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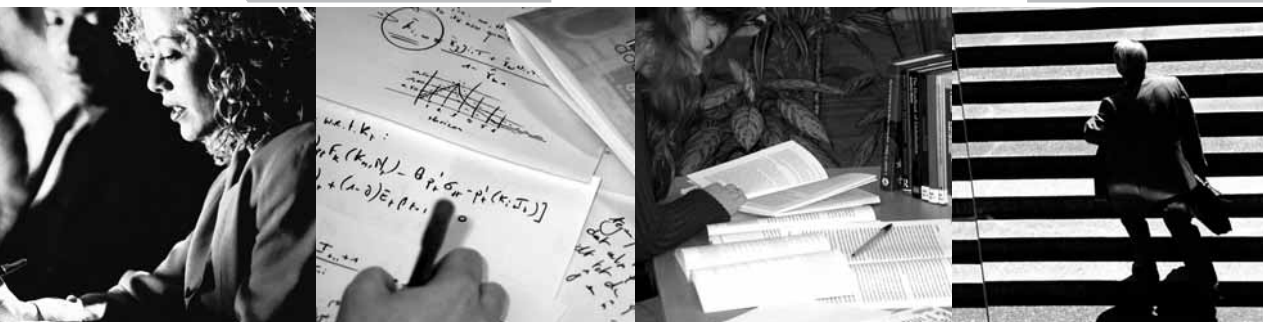
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Income uncertainty and aggregate consumption

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National Bank
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Income uncertainty and aggregate consumption

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Abstract

We investigate the relevance of aggregate and consumer-specific income uncertainty for aggregate consumption changes in the US over the period 1952-2001. Theoretically, the effect of income risk on consumption changes is decomposed into an aggregate and into a consumer-specific part. Empirically, aggregate risk is modelled through a GARCH process on aggregate income shocks and individual risk is modelled as an unobserved component and obtained through Kalman filtering. Our results suggest that aggregate income risk explains a negligible fraction of the variance of aggregate consumption changes. A more important part of aggregate consumption changes is explained by the unobserved component. The interpretation of this component as reflecting consumer-specific income risk is supported by the finding that it is negatively affected by received consumer transfers.

Keywords: income uncertainty, consumption, precaution, state space models, GARCH errors, unobserved component, Bayesian.

JEL-classification: E21.

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1   Introduction.

In this paper we investigate the effects of income uncertainty on aggregate consumption changes using quarterly data for the US over the period 1952-2001. The approach undertaken differs from the existing literature in three respects. First, using the theoretical results of Caballero (1990) as a starting point, we present a theoretical framework in which the effect of income risk on the change in aggregate consumption is decomposed into two parts: the impact of aggregate income risk and the impact of consumer-specific income risk (see for instance Banks et al. (2001) for a comparable idea but a different set-up). This decomposition is useful because limiting income risk to aggregate income risk is too restrictive. The reason is that the variance of aggregate labour income is low. As a result, in permanent income models with no habit formation nor rule-of-thumb consumption, the magnitude of the average growth in consumption (which in part can be expected to reflect the postponement of consumption to the future due to uncertainty) can only be explained by values of risk aversion that are much higher than what is widely believed. Another reason is that there is no theoretical a priori justification (see e.g. Deaton 1992, p.37) or empirical evidence (see e.g. Banks et al. 2001) to suggest that risk pooling mechanisms that effectively eliminate individual-specific income risk actually do exist. Second, instead of estimating the resulting consumption function using a micro-based income uncertainty proxy[1] we follow a pure aggregate time series approach. Aggregate income risk is modelled through a GARCH process on aggregate labour income shocks. While some previous studies have investigated the impact of aggregate income uncertainty on aggregate consumption with ARCH models (see e.g. Wilson 1998), these studies have not simultaneously taken into account the effects on private consumption of individual-specific income risk. Individual income risk is modelled as an unobserved component and identified through Kalman filtering techniques. To facilitate the identification of the unobserved component we inves-

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[1] The use of micro-based uncertainty measures is limited because of the small length of the available time series. Also, decomposing these measures into an aggregate and an idiosyncratic part is not straightforward. Further, the use of these measures can be problematic in the presence of measurement errors or "self-selection" problems (see Attanasio 1999 for a discussion).

tigate a number of determinants of income risk suggested in the literature. First, Carroll (1992) notes that "the most drastic fluctuations in household income are those associated with spells of unemployment". We investigate whether increases in the unemployment rate lead to a postponement of consumption to the future. Second, from the papers by Hubbard et al. (1995) and Engen and Gruber (2001), we know that transfers provided by the social security system (i.e. pensions, health and unemployment insurance) may reduce individual income risk by providing insurance against bad draws of labour income in certain periods. We therefore also investigate whether transfers received by consumers cause a shift from current consumption to future consumption. Note, however, that in the presence of rule-of-thumb consumers who base their consumption decisions on current income instead of permanent income consumption is excessively sensitive to total current after-tax income and transfers may affect consumption through this channel as well. The existence of rule-of-thumb consumers may be due to liquidity constraints (see e.g. Campbell and Mankiw 1990) or myopia (see e.g. Flavin 1985). We therefore check the sensitivity of our basic model to a specification where total after-tax income is added as an additional regressor in the consumption function. Third, we use a Bayesian approach to parameter estimation. A Bayesian approach allows us to incorporate prior knowledge into our estimation. Priors are particularly useful in this paper to estimate GARCH effects in state space systems in the presence of outliers in the data.

Our results suggest that aggregate income risk explains only a negligible fraction of the variance of aggregate consumption changes. The unobserved component explains a more important part of consumption changes. The interpretation of this component as (at least partially) reflecting consumer-specific income risk is supported by the finding that it is negatively affected by the trend in transfers received by consumers. We argue that, from the eighties onward, the trend change in transfers received by consumers can (partially) explain low frequency movements in consumption changes. Our extended model, which allows for a fraction of consumers that follow current income instead of permanent income, provides evidence that our results are robust when "excess sensitivity" of consumption to anticipated disposable income is taken into account.

The paper is structured as follows. In section 2 we discuss, in a non-exhaustive way, the more recent research on income risk and consumption and we discuss the relevance of the topic. In section 3 we present a consumption model with time-varying aggregate and time-varying idiosyncratic or consumer-specific income risk. In section 4 we present our basic empirical specification and we put it into state space form (which allows us to deal with the unobserved component). We discuss how to tackle GARCH errors in state space models. We also discuss Bayesian estimation of the unknown parameters in the model (with the use of importance sampling to obtain posterior parameter distributions). In section 5 we present the estimation results for our basic model. In section 6 we investigate whether our conclusions are affected by the introduction of rule-of-thumb consumers in the model. Section 7 discusses some limitations of our approach and provides concluding remarks.

2 Income risk and consumption: context and relevance.

Until recently most work on consumption and saving, both at the aggregate and at the household level, has been based on the life cycle/permanent income models. These models state that consumers base their current consumption decisions on the sum of current and discounted future income (i.e permanent or life cycle income) and smooth consumption over time and over the life cycle. The empirical evidence however has failed to support these models. More specifically, Zeldes (1989) mentions three empirical puzzles that the permanent income/life cycle models have not explained. First, consumption tracks current income too closely, i.e the excess sensitivity puzzle. Second, consumption growth in the US has been positive in periods where the interest rates were close to zero and lower than the rate of time preference. Three, the elderly fail to run down their assets after retirement as predicted by the life cycle model. As a result of these puzzles the theoretical foundations of these models have been put to the test.

One of these foundations is that only the mean of future income affects current consumption. It is now generally acknowledged that also the variance of future income may

influence consumption, savings and wealth accumulation. Precautionary savings, i.e. savings against uninsurable income risks, occur once the assumption of certainty equivalence is omitted from the original permanent income/life cycle models. This assumption usually takes the form of linear marginal utility for consumers. Once it is assumed that marginal utility is nonlinear, an increase in uncertainty about future income or consumption lowers current consumption and raises savings (see e.g. Deaton 1992, p.178). Dreze and Modigliani (1972) consider explicitly the effects of nonlinear marginal utility. Kimball (1990) proves that precautionary savings occur when the utility function exhibits "prudence", i.e when the third derivative of the utility function is positive. Examples of utility functions that satisfy this requirement are the constant absolute risk aversion (CARA) and the constant relative risk aversion (CRRA) utility functions. Caballero (1990) shows that a closed form solution for consumption can be obtained with CARA utility where consumption depends positively on permanent income (as with quadratic utility) and negatively on a term that captures precaution. With CRRA utility, on the other hand, a closed form for consumption cannot be obtained. Much research has therefore been based on simulation (see e.g. Skinner 1988, Zeldes 1989, Deaton 1991 and Carroll 1992, 1994). Apart from this type of research there is also a large literature that uses micro data to test the relevance of the precautionary motive for saving (see Browning and Lusardi (1996) and Kennickell and Lusardi 2001 for an overview). In this literature individual savings are usually related to some objective or subjective measure of income uncertainty (see e.g. Guiso et al. 1992). Other studies investigate the impact of these micro-based uncertainty measures on *aggregate* savings and consumption (Carroll 1992, 1994; Dardanoni 1991; Hahm and Steigerwald 1999; Banks et al. 2001...).

The empirical evidence provided by these different studies on precaution so far is mixed. On the one hand, Skinner (1988), Caballero (1990) and Carroll and Samwick (1998), for instance, argue that precaution could be responsible for up to 50 percent of total wealth in the US. Dynan (1993) and Guiso et al. (1992), on the other hand, find only modest precaution effects. Browning and Lusardi (1996) argue that the finding of modest precaution effects is not surprising given that, for the US, most of the saving is

done by the wealthy and elderly for whom future income shocks may not be that relevant.

The determination of the true relevance of precautionary savings is important because it bears on a large number of economic issues. First, given the importance of aggregate consumption for aggregate demand, the relevance of precautionary saving for understanding economic fluctuations must not be understated. For instance, to the extent that monetary and fiscal policy shocks affect consumers' uncertainty about their future income and consumption, the presence of precaution implies an additional channel through which policy can stabilize GDP. Second, it can explain, why the elderly and the young save more than what is predicted by the life cycle hypothesis and why saving rates differ across occupations. Third, it can explain why consumption tracks current income. This issue is tackled in the buffer-stock model of saving which has been advocated strongly by Carroll (1992, 1994). Buffer-stock consumers are impatient and want to consume now. At the same time they are uncertain about future employment and income prospects. So they hold assets as a buffer against income shocks but only in small amounts. As a result, consumption and income never drift apart for very long. Fourth, precautionary savings are one of the reasons why the Ricardian Equivalence hypothesis may fail. Barsky, Mankiw and Zeldes (1986) point out that, if consumers have a precautionary savings motive, and if taxes are an increasing function of income, then lowering taxes today and increasing them tomorrow may increase consumption. The reason is that the current tax cut provides certain wealth while the future tax increase depends on future income which is uncertain. The intertemporal transfer provided by the government lowers the uncertainty about future income and thus precautionary savings. Finally, since precautionary savings are basically a self-insurance mechanism, they may be a substitute for other types of insurance, like unemployment and health insurance. A literature spawned by Feldstein (1974) has attributed the decline in the personal saving rate in the US in the eighties to the more generous social security system. Hubbard et al. (1995) for instance argue that some social security programs discourage saving and wealth accumulation by low-income households. Engen and Gruber (2001) find, using a panel of households, that households tend to save less when the publicly provided unemployment insurance is more generous.

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3   A consumption model with time-varying aggregate and time-varying idiosyncratic income risk.

In this section we derive an expression for the change in aggregate private consumption that takes into account uncertainty with respect to aggregate labour income and uncertainty with respect to the consumer-specific component of labour income. The latter type of risk is present because insurance markets are assumed to be incomplete (i.e there is no risk pooling across consumers). The model uses the results of Caballero (1990) in a setting where consumers are heterogeneous in the sense that they experience different income draws. As a result, given the absence of insurance mechanisms, consumption trajectories and wealth levels may diverge considerably over consumers.

The economy consists of n consumers, each having an infinite planning horizon. Each consumer i (where $i = 1, ..., n$) has a utility function of the constant absolute risk aversion ($CARA$) type, namely $u(c_{it}) = (-1/\gamma)e^{-\gamma c_{it}}$ where c_{it} is real consumption of consumer i in period t and where γ is the coefficient of absolute risk aversion ($\gamma > 0$) which also equals the coefficient of absolute prudence. We use this type of utility function instead of the more usual utility function of the constant relative risk aversion ($CRRA$) type because of its analytical convenience (i.e. it facilitates aggregation[2], see below). We further assume that all consumers can freely lend and borrow, i.e. capital markets are perfect. We assume that all consumers face the same constant real interest rate r which equals their rate of time preference. Unlike capital markets, insurance markets are incomplete. That is, consumers cannot insure themselves through the use of so-called Arrow securities (see Deaton 1992 p.35-36) that could be traded among them to smooth consumption across different states of the world.[3]   In section 7 we discuss the implications for the empirical results of the

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[2]More specifically, under CRRA preferences, a specification can be obtained that gives the impact of income risk on consumption growth rather than on the first difference of consumption as is the case with CARA utility. Since the impact of income risk on consumption growth varies inversely with the wealth level (see e.g. Banks et al. 2001) the consumption growth equation has a multiplicative structure which makes aggregation over consumers difficult.

[3]This assumption is consistent with reality. The existence of complete insurance markets is unlikely

somewhat restrictive assumptions that utility is of the $CARA$ type and that the interest rate is constant.

Given the stated assumptions, the first-order condition in period $t+1$ for consumer i is,

$$E_{it}(e^{-\gamma \Delta c_{it+1}}) = 1 \tag{1}$$

Using a second-order Taylor expansion of $e^{-\gamma \Delta c_{it+1}}$ around $E_{it}\Delta c_{it+1}$ we rewrite eq.(1) as,

$$\Delta c_{it+1} = \frac{\gamma}{2} E_{it}\varepsilon_{cit+1}^2 + \varepsilon_{cit+1} \tag{2}$$

where $\varepsilon_{cit+1} = c_{it+1} - E_{it}c_{it+1}$ (see appendix A).

The period $t+1$ budget constraint under which the optimization takes place is given by,

$$w_{it+1}^f = (1+r)w_{it}^f + y_{it+1} - c_{it+1} \tag{3}$$

where the variable w_{it}^f is consumer i's financial wealth at the end of period t and where y_{it+1} is consumer i's after-tax labour income. Following Demery and Duck (2000) we model y_{it+1}, which is the exogenous process driving the model, as consisting of an aggregate component and an individual-specific component. Both components are modelled as $ARIMA$ processes. Aggregate after-tax labour income y_{t+1} is modelled as an $ARIMA(p1, 1, q1)$ process giving,

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due to e.g. moral hazard problems.

$$\pi(L)(\Delta y_{t+1} - \mu) = \pi^*(L)\varepsilon_{yt+1} \tag{4}$$

where $\pi(L)$ and $\pi^*(L)$ are polynomials in the lag operator L of respectively order $p1$ and $q1$, where μ is the mean and ε_{yt+1} is the income shock which is assumed to be white noise. It follows a $GARCH(1,1)$ process,

$$\varepsilon_{yt+1}^2 = \delta_1 + \delta_2 \varepsilon_{yt}^2 + \delta_3 E_{it-1} \varepsilon_{yt}^2 + \omega_{t+1}^\varepsilon \tag{5}$$

where E_{it} is the expectations operator conditional on information set Ω_{it} available to consumer i in period t, where $\delta_1, \delta_2, \delta_3 > 0$ and where $\delta_2 + \delta_3 < 1$. The term $\omega_{t+1}^\varepsilon = \varepsilon_{yt+1}^2 - E_{it}\varepsilon_{yt+1}^2$ is white noise (bounded from below) with variance $\sigma_{\omega\varepsilon}^2$. Individual income is given by an $ARIMA(p2,1,q2)$ process,

$$\phi(L)(\Delta y_{it+1} - \Delta y_{t+1}) = \phi^*(L)\eta_{it+1} \tag{6}$$

where $\phi(L)$ and $\phi^*(L)$ are polynomials in the lag operator L of respectively order $p2$ and $q2$ and where η_{it+1} is an individual-specific income shock that is white noise. It further has a constant unconditional variance across consumers. Also, it is uncorrelated across individuals, so that it disappears on aggregation over consumers, i.e. $n^{-1}\sum_{i=1}^n \eta_{it+1} = 0$. The term η_{it+1} further follows a $GARCH(1,1)$ process,

$$\eta_{it+1}^2 = \xi_1 + \xi_2 \eta_{it}^2 + \xi_3 E_{it-1} \eta_{it}^2 + \omega_{it+1}^\eta \tag{7}$$

where $\xi_1, \xi_2, \xi_3 > 0$ and where $\xi_2 + \xi_3 < 1$. The term $\omega_{it+1}^\eta = \omega_{it+1}^\eta - E_{it}\omega_{it+1}^\eta$ is white noise (bounded from below) with variance $\sigma_{\omega\eta}^2$ (constant across consumers). We assume

that $n^{-1} \sum_{i=1}^{n} \omega_{it+1}^{\eta} = \omega_{t+1}^{\eta}$. Note finally that the errors ε_{yt+1}, η_{it+1}, $\omega_{t+1}^{\varepsilon}$ and ω_{it+1}^{η} are assumed to be mutually uncorrelated. Combining eqs.(4) and (6) we obtain,

$$\Delta y_{it+1} = \mu + A(L)\varepsilon_{yt+1} + B(L)\eta_{it+1} \tag{8}$$

where $A(L)$ and $B(L)$ are infinite order lag polynomials given by $A(L) = \pi^*(L)\pi(L)^{-1} = A_0 + A_1 L + A_2 L^2 + ...$ with $\sum_{j=0}^{\infty} |A_j| < \infty$ and $B(L) = \phi^*(L)\phi(L)^{-1} = B_0 + B_1 L + B_2 L^2 + ...$ with $\sum_{j=0}^{\infty} |B_j| < \infty$ (see e.g. Hamilton 1994, chapter 2).

After solving eq.(3) forward and imposing a transversality condition we write the intertemporal budget constraint as,

$$w_{it}^f = \sum_{j=1}^{\infty} \alpha^j c_{it+j} - \sum_{j=1}^{\infty} \alpha^j y_{it+j} \tag{9}$$

where $\alpha = (1 + r)^{-1}$ (see e.g. Deaton 1992, p.81). After adding and subtracting the term $\sum_{j=1}^{\infty} \alpha^j E_{it} y_{it+j}$ to the RHS of eq.(9) we obtain,

$$w_{it}^f = \sum_{j=1}^{\infty} \alpha^j c_{it+j} - \sum_{j=1}^{\infty} \alpha^j (y_{it+j} - E_{it} y_{it+j}) - \sum_{j=1}^{\infty} \alpha^j E_{it} y_{it+j} \tag{10}$$

With the use of eq.(8) it is straightforward to show that,

$$y_{it+j} - E_{it} y_{it+j} = \sum_{k=1}^{j} A_{j-k}^* \varepsilon_{yt+k} + \sum_{k=1}^{j} B_{j-k}^* \eta_{it+k} \tag{11}$$

with partial sums $A_0^* = A_0$, $A_1^* = A_0 + A_1$,..., $A_{j-1}^* = A_0 + A_1 + ... + A_{j-1}$ and $B_0^* = B_0$, $B_1^* = B_0 + B_1$, ..., $B_{j-1}^* = B_0 + B_1 + ... + B_{j-1}$. In appendix B we show that from eq.(2) we can derive,

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$$c_{it+j} = c_{it} + \sum_{k=1}^{j} \frac{\gamma}{2} E_{it}\varepsilon_{cit+k}^2 + \sum_{k=1}^{j} \varepsilon_{cit+k} + \sum_{k=1}^{j} \frac{\gamma}{2}(E_{it+k-1}\varepsilon_{cit+k}^2 - E_{it}\varepsilon_{cit+k}^2) \qquad (12)$$

After substituting eqs.(11) and (12) into eq.(10) we obtain,

$$
\begin{aligned}
w_{it}^f &= \sum_{j=1}^{\infty} \alpha^j \{ c_{it} + \sum_{k=1}^{j} \frac{\gamma}{2}(E_{it+k-1}\varepsilon_{cit+k}^2 - E_{it}\varepsilon_{cit+k}^2) \\
&\quad + \sum_{k=1}^{j} \frac{\gamma}{2} E_{it}\varepsilon_{cit+k}^2 + \sum_{k=1}^{j} \varepsilon_{cit+k} \\
&\quad - \sum_{k=1}^{j} A_{j-k}^* \varepsilon_{yt+k} - \sum_{k=1}^{j} B_{j-k}^* \eta_{it+k} \\
&\quad - E_{it}y_{it+j} \}
\end{aligned}
\qquad (13)
$$

Taking expectations conditional on information set Ω_{it} of the LHS and RHS of eq.(13) we obtain, after some rearrangements,

$$
\begin{aligned}
c_{it} &= \frac{1-\alpha}{\alpha}\left[w_{it}^f + \sum_{j=1}^{\infty} \alpha^j E_{it}y_{it+j}\right] \\
&\quad - \frac{1-\alpha}{\alpha}\left[\sum_{j=1}^{\infty} \alpha^j \sum_{k=1}^{j} \frac{\gamma}{2} E_{it}\varepsilon_{cit+k}^2\right]
\end{aligned}
\qquad (14)
$$

The first term is permanent income. The second term is the contribution of precaution which decreases consumption relative to the certainty equivalence result. Substituting eq.(14) back into eq.(13) gives,

$$\sum_{j=1}^{\infty} \alpha^j \{ \sum_{k=1}^{j} \frac{\gamma}{2}(E_{it+k-1}\varepsilon_{cit+k}^2 - E_{it}\varepsilon_{cit+k}^2) + \sum_{k=1}^{j} \varepsilon_{cit+k} - \sum_{k=1}^{j} A_{j-k}^* \varepsilon_{yt+k} - \sum_{k=1}^{j} B_{j-k}^* \eta_{it+k} \} = 0$$

$$(15)$$

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The aim is now to find an expression for ε_{cit+k} in terms of the 4 shocks ε_{yt+k}, η_{it+k}, $\omega^{\varepsilon}_{t+k}$ and ω^{η}_{it+k}. To this end we use the method of undetermined coefficients. We guess that

$$\varepsilon_{cit+k} = \pi_1 \varepsilon_{yt+k} + \pi_2 \eta_{it+k} + \pi_3 \omega^{\varepsilon}_{t+k} + \pi_4 \omega^{\eta}_{it+k} \tag{16}$$

and we find expressions for π_1, π_2, π_3 and π_4. In appendix C we show that for period $t+1$ this leads to the following expression,

$$\varepsilon_{cit+1} = A\varepsilon_{yt+1} + B\eta_{it+1} - \frac{\gamma}{2}A^2 \frac{\delta_2}{1-\delta_2-\delta_3}\omega^{\varepsilon}_{t+1} - \frac{\gamma}{2}B^2 \frac{\xi_2}{1-\xi_2-\xi_3}\omega^{\eta}_{it+1} \tag{17}$$

where $A = \sum_{j=0}^{\infty} A_j \alpha^j$ and $B = \sum_{j=0}^{\infty} B_j \alpha^j$ with A_j and B_j ($\forall j$) as defined above.[4] From confronting eq.(16) and eq. (17) we thus find $\pi_1 = A$, $\pi_2 = B$, $\pi_3 = -\frac{\gamma}{2}A^2 \frac{\delta_2}{1-\delta_2-\delta_3}$ and $\pi_4 = -\frac{\gamma}{2}B^2 \frac{\xi_2}{1-\xi_2-\xi_3}$.

By substituting this result into eq.(2) we obtain,

$$\begin{aligned}\Delta c_{it+1} &= \varkappa + \frac{\gamma}{2}A^2 E_{it}\varepsilon^2_{yt+1} + \frac{\gamma}{2}B^2 E_{it}\eta^2_{it+1} + A\varepsilon_{yt+1} + B\eta_{it+1} \\ &\quad -\frac{\gamma}{2}A^2 \frac{\delta_2}{1-\delta_2-\delta_3}\omega^{\varepsilon}_{t+1} - \frac{\gamma}{2}B^2 \frac{\xi_2}{1-\xi_2-\xi_3}\omega^{\eta}_{it+1}\end{aligned} \tag{18}$$

where $\varkappa = \frac{\gamma}{2}\left(\frac{\gamma}{2}A^2 \frac{\delta_2}{1-\delta_2-\delta_3}\right)^2 \sigma^2_{\omega\varepsilon} + \frac{\gamma}{2}\left(\frac{\gamma}{2}B^2 \frac{\xi_2}{1-\xi_2-\xi_3}\right)^2 \sigma^2_{\omega\eta}$. After aggregation (see appendix D) we obtain,

---

[4]It is easy to show that A and B are finite. For instance, for A, note that given that $\sum_{j=0}^{\infty}|A_j| < \infty$ the theory on convergence of series implies $\lim_{j\to\infty}\left|\frac{A_{j+1}}{A_j}\right| \leq 1$. Since $0 < \alpha < 1$ this implies that $\lim_{j\to\infty}\left|\frac{\alpha A_{j+1}}{A_j}\right| < 1$. Multiplying numerator and denominator by α^j gives $\lim_{j\to\infty}\left|\frac{A_{j+1}\alpha^{j+1}}{A_j\alpha^j}\right| < 1$. This condition implies that the series $A_0 + A_1\alpha + A_2\alpha^2 + ...$converges.

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$$\Delta c_{t+1} = \varkappa + \frac{\gamma}{2}A^2 E_t \varepsilon_{yt+1}^2 + \frac{\gamma}{2}B^2 E_t \eta_{t+1}^2 + A\varepsilon_{yt+1} \tag{19}$$
$$-\frac{\gamma}{2}A^2 \frac{\delta_2}{1-\delta_2-\delta_3}\omega_{t+1}^\varepsilon - \frac{\gamma}{2}B^2 \frac{\xi_2}{1-\xi_2-\xi_3}\omega_{t+1}^\eta$$

where $\Delta c_{t+1} = n^{-1}\sum_{i=1}^n \Delta c_{it+1}$, $E_t \varepsilon_{yt+1}^2 = E\left[\varepsilon_{yt+1}^2|\Omega_t\right]$, $E_t \eta_{t+1}^2 = E\left[\eta_{t+1}^2|\Omega_t\right]$ with $\eta_{t+1}^2 = n^{-1}\sum_{i=1}^n \eta_{it+1}^2$, and where $\omega_{t+1}^\eta = n^{-1}\sum_{i=1}^n \omega_{it+1}^\eta$. Note that Ω_t is the aggregate information set for which we have $\Omega_t \subset \Omega_{it}$ ($\forall i$). From eq. (19) we note that the change in aggregate consumption from period t to $t+1$ is determined by the shock in aggregate labour income ε_{yt+1} (as in the standard certainty equivalence case). Then, it is also determined by two income uncertainty terms. Aggregate income uncertainty is captured by the conditional variance of aggregate labour income shocks $E_t \varepsilon_{yt+1}^2$. Its effect on consumption depends on the degree of risk aversion γ and on the parameters of the aggregate income process. The effect on aggregate consumption of individual-specific income uncertainty is captured by the term $E_t \eta_{t+1}^2$. Its effect also depends on the degree of risk aversion γ and on the characteristics of the individual-specific part of income. Finally, the shocks ω_{t+1}^ε and ω_{t+1}^η capture the revisions in variance forecasts of both labour income shocks and enter the equation with a negative sign. Suppose for instance $\omega_{t+1}^\varepsilon > 0$, then the change in consumption from $t+2$ on will be higher because consumers update their expected variance $E_{t+1}\varepsilon_{yt+2}^2$. To accommodate the larger slope of the consumption path without violating the budget constraint, period $t+1$ consumption must fall. The more persistent the effect of the shocks ω_{t+1}^ε, that is the closer $\delta_2 + \delta_3$ approaches 1, the longer it will take before the consumption slope returns to its original level and the stronger is the necessary adjustment in period $t+1$ consumption.

Preliminary estimations suggest that over the sample period aggregate labour income follows a random walk (with drift), namely $y_{t+1} = \mu + y_t + \varepsilon_{yt+1}$. This implies that, in eq.(4), we have $\pi(L) = \pi^*(L) = 1$ leading to $A = 1$ so that eq.(19) now becomes,

$$\Delta c_{t+1} = \varkappa + \frac{1}{2}\gamma E_t \varepsilon_{yt+1}^2 + \frac{1}{2}\gamma B^2 E_t \eta_{t+1}^2 + \varepsilon_{yt+1} + \omega_{t+1} \tag{20}$$

where $\omega_{t+1} = -\frac{\gamma}{2}\frac{\delta_2}{1-\delta_2-\delta_3}\omega^\varepsilon_{t+1} - \frac{\gamma}{2}B^2\frac{\xi_2}{1-\xi_2-\xi_3}\omega^\eta_{t+1}$. Thus, given the random walk assumption for aggregate labour income, an income shock ε_{yt+1} leads to a one-for-one change in permanent income and thus in consumption.

4 Methodology.

4.1 Empirical specification and state space representation.

In this section we present our empirical specification. While aggregate income risk is modelled through a $GARCH(1,1)$ process on aggregate labour income shocks, the contribution to aggregate consumption of consumer-specific income risk is modelled as an unobserved component. We estimate the following system,

$$\Delta c_{t+1} = \frac{1}{2}\gamma h_{t+1} + \psi_{t+1} + \varepsilon_{yt+1} + \overline{\varepsilon}_{ct+1} \tag{21}$$

$$\Delta y_{t+1} = \mu + \varepsilon_{yt+1} \tag{22}$$

$$h_{t+1} \equiv E_t\varepsilon^2_{yt+1} = \delta_1 + \delta_2\varepsilon^2_{yt} + \delta_3 h_t \tag{23}$$

$$\psi_{t+1} \equiv \varkappa + \frac{1}{2}\gamma B^2 E_t\eta^2_{t+1} = \varphi_1 + \varphi_2\psi_t + \varphi_3 x_t \tag{24}$$

$$\overline{\varepsilon}_{ct+1} = \varepsilon_{ct+1} + \theta\varepsilon_{ct} \tag{25}$$

The consumption equation is given in eq.(21). First, the change in aggregate consumption depends positively on aggregate income risk, namely $h_{t+1} = E_t\varepsilon^2_{yt+1}$. Second,

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the change in consumption also depends on an unobserved component ψ_{t+1} which encompasses consumer-specific income uncertainty $E_t\eta_{t+1}^2$.[5] It also encompasses the constant $\varkappa$ which cannot be identified since it cannot be distinguished from the constant that is potentially present in the term $E_t\eta_{t+1}^2$. Third, given the random walk assumption for aggregate labour income given in eq.(22), the theoretical model derived in section 3 (see eq.(20)) predicts that every shock in labour income is permanent and leads to a one for one change in consumption. Therefore the error term ε_{yt+1} enters the consumption equation with coefficient equal to 1. Fourth, as far as the error term $\overline{\varepsilon}_{ct+1}$ is concerned, we note that it contains revisions in income variance forecasts ω_{t+1} but that it may also contain transitory consumption and measurement error. As can be seen in eq.(25) $\overline{\varepsilon}_{ct+1}$ is assumed to follow an MA(1) process where ε_{ct+1} is white noise and where $-1 < \theta < 1$ (i.e. if a white noise term is added to consumption in levels to capture measurement error or transitory consumption, an MA(1) term is found in the first difference of consumption, see Deaton 1992, p.97).

Eq.(23) is the $GARCH(1,1)$ specification for labour income shocks.[6] The conditional variance of the income shocks ε_{yt+1} is given by h_{t+1} and is a function of a constant, its past value h_t and the past income shock squared ε_{yt}^2. Note that for the positivity restriction $h_{t+1} > 0$ to hold (for all t) sufficient conditions are $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$. Moreover to assure that h_{t+1} is stationary, the restriction $\delta_2 + \delta_3 < 1$ must hold.

As can be seen in eq.(24) the unobserved component ψ_{t+1} is assumed to depend on a constant φ_1, on its own past ψ_t where $-1 < \varphi_2 < 1$ and on a predetermined variable x_t. Note that theory suggests that $\psi_{t+1} > 0$ for all t. As is the case for $E_t\varepsilon_{yt+1}^2$ the term $E_t\eta_{t+1}^2$ is in fact deterministic since the variance is modelled conditionally on information up to and including time t. No error term enters the conditional variance term. While $E_t\varepsilon_{yt+1}^2$ is observed however, $E_t\eta_{t+1}^2$ is unobserved because, contrary to $E_t\varepsilon_{yt+1}^2$, it has no link to an (aggregate) observable process. Note that if $\varphi_3 = 0$ the estimated state

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[5] Note that while γ is identified as the coefficient on $E_t\varepsilon_{yt+1}^2$, B is unidentified.

[6] It follows in a straightforward fashion from eq.(5) in the theoretical model. To see this note that a $GARCH(1,1)$ model can be written as an $ARCH(\infty)$ model. Eq.(5) can be written as $\varepsilon_{yt+1}^2 = \delta_1(1 - \delta_3)^{-1} + \delta_2(1 - \delta_3 L)^{-1}\varepsilon_{yt}^2 + \omega_{t+1}^\varepsilon$. From this we note that $E_{it}\varepsilon_{yt+1}^2 = E_t\varepsilon_{yt+1}^2$ given that $\Omega_t \subset \Omega_{it}$ ($\forall i$).

ψ_{t+1} is time-invariant.[7] To identify time variation in ψ_{t+1} we use previous results in the literature to choose variables to include in x_t. We include in x_t both the change in the unemployment rate (see e.g. Carroll 1992) and the change in the trend of the personal transfers to GDP ratio (see e.g. Hubbard et al. 1995 and Engen and Gruber 2001). We use the *trend* change in the transfer rate to reduce the effect of the cyclical component of transfers since this component is strongly correlated with the unemployment rate. For descriptive statistics, a description and the sources of all variables used we refer to table 1 and appendix E.

We write eqs.(21)-(25) as a Gaussian linear state space system with GARCH effects (see Harvey et al. (1992) and Kim and Nelson 1999, chapter 6) where the state vector is S_{t+1},

$$m_{t+1} = Z_{t+1}S_{t+1} + \varepsilon_{t+1} \tag{26}$$

$$S_{t+1} = T_{t+1}S_t + \pi_{t+1} \tag{27}$$

with

$$\varepsilon_{t+1|\Omega_t} \sim N(0, H_{t+1}) \tag{28}$$

$$\pi_{t+1|\Omega_t} \sim N(0, Q_{t+1}) \tag{29}$$

$$S_0 \sim N(E(S_{t+1}), V(S_{t+1})) \tag{30}$$

where

$$m_{t+1} = \begin{bmatrix} \Delta c_{t+1} & \Delta y_{t+1} \end{bmatrix}', S_{t+1} = \begin{bmatrix} 1 & \varepsilon_{yt+1} & \varepsilon_{ct+1} & \varepsilon_{ct} & \psi_{t+1} \end{bmatrix}', \varepsilon_{t+1} = \begin{bmatrix} 0 & 0 \end{bmatrix}',$$

$$H_{t+1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, Z_{t+1} = \begin{bmatrix} \frac{1}{2}\gamma h_{t+1} & 1 & 1 & \theta & 1 \\ \mu & 1 & 0 & 0 & 0 \end{bmatrix}, \pi_{t+1} = \begin{bmatrix} 0 & \varepsilon_{yt+1} & \varepsilon_{ct+1} & 0 & 0 \end{bmatrix}',$$

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[7]The state obtained from filtering then equals its initial value, i.e the unconditional mean (see below).

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$$
T_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ (\varphi_1 + \varphi_3 x_t) & 0 & 0 & 0 & \varphi_2 \end{bmatrix}, Q_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & h_{t+1} & 0 & 0 & 0 \\ 0 & 0 & \sigma_c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, E(S_{t+1}) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \left(\frac{\varphi_1 + \varphi_3 \overline{x}}{1 - \varphi_2}\right) \end{bmatrix},
$$

$$
diag(V(S_{t+1})) = \begin{bmatrix} 0 & \left(\frac{\delta_1}{1-\delta_2-\delta_3}\right) & \sigma_c^2 & \sigma_c^2 & \left(\frac{\sigma_x^2 \varphi_3^2}{1-\varphi_2^2}\right) \end{bmatrix}',
$$

where σ_c^2 is the variance of ε_{ct+1}, where $\overline{x}$ is the sample mean and σ_x^2 is the sample variance of x_t. Since all states in S_{t+1} are covariance-stationary the initial conditions given by eq.(30) are non-diffuse. The unconditional means and variances of the states, $E(S_{t+1})$ and $V(S_{t+1})$, initialize the system.[8]

The $GARCH$ effects h_{t+1} complicate the otherwise standard state space framework since h_{t+1} and thus Q_{t+1} is a function of the unobserved state ε_{yt+1}. Harvey et al. (1992) suggest to replace h_{t+1} in the system by $h_{t+1}^* = \delta_1 + \delta_2 \varepsilon_{yt}^{*2} + \delta_3 h_t^*$ where the unobserved ε_{yt}^2 is replaced by its conditional expectation $\varepsilon_{yt}^{*2} = E_t \varepsilon_{yt}^2$. Note that we can write $E_t \varepsilon_{yt}^2 = (E_t \varepsilon_{yt})^2 + E_t \left[(\varepsilon_{yt} - E_t \varepsilon_{yt})^2\right]$.[9] From the period t Kalman filter recursions[10] we obtain $E_t \varepsilon_{yt} = E_t S_t[2,1]$ and $E_t \left[(\varepsilon_{yt} - E_t \varepsilon_{yt})^2\right] = V_t S_t[2,1]$. Thus, for given parameter values, given h_t^* (which is initialized by the unconditional variance of ε_{yt+1}) and given the Kalman filter output from period t, namely $E_t(S_t)$ and $V_t(S_t)$, we can calculate h_{t+1}^* and the

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[8] Note that to apply the method proposed by Harvey et al. (1992) the conditional distributions of the errors in the state space model are assumed to be Gaussian. Given that ε_{yt+1} follows a GARCH process, its unconditional distribution is of course not normal (see Hamilton 1994, p.662).

[9] Note that the variance of a stochastic variable z can be written as $V(z) = E(z^2) - (E(z))^2$. Thus $E(z^2) = V(z) + (E(z))^2$.

[10] The Kalman filter recursions are (for period t):

$E_t(S_t) = E_{t-1}(S_t) + V_{t-1}(S_t)Z_t' F_t^{-1}(m_t - Z_t E_{t-1}(S_t))$

$V_t(S_t) = V_{t-1}(S_t) - (V_{t-1}(S_t)Z_t')F_t^{-1}(V_{t-1}(S_t)Z_t')'$

$E_t(S_{t+1}) = T_{t+1}E_t(S_t)$

$V_t(S_{t+1}) = T_{t+1}V_t(S_t)T_{t+1}' + Q_{t+1}$

where $F_t = Z_t V_{t-1}(S_t)Z_t' + H_t$

system matrices Q_{t+1} and Z_{t+1}. These make it possible to calculate $E_t(S_{t+1})$, $V_t(S_{t+1})$ and $E_{t+1}(S_{t+1})$, $V_{t+1}(S_{t+1})$, and so on... .

When reporting our results we present graphs of the unobserved component series and of the GARCH series.

4.2   Parameter estimation.

As noted by Harvey et al. (1992) the Kalman filter discussed in the previous section allows us to construct an approximate likelihood function. We use a Bayesian approach to parameter estimation by combining this likelihood with prior parameter information. By maximizing the sum of the sample log likelihood and the log of the prior parameter distributions we obtain the mode of the posterior parameter distribution. More formally, suppose that $m = \begin{bmatrix} m_1' & ... & m_T' \end{bmatrix}'$ (with m_{t+1} as defined in section 4.1) and $\Phi = \begin{bmatrix} \theta & \gamma & \mu & \delta_1 & \delta_2 & \delta_3 & \varphi_1 & \varphi_2 & \varphi_3 & \sigma_c^2 \end{bmatrix}'$ is the parameter vector. Denote the prior parameter density by $p(\Phi)$, the (sample) likelihood by $p(m|\Phi)$ and the posterior parameter distribution by $p(\Phi|m)$. Then the mode of the posterior parameter distribution is given by $\widehat{\Phi}^o = \arg\max\left[\ln p(\Phi|m)\right] = \arg\max\left[\ln p(\Phi) + \ln p(m|\Phi)\right]$. The corresponding Hessian-based parameter covariance matrix is obtained as $\widehat{V}^o = \left(\left[-\frac{\partial^2 \ln p(\Phi)}{\partial\Phi\partial\Phi'} - \frac{\partial^2 \ln p(m|\Phi)}{\partial\Phi\partial\Phi'}\right]_{\Phi=\widehat{\Phi}^o}\right)^{-1}$. The mode and Hessian form the basis of the importance sampling approach which is used to obtain (means, variances and percentiles of) posterior parameter distributions. Importance sampling is discussed in the next section.

As far as the priors are concerned we impose priors on the drift parameter μ, on the coefficient of absolute risk aversion γ and on the GARCH parameters δ_2 and δ_3. A prior for μ (mean and standard error) is obtained from a preliminary estimation of eq.(22) which gives mean 58 and standard deviation 7. Given the unrestricted range of values μ could take in theory, the prior distribution of μ is assumed to be normal.

A plausible range of values for the coefficient of *relative* risk aversion is $(0.5, 10)$. Given that the variable c_t (as defined in appendix F) varies over the sample period from 7500 to

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21000 with a mean of 13600 a plausible prior for γ (> 0) is given by a gamma distribution with mean .0003 and standard error .0001.

We also use priors for the parameters δ_2 and δ_3 because the estimation of a GARCH model for labour income shocks may be affected by the presence of two outliers in the series for labour income changes. These outliers considerably affect the tails (kurtosis) of the distribution of this series (see figure 1 and table 1). The quadratic form of the GARCH specification tends to extremely magnify outliers in the estimated conditional variance series for income shocks. One way of dealing with this problem is to decrease the weight of the most recent shock ε_{yt}^2 (i.e. impose a "low" prior on δ_2 in the estimation) and increase the weight of h_t (i.e. impose a "high" prior on δ_3 in the estimation). We therefore proceed as follows. Prior estimation of eqs.(22)-(23) separately by maximum likelihood gives a significant estimate for δ_2 of about 0.5 and a value for δ_3 of almost 0. We therefore first estimate the state space system with a prior for δ_2 with mean 0.5 (and standard deviation 0.2) and a prior for δ_3 with mean 0.1 (and standard deviation 0.05). Note that since $0 < \delta_2, \delta_3 < 1$ we use beta distributions as prior distributions. Second, we check the robustness of our results if we reduce the weight of ε_{yt}^2 in h_{t+1} by imposing a prior for δ_2 with mean 0.1 (and standard deviation 0.05) and a prior for δ_3 with mean 0.5 (and standard deviation 0.2).

For the remaining parameters we have no useful prior knowledge so we impose diffuse priors. Note that besides these priors we do not impose parameter restrictions when estimating the mode. Given an appropriate choice of starting values no numerical problems are encountered. Parameter restrictions (e.g stationarity restrictions on the parameters of the $GARCH$ process) are imposed for importance sampling however. This is discussed in the next section.

4.3   Importance sampling.

We use importance sampling with sequential updating to obtain posterior parameter distributions and posterior states (see Bauwens et al 1999 chapter 3). For given m the posterior

state distribution is determined by knowledge of the posterior parameter distribution, so that we can restrict interest to quantities $\overline{X}$ of the form,

$$\overline{X} = \int_\Phi X(\Phi)p(\Phi|m)d\Phi \tag{31}$$

where $X(\Phi)$ is some function of the parameter vector Φ. Since the posterior parameter distribution $p(\Phi|m)$ is unknown, we use $g(\Phi|m)$ as an importance density. Now we write eq.(31) as,

$$\overline{X} = \int_\Phi X(\Phi)\frac{p(\Phi|m)}{g(\Phi|m)}g(\Phi|m)d\Phi \tag{32}$$

which, by using Bayes' law, can be rewritten as,

$$\overline{X} = \frac{\int_\Phi X(\Phi)z^g(\Phi,m)g(\Phi|m)d\Phi}{\int_\Phi z^g(\Phi,m)g(\Phi|m)d\Phi} = \frac{E_g\left[X(\Phi)z^g(\Phi,m)\right]}{E_g\left[z^g(\Phi,m)\right]} \tag{33}$$

where E_g denotes the expectations operator with respect to $g(\Phi|m)$ and $z^g(\Phi,m) = \frac{p(\Phi)p(m|\Phi)}{g(\Phi|m)}$.

We set $g(\Phi|m) = N(\widehat{\Phi}^s, \xi\widehat{V}^s)$ as an importance density where $\widehat{\Phi}^s$ and $\widehat{V}^s$ are sequentially updated matrices and where ξ is a tuning constant (see Bauwens et al 1999). At the start of the sampling process we set $\widehat{\Phi}^s = \widehat{\Phi}^o$ and $\widehat{V}^s = \widehat{V}^o$ where $\widehat{\Phi}^o$ is the mode of the posterior parameter distribution and $\widehat{V}^o$ is the corresponding Hessian-based covariance matrix (see section 4.2). By taking draws Φ^i for $i = 1, ..., n$ from $g(\Phi|m)$ we estimate $\overline{X}$ by,

$$\widehat{X} = \frac{\sum_{i=1}^n X(\Phi^i)z^i}{\sum_{i=1}^n z^i} \tag{34}$$

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where $z^i = \frac{p(\Phi^i)p(m|\Phi^i)}{g(\Phi^i|m)}$. Parameter draws from $g(\Phi|m)$ that violate parameter restrictions imposed by the model are discarded.[11] Posterior parameter means are calculated from eq.(34) as $\widehat{\Phi} = \frac{\sum_{i=1}^n \Phi^i z^i}{\sum_{i=1}^n z^i}$. Posterior parameter covariance matrices are then calculated as $\widehat{V}(\Phi|m) = \frac{\sum_{i=1}^n (\Phi^i)(\Phi^i)'z^i}{\sum_{i=1}^n z^i} - \widehat{\Phi}\widehat{\Phi}'$. We then set $\widehat{\Phi}^s = \widehat{\Phi}$ and $\widehat{V}^s = \widehat{V}$ and the sampling process is repeated. We repeat this sequential sampling process until the coefficient of variation of the weights z^i is sufficiently reduced (see Bauwens et al. 1999, chapter 3). Further, the error bounds of the parameter means (see Bauwens et al. 1999 chapter 3, p78) indicate that the approximations of the parameter means obtained through the sampling process are of good quality (only for one parameter is the error bound somewhat high yet it is still below the "critical" threshold reported in Bauwens et al.). Note that in all cases convergence is achieved with 3 or 4 updates of the importance density when setting $n = $ 20 000 and $\xi = 1.2$. The final coefficients of variation of the weights and the error bounds of the parameter means are not reported but the results are available from the author upon request.

We further report the means, the variances and percentiles of the final posterior parameter distributions. Note that the $100k\%$ percentile of the posterior parameter distribution is $\Phi^{[m]}$ taken from the ordered sequence $\Phi^{[i]}$ of Φ^i for which $\frac{\sum_{i=1}^m z^{[i]}}{\sum_{i=1}^n z^{[i]}} \approx k$ where $z^{[i]}$ is the sequence of z^i associated to $\Phi^{[i]}$. Note, finally, that the distributions of the posterior states (in particular, state means and state variances) are calculated by running the Kalman filter (as described in section 4.1) using the posterior parameter means.

5   Results.

In tables 2 and 3 the estimation results are presented for different priors for δ_2 and δ_3. More specifically, we have $(\delta_2, \delta_3) = (0.5, 0.1)$ in table 2 and $(\delta_2, \delta_3) = (0.1, 0.5)$ in table 3. Note that for all cases x_t contains the first difference in the trend of personal personal current transfers to GDP Δt_t as well as the change in the unemployment rate Δu_t. The

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[11] We reject draws that violate $\gamma > 0$, $\sigma_c^2 > 0$, $-0.9 < \varphi_2 < 0.9$, $-0.9 < \theta < 0.9$, $\delta_1 > 0$, $\delta_2 > 0$, $\delta_3 > 0$ and $\delta_2 + \delta_3 < 1$

corresponding parameter vector is $\varphi_3 = \begin{bmatrix} \varphi_3^t & \varphi_3^u \end{bmatrix}$. For descriptive statistics, description and sources of these variables we refer to table 1 and appendix E.

From both tables we note first that the modes, means and medians of the posterior distributions of all parameters are of equal magnitude. This is an indication that the distributions are rather symmetric. Note that there is negative autocorrelation in the error term of the consumption function which can be indicative of "noise" in the level of consumption. If we look at the $GARCH$ part of the system we note from table 2 that the posterior means of the parameters δ_2 and δ_3 are close to the prior means while the posterior standard errors are considerably smaller. The data thus puts much weight on the ARCH term. To control whether this is due only to the outliers in labour income we also estimate the system with a high prior for δ_3 (table 3). From the estimated conditional variance series presented in figure 2 we note that the peaks are flattened considerably in the case presented in table 3 where priors are used to shift the weight from the ARCH term to the GARCH term. From a comparison of tables 2 and 3 we note however that that this has little effect on the parameters other than δ_1 , δ_2 and δ_3 . The reason for this is that the variation in the conditional variance series is insufficient to explain much of the variation in consumption changes given the estimated values of the risk aversion parameter γ. Aggregate income risk explains not even 1% of the variance of changes in aggregate consumption. Also, given the magnitude of the estimates for γ, the average conditional variance of aggregate income is much too small to be in accordance with the average change in consumption over the sample period. While somewhat disappointing these results are entirely in line with the general presumption that aggregate consumption and income growth are not volatile enough to cause consumption growth under plausible values for risk aversion (see Deaton, 1992 and Gourinchas and Parker, 2001).

Based on our theoretical model, the unobserved component is expected to reflect, at least partially, consumer-specific income uncertainty at the aggregate level. We note, first, that the constant φ_1 is positive. Second, the posterior estimates for φ_2 suggest that there is negative autocorrelation in this component justifying ex post our unobserved components approach. The mean of the posterior distribution of φ_3^u is positive (which is

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in accordance with what we expect on a theoretical basis) but its standard error is rather large so that zero values are present between the percentiles 5 and 95 of the distribution. The change in the trend of the personal transfers to GDP ratio, on the other hand, has a negative effect on the change in consumption. From table 2 we can derive that if Δt_t rises with 25% of its average value then Δc_{t+1} decreases with almost 5% of its average value (see table 1 for descriptive statistics of all variables). In terms of our model this effect seems to indicate that transfers received by consumers diminish consumer-specific uncertainty. This is in line with the literature (see sections 1 and 2). In figure 3 we present the estimated unobserved state ψ_t with 90% confidence bands. The unobserved component largely follows the change in the trend of the personal transfers to GDP ratio. In figure 4 this component is compared to the trend in the change of consumption. Both trends move together rather closely from the eighties onward suggesting that the trend in the transfers received by consumers may be a good candidate to (partially) explain lower frequency movements in the change in consumption in the US in the second part of the sample. Does our estimated unobserved component coincide with existing results on idiosyncratic income risk ? Storesletten et al. (2004) use both panel and macro data to calculate idiosyncratic risk and find evidence that it is strongly countercyclical, i.e. higher in recessions. This result is also reported in Parker and Preston (2002). Given the small effect of the change in the unemployment rate (i.e. a proxy for the business cycle) in our results we do not confirm this finding. In the next section we investigate whether this conclusion changes when we extend our model to allow for an effect of income changes on consumption changes.

So, while in the next section we tackle "excess sensitivity" of consumption to current income, in the remainder of this section we discuss "excess smoothness" (see Deaton 1992 for an extensive discussion of both puzzles). From table 1 it is clear that changes in consumption are less volatile than changes in labour income. Yet the finding that labour income is well described by a random walk process suggests that consumption should respond fully to every income shock (i.e our model suggests that consumption changes one-for-one in response to shocks in labour income). This implies that, in theory, consumption

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changes should be as volatile as income changes. As this is not the case, the variances of both sides of eq.(21) can only be reconciled if there is negative correlation between some of the variables included as regressors in this equation. We find that there is in fact a significant negative correlation (unreported) between the estimated states ε_{ct+1} and ε_{yt+1}. Since ε_{ct+1} contains the period $t+1$ shocks in the variance of labour income shocks ($\omega_{t+1}^{\varepsilon}$ and ω_{t+1}^{η}) and since these shocks enter the consumption equation with a negative sign, a positive correlation between these variance shocks and ε_{yt+1} could result in the finding of a negative correlation between ε_{ct+1} and ε_{yt+1}. We can therefore interpret the finding of negative correlation between the estimated states ε_{ct+1} and ε_{yt+1} as empirical support for Caballero's theoretical claim that "excess smoothness" is explainable when income shocks and income variance shocks are positively correlated.

6 Extension: rule-of-thumb consumption.

6.1 Extended specification.

Due to liquidity constraints (see Campbell and Mankiw 1990) or myopia (see Flavin 1985) some consumers may not consume according to the model derived in section 3. We assume that a fraction ρ (with $0 \leq \rho \leq 1$) of consumers simply consume their disposable income in each period. To make the extension of our basic model analytically tractable we make the assumption that per capita labour income is identical for both consumer types. Consider the following expression for aggregate (per capita) consumption changes,

$$\Delta c_{t+1} = \rho \Delta y_{t+1}^{d} + (1-\rho)\left[\varkappa + \frac{1}{2}\gamma E_t \varepsilon_{yt+1}^2 + \frac{1}{2}\gamma B^2 E_t \eta_{t+1}^2 + \varepsilon_{yt+1} + \omega_{t+1}\right] \tag{35}$$

where y_{t+1}^{d} is aggregate disposable income and where $\omega_{t+1} = -\frac{\gamma}{2}\frac{\delta_2}{1-\delta_2-\delta_3}\omega_{t+1}^{\varepsilon}$

$-\frac{\gamma}{2}B^2\frac{\xi_2}{1-\xi_2-\xi_3}\omega_{t+1}^{\eta}$. This equation reduces to eq.(20) if $\rho = 0$. Consistent with the model of section 3 the variable y_{t+1}^{d} can be written as the sum of aggregate labour income and aggregate capital income in the economy (i.e aggregate disposable income),

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$$y_{t+1}^d = y_{t+1} + rw_t^f \tag{36}$$

where $w_t^f = n^{-1} \sum_{i=1}^n w_{it}^f$ and where n is the total number of consumers in the economy. From this and given the random walk assumption for y_{t+1}, note that $\Delta y_{t+1}^d = E_t \Delta y_{t+1}^d + \varepsilon_{yt+1}$. Therefore we can write,

$$\Delta c_{t+1} = \rho E_t \Delta y_{t+1}^d + \varepsilon_{yt+1} + (1-\rho)\left[\varkappa + \frac{1}{2}\gamma E_t \varepsilon_{yt+1}^2 + \frac{1}{2}\gamma B^2 E_t \eta_{t+1}^2 + \omega_{t+1}\right] \tag{37}$$

Empirically, eqs.(22), (23), (24) and (25) do not change while eq.(21) is replaced by,

$$\Delta c_{t+1} = \frac{1}{2}\gamma(1-\rho)h_{t+1} + \rho E_t \Delta y_{t+1}^d + \psi_{t+1} + \varepsilon_{yt+1} + \overline{\varepsilon}_{ct+1} \tag{21'}$$

where the unobserved component is now defined as $\psi_{t+1} \equiv (1-\rho)\varkappa + \frac{1}{2}\gamma(1-\rho)B^2 E_t \eta_{t+1}^2$. The variable $E_t \Delta y_{t+1}^d$ is obtained as the fitted value from a preliminary regression of per capita disposable income changes on a number of variables that are suggested by Campbell and Mankiw (1990). We refer to appendix E for details. The changes to the state space system are minimal. Only the matrix Z_{t+1} is different. It is now given by

$$Z_{t+1} = \begin{bmatrix} \frac{1}{2}\gamma(1-\rho)h_{t+1} + \rho E_t \Delta y_{t+1}^d & 1 & 1 & \theta & 1 \\ \mu & & 1 & 0 & 0 & 0 \end{bmatrix}$$. There is one additional parameter to be estimated, namely ρ. A Bayesian prior for ρ is obtained from Campbell and Mankiw (1990, table 2 row 9). The mean of ρ is 0.41 with standard error 0.09. The prior distribution is assumed to be a beta distribution.

6.2 Results.

In tables 3 and 4 the results are presented for the estimation of eqs.(21') and (22)-(25) for different priors for δ_2 and δ_3. The conclusions drawn for the basic model remain valid for the extended model. The main difference compared to the results reported for the basic model is that the impact of the trend change of the transfers to GDP ratio, while still negative, is now smaller. Structural increases in the transfer to GDP rate seem to decrease the slope of the consumption path. Based on the model, the channel through which this occurs is through a reduction in consumer-specific income risk. Note again that the unobserved component seems to capture long-run movements of the change in consumption rather than high frequency movements. From looking at the point estimates we note that, compared to the basic case discussed in the previous section, it seems that the change in the unemployment rate now has a larger impact on the unobserved component. Thus it seems that our results are not completely in disagreement with the results of Storesletten et al.(2004) and Parker and Preston (2002). The higher frequency movements in the change of consumption are also explained by the income shock and by the anticipated changes in disposable income. Indeed, note that for the latter regressor the posterior mean of ρ is positive with a value of 0.2 which is lower than what is usually found for this excess sensitivity parameter in the literature. There are a number of potential reasons that can explain why the posterior mean is only half the prior mean. First, the sample period we consider is longer than the one considered by Campbell and Mankiw (1990) since it also contains the nineties. During this period further financial liberalization may have reduced the number of liquidity constrained consumers leading to lower excess sensitivity (see e.g. Bacchetta and Gerlach 1997). Peersman and Pozzi (2004) find that the excess sensitivity of consumption to anticipated disposable income is 0.27 in the US for the period 1969-1999. Second, most studies estimate the excess sensitivity parameter whilst improperly omitting income uncertainty terms. As noted also by Hahm and Steigerwald (1999) this produces an upward bias in the excess sensitivity parameter if the income uncertainty term and anticipated disposable income are positively correlated. Hahm and Steigerwald use survey responses to construct a proxy for income uncertainty and find, for

the US over the period 1981-1994, that income uncertainty increases consumption growth while the excess sensitivity of consumption growth to anticipated disposable income takes on a value of "only" 0.2.

7 Limitations of the approach and concluding remarks.

In the theoretical section of this paper the effect of income risk on consumption changes is decomposed into an aggregate and into a consumer-specific part. Analytical results are obtained under general $ARIMA$ processes for income and $GARCH(1,1)$ processes for income shocks. To obtain these results, like Caballero (1990), we assume that utility is of the $CARA$ type. In Caballero's paper this type of utility is necessary to obtain a closed form solution for the level of consumption. Since, in this paper, we are mainly interested in consumption changes (that is, in the Euler equation) the use of $CARA$ utility cannot be justified along these lines. However, it is easy to show that $CARA$ utility is necessary to make aggregation across consumers possible. Under $CARA$ utility consumption changes at the individual level are linear in the conditional variance of income shocks. Under $CRRA$ utility individual consumption growth is non-linear in the conditional variance of income shocks (see e.g. Banks et al. 2001). More specifically, under $CRRA$ utility the impact of the conditional variance of income shocks on individual consumption growth varies inversely with the individual-specific wealth level. This multiplicative structure makes aggregation difficult. Avoiding these problems by using $CARA$ utility instead of $CRRA$ utility comes at a price however. The fact that under $CARA$ utility the wealth level does not enter the Euler equation contradicts Carroll's (1992) model of buffer-stock savers. In Carroll's model (which uses $CRRA$ preferences) consumption growth is faster for households with low wealth (all other things equal) because they are building up a buffer against income shocks. An important implication of Carroll's model is that this mechanism gives a "precaution-based" explanation for the observed "excess sensitivity" of consumption to lagged / predicted income. He argues that when wealth is left out of the Euler equation the finding that lagged or predicted income growth positively affects

consumption growth can be explained by noting that low-wealth periods may coincide with rapid income growth periods (e.g. the periods of fastest income growth might be the early stages of a recovery when wealth is low because buffer stocks have been depleted during the downturn). The implication for our results is then that by using $CARA$ utility wealth is omitted from the Euler equation and observed "excess sensitivity" (as discussed in section 6) can partially[12] be caused by this omission. Basically the use of $CARA$ utility thus implies that the "excess sensitivity" parameter ρ need not be unrelated to precaution. While the estimates we find for ρ are lower than those found in cases where no time-varying income uncertainty terms enter the Euler equation (see section 6) they may still be too high because of the fact that, under $CARA$ utility, income uncertainty is not interacted with wealth.

Another assumption imposed to derive the theoretical results is the constancy of the real interest rate and its equality to the rate of time preference (contrary to the $CARA$ utility assumption this assumption is not strictly necessary to derive the model). This implies that intertemporal substitution effects caused by the (anticipated) interest rate are ruled out. While there is plenty of evidence that the ex ante real interest rate has no impact on consumption growth (see Hall 1988, Campbell and Mankiw 1990 or Ludvigson 1999 for more recent evidence) it is not clear whether this also holds when time-varying income uncertainty is taken into account. Parker and Preston (2002) shed some light on this issue by decomposing the predictable part of consumption growth into a part related to intertemporal substitution, a part reflecting preferences and a part due to incomplete markets (i.e. precaution and liquidity constraints). They find that there is a strong positive correlation between the incomplete markets component and the interest rate component. An implication of their finding is that adding a precautionary component to a regression of consumption growth on the anticipated real interest rate will tend to reduce rather than augment the effect of the real interest rate. Since the existing evidence suggests that this

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[12] We say "partially" because there is no evidence that the "buffer stock" model can by itself explain the magnitude of the observed "excess sensitivity" of aggregate consumption to income (see Ludvigson and Michaelides 2001)

effect is already small when precaution is not taken into account the restriction of zero intertemporal substitution in the presence of incomplete markets seems reasonable.

Empirically, the results of the $GARCH$ estimation seem to confirm the general presumption (see e.g. Gourinchas and Parker 2001) that aggregate income growth is not volatile enough to have a significant impact on consumption growth given realistic estimates for the coefficient of risk aversion. While aggregate income risk seems to have no impact on changes in consumption this is not so for the unobserved component which, according to the model, should reflect idiosyncratic risk at the aggregate level. The main problem here is of course that the unobserved component is a catch-all component. It may reflect idiosyncratic income uncertainty but it can also capture other components not included in the regression. First, the estimated constant in the unobserved component gives no information on the magnitude of idiosyncratic risk since it cannot be distinguished from the average change in consumption which may reflect components not related to risk. Second, the (negative) autocorrelation found in the unobserved component in our regression results may be due to idiosyncratic risk (i.e. idiosyncratic risk not driven by transfers and unemployment) but could as well reflect omitted variables that are unrelated to idiosyncratic risk. Besides the factors included in our estimations and besides the real interest rate we note that predictable consumption changes could be driven by nonseparabilities in the utility function. Examples are situations in which the marginal utility of consumption of non-durables and services is driven by durable consumption, by lagged consumption (habit formation) or by government consumption. While there is little evidence that these factors have an impact on consumption growth when income growth is added as a regressor (see e.g Campbell and Mankiw 1990), as in the case of the real interest rate, it is not clear whether this conclusion remains valid when time-varying income uncertainty is taken into account. The decomposition of Parker and Preston suggests that there is negative correlation between the component of predictable consumption growth related to precaution and the component related to preference shifts (which captures nonseparabilities in the utility function). An implication is then that the relevance of nonseparabilities (which are not taken into account in this paper) to explain consumption growth may be larger when a

precautionary term is added to the regression. When interpreting the results of this paper it is important to keep this caveat in mind.

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Appendix A: derivation of eq.(2).

We take a second-order Taylor expansion of $e^{-\gamma \Delta c_{it+1}}$ around $E_{it}\Delta c_{it+1}$ which gives the result (after taking expectations),

$$E_{it}e^{-\gamma \Delta c_{it+1}} = e^{-\gamma E_{it}\Delta c_{it+1}}\left[1 + \frac{\gamma^2}{2}E_{it}(c_{it+1} - E_{it}c_{it+1})^2\right] \tag{A1}$$

Substituting this into eq.(1) and then taking logs gives, after some rearrangements, eq.(2) in the text.

Appendix B: derivation of eq.(12).

We write eq.(2) for period $t+j$ as,

$$c_{it+j} = c_{it+j-1} + \frac{1}{2}\gamma E_{it+j-1}\varepsilon^2_{cit+j} + \varepsilon_{cit+j} \tag{B1}$$

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Writing eq.(B1) for period $t+j-1$, substituting this into eq.(B1) and re-iterating until period t gives,

$$c_{it+j} = c_{it} + \sum_{k=1}^{j} \varepsilon_{cit+k} + \sum_{k=1}^{j} \frac{\gamma}{2} E_{it+k-1} \varepsilon_{cit+k}^2 \qquad (B2)$$

Eq.(12) is obtained by adding to and subtracting from the RHS of eq.(B2) the term $\sum_{k=1}^{j} \frac{\gamma}{2} E_{it} \varepsilon_{cit+k}^2$.

Appendix C: derivation of eq.(17).

First, using the assumption that the errors ε_{yt+1}, η_{it+1}, $\omega_{t+1}^{\varepsilon}$ and ω_{it+1}^{η} are mutually uncorrelated and that $\omega_{t+1}^{\varepsilon}$ and ω_{it+1}^{η} have variances $\sigma_{\omega\varepsilon}^2$ and $\sigma_{\omega\eta}^2$ respectively we can, using eq.(16), write $E_{it+k-1} \varepsilon_{cit+k}^2 = E_{it+k-1} \left(\pi_1 \varepsilon_{yt+k} + \pi_2 \eta_{it+k} + \pi_3 \omega_{t+k}^{\varepsilon} + \pi_4 \omega_{it+k}^{\eta} \right)^2 =$

$\pi_1^2 E_{it+k-1} \varepsilon_{yt+k}^2 + \pi_2^2 E_{it+k-1} \eta_{it+k}^2 + \pi_3^2 \sigma_{\omega\varepsilon}^2 + \pi_4^2 \sigma_{\omega\eta}^2$. Similarly we can write $E_{it} \varepsilon_{cit+k}^2 =$

$\pi_1^2 E_{it} \varepsilon_{yt+k}^2 + \pi_2^2 E_{it} \eta_{it+k}^2 + \pi_3^2 \sigma_{\omega\varepsilon}^2 + \pi_4^2 \sigma_{\omega\eta}^2$. After subtracting the second result from the first we obtain

$$E_{it+k-1} \varepsilon_{cit+k}^2 - E_{it} \varepsilon_{cit+k}^2 = \pi_1^2 \left(E_{it+k-1} \varepsilon_{yt+k}^2 - E_{it} \varepsilon_{yt+k}^2 \right) + \pi_2^2 \left(E_{it+k-1} \eta_{it+k}^2 - E_{it} \eta_{it+k}^2 \right)$$
$$(C1)$$

Second, we find expressions for $E_{it+k-1} \varepsilon_{yt+k}^2 - E_{it} \varepsilon_{yt+k}^2$ and $E_{it+k-1} \eta_{it+k}^2 - E_{it} \eta_{it+k}^2$. We only present the derivation of $E_{it+k-1} \varepsilon_{yt+k}^2 - E_{it} \varepsilon_{yt+k}^2$ as the derivation of $E_{it+k-1} \eta_{it+k}^2 - E_{it} \eta_{it+k}^2$ is completely identical. Note that we can write eq.(5) as $\varepsilon_{yt+1}^2 = \delta_1 + (\delta_2 + \delta_3) \varepsilon_{yt}^2 - \delta_3 \omega_t^{\varepsilon} + \omega_{t+1}^{\varepsilon}$ and for period $t+k$ as $\varepsilon_{yt+k}^2 = \delta_1 + (\delta_2 + \delta_3) \varepsilon_{yt+k-1}^2 - \delta_3 \omega_{t+k-1}^{\varepsilon} + \omega_{t+k}^{\varepsilon}$. After repeated backward substitution we obtain,

$$\begin{aligned}
\varepsilon_{yt+k}^2 \;=\;& \delta_1\left(1+(\delta_2+\delta_3)+(\delta_2+\delta_3)^2+...+(\delta_2+\delta_3)^{k-1}\right) \qquad\qquad\text{(C2)}\\
&+\omega_{t+k}^\varepsilon+\omega_{t+k-1}^\varepsilon(\delta_2+\delta_3)^0\delta_2+\omega_{t+k-2}^\varepsilon(\delta_2+\delta_3)^1\delta_2\\
&+\omega_{t+k-3}^\varepsilon(\delta_2+\delta_3)^2\delta_2+...+\omega_{t+1}^\varepsilon(\delta_2+\delta_3)^{k-2}\delta_2\\
&-\omega_t^\varepsilon(\delta_2+\delta_3)^{k-1}\delta_3+(\delta_2+\delta_3)^k\varepsilon_{yt}^2
\end{aligned}$$

Taking expectations of eq.(38) with respect to info set Ω_{it+k-1} and info set Ω_{it} and subtracting the last result from the first we obtain,

$$E_{it+k-1}\varepsilon_{yt+k}^2-E_{it}\varepsilon_{yt+k}^2=\sum_{h=1}^{k-1}\delta_2(\delta_2+\delta_3)^{k-1-h}\omega_{t+h}^\varepsilon \qquad\qquad\text{(C3)}$$

Similarly we can write

$$E_{it+k-1}\eta_{it+k}^2-E_{it}\eta_{it+k}^2=\sum_{h=1}^{k-1}\xi_2(\xi_2+\xi_3)^{k-1-h}\omega_{it+h}^\eta \qquad\qquad\text{(C4)}$$

Using eqs.(C3) and (C4) into eq.(C1) and the result into eq.(15) we can write,

$$\sum_{j=1}^{\infty}\alpha^j\Big\{(j>2)\sum_{k=2}^{j}\frac{\gamma}{2}\pi_1^2\sum_{h=1}^{k-1}\delta_2(\delta_2+\delta_3)^{k-1-h}\omega_{t+h}^\varepsilon+(j>2)\sum_{k=2}^{j}\frac{\gamma}{2}\pi_2^2\sum_{h=1}^{k-1}\xi_2(\xi_2+\xi_3)^{k-1-h}\omega_{it+h}^\eta$$

$$+\sum_{k=1}^{j}\varepsilon_{cit+k}-\sum_{k=1}^{j}A_{j-k}^*\varepsilon_{yt+k}-\sum_{k=1}^{j}B_{j-k}^*\eta_{it+k}\Big\}=0 \qquad\qquad\text{(C5)}$$

This condition should be satisfied period-by-period since ε_{yt+1}, η_{it+1}, ω_{t+1}^ε, ω_{it+1}^η and ε_{cit+1} are white noise terms. This means that the sum of the terms in ε_{yt+1}, η_{it+1}, ω_{t+1}^ε,

ω^η_{it+1} and ε_{cit+1} equals 0 and so on for $t+2,...$ The sum of terms containing ω^ε_{t+1} in eq.(C5) is

$$\sum_{j=1}^{\infty} \alpha^j \left[\frac{\gamma}{2} \delta_2 \pi_1^2 \left((\delta_2 + \delta_3)^0 + (\delta_2 + \delta_3)^1 + (\delta_2 + \delta_3)^2 ... \right) \omega^\varepsilon_{t+1} \right]$$

or $\frac{\alpha}{1-\alpha} \frac{\gamma}{2} \pi_1^2 \frac{\delta_2}{1-\delta_2-\delta_3} \omega^\varepsilon_{t+1}$.

Similarly for ω^η_{it+1} we have $\frac{\alpha}{1-\alpha} \frac{\gamma}{2} \pi_2^2 \frac{\xi_2}{1-\xi_2-\xi_3} \omega^\eta_{it+1}$.

The sum of terms in ε_{cit+1} is given by $\sum_{j=1}^{\infty} \alpha^j \varepsilon_{cit+1} = \frac{\alpha}{1-\alpha} \varepsilon_{cit+1}$.

The terms in ε_{yt+1} are given by $\sum_{j=1}^{\infty} \alpha^j A^*_{j-1} \varepsilon_{yt+1}$ where we note, from the definition of the partial sum A^*_{j-1} in the main text, that $\sum_{j=1}^{\infty} \alpha^j A^*_{j-1} = \sum_{j=1}^{\infty} \alpha^j (A_0 + A_1 + ... + A_{j-1})$. It is easy to show that this expression can be written as $\frac{\alpha}{1-\alpha} \left(A_0 + A_1 \alpha + A_2 \alpha^2 + ... \right)$ so that for the terms in ε_{yt+1} we have $\frac{\alpha}{1-\alpha} \sum_{j=0}^{\infty} \alpha^j A_j \varepsilon_{yt+1} = \frac{\alpha}{1-\alpha} A \varepsilon_{yt+1}$ where $A = \sum_{j=0}^{\infty} \alpha^j A_j < \infty$ since $\sum_{j=0}^{\infty} A_j < \infty$.

Similarly for the terms in η_{it+1} we find $\frac{\alpha}{1-\alpha} \sum_{j=0}^{\infty} \alpha^j B_j \eta_{it+1} = \frac{\alpha}{1-\alpha} B \eta_{it+1}$ where $B = \sum_{j=0}^{\infty} \alpha^j B_j < \infty$ since $\sum_{j=0}^{\infty} B_j < \infty$.

Adding the terms in ε_{yt+1}, η_{it+1}, ω^ε_{t+1}, ω^η_{it+1} and ε_{cit+1} and setting equal to zero gives the result presented in eq.(17).

Appendix D: derivation of eq.(19).

Averaging eq.(18) over the n consumers gives,

$$
\begin{aligned}
\Delta c_{t+1} &= \varkappa + \frac{\gamma}{2} A^2 n^{-1} \sum_{i=1}^{n} E\left[\varepsilon_{yt+1}^2 | \Omega_{it}\right] \\
&\quad + \frac{\gamma}{2} B^2 n^{-1} \sum_{i=1}^{n} E\left[\eta_{it+1}^2 | \Omega_{it}\right] + A\varepsilon_{yt+1} \\
&\quad - \frac{\gamma}{2} A^2 \frac{\delta_2}{1-\delta_2-\delta_3} \omega^\varepsilon_{t+1} - \frac{\gamma}{2} B^2 \frac{\xi_2}{1-\xi_2-\xi_3} \omega^\eta_{t+1}
\end{aligned}
\tag{D1}
$$

where $\Delta c_{t+1} = n^{-1} \sum_{i=1}^{n} \Delta c_{it+1}$ and where we use $n^{-1} \sum_{i=1}^{n} \eta_{it+1} = 0$

and $n^{-1} \sum_{i=1}^{n} \omega_{it+1}^{\eta} = \omega_{t+1}^{\eta}$ to obtain the result. Note that for the aggregate information set in period t, Ω_t, we have $\Omega_t \subset \Omega_{it}$ $(\forall i)$. By taking expectations of the LHS and of the RHS of eq.(38) conditional on the information set Ω_t we obtain, after using the law of iterated expectations,

$$
\begin{aligned}
E\left[\Delta c_{t+1} | \Omega_t\right] &= \varkappa + \frac{\gamma}{2} A^2 n^{-1} \sum_{i=1}^{n} E\left[\varepsilon_{yt+1}^2 | \Omega_t\right] \\
&\quad + \frac{\gamma}{2} B^2 n^{-1} \sum_{i=1}^{n} E\left[\eta_{it+1}^2 | \Omega_t\right]
\end{aligned}
\tag{D2}
$$

Note that this result follows from the fact that we assume that ε_{yt+1} and $\omega_{t+1}^{\varepsilon}$ cannot be predicted with info set Ω_{it} for $i = 1, ..., n$. Since $\Omega_t \subset \Omega_{it}$ $(\forall i)$ these terms cannot be predicted with info set Ω_t either. Moreover, given that Ω_{it} cannot be used to forecast ω_{it+1}^{η}, Ω_t is of no use to forecast ω_{it+1}^{η} nor its sum over all consumers ω_{t+1}^{η}. Note further that the difference between Δc_{t+1} and $E\left[\Delta c_{t+1} | \Omega_t\right]$ (i.e. the period t "surprise" in the aggregate change in consumption) equals $A\varepsilon_{yt+1} - \frac{\gamma}{2} A^2 \frac{\delta_2}{1-\delta_2-\delta_3} \omega_{t+1}^{\varepsilon} - \frac{\gamma}{2} B^2 \frac{\xi_2}{1-\xi_2-\xi_3} \omega_{t+1}^{\eta}$. So, after adding $\Delta c_{t+1} - E\left[\Delta c_{t+1} | \Omega_t\right]$ to the RHS and LHS of eq.(38) and forcing the summation signs through the expectations operators we obtain eq.(19) in the text.

Appendix E: data.

Data are quarterly and the sample period is 1952:01-2001:02 (and 1953:01-2001:02 for the estimations with fitted disposable income in section 6). The beginning of the sample is determined by data availability. We take 2001:02 as the last data point because of outliers in the series for after-tax labour income and the unemployment rate in 2001:03 and 2001:04.

Data description.

c_t: per capita consumption on nondurables and services excluding shoes and clothing, seasonally adjusted, at annual rates, in 1996 dollars.

y_t: per capita after-tax labour income, seasonally adjusted, at annual rates, in 1996 dollars.

t_t: trend obtained from Hodrick-Prescott filter applied to personal current transfer receipts (current prices, seasonally adjusted, annual rates) to gdp (current prices, seasonally adjusted, annual rates) rate *in percent*.

u_t: unemployment rate *in percent*, seasonally adjusted.

y_t^d: per capita after-tax *total* personal income, seasonally adjusted, at annual rates, in 1996 dollars.

i_t: nominal 3 month T-bill rate, annual rate.

Δc_t: first difference in c_t (see table 1 for descriptive statistics).

Δy_t: first difference in y_t (see table 1 for descriptive statistics, see figure 1).

Δt_t: first difference in t_t (see table 1 for descriptive statistics).

Δu_t: first difference in u_t (see table 1 for descriptive statistics).

$E_{t-1}\Delta y_t^d$: fitted series obtained from a least squares regression (with R^2=0.161) of the first difference of y_t^d , namely Δy_t^d, on a constant, on lags 1-3 of Δc_t, on lags 1-3 of Δy_t^d, on lags 1-3 of the first difference of i_t, namely Δi_t, and on lag 1 of the error correction term $c_t - y_t^d$. We refer to Campbell and Mankiw (1990) for a justification of these explanatory variables for Δy_t^d (see table 1 for descriptive statistics).

Data sources.

c_t, y_t: taken directly from updated datset for paper Lettau and Ludvigson (2001) on http://www.econ.nyu.edu/user/ludvigsons.

t_t: personal current transfer receipts (from table 2.1: personal income and its disposition) and gdp taken from US Department of Commerce (Bureau of Economic Analysis).

u_t: from Bureau of Labor Statistics (Economagic website).

y_t^d: after-tax total personal income in current prices, seasonally adjusted at annual rates, is taken from US Department of Commerce (Bureau of Economic Analysis). Deflator used is deflator for nondurables and services (minus clothing and shoes), seasonally adjusted, constructed from tables 2.3.4 and 2.3.5 US Department of Commerce (Bureau of Economic Analysis) with baseyear adjustment (from baseyear 2000 to baseyear 1996). Population is taken from Bureau of Labor Statistics (Economagic website).

i_t: from IMF, International Financial Statistics.

Tables.

Table 1: Descriptive statistics, US data, 1952:01-2001:02 (see appendix C for description and sources).

	mean	std. dev.	maximum	minimum	skewness	kurtosis
Δc_t	66.953	62.502	220.927	-186.841	-.627	4.379
Δy_t	58.489	96.684	512.242	-410.224	-.073	7.862
Δt_t	.041	.048	.131	-.038	.209	2.150
Δu_t	.005	.380	1.667	-.966	1.296	6.614
$E_{t-1}\Delta y_t^d$	74.868	62.656	240.283	-153.493	-.682	4.594

Note: descriptive statistics for the series $E_{t-1}\Delta y_t^d$ are calculated over the sample period 1953:01-2001:02.

Table 2: Estimation results, eqs.(21)-(25), US data, 1952:01-2001:02 ("high" prior for δ_2 and "low" prior for δ_3).

coeff	prior distribution			posterior distribution					
	type	mean	sdv	mode	mean	sdv	percentiles		
							5	50	95
θ	diffuse	-	-	-0.4141	-0.4278	0.1544	-0.7382	-0.4037	-0.2138
γ	gamma	.0003	.0001	.00027	.00028	.0001	.00014	.00027	.00046
μ	normal	58	7	59.228	59.082	4.6945	51.381	59.127	66.735
δ_1	diffuse	-	-	5271.5	5345.9	857.69	4045.2	5280.7	6856.9
δ_2	beta	.5	.2	0.5142	0.5155	0.1429	0.2911	0.5118	0.7606
δ_3	beta	.1	.05	0.0511	0.0709	0.0395	0.0200	0.0634	0.1467
φ_1	diffuse	-	-	109.79	108.66	26.467	66.178	108.27	153.41
φ_2	diffuse	-	-	-0.4606	-0.5446	0.2349	-0.865	-0.581	-0.109
φ_3^t	diffuse	-	-	-312.15	-323.78	138.91	-558.22	-318.66	-103.18
φ_3^u	diffuse	-	-	7.9867	6.742	16.928	-21.352	6.810	34.635
σ_c^2	diffuse	-	-	5614.2	5762.8	874.36	4307.3	5760.8	7208.8

Note: sdv denotes standard deviation.

Table 3: Estimation results, eqs.(21)-(25), US data, 1952:01-2001:02 ("low" prior for δ_2 and "high" prior for δ_3).

coeff	prior distribution			posterior distribution					
	type	mean	sdv	mode	mean	sdv	percentiles		
							5	50	95
θ	diffuse	-	-	-0.4114	-0.4221	0.1512	-0.7210	-0.3999	-0.2104
γ	diffuse	.0003	.0001	.00027	.00028	0.0001	.00014	.00027	.00046
μ	normal	58	7	58.993	58.934	4.9361	50.787	58.919	67.179
δ_1	diffuse	-	-	3965.4	4691.7	1433.5	2374.0	4683.8	7083.6
δ_2	beta	0.1	0.05	0.1400	0.1577	0.0600	0.0696	0.1518	0.2660
δ_3	beta	0.5	0.2	0.4209	0.3398	0.1600	0.0949	0.3270	0.6203
φ_1	diffuse	-	-	109.73	108.18	26.681	66.594	107.36	153.97
φ_2	diffuse	-	-	-0.4621	-0.5406	0.2347	-0.8646	-0.5718	-0.1101
φ_3^t	diffuse	-	-	-309.81	-321.75	138.20	-555.77	-316.86	-103.12
φ_3^u	diffuse	-	-	8.574	7.118	16.873	-20.731	7.1915	34.458
σ_c^2	diffuse	-	-	5625.3	5784.5	862.04	4350.8	5772.5	7226.5

Note: sdv denotes standard deviation.

Table 4: Estimation results, eqs.(21')-(25), US data, 1953:01-2001:02 ("high" prior for δ_2 and "low" prior for δ_3).

coeff	prior distribution			posterior distribution					
	type	mean	sdv	mode	mean	sdv	percentiles		
							5	50	95
θ	diffuse	-	-	-0.4318	-0.4375	0.1495	-0.7308	-0.4155	-0.2273
γ	diffuse	-	-	.00027	.00028	.0001	.00014	.00027	.00045
μ	normal	58	7	59.083	59.010	4.7225	51.335	58.971	66.745
ρ	beta	0.41	0.09	0.2052	0.2077	0.0483	0.1327	0.2050	0.2904
δ_1	diffuse	-	-	5192.6	5305.8	903.02	3944.1	5238.6	6873.6
δ_2	beta	0.5	0.2	0.5296	0.5229	0.1434	0.2920	0.5187	0.7659
δ_3	beta	0.1	0.05	0.0512	0.0716	0.0401	0.0196	0.0651	0.1481
φ_1	diffuse	-	-	83.655	72.032	25.878	31.669	70.696	116.57
φ_2	diffuse	-	-	-0.4277	-0.5309	0.2317	-0.8571	-0.5601	-0.1094
φ_3^t	diffuse	-	-	-230.32	-236.51	140.47	-472.42	-233.02	-12.278
φ_3^u	diffuse	-	-	19.0678	16.944	17.722	-12.311	17.079	45.808
σ_c^2	diffuse	-	-	5845.8	6062.2	934.67	4522.2	6040.3	7648.6

Note: sdv denotes standard deviation.

Table 5: Estimation results, eqs.(21')-(25), US data, 1953:01-2001:02 ("low" prior for δ_2 and "high" prior for δ_3).

coeff	prior distribution			posterior distribution					
	type	mean	sdv	mode	mean	sdv	percentiles		
							5	50	95
θ	diffuse	-	-	-0.4295	-0.4364	0.1536	-0.7453	-0.4120	-0.2229
γ	diffuse	-	-	.00027	.00028	.0001	.00014	.00027	.00045
μ	normal	58	7	58.814	58.908	4.8734	50.858	58.945	66.828
ρ	beta	0.41	0.09	0.2053	0.2087	0.0489	0.133	0.206	0.2928
δ_1	diffuse	-	-	4298.9	4976.3	1526.6	2466.8	4998.6	7500.6
δ_2	beta	0.1	0.05	0.1466	0.1641	0.0646	0.0692	0.1575	0.2784
δ_3	beta	0.5	0.2	0.3774	0.3042	0.1693	0.0610	0.2855	0.6097
φ_1	diffuse	-	-	83.290	71.248	25.603	31.611	70.004	116.01
φ_2	diffuse	-	-	-0.4258	-0.5278	0.2331	-0.8579	-0.5548	-0.1067
φ_3^t	diffuse	-	-	-227.73	-234.81	142.83	-475.09	-231.56	-4.145
φ_3^u	diffuse	-	-	19.504	17.274	17.680	-11.883	17.687	45.840
σ_c^2	diffuse	-	-	5855.9	6059.6	945.73	4462.9	6055.9	7615.0

Note: sdv denotes standard deviation.

Figures.

Figure 1: First difference of real per capita after-tax labour income, US, 1952:01-2001:02 (source: see appendix C)
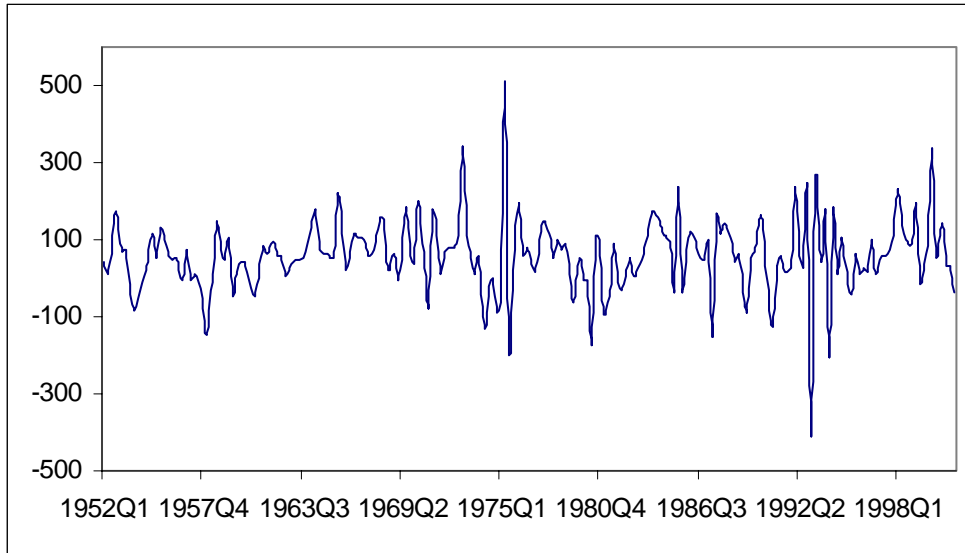

Figure 2: GARCH series with different priors on δ_2 and δ_3 (case tables 2 and 3)
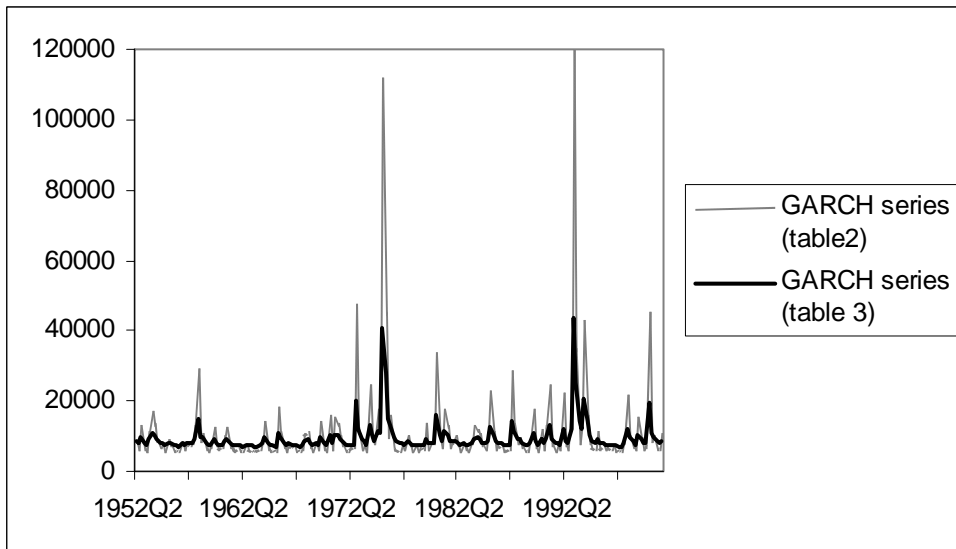
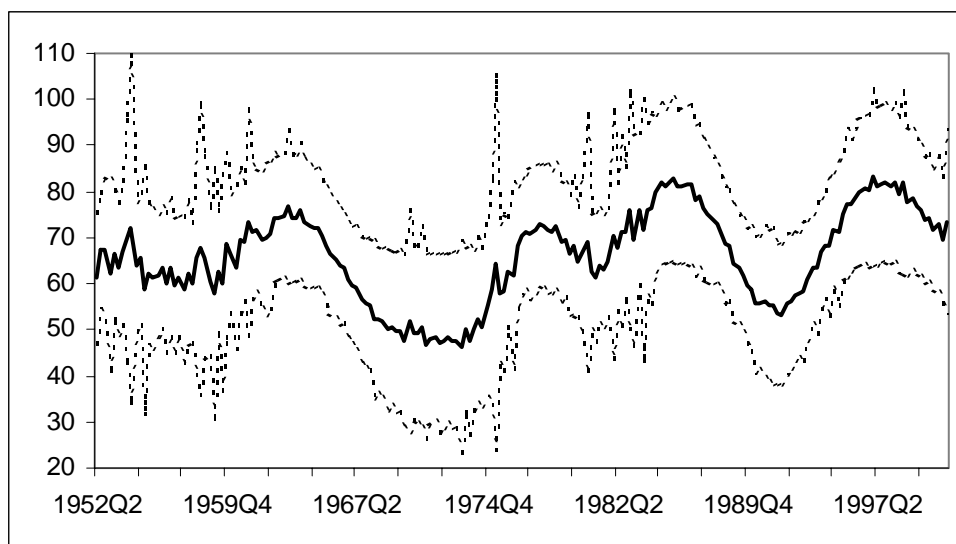Figure 3: Unobserved state with 90% confidence bands (case table 2)

Figure 4: Real per capita consumption changes, trend and unobserved state (case table 2)
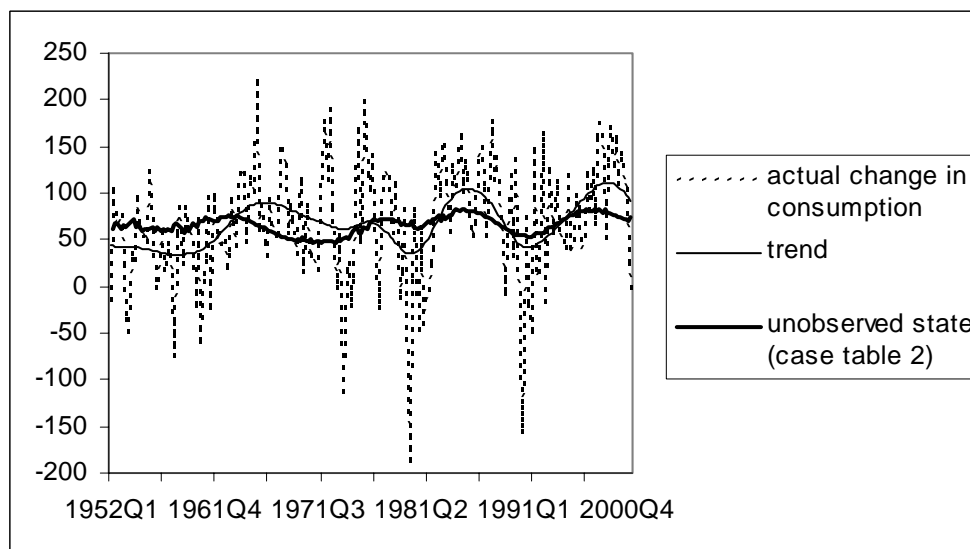
Figure 5: Unobserved state with 90% confidence bands (case table 4)
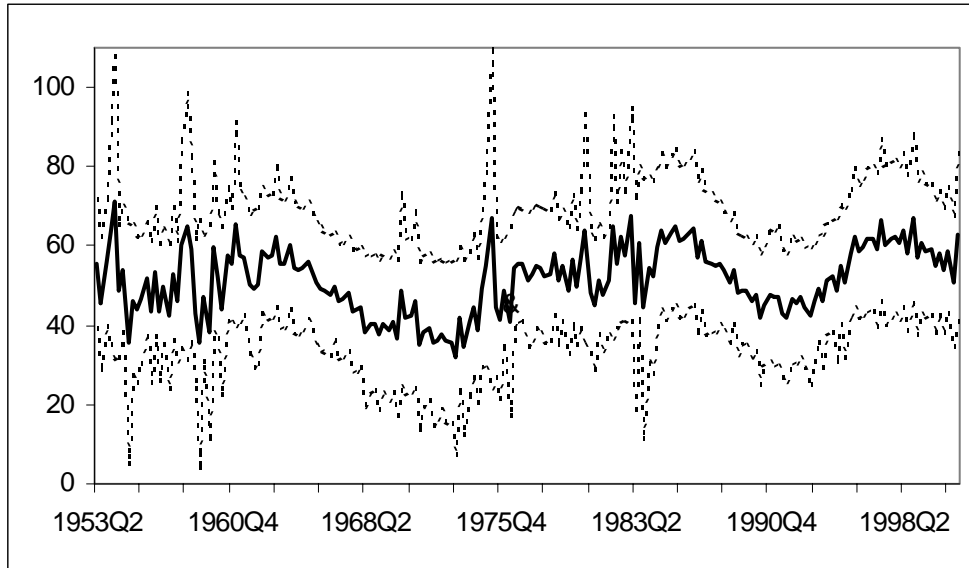

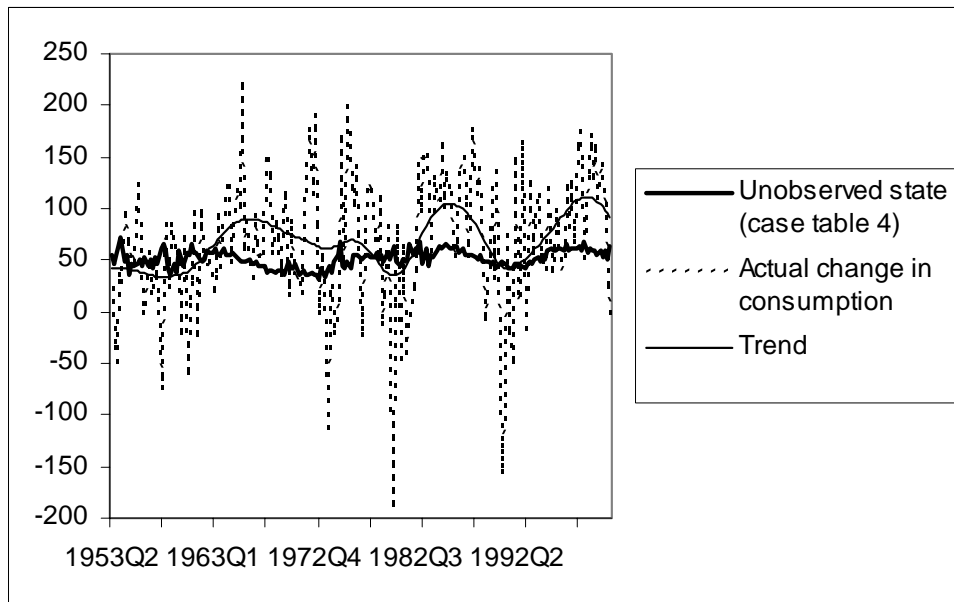Figure 6: Real per capita consumption changes, trend and unobserved state (case table 4)

Figure 7:

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