimensional approach combining inferential and exploratory methods. Computational and graphical potentiality of the proposed approach guarantees its use also in case of many observations where classical Sensitivity Analysis requires an individual inspection of the factors and units.

Exploration of the final visualization can be considered as a decision support tool for analysts and especially politicians as they can easily verify the effects of a given policy adopted to construct a CI.

Further developments will regard the inclusion of additional uncertainty factors in the model such as the *inclusion/exclusion* of each indicators, but also the analysis of more complex CI where indicators are structured in subgroups (dimensions) and the role of such dimensions must be evaluated too.

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