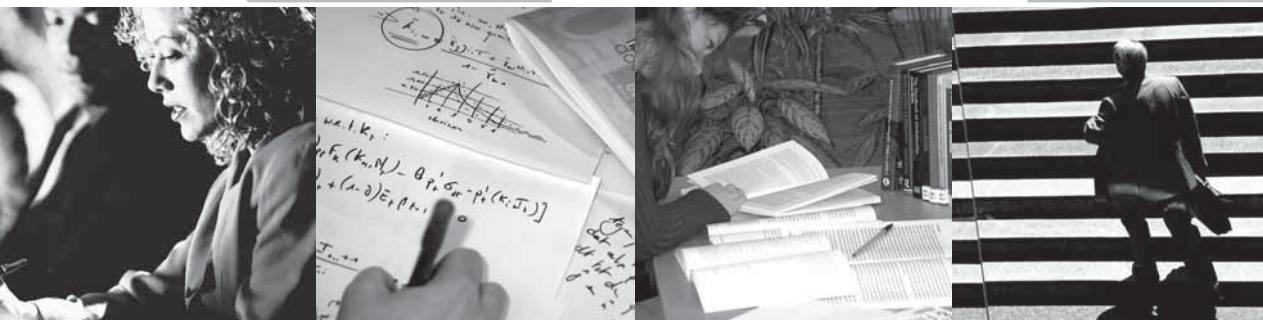


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## Noname – A new quarterly model for Belgium

Philippe Jeanfils Koen Burggraeve



# **NATIONAL BANK OF BELGIUM**

## **WORKING PAPERS - RESEARCH SERIES**

**NONAME**

### **A NEW QUARTERLY MODEL FOR BELGIUM**

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The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

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## **Abstract**

This paper gives an overview of the present version of the quarterly model for the Belgian economy built at the National Bank of Belgium (NBB). This model can provide quantitative input into the policy analysis and projection processes within a framework that has explicit micro-foundations and expectations. This new version is also compatible with the ESA95 national accounts.

This model called Noname is relatively compact. The intertemporal optimisation problem of households and firms is subject to polynomial adjustment costs, which yields richer dynamic specifications than the more usual quadratic cost function. Other characteristics are: pricing-to-market and hence flexible mark-ups and incomplete pass-through, a CES production function with an elasticity of substitution between capital and labour below one, time-dependent wage contracting à la Dotsey, King and Wollman. Most of the equations taken individually have acceptable statistical properties and diagnostic simulations suggest that the impulse responses of the model to exogenous shocks are reasonable. Its structure allows simulations to be conducted under the assumption of rational expectations as well as under alternative expectations formations.

JEL codes: C5, E2, E3, F41

Key words: Econometric modelling, Pricing-to-market, CES production function, Wage bargaining, Polynomial adjustment costs, Rational expectations.



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## 0. INTRODUCTION

The National Bank of Belgium has developed a new quarterly model for the Belgian economy. This paper gives an overview of the present version of this model, called Noname<sup>1</sup>. The model is developed as a tool for producing medium term projections along with their risk analysis and constitutes a coherent framework for analysing policy issues. To this end, it allows some compromise between theoretical structure and data matching but it meets the minimum requirement of a clear and delineated treatment of expectations. This structure allows simulations to be conducted under the assumption of rational expectations as well as under alternative expectations formations.

Noname rests on continuous re-estimation and re-specification of the original developments which took place at the NBB in the late 1990's that were published in Jeanfils (2000). Like its predecessor, the new model is, to a large extent, based on dynamic intertemporal optimisation and emphasises the importance of agents' expectations on macroeconomic outcomes. In particular, the model embodies overlapping generations of consumers, profit-maximising firms in imperfectly competitive product and labour markets, forward-looking behaviour and costly adjustment processes. However it has undergone improvements in a number of directions, including an overall re-specification of the supply-side in order to ensure that foreign trade is now theoretically consistent with the rest of the demand and supply block, to give theoretical foundations to the empirically observed flexible mark-ups and to allow for a CES production function. This new version of the model strictly respects the accounting framework to avoid possible leakages on the income accounts side. On top of that, other reasons have led to the revision of the model. Among them, recurrent changes in the database due to the rebasing of the national accounts and price indices, the availability of new input-output tables, the release of ESA95 data and new employment data series. They all created a need for a respecification of the model.

The size of the model has been kept as small as possible. This is a consequence of the belief that the cost of adding and maintaining more and more equations to an increasingly complex model outweigh the gain of more detailed and refined insights. Larger models eventually lose their original and internal logic and the results become less transparent and harder to understand. Noname consists of around 120 equations that show how different

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<sup>1</sup> Waiting for something more original, the successive prototype versions of the model were internally called "Noname". Given that our imaginations were not creative enough to get some catchier name, this acronym finally remained on the last version.



macroeconomic variables affect each other. Among these, some 25 are truly behavioural equations, 25 others consist of technical relationships defining derived prices and public finance items and the remaining ones are identities. In spite of its limited size, it still requires a significant investment in terms of time to understand a model as Noname in detail. However, even a knowledge on a fairly general level could already contribute to increase insight in the outcomes of a projection process or simulations. This paper therefore aims at familiarising the reader with the key issues of the model, rather than pretending to be an advanced user's guide.

The projections from the model reflect a plausible and internally consistent representation of the likely developments of the Belgian economy, and are based on the information available at a given time. Noname is one of the many tools that are used to produce projections, hereby serving as a guide to internally consistent thinking. It produces a forecast in a consistent structure. Such a model helps to ensure that, during the projection process, a new piece of information that is fed to the model at one specific place, is appropriately reflected in all the other parts of projection. In practice, it does not only produce an output during a projection exercise, it can also serve as a tool for an ex-post consistency check. In this context, a non-model based forecast is added as an update to the database and the model is solved for the implied residuals, also called add-factors. Where the residuals take on extreme values, this is a warning that the forecast could need some reconsideration. Such exercises, as well as the experience gained from policy simulation experiments along with new econometric evidence have also led to revisions of some parts of the model.

The next section of the paper proceeds by considering the theoretical underpinnings of the model and its steady state properties. In section 2 the dynamic adjustments are derived and estimated. This section also illustrates the impact of expectations on simulation results. Then, section 3 presents some diagnostic simulations. The emphasis will be on the properties of the overall system. This is the domain where macroeconomic models have more on offer than a focus on individual equations or partial equilibrium properties. The final section concludes.

## 1. THEORETICAL STRUCTURE OF THE MODEL

The two main groups of private agents in the model are households and firms. Households maximise utility subject to an intertemporal budget constraint. Firms maximise profits under a CES technology. Goods and labour markets are imperfectly competitive. Maximisation of their objective functions provide the long run equations of the model.

### 1.1 Households

We use a discrete time version of Blanchard's (1985) overlapping generations model of perpetual youth, which is also tractable with more than two generations.

#### 1.1.1 Consumption

##### a) Individual consumption

A consumer born in period  $t-k$  and still living in period  $t$  maximises his expected lifetime utility

$$E_t \sum_{s=0}^{\infty} \varphi^s U(c_{k+s,t+s}) \quad (1.1)$$

Uncertainty about consumption,  $c$ , at any future date, and thus the need to take expectations, comes from the possibility of death in the spirit of Blanchard's (1985) model of perpetual youth. If there is a constant probability of death each period, say  $v$ , then the probability of being alive  $s$  periods ahead is given by  $(1-v)^s$ . In case of death, utility is assumed to be zero. If alive, one will be aged  $k+s$  with a derived utility  $U(c_{k+s,t+s})$ . The objective function may then be written as

$$\sum_{s=0}^{\infty} [(1-v)\varphi]^s U(c_{k+s,t+s}) \quad (1.2)$$

A constant probability of death increases the individual's rate of time preference. It is assumed that individuals maximise expected lifetime utility with no intergenerational altruism.

The presence of a positive probability of death affects both the rate at which future utility is discounted ( $(1-v)\varphi = (1-v)/(1+\Theta)$ ) instead of  $\varphi$ , where  $\Theta$  represents the subjective rate

of time preference ) and the effective rate of interest an individual is facing (  $(1+r)/(1-v)$  instead of  $(1+r)$ ). Accordingly, the flow budget constraint of an individual of age  $k$  is written as

$$fw_{k+s,t+s} = \frac{(1+r_{t+s-1,t+s})}{(1-v)} \{fw_{k+s-1,t+s-1} + yl_{k+s-1,t+s-1} - P_{c,t+s-1}c_{k+s-1,t+s-1}\} \quad (1.3)$$

where  $c_{k+s,t+s}$  is consumption at time  $t+s$  of a consumer aged  $k+s$  priced  $P_{c,t+s}$ ,  $fw_{k+s,t+s}$  is begin-of-period asset holdings which earn real return  $r_{t+s,t+s+1}$ , and  $yl_{t+s}$  is the after-tax labour income "sensu lato", i.e. inclusive of transfer payments.

We also need an additional condition to prevent the consumer from choosing a path with an exploding debt, while allowing him to be temporarily indebted. This is the so called no-Ponzi-game condition, implying that asymptotically assets holdings should be non-negative:

$$\lim_{s \rightarrow \infty} \left( \frac{(1-v)^{s+1}}{\prod_{i=0}^s (1+r_{t+i,t+i+1})} \right) fw_{k+s+1,t+s+1} = 0 \quad (1.4)$$

and it allows to iterate (1.3) forward to obtain the intertemporal budget constraint

$$E_t \sum_{s=0}^{\infty} \left( \frac{(1-v)^s}{\prod_{i=0}^{s-1} (1+r_{t+i,t+i+1})} \right) P_{c,t+s} c_{k+s,t+s} =$$

$$fw_{k,t} + E_t \sum_{s=0}^{\infty} \left( \frac{(1-v)^s}{\prod_{i=0}^{s-1} (1+r_{t+i,t+i+1})} \right) yl_{k+s,t+s} \quad (1.5)$$

which states that the expected present value of consumption at time  $t$  equals expected present value of disposable labour income, i.e. expected human wealth, and initial non-human wealth.

The optimal solution is given by the intertemporal Euler equation:

$$U'(c_{k+s,t+s}) = \phi (1+r_{t+s,t+s+1}) \left( \frac{P_{c,t+s}}{P_{c,t+s+1}} \right) U'(c_{k+s+1,t+s+1}) \quad (1.6)$$

To provide a closed-form solution we assume that the instantaneous utility function exhibits constant relative risk aversion, where the elasticity of substitution between consumption at any two points in time is constant and equal to  $\zeta$ , that is:

$$U(c) = \left( \frac{\zeta}{(\zeta-1)} \right) c^{\frac{(\zeta-1)}{\zeta}} \quad (1.7)$$

If  $\pi_{t,t+1}$  denotes inflation during period  $t$  then (1.6) can be rewritten as

$$c_{k+s,t+s} = \prod_{i=0}^{s-1} \left( \frac{1+r_{t+i,t+i+1}}{1+\pi_{t+i,t+i+1}} \right) \varphi^{s\zeta} c_{k,t} \quad (1.8)$$

The term between brackets can be viewed as a real rate. Let it be denoted by  $(1+rr_t)$ . If households do not expect it to vary a lot, a closed form solution can be obtained. Provided that the stability condition  $(1-v)(1+rr_t)^{\zeta-1}\varphi^\zeta < 1$  holds, we obtain the following consumption function:

$$c_{k,t} = \Omega_t \left( \frac{fw_{k,t}}{P_{c,t}} + \frac{E_t hw_{k,t}}{P_{c,t}} \right) \quad (1.9)$$

$$\Omega_t = 1 - (1-v)(1+rr_t)^{\zeta-1}\varphi^\zeta \quad (1.10)$$

where  $\Omega_t$ , the propensity to consume out of total wealth, depends on the real rate of return, on the intertemporal elasticity of substitution, on the probability of death and on the subjective rate of time preference.  $\Omega$  is constant, i.e. independent of the real rate, in the particular case of logarithmic utility ( $\zeta = 1$ ). Human wealth ( $hw$ ) is defined as the sum of discounted future labour income:

$$hw_{k,t} = \sum_{s=0}^{\infty} \left( \frac{(1-v)^s}{\prod_{i=0}^{s-1} (1+r_{t+i,t+i+1})} \right) y_{k+s,t+s} \quad (1.11)$$

### *b) Aggregation*

After the description of consumption behaviour of one generation, it is necessary to sum over the generations to obtain the aggregate variables. Denote aggregate consumption, labour income, financial wealth and human wealth by  $C$ ,  $YL$ ,  $FW$ ,  $HW$ . Since we want to deal with a growing economy, or at least prevent the country from disappearing, we need

to introduce a birth rate, say  $\iota$ , that will be assumed constant. Recall that for any number of individuals, only a proportion  $(1-v)$  remains at the end of the period. This means that the rate of growth of the population,  $n$ , is given by

$$1+n = (1+\iota)(1-v) \quad (1.12)$$

Normalising the population at time zero to one, the size of a generation born at time  $t-k$  in period  $t$ , i.e. the size of the population aged  $k$  in  $t$ , is given by the probability of being born times the size of the population prior to birth times the probability of surviving in period  $t$ :

$$\iota (1+n)^{t-k} (1-v)^k \quad (1.13)$$

Summing over all generations gives the total size of the population. This leads to the following definitions for aggregate variables.

- aggregate consumption:

$$C_t \equiv \sum_{k=0}^{\infty} \iota (1+n)^{t-k} (1-v)^k c_{k,t} \quad (1.14)$$

- aggregate labour income:

$$YL_t \equiv \sum_{k=0}^{\infty} \iota (1+n)^{t-k} (1-v)^k y_{k,t} \quad (1.15)$$

- aggregate human wealth:

$$HW_t = \sum_{s=0}^{\infty} \left( \frac{(1-v)^s}{\prod_{i=0}^{s-1} (1+r_{t+i,t+i+1})} \right) YL_{t+s} \quad (1.16)$$

- aggregate financial wealth:

$$FW_t = (1+r_t) \{FW_{t-1} + YL_{t-1} - P_{CD,t-1} C_{t-1}\} \quad (1.17)$$

Note that although individual financial wealth accumulates in (1.3) at a rate  $(1+r)/(1-v)$  if an individual remains alive, aggregate wealth accumulates only at the market rate  $(1+r)$ .

The probability of death causes future income flows to be discounted at a rate above the market interest rate. Such an over-discounting implies that the households' horizon implicit in calculating the present value of future income flows is shortened and, consequently, the strong wealth effects peculiar to "infinite" horizon models are reduced while the influence

of current income is strengthened. As a consequence, the extreme version of the Ricardian equivalence does not hold since the present value of future tax changes does not completely match current adjustments in tax payments.

Since the propensity to consume was independent of age, the aggregate consumption function can now be written as:

$$C_t = \left[ 1 - (1 - \nu)(1 + rr_t)^{\xi-1} \varphi^\xi \right] \left[ \frac{FW_t}{P_{CD,t}} + \frac{E_t HW_t}{P_{CD,t}} \right] \quad (1.18)$$

which is the aggregate version of (1.9) and (1.10)

The estimation is based on a log-linear approximation of this consumption function in which the proportionality of consumption to total wealth is ensured by imposing that the coefficients of human and financial wealth sum to one:

$$c_t = 0.95 (hw_t - p_{CD,t}) + 0.05 (fw_t - p_{CD,t}) - 1.2 rr_t \quad (1.19)$$

This life-cycle model determines the desired level of consumption. It depends on the one hand on the financial wealth which equals the market value of financial assets. On the other hand it also depends on human wealth which corresponds to the present value of expected future labour income -defined net of taxes and inclusive of transfer payments-. The magnitude of the coefficient on financial wealth is low as compared to that on human wealth: a 10 p.c. increase in financial wealth only raises consumption by 0.5 p.c. against 9.5 p.c. for human wealth.

Finally, the optimal consumption is a negative function of a real short-term interest rate reflecting intertemporal substitution in consumption, i.e. the effect the interest rate exerts on the propensity to consume out of total wealth. Empirically, this negative sign is probably also a consequence of the inclusion of durable goods in the consumption data. According to the estimated coefficient, a 100 basis point cut in the annualised real rate would cause a 0.3 p.c. hike in desired consumption. If interest payments have a positive income effect, they will be accounted for by the financial wealth variable which incorporates capital incomes.

### 1.1.2 Household's net financing capacity

Households can also invest in housing. The equilibrium ratio of housing investment, IHR, to consumption is a function of the relative price of consumption to the construction cost of new houses, IHXN, and on a mortgage interest rate, RM:

$$ihr_t = c_t + \tau_1 RM_t + \tau_2 (ihxn_t - p_{CD,t}) \quad (1.20)$$

Stock-flow consistency is ensured through household net financing capacity, PLN, which is obtained by subtracting consumption and housing investment from disposable income. In turn, this balance is added to previous financial wealth augmented with positive or negative capital gains, kg, to determine a new end-of-period value for financial wealth, which subsequently will affect next period's consumption. Equation (1.17) can be written as:

$$FW_t = (1 + kg_t) FW_{t-1} + PLN_t \quad (1.21)$$

## 1.2 Goods market structure

There are two types of monopolistically competitive intermediate goods suppliers. Type one firms produce domestic intermediate goods. Type two firms import foreign intermediate goods to have them reselled in the domestic market. The intermediate goods firms' output is sold in the domestic market to retailers or sold to exporters. These retailers combine domestic intermediate goods with imports of intermediate goods to sell final goods on perfectly competitive markets for consumption and investment. Exporters proceed in the same manner to sell final goods on foreign markets. There is also imperfect competition in the market for imported (intermediate) goods, which are imperfect substitutes for each other in the production of the composite imported good produced by a representative competitive firm.

### 1.2.1 Intermediate Goods Firms

There is a large number of monopolistically competitive firms that produce differentiated varieties of domestic intermediate goods indexed by  $i = 1, K, N$ . Each firm produces a product that is slightly different from any other good, so that each good is unique but has many close substitutes. A fraction  $\Phi$  of all goods produced in the country is not tradable internationally. More specifically, non-tradable home goods will be indexed  $i = 1, K, \Phi N$  and tradable goods  $i = \Phi N + 1, K, N$ . Goods produced in the rest of the world (called foreign goods) are indexed analogously and will be denoted by an asterisk. Thus there are

$\tilde{N} = N + (1 - \Phi)N^*$  varieties demanded at home. The markets for internationally traded goods are segmented by country, so that firms have the ability to set distinct prices in each national market<sup>2</sup>. This last feature is typically called "Pricing To Market". Following Bergin and Feenstra (2001) and Bergin (2003) the final goods retailers aggregate over the differentiated goods with a translog functional form. Unlike the usual CES aggregator used in most new open economy models, the translog specification does not restrict the elasticity of substitution between goods to be constant but allows it to vary with the prices of competing goods. When demand is elastic, -a feature necessary to have monopolistic competition-, a fall in competing prices raises the elasticity leading to a pro-competitive reduction in mark-ups. This feature is important in generating pricing-to-market also in equilibrium<sup>3</sup> since the demand curve faced by the firms has an elasticity that depends on their price-setting decisions which may be different for their different markets, e.g. domestic or foreign.

### 1.2.1.1 Domestic producers

#### a) *Representative firm*

The nominal price index is defined as the dual expenditure function of the final good producers, which is assumed to have a translog form. This unit expenditure function is defined by:

$$\ln P_t \equiv \sum_{i=1}^{\tilde{N}} \alpha_i \ln P_{it} + \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln P_{it} \ln P_{jt} \quad (1.22)$$

where  $P_{it}$  is the home-currency price of good "i".

We consider the special case where all goods enter symmetrically

$$\alpha_i = \frac{1}{\tilde{N}} \quad , \quad \gamma_{ii} = \frac{-\gamma}{\tilde{N}} \quad , \quad \gamma_{ij} = \frac{\gamma}{\tilde{N}(\tilde{N}-1)} \quad \text{for } i \neq j$$

The expenditure shares are obtained by differentiating the expenditure function

$$s_{it} = \frac{\partial \ln P_t}{\partial \ln P_{it}} = \frac{1}{\tilde{N}} + \sum_{j=1(j \neq i)}^{\tilde{N}} \frac{\gamma}{\tilde{N}(\tilde{N}-1)} \ln P_{jt} - \frac{\gamma}{\tilde{N}} \ln P_{it} \quad (1.23)$$

---

<sup>2</sup> This assumption can be justified with a system of selective distribution in which producers can choose their dealers and prevent them from reselling to anyone but the end-users.



The domestic demand for each product may then be written

$$Y_{it}^H = s_{it} \frac{P_t Y_t^H}{P_{it}} \quad (1.24)$$

The positive elasticity of demand for each of the differentiated intermediate goods in the home country is computed as

$$\eta_{it} = 1 - \frac{\partial \ln s_{it}}{\partial \ln P_{it}} = 1 - \frac{\gamma_{ii}}{s_{it}} = 1 + \frac{\gamma}{\tilde{N} s_{it}}, \quad \gamma > 0$$

is required for demand to be elastic. In steady state, prices are all equal and the shares are thus simply given by  $s_i = 1/\tilde{N}$ . The demand price elasticity is then  $1 + \gamma$ .

Let the domestic unit cost be denoted by  $C_t(w, ucc)$ , the precise definition of which will be given later on, then nominal home firm profits for non-traded and traded goods may be defined as:

$$\Pi_{it} = (P_{it} - C_t(w, ucc)) Y_{it}^H \quad \text{for } i = 1, \dots, \Phi N \quad (1.25-a)$$

$$\Pi_{it} = (P_{it} - C_t(w, ucc)) Y_{it}^H + (P_{it}^* - C_t(w, ucc)) X_{it}^* \quad \text{for } i = \Phi N + 1, \dots, N \quad (1.25-b)$$

where  $P_{it}^*$  is the price of a home product sold to an exporter. Demand by exporters,  $X_{it}^*$ , are determined in the same manner as the domestic demands but from a foreign perspective.

Profit maximisation yields to the following optimal price setting rules<sup>4</sup>:

$$P_{it} = \left(1 - \frac{s_{it}}{\gamma_{ii}}\right) C_t(w, ucc) \quad \text{for } i=1, \dots, N \quad (1.26-a)$$

for home goods sold in the home market and

$$P_{it}^* = \left(1 - \frac{s_{it}^*}{\gamma_{ii}}\right) C_t(w, ucc) \quad \text{for } i = \Phi N + 1, \dots, N \quad (1.26-b)$$

for sales to exporters.

<sup>3</sup> A time-varying mark-up is also possible with a CES aggregator because of price sluggishness. However in this instance, the mark-up would be constant if prices adjusted instantly

<sup>4</sup> Under flexible prices, we obtain the standard result that a producer with market power sets its price as a mark-up over marginal cost, with the size of the mark-up determined by the elasticity of demand. However, in our case, the elasticity depends on the share of intermediate good "i" in the total expenditure of final good producer and is therefore not constant.

The expenditure share can be substituted from (1.23) so that (1.26-a) can be solved for the optimal price in terms of marginal cost and prices of competitors. This expression is nonlinear, involving  $P_{it}$  and  $\ln P_{it}$ , so we will use an approximation to obtain a simple solution for the price. In this simple linearised form, the price setting equation implies that domestic prices for both traded and nontraded goods behave as

$$p_{it} = (1-\lambda) c_t(w, ucc) + \lambda \sum_{j=1(j \neq i)}^{\tilde{N}} \frac{1}{\tilde{N}-1} p_{jt} \quad \text{for } i=1, \dots, N \quad (1.27-a)$$

where small case letters denote the logarithm of the variable and parameter  $\lambda = \gamma/(1+2\gamma)$  indicates the degree to which an individual firm's price setting decision is influenced by the price of other competitors in the market, while the remainder is determined by its own costs. And when goods are sold to exporters, their price is similarly given by

$$p_{it}^* = (1-\lambda) c_t(w, ucc) + \lambda \sum_{j=1(j \neq i)}^{\tilde{N}} \frac{1}{\tilde{N}-1} p_{jt}^* \quad \text{for } i = \Phi N + 1, \dots, N \quad (1.27-b)$$

#### *b) Aggregation*

To further simplify these price equations, consider the case of two equally-sized groups of differentiated products at home and abroad,  $N = N^*$ . The total number of products demanded in the home country is thus  $(2-\Phi)N$ , a proportion  $\frac{1}{2-\Phi}$  of which consists of domestically produced intermediate goods. Let  $p_{H,i}$  denote the price of home goods sold in the home market,  $p_j^M$  the price of foreign goods sold in the home market. Then applying expression (1.27-a) for all firms  $i = 1, \dots, N$  and noting that in equilibrium  $p_{H,i} = p_H$  and  $p_j^M = p^M = mtd_t$ . Then assuming  $N$  is large one can solve for  $p_H$  as

$$p_{Ht} = \frac{(1+\gamma)(2-\Phi)}{(2-\Phi)+\gamma(3-2\Phi)} c_t(w, ucc) + \frac{(1-\Phi)\gamma}{(2-\Phi)+\gamma(3-2\Phi)} mtd_t \quad (1.28)$$

Similarly denoting the price of home goods for the foreign market by  $xt_{d,i}$ , and the price of foreign goods sold in the foreign market by  $p_{F,i}^*$  and noting that in equilibrium  $xt_{d,i} = xtd$  and  $p_{F,i}^* = p_F^*$ , equation (1.27-b) may be compactly written as:

$$xt_{d,t} = \frac{(1+\gamma)(2-\Phi)}{(2-\Phi)+\gamma(3-\Phi)} c_t(w, ucc) + \frac{\gamma}{(2-\Phi)+\gamma(3-\Phi)} p_{Ft}^* \quad (1.29)$$

This shows that firms set their price not only in response to changes in their own costs but also in response to the prices set by their competitors. The relative weight depends on the share of non-traded goods which determines the proportion of their competitors which are foreign.

In linearised form, a definition of the price index for the domestic economy is

$$p_t = \frac{1}{(2-\Phi)} p_{Ht} + \frac{(1-\Phi)}{(2-\Phi)} mtd_t \quad (1.30)$$

and domestic output sold in the home country is obtained by summing over the "domestic" demand functions for all home goods  $i$ :

$$y_t^H = \frac{1}{(2-\Phi)} [(1+\gamma)(p_t - p_{Ht}) + d_t] \quad (1.31)$$

while exported output,  $xtr$ , is obtained in a similar manner

$$xtr_t = \frac{(1-\Phi)}{(2-\Phi)} [(1+\gamma)(p_t^* - xtd_t)] + \frac{(1-\Phi)}{(2-\Phi)} d_t^* \quad (1.32)$$

To obtain an expression similar to (1.30) for the foreign price index, consider a two-country world. Then the foreign pricing-to-market and the relative price of non-tradables would be the inverse of their home counterpart so that the foreign price index would be given by

$$p_t^* = \frac{1}{(2-\Phi)} p_{Ft}^* + \frac{(1-\Phi)}{(2-\Phi)} xtd_t \quad (1.33)$$

Thanks to this definition, the demand for exports can be expressed in terms of  $p_F^*$  and  $xtd$ .

Using  $Y$  to denote production, market clearing for domestic intermediate goods requires

$$Y_{it} = Y_{it}^H \quad \text{for } i = 1, \dots, \Phi N \quad (1.34)$$

$$Y_{it} = Y_{it}^H + X_{it}^* \quad \text{for } i = \Phi N + 1, \dots, N \quad (1.35)$$

Producing each variety of intermediate goods involves labour measured in hours, capital and technical progress within a common CES production function with constant return to scale:

$$Y_{it} = \left[ \alpha (\Lambda_L L_{it})^{-p} + (1-\alpha) (\Lambda_K K_{it})^{-p} \right]^{-\frac{1}{p}} \quad (1.36)$$

where  $\sigma = \frac{1}{1+\rho}$  is the elasticity of technical substitution between labour and capital,  $\alpha$ ,  $\Lambda_L$ ,  $\Lambda_K$  are a share parameter, labour- and capital-augmenting technical progress respectively<sup>5</sup>.

Static profit maximisation subject to the firm's production function (1.36) and to the derived demand for the firm's output (1.24) yields the following FOC

$$W_{it} = \kappa_{it} P_{it} \frac{\partial Y_{it}}{\partial L_{it}} \quad (1.37.a)$$

$$UCC_{it} = \kappa_{it} P_{it} \frac{\partial Y_{it}}{\partial K_{it}} \quad (1.37.b)$$

where

$$\frac{1}{\kappa_{it}} = \left( 1 - \frac{s_{it}}{\gamma_{it}} \right) \text{ is the gross mark-up;}$$

$$\frac{\partial Y_{it}}{\partial L_{it}} = \alpha \Lambda_L \left( \frac{Y_{it}}{\Lambda_L L_{it}} \right)^{1+\rho}; \quad (1.38.a)$$

$$\frac{\partial Y_{it}}{\partial K_{it}} = (1-\alpha) \Lambda_K \left( \frac{Y_{it}}{\Lambda_K K_{it}} \right)^{1+\rho} \quad (1.38.b)$$

Substituting equation (1.38.a) into (1.37.a) gives the optimal labour demand. Since it is expressed in volume, i.e. total hours, in order to obtain employment we added a relation for the average hours per worker. These are cyclical around a trend which is specified as an increasing function of the ratio of full-time to part-time work and of conventional working time.

Equations (1.37.b) and (1.38.b) determine the optimal demand for capital. Since the ratio of long-run equilibrium investment (IOR) to target capital equals the sum of the depreciation rate and the steady state growth rate of output (the latter being the sum of the rate of technical progress,  $\zeta$ , augmented with the rate of population growth,  $n$ ), the following steady state investment rate equation holds:

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<sup>5</sup> Note that without loss of generality we can assume that  $\alpha$  remains constant since, if  $\sigma \neq 1$ , the precise value of  $\alpha$  is arbitrary and any change in it can be represented through biased technical progress which is reflected in a change in the ratio  $\frac{\Lambda_L}{\Lambda_K}$ .

$$\log_t^* - k_{t-1}^* = \ln(\delta + g_{\Lambda_L} + n) \quad (1.39)$$

This relationship shows the investment flow necessary to make the capital stock growing at the steady state growth rate of the economy.

Knowing the optimal factor demands, real unit production cost is given as

$$c^R(w, ucc) = \left[ \alpha^\sigma \left( \frac{w}{\Lambda_L} \right)^{\sigma\rho} + (1-\alpha)^\sigma \left( \frac{ucc}{\Lambda_K} \right)^{\sigma\rho} \right]^{\frac{1}{\sigma\rho}} \quad (1.40)$$

This is the minimum cost of obtaining the unit output level given that the "real" unit input prices are  $w$  and  $ucc$ . This function is homogenous of degree one so that the nominal unit cost is simply given by  $C(w, ucc) = P \cdot c^R(w, ucc)$

### 1.2.1.2 Importing firms

The importing firms choose their resale price in our home market  $p_{jt}^M$  to maximise their profits

$$\Pi_{jt}^M = (p_{jt}^M - p_{Fjt}^*) Y_{jt}^M \quad (1.41)$$

where the cost of imported intermediates in domestic currency is given by the price of foreign goods sold in the foreign market  $p_{F,j}^*$ .

Profit maximisation implies

$$p_{jt}^M = \left( 1 - \frac{s_{jt}^M}{\gamma_{jj}} \right) p_{Fjt}^* \quad \text{for } j = \Phi N + 1, \dots, N \quad (1.42)$$

and substituting for the expenditure shares from (1.23) and following the same steps as for domestic goods in (1.28), one finally gets

$$p_t^M = mtd_t = \frac{(1+\gamma)(2-\Phi)}{(2-\Phi) + \gamma(3-\Phi)} p_{Ft}^* + \frac{\gamma}{(2-\Phi) + \gamma(3-\Phi)} p_{Ht} \quad (1.43)$$

From (1.43) and (1.28), we can obtain a price index for intermediate goods sold on the home market in terms of unit cost and foreign price

$$p_{Ht} = \left( 1 - \frac{\gamma}{1+2\gamma} \frac{(1-\Phi)}{(2-\Phi)} \right) c_t(w, ucc) + \frac{\gamma}{1+2\gamma} \frac{(1-\Phi)}{(2-\Phi)} p_{Ft}^* \quad (1.44)$$

Comparing (1.44) to (1.29), rather than to (1.28) highlights more clearly how prices charged by home firms in the domestic and export markets can be different.

### 1.2.2 Production of final goods

The composite final goods,  $Z$ , are obtained by aggregating over intermediate home goods along with aggregating over imported goods<sup>6</sup>:

$$Z_t = (Y_t^H)^\delta (Y_t^M)^{1-\delta} \quad (1.45)$$

where  $Y^H$  is an aggregate of the individual home goods sold in the domestic economy,  $Y_i^H$  and  $Y^M$  is an aggregate of the imported goods,  $Y_i^M$ .

Final goods producers (or aggregators) behave competitively, maximising profits each period, taking the price  $P_{H,t}$  of each intermediate home good  $Y_i^H$  and the price,  $mtd_t$ , of each imported foreign goods,  $Y_i^M$ , as given:

$$\pi_t^Z = P_t Z_t - P_{H,t} Y_t^H - mtd_t Y_t^M \quad (1.46)$$

where  $P$  is the overall price index of the final goods,  $P_H$ , the price index of home goods and  $mtd$ , the import price of foreign goods. Given the aggregation function defined in (1.45), the conditional aggregate demand for home and foreign goods will be

$$Y_t^H = \delta \left( \frac{P_{H,t}}{P_t} \right)^{-1} Z_t \quad (1.47)$$

$$Y_t^M = (1-\delta) \left( \frac{mtd_t}{P_t} \right)^{-1} Z_t \quad (1.48)$$

with the demand for individual goods given by (1.24) and its analogues for imports. The home price index may be written as

$$P_t = \delta^{-\delta} (1-\delta)^{\delta-1} (P_{H,t})^\delta (mtd_t)^{1-\delta} \quad (1.49)$$

which in log-linearised form is the analogue to (1.30) with  $\delta = 1/(2 - \Phi)$ .

This could in principle be the end of the story. However, since we are interested in a decomposition of aggregate demand in expenditure categories, we will consider that the composite final good is then transformed without cost into differentiated goods which are

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<sup>6</sup> A CES aggregator is often used in the literature. However since this approach will serve for each expenditure category it is assumed that the elasticity is unitary for all categories. Due to a lack of data on the various categories of both import quantities and prices, it seems hazardous to try to identify each elasticity of substitution between domestic and imported categories of goods.

either sold as consumption goods, investment goods, housing investment and government goods to final goods aggregators. The exporters proceed in the same way so that the demand for home good results from the sum:

$$Y_t^H = C_{H,t} + I_{H,t} + I_{H,t}^h + G_{H,t} + X_{H,t} \quad (1.50)$$

where  $C_{H,t}$ ,  $I_{H,t}$ ,  $I_{H,t}^h$ ,  $G_{H,t}$ ,  $X_{H,t}$  denote the amount of domestic goods used in the production of consumption, investment, housing investment, government purchases of goods and services and exports respectively. These demands are obtained by aggregating the individual demand (1.24) for each specific category. Proceeding along the same line for imports we obtain aggregate imports as:

$$Y_t^M = MTR_t = C_{M,t} + I_{M,t} + I_{M,t}^h + G_{M,t} + X_{M,t} \quad (1.51)$$

For instance, for consumption, replacing  $Z$  by  $PCR$ ,  $Y^H$  by  $C_H$  and  $Y^M$  by  $C_M$

$$PCR_t = (C_{H,t})^{\delta^C} (C_{M,t})^{(1-\delta^C)} \quad (1.52)$$

where  $C_{M,t}$  refers to imported consumption goods. The aggregator sells the final good to households at a price  $P_{CD,t}$  which may be interpreted as the consumption price index. Profit maximisation implies the following (log)price indice:

$$p_{CD,t} = \ln(1 + t_t^i) + \delta^C (p_{H,t}) + (1 - \delta^C)(mtd_t) \quad (1.53)$$

where  $t_t^i$  stands for indirect taxes (less subsidies) rate.

This demand will be allocated between home and foreign goods according to

$$C_{H,t} = \delta^C \left( \frac{P_{H,t}}{P_{CD,t}} \right)^{-1} PCR_t \quad (1.54)$$

$$C_{M,t} = (1 - \delta^C) \left( \frac{mtd_t}{P_{CD,t}} \right)^{-1} PCR_t \quad (1.55)$$

Production of the final investment goods, IOR, and of government purchases of goods and services, GCR1, are modelled analogously. Finally, exporters also combine traded domestic intermediate goods and imported brands to produce export goods, XTR. Therefore, we have the analogues to (1.53), (1.54), (1.55) for each GDP category with specific weights for domestic and imported goods reflecting information from input-output tables.

From (1.49) and (1.53), it can be seen that the domestic and foreign price are common across final expenditure categories. Consequently, differences in their respective deflator will only reflect the differences in their import share.

Note that, in line with available input-output tables, housing investment is produced from domestic materials only, i.e.  $I_M^h = 0$ .

### 1.2.3 Aggregate imports

Substituting for the import demand by categories of expenditures, (1.51) becomes:

$$\begin{aligned}
 MTR_t = & (1 - \delta^C) \left( \frac{M_{TD,t}}{P_{CD,t}} \right)^{-1} PCR_t + (1 - \delta^I) \left( \frac{M_{TD,t}}{I_{OD,t}} \right)^{-1} IOR_t + (1 - \delta^G) \left( \frac{M_{TD,t}}{G_{CD1,t}} \right)^{-1} GCR1_t \\
 & + (1 - \delta^X) \left( \frac{M_{TD,t}}{X_{TD,t}} \right)^{-1} XTR_t
 \end{aligned}
 \tag{1.56}$$

where  $\delta^C, \delta^I, \delta^G, \delta^X$  are the share of home produced goods in private consumption, investment, public procurement, GCR1, and exports respectively, which can be derived from input-output tables.

This section has described the central role played by the price of home domestic intermediate goods (also labeled price of domestic output) in the derivation of the main deflators of final demand. A comparison of the empirical versions of the price set by domestic firms on the home and foreign markets indicates the degree to which firm's price setting decision is influenced by the price of competitors in that market:

$$p_{Ht} = (4/5) c_t(w, ucc) + (1/5) p_{Ft}^*$$

$$xtd_t = (2/3) c_t(w, ucc) + (1/3) p_{Ft}^*$$

Not surprisingly home firms are more sensitive to competitors' prices on the foreign market than on the domestic market. Since imported goods compete with domestic ones, the price set by importers is also an average of their own costs, represented by the price they have to pay to acquire the foreign goods, and of the price of domestic intermediate goods:

$$mtd_t = (1/3) p_{Ht} + (2/3) p_{Ft}^* = (4/15) c_t(w, ucc) + (11/15) p_{Ft}^*$$

These relationships show that exchange rate pass-through is far from complete even in the long term because modifications of importers' mark-ups partially offset exchange rate



changes. On top of that, the composition of  $p_{Ft}^*$ , which is an indicator of the price of both intra- and extra-euro area competitors, reinforces this incompleteness of the pass-through.

In order to account for the different expenditure categories, the final demand has been subdivided into private consumption, business investment, government procurement and exports, all of which have a domestic and an imported component, housing which is made of domestic inputs only and inventories which are exogenous.

Finally, from the production function and the factor demands, we have obtained the labour-augmenting technical progress and the elasticity of substitution between capital and labour. The former is supposed to grow at a rate around 1.5 p.c. a year and the estimate of the latter is close to 0.5. A low elasticity of substitution helps somewhat to match the empirical findings of a small response of business investment to interest rate changes since it lowers the response of capital formation to variations in the user cost. In addition, with an elasticity of substitution below one, capital accumulation creates employment while growth in the labour supply and the labour-augmenting technical progress will cause a rise in unemployment unless they are offset by increased investment.

### **1.3 Labour market structure**

In Belgium, the government has intervened quite regularly in the course of the wage formation process. This may be a source of concern when trying to analyse that process econometrically. However, for simulation purposes, we will consider that in the long run wages can be explained according to a bargaining model between firms and unions and that, in practice, the correction mechanism is not market determined but sometimes imposed by government interventions. Of course, forecasting exercises need to respect the law of July 1996 for the promotion of employment and the safeguarding of firms' competitiveness which guarantees that the principle of automatic indexation of wages to the *"health" consumption price index* is maintained but that nominal wages do not grow faster than the weighted average wage growth in France, Germany and The Netherlands. Note that oil, tobacco and alcohol are excluded from the basket used to calculate the "health" consumption price index. This feature may be important in understanding the transmission of oil price shocks. In the rest of the model, the labour supply is treated exogenous.

Each intermediate good firm negotiates wage with a single union according to a "right to manage" bargaining model. By organising themselves in trade unions, households can extract some producer surplus. Once wages are fixed, the firm decides on employment according to its labour demand curve (1.37.a). Each representative union seeks to maximise the average real "consumption" income of "insiders" which is equal to

$$V_i = S_i W_i^N + (1 - S_i) \frac{A_i}{P_{CD}} \quad (1.57)$$

where  $S_i$  is the proportion of insiders who will keep their job following the wage settlement which will yield the wage  $W_i^N$  and  $A_i$  is the reservation wage or expected income for those who will lose their job. Actually, the relation between gross nominal wage,  $W^B$ , net real "consumption" wage,  $W^N$ , and real "producer" wage cost,  $W^C$ , are given by

$$W^C = \frac{W^B(1 + t_{w1})}{P_H} \quad \text{and} \quad W^N = \frac{W^B(1 - t_{w2} - t_{w3})}{P_{CD}}$$

and the "tax and price wedge" is given by  $z = \frac{W^C}{W^N}$ . In these formulae, the average tax rates  $t_{w1}, t_{w2}, t_{w3}$  refer respectively to the social security contributions of employers, of employees and withholding tax on earned income. The expected nominal income available outside the firm is assumed to be an average of the wage in other firms,  $W^B(1 - t_{w2} - t_{w3})$ , and of the unemployment benefit,  $B$ :

$$A = [1 - \psi u] W^{B^e} (1 - t_{w2} - t_{w3}) + \psi u B \quad (1.58)$$

where  $u$  is the unemployment rate and  $\psi$  is a constant.

The outcome of the bargaining process is the wage rate that maximises a Nash product of the type,

$$\Omega_i = (V_i - \bar{V}_i)^\varpi (\Pi_i - \bar{\Pi}_i) \quad (1.59)$$

where  $\varpi$  is an index of relative bargaining power, and  $V$  and  $\Pi$  are the utility functions of the unions and firms respectively. The bar above a variable indicates the outside options available to the parties if negotiations collapse and the firm shuts down. It is assumed that

$$\bar{V}_i = \frac{A}{P_{CD}}, \quad \bar{\Pi}_i = 0.$$

Then the Nash product can be rewritten as

$$\Omega_i = \left( \frac{W_i^B(1 - t_{w2} - t_{w3}) - A}{P_{CD}} \right)^\varpi \cdot S_i (W_i^B(1 + t_{w1}))^\varpi \cdot \Pi_i \quad (1.60)$$

with  $S_i$  assumed to depend on the wage cost. The product is maximised (by choosing  $W_i^B$ ) when

$$\frac{W_i^B(1+t_{w1})L_i}{\Pi_i} = \varpi \frac{W_i^B(1-t_{w2}-t_{w3})}{W_i^B(1-t_{w2}-t_{w3})-A} - \varpi \varepsilon_{SN} \left\{ (1+\rho) - \frac{(\kappa_t + \rho)\alpha}{\alpha + (1-\alpha) \left( \frac{\Lambda_k K_i}{\Lambda_L L_i} \right)^{-\rho}} \right\} \quad (1.61)$$

where  $\varepsilon_{SN}$  is a constant reflecting the vulnerability of insiders to job loss.

From the definition of alternative income (1.58), it follows that

$$\frac{W_i^B(1-t_{w2}-t_{w3})}{W_i^B(1-t_{w2}-t_{w3})-A} = \frac{W_i^B(1-t_{w2}-t_{w3})}{(1-t_{w2}-t_{w3})(W_i^B - W^B) + \psi u \left( 1 - \frac{B}{W^{B^e}(1-t_{w2}-t_{w3})} \right) W^B(1-t_{w2}-t_{w3})} \quad (1.62)$$

In a symmetric equilibrium all intermediate goods firms and unions make identical decisions so that

$$W_i^B = W^B, \quad P_{Hi} = P_H, \quad S_i = S, \quad L_i = L/N, \quad K_i = K/N, \quad \text{for all } i = 1, \dots, N$$

Thus one gets

$$\frac{W^B(1-t_{w2}-t_{w3})}{W^B(1-t_{w2}-t_{w3})-A} = \frac{1}{\psi u(1-b)}, \quad \text{where } b = \frac{B}{W^B(1-t_{w2}-t_{w3})} \quad (1.63)$$

Combining (1.59) with the fact that the profit rate is given by

$$\frac{\Pi}{P_H Y} = 1 - \kappa_t \quad (1.64)$$

yields a relationship that, in case  $\varepsilon_{SN} = 0$ , simplifies to

$$\frac{W^B(1+t_{w1})L}{P_H Y} = \frac{(1-\kappa_t)\varpi}{\psi u(1-b)} \quad (1.65)$$

Note that  $\varepsilon_{SN} = 0$  means that higher wages will have no effect on the employment of insiders although they will reduce the jobs available to outsiders<sup>7</sup>.

Making use of the production function to eliminate  $L/Y$ , one finally gets the aggregate wage equation<sup>8</sup>:

$$\frac{W^B(1+t_{w1})}{P_H} \Lambda_L^{-1} = \frac{\bar{\omega}(1-\kappa_t) \left[ \alpha + (1-\alpha) \left( \frac{k}{1-u} \right)^{-\rho} \right]^{-\frac{1}{\rho}}}{\psi u(1-b)} \quad (1.66)$$

where  $k = \frac{K}{\Lambda_L LF^P}$ ,  $LF^P$  being private labour force.

This relation shows that if  $k$  falls following a faster growth in the labour supply or a slower growth in the capital stock, the real production wage should also fall to prevent unemployment from rising.

Log-linearising equation (1.66) yields the following equilibrium wage setting rule for the economy:

$$w_t^{C*} = \log(\Lambda_{L,t}) + \log(1-\kappa_t) + \log\left(\frac{\beta}{\psi}\right) - \log(u_t) - \log(1-b_t) - \frac{1}{\rho} \log \left[ \alpha + (1-\alpha) \left( \frac{k_t}{1-u_t} \right)^{-\rho} \right] \quad (1.67)$$

From the definitions of  $W^C$  and  $W^B$ , one can see that employer social security contributions have a direct one for one impact on wage cost while employee's contribution and income taxes exert their effect through the replacement ratio.

Estimation of (1.67) result in the following relation:

$$w_t^{C*} = \log(\Lambda_{L,t}) + 0.05 \log(1-\kappa_t) - (0.18 - 0.2 b_t) \log(u_t) - \frac{1}{\rho} \log \left[ \alpha + (1-\alpha) \left( \frac{k_t}{1-u_t} \right)^{-\rho} \right]$$

<sup>7</sup> Actually it can be shown that when demand is non stochastic, as it is the case here,  $\varepsilon_{SN}$  equals either 0 or 1. Intermediate cases occur when demand is stochastic.

<sup>8</sup> Alternatively in (1.65), one could have expressed  $WL/PY$  in terms of  $k$ ,  $u$ , ... and obtained an implicit

$$\text{equation for equilibrium unemployment } u_t^* = \frac{\bar{\omega}(1-\kappa_t) \left[ \alpha + (1-\alpha) \left( \frac{k_t}{1-u_t^*} \right)^{-\rho} \right]}{\kappa_t \alpha (1-b_t) \phi}$$

This relation emphasises that the equilibrium unemployment rate is dependent not only on labour market conditions such as union power,  $\bar{\omega}$ , or the replacement ratio,  $b$ , as in Layard et al. (1991) but also the capital-labour ratio,  $k$ , and the (inverse) mark-up through which changes in foreign prices will pass.

It shows that the real producer wage follows trend labour productivity but can deviate from it due to:

- rent sharing: although the estimated coefficient at 0.05 is far below 1 as implied by theory;
- changes in the rate of unemployment and in the replacement ratio. Given that wage formation has not always been determined by market forces, it is not surprising that the empirical impact of these two variables is also lower than theory would predict. Actually, the replacement ratio dampens the already mild effect exerted by the rate of unemployment. When unemployment benefits are high as compared to pocket wages, the negative impact of unemployment is reduced;
- the last term on the right-hand-side. To illustrate how this term, which would not be present under a Cobb-Douglas production function, works. Imagine that for some reason capital is growing slower than the labour supply and that  $k$  is falling. To maintain a constant unemployment rate under these circumstances, the real wage cost must grow less than trend productivity or, in other words, the real wage in efficiency units must fall.

#### 1.4 Government

Many variables in this sector are either exogenous in real terms or defined through technical relations. Current expenditure is divided into interest payments on government debt and different types of primary expenditure categories. The allocation of the outstanding debt over long term and short term domestic currency and foreign currency debt is taken as given and a representative interest rate is applied to each corresponding debt category. The weighted sum of these representative rates is, in turn, used to estimate the implicit rate on government debt. In modelling primary expenditure, the following main items are distinguished:

- government wages and pensions are indexed according to the “health” CPI, real wages being treated exogenous;
- government consumption of goods and public investment are exogenous in real terms and the deflator of the former follows both the price of home produced goods, with a weight of  $(\delta^G)$ , and of imported goods with a weight of  $(1 - \delta^G)$  as explained in (1.49) while the deflator of the latter is related to the private sector investment deflator;
- most transfers to households are exogenous in real terms but are indexed to the “health” CPI. Unemployment benefits are the only business cycle sensitive component.

In the long run, we have to ensure that the debt to GDP ratio settles down to its steady state value. To achieve this goal, total transfer payments to households are used as the control variable. One possibility would be to incorporate a fiscal policy feedback rule that would adjust transfers to bring the debt to GDP ratio to its desired level. Such a convergence could be achieved by specifying a fiscal rule which imposes a targeted path for the debt and/or deficit ratios. Such a rule is highly pro-cyclical since in order to keep the debt or deficit ratio on target when output is below trend, the deficit must also be lower than in steady state, which reinforces the reduction of output. Therefore, to avoid too much cyclical variation when simulating the model with a fiscal rule, we make use of a more flexible rule which only guarantees that the debt ratio decreases at a given rate but does not strictly respect a given level; i.e. it accommodates shocks but ensures convergence to the steady state, the exact date of the convergence being different from shock to shock.

General government receipts have been split into

- direct taxes on households' earned income: due to the progressiveness of income taxes, the average tax rate is a log-linear function of the income level per head. The nominal income level is affected by both a price and a real component. Under normal circumstances, tax brackets are indexed to the rise in the consumption price of the preceding year and then price level changes do not change the average tax rate. In addition the average tax rate can also reflect changes in the tax structure;
- direct taxes on companies: the tax amount is explained by the firms' tax rate, which is a flat one, together with the taxable base which is represented by the gross operating surplus of companies;
- social security contributions are split into employers', employees' and self-employed contributions. In each case implicit contribution rates are modelled as functions of the official rates;
- indirect taxes are estimated as an aggregate of VAT and excises duties and the taxable base is nominal private consumption and housing investment

Government debt is determined by the government budget constraint which says that debt (GDN) equals previous period debt minus budget surplus (GLN):

$$GDN = GDN_{t-1} - GLN \quad (1.68)$$

## 1.5 Monetary and financial sector

Monetary policy is exogenous to the model so that whatever the outcome of shocks in terms of inflation the monetary policy stance, as measured by the nominal rate, will remain unchanged. There is no role for monetary aggregates in determining prices and output. Monetary policy affects model results through the interplay of interest rates. The model includes a 3-month interest rate, a long term bond rate, a mortgage rate and a rate for credit to companies.

## 1.6 Steady-state

The steady state growth rate of the model can be summarised as follows.

For real variables, define

$$y_{t+1} = (1 + g_y) y_t \quad (1.69)$$

where  $g_y$ , the equilibrium real growth rate of the economy, is derived from differentiating the production function (1.36) with respect to time:

$$g_y = \alpha \left( \frac{Y}{\Lambda_L NH} \right)^p (g_L + g_{\Lambda_L}) + (1 - \alpha) \left( \frac{Y}{\Lambda_K K} \right)^p (g_K + g_{\Lambda_K}) \quad (1.70)$$

which can also be written as

$$\left( \frac{Y}{\Lambda_L NH} \right)^{-p} g_y = \alpha (g_L + g_{\Lambda_L}) + (1 - \alpha) \left( \frac{\Lambda_K K}{\Lambda_L NH} \right)^{-p} (g_K + g_{\Lambda_K}) \quad (1.71)$$

or making use of the production function

$$\left( \frac{Y}{\Lambda_L NH} \right)^{-p} g_y = \alpha (g_L + g_{\Lambda_L}) + \left[ \left( \frac{Y}{\Lambda_L NH} \right)^{-p} - \alpha \right] (g_K + g_{\Lambda_K}) \quad (1.72)$$

Hence

$$g_y = \alpha \left( \frac{Y}{\Lambda_L NH} \right)^p (g_L + g_{\Lambda_L} - g_K - g_{\Lambda_K}) + (g_K + g_{\Lambda_K}) \quad (1.73)$$

Finally, making use of (1.37.a) and denoting the share of wages by  $s_L$ , this simplifies to

$$g_y = \frac{1}{\kappa} s_L (g_L + g_{\Lambda_L} - g_K - g_{\Lambda_K}) + (g_K + g_{\Lambda_K}) \quad (1.74)$$

Along a balanced growth path, if technical progress is purely labour-augmenting ( $g_{\Lambda_k} = 0$ ), the real growth rate of the economy equals the growth rate of labour in efficiency unit provided that the capital stock also grows at that rate.

Rowthorn (1996) also shows that contrary to the Cobb-Douglas technology used in Layard et al. (1991), this condition also affects the unemployment rate in equilibrium. He shows that  $u^*$  evolves through time according to:

$$\frac{du^*}{dt} = 0 \quad \text{if} \quad g_k = (g_L + g_{\Lambda_L} - g_{\Lambda_k}) \quad (1.75)$$

This gives the growth rate of physical capital required to offset the combined effects of labour supply growth and biased technical progress. Unemployment remains constant if capital grows at this rate, also called the "*natural*" rate of growth.

Note that it's the model's focus on consistent expectations that necessitates that more attention be given to equilibrium properties than is the case in traditional macro models. Solving forward looking models requires imposing terminal conditions that pin down agents' expectations beyond the simulation horizon. It is then natural to determine such end-points by making use of the model's steady state growth rates<sup>9</sup>.

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<sup>9</sup> While the steady-state *growth rates* are known and are invariant to shocks affecting the economy - other than shocks affecting directly the steady state growth rate itself-, the steady-state *levels* are conditional to their history in the simulations.



## 1.7 Long run parameterisation

The long run parameterisation is summarised in table 1.

**Table 1 - Long Term Parameters**

|                            | Parameter       | Value   | Source                    |
|----------------------------|-----------------|---------|---------------------------|
| technical progress         | $g_{\Lambda_L}$ | 0.00385 | estimated                 |
| labour supply growth       | $n$             | 0.0013  | sample mean               |
| elasticity of substitution | $\sigma$        | 0.52    | estimated                 |
| labour share in production | $\alpha$        | 0.80    | fixed                     |
| probability of death       | $v$             | 0.035   | fixed                     |
| share of non-traded        | $\Phi$          | 0.41    | sample mean               |
| demand price elasticity    | $\eta$          | 5       | fixed                     |
| gross mark-up              | $1/(\eta - 1)$  | 0.25    |                           |
| share of home goods in:    |                 |         |                           |
| - consumption              | $\delta^C$      | 0.32    | input-output              |
| - business investment      | $\delta^i$      | 0.74    | input-output              |
| - government procurement   | $\delta^G$      | 0.13    | input-output              |
| - exports                  | $\delta^X$      | 0.63    | input-output              |
| inflation                  | inflation       | 0.00475 | fixed                     |
| debt-to-gdp ratio          | $gdn/(yen*4)$   | 0.60    | growth and stability pact |

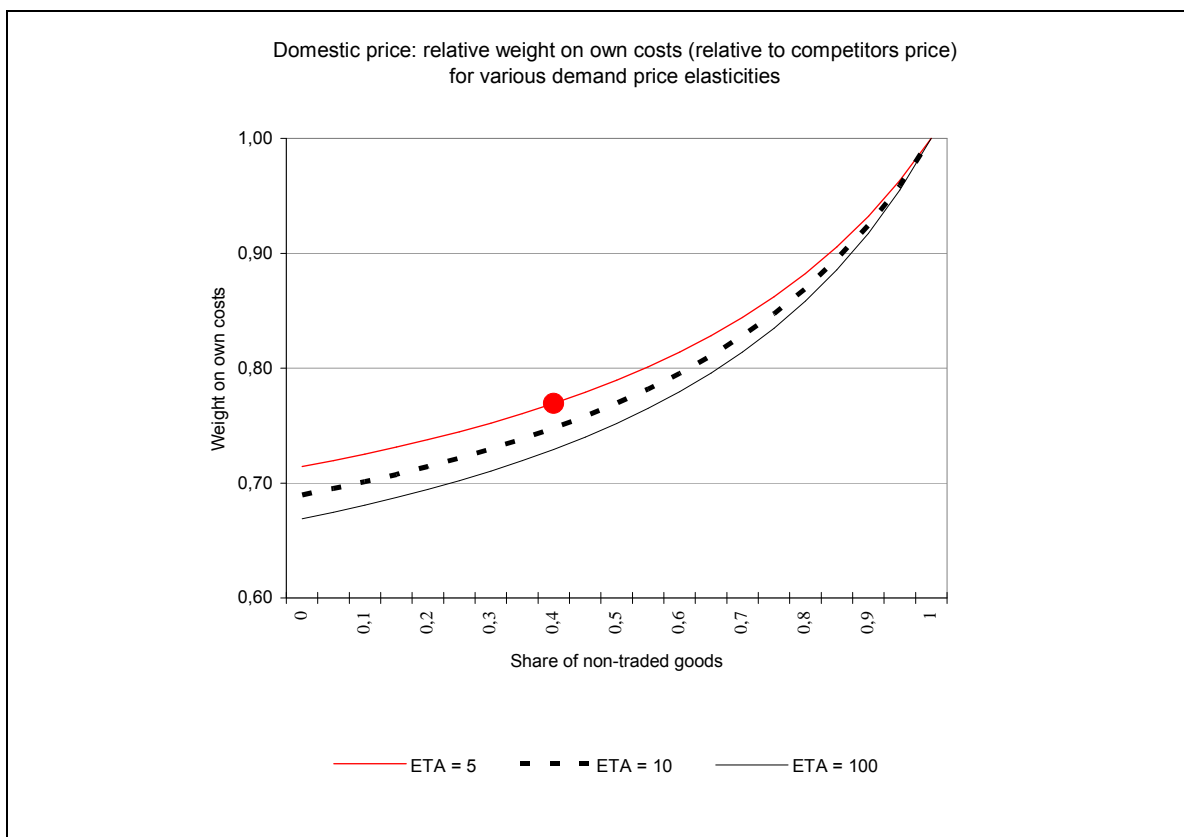
The real growth rate of the economy is given by the estimated labour-saving technical progress augmented with the average rate of growth of the labour supply. The former results from the estimation of the demand for labour and its value is 0.385 p.c. per quarter. For the latter, we assumed that no further reduction in conventional working time will occur and that it equals its average over 1980-2003, i.e. 0.13 p.c. per quarter. In calibrating the steady-state we also assumed that inflation is close to 2 p.c. per year, i.e. 0.475 p.c. per quarter. The elasticity of substitution between labour and capital,  $\sigma$ , comes from the estimation of the demand for production factors and is estimated at 0.52.

In evaluating human wealth from the future stream of labour income, we will use a probability of death of about 3.5 p.c. This is quite high and, when the discount process with the real rate of interest is also taken into account, this means that the first 5 years count for 75 p.c. in the present value sum of future incomes.

From the definition of demand elasticity under symmetry and since in a steady state all firms with the same cost charge the same price and thus have the same market share, the elasticity of demand may be written as  $\eta = 1 + \gamma$  which implies a steady-state mark-up over

unit cost of  $\frac{1}{\gamma}$ . For our selected value of 5 for the demand elasticity, this corresponds to a steady-state mark-up of 25 p.c. Figure 1 shows to what extent the domestic production price is sensitive to the share of non-traded goods for various price elasticities of demand. The dotted line reflects the calibration used in the model. It is clear from this figure that one cannot estimate accurately the demand elasticity from this price equation since a given weight on one's own cost may be compatible with a large range of estimated elasticities. For that reason, we selected a value of 5 which is at the lower bound of the elasticities usually found in the literature.

**Figure 1**



## 2. DYNAMICS

### 2.1 Theoretical considerations

Equilibrium equations are first derived and subjected to coefficient restrictions from static economic theory according to section 1. They take the following form:

$$y_t^* = \theta_0 + \sum_j^p \theta_j Z_{jt} \quad (2.1)$$

where  $y_t^*$  is the decision variable<sup>10</sup> of interest and  $Z_j$  are its  $p$  explanatory variables. These equilibrium paths for the decision targets describe the relationships between variables when all dynamic adjustments have been accomplished.

Of course, the current state of these variables should not necessarily reflect equilibrium at all points in time. It is therefore necessary to embed the equilibrium conditions into dynamic equations describing their law of motion towards these equilibrium paths. Many macroeconomic models incorporate deviations from equilibrium in *unrestricted* error correction equations:

$$\Delta y_t = c_0 - \mu(y_{t-1} - y_{t-1}^*) + a(L)\Delta y_{t-1} + \sum_{j=1}^p b_j(L)\Delta Z_{j,t} \quad (2.2)$$

where  $a(L)$  and  $b(L)$  are *unrestricted* polynomials in the lag operator added arbitrarily. Such equations may deliver nice empirical fits of the data but they are not apt for a coherent analysis of responses by rational agents reacting to news about future events. Indeed, dynamic behaviour does not solely originate from delayed responses due to the costs of adjusting variables, but also from movements induced by changes in agents' expectations about future events. To answer policy related questions appropriately, agents' expectations need to be identified. Therefore, treating expectations explicitly in estimating dynamic equations should permit to identify frictions that impede dynamic adjustments and expectations separately.

As explained in Jeanfils (2000) in order to obtain richer dynamics than the one resulting from quadratic adjustment costs we are using Polynomial Adjustment Costs (PAC). This generalisation of quadratic adjustment costs is due to Tinsley (1993). Only a brief description of his approach is presented hereafter<sup>11</sup>. Consider the following loss function:

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<sup>10</sup> In what follows, the terms 'decision variable', 'target' and 'equilibrium level' are used as equivalents.

<sup>11</sup> For a more exhaustive treatment of what follows, see Tinsley (1993) and Kozicki and Tinsley (1999).

$$C_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ b_0 (y_{t+i} - y_{t+i}^*)^2 + \sum_{k=1}^m b_k ((1-L)^k y_{t+i})^2 \right] \right\} \quad (2.3)$$

The first squared term represents the disequilibrium cost while the second represents adjustment costs and  $\beta$  is a fixed discount factor. This decision rule relaxes the assumption that it is costly to adjust only the *level* of the decision variable ( $k=1$ ) and introduces costs in modifying *differences* in the variable: the rate of growth of  $y$  corresponds to  $k=2$ , the rate of acceleration to  $k=3$ , etc. Minimisation of this loss function yields the Euler equation

$$E_t \left\{ y_t - y_t^* + \sum_{k=1}^m b_k [(1-L)(1-\beta L^{-1})]^k y_t \right\} = 0 \quad (2.4)$$

or with another notation

$$E_t \left\{ \begin{array}{l} (1 - \alpha_1 \beta L^{-1} - \dots - \alpha_m \beta^m L^{-m})(1 - \alpha_1 L - \dots - \alpha_m L) y_t \\ - (1 - \alpha_1 \beta - \dots - \alpha_m \beta^m)(1 - \alpha_1 - \dots - \alpha_m) y_t^* \end{array} \right\} = 0 \quad (2.5)$$

A solution to this Euler equation well-suited for estimation is given by the following decision rule:

$$\Delta y_t = c_0 - A(1)(y_{t-1} - y_{t-1}^*) + \sum_{i=1}^{m-1} a_i^* \Delta y_{t-i} + \sum_{i=0}^{\infty} S_i(\beta, a, m) E_{t-1} \Delta y_{t+i}^* + e_t \quad (2.6)$$

where  $S_i$  is the multiple-root discount factor, which is analogous to the inverse of the unstable root in the case of costs affecting only the level. They are non-linear functions of the discount rate  $\beta$  and of the  $m$  parameters of the lag polynomial  $A(\cdot)$ , written compactly as 'a'. Expectations of changes in the target,  $E_{t-1} \Delta y_{t+i}^*$ , are provided by an auxiliary VAR as in (2.2). Since the extent of these frictions (the size of  $m$ ) is estimated rather than imposed by an a priori choice of a particular adjustment cost function, the empirical goodness-of-fit of the dynamic model equations is far better than those obtained from usual Rational Expectations models and is comparable with time series models. In particular, high residual autocorrelation which is generally present in empirical tests of decision rules based on level adjustment costs, is strongly reduced.

Optimal adjustment today  $\Delta y_t$  depends on three factors: (i) the deviation of last period's level from its equilibrium  $y_{t-1} - y_{t-1}^*$ , (ii) past changes in  $y^{12}$ , (iii) a weighted forecast of future changes in equilibrium or target levels  $\Delta y_{t+i}^*$  for which the forecast weights  $S_i$  are declining in time since they are functions of the discount factor  $\beta$  (i.e. forecasts far in the future are less important than the forecast for tomorrow). It is the introduction of multiple lagged changes in  $y$  that enables to have a better fit for the dynamic behaviour of most macroeconomic variables than fits obtained in former empirical implementations of rational expectations.

## 2.2 Estimation

Estimation of (2.6) requires a three-step process since its coefficients are complicated nonlinear functions of both the parameters in the forecast model and the parameters in the adjustment cost polynomial. First, coefficients in the definition of the targets  $y^*$  are estimated in a cointegration framework or imposed from theoretical restrictions, cfr. Table 1. Then a forecasting VAR model for  $\Delta y^*$  is estimated. And finally the coefficients  $a_i^*$  are estimated. Since the dynamic equation (2.6) is linear in variables, its nonlinear coefficient restrictions present in the forward weights  $S_i$  can be imposed with an iterative Least Squares procedure that, at each iteration, restricts the coefficients in  $S_i$  to values determined by estimates of the adjustment coefficients from the prior estimation<sup>13</sup>. In all cases, the value of  $\beta$  has been fixed to 0.95<sup>14</sup>.

Households' decisions concerning consumption and residential investment and domestic firms' decisions (labour demand, investment, and prices) as well as exporters' and importers' pricing rules have been modelled in the polynomial adjustment costs framework. As an illustration of the results, the firms' pricing decision rule for domestic intermediate goods is given by:

$$\Delta p_{Ht} = \underbrace{-0.122}_{(0.023)} (p_{Ht-1} - p_{Ht-1}^*) + \underbrace{0.676}_{(0.078)} \Delta p_{Ht-1} + E_{t-1} \sum_{i=0}^{\infty} (S_i \Delta p_{Ht+i}^*) \quad (2.7)$$

<sup>12</sup> These lagged terms are not present if agents minimize only the costs associated with changing the level of  $y$  which was the assumption made in earlier applications of rational expectations models as estimated from (42) and (44). The parameter  $a_i^*$  are the coefficients of the lag polynomial  $A^*(L)$  implicitly defined by  $A(L) \equiv 1 - L + A(1)L - A^*(L)(1-L)L$ .

<sup>13</sup> The order of adjustment costs,  $m$ , is chosen empirically by testing for the number of significant lags of the dependent variable in an unrestricted ECM. Note that this procedure does not correct for possible generated regressor bias.

<sup>14</sup> Estimation results are not very sensitive to small variations in  $\beta$ , e.g. from 0.95 to 0.98.

This equation contains a significant error-correction term (standard deviations are given in parentheses) and one lag in output price inflation, meaning that inflation exhibits some persistence. In addition, it is augmented with expectations of the target for which the sum of weights equals 0.355<sup>15</sup>. Grouping all leads gives the following compact notation:

$$\Delta p_{Ht} = -0.122 (p_{Ht-1} - p_{Ht-1}^*) + 0.676 \text{ lags}_1 \Delta p_{Ht-1} + 0.355 \text{ leads}_\infty (E_{t-1} \Delta p_{Ht+i}^*) \quad (2.8)$$

**Table 2 - Compact view of equations**

|                         | Order of adjustment cost (m) | Mean lead of expectations of targets | Mean lag | LR test for REH <sup>1</sup> | Additional dynamics  |
|-------------------------|------------------------------|--------------------------------------|----------|------------------------------|--|
| Households              |                              |                                      |          |                              |  |
| Consumption             | 3                            | 1.7                                  | 1.6      | 0.64                         | Changes in employment                                      |
| Investment in dwellings | 4                            | 1.1                                  | 1.1      | 0.92                         |  |
| Firms                   |                              |                                      |          |                              |  |
| Labour                  | 1                            | 4.0                                  | 5.4      | 0.42                         | Accelerator + cash flow non linear output gap <sup>2</sup> |
| Investment              | 3                            | 4.4                                  | 5.6      | 0.79                         |  |
| Domestic price          | 2                            | 2.2                                  | 2.0      | 1.00                         |  |
| Exporters and importers |                              |                                      |          |                              |  |
| Export price            | 2                            | 2.1                                  | 2.5      | 0.03                         | non linear output gap <sup>2</sup>                         |
| Import price            | 1                            | 1.7                                  | 1.9      | 0.01                         | non linear output gap <sup>2</sup>                         |

<sup>1</sup> LR test (p-value) of excluding Var determinants of expected target changes,  $\Delta y^{*e}$ . A p-value of 0.05 or less indicates a rejection of REH restrictions with at least a 95 p.c. level of confidence.

<sup>2</sup> defined as the ratio of actual output growth to the steady state rate of growth.

Table 2 summarises the results obtained for the seven equations mentioned above. Column 1 gives the order of adjustment costs, m, ranging from 1 to 4. Column 2 reports the mean lead of expectations of the targets. This is a compact measure of how far ahead agents tend to look as well as how quickly a variable adjusts to expected future events. In principle, agents plan over an infinite future, but the effective length of the planning period is determined by the extent of the frictions. Actually, a quick adjustment is associated with a short expectation horizon. Column 3 gives the mean lag of the series. This is a measure of the average speed of response to past events. As shown in this table, consumption, housing investment and prices exhibit mean leads and lags of two quarters or less apparently reflecting the ability of households and price makers to adjust their decisions variables quickly. Labour shows that a low order in a variable's polynomial adjustment cost function does not necessarily imply a low mean lead or lag of its series. Column 4 of table 2 provides a Likelihood Ratio test of the rational expectations over-identifying

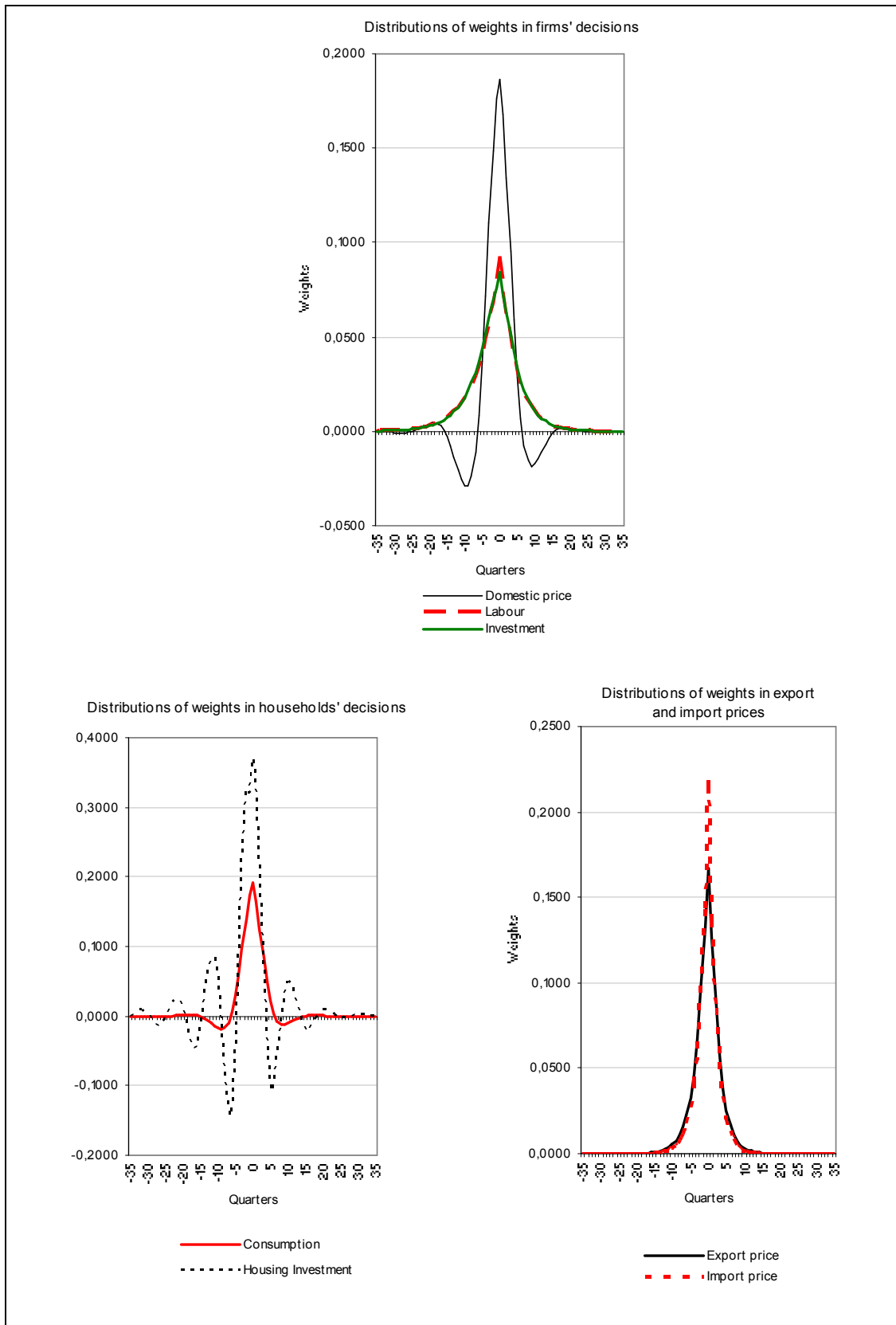
<sup>15</sup> Some diagnostic tests for the dynamic specification of the equations are given in appendix A2.

restrictions on the coefficients of the agents' VAR forecast model. If the additional regressors are statistically significant, it implies that the p-values are low which means that households or firms do not have rational expectations as defined by the VAR's forecasts in their dynamic adjustment equations. As shown by their p-values, these restrictions are, with the exception of import and export prices, never rejected at conventional levels of significance<sup>16</sup>.

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<sup>16</sup> In the unrestricted equation used in the LR test, the same lags of the variables included in the VAR are introduced as additional regressors.

Figures 2, 3 and 4





Figures 2, 3 and 4 present the distributions of weights, i.e. the contribution of past and future expected changes in the target on current decisions. The optimal current level of a variable,  $x$ , given the presence of frictions can be represented as a two-sided moving average of past and future target values,  $x^*$ :  $x_t = E_{t-1} \sum_{i=-\infty}^{\infty} \omega_i x_{t+i}^*$ . The weights depicted in figures 2, 3 and 4 sum to one for each variable. Weights for past quarters, shown to the left of the peak, indicate the importance of past equilibrium levels to current decisions. Since, as the quarters go by, older plans are revised or reach completion, the weights for past planning periods tend to zero. In the same way, the weights of future targets shown to the right of the peak diminish with the time horizon because of discounting and because more distant plans can be corrected if necessary by taking other measures in future quarters. Since long mean lead and lag are reflected in a rather flat curve, two types of weights distributions appear. One shape, exemplified by labour, investment and domestic output price, tends to be relatively flat, reflecting a strong influence of planning considerations in the evolution of the variables. The other shape which is more concentrated around the peak concerns households, exporters and importers. In particular, import price and housing investment react very quickly and there is a dampening cycle in the evolution of the latter series.

### 2.3 Wage Dynamics

In section 1.3, the optimal wage rate for a given period has been derived. However, wage contracts are not set continuously because, once signed, they will not be revised for several periods. In order to reflect this feature, we follow a simplified version of Dotsey, King and Wolman (1999) state dependent pricing model which boils down to a time dependent formulation. Assume that wages, once bargained, are set for up to a maximum time period  $J$  ( $J > 1$ ). This is different from Taylor's staggered prices (1980) in which wages are set for exactly  $J$  periods and from Calvo's model (1983) in which  $J$  is infinite. At the start of each period, there is a fraction  $\theta_{jt}$  ( $j = 1, 2, \dots, J$ ) of each vintage of contracts which has not been adjusted for  $j$  periods and thus remains equal to  $w_{t-j}^{C,*}$ . Let  $\alpha_i$  denote the probability that a contract is adjusted conditional on having remained in effect for  $i$  periods. The probability of non-adjustment is thus  $1 - \alpha_i$ . The total fraction of wages which is adjusted is equal to  $\omega_{0t} = \sum_{i=1}^J \alpha_i \theta_{it}$  and comparably a fraction  $\omega_{jt} = (1 - \alpha_j) \theta_{jt}$  in each

category  $j = 1, 2, \dots, J$  which remains equal to the wage bargained  $j$  periods ago. These fractions are related to the start-of-period fractions by

$$\theta_{j+1,t+1} = \omega_{jt} \quad \text{for } j = 0, 1, \dots, J-1 \quad (2.9)$$

so that

$$\omega_{jt} = (1 - \alpha_j) \omega_{j-1,t-1} \quad \text{for } j = 1, \dots, J-1 \quad (2.10)$$

Since all wages must be into one of the categories in terms of time span since their last change:

$$\omega_{0t} = 1 - \sum_{i=1}^{J-1} \omega_{it} \quad (2.11)$$

Then a fixed weight aggregate wage index can be calculated as

$$\left[ \sum_{j=1}^J \alpha_j \theta_j \right] w_t^* + \sum_{j=1}^J (1 - \alpha_j) \theta_j w_{t-j}^* \quad (2.12)$$

The average contract length is given by

$$D = \sum_{j=1}^J \alpha_j \cdot \prod_{h=0}^{j-1} (1 - \alpha_h) \cdot j \quad (2.13)$$

Note that in a model à la Calvo the proportion of wage contracts that are modified each period is  $\alpha_j = \alpha, \forall j$  and  $J \rightarrow \infty$ . The fraction of contracts that have not been adjusted is  $\Omega_j = \alpha (1 - \alpha)^{j-1}$  and is decreasing in  $j$  (i.e. there will be a maximum of contracts changed after one period).

The estimation of the wage equation is done in two steps. First, we estimate a "flexible" version of the log-linearised optimal wage equation and then the dynamics. For this second step, one needs to choose a maximum contract length  $J$  prior to estimation. In addition, one also needs to estimate the  $J$  conditional wage change probabilities,  $\alpha_j$ . This may be simplified by adopting the same non-linear functional form as Murchinson et al. (2004) which allows one to reduce the number of free parameters:

$$\alpha_j = \left( \frac{1}{1 + \chi} \right)^{J-j} \quad \chi > 0; \quad j = 1, 2, \dots, J-1 \quad (2.14)$$

where  $\chi$  is a (positive) parameter to be estimated and it ensures that the conditional probability of changing wage is increasing with time since the last wage change at an increasing rate ( $\partial \alpha_j / \partial j > 0, \partial^2 \alpha_j / \partial j^2 > 0$ ).

Note that wages are bargained on a real gross basis and that the government often intervenes to affect wage costs by changing  $t_{w1}$ , the employers' social security contribution rate. In addition, there is automatic ex-post indexation that comes in addition to the bargained wage. The resulting wage index can be written as:

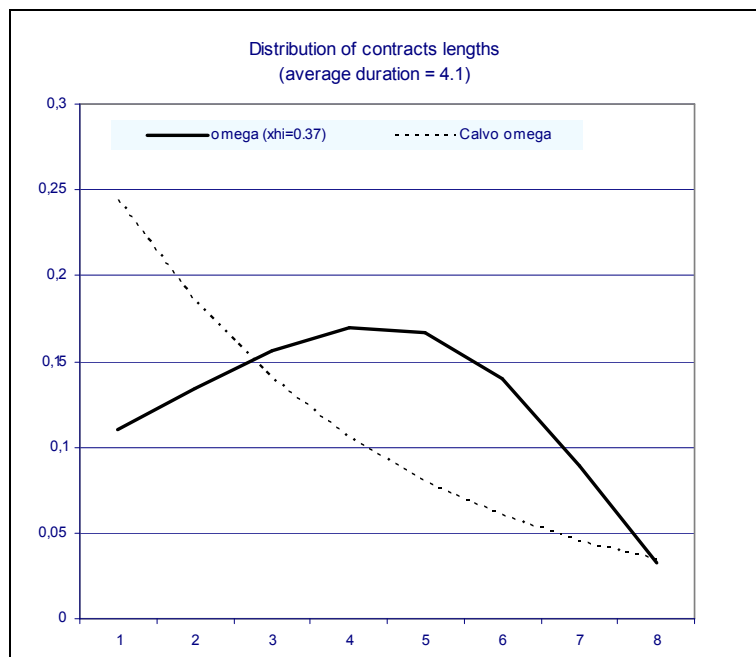
$$\left[ \sum_{j=1}^J \alpha_j \theta_j \right] w_t^* + \sum_{j=1}^{J-1} \omega_j w_{t-j}^* \left[ 1 + \sum_{q=0}^{j-1} \Delta p_{t-q} + j \cdot g_{\Lambda_L} \right] \quad (2.15)$$

The average contract length in (2.13) can be written as

$$D = \sum_{j=1}^J \omega_j \cdot j \quad (2.16)$$

We choose a maximum contract length,  $J$ , of 8 quarters and the resulting estimated  $\chi$  is then equal to 0.37. This gives an average contract length,  $D$ , of 4.1. Figure 5 shows the distribution of the contract lengths represented by the  $\omega$ 's in (2.16) and compares it to the distribution of Calvo contracts, that results in the same average contract length ( $\alpha = 0.24$ ).

**Figure 5**



## 2.4 Price Dynamics

Apart from domestic intermediate goods, exports and imports prices that result from dynamic optimisation, the dynamics of consumption, investment deflators and the price of new dwellings are all backward-looking and dynamic homogeneity has been imposed in all cases.

## 2.5 Income accounts

The income account in the model is consistent with the ESA95 accounting scheme. This structure guarantees that no primary or redistributed income is lost along the way and that all disposable sector income that is not consumed or invested gets accumulated in an appropriate wealth concept. Consequently, sector wealth can never develop a life on its own. All wealth accrual reflects an income or spending behaviour that has found its way through this accounting scheme at some point in time.

Sector income items that are not really important in magnitude, and/or where close to no information is available as to their explanation, have been regrouped in a residual item per sector. (As can be seen in table 3: OPN for households; OGN for the government sector; companies: see infra). For database purposes, this item has indeed been calculated residually as the difference between disposable sector income and the other endogenous items for that sector in the primary and secondary income account. During the projection process they are related to some aggregate concept (a fixed ratio to GDP, total wages, etc).

GDP calculated along the income or expenditures lines should of course lead to the same result. Given this equality and given the fact that the expenditures are completely accounted for in behavioural equations elsewhere in the model, at least one element in the income grid will be calculated residually. We choose it to be the firms' net operating surplus (NON).

Further constraints can be imposed inside the logic of this accounting grid. Indeed, primary income from wealth sums to zero over all sectors, which makes one of them redundant. Once again this happened in the firms' sector. Also, all re-distributional income should sum to zero over all sectors. Firms' "other income" will therefore equal the sum of all other items in the re-distributional segment of the income grid. Given that all (but the residual) items in this sub-grid have a counterpart in another sector, one can easily state the constraint for

firms' residual transfers. It equals the sum of the other sectors' residual transfers (TWN-OGN-OPN).

The transition from disposable income to financial surplus is then, for the most part, merely an accounting activity: apart from consumption, investment and changes in inventory (explained elsewhere) only exogenous elements intervene like mathematical pension reserves, capital transfers and depreciation.

Using all this information, the complete income account can then be broken down as indicated in Table 3.

Finally, net foreign assets (NFA) result from the accumulation of current account balances:

$$\text{NFA} = \text{NFA}_{t-1} + \text{XTN} - \text{MTN} + \text{TWN} + \text{NFN} \quad (2.17)$$

where XTN, MTN are export and import values respectively, TWN represents capital transfers and NFN net factor incomes.

**Table 3**

|                                |                                | Gross Domestic Product |           |              |                |
|--------------------------------|--------------------------------|------------------------|-----------|--------------|----------------|
|                                |                                | National concept: GNI  |           |              | Income ROW     |
|                                |                                | FIRMS<br>(S11/S12)     | GOV (S13) | HH (S14/S15) | ROW (S2)       |
| P7                             | imports                        |                        |           |              | MTN            |
| P6                             | exports                        |                        |           |              | -XTN           |
|                                | goods & serv. balance          |                        |           |              | -(XTN - MTN)   |
| <b>PRIMARY INCOME</b>          |                                |                        |           |              |                |
| B2n                            | net operating surplus          |                        |           |              |                |
| B3n                            | mixed income                   |                        |           | OPNI         |                |
| D1                             | wages                          |                        |           | WIN1         | WAN            |
| D2                             | indirect taxes                 |                        | TIN       |              | EUN            |
| D3                             | subsidies                      |                        | SUBS1     |              | SUBS2          |
| D4                             | income from wealth             | -NFN_INT + INN         | -INN      |              | NFN_INT        |
| <b>REDISTRIBUTIONAL INCOME</b> |                                |                        |           |              |                |
| D5                             | income taxes                   |                        |           |              |                |
|                                | direct taxes firms             | -OTN                   | OTN       |              |                |
|                                | direct taxes HH                |                        | PTN       | -PTN         |                |
| D6                             | soc.sec. HH w.r.t. Gov.        |                        |           |              |                |
|                                | personal soc.sec. contr.       |                        | PIN       | -PIN         |                |
|                                | employers' soc.sec. contr.     |                        | OSN       | -OSN         |                |
|                                | transfers from Gov to HH       |                        |           |              |                |
|                                | unemployment benefits          |                        | -TRN_UNN  | TRN_UNN      |                |
|                                | other trf. from Gov to HH      |                        | -TRN_EXO2 | TRN_EXO2     |                |
|                                | for simulation purposes only   |                        |           |              |                |
|                                | trf. resulting from fisc. rule |                        | -TRN_FR   | TRN_FR       |                |
| D7                             | other current transfers        | TWN-OGN-OPN            |           |              | -TWN           |
| B6n                            | net disposable income          | OYN                    | GYN       | PYN          | -CAN           |
| D8                             | + math. pension reserves       | -MPN                   |           | MPN          |                |
| P3                             | - consumption                  |                        | -GCN      | -PCN         |                |
| D9                             | + capital transfers'           | CON                    | CGN       | CPN          | -(CON+CGN+CPN) |
| P51                            | - investment                   | -OIN                   | -GIN      | -HIN         |                |
| P52                            | - changes in inventory         | -SCN                   |           |              |                |
| K1                             | + depreciation                 | OKN                    | GKN       | PKN          |                |
| K2                             | - non-prod. non-fin. assets    | -GLN_RES               | GLN_RES   |              |                |
| B9                             | financial surplus              |                        | GLN       | PLN          | S2LN           |

## 2.6 An illustration of the role of expectations

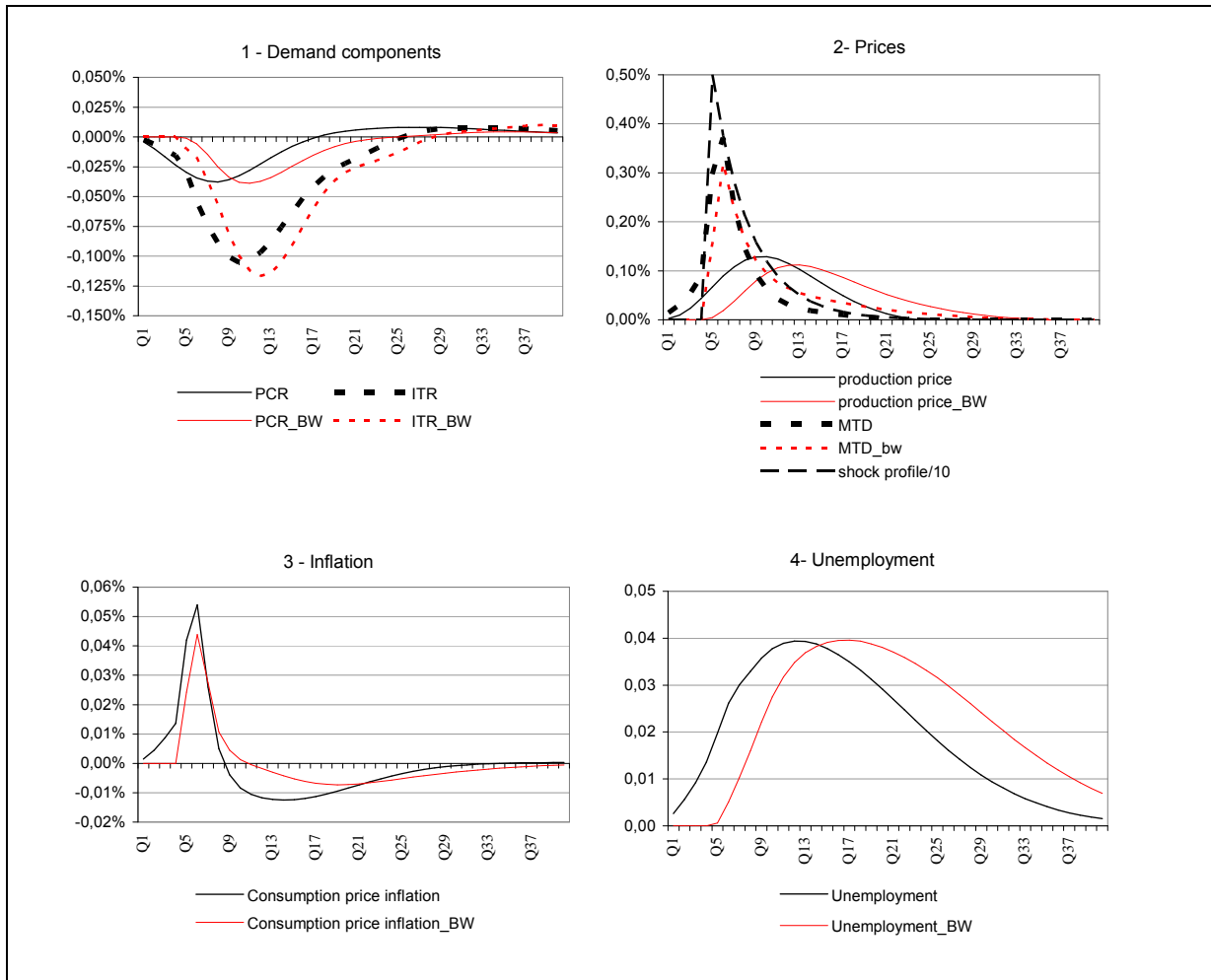
The clear delineation of expectations allows simulating the model under different expectations formations<sup>17</sup>. This is illustrated here by a very simple experiment conducted under model-consistent rationality and under limited information (confined to predetermined variables) information. The model is first solved under full model consistent expectations, referred as forward-looking expectations. This option goes some way towards addressing the Lucas critique, but it does not make the model immune to it. However, one would admit that the use of fully model-consistent expectations in policy simulations assumes too much knowledge by private agents. Alternatively, expectations are assumed to be based on the small VAR's used in estimation, i.e. according to the same limited information auxiliary forecasting models as those used by the econometrician referred as backward-looking expectations. In period  $q$ , it is announced that the import price of energy will be increased in period  $q+4$ . This increase will be temporary but with some persistence, whereby the shock gradually dies out with a decay rate of 0.75. Figures 6.1 to 6.4 compare both experiments. Results, in all instances, are displayed as deviations from a baseline. Results under model-consistent expectations are depicted by bold lines while backward-looking outcomes by thin ones.

While, in the long-run, this temporary price shock does not affect either real or nominal variables, the short-term impulse responses are quite different under both types of expectations. The first difference between both expectations schemes concerns, of course, the periods preceding the shock. Forward-looking agents anticipate the shock and its consequences right from its announcement and start reacting in advance, from  $q$  to  $q+3$ , whereas backward-looking agents are surprised by the shock, each quarter again, as long as the shock is taking place, so that no reactions occur before the shock effectively materialises.

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<sup>17</sup> Noname is coded in Troll and uses the "Stacked Time" algorithm to solve out the forward-looking solutions.

Figures 6.1, 6.2, 6.3 and 6.4



PCR: consumption, ITR: investment, MTD: import deflator

On the price side there are no major differences, except that domestic production price increases somewhat less and more slowly under backward-looking expectations. In this case, while setting prices, firms do not anticipate future increases in their marginal cost resulting from future nominal wage increase.

Private consumption falls more quickly under forward-looking mode since, in this instance, households expect their future real labour income to fall due to both an increase in unemployment and to an increase in the consumption price which, since energy products are excluded from the wage indexation mechanism, leads to a reduction of their purchasing power. Expectations of reduced future income are reflected in a reduction of human wealth, the main determinant of consumption. In the backward-looking case,



human wealth, the main determinant of consumption. In the backward-looking case, human wealth only decreases gradually as households observe the reduction in their current income (they fail to recognise immediately the size and the persistence of this reduction) so that, in this framework, aggregate consumption decreases mainly as a consequence of the reduction in employment. Investment decreases more quickly under forward-looking expectations since the future contractions of output are taken into account and thereafter it recovers somewhat more rapidly in expectation of the subsequent recovery of output.

Figure 6.4 highlights the sluggishness in the response of unemployment which returns to base more slowly than output. This is, in some way, the mirror image of the long lead and lag found in the employment equation.

### **3. DIAGNOSTIC SIMULATIONS**

#### **3.1 Preliminary remarks**

In the model, exchange rates and the short term interest rate are treated exogenously. Endogenisation of these variables would require an analysis of output, price and interest rate determination at the eurozone level. When such an analysis is necessary, we can use a DSGE model for the euroarea, such as Smets and Wouters (2003). Therefore, all the simulations presented in this paper are conditional on the assumption of constant short and long term nominal interest rates. The purpose of the following simulations is to investigate model properties. They are not intended to provide a comprehensive account of the model's multipliers. All the diagnostic simulations presented have been run for a period of 25 years<sup>18</sup>. The choice for such a long simulation period is dictated by two requirements. First, solving models under model-consistent expectations requires a terminal date sufficiently far into the future to avoid the simulation results to be affected by the choice of the terminal date. Second, simulating the model over a long period allows an inspection of the long run solution of the model. Each benchmark simulation exercise has been performed with the fiscal rule switched off over the entire period. Simulation results are reported relative to a base constructed over the future which is residual-free. This means that no add-factors are added and the baseline scenario is not calibrated to match observed data in the first quarters. As such, the base does not represent a projection in the usual meaning of the word.

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<sup>18</sup> However for readability reasons, the charts hereafter only show the first 10 years.

The model has been estimated on an equation by equation basis. As already mentioned, for Polynomial Adjustment Costs equations, this approach gives the flexibility to introduce various types of expectation formation and in that way offers an easy alternative to the model-consistent expectations scheme. A system estimation would however provide probability distributions for the parameters and allow us to generate probability distributions for simulated values of the variables. Such a system estimation would be difficult if not unfeasible with the size of our model. Since we cannot get this full characterisation of parameter uncertainty, we have conducted simple numerical experiments to explore whether the magnitudes of some relevant parameters are important for the ultimate effect of the shocks. Tables 4-a to 4-d in appendix A1 investigate how the results discussed in the next sub-sections are dependent on our benchmark parameterisation. It separately examines the repercussions on the simulations outcomes of:

- an increase in the elasticity of consumption w.r.t. financial wealth from 5 p.c. to 10 p.c. to the detriment of human wealth;
- variations of 20 p.c. in the price elasticity so that the domestic and export price elasticities vary from 4 to 6;
- variations in the dynamic coefficient of the short term impact of changes in employment on consumption from 0.25 to 0.6<sup>19</sup>;
- variations of coefficient of the cash-flow in the short term investment equation from 0.2 to 0.55;
- the introduction of the fiscal rule from the beginning of the fifth year after the shock onwards;

the remaining parameters being left unchanged.

This sensitivity analysis demonstrates that the results that follow are not too over-sensitive to these perturbations. However, the range of the distribution of the medium and long-run impact of consumption is large for the foreign demand shock. In this case, the distribution is skewed towards the minimum: the maximum effect is far above the average one because these permanent shocks have clear positive (i.e. downwards) effects on the public debt. When experiments are conducted with a fiscal rule, these positive effects are redistributed in the form of lump-sum transfers to households. These transfers increase their disposable income and, since the shock is permanent, they also increase their human wealth, being private consumption's main determinant.

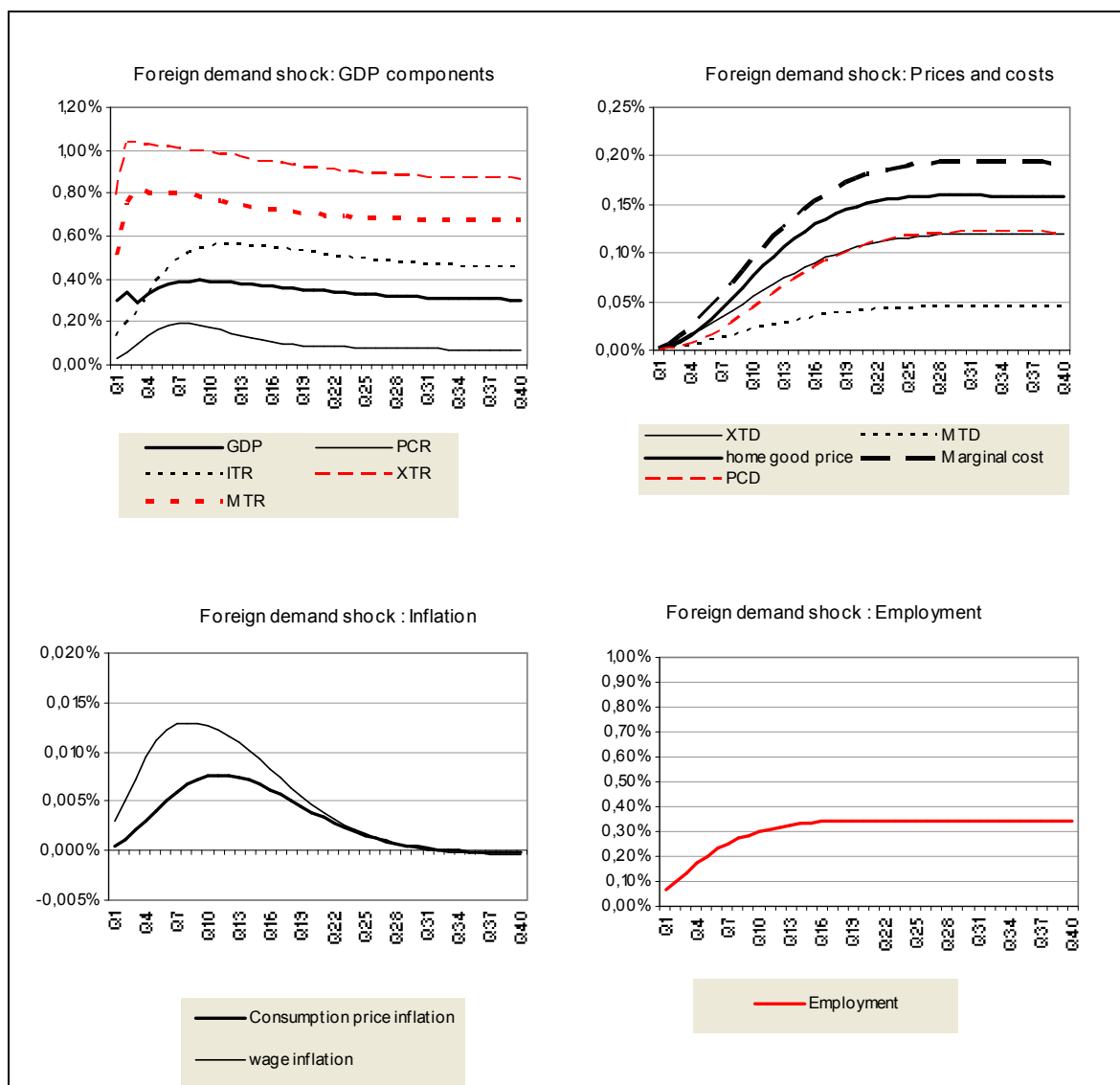
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<sup>19</sup> When this coefficient is set at 0.6, the model does not converge in the case of the labour supply shock.

### **3.2 A foreign demand shock**

This shock is characterized by a 1 p.c. permanent expansion of Belgian real export markets, both at the extra and intra euro-area levels. It has been imposed that this increase in foreign demand leaves prices inside and outside the eurozone unaffected. In response, Belgian exports increase by almost 1 p.c. which raises GDP by around 0.4 p.c. The improvement in the trade balance is limited through the large import content of exported goods. The major domestic change concerns investment which positively reacts to output as well as to improved profitability. After five years consumption increases in line with the increase in human wealth. However, in the short run, during the job creation period, consumption overshoots its long run level. The reduction of unemployment allows wages to increase, which puts upwards pressure on marginal costs as well as on prices. As nominal rates are fixed and inflation increases somewhat ex-post, the real interest rates decline. This also contributes to sustain consumption and investment. Note that this increase in domestic prices and the associated deterioration in competitiveness is responsible for the fact that exports grow less than export markets in the medium term.

Figures 7a, 7b, 7c and 7d



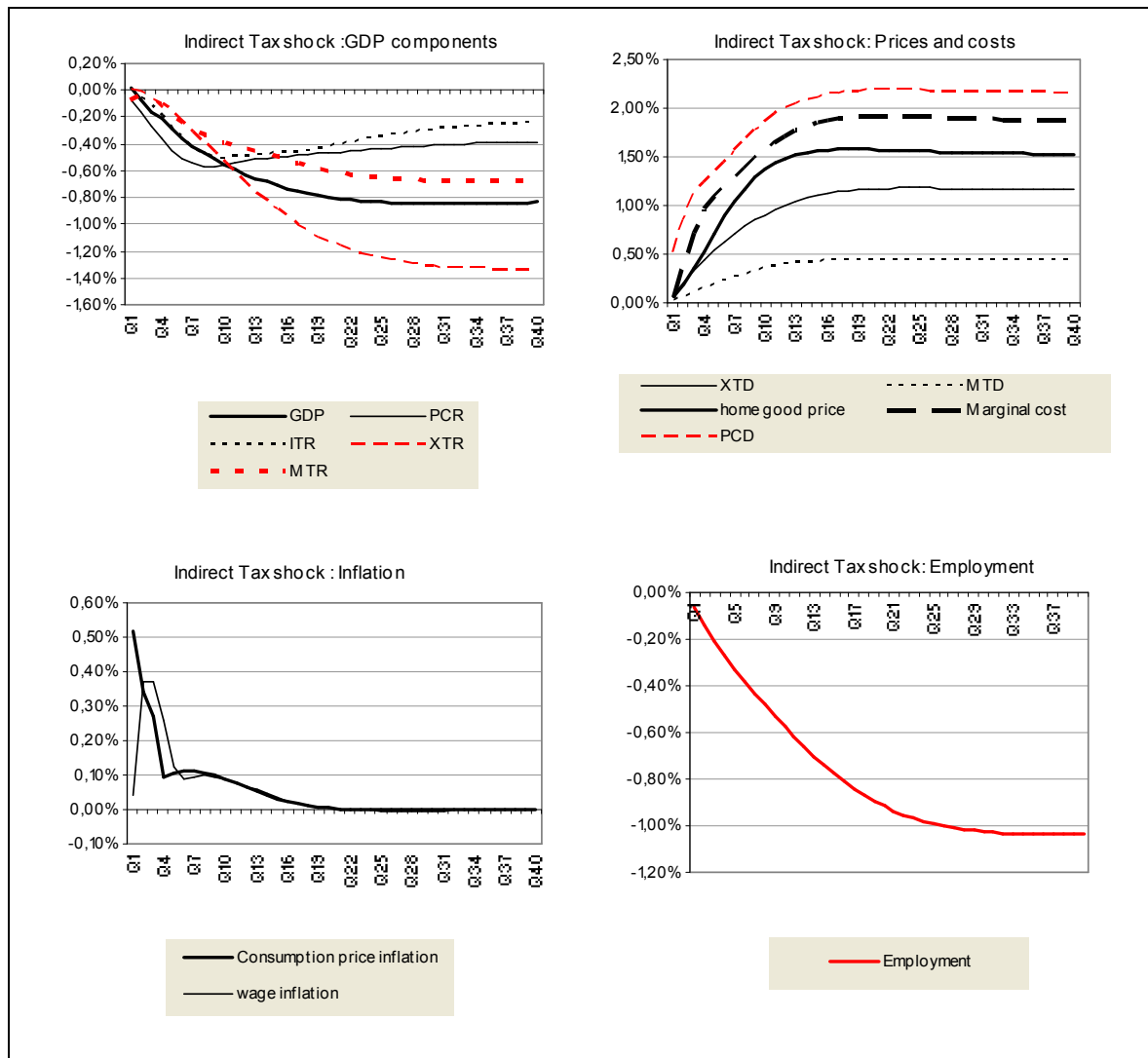
PCR: consumption, ITR: investment, XTR: exports, MTR: imports, XTD: export deflator, MTD: import deflator

### 3.3 An indirect tax shock

The average VAT rate is permanently increased by 1ppt. The transmission from changes in VAT rates to consumer prices is not immediate. Indeed, many prices are labelled VAT included and there may be menu costs to changing them. Besides, even if it is true for individual prices labelled exclusive VAT that a 1 ppt. increase in VAT rate should lead on impact to a 1 p.c. increase in the selling prices, at the aggregate macro level, it may not be the case. Since such measures are generally announced well in advance and are subject to legislative lags, consumers may bring their expenditures plans forward (in case of a VAT increase) or delay them (in case of a decrease). In the real world, there is a variety of VAT

rates and, most of the time changes to them have not been applied uniformly but rather differently to individual rates. Consequently, shocks to some rates may not lead to an immediate one-for-one impact on the consumption deflator because their weights change. For all these reasons, transmission lags are difficult to estimate empirically and there is uncertainty surrounding them. Therefore, we have fixed them as a 3-quarter distributive lag with coefficients 0.5, 0.3, 0.2.

**Figures 8a, 8b, 8c and 8d**



PCR: consumption, ITR: investment, XTR: exports, MTR: imports, XTD: export deflator, MTD: import deflator

This being said, such a measure widens the gap between output price "at factors cost" and the private consumption deflator "at market price". Given the lag structure, the latter increases on impact by 0.5 p.c. and, after 3 quarters, the pass-through is complete while the former has only changed by 0.3 p.c. within this time horizon. Thereafter, as wages are

automatically indexed with some lags to a health index (some variant of the consumer price index), marginal cost starts increasing which, in turn, pushes output price up and gives rise to a wage-price spiral. In this scenario, since the impulse stems from the consumer price, the output price is lagging: during the first 4 years, consumer price increases each successive year by 0.9, 0.6, 0.4, 0.2 on average while output price changes by 0.3, 0.7, 0.4, 0.2. Note that the pricing-to-market behaviour of importers appears clearly here since the impulse from a pure domestic price shock is partly transmitted to the price of imported tradable goods and services as shown by MTD.

On the domestic side, the main direct real impact is a reduction in private consumption caused by a lower real labour income. The decrease in real labour income stems firstly from a price effect due to a delayed indexation of wages and social benefits and thereafter from an employment effect. The latter is the consequence of the increase in nominal wages as compared to the producer price. From the second year on, investment also falls in reaction to a lower output level.

Foreign sales are negatively affected by the loss of competitiveness. While the contribution of domestic demand to output contraction in the first four years is larger than that of the trade balance, the latter dominates thereafter.

All in all, there is an improvement in the government net financing capacity of some 0.4 p.c. of GDP in the first year while as unemployment increase this surplus reduce to 0.2 p.c. thereafter.

### **3.4 A labour supply shock**

In this simulation, labour supply is permanently increased by 1 p.c. which corresponds to an immigration of some 47,000 new workers at the beginning of the simulation. These immigrants are supposed to be immediately entitled to the "average" unemployment benefit<sup>20</sup>.

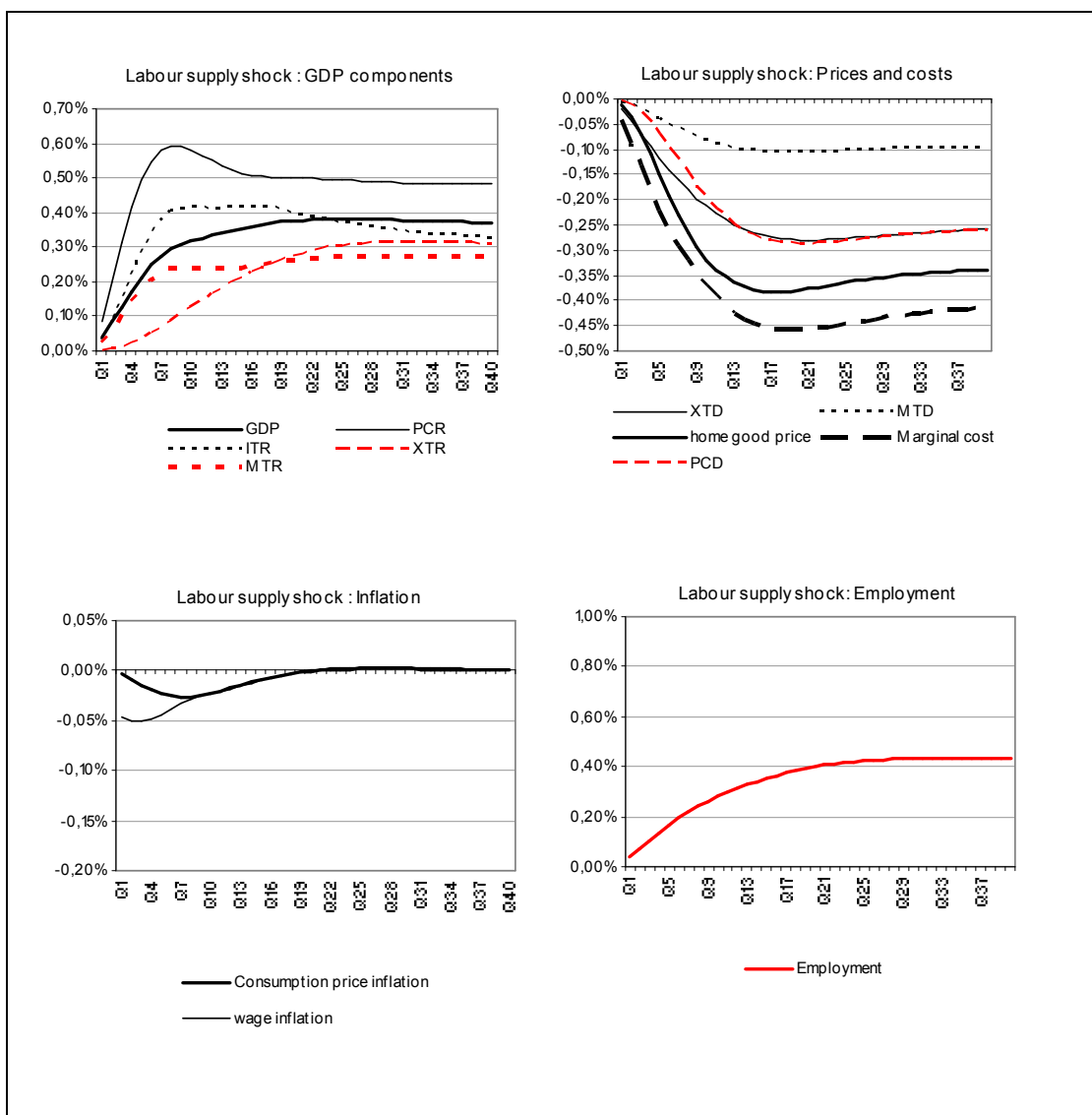
Following this shock, unemployment immediately rises by 0.9 ppt. Since the proxy used for the perceived risk concerning future expected income is the change in employment, such an increase in unemployment does not exert a discouraging effect on consumption.

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<sup>20</sup> This average unemployment benefit is exogenous in real terms and is calculated as the total amount of unemployment benefit expenditures divided by the number of "paid" unemployed.

Moreover disposable income mechanically increases by the amount of the transfer from the government to the new unemployment benefit recipients. This boosts aggregate consumption which, in turn, stimulates output and investment. Such an increase in domestic absorption leads to a deterioration of the trade balance even when competitiveness gains in both the import and the export markets show up. The increase in employment resulting from a higher output and a lower real "producer" wage falls short of the initial jump in labour supply. This stems from the lack of sufficient flexibility in wage formation which prevents wages from falling enough to absorb effectively the whole increase in labour supply. As a result, the unemployment rate rises permanently by some 0.6 p.c. This result is somewhat higher than that reported by Karanassou, Sala and Snower (2004) who obtain an increase of 0.42 p.c. for an average of 11 EU countries (including Belgium). Finally, government net financing capacity deteriorates by 0.2 p.c. of GDP.

Figures 9a, 9b, 9c and 9d



PCR: consumption, ITR: investment, XTR: exports, MTR: imports, XTD: export deflator, MTD: import deflator

### 3.5 An oil price shock

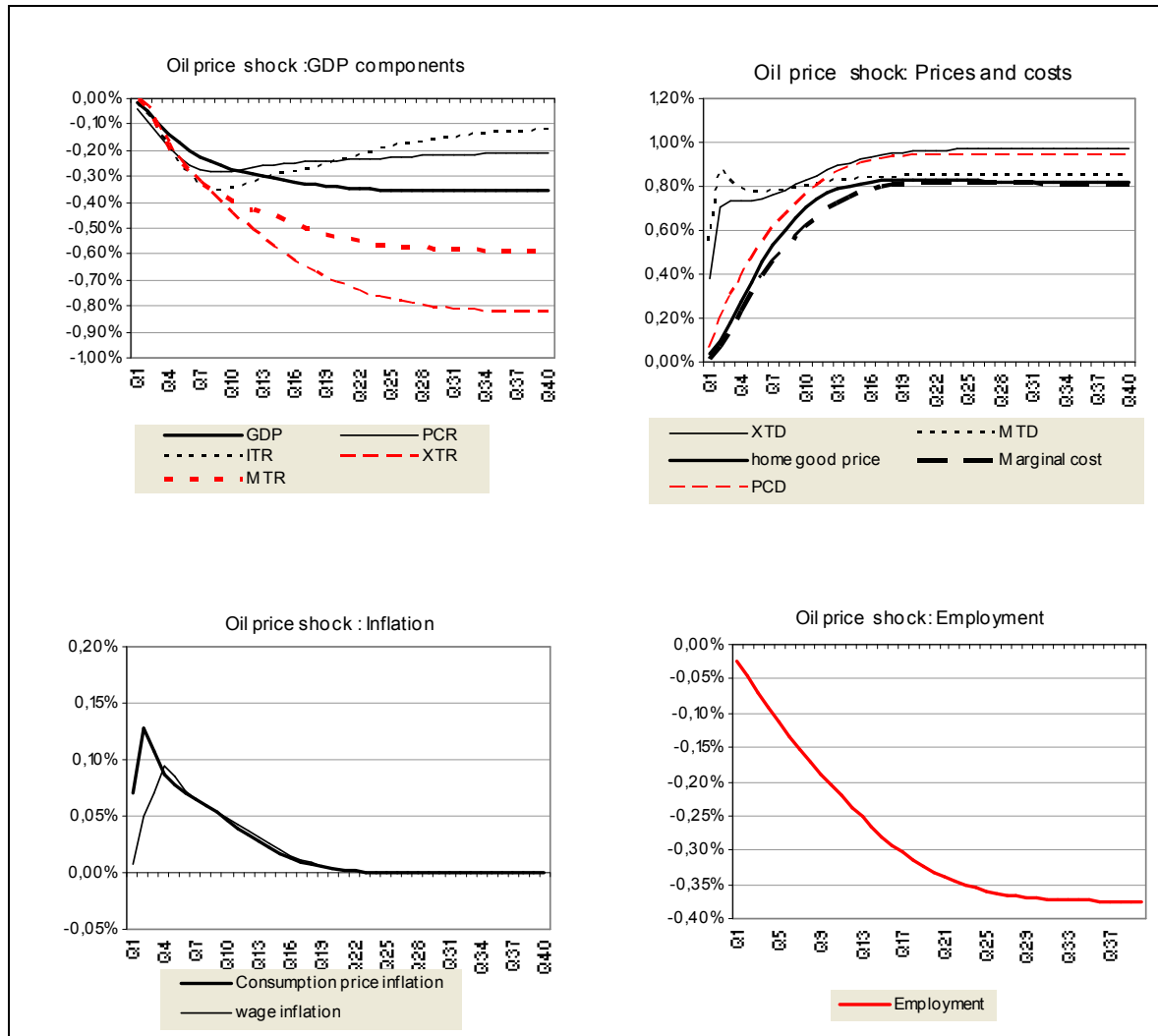
In this exercise, we analyse the effect of a **permanent** 10 p.c. increase in the price of crude oil expressed in euro.

Following this price shock, there is a quick and sustained increase in import prices which rise by almost 0.8%. Since imported items enter the private consumption directly also, its deflator also increases by around 0.25% in year one. Higher import prices also push up intermediate goods producers' price, initiating a cost-price spiral. As a result the consumption deflator increases by 0.9% in the long-run. The shock also affects export



prices but during the first year to a lesser extent than import prices, so that one observes some deterioration of the terms of trade at that horizon.

**Figures 10a, 10b, 10c and 10d**



PCR: consumption, ITR: investment, XTR: exports, MTR: imports, XTD: export deflator, MTD: import deflator

On the real side, there is a loss of GDP resulting mainly from the reduction of private consumption and exports. The former stems from decline in real disposable income, resulting in turn from both a deterioration of the labour market conditions and a loss of terms of trade for wage earners. Indeed, according to institutional arrangements, wages are indexed according to a consumer price index that excludes energy and tobacco products (characterised by high excise duties). This real "consumption" wage fall is further magnified by the presence of a lagged indexation which makes wages adjust more slowly than prices. The weaker demand addressed to firms reduces their investment needs and

the effect the price of energy exerts on costs in the building industry, combined with the decrease in households' real disposable income, causes a fall in housing investment. After three years, investment recovers somewhat in so far that firms are now able to improve their financing capacities. As time passes wage inflation catches up with consumer price and employment stops deteriorating, allowing consumption to stabilise, be it still below base. The main driving force behind the reduction in real output in the long run is the drop in exports following a loss of external competitiveness. Note that in reality such an oil price shock is likely to increase also competitors' prices reducing thereby the effect on competitiveness.

#### **4. CONCLUDING REMARKS**

In this paper, we depicted the core structure of the small open economy quarterly model of the National Bank of Belgium.

The goal was to build a model that is able to provide quantitative input into the policy analysis and projection processes with a framework that has explicit micro-foundations and expectations. Since a model as Noname obeys economic theory not only in the long-term but, for many variables, also in the dynamic adjustment process, its responses are easier to understand from an economic point of view than those of traditional backward-looking models with arbitrarily added unrestricted polynomials in the lag operator that nicely fit the data. We also believe that it is therefore more suited in the context of a coherent analysis of responses by rational agents reacting to news about future events than was the case in traditional econometric models. Another element of the strategy involves the decision to maintain the model relatively compact to keep the structure simple enough for projections and simulation purposes.

Some key differences between our chosen setup and that typically found in the literature have been focused on:

- Noname can be operated under different assumptions concerning expectation formation and whatever the option used it converges to the same long-run solution;
- In order to rationalise the introduction of dynamics, the intertemporal optimisation problem of households and firms is subjected to costs related to the adjustment of decision variables. Noname, as FRB/US, makes use of a richer dynamic specification than the quadratic cost function encountered in former testing of the rational

expectations hypothesis. Such richer dynamics are introduced by means of Polynomial Adjustment Costs (PAC);

- We have pricing-to-market and hence flexible mark-ups and incomplete pass-through even in equilibrium thanks to "translog" preferences rather than the usual and easier to work with "CES" aggregator which generates incomplete pass-through only along the transition path;
- A CES production function characterises the data better than the previous Cobb-Douglas production function, and it generates a below one elasticity of substitution between capital and labour;
- In case the model is utilised with endogenous wages, we use a simplified version of the time-dependent pricing framework à la Dootsey, King and Wollman which is able to generate hump-shaped responses contrary to the Calvo model used in most DSGE models or to an error correction model used in more traditional settings, which implies the largest response to a shock during the period the shock occurs.

Most of the equations taken individually have acceptable statistical properties although it was not our first goal to exactly match the data. The reason for not sticking to the data individually was that a model is more interesting taken as a whole and single equation and partial equilibrium analysis have hardly some value-added in this context. In addition, as Kapetanios et al. (2005) have noted in a paper on policy-oriented modelling (p.3) "Replicating the data is rarely the objective. There are many reasons for this. Data is often subject to substantial revisions for long periods of time, so one may be replicating a chimera; it can also be influenced by many special factors that are hard to specify, ...". Finally, diagnostic simulations suggest that the impulse responses of the model to exogenous shocks are reasonable.

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## Appendix A1: Impact of model parameters on simulation results

**Table 4a - Sensitivity of the results from a foreign demand shock to perturbations in selected parameters**

| YER   | benchmark | max   | min   | avg   | NetX  | benchmark | max   | min   | avg   |
|-------|-----------|-------|-------|-------|-------|-----------|-------|-------|-------|
| q=1   | 0.30%     | 0.30% | 0.29% | 0.30% | q=1   | 0.28%     | 0.28% | 0.27% | 0.28% |
| q=5   | 0.36%     | 0.40% | 0.35% | 0.37% | q=5   | 0.23%     | 0.24% | 0.19% | 0.22% |
| q=9   | 0.39%     | 0.45% | 0.38% | 0.40% | q=9   | 0.21%     | 0.22% | 0.16% | 0.20% |
| q=21  | 0.34%     | 0.43% | 0.33% | 0.35% | q=21  | 0.21%     | 0.22% | 0.13% | 0.20% |
| q=inf | 0.30%     | 0.32% | 0.28% | 0.30% | q=inf | 0.19%     | 0.20% | 0.14% | 0.19% |

| PCR   | benchmark | max   | min   | avg   | PCD   | benchmark | max   | min   | avg   |
|-------|-----------|-------|-------|-------|-------|-----------|-------|-------|-------|
| q=1   | 0.02%     | 0.04% | 0.02% | 0.03% | q=1   | 0.00%     | 0.00% | 0.00% | 0.00% |
| q=5   | 0.16%     | 0.26% | 0.13% | 0.18% | q=5   | 0.01%     | 0.01% | 0.01% | 0.01% |
| q=9   | 0.19%     | 0.33% | 0.15% | 0.20% | q=9   | 0.04%     | 0.04% | 0.03% | 0.04% |
| q=21  | 0.09%     | 0.31% | 0.08% | 0.11% | q=21  | 0.11%     | 0.13% | 0.10% | 0.11% |
| q=inf | 0.06%     | 0.16% | 0.05% | 0.07% | q=inf | 0.12%     | 0.14% | 0.11% | 0.12% |

| ITR   | benchmark | max   | min   | avg   | ULC   | benchmark | max    | min    | avg    |
|-------|-----------|-------|-------|-------|-------|-----------|--------|--------|--------|
| q=1   | 0.14%     | 0.15% | 0.14% | 0.14% | q=1   | -0.22%    | -0.22% | -0.22% | -0.22% |
| q=5   | 0.40%     | 0.45% | 0.36% | 0.40% | q=5   | -0.11%    | -0.10% | -0.11% | -0.11% |
| q=9   | 0.54%     | 0.63% | 0.48% | 0.55% | q=9   | -0.01%    | -0.01% | -0.02% | -0.01% |
| q=21  | 0.51%     | 0.65% | 0.44% | 0.53% | q=21  | 0.16%     | 0.19%  | 0.14%  | 0.17%  |
| q=inf | 0.44%     | 0.49% | 0.38% | 0.44% | q=inf | 0.18%     | 0.21%  | 0.16%  | 0.18%  |

**Table 4b - Sensitivity of the results from a indirect tax shock to perturbations in selected parameters**

| YER   | benchmark | max    | min    | avg    | NetX  | benchmark | max    | min    | avg    |
|-------|-----------|--------|--------|--------|-------|-----------|--------|--------|--------|
| q=1   | 0.02%     | 0.02%  | 0.02%  | 0.02%  | q=1   | 0.07%     | 0.07%  | 0.07%  | 0.07%  |
| q=5   | -0.29%    | -0.27% | -0.31% | -0.29% | q=5   | 0.03%     | 0.05%  | 0.01%  | 0.03%  |
| q=9   | -0.52%    | -0.49% | -0.55% | -0.52% | q=9   | -0.11%    | -0.08% | -0.13% | -0.11% |
| q=21  | -0.81%    | -0.76% | -0.87% | -0.81% | q=21  | -0.55%    | -0.52% | -0.57% | -0.55% |
| q=inf | -0.83%    | -0.78% | -0.87% | -0.83% | q=inf | -0.67%    | -0.64% | -0.69% | -0.66% |

| PCR   | benchmark | max    | min    | avg    | PCD   | benchmark | max   | min   | avg   |
|-------|-----------|--------|--------|--------|-------|-----------|-------|-------|-------|
| q=1   | -0.07%    | -0.06% | -0.08% | -0.07% | q=1   | 0.52%     | 0.52% | 0.52% | 0.52% |
| q=5   | -0.45%    | -0.40% | -0.51% | -0.45% | q=5   | 1.34%     | 1.34% | 1.33% | 1.34% |
| q=9   | -0.57%    | -0.50% | -0.65% | -0.57% | q=9   | 1.78%     | 1.79% | 1.77% | 1.78% |
| q=21  | -0.46%    | -0.39% | -0.54% | -0.46% | q=21  | 2.20%     | 2.25% | 2.16% | 2.20% |
| q=inf | -0.38%    | -0.36% | -0.44% | -0.39% | q=inf | 2.18%     | 2.25% | 2.12% | 2.18% |

| ITR   | benchmark | max    | min    | avg    | ULC   | benchmark | max    | min    | avg    |
|-------|-----------|--------|--------|--------|-------|-----------|--------|--------|--------|
| q=1   | 0.00%     | 0.00%  | -0.01% | 0.00%  | q=1   | -0.04%    | -0.04% | -0.05% | -0.04% |
| q=5   | -0.28%    | -0.26% | -0.32% | -0.28% | q=5   | 1.12%     | 1.13%  | 1.11%  | 1.12%  |
| q=9   | -0.50%    | -0.45% | -0.56% | -0.50% | q=9   | 1.52%     | 1.55%  | 1.50%  | 1.52%  |
| q=21  | -0.41%    | -0.27% | -0.52% | -0.40% | q=21  | 1.87%     | 1.94%  | 1.80%  | 1.87%  |
| q=inf | -0.21%    | -0.05% | -0.35% | -0.21% | q=inf | 1.75%     | 1.86%  | 1.66%  | 1.75%  |

Notes : the table compares the maximum and minimum as well as the average effects of the shock when the size of some parameters varies as compared to our benchmark parameterisation on GDP, YER, on private consumption, PCR, on gross fixed capital formation, ITR, on net exports, NetX, on private consumption deflator, PCD and on unit labour costs, ULC.

**Table 4c - Sensitivity of the results from a labour supply shock to perturbations in selected parameters**

| YER   | benchmark | max   | min   | avg   | NetX  | benchmark | max    | min    | avg    |
|-------|-----------|-------|-------|-------|-------|-----------|--------|--------|--------|
| q=1   | 0.04%     | 0.04% | 0.04% | 0.04% | q=1   | -0.02%    | -0.02% | -0.02% | -0.02% |
| q=5   | 0.21%     | 0.22% | 0.20% | 0.21% | q=5   | -0.14%    | -0.13% | -0.14% | -0.14% |
| q=9   | 0.31%     | 0.32% | 0.28% | 0.30% | q=9   | -0.13%    | -0.11% | -0.14% | -0.13% |
| q=21  | 0.38%     | 0.41% | 0.34% | 0.37% | q=21  | 0.02%     | 0.05%  | -0.01% | 0.02%  |
| q=inf | 0.37%     | 0.39% | 0.34% | 0.36% | q=inf | 0.03%     | 0.05%  | 0.00%  | 0.03%  |
| PCR   | benchmark | max   | min   | avg   | PCD   | benchmark | max    | min    | avg    |
| q=1   | 0.09%     | 0.09% | 0.08% | 0.09% | q=1   | 0.00%     | 0.00%  | 0.00%  | 0.00%  |
| q=5   | 0.49%     | 0.50% | 0.45% | 0.48% | q=5   | -0.07%    | -0.07% | -0.07% | -0.07% |
| q=9   | 0.59%     | 0.59% | 0.53% | 0.58% | q=9   | -0.17%    | -0.16% | -0.18% | -0.17% |
| q=21  | 0.50%     | 0.50% | 0.40% | 0.49% | q=21  | -0.29%    | -0.26% | -0.31% | -0.29% |
| q=inf | 0.49%     | 0.49% | 0.46% | 0.49% | q=inf | -0.25%    | -0.23% | -0.27% | -0.25% |
| ITR   | benchmark | max   | min   | avg   | ULC   | benchmark | max    | min    | avg    |
| q=1   | 0.04%     | 0.04% | 0.03% | 0.03% | q=1   | -0.05%    | -0.05% | -0.05% | -0.05% |
| q=5   | 0.28%     | 0.29% | 0.26% | 0.28% | q=5   | -0.28%    | -0.27% | -0.28% | -0.28% |
| q=9   | 0.41%     | 0.43% | 0.37% | 0.40% | q=9   | -0.40%    | -0.37% | -0.42% | -0.40% |
| q=21  | 0.39%     | 0.43% | 0.33% | 0.38% | q=21  | -0.45%    | -0.40% | -0.49% | -0.45% |
| q=inf | 0.32%     | 0.35% | 0.29% | 0.32% | q=inf | -0.38%    | -0.34% | -0.40% | -0.38% |

**Table 4d - Sensitivity of the results from a permanent oil price shock to perturbations in selected parameters**

| YER   | benchmark | max    | min    | avg    | NetX  | benchmark | max    | min    | avg    |
|-------|-----------|--------|--------|--------|-------|-----------|--------|--------|--------|
| q=1   | -0.02%    | -0.02% | -0.02% | -0.02% | q=1   | 0.01%     | 0.01%  | 0.01%  | 0.01%  |
| q=5   | -0.17%    | -0.15% | -0.19% | -0.17% | q=5   | 0.01%     | 0.02%  | 0.01%  | 0.01%  |
| q=9   | -0.26%    | -0.22% | -0.29% | -0.26% | q=9   | -0.03%    | -0.01% | -0.04% | -0.03% |
| q=21  | -0.35%    | -0.29% | -0.40% | -0.35% | q=21  | -0.19%    | -0.16% | -0.21% | -0.18% |
| q=inf | -0.36%    | -0.30% | -0.40% | -0.36% | q=inf | -0.24%    | -0.21% | -0.27% | -0.24% |
| PCR   | benchmark | max    | min    | avg    | PCD   | benchmark | max    | min    | avg    |
| q=1   | -0.04%    | -0.03% | -0.04% | -0.04% | q=1   | 0.07%     | 0.07%  | 0.07%  | 0.07%  |
| q=5   | -0.23%    | -0.21% | -0.26% | -0.23% | q=5   | 0.47%     | 0.47%  | 0.47%  | 0.47%  |
| q=9   | -0.28%    | -0.26% | -0.33% | -0.29% | q=9   | 0.72%     | 0.72%  | 0.72%  | 0.72%  |
| q=21  | -0.24%    | -0.21% | -0.32% | -0.25% | q=21  | 0.95%     | 0.96%  | 0.93%  | 0.95%  |
| q=inf | -0.21%    | -0.19% | -0.29% | -0.22% | q=inf | 0.95%     | 0.97%  | 0.93%  | 0.95%  |
| ITR   | benchmark | max    | min    | avg    | ULC   | benchmark | max    | min    | avg    |
| q=1   | -0.02%    | -0.02% | -0.03% | -0.02% | q=1   | 0.00%     | 0.00%  | 0.00%  | 0.00%  |
| q=5   | -0.25%    | -0.22% | -0.27% | -0.25% | q=5   | 0.36%     | 0.36%  | 0.35%  | 0.36%  |
| q=9   | -0.35%    | -0.30% | -0.40% | -0.36% | q=9   | 0.62%     | 0.63%  | 0.62%  | 0.62%  |
| q=21  | -0.23%    | -0.14% | -0.31% | -0.24% | q=21  | 0.82%     | 0.84%  | 0.79%  | 0.82%  |
| q=inf | -0.11%    | 0.00%  | -0.20% | -0.11% | q=inf | 0.79%     | 0.83%  | 0.75%  | 0.79%  |

Notes : the table compares the maximum and minimum as well as the average effects of the shock when the size of some parameters varies as compared to our benchmark parameterisation on GDP, YER, on private consumption, PCR, on gross fixed capital formation, ITR, on net exports, NetX, on private consumption deflator, PCD and on unit labour costs, ULC.



## Appendix A2: Main dynamic behavioural equations

### 1. PRICE AND WAGE EQUATIONS IN THE MODEL

#### 1.1. Prices

##### *(i) Main driving domestic price*

The price of the domestic intermediate goods,  $p_H$ , is the main driving domestic price variable in the model.

The target or optimal price for domestic intermediate goods is given by:

$$p_{Ht}^* = 0.77 c_t(w, ucc) + 0.23 mtd_t$$

where

$c_t(w, ucc)$  is unit cost of production with "w" being the hourly wage cost and the user cost of capital being defined as  $ucc = I_{OD} (\delta + r - E_t(\Delta p_{CD,t+1}) + rp)$  with rp designing a constant risk premium arbitrarily fixed at 0.025.

This shows that firms set their price not only in response to changes in their own costs but also in response to the prices set by their competitors. The relative weight depends on the share of non-traded goods which determines the proportion of their competitors that are foreign.

The dynamic equation for domestic intermediate goods is :

$$\Delta p_{Ht} = -0.12 (p_{Ht-1} - p_{Ht-1}^*) + 0.676 \text{lags}_1 \Delta p_{Ht-1} \\ + 0.355 \text{leads}_\infty (E_{t-1} \Delta p_{Ht+i}^*) + 0.001 \left( \frac{\Delta y_{t-1}}{\Delta y^{ss}} - 1 \right)$$

$R^2: 0.34$ ;  $dw: 2.64$ ;  $LM(4): 24.6 (0.0)$ ;  $JB: 1.61 (0.44)$ ;  $Arch(4): 6.16(0.18)$

where  $\Delta y$  is the growth rate of private output and  $\Delta y^{ss}$  its steady-state value.

##### *(ii) Consumption deflator*

Since final goods result from the combination of domestic products and imported goods, price indices for consumption (PCD), and also for investment by companies and exported goods are weighted functions of the price set by domestic suppliers ( $p_H$ ) and the price of imported goods (mtd). Given that  $p_H$  is defined at factor costs, the consumption deflator is modelled by adding indirect tax rates. In addition the energy price is added to account for the direct price effect on fuel.

Thus consumption price inflation evolves according to:

$$\Delta pcd_t - \Delta ti_t = 0.45 (\Delta pcd_{t-1} - \Delta ti_{t-1}) + 0.15 (\Delta pcd_{t-2} - \Delta ti_{t-2}) \\ + 0.4 (0.68 \Delta p_{Ht} + 0.307 \Delta mtd_t + 0.013 \Delta pei_t) \\ \text{[ mean lag (inflation): 1.9 ]}$$

where the import price of energy, PEI, and the indirect tax rate, ti, are exogenous and the rate of growth of the price of imports, MTD, is given by

$$\Delta mtd_t = -0.34 (mtd_{t-1} - mtd_{t-1}^*) + 0.91 \text{leads}_\infty (E_{t-1} \Delta mtd_{Ht+i}^*) + 0.001 \left( \frac{\Delta y_{t-1}}{\Delta y^{ss}} - 1 \right)$$

$R^2: 0.51$ ;  $dw: 2.03$ ;  $LM(4): 4.65 (0.32)$ ;  $JB: 0.16 (0.92)$ ;  $Arch(4): 3.58(0.46)$   
and the target level is

$$mtd_t^* = 0.67 p_{Ft} + 0.06 pei_t + 0.27 p_{Ht}$$

And finally, the price that serves in the indexation mechanism of wages and transfers evolves according to:

$$\Delta pcd2_t = 1.005025 (\Delta pcd_t - 0.013 \Delta pei_t + 0.008 \Delta pmazout_t)$$

where pmazout stands for heating oil which is not excluded from the health index.

(iii) Exports and GDP deflators

The rate of growth of the price of exports, XTD, is given by

$$\Delta xtd_t = -0.24 (xtd_{t-1} - xtd_{t-1}^*) + 0.15 \Delta xtd_{t-1} + 0.76 \text{ leads}_{\infty} (E_{t-1} \Delta xtd_{t+i}^*)$$

R<sup>2</sup>: 0.59; dw: 1.79; LM(4): 12.1 (0.02); JB: 0.28 (0.86); Arch(4): 0.41(0.98)

where the target level is

$$xtd_t^* = 0.624 c(w_t, ucc_t) + 0.046 pei_t + 0.25 p_{Ft}^*$$

The GDP deflator is defined by an accounting identity that divides the value of GDP, YEN, by its volume, YER:

$$YED = \frac{YEN}{YER}$$

and it has no feedback on other prices.

## 1.2. Wages

Note that fuel, tobacco and alcohol are excluded from the basket used to calculate the "health" consumption price index, PCD2. Therefore, for forecasts, the wage rate excluding indexation (wrh\_R) is assumed to be exogenous and the endogenous nominal wage is obtained by indexing it according to:

$$wrh_t = wrh\_R_t * (1/12 * PCD2_t + 8/12 * PCD2_{t-1} + 3/12 * PCD2_{t-2})$$

However if necessary for simulations, an endogenous "real" wage may be used. This is a "flexible" version of (1.67) :

$$w_t^{C^*} = \log(\Lambda_{L,t}) + 0.05 \log(1 - \kappa_t) - (0.18 - 0.2 b_t) \log(u_t) - \frac{1}{\rho} \log \left[ \alpha + (1 - \alpha) \left( \frac{k_t}{1 - u_t} \right)^{-\rho} \right]$$

and after indexation the aggregate wage index is calculated as in (2.12)

## 2. DEMAND COMPONENTS

### 2.1. Consumption

Due to habit formation consumption does not immediately adjust to changes in the desired level. In addition to taking into account these delayed adjustments, the dynamic consumption equation also depends on changes in employment to capture uncertainty in future income:

$$\Delta c_t = -0.178 (c_{t-1} - c_{t-1}^*) + 0.538 \text{ lags}_2 \Delta c_{t-1} + 0.485 \text{ leads}_{\infty} (E_{t-1} \Delta c_{t+i}^*) + 0.42 * \Delta \ln \left( \sum_{i=0}^{-3} L_{t+i} / 4 \right)$$

R<sup>2</sup>: 0.39; dw: 1.83; LM(4): 6.72 (0.15); JB: 0.04(0.98); Arch(4): 3.64(0.45)

where C\* is given in (1.19)

### 2.2. Housing investment

The equilibrium ratio of housing investment to consumption is a function of the relative price of consumption to the cost of new houses and on a mortgage interest rate.

$$ihr_t^* - c_t = -1.2 (rmt_t - E_t(\Delta pcd_{t+1}) - rkrh_t) - 0.4(ihxn_t - pcd_t)$$

$$\Delta ihr_t = -0.19 (ihr_{t-1} - ihr_{t-1}^*) + 0.604 \text{lags}_3 \Delta ihr_{t-1} + 0.396 \text{leads}_\infty (E_{t-1} \Delta ihr_{t+i}^*)$$

R<sup>2</sup>: 0.74; dw: 1.75; LM(4): 4.46 (0.34); JB: 0.26 (0.87); Arch(4): 3.97(0.41);  
sample restricted to 1990:1 to 2003:4

### 2.3. Business investment

$$\Delta ior_t = -0.12 (ior_{t-1} - kre_{t-2} - \ln(g_L + n + \delta)) + 0.21 \text{lags}_3 \Delta ior_{t-1} + 0.65 \text{leads}_\infty (E_{t-1} \Delta kre_{t+i}^*)$$

$$+ 0.39 \sum_{i=1}^4 (NON_{t-i} - OTN_{t-i}) / (KRE_{t-1} IOD_{t-1})$$

R<sup>2</sup>: 0.42; dw: 2.54; LM(4): 12.2 (0.02); JB: 2.5 (0.28); Arch(4): 4.4(0.35)

### 2.4. Exports

$$\Delta xtr_t = -0.15 (xtr_{t-1} - xtr_{t-1}^*) + 0.68 (0.7 \Delta wdr\_in_t + 0.23 \Delta wdr\_in_{t-1} + 0.07 \Delta wdr\_in_{t-2})$$

$$+ 0.32 \Delta wdr\_ex_t + 0.3 (0.6 (\Delta p_{ft}^* - \Delta xtd_t) + 0.4 (\Delta p_{ft-1}^* - \Delta xtd_{t-1}))$$

R<sup>2</sup>: 0.34; dw: 1.57; LM(4): 2.96 (0.56); JB: 3.26 (0.19); Arch(4): 3.46 (0.48)

where  $xtr^*$  is given by combining (1.32) and (1.33).

### 2.5. Imports

$$\Delta mtr_t = -0.40 (mtr_{t-1} - mtr_{t-1}^*) + 0.79 \Delta mtr_t^* + 0.49 \Delta duc_{t-1}$$

R<sup>2</sup>: 0.54; dw: 2.26; LM(4): 10.5 (0.03); JB: 1.82 (0.40); Arch(4): 25.7 (0.00)

where  $mtr^*$  is given by (1.56).

## 3. EMPLOYMENT

$$\Delta \langle lpn_t + hours_t \rangle = -0.16 ( \langle lpn_{t-1} + hours_{t-1} \rangle - \langle lpn_{t-1} + hours_{t-1} \rangle^* ) + 0.16 \Delta y_t$$

$$+ 0.79 \text{leads}_\infty (E_{t-1} \Delta \langle lpn_{t+i} + hours_{t+i} \rangle^*)$$

R<sup>2</sup>: 0.61; dw: 1.97; LM(4): 6.21 (0.18); JB: 3.64 (0.16); Arch(4): 4.80 (0.30)

where the optimal level comes from log-linearisation of (1.28a) :

$$\langle lpn_t + hours_t \rangle^* = y_t - 0.52 \cdot \kappa_t - 0.52 (wrh_t - p_{Ht}) - 0.48 (0.003845 \cdot t)$$

and hours per employee evolve according to :

$$\Delta hours_t = -0.38 (hours_{t-1} - durcon_{t-1} - 0.1 \text{etpl}_{t-1} - 0.17 (y_{t-1} - kre_{t-1}))$$

$$+ 0.13 \Delta (y_t - kre_{t-1}) + 0.17 \Delta hours_{t-1}$$

R<sup>2</sup>: 0.43; dw: 2.25; LM(4): 6.61 (0.15); JB: 0.66 (0.71); Arch(4): 4.65 (0.32)

where  $durcon$  is conventional working time and  $etpl$  reflects the proportion full-time workers.

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