

# Convergence and Cointegration

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## Abstract

This paper provides a new, unified, and flexible framework to measure and characterize a convergence process. Specifically, we formally define the notion of asymptotic price convergence and propose a model to represent a wide range of transition paths that converge to a common steady-state. Our framework enables the econometric measurement of such transitional behaviors and the development of testing procedures. In particular, we derive a statistical test to determine whether asymptotic convergence exists and, if so, of which type: strong or weak and as catching-up or steady-state. The application of this methodology to historic wheat prices results in a novel explanation about the event that triggered the convergence process experienced during the 19<sup>th</sup> century.

**Keywords:** Convergence, cointegration, law of one price, unit root.

**JEL:** C22, C32, N70, F15

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# 1 Introduction

Most articles on convergence in economics are based on the study of the steady state of the underlying processes, i.e., the state in which convergence was already reached. In this context, common cointegration analysis fails to answer some questions about the character of the convergence process, concerning, e.g., when the process started or ended and the shape or speed of convergence. Specifically, this happens when the convergence process is known as catching-up. The previous questions are not only of theoretical interest, but also of major importance in some fields, such as macroeconomics, economic history, international economics or financial econometrics.

Particularly, our paper looks at price convergence from a time-series perspective, but some straightforward modifications make its results general enough to be applied to convergence in output or any other variable of interest. We present four contributions (three theoretical and one empirical) in this framework. First, we formally define a generalized notion of asymptotic price convergence, based on the property of cointegration, which adapts to both existing types of convergence –as catching-up and steady-state– and relates with the Law of One Price. To our knowledge, this is done for the first time in the literature. Second, we provide a model where the transition or catching-up phase is represented by an exogenous deterministic input. Therefore, this model allows one to identify from the data many different and flexible transition paths (even one for each relative price, if necessary), and fully describes a general convergence process. Third, we show how to test appropriately the parameter restrictions implied by the definitions proposed in the model previously built. Thus, the methodology proposed is self-contained, presenting the steps and tools required to fully analyze a price convergence process using a time-series approach. The last contribution is of empirical and historical character. Our study suggests a novel explanation for the convergence process experienced by wheat prices during the second half of the 19<sup>th</sup> century. We find that prices' transition to parity began just after the elimination, mainly in Britain and about 1846, of the import tariffs on grain and were almost completed just before the American Civil War.

The number of works that relates convergence and cointegration in a time-series approach is extensive. The pioneer work by Bernard and Durlauf (1995, 1996) was the first in stating this relationship with two different definitions for convergence: as catching-up or steady-state. They concluded that only the second one can be linked to the concept of cointegration. Our definition is more general and encompasses these two type of convergences. Since then, several authors have contributed to this literature. Hobijn and

Franses (2000) redefine the term, derive its necessary and sufficient conditions and introduce a cluster algorithm that allows for the endogenous selection of converging countries. Nahar and Inder (2002) prove that stationarity is not a necessary condition in Bernard and Durlauf's steady-state convergence definition. These authors propose a new test for convergence and highlight the inappropriateness of tests for unit roots and cointegration as an indicator of the presence of convergence. Other authors test the hypothesis of convergence using more complex and recent cointegration models by relaxing some assumptions in the original framework. Specifically, Datta (2003) and Bentzen (2005) relax structural stability, while Chong et al. (2008) and King and Ramlogan-Dobson (2011) test for nonlinear convergence. The papers mentioned above were originally devised to analyze convergence in output, but their theoretical contributions have also been used to study convergence in prices (see, e.g., Robinson, 2007). However, none of them generalizes the definition of convergence or presents a model that fully represents it. Particularly focused on relative prices and inflation convergence, Busetti et al. (2006) show how the joint use of unit-root and stationarity tests in levels and first differences allows one to distinguish between catching-up and steady-state convergence.

An expanded and slightly different approach is to analyze convergence using panel data methods. A very partial list of some recent contributions to this focus includes Cecchetti et al. (2002), Goldberg and Verboven (2005), Fan and Wei (2006), Phillips and Sul (2007, 2009) or Lan and Sylwester (2010). Unfortunately, available data does not always fulfill the characteristics required by panel-data analysis, e.g., a large enough number of cross-section observations. This is often the case of data used by economic historians. In other cases, one could be interested in testing the hypothesis of convergence of a restricted and small number of goods, cities or countries. It is in those situations where our time-series procedure seems to be the adequate approach.

The paper is organized as follows. Section 2 introduces our theoretical framework and two definitions of asymptotic price convergence. Section 3 describes the model and illustrates different types of convergence in prices. In Section 4 the econometric representation and the hypothesis testing are presented. Section 5 shows the empirical results on wheat price convergence in the second half of the 19<sup>th</sup> century, while Section 6 concludes.

## 2 Theoretical framework

We assume that (log) nominal prices need a difference to be stationary. This reflects the idea that some shifts in supply (e.g., due to technological breakthroughs, changes

in wages, etc.) or in demand (e.g., due to changes in consumer preferences, population growth, etc) imply price adjustments are necessary to clear the market in the long run. Specifically, we consider that the (log) price series satisfy an ARIMA( $p, 1, q$ ) model as:

$$\phi_{i,p}(B)\Delta p_{i,t} = \theta_{i,q}(B)a_{i,t}, \quad (1)$$

where  $\phi_{i,p}(B) = 1 - \phi_{i,1}B - \dots - \phi_{i,p}B^p$ ,  $\theta_{i,q}(B) = 1 - \theta_{i,1}B - \dots - \theta_{i,q}B^q$ ,  $\Delta p_{i,t} = (1 - B) \log P_{i,t} = \log(P_{i,t}/P_{i,t-1})$ ,  $a_{i,t}$  is a sequence of zero-mean uncorrelated random variables with finite variance, from now on Weak White Noise (WWN), and  $i, j = 1, 2, \dots, m$  for  $i \neq j$ . We assume that the process  $\Delta p_{i,t}$  is strictly stationary and invertible –i.e., the autoregressive and moving average polynomials have all their zeros lying outside the unit circle– and there are no common factors between  $\phi_{i,p}(B)$  and  $\theta_{i,q}(B)$ . Our view of market efficiency follows the line proposed by Lo (2004, 2005) in the sense of the Adaptive Market Hypothesis, which means that model (1) permits transitory arbitrage situations under uncertainty.<sup>1</sup>

We establish now some assumptions about the relationship between the goods whose prices will be analyzed. The paper considers price convergence of perfectly homogeneous and quasi-homogeneous goods. We assume that price similarities or dissimilarities (generated by quality, brand, and consumer perception) are time-invariant. Then arbitrage should prevent prices for those goods from moving independently of each other. This idea can be expressed as  $p_{i,t} = \alpha p_{j,t} + \varepsilon_t$ , where  $p_{k,t}$  ( $k = i, j$ ) is as in (1),  $\varepsilon_t$  is a stationary stochastic process, and  $\alpha > 0$  models the (time-invariant) degree of homogeneity between both prices. Thus, if two prices have converged, they should be cointegrated of order CI(1,1), with cointegrating vector  $[1, -\alpha]$ . Specifically, when two goods are perfectly homogeneous, their elasticity of substitution should be extremely large, and a change in the price of one good will lead to a proportional change in the other's. In that case,  $\alpha$  should be equal to the unity.<sup>2</sup>

From this reasoning based on arbitrage and following the stochastic definitions of convergence in output presented by Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000), we state the two following definitions, where  $\mathcal{F}_t$  denotes all information available to the agents at  $t$ .

**Definition 1** *Asymptotically Strong Price Convergence (ASPC). Prices of goods  $i$  and  $j$*

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<sup>1</sup>Other market hypothesis can be used. For instance, the Efficient Market Hypothesis can be considered by assuming that prices follow a random walk.

<sup>2</sup>It is easy to see that  $(\beta P_{i,t})/(\beta P_{j,t})^\alpha = P_{i,t}/P_{j,t}^\alpha \iff \alpha = 1$ , where  $\beta$  is a multiplicative variation of prices.

converge asymptotically strongly, if:

$$\lim_{k \rightarrow \infty} \mathbb{E}[p_{i,t+k} - \alpha p_{j,t+k} | \mathcal{F}_t] = 0.$$

This definition coincides with Bernard and Durlauf's definition of convergence in output but allows  $\alpha$  to be different from the unity. Our definition also allows asymptotically strong price convergence in non-perfectly homogeneous goods. When  $\alpha = 1$ , Definition 1 is interpreted as the asymptotic and stochastic representation of the strict form of the law of one price, which assumes perfect competition, no trade barriers and no transport costs. In such a case the long-term forecast of (log) price differentials is zero-mean stationary. However, the assumptions made in the strict law of one price can be broken in many different situations in practice. Hence, it would be worthwhile to look at a definition of convergence that relaxes those assumptions, permitting a price differential to converge to a nonzero constant,  $\tau_{ij}$ , that precisely reflects those transaction costs. Thus, we complement Definition 1 with the following one.

**Definition 2** *Asymptotically Weak Price Convergence (AWPC).* Prices of goods  $i$  and  $j$  converge asymptotically weakly, if:

$$\lim_{k \rightarrow \infty} \mathbb{E}[p_{i,t+k} - \alpha p_{j,t+k} | \mathcal{F}_t] = \tau_{ij}.$$

Note that this requirement is weaker than those required by cointegration, as it allows the price differential to present a deterministic trend. The relations between these two definitions and the notion of cointegration are straightforward: zero-mean stationary price differential (zero-mean cointegrated nominal prices)  $\Rightarrow$  ASPC  $\Rightarrow$  AWPC, while stationary price differential (cointegrated nominal prices)  $\Rightarrow$  AWPC. However, as we will see further on, the reverse does not hold true.

### 3 The model

Once the framework has been set up, we introduce a deterministic input that will be used to represent the convergence process in our model. Let  $\xi_t^{t^*}$  describe the effects of an event that will last permanently after time  $t^*$ , as unity whenever  $t > t^*$  and zero otherwise. We use this step-at-time- $t^*$  sequence to formally define the transition path as:

$$\nu(B)\xi_t^{t^*} := \frac{\omega_s(B)}{\delta_r(B)} B^b \xi_t^{t^*} \quad (2)$$

where  $\omega_s(B) = \omega_0 - \omega_1 B - \dots - \omega_s B^s$ ,  $\delta_r(B) = 1 - \delta_1 B - \dots - \delta_r B^r$ , there are no common factors between  $\omega_s(B)$  and  $\delta_r(B)$ , and  $s, r, b$  are non-negative integers. The concept

of convergence is closely linked to stability and, consequently, we assume that  $\delta_r(B)$  is stable, i.e., the roots of the characteristic equation  $\delta_r(B) = 0$  lie outside the unit circle. Two interesting parameters can be estimated from the stable convergence process defined in (2): (i) the steady-state gain,  $g$ , defined as  $g := \sum_{k=0}^{\infty} \nu_k = \nu(1) < \infty$ , and (ii) the mean lag of response,  $l := \nu'(B)/\nu(B)|_{B=1}$ , where  $\nu'(B)$  is the derivative of  $\nu(B)$  with respect to  $B$ , which measures the speed of convergence when the response is monotone. Note that our transition path can easily be related to the literature of level shifts. For  $s = 0$  and  $r = 0$  the convergence path (2) results in an abrupt shift in the level, known as an additive outlier, while for  $s = 0$  and  $r > 0$  it allows for a smooth shift from the initial level to a new level, known as an innovational outlier.  $b > 0$  just introduces a number of time delays or dead times.

An example of a convergence process can be represented by a smooth monotone response, with  $\nu(B) = \omega_0/(1 - \delta_1 B)$  and  $0 < \delta_1 < 1$ , reflecting the fact that agents are not likely to react all at once, for instance, due to market inefficiencies. In this case,  $\delta_1 = 1$  would imply a  $\omega_0$ -slope linear transition. This linearity is commonly used in the literature –see Razzaque et al. (2007); Robinson (2007); Chong et al. (2008) among others– but has two important drawbacks, as it: (i) is very restrictive, (ii) very abrupt, and (iii) requires one to provide not only the date when the convergence started but also when it ended, otherwise no convergence is possible. Figure 1 shows two examples of (2) that represent a gradual monotone, and a damped quasi-cyclical convergence path.

Figure 1 should be around here

Using equations (1) and (2), our model for the (log) price differential is written as:

$$\begin{aligned}
 p_{i,t} - \alpha p_{j,t} &= D_{ij,t} + S_{ij,t}, \\
 D_{ij,t} &= \nu_{ij}(B) \xi_t^{t*} + \mu_{ij}, \\
 \phi_{ij,p}(B) S_{ij,t} &= \theta_{ij,q}(B) a_{ij,t}
 \end{aligned} \tag{3}$$

where  $\mu_{ij}$  is a constant mean,  $S_{ij,t}$  is a stochastic process and  $a_{ij,t}$  is a WWN. From the asymptotic price convergence definitions previously stated, we relate the concepts of catching-up and steady-state convergence with the existence or non-existence of a transition path in model (3).

**Definition 3** *Steady-state convergence.* Prices of goods  $i$  and  $j$  converge in steady-state, if they converge asymptotically (weakly or strongly) when  $\nu_{ij}(B) = 0$ .

**Definition 4** *Catching-up convergence.* Prices of goods  $i$  and  $j$  converge in catching-up, if they converge asymptotically (weakly or strongly) when  $\nu_{ij}(B) \neq 0$ .

The relations between model (3) and Definitions 1-4 are stated in the following proposition that makes them more easily testable.

**Proposition 1** *Let  $p_{i,t} - \alpha p_{j,t}$  be represented by model (3), then  $p_{i,t}$  and  $p_{j,t}$  converge in:*

1. *AWPC as steady-state if  $p_{i,t} - \alpha p_{j,t}$  is a stationary process.*
2. *AWPC as catching-up if  $\nu_{ij}(B) \neq 0$  and  $S_{ij,t}$  is a stationary process.*
3. *ASPC as steady-state if  $p_{i,t} - \alpha p_{j,t}$  is a zero-mean stationary process.*
4. *ASPC as catching-up if  $\nu_{ij}(B) \neq 0$ ,  $S_{ij,t}$  is a stationary process and  $\tau_{ij} := g_{ij} + \mu_{ij} = 0$ .*

Proofs are given in the Appendix. Clearly, log-price differential (corrected by the convergence path in the case of catching-up) stationarity is a necessary and sufficient condition so that AWPC holds, but only a necessary condition for ASPC. Propositions 1.3 and 1.4 also require the transition path steady-state gain to be equal to the mean of the log-price differential corrected by the transition. In the following examples we will explain the implications of ASPC and why it fits both types of convergence with three illustrative cases.<sup>3</sup> For simplicity, we will assume that  $\alpha = 1$ ,  $S_{ij,t}$  is stationary and  $\tau_{ij} = 0$  in all of them. Examples 2 and 3 are depicted in Figure 2.

Figure 2 should be around here

**Example 1.** ASPC as steady-state. This coincides with Bernard and Durlauf's original interpretation. Prices converged at some  $t < t_0$  and the relationship has been, since then, in its steady-state.  $p_{i,t} - p_{j,t}$  is stationary and  $\tau_{ij} = 0$  implies  $\mu_{ij} = 0$ , therefore (log) nominal prices converge in ASPC as steady-state.

**Example 2.** ASPC as catching-up. The relative price started the transition to its steady-state before or at the beginning of the sample and (almost<sup>4</sup>) reached it at some point before its end.  $p_{i,t} - p_{j,t} - \nu_{ij}(B)\xi_t^{t^*}$  requires  $\nu_{ij}(B) \neq 0$  to be stationary. Further, as  $\tau_{ij} = \mu_{ij} + g_{ij} = 0$ , they converge in ASPC.

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<sup>3</sup>Implications of AWPC are straightforwardly derived from those of Definition ASPC.

<sup>4</sup>The steady-state is only strictly reached when  $t \rightarrow \infty$ . With *almost* we mean close enough to the steady-state.

**Example 3.** ASPC as catching-up. Prices shared the same long-run trend with a nonzero mean from the beginning of the sample, but a transition to the zero mean steady-state began at some  $t^*$  and was (almost) completed before the sample ended. Again,  $p_{i,t} - p_{j,t}$  is nonstationary, but  $p_{i,t} - p_{j,t} - \nu(B)\xi_t^{t^*}$  is a stationary process. As  $\tau_{ij} = \mu_{ij} + g_{ij} = 0$  prices converge in ASPC.

## 4 Representation and hypothesis testing

Testing the type of convergence that two price series present or whether the definitions above hold, requires an appropriate representation. To do so, univariate and multivariate techniques should be employed.

### 4.1 Testing the Asymptotically Weak Price Convergence

The AWPC only requires the log-price differential, corrected or uncorrected by the transition path, to be stationary. Thus, we suggest to use the common two-stages procedure proposed by Engle and Granger (1987) that consists of running a regression and testing the nonstationarity of its residuals. We test AWPC as follows:

1. Run the regression:  $p_{i,t} = \alpha p_{j,t} + \mu_{ij} + S_{ij,t}$ .
2. Get the residuals,  $\hat{S}_{ij,t}$ , and test whether they are stationary. To do so, common Augmented Dickey-Fuller (ADF) type tests can be employed. In this respect, MacKinnon (1991) provides appropriate tables (distinct from usual Dickey-Fuller's) for different sample sizes and number of regressors. Here the constant-and-two-regressors row should be used.
  - 2.1. When the null of nonstationary is rejected, there is evidence in favor of AWPC as steady-state. The next step will be testing the ASPC (see section 4.2).
  - 2.2. When the nonstationary is not rejected, there is no evidence in favor of steady-state convergence and a transition path should be introduced in the regression in step 1.
3. Run the regression:  $p_{i,t} = \alpha p_{j,t} + \mu_{ij} + \nu_{ij}(B)\xi_t^{t^*} + S_{ij,t}^{t^*}$ . When  $r = 0$ ,  $\nu_{ij}(B) = \omega_s(B)$  and the previous model can be estimated by ordinary least squares. When  $r > 0$ , the estimation requires a nonlinear iterative procedure.
4. Get the residuals,  $\hat{S}_{ij,t}^{t^*}$ , and test whether they are stationary as in step 2.
  - 4.1. When the null of nonstationary is rejected, there is evidence in favor of AWPC as catching-up. The next step will be testing the ASPC (see section 4.2).



4.2. When the nonstationary is not rejected, we will conclude that there is no evidence in favor of convergence as steady-state or catching-up.

## 4.2 Testing the Asymptotically Strong Price Convergence

When  $\alpha$  is jointly estimated with the rest of the parameters, estimates from steps 1 or 3, in the previous section, are generally not asymptotically normally distributed, and hence standard inference cannot be applied (see Phillips, 1991). To properly test the ASPC we use a modified version of Phillips' Triangular Error Correction Mechanism (TECM, Phillips, 1991). This representation has two remarkable advantages for our purpose. First, maximum likelihood estimation of the multivariate model makes that an optimal asymptotic theory of inference applies, and so ASPC can be tested using standard asymptotic tests. Second, some other interesting joint null hypotheses, e.g.,  $g_i = g_m$  or  $\mu_i = g_j$ , for  $i, j = 2, \dots, m - 1$ , can now be tested in the multivariate model (4), which is presented below. Taking  $p_{1t}$  as *numéraire* without loss of generality, we formalize:

$$\Phi(B)[\Psi(B)\mathbf{Z}_t - \boldsymbol{\mu}] = \Theta(B)\mathbf{a}_t, \quad (4)$$

with:

$$\Phi(B) = \begin{bmatrix} \phi_{11}(B) & 0 & \cdots & 0 \\ 0 & \phi_{22}(B) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_{mm}(B) \end{bmatrix}; \quad \Psi(B) = \begin{bmatrix} \Delta & \mathbf{0} & 0 \\ -\boldsymbol{\alpha} & \mathbf{I}_{m-1} & -\boldsymbol{\nu}(B) \end{bmatrix};$$

$$\Theta(B) = \begin{bmatrix} \theta_{11}(B) & 0 & \cdots & 0 \\ 0 & \theta_{22}(B) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \theta_{mm}(B) \end{bmatrix} \begin{bmatrix} 1 & \theta_{12}(B) & \cdots & \theta_{1m}(B) \\ \theta_{21}(B) & 1 & \cdots & \theta_{2m}(B) \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{m1}(B) & \theta_{m2}(B) & \cdots & 1 \end{bmatrix},$$

where  $\phi_{ii}(B) = 1 - \phi_{1,ii}B - \dots - \phi_{p,ii}B^p$ ,  $\theta_{ii}(B) = 1 - \theta_{1,ii}B - \dots - \theta_{q,ii}B^q$  for  $i = 1, \dots, m$  and  $\theta_{ij}(B) = -\theta_{1,ij}B - \dots - \theta_{\bar{q},ij}B^{\bar{q}}$ , for  $i, j = 1, \dots, m$  and  $i \neq j$ . We assume  $\mathbf{Z}_t$  to be strictly stationary and strictly invertible, i.e., all the zeros of the determinantal polynomials  $|\Phi(B)|$  and  $|\Theta(B)|$  are outside the unit circle, and  $[\Phi(B) \quad \Theta(B)]$  to be left prime. Moreover,  $\boldsymbol{\alpha} := [\alpha_2, \alpha_3, \dots, \alpha_m]^T$ ,  $\mathbf{I}_{m-1}$  is a  $m - 1 \times m - 1$  identity matrix,  $\boldsymbol{\nu}(B) := [\nu_2(B), \nu_3(B), \dots, \nu_m(B)]^T$ ,  $\mathbf{Z}_t := [p_{1t}, p_{2t}, \dots, p_{mt}, \xi_t^{t*}]^T$  and  $\boldsymbol{\mu} := [0, \mu_2, \dots, \mu_m]^T$ . Vector  $\mathbf{a}_t := [a_{1t}, a_{2t}, \dots, a_{mt}]^T$  is a  $(\mathbf{0}_m, \boldsymbol{\Sigma}_a)$  WWN, where  $\boldsymbol{\Sigma}_a$  is a  $m \times m$  symmetric positive definite matrix. Although model (4) is somehow restrictive, compared to a general VARMA representation, it is general enough for our purpose.

For simplicity's sake, in what follows we will assume  $m = 2$  in (4), and then  $\mathbf{Z}_t := [p_{1t}, p_{2t}, \xi_t^{t*}]^T$ . Assuming normality, we use the likelihood function  $l(\Theta_1|p_{1,t}, p_{2,t}, \xi_t^{t*})$ , where  $\Theta_1 = \{\alpha, \mu_2, \omega_0, \dots, \omega_s, \delta_1, \dots, \delta_r, \phi_{1,ii}, \dots, \phi_{p,ii}, \theta_{1,ii}, \dots, \theta_{q,ii}, \theta_{ij}\}$  for  $i, j = 1, 2$  and  $i \neq j$  that can be derived from Mauricio (2005), to estimate model (4). When there is evidence of AWPC, as steady-state or catching-up (see steps 2.1 or 4.1, respectively, in the previous section), testing ASPC consists of testing the null hypothesis  $\tau_{12} = 0$ . To do so, one could use the following lemma which is proved in the Appendix.

**Lemma 1** *Let  $\hat{g}_{ij}$  and  $\hat{\mu}_{ij}$  be consistent and asymptotically normally distributed estimators of  $g_{ij}$  and  $\mu_{ij}$ , respectively. We have that  $\sqrt{T}(\hat{\tau}_{ij} - \tau_{ij})/\hat{\sigma}_\tau \xrightarrow{d} N(0, 1)$ , where  $\hat{\tau}_{ij}$  and  $\hat{\sigma}_\tau$  are defined in the Appendix.*

For the same purpose the statistic  $-2 \log l(\Theta_2|p_{1,t}, p_{2,t}, \xi_t^{t*})/l(\Theta_1|p_{1,t}, p_{2,t}, \xi_t^{t*})$ , where  $\Theta_2 = \{\alpha, \omega_0, \dots, \omega_s, \delta_1, \dots, \delta_r, \phi_{1,ii}, \dots, \phi_{p,ii}, \theta_{1,ii}, \dots, \theta_{q,ii}, \theta_{ij}\}$ , that asymptotically follows a  $\chi^2$  distribution with 1 degree of freedom can be applied. Whatever the test employed, when AWPC and  $\tau_{12} = 0$  cannot be rejected then  $p_{1,t}$  and  $p_{2,t}$  converge in ASPC. This Likelihood Ratio (LR) test will also be used to test some other joint null hypotheses as  $g_i = g_m$  or  $\mu_i = g_j$ , for  $i, j = 2, \dots, m - 1$ , in the multivariate model (4).

On the other hand, in many situations one would be interested in fixing the parameter  $\alpha$  a priori, according to the economic theory, e.g., the goods whose prices are analyzed are identical and so  $\alpha$  is restricted to unity. This not only makes the analysis much simpler but also has gains in terms of power in the unit root tests as the critical values are closer to zero. In this case, Saikkonen and Lütkepohl (2002), hereafter SL-GLS, present a test for unit root with different level shifts that includes our transition path (2), where they proof that the convergence parameters in  $\nu_{ij}(B)$  or the time at which the convergence begins,  $t^*$ , do not affect the limiting distribution of the nonstationarity test. Further, Shin and Fuller (1998) test, SF, which is more powerful than ADF-type tests in the case of ARMA structures, can also be employed. Moreover, when the nonstationary hypothesis is rejected standard inference applies and Phillips' TECM is no required. Table 1 summarizes the methodology for price convergence analysis.

Table 1 should be around here

## 5 Empirical results on price convergence

Our empirical analysis considers annual series of wheat prices in Arnhem (A), London and Southern England (L), Vienna (V), Strasbourg (S) and Pennsylvania (P), measured

in gram of silver per liter. All of them cover a common period from 1720 to 1875.<sup>5</sup> This sample includes most of O’Rourke and Williamson’s (1999) canonical period of globalization, i.e., when these prices went to parity in some moment after 1840. The series are depicted in Figure 3.

Figure 3 should be around here

The main goal of this exercise is to test whether the wheat prices mentioned above converge at some point in the 19<sup>th</sup> century and, if so, find out the type of convergence and describe the process. The historical literature does not reach a consensus on when this process could have started and what the main cause was that preceded it. We suggest that two events constituted the spark that triggered this convergence process: (i) the ending of the protectionist trade policy in Britain and other countries -denominated “Corn Laws”- from 1846<sup>6</sup> and (ii) the rapid decline in transaction costs experienced some years later.

All nominal prices show similar statistical properties. They: (i) are integrated of order one, (ii) need to be transformed into natural logarithms to avoid heteroskedasticity, non-normality and non-linearity, (iii) fit a zero-mean ARIMA(2,1,1) model, and (iv) have a small number of impulse interventions due to wars and revolutions.<sup>7</sup> The AR(2) structures have two conjugate imaginary roots, giving rise to damped oscillations with a period of 5-13 years and a damping factor of around 0.5, which represent a quasi-cyclical behavior where the period describes the time elapsed (in years) from peak to trough. There is no evidence of over-differentiation in the univariate models of the nominal prices, as the null hypothesis of MA(1) noninvertibility is clearly rejected by the Generalized Likelihood Ratio (GLR) test by Davis et al. (1995). Moreover, SF does not reject the null hypothesis of nonstationarity in an alternative ARIMA(3,1) model. Consequently, I(1) is confirmed in all cases. These results are summarized in Table 2.

Table 2 should be around here

Figure 3 also shows the standardized relative prices with respect to London’s wheat price. We fix London as numéraire as it was considered the linking-wheat price between European and American trade activities by that time, although the results are robust

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<sup>5</sup>L covers 1700-1896, P covers 1720-1896 and A, V and D cover 1700-1875

<sup>6</sup>The import tariffs for foreign imports of grains were abolish gradually between 1846-1848.

<sup>7</sup>All the interventions are of impulse type and do not significantly affect the results. The information about the intervention analysis is available from the authors upon request.

regardless of the numéraire chosen. The relative prices look stationary except for the last part of each sample. It was only around 1847 that prices seem to converge toward parity.

In all the analyses performed from now on, we fix  $t^* = 1847$  the year at which the convergence could have started. Despite historical reasons that justify the use of this year as an initial point for the convergence process, we carried out a thorough search looking for alternative starting dates. For each case presented in this section, we estimate several models with different transition paths and different starting dates,  $t^* = 1830, 1831, \dots, 1850$ , without finding any other satisfactory result. A more sophisticated method to determine where the convergence processes begin, when there is no extra-sample information available, could be the subject of future research.

## 5.1 Results imposing the perfect homogeneity restriction

Here we assume perfect homogeneity of wheat across markets as it simplifies the analysis, improves the performance of the unit root tests and seems a realistic restriction. We will relax this constraint later on. We carry out the steps presented in Section 4 to every wheat-relative-price series, with  $\alpha = 1$  and employing  $L$  as numéraire. Table 3 Panel A shows the results of the unit root tests previously mentioned. The tests generally do not reject the nonstationarity at a 5% level (except, maybe, for S/L) when there is no convergence input in the model. A first conclusion is that nominal wheat prices do not converge as steady-state when  $\alpha = 1$  is imposed.<sup>8</sup> However, the nonstationarity is clearly rejected in all the cases when a transition term is introduced from 1847, which reveals a strong evidence of asymptotic convergence as catching-up.

Table 3 should be around here

The estimation results for the ratios are reported in Table 4, Panel A. The model identified is relatively simple: (i) an order-one autoregressive for the stochastic part, and (ii) a mean,  $\mu$ , and a gradual and monotone convergence path,  $\omega_0/(1 - \delta_1 B)$ , for the deterministic component.<sup>9</sup> The estimated parameters and some diagnostic tools are also presented. All the parameters are statistically different from zero, including the steady-state gain  $g$ , and the convergence operator is stable.  $Q$  statistics by Ljung and Box (1978) show no sign of poor fit, except for the case  $V/L$ , where a second-order AR

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<sup>8</sup>Again S and L could be an exception to this statement, although we did not find any economic or historical argument in favor of this fact.

<sup>9</sup>The initial specification for the stochastic part is according to the correlogram, AIC (Akaike, 1974) and HQ (Hannan and Quinn, 1979) criteria, which agree with the same initial specification.

operator could better fit the data.<sup>10</sup> The speed of convergence, measured by  $l$ , is not statistically different between cases and more than 90% of the transition path long-run gain was already reached in 1860 for every ratio.

Table 4 should be around here

Results of the test for ASPC ( $H_0: \tau = 0$ ) is presented in Table 5. Both,  $t$ -student and LR test, clearly confirm that wheat series converge in ASPC. In 1875 the remaining gaps between Arnhem, Strasbourg, and Pennsylvania with London prices do not exceed five percent, being less than the respective relative price standard error.

Table 5 should be around here

## 5.2 Results relaxing the perfect homogeneity restriction

Results of the unit root tests when the perfect homogeneity restriction is relaxed are even more convincing. Table 2 Panel B shows that none of the tests rejects the nonstationarity in the model without the convergence input while the opposite occurs when a transition path is included from 1847.

To study the robustness of the results when  $\alpha$  is jointly estimated with the rest of the parameters, we employ the TECM introduced in (4). The two equations in the baseline bivariate model are the univariate model for  $L$  (the *numéraire* equation) and the ratio univariate model,  $i/\alpha L$  where  $i = A, V, S, P$ .

The results are reported in Table 4 Panel B, that does not present information about the *numéraire* equation,  $L$ , as it is virtually the same as in Table 2. Estimated parameters in the TECMs do not differ substantially from those in the respective univariate relative price models. The cointegration coefficient  $\alpha$  is estimated to be significantly different from zero and close to one in all cases. Only in  $A/\alpha L$  the null hypothesis of perfect homogeneity is rejected at 5% level. The estimates  $\hat{\theta}_{12}(B)$  and  $\hat{\theta}_{21}(B)$ , interpreted as the percentage response of lagged effects of an unexpected unitary shock from one side of the market to the other, indicate positive relationships in both directions for each pair of markets. A unitary shock in London had a two-lag effect in Arnhem (34% and 32%) and in Philadelphia (16% and 9%), a one-lag effect in Strasbourg (24%), and no appreciable effects in Vienna. The feedback is of order one in all cases. The speed of convergence

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<sup>10</sup>For simplicity's sake only first order AR models are shown. Conclusions do not change significantly if a second-order representation is employed.

is very similar in both types of model. The only exception is  $V/L$  since its convergence velocity in the bivariate model is a little slower than in the univariate counterpart. This could be due to the absence of the estimated feedback in the direction  $L \rightarrow V/L$ . The estimated value of  $\tau$  is close to zero in all cases, independently of the model specification, confirming the ASPC as catching-up of nominal prices (see Table 5). Figure 4 shows the estimated convergence paths for this case.

Figure 4 should be around here

Finally, we estimate a multivariate TECM, which includes all the series, and we test the joint null hypothesis  $H_0: \tau_i = 0$ , with  $i = A, V, S, P$ . The estimated values of  $\tau_i = g_i + \mu_i$  are again close to zero in all cases:  $\hat{\tau}_A = 0.04 \pm 0.26$ ,  $\hat{\tau}_V = -0.18 \pm 0.24$ ,  $\hat{\tau}_S = 0.02 \pm 0.24$  and  $\hat{\tau}_P = 0.14 \pm 0.32$  in a 95% confidence interval, where  $\hat{\tau}_i$  corresponds to  $i/L$  and  $\pm$  values are twice the standard errors from the estimated values. Further, the values of the LR statistic for the joint null hypothesis  $H_0: \tau_i = 0, \forall i$  and its corresponding p-value are 1.5 and 0.68, respectively, clearly not rejecting  $H_0$ .<sup>11</sup>

The main conclusions of the analysis are: (i) nominal wheat prices in Arnhem, Vienna, Strasbourg and Pennsylvania converge to London's in ASPC as catching-up, (ii) the catching-up process began around 1847, lasted about 14 years, and was gradual and monotone, and (iii) the estimated parameters of the transition path are robust across model specifications. In this sense, the univariate analyses of the ratios seem to be sophisticated enough to draw conclusions about the convergence to parity by price pairs. However, the multivariate models are more helpful to understand how the system works and whether the perfect homogeneity hypothesis has an empirical basis.

## 6 Concluding remarks

This paper presents a general framework for the analysis of price convergence according to the econometric tradition, i.e., including assumptions, definitions, model building, econometric representations, and hypothesis testing. Our work is based on cointegration analysis but is very flexible and, consequently, compatible with steady-state or catching-up convergence. Further, it enables one to distinguish between asymptotic weak or strong convergence, as steady-state or catching-up, and describe completely a convergence process, by representing its transition path and measuring its speed.

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<sup>11</sup>Parameter estimates of this model are available from the authors upon request.

The empirical study shows how to use the proposed methodology, coming to an interesting conclusion for economic historians: the end of Britain's protectionist trade policy about 1846 triggered the price convergence process as catching-up experienced during the second half of the 19<sup>th</sup> century.

Finally, at least two main subjects related to this paper could be the object of future research. First, the methodology is flexible enough to different data frequencies and so it has great potential not only in prices, but also in output, productivity or finance. Second, a procedure to endogenously identify the time when the convergence process begins would be very helpful for users who have no extra-sample information. This last issue is clearly related with the existing literature about unit roots with shifts at unknown dates.

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# Technical Appendix

## Proof of Proposition 1:

We will first prove that any convergent sequence can be *good enough* approximated by  $\nu(B)\xi_t^{t^*}$ , defined in (2). We assume, for simplicity and without loss of generality,  $b = 0$ ,  $s = r$  and  $t^* = 1$  in  $\nu(B)\xi_t^{t^*} = [\omega_s(B)/\delta_r(B)]B^b\xi_t^{t^*}$ .

Let define a convergent sequence  $\{x_t: t \in \mathbb{N}\}$  with  $\lim_{t \rightarrow \infty} x_t = L$  and  $L \in \mathbb{R}$ . Now we will prove that  $\nu(B)\xi_t^{t^*}$  can approximate  $x_t$  as close as necessary by choosing adequately  $s$ ,  $\omega_s$  and  $\delta_r$ . To do so, let, rewrite  $\nu(B)\xi_t^{t^*}$  as the sequence  $\{z_t: t \in \mathbb{N}\}$  with general term:

$$\begin{aligned} z_t &= \sum_{i=0}^{t-1} \omega_i \left( \sum_{n_1=1}^t \delta_1^{n_1-1} + \sum_{n_2=2}^{t-1} \delta_2^{n_2-1} + \dots + \sum_{n_r=r}^{t-r} \delta_r^{n_r-1} \right), \quad \text{for } t \leq s, \\ z_t &= \sum_{i=0}^s \omega_i \left( \sum_{n_1=1}^t \delta_1^{n_1-1} + \sum_{n_2=2}^{t-1} \delta_2^{n_2-1} + \dots + \sum_{n_r=r}^{t-r} \delta_r^{n_r-1} \right), \quad \text{for } t > s. \end{aligned} \quad (5)$$

On the one hand, by choosing appropriate  $w_i$  in (5), the first  $s$  terms of  $x_t$  will be perfectly approximated by  $z_t$ , i.e., there exists an integer  $s$  such that  $|x_t - z_t| = 0$ , whenever  $t \leq s$ . On the other hand, from the definition of limit, we have that for all  $\varepsilon > 0$  there exists  $s > 0$  such that  $|x_t - L| < \varepsilon$ , whenever  $t > s$ . As  $z_t$  is also a convergent sequence (recall we assume the roots of the characteristic equation  $\delta_r(B) = 0$  to lie outside the unit circle), then the subsequence  $x_{s+1}, x_{s+2}, \dots$  will be good enough approximated by the sum of summations in parenthesis in equation (5) when the integer  $r$  and the coefficients  $\delta_r$  are appropriately choosing.

The second part of the proof arises directly from model (3). By adding  $k$  periods and taking conditional expectations we have:

$$\mathbb{E}[p_{i,t+k} - \alpha p_{j,t+k} | \mathcal{F}_t] = \mathbb{E}[D_{ij,t+k} + S_{ij,t+k} | \mathcal{F}_t] = \nu_{ij}(B)\xi_{t+k}^{t^*} + \mu_{ij} + \mathbb{E}[S_{ij,t+k} | \mathcal{F}_t], \quad (6)$$

and applying limits to (6) yields:

$$\lim_{k \rightarrow \infty} \mathbb{E}[p_{i,t+k} - \alpha p_{j,t+k} | \mathcal{F}_t] = \lim_{k \rightarrow \infty} \nu_{ij}(B)\xi_{t+k}^{t^*} + \mu_{ij} + \lim_{k \rightarrow \infty} \mathbb{E}[S_{ij,t+k} | \mathcal{F}_t]. \quad (7)$$

We will use equation (7) in the proof of the four points in Proposition 1:

1. From Definition 3,  $\nu_{ij}(B) = 0$ . It is then easy to see in (7) that  $S_{ij,t}$  must be stationary so that AWPC holds. Further, from model (3), if  $\nu_{ij}(B) = 0$  and  $S_{ij,t}$  is stationary then  $p_{i,t} - \alpha p_{j,t}$  is necessarily stationary, as it has no deterministic and/or stochastic trends.

2. From Definition 4,  $\nu_{ij}(B) \neq 0$ . As  $\nu_{ij}$  is stable, then  $\sum_{k=0}^{\infty} \nu_{ij,k} = g_{ij}$ . So again  $S_{ij,t}$  must be stationary so that AWPC holds in (7).

3. Identical to 1, but in this case  $\mu_{ij} = 0$  must hold, so that ASPC holds in (7).

4. Identical to 2, but in this case  $\mu_{ij} = g_{ij}$  must hold, so that ASPC holds in (7).

□

**Proof of Lemma 1:**

As  $g \equiv \sum_{k=0}^{\infty} \nu_k = \nu(1)$  is the long-run gain of the rational transfer function, by employing the polynomial approximation for  $g$ , we get  $g = \left(\omega_0 - \sum_{i=1}^s \omega_i\right) / \left(1 - \sum_{i=1}^r \delta_i\right) = \omega(1) / \delta(1)$ .

Replacing the parameters  $\omega_i$ ,  $i = 0, 1, \dots, s$  and  $\delta_i$ ,  $i = 1, 2, \dots, r$  with their consistent and asymptotically normally distributed maximum likelihood estimates, leads to consistent estimate of  $g$ . Similarly, a consistent estimate for the long-run gap,  $\hat{\tau}$ , is obtained by replacing the parameters in  $\tau = g + \mu$  with  $\hat{g}$  and  $\hat{\mu}$ . Further, an approximate linear expansion of  $\hat{\tau}$  can be got as:

$$\hat{\tau} = \tau + \frac{1}{\delta(1)}(\hat{\omega}_0 - \omega_0) - \frac{1}{\delta(1)} \sum_{i=1}^s (\hat{\omega}_i - \omega_i) + \frac{\omega(1)}{\delta(1)^2} \sum_{i=1}^r (\hat{\delta}_i - \delta_i) + (\hat{\mu} - \mu) + O_p(n^{-1}). \quad (8)$$

Taking variances on (8) leads to:

$$\begin{aligned} \sigma_{\hat{\tau}}^2 \simeq & \frac{1}{\delta(1)^2} \sum_{i=0}^s \sigma_{\omega_i}^2 + \frac{\omega(1)^2}{\delta(1)^4} \sum_{i=1}^r \sigma_{\delta_i}^2 + \sigma_{\mu}^2 + -\frac{2}{\delta(1)^2} \sum_{i=1}^s \sigma_{\omega_0, \omega_i} + \frac{2}{\delta(1)^2} \sum_{i=1}^{s-1} \sum_{j=i+1}^s \sigma_{\omega_i, \omega_j} \\ & + \frac{2\omega(1)}{\delta(1)^3} \sum_{i=1}^r \sigma_{\omega_0, \delta_i} - \frac{2\omega(1)}{\delta(1)^3} \sum_{i=1}^s \sum_{j=1}^r \sigma_{\omega_i, \delta_j} + 2\frac{\omega(1)^2}{\delta(1)^4} \sum_{i=1}^{r-1} \sum_{j=i+1}^r \sigma_{\delta_i, \delta_j} + \frac{2}{\delta(1)} \sigma_{\mu, \omega_0} \\ & - \frac{2}{\delta(1)} \sum_{i=1}^s \sigma_{\mu, \omega_i} + \frac{2\omega(1)}{\delta(1)^2} \sum_{i=1}^r \sigma_{\mu, \delta_i} \end{aligned} \quad (9)$$

where  $\sigma_a^2$  and  $\sigma_{a,b}$  denote, respectively, the variance of  $\hat{a}$  and the covariance between  $\hat{a}$  and  $\hat{b}$ . From this result and appealing to Slutsky's Theorem, it follows that  $\sqrt{T}(\hat{\tau} - \tau) \xrightarrow{d} N(0, \sigma_{\hat{\tau}}^2)$ . The square root of the value obtained by replacing  $\omega(1)$ ,  $\delta(1)$ , the variances and covariances in (9) with its consistent estimates is a consistent estimate of  $\sigma_{\hat{\tau}}$ , and therefore  $\sqrt{T}(\hat{\tau} - \tau) / \hat{\sigma}_{\hat{\tau}} \xrightarrow{d} N(0, 1)$ .

The estimated variance of  $\hat{g}$  can similarly be obtained by removing the terms associated to  $\mu$  in (9). □

## Data Appendix

Sources of data and methods of conversion from local units into grams of silver per liter are:

**Arnhem, 1700-1875:** Wheat prices in grams of silver per liter from series elaborated by Robert C. Allen:

[http://www.history.ubc.ca/faculty/unger/ECPdb/xls/Wheat/Arnhem\\_Wheat.xls](http://www.history.ubc.ca/faculty/unger/ECPdb/xls/Wheat/Arnhem_Wheat.xls)

The original source is C.A. Verrijn Stuart "Marktprijzen van Granen te Arnhem in de jaren 1544-1901," "Bijdragen tot de Statistiek van Nederland", new series, vol. 26 (1903), pp. 19-25.

**London and Southern England, 1700-1896:** Wheat prices in grams of silver per liter from series elaborated by Robert C. Allen:

<http://www.nuff.ox.ac.uk/users/allen/studer/london.xls>

**Pennsylvania, 1720-1896:** Wheat prices in grams of silver per kilo from the GPIHG:

[http://gpih.ucdavis.edu/files/Penn\\_spliced\\_1720-1896.xls](http://gpih.ucdavis.edu/files/Penn_spliced_1720-1896.xls). The kilo/liter ratio used is 0.772: [http://gpih.ucdavis.edu/files/Weight\\_vs\\_volume.xls](http://gpih.ucdavis.edu/files/Weight_vs_volume.xls).

**Strasbourg, 1700-1875:** Wheat prices in grams of silver per liter from series elaborated by Robert C. Allen: <http://www.nuff.ox.ac.uk/users/Allen/studer/strasbourg.xls>. Missing observations are 1794-1795.

**Vienna, 1700-1875:** Wheat prices in grams of silver per liter from series elaborated by Robert C. Allen: <http://www.nuff.ox.ac.uk/users/allen/studer/vienna.xls>.

**Table 1:** Methodology for price convergence analysis.

Model without transition path (a)	Unit root decision on $\hat{S}_t$ (b)	Model with transition path (c)	Unit root decision on $\hat{S}_t^*$ (d)	Inference result on $\tau$ (e)	Convergence conclusion
$p_{1,t} = \hat{\alpha}p_{2,t} + \hat{\mu} + \hat{S}_t$	I(0)	-	-	$\tau = 0$ ( $\mu = 0$ )	ASPC as SS
	I(1)	-	-	$\tau \neq 0$ ( $\mu \neq 0$ )	AWPC as SS
$p_{1,t} = \hat{\alpha}p_{2,t} + \hat{\mu} + \hat{\nu}(B)\xi_t^* + \hat{S}_t^*$	I(1)		I(0)	$\tau = 0$ ( $\mu = g$ )	ASPC as CU
			I(1)	$\tau \neq 0$ ( $\mu \neq g$ )	AWPC as CU
				-	NC

*Notes:* When  $\alpha$  is estimated, we use a two-stage estimation procedure so that ADF-type tests can be applied in (b) and (d) to test residuals nonstationarity. In (e), standard inference on  $\tau$  is employed after reestimating the parameters in (c) using Phillips' TECM. Fixing  $\alpha$  makes the procedure simpler and the tests more powerful. When  $\alpha$  is fixed, residuals in (b) can also be tested using SF and in (d) using SF and SL-GLS. If nonstationarity is rejected, Phillips TECM is not then require and standard inference is applied to model (c).

Acronyms: ADF, Augmented Dickey-Fuller test; SF, Shin-Fuller test; SL-GLS, Saikonnen and Lütkepohl test; ASPC, Asymptotically Strong Price Convergence; AWPC, Asymptotically Weak Price Convergence; SS, Steady-State; CU, Catching-up; NC, No Convergence.

**Table 2:** Estimated univariate models of wheat prices series in log differences.<sup>(1)</sup>

Sample	Variable (Acronym)	AR(2)		MA(1)	Resid.	ACF <sup>(2)</sup>	SF <sup>(3)</sup>	GLR <sup>(4)</sup>
		$\hat{\phi}_1$ (s.e.)	$\hat{\phi}_2$ (s.e.)	$\hat{\theta}$ (s.e.)	Std.Dev. (%)	$Q_{(9)}$	$H_0 : \phi = 1$	$H_0 : \theta = 1$
1700-1896	London	.63	-.26	.78	16.6	6.3	0.0	23.0*
	(L)	(.09)	(.07)	(.07)				
1700-1875	Arnhem	.64	-.11	.86	18.9	9.4	0.0	19.2*
	(A)	(.09)	(.07)	(.06)				
	Vienna	.70	-.18	.88	22.4	7.9	0.0	18.54*
	(V)	(.09)	(.08)	(.05)				
	Strasbourg	.74	-.44	.71	16.9	6.9	0.1	30.8*
	(S)	(.12)	(.07)	(.13)				
1720-1896	Pennsylvania	.66	-.40	.67	16.9	3.3	0.0	32.0*
	(P)	(.12)	(.08)	(.12)				

**Notes:** (1) 18<sup>th</sup> and 19<sup>th</sup> centuries yearly prices in gr.Ag./liter. (2) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF).  $H_0$ : there is no autocorrelation in the first nine lags. (3) We estimate an alternative ARIMA(3,0,1) model and test the null hypothesis with Shin and Fuller (1998) test. (4) GLR: Generalized Likelihood Ratio (GLR) test of Davis, Chen and Duismuir (1995) for the null hypothesis of noninvertibility of an MA(1) operator.

\*Rejects  $H_0$  at 5% level.

**Table 3:** Results of the unit root tests for wheat price series: Testing asymptotic convergence as steady-state or catching-up.

Model	Test	$A/\alpha L$	$P/\alpha L$	$S/\alpha L$	$V/\alpha L$
Panel A: $\alpha$ restricted to 1 (Relative prices)					
Without transition path	ADF	-1.67	-1.60	-3.09*	-1.50
	SL-GLS	-0.92	-1.67	-2.24*	-1.69
	SF	0.00	0.20	1.60	0.00
With transition path	ADF	-2.47*	-3.39*	-4.20*	-3.70*
	SL-GLS	-1.16	-2.99*	-2.46*	-2.87*
	SF	55.5*	53.0*	29.8*	44.3*
Lags in test regressions	-	6	10	5	10
*rejects $H_0$ at 5%. Critical values: -1.94 (Fuller, 1996) for ADF and SL-GLS, and 1.75 for SF (Shin and Fuller, 1998).					
Panel B: $\alpha$ estimated					
Without transition path	ADF	-2.78	-2.36	-2.62	-2.96
	SL-GLS	-1.26	-2.00	-2.89	-2.71
With transition path	ADF	-4.70*	-4.14*	-3.96*	-4.64*
	SL-GLS	-1.02	-3.42*	-3.37*	-4.39*
Lags in test regressions	-	5	4	3	4

\*rejects  $H_0$  at 5%. Critical value: -3.37 (MacKinnon, 1991).

**Table 4:** Relative price and bivariate convergence model estimates for wheat price series with London as numéraire.

Ratio	Stochastic Component					Deterministic Component					Resid. Std.Dev. (%)	Diagnostics	
	$\hat{\alpha}$ (s.e.)	AR $\hat{\phi}_1$ (s.e.)	MA			Convergence Op.				Mean $\hat{\mu}$ (s.e.)		ACF <sup>(1)</sup> $Q_{(9)}$	CCF <sup>(1)</sup> $Q_{(9)}$
			$\theta_{21}(B)$ $\hat{\theta}_{1,21}$ (s.e.)	$\theta_{2,21}$ $\hat{\theta}_{2,21}$ (s.e.)	$\theta_{12}(B)$ $\theta_{12}$ (s.e.)	$\nu(B) = \omega/(1 - \delta B)$							
$\hat{\omega}$ (s.e.)	$\hat{\delta}$ (s.e.)	$\hat{l}$ (s.e.)	$\hat{g}$ (s.e.)										

Panel A: Univariate models for relative prices ( $\alpha = 1$ )

$A/L$	1	.46 (.07)	-	-	-	.074 (.040)	.79 (.12)	3.6 (2.2)	.35 (.11)	-.32 (.04)	23.4	8.5	-
$V/L$	1	.56 (.06)	-	-	-	.13 (.05)	.77 (.10)	3.3 (1.8)	.56 (.13)	-.70 (.06)	24.7	21.8*	-
$S/L$	1	.70 (.05)	-	-	-	.13 (.06)	.66 (.10)	4.0 (0.9)	.39 (.15)	-.39 (.05)	20.7	7.0	-
$P/L$	1	.54 (.06)	-	-	-	.086 (.048)	.82 (.11)	4.6 (3.5)	.48 (.08)	-.43 (.03)	18.9	12.9	-

Panel B: Bivariate models in TECM ( $\alpha$  estimated)

$A/\alpha L$	.67 (.07)	.50 (.07)	-.34 (.09)	-.32 (.08)	-.13 (.07)	.10 (.06)	.75 (.16)	3.1 (2.6)	.40 (.07)	-.38 (.03)	18.3	7.8	7.3
$V/\alpha L$	.93 (.11)	.57 (.06)	-	-	-.14 (.06)	.094 (.004)	.84 (.05)	5.3 (1.8)	.59 (.15)	-.70 (.04)	24.3	20.6*	7.6
$S/\alpha L$	.85 (.13)	.73 (.05)	-.24 (.09)	-	-.22 (.07)	.14 (.02)	.65 (.13)	3.8 (1.0)	.41 (.13)	-.41 (.05)	19.0	12.6	8.3
$P/\alpha L$	1.10 (.08)	.55 (.08)	-.16 (.11)	-.091 (.084)	-.23 (.07)	.090 (.006)	.80 (.15)	4.1 (3.8)	.46 (.07)	-.41 (.03)	19.4	12.0	6.9

**Notes:** (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF) and the cross correlation function (CCF).  $H_0$ : there is no autocorrelation or cross correlation in the first nine lags.

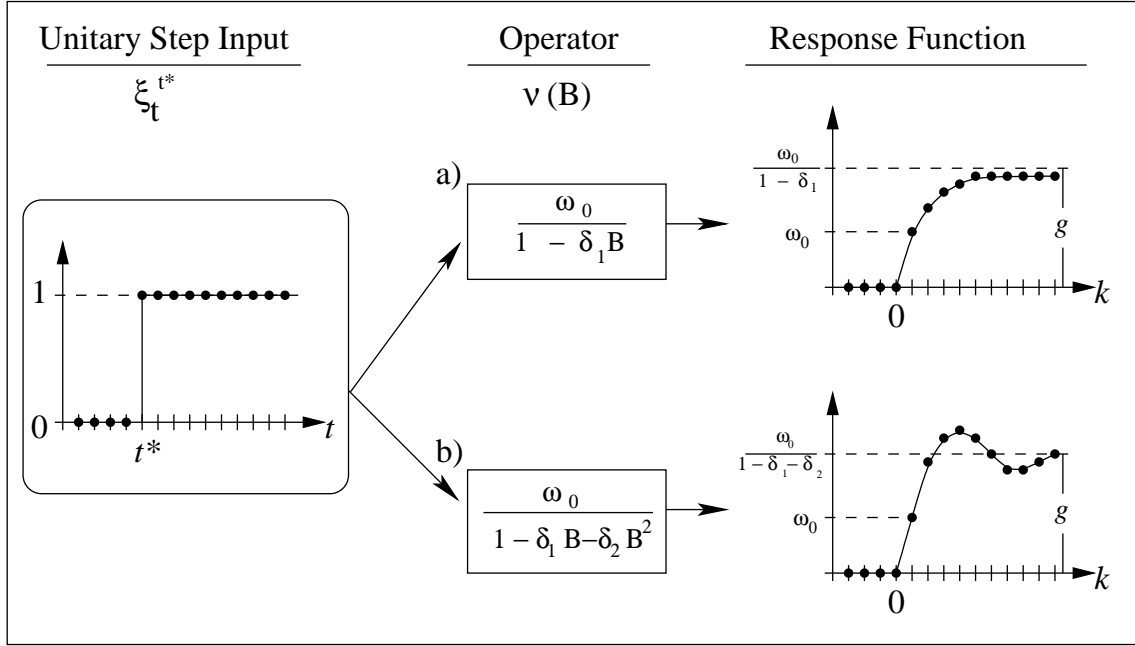
\*Rejects  $H_0$  at 5% level.

**Table 5:** Results of the statistical test for ASPC.

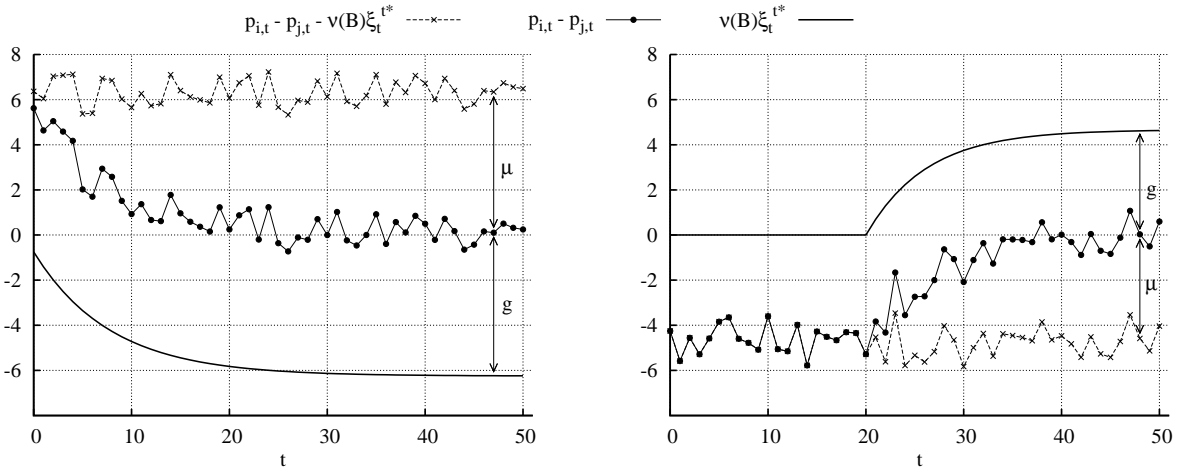
	Relative prices ( $\alpha = 1$ )			Bivariate TECM ( $\alpha$ estimated)		
	$\hat{\tau}_{i,L}$ (s.e.)	LR	p-value	$\hat{\tau}_{i,L}$ (s.e.)	LR	p-value
$A/\alpha L$	.03 (.11)	.1	.75	.03 (.07)	.1	.75
$V/\alpha L$	-.14 (.13)	.3	.58	-.12 (.15)	.1	.75
$S/\alpha L$	.00 (.15)	.0	1.0	-.01 (.13)	.0	1.0
$P/\alpha L$	.05 (.08)	.6	.43	.05 (.07)	.6	.44

**Notes:**  $\hat{\tau}_{i,L} = \hat{g}_{i,L} + \hat{\mu}_{i,L}$  is the estimated remaining gap. LR is the likelihood ratio test with  $H_0: \tau_{i,L} = 0$  for  $i = A, V, S, P$ .

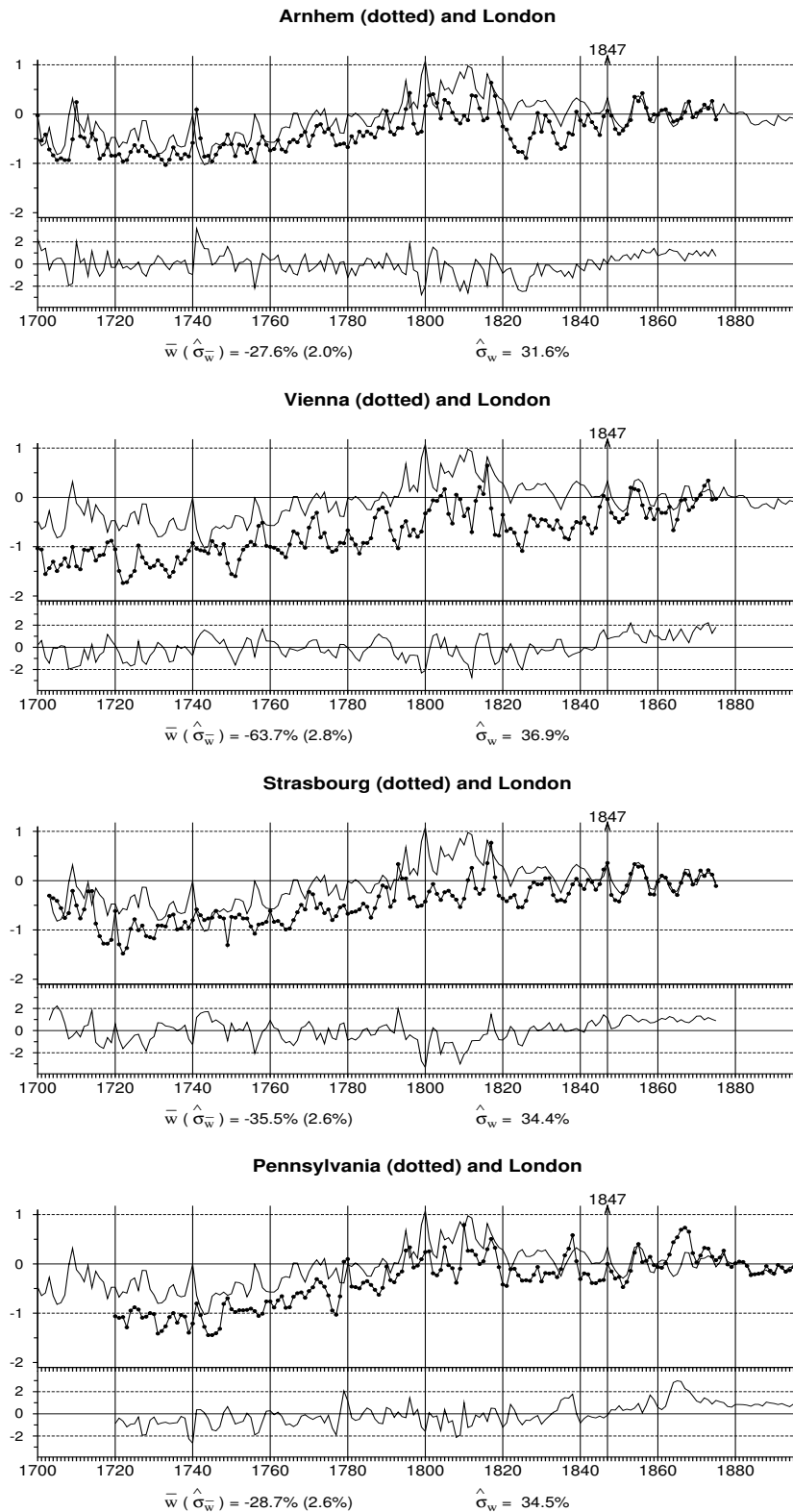




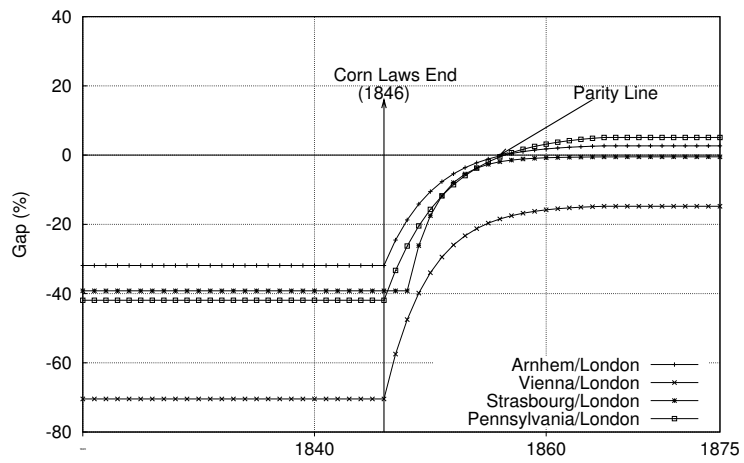
**Figure 1:** Examples of transitional paths: a) gradual monotone; b) damped quasi-cyclical. Case a) subject to  $\omega_0 > 0$  and  $0 < \delta_1 < 1$ . Case b) subject to  $\omega_0 > 0$ ,  $\delta_1^2 + 4\delta_2 < 0$ ,  $\delta_2 + \delta_1 < 1$ ,  $\delta_2 - \delta_1 < 1$  and  $|\delta_2| < 1$ .



**Figure 2:** Examples of two relative prices ( $\alpha = 1$ ) following ASPC as catching-up. Left: Case 2. The relative price started the transition to its steady-state before or at the beginning of the sample and reached it at some point before its end.  $p_{i,t} - p_{j,t}$  is nonstationary, but  $p_{i,t} - p_{j,t} - v(B)\xi_t^{t^*}$  is a  $\mu$ -mean stationary process and  $g + \mu = 0$ . Right: Case 3. Prices shared the same long-run trend from the begin of the sample, but a transition to the zero-mean steady-state began at some  $t^*$  and was completed before the sample ended. Again,  $p_{i,t} - p_{j,t}$  is nonstationary, but  $p_{i,t} - p_{j,t} - v(B)\xi_t^{t^*}$  is a  $\mu$ -mean stationary process and  $g + \mu = 0$ .



**Figure 3:** Yearly wheat price (gr.Ag/liter in logs) in Europe and Pennsylvania during the 18<sup>th</sup> and 19<sup>th</sup> centuries and standardized relative price with London as Numraire (plot below) in  $\text{Logs}.\bar{w}$  and  $\sigma_w$  are, respectively, the sample mean and standard deviation of the relative price. The Corn Laws were repealed from 1846.



**Figure 4:** Estimated transition paths in the multivariate TECMs.