Variance Swaps and Intertemporal Asset Pricing

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Abstract

This paper proposes an ICAPM in which the risk premium embedded in variance swaps is the factor mimicking portfolio for hedging exposure to changes in future investment conditions. Recent empirical evidence shows that the fears by investors to deviations from Normality in the distribution of returns are able to explain time-varying financial and macroeconomic risks in addition to being a determinant of the variance risk premium. Moreover, variance swaps hedges unfavorable changes in the stochastic investment opportunity set, and is not a redundant asset because significantly expands the efficient mean-variance frontier. Thence, we should expect the variance swap risk premium to be priced in the market. We report relatively favorable evidence on the incremental pricing information associated with the variance risk premium, particularly at shorter horizons.

1. Introduction

As shown by Carr and Wu (2009), Todorov (2010), and Egloff, Leippold and Wu (2010) among many others, the average variance risk premium is negative and sizeable for all available horizons. Since the payoff of a variance swap contract is the difference between the realized variance and the variance swap rate, the observed negative returns for long positions on variance swap contracts for all time horizons suggest that investors are willing to accept negative returns for insuring against future realized variance. Recently, Nieto, Novales and Rubio (2010) use the implications of an asset pricing model proposed by Chabi-Yo (2009) to find evidence that as it is the case with standard indicators of different types of macroeconomic and financial risks, the variance risk premium responds to changes in higher order moments of the conditional distribution of market returns.[1] This common dependence suggests that the variance swap may offer hedging against a variety of risks and, consequently, the variance risk premium could be capturing the market willingness to pay for such a hedge.

A natural question then refers to whether the fluctuations in the variance risk premium may act as a sufficient statistic summarizing the information contained in a variety of macroeconomic and financial risk indicators which is relevant for asset valuation. In the continuous-time Intertemporal Asset Pricing Model (ICAPM hereafter) of Merton (1973), the value function depends not only on aggregate wealth, but also on the innovations to some state variables that describe the stochastic behavior of the investment opportunity set. These additional variables may hint at ways to design an appropriate hedge against unfavorable changes in the stochastic investment opportunities and the optimal portfolio should be made up by a combination of the market and the hedging portfolios. In this paper, we employ the payoff of the variance

[1] See the related evidence reported by Bondareko (2004) who shows that the variance risk premium explains returns that exhibit significant skewness.

swap as the hedging variable for alternative investment horizons. Therefore, we take the ICAPM as the natural framework to investigate whether the variance risk premium may add information to the return on market wealth as an aggregate risk factor explaining the cross-section of expected returns.[2]

Specifically, the stochastic discount factor (*SDF* hereafter) is specified as a power function of the return on the market portfolio, expanded with an exponential function of the excess return of the variance swap contract as hedging variable. We perform several empirical tests of the model that suggest that the variance risk premium contains relevant information that helps pricing average stock returns. The measures of the global fit indicate that the model performs better when it includes the variance risk premium factor than when it only incorporates the market return portfolio. This evidence is generally observed for both the non-linear specification and the beta (linear) specification of the model. Specifically, the comparison between the one-factor model and the two-factor model at the one-month investment horizon reveals that the mean absolute pricing error decreases from 0.343 to 0.288 in the non-linear specification, and that the pseudo cross-sectional *R*-square used in the estimation increases from 0.278 to 0.412 in the beta specification. Moreover, we also show that, on average, test portfolio betas relative to the variance risk premium factor are strongly negative when we allow for regressions with two regimes based on a market return threshold. The relatively favorable evidence on the variance risk premium being a financial factor that is priced in the market is consistent with the result in Nieto, Novales and Rubio (2010), who show that the variance swap is not being spanned by a set of assets composed of government and corporate bonds and the stock market portfolio.[3]

[2] Malkhozov (2009) shows how the variance risk premium arises in asset pricing models with stochastic volatility and production economies with dynamic hedging effects.
[3] See also the related evidence reported by Chabi-Yo (2008).

The paper is organized as follows. Section 2 briefly describes the variance swap contract and defines the variance risk premium. Section 3 contains a description of the data. The two-factor asset pricing model is presented in Section 4, while Section 5 reports the results of the empirical tests. Section 6 concludes with a summary of our findings.

2. Variance Swap Contracts and the Variance Risk Premium

A variance swap is an over-the-counter financial instrument that pays the difference between a standard estimate of the realized variance of the return on a given asset (a stock market index in this case) and the fixed variance swap rate. More in detail, one leg of the variance swap pays an amount based upon the realized variance of the price changes of the underlying asset. Conventionally, these price changes will be daily log returns, based upon the most commonly used closing price. The other leg of the swap pays a fixed amount, the strike, quoted at the deal's inception. Thus the net payoff to the counterparties is the difference between these two values. It is settled in cash at the expiration of the deal, though some cash payments are likely to be made along the way by one or the other counterpart to maintain an agreed upon margin. The payoff of a variance swap issued at time *t* and maturing at time $t+\tau$ is therefore given by,

$$N_{var}\left(RV_{t,t+\tau} - SW_{t,t+\tau}\right), \tag{1}$$

where N_{var} denotes variance notional, also called variance units, $RV_{t,t+\tau}$ is the annualized realized variance over the life of the contract, and $SW_{t,t+\tau}$ is the delivery price quoted at time *t* for the variance of the asset between *t* and $t+\tau$, also known as the variance swap rate. Hence, profits and losses from a variance swap depend directly on the difference between realized and implied variance.

Since variance swaps cost zero at entry, no arbitrage requires that the variance swap rate must be equal to the risk-neutral expected value of the realized variance. Therefore,

$$SW_{t,t+\tau} = E_t^Q\left(RV_{t,t+\tau}\right), \tag{2}$$

where $E_t^Q(.)$ is the time-*t* conditional expectation operator under some risk-neutral measure *Q*. The variance risk premium at period *t* is then defined as,

$$VRP_t^{t+\tau} = E_t^P\left(RV_{t,t+\tau}\right) - SW_{t,t+\tau}, \tag{3}$$

where $E_t^P(.)$ is the time-*t* conditional expectation operator under the physical probability measure *P*. If investors price variance risk, the variance swap rate will differ from the expected realized variance under *P* at the corresponding horizon, the difference being the variance risk premium.

3. Data and Descriptive Statistics

In this paper we analyze variance swap contracts on the S&P 500 index. Daily variance swap rates on five different maturities from January 4, 1996 to January 31, 2007 are obtained from Bank of America. We get monthly data by using the quotes on the last day of each month. Our estimation of the realized variance employs intra-daily data observed at 30-minute intervals, from 9 a.m. to 3 p.m., on the S&P 500 index returns provided by the Institute of Financial Markets. For each month in our sample, we compute the realized variance for each maturity τ of a variance swap contract ($\tau = 1$, 2, 3, 6, and 12 months) using quadratic changes on the value of the S&P 500 index, as given by

$$RV_{t,t+\tau} = L\sum_{l=1}^{L}\left(\frac{P_{t,l} - P_{t,l-1}}{P_{t,l-1}}\right)^2, \tag{4}$$

where P_t is the level of the index at time *t*, *L* is the number of 30-minute intervals comprised in the interval $(t, t+\tau)$. We work with variance swap rates and realized variances in percent numbers.

For each month *t* and each maturity τ we compute the log variance risk premium as the logarithm of the ratio between realized variance and the swap rate,

$$VRP_t^{t+\tau} = log\left(\frac{RV_{t,t+\tau}}{SW_{t,t+\tau}}\right), \tag{5}$$

which can be read as the excess rate of return of the variance swap contract. Clearly, $VRP_t^{t+\tau}$ is known only at time $t+\tau$. Figure 1 displays variance swap rates and realized variance for 1-, 3- and 6-month maturities. As expected, the swap rate is most often above the level of realized variance, especially for longer maturities. This evidence is similar to that shown by Carr and Wu (2009) for stock market indices and, to a lesser extent, for individual stocks.[4] It is clear that investors are willing to accept a significantly negative return to long variance swaps on the S&P index in exchange for being hedged against future unexpected volatility shocks. Therefore, shorting variance swap contracts in the S&P index generates significantly positive average excess returns during our sample period, since the variance risk premium can be seen as the return on holding the variance swap contract.

Panel A of Table 1 reports the variance risk premium descriptive statistics for alternative maturities. The variance risk premium is always negative on average, and it becomes more negative with maturity. Panel B of Table 1 reports the correlation coefficients between the variance risk premia at any two different maturities.

[4] Driessen, Maenhout, and Vilkov (2009), and Vilkov (2008) show that the variance risk premium for stock indices are systematically larger, i.e., more negative, than for individual securities. They argue that the variance risk premium can in fact be interpreted as the price of time-varying correlation risk. Antón (2010) replicates their analysis using Eurostoxx50 and, contrary to the previous results, he reports individual variance risks different form zero.

Correlations between variance risk premia at adjacent maturities are high, debilitating for faraway maturities. The correlation matrix suggests the existence of at least two factors in the structure of variance risk premium.[5]

Monthly data on value-weighted stock market portfolio returns (R_W) and the risk-free rate (R_f) are taken from Kenneth French´s web page. We also collect the excess returns of 25 size/book-to-market value-weighted portfolios and 17 industry value-weighted portfolios. We compute monthly series of cumulative returns corresponding to the five maturity intervals of the variance swap rates for the market return, the risk-free rate, and the 25 and 17 portfolio returns.

4. A Two-Factor Intertemporal Asset Pricing Model

Evidence presented in Nieto, Novales and Rubio (2010) indicates that the variance risk premium is able to anticipate different kinds of risk embedded in traditional state variables, such as the stock market risk, risk of default, illiquidity risk or consumption and employment growth risk. On the other hand, previous empirical literature about the ICAPM shows that innovations to state variables that forecast future investment opportunities seem to be priced by investors.[6] It may therefore be the case that the ICAPM holds as a two factor model with the excess return of the variance swap contract as the hedging factor. Bollerslev and Todorov (2010) show that, even though the equity market risk premium and the variance risk premium share similarities in the general dynamic dependencies in jump risk premia, they maintain important differences

[5] This is consistent with the formal analysis contained in Egloff, Leippold, and Wu (2010), and Amengual (2009). They show that two factors are needed to capture the term structure variation of the variance swap rates. The first factor might control the instantaneous change in the variance rate, while the second could represent the level to which the variance reverts.

[6] See Brennan, Wang and Xia (2004), Hahn and Lee (2006), and Petkova (2006).

in the way how they capture the compensations for rare events (tail events).[7] Their results imply that any satisfactory model explaining the cross-sectional variation of expected returns should be able to generate a large and time-varying compensation for fears of economic recessions. This is precisely the role that the variance risk premium may be playing in the ICAPM framework.

It is well known that, assuming no arbitrage opportunities, a positive *SDF* (m_t) exists such that,

$$E_t\left[m_{t+1}R^e_{j,t+1}\right]=0 \quad , \tag{6}$$

where $R^e_{j,t+1}$ is the excess return on asset *j* from *t* to *t+1*. The alternative asset pricing models are generated by specifying different *SDFs*; that is, assuming different preferences for investors or different stochastic processes for asset prices. For example, under the ICAPM, the *SDF* contains, in addition to the aggregate wealth return, variables that capture time variation in future investment opportunities. Although the model is generally accepted because evidence shows that state variables other than the market index are important for pricing stock returns, the debate about which state variables must enter in the *SDF* remains open. Therefore, a natural question to ask is whether the information embedded in fluctuations in the variance risk premium may act as a sufficient statistic summarizing relevant information for asset valuation.

To explore this possibility, we use five time horizons corresponding to the five maturities of the swap contracts and data sampled at monthly frequency, to estimate the following ICAPM specification

$$E_{t+\tau-1}\left[\left(\left(R_{W,t+\tau}\right)^{-\gamma}\exp\left\{c_1 VRP_t^{t+\tau}\right\}\right)R^e_{j,t+\tau}\right]=0\,, \tag{7}$$

[7] Similarly, Bondareko (2004) shows that the variance risk premium has a component that is independent of the risk premium on primitive assets.

where $R_{W,t+\tau}$ is the gross cumulative return on wealth between *t* and $t+\tau$, $VRP_t^{t+\tau}$ is the variance risk premium, i.e.; the log-difference between the variance swap rate at month *t* with maturity on $t+\tau$ and the realized variance of the market index between *t* and $t+\tau$, as defined by expression (5), $R^e_{j,t+\tau}$ is the excess cumulative return between *t* and $t+\tau$ on asset *j*, and τ = 1, 2, 3, 6 and 12 months.

This *SDF* specification is consistent with Brennan, Wang and Xia (2004), and it ensures a positive *SDF*. These authors argue that if the interest rate and the maximal Sharpe ratio follow a joint Markov process, the investment opportunity set is fully described by their joint dynamics. Accordingly, they propose a three-factor intertemporal model in which the *SDF* is the product of an exponential function of the innovations of these two variables and a power function of the aggregate wealth return.[8]

The basic idea behind equation (7) relies on focusing on the two key risk premia in financial markets: *i*) the equity risk premium for holding the market portfolio, and *ii*) the variance risk premium for holding the variance of the market portfolio. It is clear that both risk premia should be correlated. Bollerslev and Todorov (2010) show that roughly 60 percent of the equity risk premium is due to fears of rare events, while half of the variance risk premium is also due to investors fears. Then, in the empirical estimation of equation (7), rather than using directly the variance risk premium, it may be advisable to employ the residuals of a linear projection of the variance risk premium on the market excess portfolio return. Our aim is therefore to test whether the variance risk premium has incremental explanatory power over and above the market portfolio return within an ICAPM framework.

[8] More recently, Brennan, Liu and Xia (2006) include market volatility as the third state variable into their exponential pricing kernel

5. Asset Pricing Model Performance

5.1 The Non-linear Version of the Two-factor ICAPM Model

Panel A in Table 2 reports estimates of the coefficients of the iso-elastic *SDF*, obtained by applying first-stage GMM to Euler equation (7), which amounts to minimizing the Euclidean norm of the average vector of pricing errors.[9] The test assets are the 25 Fama-French portfolios and 17 industry portfolios. Below each estimate, in parentheses, we report the standard errors that are computed taking into account the fact that pricing errors have different variances and nontrivial covariances. The *J*-test statistic for overidentifying restrictions, given by *T* times the sum of the squared pricing errors, $T(g'g)$, is reported in the fourth column, with its *p*-value in parenthesis. The last column of the table (*MAPE*) is the mean absolute pricing error across portfolios. We estimate model (7) twice, with and without the exponential factor for the variance risk premium, and for the for five time horizons available in our database. The sample frequency is always monthly, from January 2006 to January 2007, which permits the comparison between results across the different horizons and panels of Table 2.

When we use the identity matrix as the weighting matrix in Panel A, the results for the one-month horizon show that the *J*-test fails to reject both pricing specifications. Estimates of risk aversion look reasonable, between 2.5 and 3.6, although estimated standard errors are relatively large. The coefficient of the variance risk premium (c_1) is positive, as expected, but it is also estimated with low precision.[10] Apart from that, both the *J*-statistic and the *MAPE* become lower when adding the variance risk premium to

[9] It is basically the Hansen-Jagannathan (1997) distance (*HJ*-distance) with the identity matrix as the weighting matrix. See the Appendix for a brief description of the estimation method and the calculation of the *p*-value for the test of overidentification restrictions.

[10] To understand the positive sign of the coefficient associated with the variance risk premium in the proposed SDF, it should be noted that if the variance risk premium increases and becomes positive, the marginal utility of wealth would decrease. One additional unit of wealth would then not be highly valued, because we would already be hedged by going long in the variance swap contract. Hence, the estimate associated with the variance risk premium should be positive, as it is the case in Table 2.

the market factor, reflecting an improvement in the fit of the model. Hence, the variance risk premium, as a second factor in an ICAPM framework, seems to contain some relevant information for explaining the cross-section of average returns.

For all other horizons, the two pricing specifications are rejected by the *J*-test at the standard 5% significance level, although the enlarged specification at the 2-month horizon presents a p-value of 0.04. The risk aversion estimate increases with the horizon. The estimated coefficient for the variance risk premium is always positive, with a relatively low standard error for maturities of six and twelve months. The monthly average pricing errors of the CAPM and the two-factor model are higher than those obtained for the shortest horizon. The reduction in *MAPE* by introducing the *VRP* as a second factor for asset pricing is negligible at 2- and 3-month horizons, but it is around 16 percent at the 1-month horizon, and 18 percent at the 6- and 12-month horizons.

Panel B of Table 2 displays estimation results using the inverse of the matrix of second order moments of excess returns as weighting matrix. Therefore, the pricing specification tests are now based on the traditional *HJ*-distance.[11] Neither one of the two alternative pricing specifications are rejected at the one-month horizon at the 1% significance level. On the contrary, both specifications are rejected at conventional significance levels for all other horizons. As before, the relative risk aversion coefficient increases with the horizon, but it is uniformly lower than in Panel A. The coefficient of the variance risk premium is smaller than in Panel A, close to zero except at the one-

[11] We could have also used the optimal GMM weighting matrix; that is, the variance-covariance matrix of pricing errors, instead of a pre-specified matrix. However, that choice would have precluded the comparison between the values of the objective function for different specifications of the *SDF*. To establish that comparison, we need to use the same weighting matrix for each *SDF* specification. On the other hand, we are also specifically interested in pricing the original portfolios, which is why we also emphasize the use of the identity as weighting matrix. In any case, the correlations among the pricing errors are taken into account when computing the standard errors of the parameter estimates, as shown in the Appendix. See Cochrane (2005) for a detailed discussion of these issues.

month horizon, and it is estimated with large standard errors. As a consequence, the contribution of the variance risk premium is now much smaller than when estimating with the identity matrix. Asset prices in our sample are much better fitted under the first-step GMM estimates. In fact, *MAPE* is lower for all horizons by at least 25%, relative to estimates obtained under the *HJ*-metric.

As an alternative way to compare the two model specifications, we now compute the time series for the *SDFs* obtained with the parameters estimated with an identity weighting matrix. To capture the strong cross-sectional and time-series variation of expected returns, we need a *SDF* with enough volatility. Moreover, its volatility should be high at the beginning of recessions and low when expansion periods begin. Figure 2 shows the time-series of estimated *SDFs* for the two asset pricing models, for the one- and six-month horizons. At the shortest maturity, the *SDF* for the one-factor model becomes more volatile and with higher peaks in declining stock market periods once we add the variance risk premium as a second factor. This contribution of the variance risk premium is consistent with the relatively best results provided by the variance risk premium-based ICAPM relative to the one factor model in Table 2. At the six-month horizon, adding the variance risk premium again increases the volatility of the estimated *SDF*, relative to the one-factor model. This extensive representation of the *SDF* over the whole sample seems quite revealing of the difference between the two specifications. Furthermore, the reduction in *MAPE* indicates that the increased volatility in *SDF* actually helps pricing the portfolios in our sample.

Independently of the non-concluding global evaluation of the model through the *J*-test, it is worthwhile to examine the model ability to explain portfolio prices in detail. We now describe which specific portfolios the model is more able to price correctly. Figure 3 shows the average over time of the absolute pricing errors (*APE*) for each of

the 42 original portfolios at the one- and six-month horizons, under the CAPM as well as under the ICAPM specification that incorporates the variance risk premium. When we add the variance risk premium to the one-factor model, the *APE* is reduced for most of the 42 portfolios considered. More specifically, the two-factor model reduces the *APE* for three out of the five extreme growth portfolios, *FF31*, *FF41*, and *FF51* at both horizons. Interestingly, this is not the case for *FF11*, the portfolio of growth and small assets, whose performance shows a higher *APE*, or for the *FF21* portfolio, whose pricing errors are essentially equal under the two specifications. It is also important to point out that the variance risk premium consistently helps pricing the extreme value Fama-French portfolios (*FF15*, *FF25*, *FF35*, and *FF45*).[12] Finally, at the one-month horizon, the ICAPM model achieves a better fit for portfolios *FF12* throughout *F15* (smallest assets) than the one-factor model. This evidence therefore suggests that the *VRP* factor contributes to an improvement in pricing extreme value, extreme growth and small-firm portfolios. Regarding industry portfolios, it turns out that adding the variance risk premium leads to a smaller *APE* for Mines, Oil, Machinery and Utilities at both horizons. Uncovering the characteristics of these sectors that provide a better fit in prices remains an interesting issue for further research.

5.2 The Linear Version of the Two-factor ICAPM Model

Estimating a tight theoretical model with a relatively short time series data can easily lead to a significant loss of efficiency in estimation that may condition the results of the tests for model adequacy. Despite the fact that the *VRP* seems to contain significant incremental information when pricing the cross-section of our test portfolios, especially at the shortest horizon, it should be recognized that the estimated coefficient

[12] With the exception of the largest *FF55* portfolio.

of the *VRP* at this horizon is obtained with a large standard error. This consideration moves us to analyze in this section the pricing results obtained for the 25 Fama-French and the 17 industry portfolios under the linear beta representation of equation (7) for the one-month horizon. We therefore perform the well known Fama-MacBeth (1973) two-pass cross sectional analysis in which the monthly cross-sectional regressions are given by:

$$R^{e}_{jt} = \lambda_0 + \lambda_m \beta_{jmt} + \lambda_{vrp} \beta_{jvrpt} + u_{jt} \tag{8}$$

The results are presented in Table 3. Columns 1 to 3 report the risk premia estimates, together with the Fama-MacBeth and Shanken´s (1992) standard errors. Columns 4 and 5 provide two pseudo-R^2 statistics based on the residual sum of squares of the cross-sectional regressions. The coefficient associated with the variance risk premium beta turns out not to be statistically different from zero.[13] But as in the non-linear model, it looks as if this could be more a consequence of estimating the risk premium for the variance swap payoff with low precision, since the incorporation of the variance risk premium as hedging factor leads to an increase in the cross-sectional overall goodness of fit from 0.237 to 0.358, or from 0.334 to 0.462, depending upon the statistical measure we may employ. The better fit of the linear model after incorporating the variance risk premium can be clearly appreciated in the two graphs of Figure 4, that contain fitted expected returns versus average realized returns for the 42 portfolios for the CAPM and the ICAPM. The largest revisions occur for the *FF25* portfolio, and for the Steel and Mine industry portfolios. The variance risk premium also improves average pricing for the small-value Fama-French portfolios, *FF14* and *FF15*, which is consistent with the evidence reported on the GMM estimates.

[13] As expected, under the linear specification, the sign of the coefficient associated with the variance risk premium is negative.

5.3 The Linear Version of the Two-factor ICAPM Model with Two Regimes based on a Market Return Threshold

As suggested by our proposed *SDF*, stock returns should react very differently to the variance risk premium depending upon the state of the economy. In fact, as we already pointed out, the variance risk premium has very distinct compensation behaviour for negative tail events. The previous non-significant cross-sectional results ignore the possibility of different conditional sensitivities of stock returns to the variance swap payoffs on "bad" versus "good" scenarios. We now want to analyze whether the actual information content of the variance risk premium occurs mainly during recessions

In order to investigate this issue, we allow for market and variance risk premium betas to change over time as a function of the market state. We define factor regression regimes as a function of a given level of market returns, and estimate such threshold simultaneously with the betas for the market and the variance risk premium in each regime. In each regime we use the pooled data for the 42 portfolios for the corresponding periods. This is a Threshold Regression Model, which we estimate under the assumption of a Normal error term. The maximum likelihood estimate is the threshold level for which the least squares estimates of the regressions for the good and bad regimes lead to the lowest aggregate residual sum of squares:

$$\underset{\{u,\alpha,\beta_m,\beta_{vrp}\}}{Min}\left\{\begin{array}{l}\sum_{j=1}^{42}\sum_{t=1}^{T_1(u)}\left(R_{jt}^{e}-\alpha_1(u)-\beta_{m1}(u)R_{mt}^{e}-\beta_{vrp1}(u)VRP_t^{t+1}\right)^2 \\ +\sum_{j=1}^{42}\sum_{t=1}^{T_2(u)}\left(R_{jt}^{e}-\alpha_2(u)-\beta_{m2}(u)R_{mt}^{e}-\beta_{vrp2}(u)VRP_t^{t+1}\right)^2\end{array}\right\}, \quad (9)$$

where u is the market return threshold, and β_{m1}, β_{vrp1}, and $\beta_{m2}\ \beta_{vrp2}$ are the market and variance risk premium betas for the regimes above and below the threshold respectively.

The maximum likelihood estimate of the market return threshold is -7.20 percent. This is an extreme return that splits the sample into "good/regular" regime for the 95 percent of the sample, and a "very low/very bad" regime that includes 5 percent of the sample. Given this partition, the results for the two-regime betas are reported in Table 4. The difference on the overall variance risk premium betas between both regimes are striking. The variance risk premium beta is highly significant and equal to -0.067. Since the variance risk premium is negative for most periods, long positions on variance swaps have positive payoffs only in those states in which the realized volatility is high enough to compensate the fears embedded in the risk neutral expectation of volatility. Moreover, it is also well known that volatility increases in periods of extremely low returns. This explains the large negative and highly significant variance risk premium beta in bad states. On the other hand, the variance risk premium beta for periods with positive or relatively small negative returns becomes practically zero. Even more illustrative is the evidence contained in Figure 5 in which we present the variance risk premium betas for both regimes for each portfolio separately. For most portfolios, the variance risk premium betas become negative and large in bad states. However, they are practically zero in good and regular states. Interestingly, the extreme small-growth portfolios and construction have positive variance risk premium betas in bad states. This implies that the variance swap does not play its hedging role relative to these portfolios. It should be recalled that our sample period coincides with the boom in the real estate industry.

Given this evidence, we now run the Fama-MacBeth two-pass cross sectional regressions using for the market return and the variance risk premium the appropriate betas for the market state in each period:

$$R_{jt}^{e} = \lambda_0 + \lambda_m \beta_{jmt}^{+/-} + \lambda_{vrp} \beta_{jvrpt}^{+/-} + u_{jt} \ , \qquad (10)$$

where $\beta_{jmt}^{+/-}$ and $\beta_{jvrpt}^{+/-}$ denote the betas in the appropriate "good" or "bad" states. As before, Figure 6 shows a clear improvement in fit when we include the two-regime variance risk premium betas relative to the CAPM. More precisely, Table 5 reports the risk premia coefficients from the cross-sectional regression of expression (10). The compensation for the variance risk premium beta becomes much more negative than in Table 3 moving from -0.083 to -0.265 with a clear increase in precision. Moreover, the two measures of goodness of fit employed in the paper increase from 0.188 to 0.274 and from 0.278 to 0.412 when we add to the cross-sectional regression the variance risk premium betas conditional on the market threshold.

To summarize our findings, Figure 7 contains the monthly differences between the adjusted R^2 statistic from each Fama-MacBeth cross-sectional regressions with and without the variance risk beta as an explanatory variable. Independently of using a market threshold in the estimation of betas, we find an increase in the explanatory power of the two-factor ICAPM model relative to the one-factor CAPM model in all months of our sample. We may therefore conclude that the variance risk premium contains incremental information for asset pricing over and above the market portfolio.

6. Conclusions

Recent available evidence show that the excess return on the variance swap contract hedges equity market risks, interest rate and business cycle risks. This evidence

motivates the consideration of a two-factor ICAPM with the variance risk premium playing the role of a hedging portfolio. The question is whether the variance risk premium acts as a sufficient statistic summarizing the information contained in a variety of risk indicators that might be potentially relevant for asset valuation.

Specification tests based on GMM estimates using the identity matrix as metric do not reject the model at one- and two-month horizons at conventional significance level, although the opposite is obtained at the remaining horizons. The time-varying behavior of the estimated *SDF* under the two-factor model presents a relatively more volatile behavior than the simple one-factor model, and pricing errors on individual portfolios are generally lower when the variance risk premium is incorporated into the model. More specifically, and relative to the one-factor model, the variance risk premium seems to explain small and value stocks, as well as Mines, Steel, Oil, Machinery, and Utilities. This is reflected in a reduction in global measures of fit between 16 and 18 percent for 1-, 6- and 12-month horizons, even though the reduced size of pricing errors does not seem to be small enough to not reject the model at these longer horizons according to the standard test for over-identification constraints. The linearized version of the model supports these results by providing a clearly improved fit to observed returns for the 25 Fama-French portfolios and the 17 industry portfolios, always at the one-month horizon.

Although it is standard practice, considering time-invariant parameter values for the full sample period might be too strong an assumption to make the model compatible with the data. When we include a recession threshold in the estimation of the variance risk premium betas in the linearized version of the model, we obtain that the compensation to the variance risk premium beta in asset pricing is limited to recession periods. Hence, the role of the variance risk premium as a pricing factor seems to

concentrate on periods of significant market downturns. The cross-sectional overall measures of fit for the two-factor ICAPM model relative to the one-factor CAPM specification increase independently of using conditional bad state betas or constant betas. The increase in monthly adjusted R^2 in the cross sectional regression from adding the variance risk premium beta is often sizeable. Overall, our results suggest that the premium in variance swaps contains relevant information for asset pricing, possibly because summarizes information contained in a variety of macroeconomic and financial risk indicators. Analyzing the distinct gains in fitting prices of the different portfolios remains as an interesting issue for further research.

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Appendix

Let R_t^e be the *N* x 1 vector of excess return of the *N* assets at time *t* and $m_t(\theta)$ be one out of the two specifications of the *SDFs* described in Section 4, where θ is the vector of the preference parameters for each particular specification. We define an *N* x 1 vector of moment conditions containing the pricing errors generated by the model at time *t*,

$$f_t(\theta) = \left[R_t^e - m_t(\theta) R_t^e \right], \tag{A.1}$$

and the corresponding sample averages,

$$g_T(\theta) = \frac{\sum_{t=1}^{T} f_t(\theta)}{T} \tag{A.2}$$

Then GMM estimator minimizes the following quadratic form

$$g_T(\theta)' W_T g_T(\theta) \tag{A.3}$$

where W_T is a weighting *N* x *N* matrix.

For estimation, we could use the optimal weighting matrix in Hansen (1982), S_T^{-1}, where

$$S_T = \frac{\sum_{t=1}^{T} f_t(\theta) f_t(\theta)'}{T} \tag{A.4}$$

Instead of that, we employ a pre-specified weighting matrix which is either the identity matrix (for the results of Panel A of Table 4) or the matrix of the second moments of excess returns (for the results of Panel B of Table 4).

The asymptotic variance-covariance matrix of the GMM estimates is given by

$$V = \frac{1}{T}\left(D_T{}'W_T D_T\right)^{-1} D_T{}'W_T S_T W_T D_T \left(D_T{}'W_T D_T\right)^{-1}, \tag{A.5}$$

where D_T is a matrix of partial derivatives defined by

$$D_T = \frac{\sum_{t=1}^{T} \partial f_t(\theta)/\partial(\theta)}{T} \tag{A.6}$$

Then, the standard errors of the estimated coefficients $\hat{\theta}$ are computed from the estimated variance:

$$\hat{V} = \frac{1}{T}\left(\hat{D}_T{}'W_T \hat{D}_T\right)^{-1} \hat{D}_T{}'W_T \hat{S}_T W_T \hat{D}_T \left(\hat{D}_T{}'W_T \hat{D}_T\right)^{-1}, \tag{A.7}$$

where $\hat{D}_T$ and $\hat{S}_T$ are obtained replacing θ by $\hat{\theta}$ in D_T and S_T, respectively.

The evaluation of the model performance is carried out by testing the null hypothesis:

$$H_{\cdot}: T\left[Dist(\theta)\right] = 0, \tag{A.8}$$

with $Dist = \sqrt{g_{\cdot}(\theta)' W g_{\cdot}(\theta)}$ where, as mentioned above, the weighting matrix, *W*, is either the identity matrix or the second moment matrix of excess returns.

If the weighting matrix is optimal, $T\left[Dist\left(\hat{\theta}\right)\right]^2$ is asymptotically distributed as a Chi-square with *N-P-1* degrees of freedom, where *P* is the number of parameters. However, for any other weighting matrix (including the identity matrix), the distribution of the test statistic is unknown. Jagannathan and Wang (1996) show that, in this case, $T\left[Dist\left(\hat{\theta}\right)\right]^2$ is asymptotically distributed as a weighted sum of *N-P-1* independent Chi-squares random variables with one degree of freedom. That is

$$T\left[Dist\left(\hat{\theta}\right)\right]^2 \xrightarrow{d} \sum_{i=1}^{N-P-1} \lambda_i \chi_i^2(1), \tag{A.9}$$

where λ_i, for $i = 1,2,...,N-P-1$, are the positive eigenvalues of the following matrix:

$$A = S_T^{1/2} W_T^{1/2} \left[I_N - \left(W_T^{1/2}\right)^{-1} D_T \left(D_T' W_T D_T\right)^{-1} D_T' W_T^{1/2} \right] \left(W_T^{1/2}\right)' \left(S_T^{1/2}\right)' \tag{A.10}$$

in which $X^{1/2}$ means the upper-triangular matrix from the Choleski decomposition of X, and I_N is a *N*-dimensional identity matrix.

Therefore, in order to test the different models we estimate, we proceed in the following way. First, we estimate the matrix A by

$$\hat{A} = \hat{S}_T^{1/2} W_T^{1/2} \left[I_N - \left(W_T^{1/2}\right)^{-1} \hat{D}_T \left(\hat{D}_T' W_T \hat{D}_T\right)^{-1} \hat{D}_T' W_T^{1/2} \right] \left(W_T^{1/2}\right)' \left(\hat{S}_T^{1/2}\right)' \tag{A.11}$$

and compute its nonzero *N-P-1* eigenvalues. Second, we generate $\{v_{hi}\}$, *h* = 1,2,…,100, *i* = 1,2,…,*N*+1-*P*, independent random draws from a $\chi^2(1)$ distribution. For each *h*, $u_h = \sum_{i=1}^{N-P-1} \lambda_i v_{hi}$ is computed. Then we compute the number of cases for which $u_h > T\left[Dist\left(\hat{\theta}\right)\right]^2$. Let *p* denote the percentage of this number. We repeat this procedure 1000 times. Finally, the *p*-value for the specification test of the model is the average of the *p* values for the 1000 replications.

Table 1
Variance Risk Premia
Descriptive Statistics

Panel A: Descriptive Statistics					
	VRP1	*VRP2*	*VRP3*	*VRP6*	*VRP12*
Mean	-0.646	-0.635	-0.659	-0.694	-0.736
Median	-0.697	-0.682	-0.719	-0.751	-0.734
Maximun	0.834	0.952	0.841	0.706	0.441
Minimum	-1.556	-1.612	-1.631	-1.576	-1.600
Panel B: Linear Correlations					
	VRP1	*VRP2*	*VRP3*	*VRP6*	*VRP12*
VRP1	1	0.793	0.659	0.402	0.224
VRP2		1	0.910	0.650	0.453
VRP3			1	0.798	0.574
VRP6				1	0.793
VRP12					1

VRP is the variance risk premium associated with the alternative horizons of the variance swap contract between 1 and 12 months. It is computed as the difference between the ex-post realized variance at the end of the swap contract and the observed variance swap rate.

Table 2
GMM Estimation
25 Size/Book-to-Market Portfolios and 17 Industry Portfolios
Monthly Data, January 1996-January 2007

	Panel A: First Step				*Panel B: Hansen-Jagannathan Distance*			
Horizon	γ	c_1	$T(g'g)$	*MAPE*	γ	c_1	$TDist^2$	*MAPE*
1 month	3.57		0.0992	0.3425	2.07		64.895	0.4588
	(2.11)		(0.092)		(1.97)		(0.010)	
	2.54	0.90	0.0758	0.2881	1.95	0.29	63.689	0.4389
	(2.15)	(0.65)	(0.286)		(1.98)	(0.43)	(0.018)	
2 months	5.78		0.1024	0.3480	3.02		92.063	0.4609
	(2.95)		(0.008)		(2.74)		(0)	
	4.10	1.12	0.0893	0.3298	2.93	0.11	91.866	0.4595
	(2.97)	(0.67)	(0.041)		(2.74)	(0.44)	(0)	
3 months	8.37		0.1038	0.3514	4.27		105.805	0.4542
	(3.76)		(0.000)		(3.42)		(0)	
	6.48	1.08	0.0948	0.3406	4.30	-0.03	105.790	0.4542
	(3.87)	(0.84)	(0.001)		(3.41)	(0.39)	(0)	
6 months	16.76		0.1202	0.3903	8.19		118.208	0.4702
	(5.68)		(0)		(5.16)		(0)	
	12.22	1.24	0.0960	0.3292	8.19	0.00	118.208	0.4702
	(5.63)	(0.54)	(0)		(5.13)	(0.31)	(0)	
12 months	25.31		0.1340	0.4261	12.53		121.344	0.4732
	(7.00)		(0)		(6.60)		(0)	
	20.08	1.21	0.1048	0.3569	12.57	-0.02	121.336	0.4745
	(6.92)	(0.41)	(0)		(6.59)	(0.26)	(0)	

We estimate the standard version and an intertemporal version of the CAPM using the variance risk premium as the hedging factor in the intertemporal specification. The vector of moment conditions is

$$E\left[\left(\left(R_{W,t+\tau}\right)^{-\gamma} exp\left(c_1 VRP_t^{t+\tau}\right)\right)R^e_{j,t+\tau} \mid I_{t+\tau-1}\right]=0$$

where $\tilde{R}_w$ is the gross return on wealth, γ is the relative risk aversion coefficient, R^e_j is the excess return on portfolio *j* and *VRP* represents the variance risk premium, computed as the log difference between the realized variance at the end of the swap contract ($t+\tau$) and the variance swap rate at the beginning of the contract (*t*) . We use a linear projection to compute the component of the variance risk premium that is orthogonal to the market return. The estimation is made for different investment horizons (τ), from 1- to 12-months, using always monthly data. Results reported on Panel A refer to the first step GMM estimation while the estimates shown in Panel B have been obtained using the inverse of covariance matrix of the portfolio excess returns as weighting matrix. Columns 2, 3, 6, and 7 contain the estimated coefficients. Associated standard errors are shown below, in brackets. Column 4 provides the value of *T* times the sum of squared pricing errors. The p-value for the test of overidentifying conditions is shown in brackets, while in Panel B the specification test of the model is performed using the Hansen-Jagannathan distance (column 9). Finally, *MAPE* indicates the mean absolute pricing error across portfolios, in percentage terms.

Table 3
Estimates and Standard Errors of Intercepts and Risk Premia for the Traditional Fama-MacBeth Two-Pass Cross-Sectional Regressions, Monthly Data, January 1996-January 2007

	$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{vrp}$	*Statistic 1*	*Statistic 2*
FM Estimate	0.00445	0.00218		0.237	0.334
FM St. Error	(0.00475)	(0.00647)			
SH St. Error	[0.00475]	[0.00682]			
FM Estimate	0.00444	0.00169	-0.08337	0.358	0.462
FM St. Error	(0.00467)	(0.00628)	(0.08017)		
SH St. Error	[0.00467]	[0.00673]	[0.08670]		

This table presents the Fama-MacBeth two-step cross-sectional estimation results for the one-factor (CAPM) and two-factor (ICAPM) capital asset pricing models using the variance risk premium as the hedging factor:

$$R_{jt}^{e} = \lambda_0 + \lambda_m \beta_{jmt} + \lambda_{vrp} \beta_{jvrpt} + u_{jt}$$

The test assets are the returns on the 25 *FF* portfolios plus 17 industry portfolios in excess of the T-bill rate. We report risk premium parameter estimates ($\hat{\lambda}$), standard errors under the Fama-MacBeth (FM) methodology in parenthesis, and the Shanken (SH) errors-in-variable-robust standard errors in brackets. The overall goodness of model fit is measured by the two following statistics:

$$\textit{Statistic 1: } \frac{\sum_{t=1}^{T} \left(TSS_t - RSS_t \right)}{\sum_{t=1}^{T} TSS_t} \; ; \; \textit{Statistic 2: } \frac{1}{T} \sum_{t=1}^{T} \left(1 - \frac{RSS_t}{TSS_t} \right)$$

TSS and *RSS* denote the Total Sum of Squares and the Residual Sum of Squares, respectively.

Table 4
Estimates and Standard Errors of Alphas and Betas from a Pooled OLS Regression with Two Regimes Based on a Market Return Threshold
January 1996-January 2007

	Extremely Low Market Return			*Regular Market Return*		
	$\hat{\alpha}$	$\hat{\beta}_m$	$\hat{\beta}_{vrp}$	$\hat{\alpha}$	$\hat{\beta}_m$	$\hat{\beta}_{vrp}$
Estimates	0.0093	1.0523	-0.0677	0.0022	0.8814	-0.0007
St. Errors	0.0161	0.1467	0.0073	0.0011	0.0162	0.0015

This table reports the overall market beta and the variance risk premium beta from a pooled OLS time-series regression under a two-regime specification defined by a given market return. The market return threshold is simultaneously estimated with the two regressions. The test assets are the 25 *FF* portfolios and 17 industry portfolios, with the returns in excess of the T-bill rate. The maximum likelihood estimate is the threshold level for which the least squares estimates of the regressions for the good and bad regimes lead to the lowest aggregate residual sum of squares:

$$\underset{\{u,\alpha,\beta_m,\beta_{vrp}\}}{Min}\left\{\begin{array}{l}\sum_{j=1}^{42}\sum_{t=1}^{T_1(u)}\left(R_{jt}^{e}-\alpha_1(u)-\beta_{m1}(u)R_{mt}^{e}-\beta_{vrp1}(u)VRP_t^{t+1}\right)^2 \\ +\sum_{j=1}^{42}\sum_{t=1}^{T_2(u)}\left(R_{jt}^{e}-\alpha_2(u)-\beta_{m2}(u)R_{mt}^{e}-\beta_{vrp2}(u)VRP_t^{t+1}\right)^2\end{array}\right\}$$

where u is the market return threshold, and β_{m1}, β_{vrp1}, and β_{m2}, β_{vrp2} are the market and the variance risk premium betas for the regime with the market return above and below the threshold respectively

Table 5
Estimates and Standard Errors of Intercepts and Risk Premia for the Market Threshold Two-Regimes Fama-MacBeth Two-Pass Cross-Sectional Regressions, January 1996-January 2007

	$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{vrp}$	*Statistic 1*	*Statistic 2*
FM Estimate	0.00438	0.00377		0.188	0.278
FM St. Error	(0.00526)	(0.00613)			
SH St. Error	[0.00526]	[0.00746]			
FM Estimate	0.00247	0.00437	-0.26525	0.274	0.412
FM St. Error	(0.00467)	(0.00642)	(0.19408)		
SH St. Error	[0.00467]	[0.00739]	[0.21250]		

This table presents the Fama-MacBeth two-step cross-sectional estimation results for the one-factor (CAPM) and two-factor (ICAPM) capital asset pricing models using the variance risk premium as the hedging factor:

$$R_{jt}^{e} = \lambda_0 + \lambda_m \beta_{jmt}^{+/-} + \lambda_{vrp} \beta_{jvrpt}^{+/-} + u_{jt},$$

where $\beta_{jmt}^{+/-}$ and $\beta_{jvrpt}^{+/-}$ denote the betas in the corresponding states.

The test assets are the 25 *FF* portfolios and 17 industry portfolios, with returns in excess of the T-bill rate. We report risk premia parameter estimates ($\hat{\lambda}$), standard errors under the Fama-MacBeth (FM) methodology in parenthesis, and the Shanken (SH) errors-in-variable-robust standard errors in brackets. The overall goodness of model fit is measured by two statistics:

$$\textit{Statistic 1: } \frac{\sum_{t=1}^{T}\left(TSS_t - RSS_t\right)}{\sum_{t=1}^{T} TSS_t} \textit{ ; Statistic 2: } \frac{1}{T}\sum_{t=1}^{T}\left(1 - \frac{RSS_t}{TSS_t}\right)$$

TSS and *RSS* denote the Total Sum of Squares and the Residual Sum of Squares, respectively.

Figure 1. Monthly Variance Swap Rate and Realized Variance for Different Maturities
January 1996-January 2007

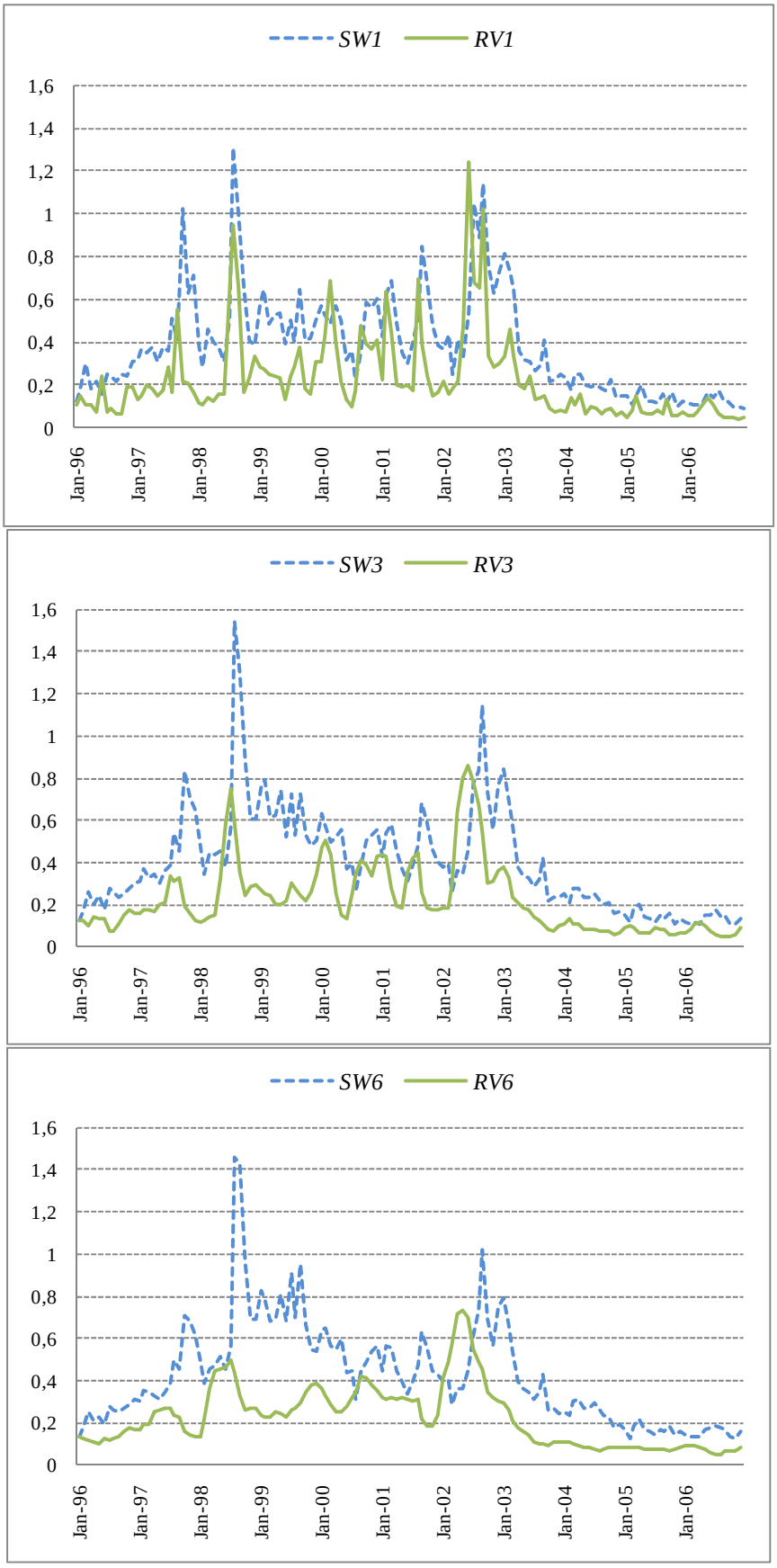
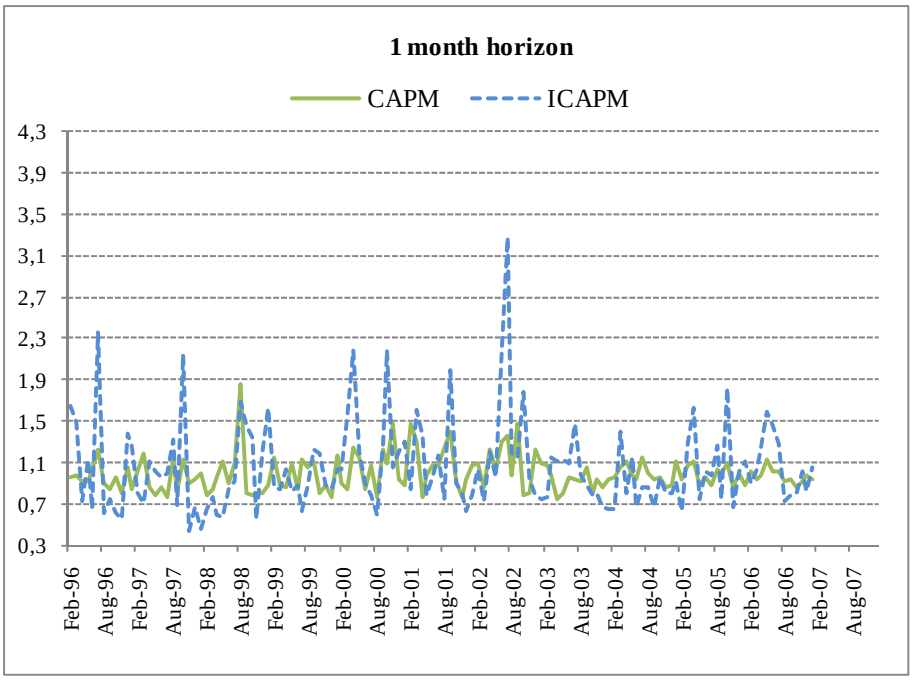


Figure 2. Time Series of Estimated Stochastic Discount Factors from GMM Parameter Estimates Obtained Using the Identity Matrix as Weighting Matrix.
January 1996-January 2007



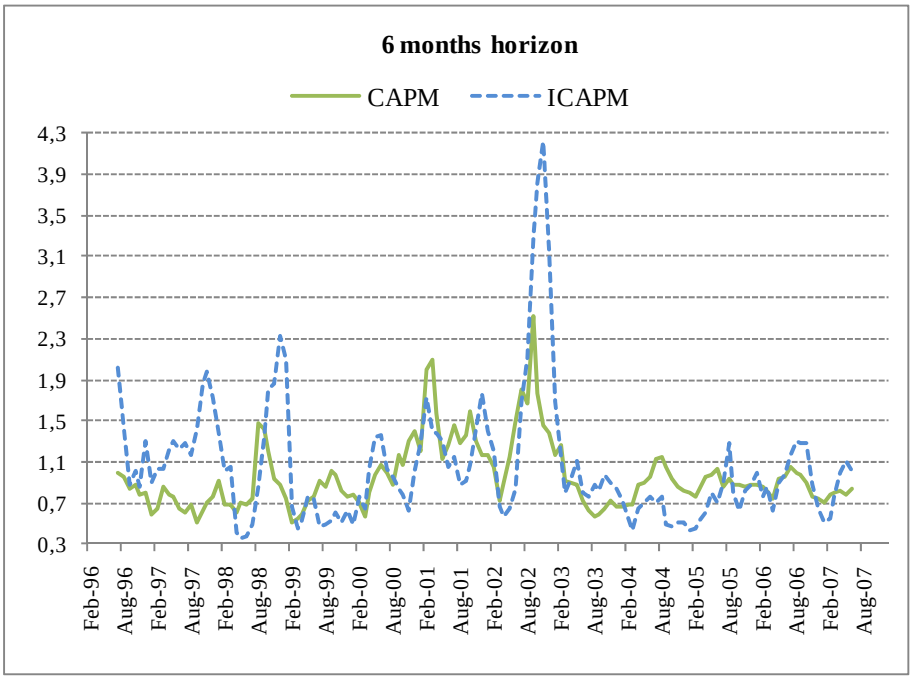
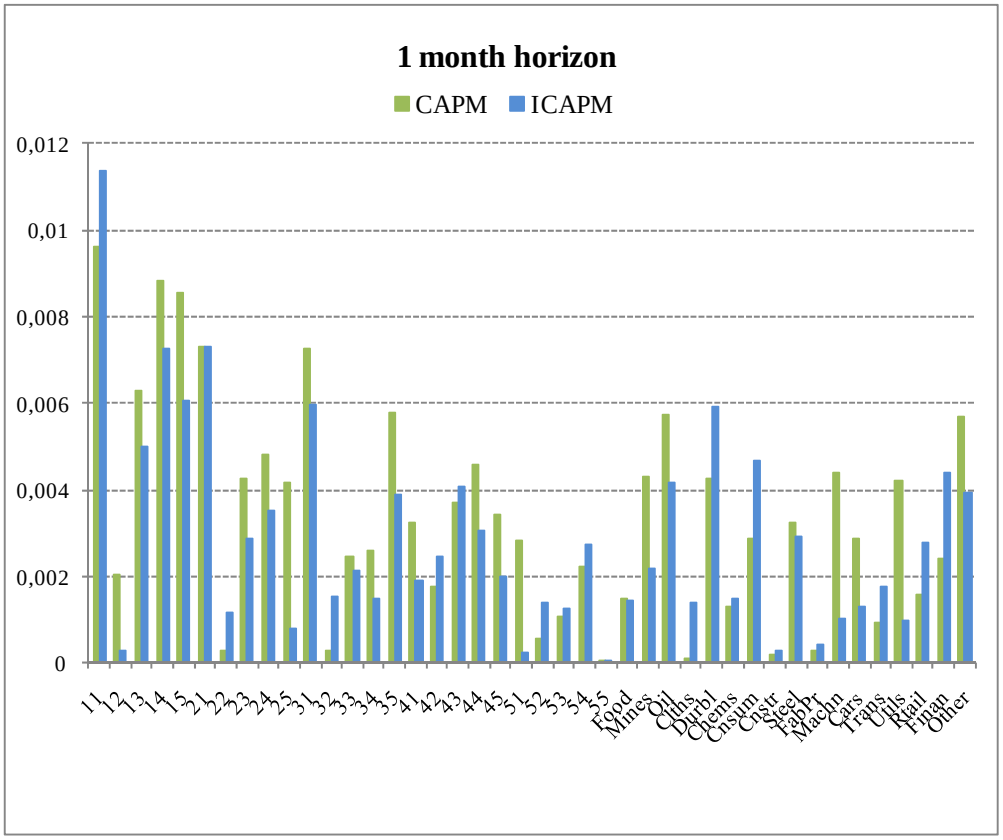


Figure 3. Fama-French and Industry Portfolio Absolute Pricing Errors from GMM Parameter Estimates Obtained Using the Identity Matrix as Weighting Matrix. January 1996-January 2007



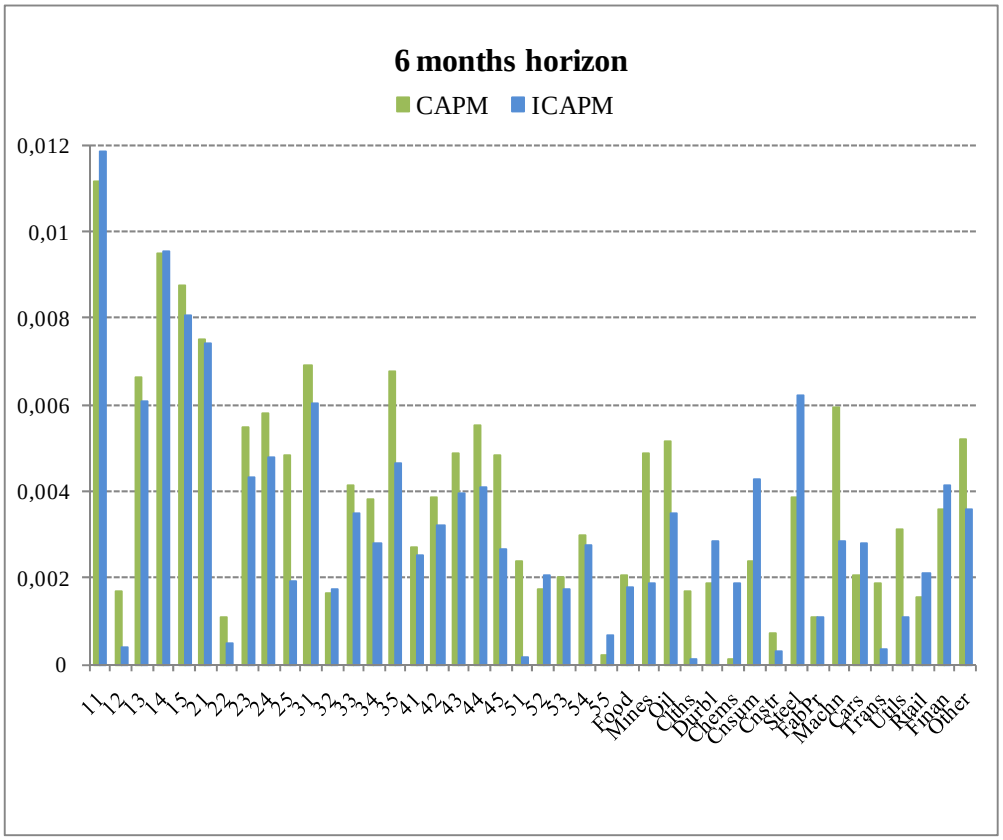


Figure 4. Fitted Expected Returns vs. Average Realized Returns
January 1996-January 2007

This figure shows realized returns on the horizontal axis and fitted expected returns on the vertical axis for 25 size and book-to-market sorted portfolios and 17 industry portfolios. For each portfolio, the realized average return is the time-series average of the portfolio return, while the fitted expected return is the fitted value for the expected return from the corresponding model.

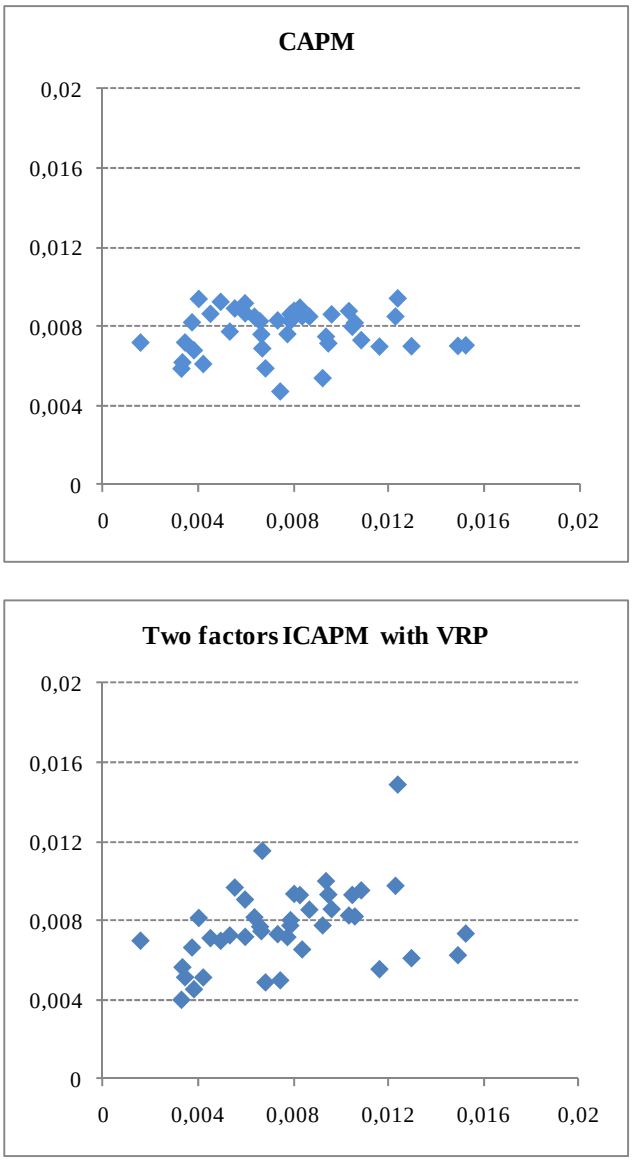


Figure 5. Variance Risk Premium Betas across 25 Fama-French and 17 Industry Portfolios from a Pooled OLS Regression with Two Regimes Based on a Market Return Threshold
January 1996-January 2007

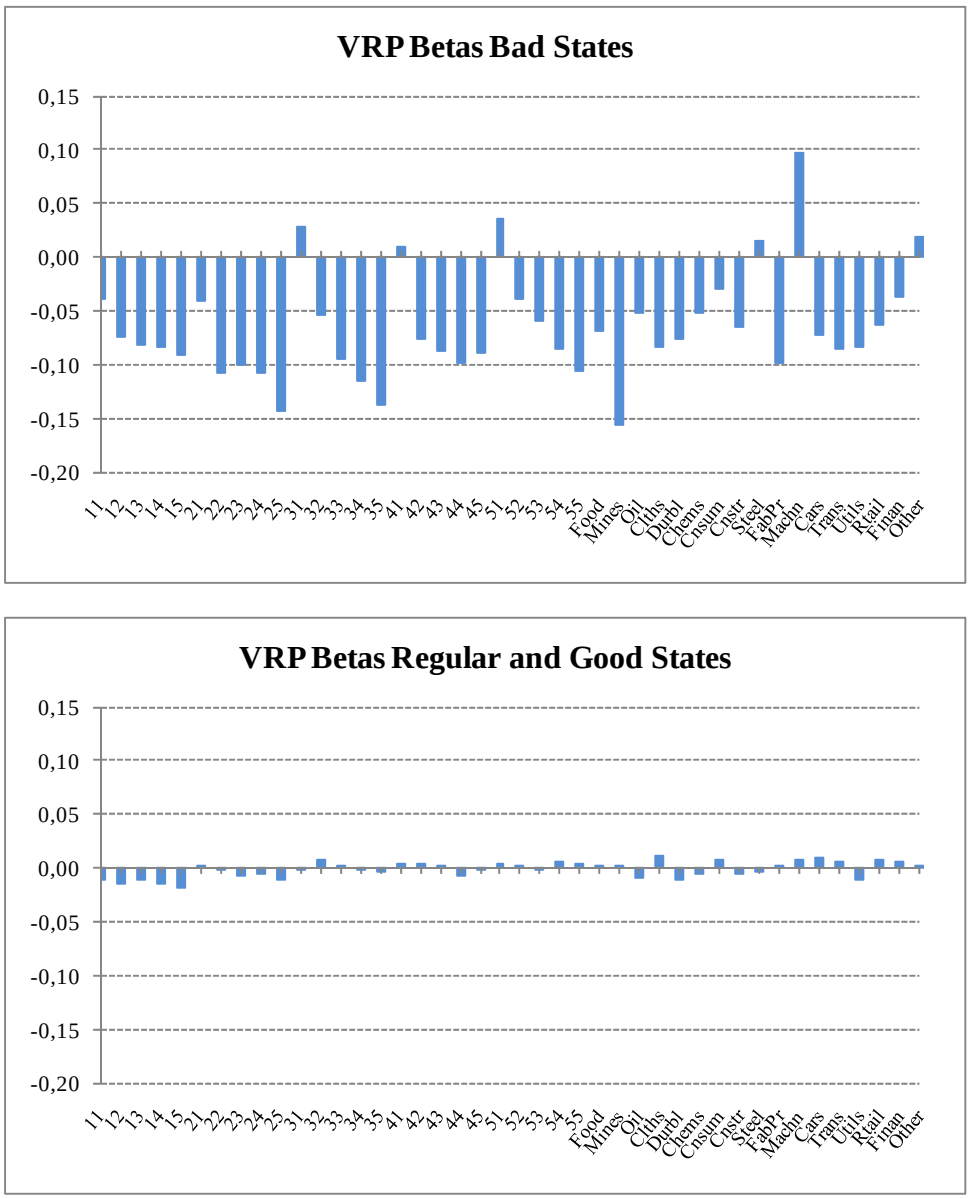
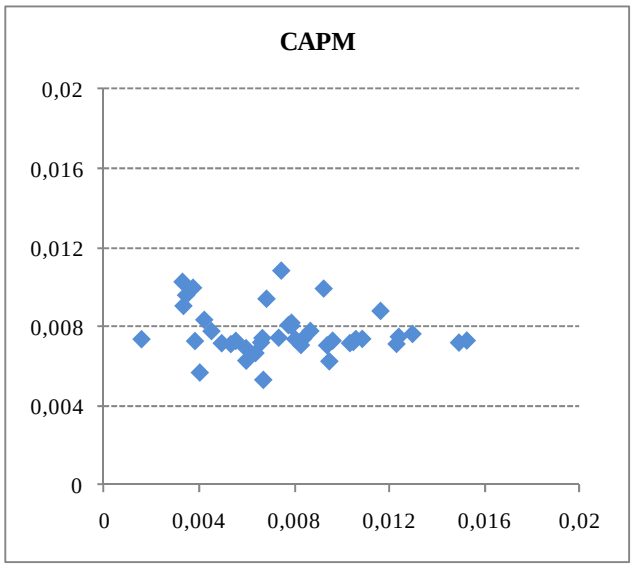


Figure 6. Fitted Expected Returns vs. Average Realized Returns from Cross-sectional Regressions under Two-Regime Betas Based on a Market Return Threshold January 1996-January 2007

This figure shows realized returns on the horizontal axis and fitted expected returns on the vertical axis for 25 size and book-to-market sorted portfolios and 17 industry portfolios. For each portfolio, the realized average return is the time-series average of the portfolio return, while the fitted expected return is the fitted value for the expected return from the corresponding model.



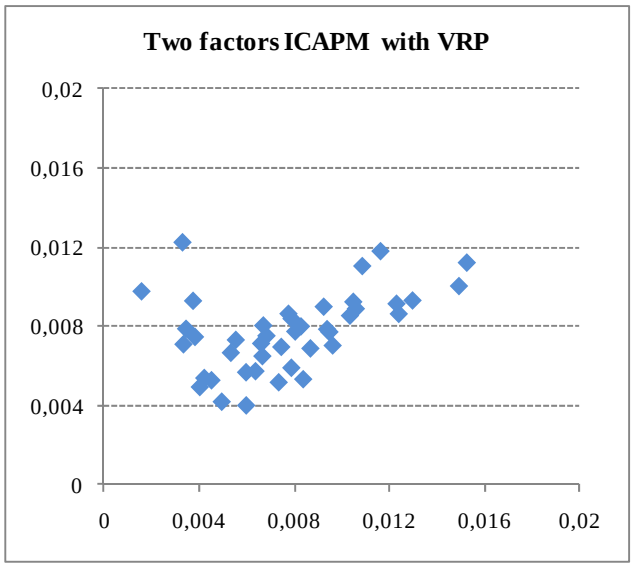


Figure 7. Differences between Adjusted *R*-squares from Monthly Cross-Sectional Regressions under Rolling OLS and Two-Regime Beta Estimates
January 1996-January 2007