## Working paper research

n ${ }^{\circ} 94$ October 2006
Simulation, estimation and
welfare implications of monetary policies in a 3-country NOEM model

Joseph Plasmans Tomasz Michalak Jorge Fornero



## NATIONAL BANK OF BELGIUM

## WORKING PAPERS - RESEARCH SERIES

## SIMULATION, ESTIMATION AND WELFARE IMPLICATIONS OF MONETARY POLICIES IN A 3-COUNTRY NOEM MODEL

Joseph Plasmans (*)
Tomasz Michalak Jorge Fornero

The authors thank Bas van Aarle (University of Maastricht) and Raf Wouters (National Bank of Belgium) for interesting comments on a previous version of the paper.

[^0]
## Editorial Director

Jan Smets, Member of the Board of Directors of the National Bank of Belgium

## Statement of purpose:

The purpose of these working papers is to promote the circulation of research results (Research Series) and analytical studies (Documents Series) made within the National Bank of Belgium or presented by external economists in seminars, conferences and conventions organised by the Bank. The aim is therefore to provide a platform for discussion. The opinions expressed are strictly those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

## The Working Papers are available on the website of the Bank:

http://www.nbb.be

## Individual copies are also available on request to:

## NATIONAL BANK OF BELGIUM

## Documentation Service

boulevard de Berlaimont 14
BE - 1000 Brussels

Imprint: Responsibility according to the Belgian law: Jean Hilgers, Member of the Board of Directors, National Bank of Belgium.
Copyright © fotostockdirect - goodshoot
gettyimages - digitalvision
gettyimages - photodisc
National Bank of Belgium
Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.
ISSN: 1375-680X

## Editorial

On October 12-13, 2006 the National Bank of Belgium hosted a Conference on "Price and Wage Rigidities in an Open Economy". Papers presented at this conference are made available to a broader audience in the NBB Working Paper Series (www.nbb.be).

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium.


#### Abstract

In this paper we derive a microfounded macro New Keynesian model for open economies, be them large or small. We consider habit formation in consumption, sectoral linkages, domestic and foreign governments, tradable and non-tradable final and intermediate goods and imperfect pass-through in these sectors. Sticky nominal prices and wages are modeled in a Calvo way. The model economy is composed of a continuum of infinitely-lived consumers and producers for three regions (countries). Numerical simulations and econometric estimations are presented with a focus on a small open economy member of the EMU. Welfare implications of the involved price and wage rigidities are discussed.


JEL-code: E31, D21, F41, P24.

Keywords: New Keynesian open economy model, tradable and non-tradable sectors, final and intermediate goods, monetary policy rules, numerical simulations, Bayesian estimation, welfare implications.

## TABLE OF CONTENTS

1. Introduction ..... 1
2. A literature review ..... 2
3. Households ..... 3
3.1 Domestic households ..... 3
3.2 The domestic household optimization ..... 4
4. Governments ..... 5
5. Firms ..... 6
6. Equilibrium conditions ..... 7
7. Calvo staggered price and wage setting ..... 8
8. Net Foreign Assets ..... 10
9. Monetary policies ..... 12
10. Social welfare maximizing monetary policy rules ..... 14
11. Numerical simulations ..... 16
11.1 Technological shocks ..... 16
11.1.1 Technological shock to tradable final goods sector ..... 16
11.1.2 Technological shock to intermediate goods sectors ..... 17
11.2 Exchange rate shock ..... 17
11.3 Policy shock ..... 19
11.4 A preference shock to consumption ..... 19
12. Econometric estimations ..... 19
13. Concluding remarks ..... 22
Appendixes ..... 25
References ..... 63
National Bank of Belgium Working Paper Series ..... 67

## 1 Introduction

During the recent years the theoretical and empirical research in New Keynesian (NK) macroeconomics has been extended steadily and produced a whole new series of results and insights about the workings of the macroeconomy. Essential starting point of the NK approach is the explicit derivation of macroeconomic relationships from underlying microeconomic foundations. This principle is shared with New Classical macroeconomics, although the former includes a great deal of imperfections in the goods and labor markets.

Recently, much interest has also been devoted to modeling and testing effects and interactions produced by the foreign sector, e.g. rigidities of import and export pricing that might be of significant importance in an open economy. Moreover, fluctuations in import prices of intermediate goods such as oil and steel will have strong effects on domestic firms of an open economy. In that perspective, the exchange rate will play an important role in the transmission of such price fluctuations to the domestic economy. NK models with a worked out foreign sector are often referred to as New Open Economy Macroeconomics (NOEM) models.

This paper studies numerical simulations, econometric estimations and welfare implications of monetary policies taking account of nominal price and wage rigidities both on a small open economy being a member of a monetary union (MU), on the rest of that monetary union (RoMU) and on the rest of the world (RoW).

More specifically, we extend the NOEM model of Plasmans et al. (2006a), characterized by a detailed modeling of the consumer and producer decisions, to a three country setting. In particular, firms in our model produce two types of goods: final and intermediate. Both types of goods can be either tradable or non-tradable, depending on whether they can be traded internationally. Firms in every sector are assumed to be characterized by nested CES production functions whose arguments are technology, labor force and intermediate goods. ${ }^{1}$ Each firm is assumed to have some price-setting power on relevant markets, reflecting monopolistic competition. Consumers are assumed to purchase a bundle of domestically produced tradable and non-tradable final goods and imported final goods. Consumption allocation is assumed to be shaped by habit formation. Labor markets with sticky wages and financial markets complete the structure of the model of the domestic (small open) economy. Altogether, we distinguish 12 (final and intermediate goods) markets with which domestic firms are confronted. Foreign economies are modeled in a parallel manner so that import/export prices and quantities and other relevant variables are endogenized in our approach.

Such an extensive modeling of intermediate goods sectors is important especially in the context of exchange rate policies. Dellas (2005) points out that the presence of intermediate goods has vital consequences for the ability of monetary authorities to manipulate nominal exchange rates. When there is no production interdependence between countries (i.e. only consumption goods are traded) changes in a nominal exchange rate do not affect production costs. However, when there is trade in intermediate goods as in the real world, an exchange rate depreciation has adverse direct effects on the cost of domestic production. Consequently, it makes the exchange rate instrument less useful. ${ }^{2}$

As an application, we consider the case of The Netherlands (NL) as a member of the European Monetary Union (EMU), the latter being considered as the first large economy. Our choice is

[^1]driven by the availability of time series data of intermediate goods inputs and prices and also by the fact that NL has strong economic relationships with the RoW (second large economy, proxied by the USA). In this setting we compare possible European Central Bank (ECB) policies under the form of different specifications of a Taylor rule.

Numerical simulations and econometric estimations (e.g. Bayesian methods) will be analyzed to study the impact of shocks in such a three-country setting. More in detail, we study the impulse response functions (IRFs) to the following shocks: common and asymmetric positive technological shocks in the tradable (non-tradable) final goods sectors in different economies, an exchange rate shock, a monetary policy shock and a taste shock to consumption.

## 2 A literature review

There are not many papers that consider a NOEM setting with an MU. Pytlarczyk (2005) develops a two-region dynamic stochastic general equilibrium (DSGE) model for the euro area with a particular focus on analyzing the German economy within the EMU. The interesting elements of his work are (i) the utilization of disaggregated information in the estimation, i.e. national accounts data, along with the "synthetic" euro area data; (ii) the utilization of regime-switching models in a DSGE framework, which allows to use a longer data sample (prior to and during the EMU). He applies Bayesian techniques in estimation and explicitly takes account of the change in the monetary regime in the EMU.

Pierdzioch (2004) uses a two-country NOEM model of a monetary union to analyze the consequences of international financial market integration for the propagation of asymmetric productivity shocks. His model implies that business cycle volatility is higher the more integrated the capital markets of the member countries of the MU are.

In this MU setting we could study, for instance, the issue of welfare implications of monetary policies for a small open economy, which is a member of the MU, both considering different levels of price and wage rigidities between the small open economy and the RoMU. The most interesting reference for us is Ortega and Rebei (2006), since they treat the welfare implications in a multisectoral two-country setting. Following their spirit we characterize simple Taylor-type monetary policy reaction functions.

The welfare costs of nominal inertia are directly related to the welfare costs of business cycle fluctuations which have been at the center of macroeconomic research since the seminal paper by Lucas (1987). Lucas (2003) focused on the variability of consumption and has argued that the costs of these fluctuations are very small, roughly 0.07 percent of steady state consumption. His conclusion is that macroeconomists should set their research priorities on improving economic growth rather than on fine-tuning of the cycle. Recently, Galí et al. (2003) have built a measure of the costs of business cycle fluctuations centering around the gap between the marginal product of labor and the marginal rate of substitution between consumption and leisure, the so-called "efficiency gap". They calibrate a small number of parameters and take that measure to US data. Galí et al. (2003) show that the average welfare costs of business cycle fluctuations could very well be higher than what Lucas (2003) computes. Costs in the benchmark calibration of Galí et al. (2003) are 0.28 percent of steady state consumption and range up to 0.75 per cent. Variations in the efficiency gap arise endogenously in models with wage and price stickiness such as in the seminal work by Erceg et al. (2000). As pointed out by Galí et al. (2003), there are however a number of other frictions that could also contribute to the variance of this gap. Canzoneri et al. (2005) follow this discussion and calculate the welfare cost of nominal inertia in a New Neoclassical

Synthesis model with wage and price stickiness, capital formation, and empirically estimated rules for government spending and the central bank's interest rate policy. Their model is calibrated to US data and the cost of nominal inertia is calculated under two specifications of monetary policy.

Paustian (2004) considers the share of nominal rigidities in wages and prices in the average welfare costs of business cycle fluctuations. He computes a quadratic approximation to agents' expected lifetime utility and evaluate welfare for different modeling schemes of nominal rigidities. His results indicate that the welfare loss depends on the way of modeling the stickiness, even with the same average duration of contracts. Calvo (1983) contracts can imply welfare costs that are up to 4 times higher than those implied by overlapping contracts in the spirit of Taylor (1980) or Wolman (1999). Furthermore, the sticky information framework of Mankiw and Reis (2002) may generate welfare costs that are smaller.

This paper is organized as follows. In Section 3 we model both domestic and foreign households. The households' decisions feature habit formation in consumption of tradable and non-tradable goods. Section 4 introduces domestic and foreign governments. In Section 5 we study the firms' decisions considering supply linkages in production by distinguishing intermediate goods from final goods and tradable from non-tradable goods. Equilibrium conditions are proposed in Section 6, while a discussion on net foreign assets follows in Section 7. Regarding price formation we consider the cases of price stickiness with Calvo-type staggered price and wage settings in Section 8. In Section 9, we discuss the institutional setup of the monetary policy both in the MU and the RoW. Section 10 concentrates on welfare maximizing monetary policy rules utilizing a second order approximation. Some numerical simulations demonstrating the functioning of the model will be discussed in Section 11, while econometric estimations are reported in Section 12. Finally, some concluding remarks are contained in Section 13.

## 3 Households

### 3.1 Domestic households

In the model to be constructed the following notation is used: the country of origin is always denoted as a subscript, whereas the country of destination (consumption) is denoted as a superscript. In this paper, $h$ stands for the home country (the small open economy) being a member of the MU, $U$ stands for the RoMU (first foreign country) and $W$ stands for the RoW (second foreign country).

Each household $i$ in home country $h$ is assumed to face a consumer budget constraint (CBC) of the following kind:

$$
\begin{align*}
& P_{t}^{C, h}(i) C_{t}^{h}(i)+E_{t}\left[Q_{h, t, t+1}^{h} B_{h, t+1}^{h}(i)+\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}(i)+S_{t} \hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}(i)\right]+M_{t+1}^{h}(i) \\
& \leq\left(1-\tau_{h}\right) W_{h, t}(i) L_{h, t}(i)+T_{h, t}^{h}(i)+B_{h, t}^{h}(i)+B_{U, t}^{h}(i)+S_{t} B_{W, t}^{h}(i)+M_{t}^{h}(i)+\mathcal{B}_{h, t}(i), \tag{1}
\end{align*}
$$

where $\tau_{h}$ is a proportional tax rate on nominal labor income $W_{h, t}(i) L_{h, t}(i) \equiv \sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}$, $T_{h, t}^{h}(i)$ is a nominal lump-sum transfer from the government to household $i$ and $\mathcal{B}_{h, t}(i)$ are consumer $i$ 's net benefits from firms' ownership, where it is assumed that the whole entrepreneurial benefits accrue to domestic consumers, $M_{t}^{h}(i)$ are nominal money balances at the beginning of period $t$, $C_{t}^{h}(i)$ is total consumption and $P_{t}^{C, h}(i)$ is its aggregate price. Moreover, according to Woodford (2003) and Ambler et al. (2003, 2004), $Q_{h, t, t+1}^{h} \equiv\left(1+r_{t, t+1}\right)^{-1}, \hat{Q}_{U, t, t+1}^{h} \equiv\left(1+\hat{r}_{U, t, t+1}^{h}\right)^{-1}$ and
$\hat{Q}_{W, t, t+1}^{h} \equiv\left(1+\hat{r}_{W, t, t+1}^{h}\right)^{-1}$ are for domestic consumers the one-period ahead stochastic discount factors for (nominal) domestic and foreign asset payoffs, respectively. ${ }^{3}$ Hence, the expected (nominal) market price at date $t$ of an asset portfolio, held at the beginning of period $t+1$, is given by $E_{t}\left[Q_{h, t, t+1}^{h} B_{h, t+1}^{h}(i)+\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}(i)+S_{t} \hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}(i)\right]$, with $S_{t}$ the nominal exchange rate (the price of one unit of foreign currency in domestic currency) in that period. ${ }^{4}$

The specification of CBCs for $U$ and $W$ can be found in Appendix A (see (42) and(43)).

### 3.2 The domestic household optimization

We specify household $i$ 's (from country $h$ ) intertemporal utility function as: ${ }^{5}$

$$
\begin{equation*}
E_{0}\left[\sum_{t=0}^{\infty}\left(\beta_{h}\right)^{t}\left[\frac{1}{1-\sigma_{h}}\left(\frac{C_{t}^{h}(i)}{\left(C_{t-1}^{h}(i)\right)^{\kappa_{h}}}\right)^{1-\sigma_{h}}-\frac{\left(\sum_{m=F T, F N, V T, V N} L_{h, t}^{m}(i)\right)^{1+\phi_{h}}}{1+\phi_{h}}+\chi_{M, h} \frac{\left(\frac{M_{t+1}^{h}(i)}{P_{t}^{C, h}(i)}\right)^{1-\frac{1}{\chi_{h}}}}{\left(1-\frac{1}{\chi_{h}}\right)}\right]\right] \tag{2}
\end{equation*}
$$

where $\sigma_{h}>0$ is a parameter of constant relative risk aversion (CRRA) of domestic households in the home country being equal to the inverse of the intertemporal elasticity of substitution in consumption, $\kappa_{h}$ is the home consumers' habit persistence parameter and $\phi_{h}$ is the inverse of the intertemporal elasticity of work effort with respect to the real wage, $\frac{M_{t+1}^{h}(i)}{P_{t}^{C, h}(i)}$ is household $i$ 's real money balance at the beginning of period $t+1, \chi_{M, h}$ is a constant and $\chi_{h}$ is the elasticity of substitution of real money balances in the home country. ${ }^{6}$

Maximization of the expected discounted sum (2) of household $i$ 's utility flows subject to its budget constraint (1) and the relevant demand for labor with respect to $C_{t}^{h}(i), L_{h, t}^{m}(i)$ for $m=F T, F N, V T, V N, M_{t+1}^{h}(i)$, and household $i$ 's nominal domestic and foreign asset portfolios (consisting of bonds and stocks) at the beginning of period $t+1, B_{h, t+1}^{h}(i), B_{U, t+1}^{h}(i)$ and $B_{W, t+1}^{h}(i),{ }^{7}$ results in the following first order conditions (FOCs) (see Appendix A in Plasmans et al. (2006a)):

[^2]\[

$$
\begin{gather*}
\Gamma_{t}^{h}(i)=\frac{U_{C_{t}^{h}(i)}}{P_{t}^{C, h}(i)}=\frac{\left(C_{t}^{h}(i)\right)^{-\sigma_{h}}}{P_{t}^{C, h}(i)\left(C_{t-1}^{h}(i)\right)^{\kappa_{h}\left(1-\sigma_{h}\right)}}-E_{t}\left[\beta_{h} \kappa_{h} \frac{\left(C_{t+1}^{h}(i)\right)^{1-\sigma_{h}}}{P_{t}^{C, h}(i)\left(C_{t}^{h}(i)\right)^{\left[\kappa_{h}\left(1-\sigma_{h}\right)+1\right]}}\right],  \tag{3}\\
\frac{\varrho_{L m, h}}{\varrho_{L m, h}-1}\left(L_{h, t}(i)\right)^{\phi_{h}}=\Gamma_{t}^{h}(i)\left(1-\tau_{h, t}\right) W_{h, t}^{m}(i) \text { for } m=F T, F N, V T, V N,  \tag{4}\\
E_{t}\left[\Gamma_{t}^{h}(i) Q_{h, t, t+1}^{h}-\beta_{h} \Gamma_{t+1}^{h}(i)\right]=0,  \tag{5}\\
E_{t}\left[\Gamma_{t}^{h}(i) \hat{Q}_{U, t, t+1}^{h}-\beta_{h} \Gamma_{t+1}^{h}(i)\right]=0,  \tag{6}\\
E_{t}\left[\Gamma_{t}^{h}(i) \hat{Q}_{W, t, t+1}^{h} S_{t}-\beta_{h} \Gamma_{t+1}^{h}(i) S_{t+1}\right]=0,  \tag{7}\\
\frac{\chi_{M, h}}{P_{t}^{C, h}(i)}\left[\frac{M_{t+1}^{h}(i)}{P_{t}^{C, h}(i)}\right]^{-\frac{1}{\chi_{h}}}=-\Gamma_{t}^{h}(i)+\beta_{h} E_{t}\left[\Gamma_{t+1}^{h}(i)\right], \tag{8}
\end{gather*}
$$
\]

where the Lagrange multiplier $\Gamma_{t}^{h}(i)$ of the budget constraint is the marginal utility of household $i$ 's wealth and $\varrho_{L m, h}$ is the intertemporal elasticity of substitution for different types of labor demanded by firms in sector $m=F T, F N, V T, V N$ and which should be larger than one. Equation (3) implies that the marginal utility of a particular household's consumption good equals the marginal utility of its wealth. Equations (4) relate the household $i$ 's marginal utility of leisure to its marginal utility of the nominal wage in every sector. Equations (5), (6) and (7) refer to the household $i$ 's intertemporal decision involving decisions on home and foreign financial assets and (8) is the optimal real money balances demand.

Each foreign consumer in countries $U$ and $W$ solves a similar problem as presented above. More in detail, foreign consumers must satisfy a set of FOCs analogous to (3-8) in the optimum, where $\Gamma_{t}^{U}(i)$ and $\Gamma_{t}^{W}(i)$ are then the marginal utilities of foreign household $i$ 's (in country $U$ and in country $W$, respectively) nominal consumption and where we can derive foreign stochastic consumption Euler relationships.

## 4 Governments

The government budget constraint (GBC) in the home country is given by:

$$
\begin{align*}
& \int_{0}^{1}\left(M_{t+1}^{h}(i)-M_{t}^{h}(i)\right) d i+\tau_{h}\left[\sum_{m=F T, F N, V T, V N} \int_{0}^{1} W_{h, t}^{m}(i) L_{h, t}^{m}(i) d i\right]-\int_{0}^{1} T_{h, t}^{h}(i) d i \\
&+E_{t} {\left[Q_{h, t, t+1}^{h} \int_{0}^{1} B_{h, t+1}^{h}(i)\right]-\int_{0}^{1} B_{h, t}^{h}(i) d i+E_{t}\left[\hat{Q}_{h, t, t+1}^{U} \int_{0}^{1} B_{h, t+1}^{U}(i) d i\right] } \\
&-\int_{0}^{1} B_{h, t}^{U}(i) d i+E_{t}\left[\hat{Q}_{h, t, t+1}^{W} \int_{0}^{1} B_{h, t+1}^{W}(i) d i\right]-\int_{0}^{1} B_{h, t}^{W}(i) d i=0 . \tag{9}
\end{align*}
$$

Government revenues include personal tax, money creation and domestic and foreign bond incomes whereas expenditures include nominal lump-sum transfers.

Foreign governments solve a similar problem as the domestic one.

## 5 Firms

Following Smets and Wouters (2003), Galí and Monacelli (2002), Choi and Jung (2003), Jung (2004) and Lindé et al. (2004), it is assumed that suppliers of inputs are price setters under profit maximization and demanders for inputs are price takers under cost minimization. Goods markets are characterized by monopolistic competition and profit maximization in the output markets. An alternative interpretation is that the firms' problem in open economies can be disentangled in two stages: one for final goods and one for intermediate goods. The inspiration for an intermediate goods versus final goods distinction can be found in Clarida et al. (2002), Smets and Wouters (2003) and Plasmans et al. (2006a). ${ }^{8}$ We assume that intermediate and final goods and tradable and non-tradable goods are produced in a sectoral framework. Domestic goods are assumed to be produced in each sector by a continuum of monopolistically competitive firms, indexed by $j \in[0,1]$, while imported final and intermediate goods are bought (at marginal cost) in the foreign market by importing firms (in import sectors $M F$ and $M V$ ), repacked and sold in the domestic market, also under monopolistic competition. Hence, firms in the monopolistically competitive import goods sectors turn the foreign goods, bought at their given world price marginal cost, into differentiated final and intermediate import goods.

Each domestic firm $j$ is assumed to produce one differentiated final or intermediate good which can be either tradable or non-tradable. Six main sectors are distinguished: 1. the tradable final goods sector $(F T)$; 2. the non-tradable final goods sector $(F N) ; 3$. the tradable intermediate goods sector $(V T) ; 4$. the non-tradable intermediate goods sector $(V N)$. Additionally, we assume that all tradable goods are imported, repacked or rebranded by import sectors $M F$ and $M V$ for final and intermediate goods, respectively.

In final goods production sectors, $m=F T, F N$, we assume variable returns to scale and a nested CES production technology with labor, $L_{t}^{m}(j)$, and intermediate goods, $V_{t}^{m}(j)$, as inputs: ${ }^{9}$

$$
\begin{align*}
Y_{h, t}^{m}(j) & \equiv\left(\Omega_{h, t}^{m} Z_{t}^{m, h}(j)\right)^{\varpi_{m, h}} \\
& \equiv\left\{\Omega_{h, t}^{m}\left[v_{L m, h}^{\frac{1}{\gamma_{m, h}}}\left(L_{h, t}^{m}(j)\right)^{\frac{\gamma_{m, h}-1}{\gamma_{m, h}}}+\left(1-v_{L m, h}\right)^{\frac{1}{\gamma_{m, h}}}\left(V_{t}^{m, h}(j)\right)^{\frac{\gamma_{m, h}-1}{\gamma_{m, h}}}\right]^{\frac{\gamma_{m, h}}{\gamma_{m, h}-1}}\right\}^{\varpi_{h, m}} \tag{10}
\end{align*}
$$

where
$V_{t}^{m, h}(j) \equiv\left[\begin{array}{c}\nu_{m 1, h}^{\frac{1}{\chi_{m, h}}}\left(V_{h, t}^{T, m, h}(j)\right)^{\frac{\chi_{m, h}-1}{\chi_{m, h}}}+\nu_{m 2, h}^{\frac{1}{\chi_{m, h}}}\left(V_{h, t}^{N, m, h}(j)\right)^{\frac{\chi_{m, h}-1}{\chi_{m, h}}} \\ +\nu_{m 3, h}^{\frac{1}{\chi_{m, h}}}\left(V_{U, t}^{T, m, h}(j)\right)^{\frac{\chi_{m, h}-1}{\chi_{m, h}}}+\left(1-\nu_{m 1, h}-\nu_{m 2, h}-\nu_{m 3, h}\right)^{\frac{1}{\chi_{m, h}}}\left(V_{W, t}^{T, m, h}(j)\right)^{\frac{\chi_{m, h}-1}{\chi_{m, h}}}\end{array}\right]^{\frac{\chi_{m, h}}{\chi_{m, h}-1}}$,
for $j \in[0,1]$, where $\gamma_{m, h}$ and $\chi_{m, h}$ are the (home country) intertemporal elasticities of substitution between the final goods inputs and different intermediate goods inputs, respectively. Both must be larger than one.

In (10-11) $\nu_{L m, h}$ is the share of labor input in total input, while $\nu_{m 1, h}, \nu_{m 2, h}$ and $\nu_{m 3, h}$ are the shares of domestically produced tradable and non-tradable intermediate goods and imported

[^3]intermediate goods produced in $U$ in total intermediate goods input of that firm in sector $m ; \Omega_{h, t}^{m}$ is a domestic technology shock in period $t$, which, according to learning characteristics, is assumed to satisfy an $A R(1)$ process: $\ln \Omega_{h, t}^{m} \equiv \omega_{h, t}^{m}=\rho_{m, \omega, h} \omega_{m, t-1}^{h}+\xi_{m, \omega, h, t}$ with $-1<\rho_{m, \omega, h}<1$ and $\xi_{m, \omega, h, t}$ an independently and identically distributed (iid) error term; in addition, $\varpi_{m, h}$ is the returns to scale parameter in sector $m$ in the home country. ${ }^{10}$

Intermediate goods firms $m=V T, V N$ use labor according to the production function:

$$
\begin{equation*}
V_{h, t}^{m}(j) \equiv\left(\Omega_{h, t}^{m} Z_{t}^{m, h}(j)\right)^{\varpi_{m, h}}=\left\{\Omega_{h, t}^{m} L_{h, t}^{m}(j)\right\}^{\varpi_{m, h}} \tag{12}
\end{equation*}
$$

where $\varpi_{m, h}$ is the returns to scale parameter in the production of intermediate goods in sector $m$ of the home country.

It is assumed that foreign producers in countries $U$ and $W$ face production functions similar to (10-12).

## 6 Equilibrium conditions

Since sectoral firms are able to exert monopolistic power, they are able to charge different prices according to the market they serve. First, domestic producers of tradable final goods sell their output at 3 markets: domestic market of home tradable consumption and foreign (wholesale) markets of home consumption goods. Hence, a tradable final goods company sells its output by charging the vector of prices $\mathbf{P}_{h, t}^{F T}(j)=\mathbf{P}_{h, t}^{C T}(j) \equiv\left[P_{h, t}^{C T, h}(j), P_{h, t}^{X, C T, U}(j), P_{h, t}^{X, C T, W}(j)\right]^{\prime}$, where $P_{h, t}^{X, C T, U}(j)$ and $P_{h, t}^{X, C T, W}(j)$ are export prices of domestic tradable consumption goods. Second, a domestic producer of non-tradable final goods sells her products only to domestic consumers charging a price $\mathbf{P}_{h, t}^{F N}(j) \equiv P_{h, t}^{C N}(j)$. Third, an intermediate tradable goods company sells its output to 2 markets $(m=F T, F N)$ in each of the three economies $h, U$ and $W$, charging the following set of prices: $\mathbf{P}_{h, t}^{V T}(j) \equiv\left[P_{h, t}^{V T, F T, h}(j), P_{h, t}^{V T, F N, h}(j), P_{h, t}^{X, V T, F T, U}(j), P_{h, t}^{X, V T, F N, U}(j), P_{h, t}^{X, V T, F T, W}(j)\right.$, $\left.P_{h, t}^{X, V T, F N, W}(j)\right]^{\prime}$, where, similarly to the case of final goods, $X$ denotes exports of domestic tradable intermediate goods. Fourth, an intermediate non-tradable goods company sells its output to 2 sectors, $m=F T, F N$, of the home country, or $\mathbf{P}_{h, t}^{V N}(j) \equiv\left[P_{h, t}^{V N, F T, h}(j), P_{h, t}^{V N, F N, h}(j)\right]^{\prime}$. Finally, a firm in the domestic import sector of final tradable goods charges $\mathbf{P}_{t}^{M F, h}(j) \equiv\left[P_{U, t}^{C T, h}(j), P_{W, t}^{C T, h}(j)\right]^{\prime}$ and a firm in the domestic import sector of intermediate tradable goods charges $\mathbf{P}_{t}^{M V, h}(j) \equiv$ $\left[P_{U, t}^{V T, F T, h}(j), P_{U, t}^{V T, F N, h}(j), P_{W, t}^{V T, F T, h}(j), P_{W, t}^{V T, F N, h}(j)\right]^{\prime} .^{11}$

Moreover, we require market clearing equilibrium conditions for all firms. The demand for the tradable final good produced by company $j$ is the sum of domestic and foreign consumption demands for this good. The demand for the non-tradable final good is equal to domestic demand for this good. The demand for the tradable intermediate good is the sum of the domestic and foreign demands for this good in the final goods sectors $m=F T, F N$ and the demand for the non-tradable intermediate good is the sum of the domestic demands for this good in all sectors.

Let aggregate demand for tradable goods produced by domestic company $j$ be defined as $C_{h, t}^{T, h}(j) \equiv \int_{0}^{1} C_{h, t}^{T, h}(j, i) d i, C_{h, t}^{T, U}(j) \equiv \int_{0}^{1} C_{h, t}^{T, U}(j, i) d i$ and $C_{h, t}^{T, W}(j) \equiv \int_{0}^{1} C_{h, t}^{T, W}(j, i) d i$. Hence, we can define the vector of relevant demands for the products of company $j$ in sector $F T$ as $\mathbf{D}_{h, t}^{F T}(j) \equiv$ $\left[C_{h, t}^{T, h}(j), C_{h, t}^{T, U}(j), C_{h, t}^{T, W}(j)\right]^{\prime}$. Analogously, $\mathbf{D}_{h, t}^{F N}(j) \equiv\left[C_{h, t}^{N}(j)\right], \mathbf{D}_{h, t}^{V T}(j) \equiv\left[V_{h, t}^{T, F T, h}(j), V_{h, t}^{T, F N, h}(j)\right.$,

[^4]$\left.V_{h, t}^{T, F T, U}(j), V_{h, t}^{T, F N, U}(j), V_{h, t}^{T, F T, W}(j), V_{h, t}^{T, F N, W}(j)\right]^{\prime}$ and $\mathbf{D}_{h, t}^{V N, h}(j) \equiv\left[V_{h, t}^{N, F T, h}(j), V_{h, t}^{N, F N, h}(j)\right]^{\prime}$, where, for example, $V_{h, t}^{T, m, h}(j)$ represents the (domestic) demand for tradable intermediate goods produced by company $j$ demanded by firms in sector $m$ in the home country.

Consequently, the equilibrium or market clearing conditions can be written as (see also Natalucci and Ravenna (2003)): ${ }^{12}$

$$
\begin{align*}
Y_{h, t}^{F T}(j) & =\left[\mathbf{D}_{h, t}^{F T}(j)\right]^{\prime} \iota^{F T},  \tag{13}\\
Y_{h, t}^{F N}(j) & =\left[\mathbf{D}_{h, t}^{F N}(j)\right]^{\prime} \iota^{F N},  \tag{14}\\
Y_{h, t}^{V T}(j) & =\left[\mathbf{D}_{h, t}^{V T}(j)\right]^{\prime} \iota^{V T} \text { and }  \tag{15}\\
Y_{h, t}^{V N}(j) & =\left[\mathbf{D}_{h, t}^{V N}(j)\right]^{\prime} \iota^{V N}, \tag{16}
\end{align*}
$$

where $\iota^{m}$ is the unity vector containing an appropriate number of ones being equal to the number of markets in sector $m$. Furthermore, we disaggregate the total output of company $j$ in every domestic sector as $\mathbf{Y}_{h, t}^{F T}(j) \equiv\left[Y_{h, t}^{C T, h}(j), Y_{h, t}^{C T, U}(j), Y_{h, t}^{C T, W}(j)\right]^{\prime}$, and, analogously, $Y_{h, t}^{F N}(j) \equiv\left[Y_{h, t}^{C N}(j)\right] ; \mathbf{Y}_{h, t}^{V T}(j)$ $\equiv\left[Y_{h, t}^{V T, F T, h}(j), Y_{h, t}^{V T, F N, h}(j), Y_{h, t}^{V T, F T, U}(j), Y_{h, t}^{V T, F N, U}(j), Y_{h, t}^{V T, F T, W}(j), Y_{h, t}^{V T, F N, W}(j)\right]^{\prime}$ and $Y_{h, t}^{V N}(j) \equiv$ $\left[Y_{h, t}^{V N, F T, h}(j), Y_{h, t}^{V N, F N, h}(j)\right]^{\prime}$. At equilibrium we assume that $\mathbf{Y}_{h, t}^{F T}(j)=\mathbf{D}_{h, t}^{F T}(j), \mathbf{Y}_{h, t}^{F N}(j)=\mathbf{D}_{h, t}^{F N}(j)$, $\mathbf{Y}_{h, t}^{V T}(j)=\mathbf{D}_{h, t}^{V T}(j)$ and $\mathbf{Y}_{h, t}^{V N}(j)=\mathbf{D}_{h, t}^{V N}(j)$.

Analogous equilibrium conditions for the RoMU and the RoW hold.

## 7 Calvo staggered price and wage setting

Following Calvo (1983), we assume that domestic firms (suppliers) adjust their price(s) infrequently and that the opportunity to adjust follows an exogenous Poisson process. In each period, there is a constant probability $1-\varphi_{n_{m}, h}^{(j)}$ that firm $j$ in sector $m$ in the home country will be able to adjust its price on its market $n_{m}$, independently of past history. Hence, the average duration between two subsequent price adjustments can be computed as $\frac{1}{1-\varphi_{n_{m}, h}^{(j)}}$, since $\varphi_{n_{m}, h}^{(j)}$ lies between 0 and $1 .{ }^{13}$

Assuming that there is a continuum of firms, then each firm's probability to adjust its price is more or less similar so that, if the law of large numbers holds, this implies that the fraction of firms in sector $m$ at market $n_{m}$ not setting prices at $t$ is $\varphi_{n_{m}, h}$. Moreover, price signals are drawn randomly and independently of the time that suppliers were last offered to adjust their price. In particular, a fraction $1-\varphi_{n_{m}, h}$ of monopolistically competitive firms in the (domestic) product markets are assumed to set their own prices in advance by maximizing the present discounted value of profits subject to appropriate demand functions, which have to be determined first.

The newly set vector of domestic prices is derived for every producer $j$ in sector $m$ in the home country from the solution of the expected profit maximization problem (see Ascari (2004)): ${ }^{14}$

$$
\begin{equation*}
\max _{\mathbf{P}_{h, t}^{m}(j)} E_{t}\left\{\sum_{a=0}^{\infty} \Delta_{h, t, t+a}(j)\left(\varphi_{\mathbf{m}, \mathbf{h}}\right)^{a}\left[\left[\mathbf{P}_{h, t}^{m}(j)\right]^{\prime} \mathbf{Y}_{h, t+a}^{m}(j)-T C_{t+a}^{m, h}(j)\left(\left[\mathbf{Y}_{h, t+a}^{m}(j)\right]^{\prime} \iota^{m}\right)\right]\right\} \tag{17}
\end{equation*}
$$

[^5]s.t. relevant demand functions where the Calvo-assumption is applied, i.e. the producer assumes that the price remains constant; for example, taking account of equilibrium conditions in section 6 , the aggregate optimal demand for a domestic consumption tradable good becomes (see Plasmans et al. (2006a), Subsection 5.5.1):
\[

$$
\begin{equation*}
Y_{h, t+a}^{T}(j)=\left(\frac{P_{h, t}^{C T, h}(j)}{P_{h, t+a}^{C T, h}}\right)^{-\theta_{C T}} Y_{h, t+a}^{C T} \tag{18}
\end{equation*}
$$

\]

In (17) $\Delta_{h, t, t+a}(j)$ represents the (nominal) discount factor from $t$ to $t+a$ applied by firm $j$ to the stream of future profits and $T C_{t+a}^{m, h}(j)$ is the (nominal) total cost of production at period $t+a$ of firm $j$ in sector $m$ in the home country. Moreover, $\iota^{m}$ is the unity vector containing of an appropriate number of ones which equals the number of markets and $\left(\varphi_{\mathbf{m}, \mathbf{h}}\right)^{a}$ is a vector of probabilities of price changes for a producer in sector $m$ in the home country. Elements of this vector correspond to elements of relevant price vectors $\mathbf{P}_{h, t}^{F T}(j), \mathbf{P}_{h, t}^{F N}(j), \mathbf{P}_{h, t}^{V T}(j), \mathbf{P}_{h, t}^{V N}(j), \mathbf{P}_{h, t}^{M F}(j)$ and $\mathbf{P}_{h, t}^{M V}(j)$. For instance, $\varphi_{F T, h} \equiv\left[\varphi_{C T, h}, \varphi_{X, C T, U, h}, \varphi_{X, C T, W, h}\right]^{\prime}$ and $\varphi_{V N, h} \equiv\left[\varphi_{V N, F T ; h}, \varphi_{V N, F N, h}\right]^{\prime}$.

Assuming for simplicity that $\Delta_{h, t, t+a}(j)=\left(b_{h}\right)^{a}$, we obtain the following optimality condition (see Plasmans et al. (2006a), Subsection 5.5.1 and Appendix G):
where $i$ is the number of markets with which the home firm in sector $m$ is confronted.
Since any domestic price at period $t, P_{h, t}^{n_{n}^{(2)}}$, is assumed to be determined by a CES aggregator applied to the predetermined price $\left\{P_{h, t-l}^{n_{i}^{(i)}}(j)\right\}$, and the newly set price, $\breve{P}_{h, t}^{n_{i}^{(i)}}(j)$, according to Calvo, the vector of price indices for company $j$ in sector $m$ in the home country is:

$$
\begin{equation*}
\mathbf{P}_{h, t}^{m}(j)=\left[\varphi_{\mathbf{m}, \mathbf{h}} \otimes\left(\mathbf{P}_{h, t-1}^{m}(j)\right)^{1-\theta_{m, h}}+\left(\iota^{m}-\varphi_{\mathbf{m}, \mathbf{h}}\right) \otimes\left(\breve{\mathbf{P}}_{h, t}^{m}(j)\right)^{1-\theta_{m, h}}\right]^{\frac{1}{1-\theta_{m, h}}} \tag{20}
\end{equation*}
$$

where $\otimes$ is a component-wise multiplication operator.
Given the monopolistically competitive structure of the labor market, forward-looking households set nominal wages in staggered contracts that are analogous to the price contracts described above. More specifically, wages adjust with in Calvo-type manner with a probability $\varphi_{m, h}^{W}$ in every period $t$. Hence, in any period in which household $i$ is able to reset its wage contract, it maximizes utility (2) with respect to wage rates $W_{h, t}^{m}$, s.t. the relevant demand for labor and budget constraint (1) for every production sector $m=F T, F N, V T, V N$. According to Plasmans et al. (2006a), Subsection 5.6.1 and Appendix I, this maximization leads to:

$$
\begin{equation*}
\breve{W}_{h, t}^{m}(i)=\frac{\varrho_{L m, h}}{\left(\varrho_{L m, h}-1\right)} \frac{E_{t} \sum_{a=0}^{\infty}\left(\beta \varphi_{m, h}^{W}\right)^{a}\left[\left(W_{h, t+a}^{m}\right)^{\varrho_{L m, h}} L_{h, t+a}^{m} L_{h, t+a}^{\varphi_{h}}\right]}{E_{t} \sum_{a=0}^{\infty}\left(\beta \varphi_{m, h}^{W}\right)^{a}\left[\left(W_{h, t+a}^{m}\right)^{\varrho_{L m, h}} L_{h, t+a}^{m}\left(1-\tau_{h}\right) \Gamma_{t}^{h}(i)\right]} . \tag{21}
\end{equation*}
$$

Since any (domestic) wage at period $t, W_{h, t}^{m}(i)$, is assumed to be determined by the CES aggregator of the predetermined wage $W_{h, t-l}^{m}(i)$ and the newly set wage, $\breve{W}_{h, t}^{m}(i)$, according to Calvo a wage index for sector $m$ in the home country reads:

$$
\begin{equation*}
W_{h, t}^{m}(i)=\left[\varphi_{m, h}^{W}\left(W_{h, t-1}^{m}(i)\right)^{1-\gamma_{m, h}}+\left(1-\varphi_{m, h}^{W}\right)\left(\breve{W}_{h, t}^{m}(i)\right)^{1-\gamma_{m, h}}\right]^{\frac{1}{1-\gamma_{m, h}}} \tag{22}
\end{equation*}
$$

## 8 Net Foreign Assets

In Subsection $3.1 r_{t, t+1}$ is the nominal domestic interest rate in the whole MU between $t$ and $t+1$, while $\hat{r}_{U, t, t+1}^{h}$ and $\hat{r}_{W, t, t+1}^{h}$ are the interest rates effectively paid by domestic consumers for both kinds of foreign assets, which are defined as:

$$
\begin{align*}
\left(1+\hat{r}_{U, t, t+1}^{h}\right) & =\digamma_{U, t+1}^{h}(.)\left(1+r_{t, t+1}\right),  \tag{23}\\
\left(1+\hat{r}_{W, t, t+1}^{h}\right) & =\digamma_{W, t+1}^{h}(.)\left(1+r_{t, t+1}^{W}\right), \tag{24}
\end{align*}
$$

for the RoMU and the RoW, respectively. In (23) and (24) the factors of proportionality $\digamma_{U, t+1}^{h}($. and $\digamma_{W, t+1}^{h}($.$) are functions which, according to Benigno (2001), depend on the real holdings of$ the corresponding domestic consumer $i$ 's foreign assets; this means that domestic households take these functions as given when deciding on the optimal position in foreign assets. The functions $\digamma_{U, t+1}^{h}\left(\frac{B_{V, t+1}^{h}(i)}{P_{t+1}^{C}(i)}\right)$ and $\digamma_{W, t+1}^{h}\left(\frac{S_{t} B_{W, t+1}^{h}(i)}{P_{t+1}^{C}(i)}\right)$, generally represented by the function $\digamma_{t+1}\left(B_{t+1}^{*(R)}(i)\right)$, where $B_{t+1}^{*(R)}(i)$ are appropriately defined real foreign assets, are subject to the following restrictions: $E\left[\digamma_{t+1}(0)\right]=1$ and this function takes the value 1 only if $B_{t+1}^{*(R)}(i)=0$. Moreover $\digamma_{t+1}($.$) is a$ differentiable, decreasing function in the neighborhood of zero. Benigno (2001, p. 5) argues that function $\digamma_{t+1}($.$) can be described in two ways. First, it captures the (intermediation) costs, for$ the domestic households of undertaking positions in the international bonds (assets) market. As borrowers, they will be charged a (risk) premium on the foreign interest rate; as lenders, they will receive a remuneration lower than the foreign interest rate. Second, an alternative way to describe this cost is to assume the existence of intermediaries in the foreign bonds (assets) market (which are owned by the foreign households), who can borrow from and lend to foreign households at rates $1+r_{t, t+1}$ and $1+r_{t, t+1}^{W}$, but can borrow from and lend to domestic households at rates $\digamma_{U, t+1}^{h}\left(1+r_{t, t+1}\right)$ and $\digamma_{W, t+1}^{h}\left(1+r_{t, t+1}^{W}\right)$, respectively. ${ }^{15}$

There are many functions $\digamma_{t}($.$) that satisfy the above requirements In the spirit of Ambler$ et al. (2003, 2004), consumer $i$ 's risk premium depends on her real net foreign assets (NFAs) as follows: ${ }^{16}$

$$
\begin{equation*}
\digamma_{t}(.) \equiv \exp \left(v_{t}\left[\exp \left(\frac{\bar{S} \bar{B}^{*}(i)}{\bar{P}^{C}(i)}-\frac{S_{t-1} B_{t}^{*}(i)}{P_{t}^{C}(i)}\right)-1\right]\right) \tag{25}
\end{equation*}
$$

[^6]where $S_{t-1} B_{t}^{*}(i)$ is consumer $i$ 's nominal NFAs position with $\bar{S} \bar{B}^{*}(i)$ being her steady-state value of nominal foreign assets and $v_{t}$ is an iid process centered in a certain parameter value $\delta$. Therefore, once we allow either for perfect capital mobility or for NFAs equal to zero, the familiar uncovered interest rate parity (UIP) hypothesis holds with purely temporary deviations.

Notice that the arbitrage condition is operating for the returns of all NFAs. In particular, dividing (7) by (5) yields the UIP hypothesis between the home country and the RoW, taking account of equation (45) from Appendix A:

$$
\frac{E_{t}\left[S_{t+1}\right]}{S_{t}}=\frac{E_{t}\left[\hat{Q}_{W, t, t+1}^{h}\right]}{Q_{h, t, t+1}^{h}}=\frac{E_{t}\left[\left[\digamma_{W, t+1}^{h}(.)\left(1+r_{t, t+1}^{W}\right)\right]^{-1}\right]}{\left(1+r_{t, t+1}\right)^{-1}} .
$$

Log-linearizing with respect to an expected zero depreciation of the exchange rate, we get:

$$
\begin{equation*}
E_{t}\left[\Delta s_{t+1}\right]=r_{t, t+1}-r_{t, t+1}^{W}-\log E_{t}\left[\digamma_{W, t+1}^{h}(.)\right] \tag{26}
\end{equation*}
$$

where $\log \digamma_{W, t+1}^{h}($.$) is the (logarithmic) risk premium defined in (25).$
The methodology to state the arbitrage condition between the RoMU and the RoW is the same as that before. Dividing equation (7) by (6) and using both (44) and (45) from Appendix A:

$$
\frac{E_{t}\left[S_{t+1}\right]}{S_{t}}=\frac{E_{t}\left[\left[\digamma_{W, t+1}^{h}(.)\left(1+r_{t, t+1}^{W}\right)\right]^{-1}\right]}{E_{t}\left[\left[\digamma_{U, t+1}^{h}(.)\left(1+r_{t, t+1}\right)\right]^{-1}\right]},
$$

we can log-linearize again around the zero expected exchange depreciation rate to obtain:

$$
\begin{equation*}
E_{t}\left[\Delta s_{t+1}\right]=r_{t, t+1}-r_{t, t+1}^{W}+\log E_{t}\left[\digamma_{U, t+1}^{h}(.)\right]-\log E_{t}\left[\digamma_{W, t+1}^{h}(.)\right] . \tag{27}
\end{equation*}
$$

"A kind of UIP condition" between the home country and the RoMU can be directly derived from dividing FOC (6) by FOC (5), yielding:

$$
\begin{equation*}
\frac{E_{t}\left[\hat{Q}_{U, t, t+1}^{h}\right]}{Q_{h, t, t+1}^{h}}=\frac{\left(1+r_{t, t+1}\right)}{\left(1+r_{t, t+1}\right) E_{t}\left[\digamma_{U, t+1}^{h}(.)\right]}=1 \tag{28}
\end{equation*}
$$

from which it becomes clear that the expected risk premium implicit in asset prices must be equal to 1. Comparing (26) and (27), we observe that $\log E_{t}\left[\digamma_{U, t+1}^{h}().\right]=0$, which is consistent with (28). Moreover, the relevant UIP condition derived from the FOCs of the consumers in the RoMU and corresponding to (26) is:

$$
\begin{equation*}
E_{t}\left[\Delta s_{t+1}\right]=r_{t, t+1}-r_{t, t+1}^{W}-\log E_{t}\left[\digamma_{W, t+1}^{U}(.)\right] \tag{29}
\end{equation*}
$$

so that from (26) and (29), we get $E_{t}\left[\digamma_{W, t+1}^{h}().\right]=E_{t}\left[\digamma_{W, t+1}^{U}().\right]$.
To relate the current account balance with the trade balance, we assume that the home government has a balanced budget policy so that we can replace the $\mathrm{GBC}(9)$ into the active CBC (see (41) in Appendix A). Following Appendix A, we subtract each country's GBC from each country's active CBC to obtain three bilateral NFAs equations. In Appendix A, assuming that the conditional expectations of the stochastic discount factors are mutually pairwise equal between countries, i.e. $E_{t}\left[\hat{Q}_{U, t, t+1}^{h}\right]=E_{t}\left[\hat{Q}_{h, t, t+1}^{U}\right] \equiv E_{t}\left[\hat{Q}_{h U, t, t+1}\right], E_{t}\left[\hat{Q}_{W, t, t+1}^{h}\right]=E_{t}\left[\hat{Q}_{h, t, t+1}^{W}\right] \equiv E_{t}\left[\hat{Q}_{h W, t, t+1}\right]$ and $E_{t}\left[\hat{Q}_{W, t, t+1}^{U}\right]=E_{t}\left[\hat{Q}_{U, t, t+1}^{W}\right] \equiv E_{t}\left[\hat{Q}_{U W, t, t+1}\right]$, we ultimately obtain the NFAs equations (65),
and (68), respectively, where we used equation (62) for the domestic nominal benefits $\mathcal{B}_{h, t}$, which the consumers get from their entrepreneurial activities and similarly for the RoMU and the RoW we used foreign benefits (63) and (64). ${ }^{17}$

In Appendix B, the log-linearization is worked out for the above-mentioned NFAs equations, where the final form directly results from substitution of (134) and (135) for tradable final and intermediate goods, respectively.

In order to evaluate $\log E_{t}\left[\digamma_{W, t+1}^{U}().\right]$ in (29) we derive from (25) that:

$$
\log E_{t}\left[\digamma_{W, t+1}^{U}(.)\right]=E_{t}\left\{v_{W, t+1}^{U}\right\}\left[\exp \left(\frac{\bar{S} \bar{B}_{W}^{U}}{\bar{P}^{C}}-\frac{S_{t} B_{W, t+1}^{U}}{P_{t+1}^{C}}\right)-1\right]=\delta\left[\exp \left(\frac{\bar{S} \bar{B}_{W}^{U}}{\bar{P}^{C}}-\frac{S_{t} B_{W, t+1}^{U}}{P_{t+1}^{C}}\right)-1\right]
$$

where the term between square brackets can be approximated by $b_{W, t+1}^{U(R)} \equiv \log \frac{\bar{S} \bar{B}_{W}^{U}}{P^{C}}-\log \left(\frac{S_{t} B_{W, t+1}^{U}}{P_{t+1}^{C}}\right)$, so that (29) becomes the familiar UIP condition:

$$
\begin{equation*}
E_{t}\left[\Delta s_{t+1}\right] \simeq r_{t, t+1}-r_{t, t+1}^{W}-\delta b_{W, t+1}^{U(R)} \tag{30}
\end{equation*}
$$

## 9 Monetary policies

Designing monetary policy rules concerns the choice of (a) the monetary policy instruments, (b) the variables which are targeted and (c) their targeted values. The most important variables that are targeted by a central bank (CB) in the literature are: (1) real output (gap), (2) (change in) prices, (3) (change in) wages, (4) (change in) exchange rates, (5) change in interest rates, (6) a combination of real output and prices in the form of nominal GDP.

Kydland and Prescott (1977) claim that monetary policy effectiveness depends not only on policy actions undertaken but also on the public perception about these actions and its expectations about future actions. Consequently, policy is more effective when future actions are predictable so that a monetary authority can commit itself to a certain course of policies. As Atoian et al. (2004) argue, commitment permits the CB to distribute 'policy medicine' over time. For example, when the CB wishes to offset inflation that will result from a supply shock, under commitment, it can raise interest rates moderately provided that it maintains higher rates for a period of time. In contrast, in the case of lack of commitment, a higher initial rate increase will be necessary because of the public doubts that the CB will sustain this interest rate increase. Atoian et al. (2004) also argue that optimal commitment does not need to take the form of a reaction function with fixed coefficients. In general, an optimal commitment rule has the form of a state-contingent plan that presents the instrument setting as a function of the history of exogenous shocks. However, optimal commitment is not practical because, first, as noted by Woodford (2003), it is not feasible to provide an advance listing of all relevant contingencies and, second, it is difficult for the public to distinguish between discretion and a complicated contingency rule. Both problems are avoided when the CB commits to a rule with fixed coefficients.

Which form should such a rule with fixed coefficients take? Most CBs use the short-term nominal interest rate as their control variable, depending on economic conditions. The most famous and widely used examples of interest rate rules are those proposed by John Taylor. The log-linearized standard Taylor rule (see Taylor (1993)) relates the interest rate to inflation and (logarithmic) output gap:

[^7]\[

$$
\begin{equation*}
r_{t, t+1}=\vartheta_{0}+\vartheta_{1} \pi_{t}+\vartheta_{2} y_{t}, \tag{31}
\end{equation*}
$$

\]

where $\pi_{t}$ and $y_{t}$ are annualized inflation and (logarithmic) deviations of output w.r.t its steady state value, which are assumed to be the target variables of a monetary authority. Taylor (1993) assigns coefficient values consistent with an accurate description of Federal Reserve policy for quarterly data and annualized inflation (so instead of, $\pi_{t}$, we use $\pi_{t}^{(4)} \equiv \sum_{j=0}^{3} \pi_{t-j}$ ) as $\vartheta_{1}=1.5$ and $\vartheta_{2}=0.5$. The intuition for the value of the former reaction parameter is that the CB must raise the interest rate by more than any increase in inflation in order to raise the real rate of interest, cool the economy, and move inflation back toward its target.

We review some simple deviations from the original Taylor rule studied in the literature, more in detail in Plasmans et al. (2006b). In this paper, we consider an institutional setting in which the home country and the RoMU already agreed in a common monetary policy rule. Therefore, the common CB is going to set the nominal interest rate taking weighted aggregates into account (as, for instance, in the Maastricht Treaty). The relative weight for the home country in the MU is $\frac{n_{1}}{n_{2}}$, while $\frac{n_{2}-n_{1}}{n_{2}}$ is the relative weight of the RoMU in the MU. Moreover, considering the twolevel production functions (10)-(11) and (12) of our model, it is assumed that the CB targets only the final goods deviations from the steady state production. Since a CB is in general primarily interested in targeting CPI inflation, that is inflation of final goods production, we do not consider outputs of intermediate goods in the proposed monetary policy rules. ${ }^{18}$

Therefore, we consider the following monetary policy rules extended to an MU setting:
(I) The Henderson-McKibbin and Taylor (HMT) rule for the MU, which is a direct extension of the standard Taylor rule (31) for a setup with weighted tradable and non-tradable outputs (see Collard and Dellas (2004)):

$$
\begin{align*}
r_{t, t+1} & =\vartheta_{0, U}^{I}+\vartheta_{1, U}^{I}\left[\frac{n_{1}}{n_{2}} \pi_{h, t}^{(4)}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) \pi_{U, t}^{(4)}\right]+\vartheta_{2, U}^{I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{T}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{T}\right] \\
& +\vartheta_{3, U}^{I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{N}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{N}\right]+\varepsilon_{1 t}, \tag{32}
\end{align*}
$$

with $y_{h}^{T}$ and $y_{h}^{N}$ are the logarithmic deviations of domestic outputs of tradable and non-tradable final goods from their steady state values. Since only final goods are considered, it is expected that the sum of the values $\vartheta_{2, U}^{I}$ and $\vartheta_{3, U}^{I}$ is (much) smaller than 0.5 (as found by Taylor (1993)).
(II) Taylor (1999) suggests another alternative for the standard Taylor rule (31) that allows for interest-rate smoothing:

$$
\begin{align*}
r_{t, t+1} & =\vartheta_{0, U}^{I I}+\vartheta_{1, U}^{I I}\left[\frac{n_{1}}{n_{2}} \pi_{h, t}^{(4)}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) \pi_{U, t}^{(4)}\right]+\vartheta_{2, U}^{I I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{T}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{T}\right] \\
& +\vartheta_{3, U}^{I I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{N}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{N}\right]+\vartheta_{4, U}^{I I} r_{t-1, t}+\varepsilon_{2 t}, \tag{33}
\end{align*}
$$

where $\vartheta_{4}^{I I}>0$ is a smoothing parameter.

[^8]Finally, we are also interested in (III) a Taylor-type rule with wage inflation:

$$
\begin{align*}
r_{t, t+1} & =\vartheta_{0, U}^{I I I}+\vartheta_{1, U}^{I I I}\left[\frac{n_{1}}{n_{2}} \pi_{h, t}^{(4)}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) \pi_{U, t}^{(4)}\right]+\vartheta_{2, U}^{I I I}\left[\frac{n_{1}}{n_{2}} \pi_{h, t}^{W(4)}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) \pi_{U, t}^{W(4)}\right] \\
& +\vartheta_{3, U}^{I I I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{T}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{T}\right]+\vartheta_{4, U}^{I I I}\left[\frac{n_{1}}{n_{2}} y_{h, t}^{N}+\left(\frac{n_{2}-n_{1}}{n_{2}}\right) y_{U, t}^{N}\right]+\vartheta_{5, U}^{I I I} r_{t-1, t}+\varepsilon_{3 t}, \tag{34}
\end{align*}
$$

where $\pi_{t}^{W(4)} \equiv \sum_{j=0}^{3} \pi_{t-j}^{W}$ is the annualized wage inflation. ${ }^{19}$
The RoW's CB is committed to similar Taylor-type rules as specified in (32), (33) and (34) with the instrument being the $r_{t}^{W}$ depending on its country variables such as inflation, wage inflations and tradable and non-tradable output gaps.

Mc Callum (1997) argues that the policymakers' reaction is more accurate if it is based on lagged and not on current values of output and inflation. In response, Taylor (1999) suggests an alternative form of his rules where lagged values of output and inflation replace the current values in (33). In contrast, Clarida et al. (1998) and others argue that rules in which the CB reacts to forward looking variables are optimal in the case of a quadratic objective function for the monetary authorities, which will be also utilized in this paper. The difference between backward-looking, contermporaneous and forward-looking monetary rules relates primarily to the information set of the monetary policymakers. For instance, in the case of a contemporaneous rule the actual inflation rate, on which the CB is assumed to have adequate information, is targeted.

## 10 Social welfare maximizing monetary policy rules

As already mentioned in the previous section, we focus now on those policy rules supported by the CB that maximize welfare. Ortega and Rebei (2006) look for that Taylor rule parameterization which maximizes the unconditional expectation of households' lifetime utility. ${ }^{20}$ Unconditional maximization implies finding the parameters that satisfy:

$$
\vartheta_{0}, \vartheta_{1}, \vartheta_{2}, \vartheta_{4}, \vartheta_{5}=\arg \max \left\{E\left[U\left(C_{t}^{h}, M_{t}^{h}, L_{h, t}\right)\right]\right\}
$$

Ortega and Rebei (2006) compute this maximum using a grid-search algorithm of the utility function. The model is solved for a number of parameter combinations $\vartheta_{1}, \vartheta_{2}, \vartheta_{3}$, for different values of $\vartheta_{4}$ and $\vartheta_{5}$, while holding constant the estimated degree of inertia in the setup of the interest rate. Computed levels of welfare constitute a grid with the step in parameters of size 0.2. The optimal monetary policy is the one which maximizes the individual's utility after evaluation of all points in the grid.

Following Ortega and Rebei (2006), we measure the welfare gain associated with a particular monetary policy in terms of its compensating variation. That is, we calculate the percentage of lifetime consumption that should be added to that obtained under the estimated Taylor rule in

[^9]order to give households the same unconditional expected utility as under the new monetary policy rule scenario:
$$
E\left[U\left(C_{t}^{h}+\text { welfare gain, } M_{t}^{h}, L_{h, t}\right)\right]=\underbrace{E\left[U\left(\tilde{C}_{t}^{h}, \tilde{M}_{t}^{h}, \tilde{L}_{h, t}\right)\right]}_{\text {optimal policy }} .
$$

In the next step, we take a second order approximation of $E\left[U\left(C_{t}^{h}, M_{t}^{h}, L_{h, t}\right)\right]$, i.e.:

$$
E\left[U\left(c_{t}^{h}, m_{t}^{h}, l_{h, t}\right)\right] \simeq U\left(\bar{C}^{h}, \bar{M}^{h}, \bar{L}_{h}\right)+U^{\prime} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)+\frac{1}{2} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)^{\prime} U^{\prime \prime} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right),
$$

where $U^{\prime}\left(U^{\prime \prime}\right)$ is the first (second) order derivative of the utility function evaluated at the steady state and hats mean a deviation from the deterministic steady state.

The closed form solution of the second order approximation of (2) is:

$$
\begin{gathered}
E\left[U\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)\right] \simeq \frac{1}{1-\sigma_{h}}\left(\left(\bar{C}^{h}\right)^{1-\kappa_{h}}\right)^{1-\sigma_{h}}-\frac{\left(\sum_{m=F T, F N, V T, V N} \bar{L}_{h}^{m}\right)^{1+\phi_{h}}}{1+\phi_{h}}+\chi_{M, h} \frac{\left(\frac{\bar{M}^{h}}{P^{C, h}}\right)^{1-\frac{1}{\chi_{h}}}}{\left(1-\frac{1}{\chi_{h}}\right)}+ \\
\left(\begin{array}{cc}
\left(\bar{C}^{h}\right)^{\sigma_{h}\left(\kappa_{h}-1\right)} & \frac{\chi_{M, h}}{\bar{P}^{C, h}}\left(\frac{\bar{M}^{h}}{P^{C, h}}\right)^{-\frac{1}{\chi_{h}}} \\
\left(\bar{L}_{h}\right)^{\phi_{h}}
\end{array}\right)\left(\begin{array}{c}
E \tilde{c}_{t}^{h} \\
E \tilde{m}_{t}^{h} \\
\left.E \tilde{l}_{h, t}\right)
\end{array}\right)+ \\
\frac{1}{2}\left(\begin{array}{ccc}
\sigma_{h}\left(\kappa_{h}-1\right)\left(\bar{C}^{h}\right)^{\sigma_{h}\left(\kappa_{h}-1\right)-1} & \left(-\frac{\chi_{M, h}}{\chi_{h}\left(P^{C, h}\right)^{2}}\right)\left(\frac{\bar{M}^{h}}{P^{C, h}}\right)^{-\frac{\left(\chi_{h}+1\right)}{\chi_{h}}} & 0 \\
0 & 0 & 0 \\
0 & \phi_{h}\left(\bar{L}_{h}\right)^{\phi_{h}-1}
\end{array}\right) V A R\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right) .
\end{gathered}
$$

Following Ortega and Rebei (2006) we check variations of the Taylor rule, which assures determinacy in the system. The compensating variation can be decomposed in two effects, namely a first level effect and a second level effect. The first level is defined as the amount of consumption necessary to compensate the consumer since the monetary policy parameterization has been changed, in order to reach the same level of utility, which the consumer reached under the optimal monetary policy parameterization. The former can be defined as:

$$
E\left[U\left(C_{t}^{h}+1^{\text {st }} \text { level effect, } M_{t}^{h}, L_{h, t}\right)\right] \simeq U\left(\bar{C}^{h}, \bar{M}^{h}, \bar{L}_{h}\right)+U^{\prime} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)
$$

and the second level (order) effect as:

$$
E\left[U\left(C_{t}^{h}+2^{\mathrm{nd}} \text { level effect, } M_{t}^{h}, L_{h, t}\right)\right] \simeq U\left(\bar{C}^{h}, \bar{M}^{h}, \bar{L}_{h}\right)+\frac{1}{2} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)^{\prime} U^{\prime \prime} E\left(\tilde{c}_{t}^{h}, \tilde{m}_{t}^{h}, \tilde{l}_{h, t}\right)
$$

To get the approximated welfare gain, both effects can be added, i.e.:

$$
C_{t}^{h}(1+\text { welfare gain }) \approx C_{t}^{h}\left(1+1^{\text {st }} \text { level effect }\right)\left(1+2^{\text {nd }} \text { level effect }\right)
$$

Intuitively, the welfare gain computed can be divided in two effects that work differently. The first level concerns the mean utility level required by the equivalence compensation, while the second level concerns the volatility of the consumption. Therefore, a policy that produces a higher
level of utility but at the expense of a higher volatility in consumption may be discouraged by the CB.

Alternatively, Levin et al. (2005) estimate an historical monetary policy rule for the US and look for the welfare maximizing monetary policy rule under commitment. They construct a Lagrangian with the structural model as a set of constraints to the welfare function. ${ }^{21}$

## 11 Numerical simulations

In the simulations, we use the numerical parameters presented in Table 1 of Appendix G. Since we aim to reflect The Netherlands, the Rest of the EMU and the USA (as the RoW) and it is clear from Appendix F that the data included in our sample is quarterly from $1970^{I I}$ to $2005^{I V}$, we try to approximate the relative weights occurring in the EMU-wide variables of the monetary policy rules (32), (33) and (34) from this time series data. Since the relative weights are computed in Euros and no exchange rate data is available before 1979, ${ }^{22}$ these relative weights are computed for the (sub)sample period $1979^{I}-2005^{I V}$ and presented in Figure 16 of Appendix F, from which it is evident that these weights are relatively constant over time. Hence, the average relative weights are good approximations to the effective relative weights. Therefore, $n_{1}$ is evaluated at 0.026 and $n_{2}$ at 0.3807.

In this paper, the parameterization introduces two layers of asymmetry. First, the economies differ in size, i.e. the home economy is assumed to be $2.6 \%$ of the world, the RoMU $38.07 \%$, whereas the RoW accounts for 59.33 \%. Second, economies are slightly asymmetric in terms of the structure (in the assets market). The most obvious and important asymmetries are caused by the fact that the home economy and the RoMU create an MU; hence, they experience both a common monetary policy and a common exchange rate. Second, we took into account more sclerotic characteristics of the European economy assuming that the RoW (the USA) features faster adjustment of prices and wages (lower relevant Calvo parameters in Table 1).

In the above setting we consider various shocks: common and asymmetric positive technological shocks in the tradable final and intermediate goods sectors in different economies, an exchange rate shock, a monetary policy shock and a preference shock to consumption, which will be explained in the following subsections. All the figures are gathered in Appendix E.

### 11.1 Technological shocks

### 11.1.1 Technological shock to tradable final goods sector

First, we consider a positive technological shock in the production sector of tradable final goods hitting (1a) all three economies at the same time, (1b) only the small (open) economy, (1c) only the RoMU and (1d) only the RoW. Figures 1-3 show the adjustment of the main variables for all these shocks. ${ }^{23}$

Common shock (1a) has by far the biggest impact on aggregate consumption (see Figure 1) and most of other variables considered. This effect is expected, as the production capabilities of tradable final goods grow in every economy, increasing their consumption all over the world

[^10]via export channels (consumption disaggregated into tradable and non-tradable components is shown on Figure 1). Obviously, CPI inflation is negative in every economy (Figure 1). What is interesting (and common for all positive technological shocks in the production of tradable final goods (1a-1d)), is that increased consumption of tradable goods raises (of course, to a lesser extent) consumption of non-tradable goods in both MU economies. This indicates that the substitution effect is lower than the income and habit formation effects (in total consumption), which does not occur in the RoW. Figure 2 presents output, labor and wage inflation in all final production sectors. As expected, production of tradable final goods increases, employment decreases and wage inflation is negative.

In spite of the symmetric shock, asymmetric economies (in terms of size and partially in terms of structure) react differently, mainly due to different reactions of both CBs, which drive the exchange rate via the UIP condition out of equilibrium (Figure 3). Both monetary policies are expansionary (loosening) and rather similar.

Considering shocks (1b-1d), it is clear that asymmetric shocks, hitting only one country at a time, have considerably lower impact on the economic systems than the common shock. The main finding is that the economy of the RoW is more influenced by the asymmetric technological shock that hits its $F T$ sector than economies of the MU. As expected, the union is a net lender when one of its countries is hit by a positive technological shock and it is a net borrower when the RoW is hit by such a shock.

### 11.1.2 Technological shock to intermediate goods sectors

Second, we consider a positive technological shock either in the production of tradable intermediate goods (shock (1e)) or non-tradable intermediate goods (shock (1f)) of the home country. Lower prices of tradable intermediate inputs caused by shock (1e) decrease marginal costs and prices in both tradable and non-tradable final goods sectors so that output is cheaper and increased. Negative inflation leads to loosening of monetary policies as in the technological shock to final goods sectors in the previous subsection. After initial appreciation the exchange rate depreciates, which makes the MU goods to be more competitive in the RoW inducing export at the expense of domestic tradable consumption. This explains the increase in consumption of tradable final goods abroad and counterpart decrease in the MU. Depreciation of the MU currency has additional effect on domestic consumers who become net borrowers in spite of increasing export.

As expected, the effects of the shock to domestic non-tradable intermediate goods are the largest in the home country. As it is assumed that non-tradable intermediate goods account for one half of input demand, the shock in the $V N$ sector has a bigger impact on final goods production in both $F T$ and $F N$. In these final goods producing sectors labor is substituted by now relatively cheaper intermediate inputs (as for example capital stock).

### 11.2 Exchange rate shock

The exchange rate shock is modeled as a transitory appreciation of the MU currency. At the very beginning the MU currency appreciates by $4 \%$ and the persistence of the shock is 0.85 . Relevant IRFs are presented in Figures 7-9.

The first observation is that both countries of the MU react in a similar way, which, in turn, is opposite to the reactions of the RoW variables. From Figure 7, we conclude that an increase in relative competitiveness of foreign (RoW) goods w.r.t. the MU has the following consequences: consumption of tradable final goods grows in both countries of an MU, whereas it decreases in the

RoW, making the aggregate quantities of consumption to increase and decrease, respectively. This movement, in turn, increases non-tradable consumption due to the income and habit formation effects, which, apparently, are stronger than the substitution effect. The increased export of the RoW makes MU consumers to become net borrowers (negative NFAs positions).

Figure 8 presents the production side of the economy. As expected, decreased competitiveness results in a lower production of tradable final goods in the MU and a higher one in the RoW. The other variables also react in a way analogous to the consumption side of the economy. The above results are comparable to those obtained by Erceg et al. (2005).

We observe very interesting effects in the adjustment of labor. In spite of the fact that there is an increased demand and production of non-tradable final goods, the employment in this sector decreases (in the home country and the RoMU). That means that labor input for increased production was substituted by cheaper intermediate goods imported from the RoW.

The exchange rate adjustment and monetary policies are depicted in Figure 9. The CB of the union pursues expansionary (loosening) monetary policy to stimulate initially decreased output, whereas the RoW's CB restricts monetary policy in order to suppress inflation.

In the case of this exchange rate shock, we demonstrate the influence of the presence of tradable intermediate goods on the behavior of the model. As it was mentioned in the introduction, Dellas (2005) points out that the presence of intermediate goods has vital consequences for the ability of monetary authorities to manipulate nominal exchange rates. When there is no production interdependence between countries (i.e. only consumption goods are traded) changes in a nominal exchange rate do not affect production costs. However, when there is trade in intermediate goods as in the real world, an exchange rate depreciation has adverse direct effects on the cost of domestic production. Consequently, it makes the exchange rate instrument less useful. ${ }^{24}$

To verify whether it is also the case here, we construct a second version of our model, where no tradable intermediate goods are present. More in detail, to make both versions as similar as possible, we do not erase the $V T$ sector completely but we assume that intermediate goods produced in this sector do not cross the border, but are used in the production of final goods in the domestic country. We also keep intermediate goods import sectors as such, but their role now is not to import foreign intermediate goods, but to rebrand and repackage domestic intermediate goods. In such a way, we obtain the version of the model which can be directly compared to the original one. Additionally, also NFAs equations must be changed accordingly, as there is no more trade in intermediate goods. However, this effect was not taken into account in the reasoning of Dellas (2005), because imperfect risk sharing was out of his consideration. Without imperfect risk sharing, indeed, we can expect that the presence of trade in intermediate goods is going to reduce the response of the economy to an exchange rate shock. However, with imperfect risk sharing, no trade in intermediate goods implies a lower deviation of NFAs (see Figure 7), which should lead to a faster stabilization of the economy. Consequently, imperfect risk sharing can have a substantial influence.

Dashed lines in Figures 7-9 present the relevant adjustments of the second version of the model. Indeed, for most variables the model without trade in intermediate goods seems to be adjusting slower than the standard model, which confirms the point of Dellas (2005).

[^11]
### 11.3 Policy shock

In the next step, we consider a (negative) shock to the interest rate of the CB of the MU . More in detail, we simulate the loosening of MU (monetary) policy for the three Taylor-type rules considered: (32) (Rule (I)), (33) (Rule (II)) and (34) (Rule (III)). We assume compatibility between policies of both CBs in our model, i.e. in the case when the MU CB follows the first rule, the RoW's CB also follows the same rule. ${ }^{25}$

The general observation is that Rules (II) and (III) with interest rate smoothing (and, therefore, with a higher persistence after the shock) have a lower impact on the economic system. Moreover, Rule (III) leads to lower deviations from the equilibrium after the shock, which indicates that taking account of wage inflation acts countercyclically and may reduce the instability of the economy.

Expansionary monetary policy of the MU's monetary authority results in increased consumption in the whole MU, which induces inflation, output, employment and wage inflation (Figures 10 and 11). Increased output and inflation make an initially expansionary policy to change into a restrictive one after a while.

### 11.4 A preference shock to consumption

Next, we consider a preference shift shock which changes the relative value of consumption w.r.t. labor by increasing the consumers' marginal utility of consumption. In Figures 13-15 we present and compare two shocks: (4a) hits consumers of the home country whereas (4b) hits consumers of the RoMU. Their effects are comparable to a demand shock. ${ }^{26}$ In the case of the first shock, consumption in the home country grows and this higher demand induces inflation. Moreover, since domestic consumers have lower incentives to work we observe wage inflation, which aims to bring a decreased employment to the initial level. The increased demand in the home country is not matched by domestic firms (except for the non-tradable sector), but primarily by RoMU's firms which increase export to the market where prices become higher due to inflation. This leads even to a temporary decrease in the RoMU's consumption. To suppress inflation in both MU countries, the CB pursues a restrictive monetary policy.

## 12 Econometric estimations

The model is estimated with an institutional setup that considers a small open economy being a member of the EMU (The Netherlands), the rest of the EMU and the RoW (proxied by the USA in our case). The data series included in our sample are quarterly data on real private consumption, CPI inflation, wage inflation, nominal short run interest rate, tradable and nontradable intermediate goods running from $1970^{I I}$ to $2005^{I V}$ (see Appendix F). To get an idea of the statistical properties (the degree of integration) of the sample variables, especially those variables occurring in the monetary policy rules (32), (33) and (34), we performed augmented Dickey-Fuller tests. As put forward in Section 11, the relative weights in the EMU-wide variables in these monetary policy rules are presented in Figure 16 of Appendix F and good approximations to the relative weights are 0.026 for $n_{1}$ and 0.3807 for $n_{2}$, respectively. Applying augmented DickeyFuller tests, we find that, except for the EMU wage inflation, all other EMU-wide variables are

[^12]found to be integrated of order 1 at the $5 \%$ significance level. ${ }^{27}$ We estimate single equation monetary policy rules (32), (33) and (34) using OLS in order to obtain preliminary values of the ECB reaction parameters in setting the nominal EMU interest rate. ${ }^{28}$ This is useful for giving support and setting monetary policy parameter bounds that govern prior densities to be assumed in the simultaneous equations system Bayesian estimation (see further in this section). Our singleequation estimated rules are as follows:
(i) Rule (I), i.e. monetary policy rule (32), using detrended data and in first and fourth differences (for nominal short-term interest rates and annualized inflation rates, on the one hand, and outputs, on the other hand) to get rid of possible seasonal integration, yields:
\[

$$
\begin{align*}
\hat{r}_{t, t+1} & =\underset{(-0.26)}{-0.00004}+\underset{(13.7)}{0.24}\left[0.068 \pi_{h, t}^{(4)}+0.932 \pi_{U, t}^{(4)}\right]+\underset{(2.62)}{0.047}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right] \\
& +\underset{(2.47)}{0.038}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right], \quad \bar{R}^{2}=0.65 \tag{35}
\end{align*}
$$
\]

and

$$
\begin{align*}
\Delta \hat{r}_{t, t+1} & =\underset{(-5.0)}{-0.000898}+\underset{(2.06)}{0.089} \Delta\left[0.068 \pi_{h, t}^{(4)}+0.932 \pi_{U, t}^{(4)}\right]+\underset{(0.845)}{0.007 \Delta_{4}}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right] \\
& +\underset{(4.57)}{0.036 \Delta_{4}}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right], \quad \bar{R}^{2}=0.265 . \tag{36}
\end{align*}
$$

(ii) Rule (II) (33) is the above Taylor rule with smoothing, yielding:

$$
\begin{align*}
\hat{r}_{t, t+1} & =\underset{(-0.01)}{1.59 .10^{-6}}+\underset{(2.48)}{0.049}\left[0.068 \pi_{h, t}^{(4)}+\underset{(4.28)}{\left.0.932 \pi_{U, t}^{(4)}\right]}+\underset{(0.0125)}{0.03}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right]\right. \\
& +\underset{(11.93)}{0.045}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right]+\underset{(1.74}{0.74} r_{t-1, t}, \quad \bar{R}^{2}=0.83 \tag{37}
\end{align*}
$$

and

$$
\begin{align*}
\Delta \hat{r}_{t, t+1} & =\underset{(-2.82)}{-0.000517}+\underset{(2.77)}{0.083 \Delta} \Delta\left[0.068 \pi_{h, t}^{(4)}+0.932 \pi_{U, t}^{(4)}\right]+\underset{(0.37)}{0.003 \Delta_{4}}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right] \\
& +\underset{(2.75)}{0.022}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right]+\underset{(3.17)}{0.263 \Delta r_{t-1, t},} \quad \bar{R}^{2}=0.37 . \tag{38}
\end{align*}
$$

(iii) Rule (III) is estimated with total EMU wage inflation, since both the tradable and nontradable EMU wage inflation rates are treated as unobserved variables. Using detrended series and appropriately differenced series to estimate (34), we find:

$$
\begin{align*}
\hat{r}_{t, t+1} & =\underset{(0.016)}{1.7 .10^{-6}}+\underset{(2.47)}{0.05}\left[0.068 \pi_{h, t}^{(4)}+\underset{(2.4)}{\left.0.932 \pi_{U, t}^{(4)}\right]-\underset{(-0.13)}{0.002}\left[0.068 \pi_{h, t}^{W(4)}+0.932 \pi_{U, t}^{W(4)}\right]}\right. \\
& +\underset{(0.03}{0.0}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right]+\underset{(4.26)}{0.046}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right]+\underset{(0.44)}{0.709 r_{t-1, t},} \bar{R}^{2}=0.83 \tag{39}
\end{align*}
$$

[^13]and
\[

$$
\begin{align*}
\Delta \hat{r}_{t, t+1} & =\underset{(-2.46)}{-0.00460}+\underset{(2.73)}{0.081 \Delta}\left[0.068 \pi_{h, t}^{(4)}+\underset{\left(0.932 \pi_{U, t}^{(4)}\right]}{(1.35)} \underset{(0.024}{0.024} \Delta\left[0.068 \pi_{h, t}^{W(4)}+0.932 \pi_{U, t}^{W(4)}\right]\right. \\
& +\underset{(0.073)}{0.0005 \Delta_{4}}\left[0.068 y_{h, t}^{T}+0.932 y_{U, t}^{T}\right]+\underset{(2.66)}{0.021 \Delta_{4}}\left[0.068 y_{h, t}^{N}+0.932 y_{U, t}^{N}\right]+\underset{(2.64)}{0.228 \Delta r_{t-1, t}} \\
\bar{R}^{2} & =0.37 \tag{40}
\end{align*}
$$
\]

Similar equations are derived for the nominal short-term interest rate of the Federal Reserve Bank (Fed).

These estimators are very useful to establish the parameter bounds of the assumed prior densities. Next to the arguments put forward in Section 9, the parameter values in (35), (37) and (39) are lower than those reported in the literature, which is very probably due to the method of filtering used (Hodrick Prescott).

A large number of parameters is involved in DSGE models and, particularly, in our threecountry model. Therefore, it might not be trivial to know a priori the set of model coefficients assuring the rank condition for the solution of forward-looking (jump) variables (see Blanchard and Kahn (1980))..$^{29}$

To estimate the model, it is transformed in the state space form. The set of variables can either be observed or not: (i) observed, i.e. taken from national accounts databases and (ii) unobserved, or the rest of the model variables. In order to compute the joint sample likelihood and get the estimators, we use the Kalman filter. The paper by Ratto et al. (2005a) discusses this method in detail; moreover, their and our computations are performed using the DYNARE toolbox for Matlab (see Juillard (2005)).

In Table 1 in Appendix G, we inform about prior densities of the 'deep' structural parameters, AR coefficients of autocorrelated shocks and standard deviations of these shocks. Observe that several structural parameters are fixed rather than attributed a prior density. Following Ortega and Rebei (2006) and other authors, we calibrate these fixed parameters to those found in the literature. When this is the case, there is no standard deviation (SD) declared between parentheses in Table 1 , since it is always zero then.

In order to obtain Bayesian estimates for the whole system we formulate independent prior densities for each of the 36 model parameters in the case of the first monetary policy rule; 38 in the case of the second monetary rule and 40 in the case of the third monetary rule. ${ }^{30}$ The choice of these prior densities is based on the literature mentioned in Table 1 of Appendix G. Notice that most of these prior densities are relatively uninformative.

Overall, the posterior densities are quite sensitive to the assumed prior densities as also discussed in Onatski and Williams (2003), Smets and Wouters (2003), Lubik and Schorfheide (2005) and Levin et al. (2005). Point estimates of the posterior means and corresponding standard errors and of standard deviations of the shocks are conditional on the estimation methodology, the sample (duration) and the values of calibrated (non-stochastic) parameters.

In Table 2 in Appendix G, we report estimates using the Bayesian estimation procedure. In particular, we report estimates of the complete NOEM model taking account of the historical monetary policy rule where the ECB and the Fed systematically smooth the nominal short-term

[^14]interest rate (Rule (II)). Estimation results using model version with Rules (I) and (III) provide us contradictory results and are not reported. ${ }^{31}$

Assuming that the ECB and the Fed are committed to Rule (II), the Bayesian estimation results are reported in Table 2 in Appendix G. In general, the application illuminates on the deep parameters, mainly of The Netherlands being the small open economy studied. Our discussion restricts to the parameters being estimated significantly different from zero. The Dutch consumers' risk aversion parameter is estimated at 2.005, very close to the assumed prior. Furthermore, the degree of Dutch consumer habit persistence is estimated at 0.602 , which is significatively large than the prior mean of 0.5. ${ }^{32}$

We focus on rigidity parameters governing prices of final and intermediate goods and wages in the three countries. Assuming identical prior values and densities for the NL and the RoEMU, point estimates are quite accurate, finding evidence that the NL show more stickiness in prices than the RoEMU (on average the NL reset FT and FN prices every 2.44 quarter vs. 2.18 in the RoEMU and 1.53 quarter in the US). Regarding VT and VN stickiness, on average in the NL these intermediate goods sectors reset prices every 1.53 quarter, in the RoMU every 1.65 quarter, and in the US every 1.63 quarter. These results are consistent with the common wisdom that intermediate goods markets are more competitive than final goods markets and with a very small disparity among countries. Finally, regarding wages, the same ordering as in final goods sectors remains: on average the NL reset wages every 1.90 quarter, the RoEMU every 1.80 quarter and the US each 1.56 quarter. Regarding the reaction parameters that shape monetary policy Rule (II) for the ECB and the Fed, we find estimates of the posterior means for the CPI inflation, $\vartheta_{1, U}^{I I}$ and $\vartheta_{1, W}^{I I}$, and smoothed interest rate reaction parameters, $\vartheta_{4, U}^{I I}$ and $\vartheta_{4, W}^{I I}$, that are in the very neighborhood of the prior means with (very) low standard errors. ${ }^{33}$

The reaction parameters for tradable and non-tradable final output in the EMU (US), $\vartheta_{2, U}^{I I}$ and $\vartheta_{3, U}^{I I}\left(\vartheta_{2, W}^{I I}\right.$ and $\left.\vartheta_{3, W}^{I I}\right)$ depart from the prior means, but show extremely (and incredibly?) low standard errors. Moreover, the standard deviations of the shocks are not significantly different from zero.

## 13 Concluding remarks

In this paper, a New-Keynesian open economy (NOEM) model for three (regions of) countries was constructed with a detailed treatment of the consumption and production sectors involving tradable and non-tradable final and intermediate goods under several sources of stickiness (the most important of which are Calvo-type nominal price and wage restrictions). More in particular, in our setting we model a small open economy being a member of a monetary union, the rest of that monetary union and the rest of the world. Historical monetary policy rules, derived as direct extensions of the familiar Taylor rules, are added to the model, assuming full commitment to these rules by the two central banks involved. More in detail, we studied three types of Taylor rules: (i) a simple Taylor rule based on deviations of CPI inflation and output from the steady state, (ii)

[^15]similar to (i) but including (nominal short-term) interest rate smoothing and (iii) similar to (ii) but including wage inflation.

A full characterization of this three-country setting is derived as a set of dynamic non-linear equations. Then, the complete structure is log-linearized and the impacts of various types of shocks are analyzed and simulated: (i) common and asymmetric positive technological shocks in the tradable final (intermediate) goods sectors in different economies, (ii) an exchange rate shock, (iii) a monetary policy shock, and (iv) a preference shock to consumption. The impact of these shocks is illustrated with graphs of the impulse response functions based on numerical simulations. In most of the simulations (except for shock (iii), where we compare different rules with each other), we assume that both central banks follow the second type of Taylor rule.

The results of our simulations, conditioned on the assumed parameterization, can be summarized as follows. A positive technological shock hitting all the economies at the same time has a much stronger impact than similar country-specific technological shocks. What is interesting, in particular, is that both monetary policies are expansionary (loosening) after a positive technological shock which increases outputs. This happens because output increase is offset with deflation. A similar behavior of central banks is noticed after a positive technological shock in intermediate goods sectors.

After an exchange rate shock (appreciation of a monetary union currency), the central bank of the monetary union pursues an expansionary (loosening) monetary policy to stimulate initially decreased output, whereas the central bank of the rest of the world restricts monetary policy in order to suppress inflation.

The general observation from the comparison of different monetary policy rules is that rules with interest rate smoothing (and, therefore, with a higher persistence after the shock) have a more moderate impact on the economic system. Moreover, a rule including wage inflation leads to lower deviations from the equilibrium after the shock, which indicates that taking into account these variables acts countercyclically and may increase stability of the economy.

In the case of this exchange rate shock, we demonstrated the influence of tradable intermediate goods modeling on the behavior of the model. To do that, we constructed a very similar setting with only one difference - previously tradable intermediate goods were not tradable but were used in the production process of the origin country. Our simulations confirm the proposition of Dellas (2005) that the presence of intermediate goods can have important (negative) consequences for the ability of monetary authorities to manipulate nominal exchange rates (in our model via the UIP condition). Our model with intermediate goods appeared to be more stable after an exchange rate shock than the model without intermediate goods. All the simulations have passed a moderate robustness check w.r.t. main parameters and especially w.r.t. a relative size of countries.

Further insights about the asymmetric characteristics of the model are gained from the empirical Maximum Likelihood and Bayesian model estimations of the deep structural parameters, applied to a quarterly panel data sample (running from $1970^{I I}$ to $2005^{I V}$ ) of the Netherlands, the rest of the EMU and the rest of the world (proxied by the USA).

Rule (II) is the rule that yields the most consistent results. We find that the NL show more stickiness in prices than the RoEMU and the US. For the tradable and non-tradable final goods the prices of the NL are sticky during 2.5 quarters on the average, while these prices in the US are sticky during one quarter less. The price stickiness in the intermediate goods sectors is about one quarter less than in the final goods sectors and very similar among countries, but somewhat less sticky in the NL than in the RoEMU. In setting wages, we find the same ordering as in the final goods sectors: on average the NL reset wages every 1.90 quarter, the RoEMU every 1.80 quarter and the US each 1.56 quarter.

Finally, welfare maximizing optimal monetary policy rules are analyzed in the above mentioned theoretical three-country framework.

## Appendix

## A Net Foreign Assets in a 3-country setting

In this appendix we assume that all consumers as well as producers are symmetric and quantity variables are in per capita terms; hence, we can simplify relevant notations such that the only difference is the size of the countries.

The active consumers' budget constraint (CBC) satisfies from (1) for home country $h:^{34}$

$$
\begin{align*}
& P_{t}^{C, h} C_{t}^{h}+E_{t}\left[Q_{h, t, t+1}^{h} B_{h, t+1}^{h}\right]+E_{t}\left[\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}\right]+E_{t}\left[S_{t} \hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}\right] \\
& +M_{t+1}^{h}-\left(1-\tau_{h}\right)\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right]-T_{h, t}^{h}-B_{h, t}^{h}-B_{U, t}^{h}-S_{t} B_{W, t}^{h}-M_{t}^{h}-\mathcal{B}_{h, t}=0 \tag{41}
\end{align*}
$$

Similarly, for the RoMU and the RoW:

$$
\begin{align*}
& P_{t}^{C, U} C_{t}^{U}+E_{t}\left[Q_{U, t, t+1}^{U} B_{U, t+1}^{U}\right]+E_{t}\left[\hat{Q}_{h, t, t+1}^{U} B_{h, t+1}^{U}\right]+E_{t}\left[S_{t} \hat{Q}_{W, t, t+1}^{U} B_{W, t+1}^{U}\right] \\
& +M_{t+1}^{U}-\left(1-\tau_{U}\right)\left[\sum_{m=F T, F N, V T, V N} W_{U, t}^{m} L_{U, t}^{m}\right]-T_{U, t}^{U}-B_{U, t}^{U}-B_{h, t}^{U}-S_{t} B_{W, t}^{U}-M_{t}^{U}-\mathcal{B}_{U, t}=0 \tag{42}
\end{align*}
$$

and

$$
\begin{align*}
& P_{t}^{C, W} C_{t}^{W}+E_{t}\left[Q_{W, t, t+1}^{W} B_{W, h, t+1}^{W}\right]+E_{t}\left[S_{t}^{-1} \hat{Q}_{h, t, t+1}^{W} B_{h, t+1}^{W}\right]+E_{t}\left[S_{t}^{-1} \hat{Q}_{U, t, t+1}^{W} B_{U, t+1}^{W}\right] \\
& +M_{t+1}^{W}-\left(1-\tau_{W}\right)\left[\sum_{m=F T, F N, V T, V N} W_{W, t}^{m} L_{W, t}^{m}\right]-T_{W, t}^{W}-B_{W, t}^{W}-S_{t}^{-1} B_{h, t}^{W}-S_{t}^{-1} B_{U, t}^{W}-M_{t}^{W}-\mathcal{B}_{W, t}=0 \tag{43}
\end{align*}
$$

respectively, where, according to the risk premium definitions, we have:

$$
\begin{align*}
\hat{Q}_{U, t, t+1}^{h} & \equiv\left(1+\hat{r}_{U, t, t+1}^{h}\right)^{-1}=\left[\digamma_{U, t+1}^{h}(.)\left(1+r_{t, t+1}\right)\right]^{-1}  \tag{44}\\
\hat{Q}_{W, t, t+1}^{h} & \equiv\left(1+\hat{r}_{W, t, t+1}^{h}\right)^{-1}=\left[\digamma_{W, t+1}^{h}(.)\left(1+r_{t, t+1}^{W}\right)\right]^{-1}  \tag{45}\\
\hat{Q}_{h, t, t+1}^{U} & \equiv\left(1+\hat{r}_{h, t, t+1}^{U}\right)^{-1}=\left[\digamma_{h, t+1}^{U}(.)\left(1+r_{t, t+1}\right)\right]^{-1}  \tag{46}\\
\hat{Q}_{W, t, t+1}^{U} & \equiv\left(1+\hat{r}_{W, t, t+1}^{U}\right)^{-1}=\left[\digamma_{W, t+1}^{U}(.)\left(1+r_{t, t+1}^{W}\right)\right]^{-1}  \tag{47}\\
\hat{Q}_{h, t, t+1}^{W} & \equiv\left(1+\hat{r}_{h, t, t+1}^{W}\right)^{-1}=\left[\digamma_{h, t+1}^{W}(.)\left(1+r_{t, t+1}\right)\right]^{-1}  \tag{48}\\
\hat{Q}_{U, t, t+1}^{W} & \equiv\left(1+\hat{r}_{U, t, t+1}^{W}\right)^{-1}=\left[\digamma_{U, t+1}^{W}(.)\left(1+r_{t, t+1}\right)\right]^{-1} \tag{49}
\end{align*}
$$

with $\digamma_{U, t+1}^{h}(),. \digamma_{W, t+1}^{h}(),. \digamma_{h, t+1}^{U}(),. \digamma_{W, t+1}^{U}(),. \digamma_{h, t+1}^{W}($.$) and \digamma_{U, t+1}^{W}($.$) relevant risk premia and r_{t, t+1}$ and $r_{t, t+1}^{W}$ the MU and RoW nominal interest rates, respectively. ${ }^{35}$

Introducing balanced governments' budget constraints (GBCs) from (9):

$$
\begin{gather*}
M_{t+1}^{h}-M_{t}^{h}+\tau_{h}\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right]-T_{h, t}^{h}+E_{t}\left[Q_{h, t, t+1}^{h} B_{h, t+1}^{h}\right] \\
-B_{h, t}^{h}+E_{t}\left[\hat{Q}_{h, t, t+1}^{U} B_{h, t+1}^{U}\right]-B_{h, t}^{U}+E_{t}\left[\hat{Q}_{h, t, t+1}^{W} B_{h, t+1}^{W}\right]-B_{h, t}^{W}=0,  \tag{50}\\
M_{t+1}^{U}-M_{t}^{U}+\tau_{U}\left[\sum_{m=F T, F N, V T, V N} W_{U, t}^{m} L_{U, t}^{m}\right]-T_{U, t}^{U}+E_{t}\left[Q_{U, t, t+1}^{U} B_{U, t+1}^{U}\right] \\
-B_{U, t}^{U}+E_{t}\left[\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}\right]-B_{U, t}^{h}+E_{t}\left[\hat{Q}_{U, t, t+1}^{W} B_{U, t+1}^{W}\right]-B_{U, t}^{W}=0 \tag{51}
\end{gather*}
$$

[^16]and
\[

$$
\begin{gather*}
M_{t+1}^{W}-M_{t}^{W}+\tau_{W}\left[\sum_{m=F T, F N, V T, V N} W_{W, t}^{m} L_{W, t}^{m}\right]-T_{W, t}^{W}+E_{t}\left[Q_{W, t, t+1}^{W} B_{W, t+1}^{W}\right] \\
-B_{W, t}^{W}+E_{t}\left[\hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}\right]-B_{W, t}^{h}+E_{t}\left[\hat{Q}_{W, t, t+1}^{U} B_{W, t+1}^{U}\right]-B_{W, t}^{U}=0, \tag{52}
\end{gather*}
$$
\]

we may subtract each country's balanced GBC from each country's active CBC to obtain three bilateral NFAs equations:

$$
\begin{align*}
0 & =P_{t}^{C, h} C_{t}^{h}-\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right] \\
& +\left(E_{t}\left[\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}\right]-E_{t}\left[\hat{Q}_{h, t, t+1}^{U} B_{h, t+1}^{U}\right]\right)+\left(B_{h, t}^{U}-B_{U, t}^{h}\right) \\
& +\left(E_{t}\left[S_{t} \hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}\right]-E_{t}\left[\hat{Q}_{h, t, t+1}^{W} B_{h, t+1}^{W}\right]\right)+\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right)-\mathcal{B}_{h, t}  \tag{53}\\
0 & =P_{t}^{C, U} C_{t}^{U}-\left[\sum_{m=F T, F N, V T, V N} W_{U, t}^{m} L_{U, t}^{m}\right] \\
& +\left(E_{t}\left[\hat{Q}_{h, t, t+1}^{U} B_{h, t+1}^{U}\right]-E_{t}\left[\hat{Q}_{U, t, t+1}^{h} B_{U, t+1}^{h}\right]\right)+\left(B_{U, t}^{h}-B_{h, t}^{U}\right) \\
& +\left(E_{t}\left[S_{t} \hat{Q}_{W, t, t+1}^{U} B_{W, t+1}^{U}\right]-E_{t}\left[\hat{Q}_{U, t, t+1}^{W} B_{U, t+1}^{W}\right]\right)+\left(B_{U, t}^{W}-S_{t} B_{W, t}^{U}\right)-\mathcal{B}_{U, t} \tag{54}
\end{align*}
$$

and

$$
\begin{align*}
0 & =P_{t}^{C, W} C_{t}^{W}-\left[\sum_{m=F T, F N, V T, V N} W_{W, t}^{m} L_{W, t}^{m}\right] \\
& +\left(E_{t}\left[S_{t}^{-1} \hat{Q}_{h, t, t+1}^{W} B_{h, t+1}^{W}\right]-E_{t}\left[\hat{Q}_{W, t, t+1}^{h} B_{W, t+1}^{h}\right]\right)+\left(B_{W, t}^{h}-S_{t}^{-1} B_{h, t}^{W}\right) \\
& +\left(E_{t}\left[\hat{Q}_{W, t, t+1}^{U} B_{W, t+1}^{U}\right]-E_{t}\left[S_{t}^{-1} \hat{Q}_{U, t, t+1}^{W} B_{U, t+1}^{W}\right]\right)+\left(S_{t} B_{U, t}^{W}-S_{t} B_{W, t}^{U}\right)-\mathcal{B}_{W, t} . \tag{55}
\end{align*}
$$

Assuming that all domestic firms are symmetric, we derive from Sections 5 and 6 and from Plasmans et al. (2006a) that the nominal benefits $\mathcal{B}_{h, t}, \mathcal{B}_{U, t}$ and $\mathcal{B}_{W, t}$ in (53), (54) and (55), which the consumers get from their entrepreneurial activities, can be expressed as follows, e.g. for the home benefits:

$$
\begin{align*}
\mathcal{B}_{h, t} & =\left[\mathbf{P}_{h, t}^{C T}\right]^{\prime} \mathbf{Y}_{h, t}^{C T}+P_{h, t}^{C N} Y_{h, t}^{C N}+\left[\mathbf{P}_{h, t}^{V T}\right]^{\prime} \mathbf{Y}_{h, t}^{V T}+\left[\mathbf{P}_{h, t}^{V N}\right]^{\prime} \mathbf{Y}_{h, t}^{V N}-\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right]-P_{V, t}^{F T, h} V_{t}^{F T, h}-P_{V, t}^{F N, h} V_{t}^{F N, h} \\
& +\left(\left[\mathbf{P}_{t}^{M F, h}\right]^{\prime}-\left[P_{U, t}^{X, C T, h}, S_{t} P_{W, t}^{X, C T, h}\right]\right)\left[C_{U, t}^{T, h}, C_{W, t}^{T, h}\right]^{\prime} \\
& +\left(\left[\mathbf{P}_{t}^{M V, h}\right]^{\prime}-\left[P_{U, t}^{X, V T, F T, h}, P_{U, t}^{X, V T, F N, h}, S_{t} P_{W, t}^{X, V T, F T, h}, S_{t} P_{W, t}^{X, V T, F N, h}\right]\right)\left[V_{U, t}^{T, F T, h}, V_{U, t}^{T, F N, h}, V_{W, t}^{T, F T, h}, V_{W, t}^{T, F N, h}\right]^{\prime} \tag{56}
\end{align*}
$$

We can disaggregate the intermediate tradable goods terms in (56) in quantities from the CES intermediate input index (11) and the corresponding prices also defined in Subsection 5.3.1 of Plasmans et al. (2006a):

$$
\begin{gather*}
{\left[\mathbf{P}_{h, t}^{V T}\right]^{\prime} \mathbf{Y}_{h, t}^{V T}-P_{V, t}^{F T, h} V_{t}^{F T, h}=} \\
=P_{h, t}^{V T, F T, h} Y_{h, t}^{V T, F T, h}+P_{h, t}^{V T, F N, h} Y_{h, t}^{V T, F N, h}+P_{h, t}^{X, V T, F T, U} Y_{h, t}^{V T, F T, U}+P_{h, t}^{X, V T, F N, U} Y_{h, t}^{V T, F N, U}+P_{h, t}^{X, V T, F T, W} Y_{h, t}^{V T, F T, W} \\
+P_{h, t}^{X, V T, F N, W} Y_{h, t}^{V T, F N, W}-P_{h, t}^{V T, F T, h} V_{h, t}^{T, F T, h}-P_{h, t}^{V N, F T, h} V_{h, t}^{N, F T, h}-P_{U, t}^{V T, F T, h} V_{U, t}^{T, F T, h}-P_{W, t}^{V T, F T, h} V_{W, t}^{T, F T, h} \tag{57}
\end{gather*}
$$

and, similarly, for the intermediate non-tradable goods terms in (56):

$$
\begin{gather*}
{\left[\mathbf{P}_{h, t}^{V N}\right]^{\prime} \mathbf{Y}_{h, t}^{V N}-P_{V, t}^{F N, h} V_{t}^{F N, h}=} \\
=P_{h, t}^{V N, F T, h} Y_{h, t}^{V N, F T, h}+P_{h, t}^{V N, F N, h} Y_{h, t}^{V N, F N, h} \\
-P_{h, t}^{V T, F N, h} V_{h, t}^{T, F N, h}-P_{h, t}^{V N, F N, h} V_{h, t}^{N, F N, h}-P_{U, t}^{V T, F N, h} V_{U, t}^{T, F N, h}-P_{W, t}^{V T, F N, h} V_{W, t}^{T, F N, h} . \tag{58}
\end{gather*}
$$

Adding expressions (57), (58) and the two last terms of (56), which constitute benefits of import sectors, and since, at the home country equilibrium, we assume from equilibrium conditions (15) and (16) that $\left[Y_{h, t}^{V T, F T, h}, Y_{h, t}^{V T, F N, h}\right.$, $\left.Y_{h, t}^{V T, F T, U}, Y_{h, t}^{V T, F N, U}, Y_{h, t}^{V T, F T, W}, Y_{h, t}^{V T, F N, W}\right]^{\prime}=\left[V_{h, t}^{T, F T, h}, V_{h, t}^{T, F N, h}, V_{h, t}^{T, F T, U}, V_{h, t}^{T, F N, U}, V_{h, t}^{T, F T, W}, V_{h, t}^{T, F N, W}\right]^{\prime}$ and $\left[Y_{h, t}^{V N, F T, h}, Y_{h, t}^{V N, F N, h}\right]^{\prime}=\left[V_{h, t}^{N, F T, h}, V_{h, t}^{N, F N, h}\right]^{\prime}$, and similarly for the RoMU and the RoW, we may rewrite this sum, taking account of CES input aggregator (11), as:

$$
\begin{gather*}
\mathrm{S}_{h, t} \equiv\left[\mathbf{P}_{h, t}^{V T]^{\prime}} \mathbf{V}_{h, t}^{T}-P_{V, t}^{F T, h} V_{t}^{F T, h}+\left[\mathbf{P}_{h, t}^{V N}\right]^{\prime} \mathbf{V}_{h, t}^{N}-P_{V, t}^{F N, h} V_{t}^{F N, h}\right. \\
\\
+\left(\left[\mathbf{P}_{t}^{M F, h}\right]^{\prime}-\left[P_{U, t}^{X, C T, h}, S_{t} P_{W, t}^{X, C T, h}\right]\right)\left[C_{U, t}^{T, h}, C_{W, t}^{T, h}\right]^{\prime} \\
+\left(\left[\mathbf{P}_{t}^{M V, h}\right]^{\prime}-\left[P_{U, t}^{X, V T, F T,, h}, P_{U, t}^{X, V T, F N, h}, S_{t} P_{W, t}^{X, V T, F T, h}, S_{t} P_{W, t}^{X, V T, F N, h}\right]\right)\left[V_{U, t}^{T, F T, h}, V_{U, t}^{T, F N, h}, V_{W, t}^{T, F T, h}, V_{W, t}^{T, F N, h}\right]^{\prime} \\
=P_{h, t}^{V T, F T, h} V_{h, t}^{T, F T, h}+P_{h, t}^{V T, F N, h} V_{h, t}^{T, F N, h}+P_{h, t}^{X, V T, F T, U} V_{h, t}^{T, F T, U}+P_{h, t}^{X, V T, F N, U} V_{h, t}^{T, F N, U}+P_{h, t}^{X, V T, F T, W} V_{h, t}^{T, F T, W} \\
+P_{h, t}^{X, V T, F N, W} V_{h, t}^{T, F N, W}-P_{h, t}^{V T, F T, h} V_{h, t}^{T, F T, h}-P_{h, t}^{V N, F T, h} V_{h, t}^{N, F T, h}-P_{U, t}^{V T, F T, h} V_{U, t}^{T, F T, h}-P_{W, t}^{V T, F T, h} V_{W, t}^{T, F T, h} \\
\quad+P_{h, t}^{V N, F T, h} Y_{h, t}^{V N, F T, h}+P_{h, t}^{V N, F N, h} Y_{h, t}^{V N, F N, h}
\end{gather*}
$$

where we used the export price notations for final and intermediate goods produced in the RoW from Plasmans et al. (2006a).

Canceling terms in (59), taking account of equilibrium conditions (16), only international trade terms are remaining:

$$
\begin{gather*}
\mathrm{S}_{h, t}=P_{U, t}^{C T, h} C_{U, t}^{T, h}+P_{W, t}^{C T, h} C_{W, t}^{T, h}-P_{U, t}^{X, C T, h} C_{U, t}^{T, h}-S_{t} P_{W, t}^{X, C T, h} C_{W, t}^{T, h} \\
P_{h, t}^{X, V T, F T, U} V_{h, t}^{T, F T, U}+P_{h, t}^{X, V T, F N, U} V_{h, t}^{T, F N, U}+P_{h, t}^{X, V T, F T, W} V_{h, t}^{T, F T, W}+P_{h, t}^{X, V T, F N, W} V_{h, t}^{T, F N, W} \\
-P_{U, t}^{X, V T, F T, h} V_{U, t}^{T, F T, h}-P_{U, t}^{X, V T, F N, h} V_{U, t}^{T, F N, h}-S_{t} P_{W, t}^{X, V T, F T, h} V_{W, t}^{T, F T, h}-S_{t} P_{W, t}^{X, V T, F N, h} V_{W, t}^{T, F N, h} \\
=P_{U, t}^{C T, h} C_{U, t}^{T, h}+P_{W, t}^{C T, h} C_{W, t}^{T, h}-P_{U, t}^{X, C T, h} C_{U, t}^{T, h}-S_{t} P_{W, t}^{X, C T, h} C_{W, t}^{T, h}+N X_{h, t}^{V T}, \tag{60}
\end{gather*}
$$

where

$$
\begin{align*}
N X_{h, t}^{V T} & \equiv P_{h, t}^{X, V T, F T, U} V_{h, t}^{T, F T, U}+P_{h, t}^{X, V T, F N, U} V_{h, t}^{T, F N, U}+P_{h, t}^{X, V T, F T, W} V_{h, t}^{T, F T, W}+P_{h, t}^{X, V T, F N, W} V_{h, t}^{T, F N, W} \\
& -P_{U, t}^{X, V T, F T, h} V_{U, t}^{T, F T, h}-P_{U, t}^{X, V T, F N, h} V_{U, t}^{T, F N, h}-S_{t} P_{W, t}^{X, V T, F T, h} V_{W, t}^{T, F T, h}-S_{t} P_{W, t}^{X, V T, F N, h} V_{W, t}^{T, F N, h} \tag{61}
\end{align*}
$$

is the home country's net export of tradable intermediate goods in period $t$.
Finally, using the equilibrium conditions (13) and (14) for the home country such that $\left[Y_{h, t}^{C T, h}, Y_{h, t}^{C T, U}, Y_{h, t}^{C T, W}\right]^{\prime}$ $=\left[C_{h, t}^{T, h}, C_{h, t}^{T, U}, C_{h, t}^{T, W}\right]^{\prime}$ and $Y_{h, t}^{C N}=C_{h, t}^{N}$, we can rewrite total benefits (56) of households in the home country as:

$$
\begin{align*}
\mathcal{B}_{h, t} & =P_{h, t}^{C T, h} C_{h, t}^{T, h}+P_{h, t}^{X, C T, U} C_{h, t}^{T, U}+P_{h, t}^{X, C T, W} C_{h, t}^{T, W}+P_{U, t}^{C T, h} C_{U, t}^{T, h}+P_{W, t}^{C T, h} C_{W, t}^{T, h}-P_{U, t}^{X, C T, h} C_{U, t}^{T, h}-S_{t} P_{W, t}^{X, C T, h} C_{W, t}^{T, h} \\
& +P_{h, t}^{C N} C_{h, t}^{N}-\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right]+N X_{h, t}^{V T} . \tag{62}
\end{align*}
$$

Similarly, all firms in the RoMU and the RoW are also assumed to be symmetric with benefits:

$$
\begin{align*}
\mathcal{B}_{U, t} & =P_{U, t}^{C T, U} C_{U, t}^{T, U}+P_{U, t}^{X, C T, h} C_{U, t}^{T, h}+P_{U, t}^{X, C T, W} C_{U, t}^{T, W}+P_{h, t}^{C T, U} C_{h, t}^{T, U}+P_{W, t}^{C T, U} C_{W, t}^{T, U}-P_{h, t}^{X, C T, U} C_{h, t}^{T, U} \\
& -S_{t} P_{W, t}^{X, C T, U} C_{W, t}^{T, U}+P_{U, t}^{C N} C_{U, t}^{N}-\left[\sum_{m=F T, F N, V T, V N} W_{U, t}^{m} L_{U, t}^{m}\right]+N X_{U, t}^{V T} \tag{63}
\end{align*}
$$

and

$$
\begin{align*}
\mathcal{B}_{W, t} & =P_{W, t}^{C T, W} C_{W, t}^{T, W}+P_{W, t}^{X, C T, h} C_{W, t}^{T, h}+P_{W, t}^{X, C T, U} C_{W, t}^{T, U}+P_{h, t}^{C T, W} C_{h, t}^{T, W}+P_{U, t}^{C T, W} C_{U, t}^{T, W}-S_{t}^{-1} P_{h, t}^{X, C T, W} C_{h, t}^{T, W} \\
& -S_{t}^{-1} P_{U, t}^{X, C T, W} C_{U, t}^{T, W}+P_{W, t}^{C N} C_{W, t}^{N}-\left[\sum_{m=F T, F N, V T, V N} W_{W, t}^{m} L_{W, t}^{m}\right]+N X_{W, t}^{V T} . \tag{64}
\end{align*}
$$

Now, substituting these benefits (62), (63) and (64) into the NFAs equations (53), (54) and (55), and assuming that the conditional expectations of the stochastic discount factors are all mutually pairwise equal between countries, i.e. $E_{t}\left[\hat{Q}_{U, t, t+1}^{h}\right]=E_{t}\left[\hat{Q}_{h, t, t+1}^{U}\right] \equiv E_{t}\left[\hat{Q}_{h U, t, t+1}\right], E_{t}\left[\hat{Q}_{W, t, t+1}^{h}\right]=E_{t}\left[\hat{Q}_{h, t, t+1}^{W}\right] \equiv E_{t}\left[\hat{Q}_{h W, t, t+1}\right]$ and $E_{t}\left[\hat{Q}_{W, t, t+1}^{U}\right]=E_{t}\left[\hat{Q}_{U, t, t+1}^{W}\right] \equiv E_{t}\left[\hat{Q}_{U W, t, t+1}\right]$, we get for the home country: ${ }^{36}$

$$
\begin{aligned}
0 & =P_{t}^{C, h} C_{t}^{h}-\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right] \\
& +E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{U, t+1}^{h}-B_{h, t+1}^{U}\right)+\left(B_{h, t}^{U}-B_{U, t}^{h}\right) \\
& +E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{h}-B_{h, t+1}^{W}\right)+\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right) \\
& -\left(P_{h, t}^{C T, h} C_{h, t}^{T, h}+P_{h, t}^{X, C T, U} C_{h, t}^{T, U}+P_{h, t}^{X, C T, W} C_{h, t}^{T, W}+P_{U, t}^{C T, h} C_{U, t}^{T, h}+P_{W, t}^{C T, h} C_{W, t}^{T, h}\right) \\
& -\left(P_{h, t}^{C N} C_{h, t}^{N}-P_{U, t}^{X, C T, h} C_{U, t}^{T, h}-S_{t} P_{W, t}^{X, C T, h} C_{W, t}^{T, h}-\left[\sum_{m=F T, F N, V T, V N} W_{h, t}^{m} L_{h, t}^{m}\right]+N X_{h, t}^{V T}\right),
\end{aligned}
$$

or, recalling that $P_{t}^{C, h} C_{t}^{h} \equiv P_{h, t}^{C T, h} C_{h, t}^{T, h}+P_{U, t}^{C T, h} C_{U, t}^{T, h}+P_{W, t}^{C T, h} C_{W, t}^{T, h}+P_{h, t}^{C N} C_{h, t}^{N}$ and cancelling terms, we get:

$$
\begin{align*}
0 & =E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{U, t+1}^{h}-B_{h, t+1}^{U}\right)+\left(B_{h, t}^{U}-B_{U, t}^{h}\right) \\
& +E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{h}-B_{h, t+1}^{W}\right)+\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right) \\
& -\left(N X_{h, t}^{F T}+N X_{h, t}^{V T}\right) \tag{65}
\end{align*}
$$

where

$$
\begin{equation*}
N X_{h, t}^{F T} \equiv P_{h, t}^{X, C T, U} C_{h, t}^{T, U}+P_{h, t}^{X, C T, W} C_{h, t}^{T, W}-\left(P_{U, t}^{X, C T, h} C_{U, t}^{T, h}+S_{t} P_{W, t}^{X, C T, h} C_{W, t}^{T, h}\right) \tag{66}
\end{equation*}
$$

is the home country's net export of tradable final goods in period $t$.
Notice that
$E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{U, t+1}^{h}-B_{h, t+1}^{U}\right)+E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{h}-B_{h, t+1}^{W}\right)+\left(B_{h, t}^{U}-B_{U, t}^{h}\right)+\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right)$
is the variation in the total NFAs position that, in equilibrium, is equal to the sum of tradable final and intermediate net exports.

Similarly, for the RoMU and the RoW:

[^17]\[

$$
\begin{align*}
0 & =E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{h, t+1}^{U}-B_{U, t+1}^{h}\right)+\left(B_{U, t}^{h}-B_{h, t}^{U}\right) \\
& +E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{U}-B_{U, t+1}^{W}\right)+\left(B_{U, t}^{W}-S_{t} B_{W, t}^{U}\right) \\
& -\left(N X_{U, t}^{F T}+N X_{U, t}^{V T}\right) \tag{67}
\end{align*}
$$
\]

and

$$
\begin{align*}
0 & =E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t}^{-1} B_{h, t+1}^{W}-B_{W, t+1}^{h}\right)+\left(B_{W, t}^{h}-S_{t}^{-1} B_{h, t}^{W}\right) \\
& +E_{t}\left[\hat{Q}_{U W, t, t+1}\right]\left(S_{t}^{-1} B_{U, t+1}^{W}-B_{W, t+1}^{U}\right)+\left(B_{W, t}^{U}-S_{t}^{-1} B_{U, t}^{W}\right) \\
& -\left(N X_{W, t}^{F T}+N X_{W, t}^{V T}\right) \tag{68}
\end{align*}
$$

Recalling that, all over the world, the sum of exports is equal to the sum of imports, or:

$$
N X_{h, t}^{F T}+N X_{h, t}^{V T}+N X_{U, t}^{F T}+N X_{U, t}^{V T}+N X_{W, t}^{F T}+N X_{W, t}^{V T}=0
$$

the sum of NFAs equations (65), (67) and (68) becomes:

$$
\begin{gathered}
0=E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{U, t+1}^{h}-B_{h, t+1}^{U}\right)+E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{h}-B_{h, t+1}^{W}\right)+\left(B_{h, t}^{U}-B_{U, t}^{h}\right)+\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right)+ \\
E_{t}\left[\hat{Q}_{h U, t, t+1}\right]\left(B_{h, t+1}^{U}-B_{U, t+1}^{h}\right)+E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t} B_{W, t+1}^{U}-B_{U, t+1}^{W}\right)+\left(B_{U, t}^{h}-B_{h, t}^{U}\right)+\left(B_{U, t}^{W}-S_{t} B_{W, t}^{U}\right)+ \\
E_{t}\left[\hat{Q}_{h W, t, t+1}\right]\left(S_{t}^{-1} B_{h, t+1}^{W}-B_{W, t+1}^{h}\right)+E_{t}\left[\hat{Q}_{U W, t, t+1}\right]\left(S_{t}^{-1} B_{U, t+1}^{W}-B_{W, t+1}^{U}\right) \\
+\left(B_{W, t}^{h}-S_{t}^{-1} B_{h, t}^{W}\right)+\left(B_{W, t}^{U}-S_{t}^{-1} B_{U, t}^{W}\right)
\end{gathered}
$$

## B Net Foreign Assets log-linearization

Rewriting NFAs equation (65) for the home economy in real terms as:

$$
\begin{align*}
0 & =E_{t}\left[\hat{Q}_{h U, t, t+1}\right] \frac{\left(B_{U, t+1}^{h}-B_{h, t+1}^{U}\right)}{P_{t+1}^{C, h}}+\frac{\left(B_{h, t}^{U}-B_{U, t}^{h}\right)}{P_{t+1}^{C, h}} \\
& +E_{t}\left[\hat{Q}_{h W, t, t+1}\right] \frac{\left(S_{t} B_{W, t+1}^{h}-B_{h, t+1}^{W}\right)}{P_{t+1}^{C, h}}+\frac{\left(B_{h, t}^{W}-S_{t} B_{W, t}^{h}\right)}{P_{t+1}^{C, h}} \\
& -\frac{1}{P_{t+1}^{C, h}}\left(N X_{h, t}^{F T}+N X_{h, t}^{V T}\right), \tag{69}
\end{align*}
$$

we can log-linearize it by considering the behavior of domestic and foreign assets in the steady state.
To this end, we rewrite the aggregated first order conditions (6-7) for foreign assets in the steady state, for symmetric consumers and taking account of risk premium definitions (44) and (45), as:

$$
\begin{gathered}
\frac{\bar{\Gamma}_{h} \beta_{U}}{\digamma_{U}^{h}\left(\frac{\bar{B}_{U, h}^{h}}{P^{C, h}}\right)}-\beta_{h} \bar{\Gamma}_{h}=0 \\
\frac{\bar{\Gamma}_{h} \beta_{W} \bar{S}}{\digamma_{W}^{h}\left(\frac{\bar{S} \bar{B}_{W}^{h}}{P^{C, h}}\right)}-\beta_{h} \bar{\Gamma}_{h} \bar{S}=0
\end{gathered}
$$

Defining the real asset values as $B_{U, t}^{h(R)} \equiv \frac{B_{U, t}^{h}}{\bar{P}_{t}^{\overline{C, h}}}$ and $B_{W, t}^{h(R)} \equiv \frac{S_{t-1} B_{W, t}^{h}}{P_{t}^{C, h}}$, and considering that $\beta_{U}=\frac{1}{1+\bar{r}}$ and $\beta_{W}=\frac{1}{1+\bar{r} W}$ hold together with the assumption that the intertemporal discount rates are equal over the world in the steady state,

$$
\begin{equation*}
\beta_{h}=\beta_{U}=\beta_{W} \equiv \beta \tag{70}
\end{equation*}
$$

(see Lindé et al. (2004)), it follows that the risk premia in the steady state are one, or:

$$
\begin{align*}
\digamma_{U}^{h}\left(\bar{B}_{U}^{h(R)}\right) & =1 \text { and }  \tag{71}\\
\digamma_{W}^{h}\left(\bar{B}_{W}^{h(R)}\right) & =1 . \tag{72}
\end{align*}
$$

The assumptions concerning the risk functions $\digamma_{U}^{h}($.$) and \digamma_{W}^{h}($.$) imply that \bar{B}_{U}^{h(R)}=0$ and $\bar{B}_{W}^{h(R)}=0$, or foreign assets owned by domestic consumers are zero in the steady state. This is also so for the foreign consumers in the RoMU and the RoW. Hence, the steady state of the real NFAs equation (69) is:

$$
\begin{equation*}
0=\overline{N X}_{h}^{F T}+\overline{N X}_{h}^{V T} \tag{73}
\end{equation*}
$$

Applying the log-linearization rules from Plasmans et al. (2006a), Appendix B.1, (69) can be log-linearized around its steady state (73), taking account of (70), as:

$$
\begin{align*}
& \beta b_{U, t+1}^{h(R)}+\beta b_{h, t+1}^{U(R)}+\beta b_{W, t+1}^{h(R)}+\beta b_{h, t+1}^{W(R)}-b_{U, t}^{h(R)}-b_{h, t}^{U(R)}-b_{W, t}^{h(R)}-b_{h, t}^{W(R)} \\
& \simeq \frac{\overline{N X}_{h}^{F T}}{\bar{P}^{C, h}}\left(n x_{h, t}^{F T}-p_{t+1}^{C, h}\right)+\frac{\overline{N X_{h}^{V T}}}{\bar{P}^{C, h}}\left(n x_{h, t}^{V T}-p_{t+1}^{C, h}\right), \tag{74}
\end{align*}
$$

where from definitions (66) and (61), we find the log-linearized net export terms:

$$
\begin{align*}
\frac{\overline{N X}}{h} \bar{P}^{F T} \\
\bar{P}^{C, h} \tag{75}
\end{align*}\left(n x_{h, t}^{F T}-p_{t+1}^{C, h}\right)=\frac{\bar{P}_{h}^{X, C T, U}}{\bar{P}^{C, h}} \bar{C}_{h}^{T, U}\left(p_{h, t}^{X, C T, U}+c_{h, t}^{T, U}-p_{t+1}^{C, h}\right)+\frac{\bar{P}_{h}^{X, C T, W}}{\bar{P}^{C, h}} \bar{C}_{h}^{T, W}\left(p_{h, t}^{X, C T, W}+c_{h, t}^{T, W}-p_{t+1}^{C, h}\right)
$$

and

$$
\begin{align*}
\frac{\overline{N X}_{h}^{V T}}{\bar{P}^{C, h}}\left(n x_{h, t}^{V T}-p_{t+1}^{C, h}\right)= & \frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} \bar{V}_{h}^{T, F T, U}\left(p_{h, t}^{X, V T, F T, U}+v_{h, t}^{T, F T, U}-p_{t+1}^{C, h}\right) \\
& +\frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}} \bar{V}_{h}^{T, F N, U}\left(p_{h, t}^{X, V T, F N, U}+v_{h, t}^{T, F N, U}-p_{t+1}^{C, h}\right) \\
& +\frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}^{C, h}} \bar{V}_{h}^{T, F T, W}\left(p_{h, t}^{X, V T, F T, W}+v_{h, t}^{T, F T, W}-p_{t+1}^{C, h}\right) \\
+ & \frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}^{C, h}} \bar{V}_{h}^{T, F N, W}\left(p_{h, t}^{X, V T, F N, W}+v_{h, t}^{T, F N, W}-p_{t+1}^{C, h}\right) \\
- & \frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}^{C, h}} \bar{V}_{U}^{T, F T, h}\left(p_{U, t}^{X, V T, F T, h}+v_{U, t}^{T, F T, h}-p_{t+1}^{C, h}\right) \\
- & \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}^{C, h}} \bar{V}_{U}^{T, F N, h}\left(p_{U, t}^{X, V T, F N, h}+v_{U, t}^{T, F N, h}-p_{t+1}^{C, h}\right) \\
& -\frac{\bar{S} \bar{P}_{W}^{X, V T, F T, h}}{\bar{P}^{C, h}} \bar{V}_{W}^{T, F T, h}\left(s_{t}+p_{W, t}^{X, V T, F T, h}+v_{W, t}^{T, F T, h}-p_{t+1}^{C, h}\right) \\
& -\frac{\bar{S} \bar{P}_{W}^{X, V T, F N, h}}{\bar{P}^{C, h}} \bar{V}_{W}^{T, F N, h}\left(s_{t}+p_{W, t}^{X, V T, F N, h} v_{W, t}^{T, F N, h}-p_{t+1}^{C, h}\right), \tag{76}
\end{align*}
$$

where all the time-dependent variables in lower-case letters are logarithmic deviations of the original variables from their own steady state, i.e. for any variable $X_{t}$ we have $x_{t} \equiv \ln X_{t}-\ln \bar{X}$.

In (75-76), all the steady state terms, as e.g. $\frac{\bar{P}_{h}^{X, C T, U} \bar{C}_{h}^{T, U}}{\bar{P}^{C, h}}$ and $\frac{\bar{P}_{h}^{X, V T, F T, U} \bar{Y}_{h}^{V T, F T, U}}{\bar{P}^{C, h}}$, must be expressed in terms of model parameters. In the following two subsections we will derive these steady state terms.

## B. 1 Steady states of relative final goods prices

From Subsection 5.4 about marginal costs in Plasmans et al. (2006a), the steady state tradable final goods prices for the non-producing importers in the RoMU and the RoW can be written from (65) in the above-mentioned paper, as:

$$
\begin{align*}
\bar{P}_{h}^{C T, U} & =\frac{\theta_{C T, h}^{U}}{\left(\theta_{C T, h}^{U}-1\right)} \bar{P}_{h}^{X, C T, U} \Rightarrow \bar{P}_{h}^{X, C T, U}=\frac{\left(\theta_{C T, h}^{U}-1\right)}{\theta_{C T, h}^{U}} \bar{P}_{h}^{C T, U} \text { and }  \tag{77}\\
\bar{P}_{h}^{C T, W} & =\frac{\theta_{C T, h}^{W}}{\left(\theta_{C T, h}^{W}-1\right)} \bar{S}^{-1} \bar{P}_{h}^{X, C T, W} \Rightarrow \bar{P}_{h}^{X, C T, W}=\frac{\left(\theta_{C T, h}^{W}-1\right)}{\theta_{C T, h}^{W}} \bar{S} \bar{P}_{h}^{C T, W} \tag{78}
\end{align*}
$$

where $\bar{P}_{h}^{X, C T, U}$ and $\bar{P}_{h}^{X, C T, W}$ are the marginal costs, $\overline{M C}_{h}^{M F, U}$ and $\overline{M C}_{h}^{M F, W}$, of the non-producing importers in the RoMU and the RoW, respectively, and $\theta_{C T, h}^{U}$ and $\theta_{C T, h}^{W}$ are the foreign households' intertemporal elasticities of substitution between two different consumption goods produced in the home country ( $\theta_{C T, h}^{U}, \theta_{C T, h}^{W}>1$ ).

Similarly, for the tradable final goods producer, we observe from (67) in Plasmans et al. (2006a) that the steady state of her marginal costs satisfies:

$$
\overline{M C}_{h}^{F T}=\frac{1}{\varpi_{F T, h}} \bar{P}_{Z}^{F T, h} \frac{\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\bar{w}_{F T, h}}-1}}{\bar{\Omega}_{F T, h}}
$$

Since we assume that in the steady state there is no technological growth, or $\bar{\Omega}_{F T, h}=1$, we may rewrite the above equation as:

$$
\begin{align*}
\overline{M C}_{h}^{F T} & =\frac{1}{\varpi_{F T, h}} P_{Z}^{F T, h}\left[Y_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1}=\frac{\left(\theta_{C T, h}^{h}-1\right)}{\theta_{C T, h}^{h}} \bar{P}_{h}^{C T, h} \\
& =\frac{\left(\theta_{C T, h}^{U}-1\right)}{\theta_{C T, h}^{U}} \bar{P}_{h}^{X, C T, U}=\frac{\theta_{C T, h}^{W}-1}{\theta_{C T, h}^{W}} \bar{P}_{h}^{X, C T, W} \tag{79}
\end{align*}
$$

so that from (77-79), we get:

$$
\frac{\left(\theta_{C T, h}^{h}-1\right)}{\theta_{C T, h}^{h}} \bar{P}_{h}^{C T, h}=\frac{\left(\theta_{C T, h}^{U}-1\right)}{\theta_{C T, h}^{U}} \bar{P}_{h}^{X, C T, U}=\left(\frac{\theta_{C T, h}^{U}-1}{\theta_{C T, h}^{U}}\right)^{2} \bar{P}_{h}^{C T, U}=\frac{\theta_{C T, h}^{W}-1}{\theta_{C T, h}^{W}} \bar{P}_{h}^{X, C T, W}=\left(\frac{\theta_{C T, h}^{W}-1}{\theta_{C T, h}^{W}}\right)^{2} \bar{S} \bar{P}_{h}^{C T, W} \text {. Hence, for }
$$

the three countries we have:

$$
\begin{align*}
\mathrm{\eta}_{C T, h}^{h} \bar{P}_{h}^{C T, h} & =\mathrm{\eta}_{C T, h}^{U} \bar{P}_{h}^{X, C T, U}=\left(\mathrm{\eta}_{C T, h}^{U}\right)^{2} \bar{P}_{h}^{C T, U}=\mathrm{\eta}_{C T, h}^{W} \bar{P}_{h}^{X, C T, W}=\left(\mathrm{\eta}_{C T, h}^{W}\right)^{2} \bar{S} \bar{P}_{h}^{C T, W},  \tag{80}\\
\mathrm{\eta}_{C T, U}^{U} \bar{P}_{U}^{C T, U} & =\mathrm{\eta}_{C T, U}^{h} \bar{P}_{U}^{X, C T, h}=\left(\mathrm{m}_{C T, U}^{h}\right)^{2} \bar{P}_{U}^{C T, h}=\mathrm{\eta}_{C T, U}^{W} \bar{P}_{U}^{X, C T, W}=\left(\mathrm{\eta}_{C T, U}^{W}\right)^{2} \bar{S}^{C} \bar{P}_{U}^{C T, W} \text { and }  \tag{81}\\
\mathrm{\eta}_{C T, W}^{W} \bar{P}_{W}^{C T, W} & =\mathrm{y}_{C T, W}^{h} \bar{P}_{W}^{X, C T, h}=\left(\mathrm{y}_{C T, W}^{h}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{C T, h}=\mathrm{\eta}_{C T, W}^{U} \bar{P}_{W}^{X, C T, U}=\left(\mathrm{\eta}_{C T, W}^{U}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{C T, U}, \tag{82}
\end{align*}
$$

where the y s are appropriately defined inverted mark-ups and (80-82) represent relationships between tradable final goods prices. Moreover, from the home country dual price relationship specification (13) in Plasmans et al. (2006a), a relationship between the relative tradable and non-tradable final goods prices, $\left(\frac{\bar{P}^{C T, h}}{\bar{P}^{C, h}}\right)$ and $\left(\frac{\bar{P}_{h}^{C N}}{\bar{P}^{C, h}}\right)$, has to be established. Rewriting (13) in Plasmans et al. (2006a):

$$
\begin{align*}
\bar{P}^{C, h} & =\left[\alpha_{T, h}\left(\bar{P}^{C T, h}\right)^{1-\eta_{C, h}}+\left(1-\alpha_{T, h}\right)\left(\bar{P}_{h}^{C N}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}} \text { with }  \tag{83}\\
\bar{P}^{C T, h} & =\left[n_{h}^{h}\left(\bar{P}_{h}^{C T, h}\right)^{1-\eta_{h}}+\left(n_{U}^{h}-n_{h}^{h}\right)\left(\bar{P}_{U}^{C T, h}\right)^{1-\eta_{h}}+\left(1-n_{U}^{h}\right)\left(\bar{P}_{W}^{C T, h}\right)^{1-\eta_{h}}\right]^{\frac{1}{1-\eta_{h}}}, \tag{84}
\end{align*}
$$

where $n_{h}^{h}$ represents the part of the home country in the world, while $n_{U}^{h}$ represents the part of the whole MU in the world (including the small open home economy).

We need to obtain the following four steady state price ratios:

$$
\frac{\bar{P}_{h}^{X, C T, U}}{\bar{P}^{C, h}}, \frac{\bar{P}_{h}^{X, C T, W}}{\bar{P}^{C, h}}, \frac{\bar{P}_{U}^{X, C T, h}}{\bar{P}^{C, h}}, \frac{\bar{S} \bar{P}_{W}^{X, C T, h}}{\bar{P}^{C, h}},
$$

which appear in the net export definition (75) for final goods, in terms of model parameters. To this end, we need to define steady state "terms of trade" for final goods in the domestic economy:

$$
\begin{equation*}
\bar{\tau}_{U}^{h} \equiv \frac{\bar{P}_{U}^{C T, h}}{\bar{P}_{h}^{C T, h}} \quad \text { and } \quad \bar{\tau}_{W}^{h} \equiv \frac{\bar{P}_{W}^{C T, h}}{\bar{P}_{h}^{C T, h}} \tag{85}
\end{equation*}
$$

and the ratio of the steady state of tradable and non-tradable goods prices, $\overline{\mathcal{R}}_{h} \equiv \frac{\bar{P}^{C T, h}}{\bar{P}_{h}^{C N}}$.
First, we derive the steady state price ratios $\frac{\bar{P}_{h}^{C T, h}}{P^{C, h}}$ and $\frac{\bar{P}_{h}^{C N}}{\bar{P}^{C, h}}$, which can be obtained from (84), as:

$$
\begin{align*}
\frac{\bar{P}_{h}^{C T, h}}{\bar{P}^{C, h}} & =\frac{\bar{P}_{h}^{C T, h}}{\left[\alpha_{T, h}\left(\bar{P}^{C T, h}\right)^{1-\eta_{C, h}}+\left(1-\alpha_{T, h}\right)\left(\bar{P}_{h}^{C N}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}} \\
& =\frac{\bar{P}_{h}^{C T, h}}{\left\{\bar{P}^{C T, h}\left[\alpha_{T, h}+\left(1-\alpha_{T, h}\right)\left(\frac{1}{\mathcal{R}_{h}}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}\right\}} \\
& =\frac{1}{\bar{P}_{h}^{C T, h}} \frac{\bar{P}^{C T, h}}{\left\{\left[\alpha_{T, h}+\left(1-\alpha_{T, h}\right)\left(\frac{1}{\overline{\mathcal{R}}_{h}}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}\right\}} \tag{86}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{\bar{P}_{h}^{C N}}{\bar{P}^{C, h}}=\left(\left[\alpha_{T, h}\left(\overline{\mathcal{R}}_{h}\right)^{1-\eta_{C, h}}+1-\alpha_{T, h}\right]^{\frac{1}{1-\eta_{C, h}}}\right)^{-1} \equiv \Theta_{h, 1}^{C N} . \tag{87}
\end{equation*}
$$

Second, from relationship (80) it follows that $\frac{\bar{P}_{h}^{X, C T, U}}{\bar{P}^{C, h}}=\frac{\mathfrak{\eta}_{C T, h}^{h} \bar{P}_{h}^{C T, h}}{\eta_{C T, h}^{U} \bar{P}^{C, h}}$ and noting that $\frac{\bar{P}_{h}^{C T, h}}{\bar{P}_{U}^{C T, h}}=\frac{1}{\bar{\tau}_{U}^{h}}$ and $\frac{\bar{P}_{h}^{C T, h}}{\bar{P}_{W}^{C T, h}}=\frac{1}{\bar{\tau}_{W}^{h}}$, we can obtain $\frac{\bar{P}_{h}^{C T, h}}{P^{C T, h}}$ as:

$$
\begin{align*}
\frac{\bar{P}_{h}^{C T, h}}{\bar{P}^{C T, h}} & =\frac{\bar{P}_{h}^{C T, h}}{\left[n_{h}^{h}\left(\bar{P}_{h}^{C T, h}\right)^{1-\eta_{h}}+\left(n_{U}^{h}-n_{h}^{h}\right)\left(\bar{P}_{U}^{C T, h}\right)^{1-\eta_{h}}+\left(1-n_{U}^{h}\right)\left(\bar{P}_{W}^{C T, h}\right)^{1-\eta_{h}}\right]^{\frac{1}{1-\eta_{h}}}}= \\
& =\frac{1}{\left[n_{h}^{h}+\left(n_{U}^{h}-n_{h}^{h}\right)\left(\bar{\tau}_{U}^{h}\right)^{1-\eta_{h}}+\left(1-n_{U}^{h}\right)\left(\bar{\tau}_{W}^{h}\right)^{1-\eta_{h}}\right]^{\frac{1}{1-\eta_{h}}}} \equiv \Lambda_{h, 1} \tag{88}
\end{align*}
$$

Hence, after substituting (88) in (86), we obtain:

$$
\begin{equation*}
\frac{\bar{P}_{h}^{C T, h}}{\bar{P}^{C, h}}=\frac{\Lambda_{h, 1}}{\left[\alpha_{T, h}+\left(1-\alpha_{T, h}\right)\left(\frac{1}{\mathcal{R}_{h}}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}} \equiv \Theta_{h, 1}^{C T} \tag{89}
\end{equation*}
$$

and, therefore, from (80),

$$
\begin{equation*}
\frac{\bar{P}_{h}^{X, C T, U}}{\bar{P}^{C, h}}=\frac{\mathfrak{\eta}_{C T, h}^{h} \bar{P}_{h}^{C T, h}}{\mathfrak{\eta}_{C T, h}^{U} \bar{P}^{C, h}}=\frac{\mathfrak{\eta}_{C T, h}^{h}}{\eta_{C T, h}^{U}} \Theta_{h, 1}^{C T} . \tag{90}
\end{equation*}
$$

Likewise, we derive $\frac{\bar{P}_{h}^{X, C T, W}}{P^{C, h}}$. Taking (80) into account, we get:

$$
\begin{equation*}
\frac{\bar{P}_{h}^{X, C T, W}}{\bar{P}^{C, h}}=\frac{\eta_{C T, h}^{h} \bar{P}_{h}^{C T, h}}{\eta_{C T, h}^{W} \bar{P}^{C, h}}=\frac{\eta_{C T, h}^{h}}{\eta_{C T, h}^{W}} \Theta_{h, 1}^{C T} . \tag{91}
\end{equation*}
$$

Ratios of export prices to home prices, $\frac{\bar{P}_{U}^{X, C T, h}}{\bar{P}^{C, h}}$ and $\frac{\overline{S_{\bar{S}} \bar{P}_{W}^{X, C T, h}}}{P^{C, h}}$, are derived using relationships (80-82),

$$
\begin{align*}
& \frac{\bar{P}_{U}^{X, C T, h}}{\bar{P}^{C, h}}=\frac{\bar{P}_{U}^{X, C T, h}}{\bar{P}_{U}^{C T, h}} \frac{\bar{P}_{U}^{C T, h}}{\bar{P}_{h}^{C, h}} \frac{\bar{P}_{h}^{C, h}}{\bar{P}^{C, h}}=\mathrm{y}_{C T, U}^{h} \bar{\tau}_{U}^{h} \Theta_{h, 1}^{C T} \text { and }  \tag{92}\\
& \frac{\bar{S} \bar{P}_{W}^{X, C T, h}}{\bar{P}^{C, h}}=\frac{\bar{S} \bar{P}_{W}^{X, C T, h}}{\bar{P}_{W}^{C T, h}} \frac{\bar{P}_{W}^{C T, h}}{\bar{P}_{h}^{C, h}} \frac{\bar{P}_{h}^{C, h}}{\bar{P}^{C, h}}=\mathrm{y}_{C T, W}^{h} \bar{\tau}_{W}^{h} \Theta_{h, 1}^{C T} . \tag{93}
\end{align*}
$$

## B. 2 Steady states of relative intermediate goods prices

Proceeding in a similar way as in Subsection B. 1 and using the marginal costs specification in Subsection 5.4 in Plasmans et al. (2006a), the steady state prices for the non-producing importers of tradable intermediate goods in the RoMU and the RoW can be written as:

$$
\begin{align*}
\bar{P}_{h}^{V T, F T, U}= & \frac{\varrho_{V T, F T, h}^{U}}{\left(\varrho_{V T, F T, h}^{U}-1\right)} \bar{P}_{h}^{X, V T, F T, U} \Rightarrow \bar{P}_{h}^{X, V T, F T, U}=\frac{\left(\varrho_{V T, F T, h}^{U}-1\right)}{\varrho_{V T, h}^{U}} \bar{P}_{h}^{V T, F T, U}  \tag{94}\\
\bar{P}_{h}^{V T, F N, U}= & \frac{\varrho_{V T, F N, h}^{U}}{\left(\varrho_{V T, F N, h}^{U}-1\right)} \bar{P}_{h}^{X, V T, F N, U} \Rightarrow \bar{P}_{h}^{X, V T, F N, U}=\frac{\left(\varrho_{V T, F N, h}^{U}-1\right)}{\varrho_{V T, F N, h}^{U}} \bar{P}_{h}^{V T, F N, U}  \tag{95}\\
\bar{P}_{h}^{V T, F T, W}= & \frac{\varrho_{V T, F T, h}^{W}}{\left(\varrho_{V T, F T, h}^{W}-1\right)} \bar{S}^{-1} \bar{P}_{h}^{X, V T, F T, W} \Rightarrow \bar{P}_{h}^{X, V T, F T, W}=\frac{\left(\varrho_{V T, F T, h}^{W}-1\right)}{\varrho_{V T, F T, h}^{W}} \bar{S} \bar{P}_{h}^{V T, F T, W} \text { and }  \tag{96}\\
\bar{P}_{h}^{V T, F N, W}= & \frac{\varrho_{V T, F N, h}^{W}}{\left(\varrho_{V T, F N, h}^{W}-1\right)} \bar{S}^{-1} \bar{P}_{h}^{X, V T, F N, W} \Rightarrow \bar{P}_{h}^{X, V T, F N, W}=\frac{\left(\varrho_{V T, F N, h}^{W}-1\right)}{\varrho_{V T, F N, h}^{W}} \bar{S} \bar{P}_{h}^{V T, F N, W} \tag{97}
\end{align*}
$$

where $\bar{P}_{h}^{X, V T, F T, U}, \bar{P}_{h}^{X, V T, F N, U}, \bar{S}^{-1} \bar{P}_{h}^{X, V T, F T, W}$ and $\bar{S}^{-1} \bar{P}_{h}^{X, V T, F N, W}$ are the marginal costs for the foreign nonproducing importers of tradable intermediate goods from the home country, who, then, sell these goods on the respective tradable and non-tradable final goods markets in these countries. Moreover, the $\varrho s$ in (94-97) are the foreign producers' intratemporal elasticities of substitution between production factors in the foreign production functions.

For the domestic tradable intermediate goods producer we observe from (72) in Plasmans et al. (2006a) that the steady state of her marginal costs satisfies for $\bar{\Omega}_{V T, h}=1$ :

$$
\begin{gathered}
\overline{M C}_{h}^{V T}=\frac{1}{\varpi_{V T, h}} P_{Z}^{V T, h}\left[Y_{h}^{V T}\right]^{\frac{1}{\omega_{V T, h}}-1}=\frac{\varrho_{V T, F T, h}^{h}-1}{\varrho_{V T, F T, h}^{h}} \bar{P}_{h}^{V T, F T, h}=\frac{\varrho_{V T, F N, h}^{h}-1}{\varrho_{V T, F N, h}^{h}} \bar{P}_{h}^{V T, F N, h} \\
=\frac{\varrho_{V T, F T, h}^{U}-1}{\varrho_{V T, F T, h}^{U}} \bar{P}_{h}^{X, V T, F T, U}=\frac{\varrho_{V T, F N, h}^{U}-1}{\varrho_{V T, F N, h}^{U}} \bar{P}_{h}^{X, V T, F N, U}=\left(\frac{\varrho_{V T, F T, h}^{U}-1}{\varrho_{V T, F T, h}^{U}}\right)^{2} \bar{P}_{h}^{V T, F T, U} \\
=\left(\frac{\varrho_{V T, F N, h}^{U}-1}{\varrho_{V T, F N, h}^{U}}\right)^{2} \bar{P}_{h}^{V T, F N, U}=\frac{\varrho_{V T, F T, h}^{W}-1}{\varrho_{V T, F T, h}^{W}} \bar{P}_{h}^{X, V T, F T, W}=\frac{\varrho_{V T, F N, h}^{W}-1}{\varrho_{V T, F N, h}^{W}} \bar{P}_{h}^{X, V T, F N, W} \\
=\left(\frac{\varrho_{V T, F T, h}^{W}-1}{\varrho_{V T, F T, h}^{W}}\right)^{2} \bar{S} \bar{P}_{h}^{V T, F T, W}=\left(\frac{\varrho_{V T, F N, h}^{W}-1}{\varrho_{V T, F N, h}^{W}}\right)^{2} \bar{S} \bar{P}_{h}^{V T, F N, W}
\end{gathered}
$$

where $P_{Z}^{V T, h}$ is an aggregate price index for intermediate inputs, in our case equal to domestic nominal wages. Defining y s as the inverted mark-ups for intermediate goods, we get:

$$
\begin{gather*}
\mathrm{y}_{V T, F T, h}^{h} \bar{P}_{h}^{V T, F T, h}=\mathrm{y}_{V T, F N, h}^{h} \bar{P}_{h}^{V T, F N, h}=\mathrm{y}_{V T, F T, h}^{U} \bar{P}_{h}^{X, V T, F T, U}=\mathrm{y}_{V T, F N, h}^{U} \bar{P}_{h}^{X, V T, F N, U}=\left(\mathrm{n}_{V T, F T, h}^{U}\right)^{2} \bar{P}_{h}^{V T, F T, U} \\
=\left(\mathrm{\eta}_{V T, F N, h}^{U}\right)^{2} \bar{P}_{h}^{V T, F N, U}=\mathrm{\eta}_{V T, F T, h}^{W} \bar{P}_{h}^{X, V T, F T, W}=\mathrm{\eta}_{V T, F N, h}^{W} \bar{P}_{h}^{X, V T, F N, W} \\
=\left(\mathrm{\eta}_{V T, F T, h}^{W}\right)^{2} \bar{S} \bar{P}_{h}^{V T, F T, W}=\left(\mathrm{y}_{V T, F N, h}^{W}\right)^{2} \bar{S} \bar{P}_{h}^{V T, F N, W} \tag{98}
\end{gather*}
$$

$$
\begin{align*}
& \mathrm{\eta}_{V T, F T, U}^{U} \bar{P}_{U}^{V T, F T, U}= \mathrm{\eta}_{V T, F N, U}^{U} \bar{P}_{U}^{V T, F N, U}=\mathrm{\eta}_{V T, F T, U}^{h} \bar{P}_{U}^{X, V T, F T, h}=\mathrm{\eta}_{V T, F N, U}^{h} \bar{P}_{U}^{X, V T, F N, h}=\left(\mathrm{\eta}_{V T, F T, U}^{h}\right)^{2} \bar{P}_{U}^{V T, F T, h} \\
&=\left(\mathrm{\eta}_{V T, F N, U}^{h}\right)^{2} \bar{P}_{U}^{V T, F N, h}=\mathrm{\eta}_{V T, F T, U}^{W} \bar{P}_{U}^{X, V T, F T, W}=\mathrm{\eta}_{V T, F N, U}^{W} \bar{P}_{U}^{X, V T, F N, W} \\
&=\left(\mathrm{\eta}_{V T, F T, U}^{W}\right)^{2} \bar{S} \bar{P}_{U}^{V T, F T, W}=\left(\mathrm{\eta}_{V T, F N, U}^{W}\right)^{W} \bar{S} \bar{P}_{U}^{V T, F N, W}, \tag{99}
\end{align*}
$$

and

$$
\begin{align*}
\eta_{V T, F T, W}^{W} \bar{P}_{W}^{V T, F T, W} & =\eta_{V T, F N, W}^{W} \bar{P}_{W}^{V T, F N, W}=\eta_{V T, F T, W}^{U} \bar{P}_{W}^{X, V T, F T, U}=\eta_{V T, F N, W}^{U} \bar{P}_{W}^{X, V T, F N, U}= \\
\left(\mathfrak{\eta}_{V T, F T, W}^{U}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{V T, F T, U} & =\left(\eta_{V T, F N, W}^{U}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{V T, F N, U}=\eta_{V T, F T, W}^{h} \bar{S}_{W}^{X, V T, F T, h}=\eta_{V T, F N, W}^{h} \bar{S} \bar{P}_{W}^{X, V T, F N, h} \\
& =\left(\mathrm{\eta}_{V T, F T, W}^{h}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{V T, F T, h}=\left(\mathrm{\eta}_{V T, F N, W}^{h}\right)^{2} \bar{S}^{-1} \bar{P}_{W}^{V T, F N, h} . \tag{100}
\end{align*}
$$

Consider the log-linearized form (76) of $N X_{h}^{V T}$, we need to find the following real export and import prices:

$$
\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}}, \frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}}, \frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}^{C, h}}, \frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}^{C, h}}
$$

and

$$
\frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}^{C, h}}, \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}^{C, h}}, \frac{\bar{S} \bar{P}_{W}^{X, V T, F T, h}}{\bar{P}^{C, h}}, \frac{\bar{S} \bar{P}_{W}^{X, V T, F N, h}}{\bar{P}^{C, h}}
$$

in terms of model parameters. To this end, we use equations (98), (99) and (100). First, notice that $\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}}$ can be rewritten as a product of relative prices:

$$
\begin{align*}
\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} & =\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}_{h}^{V T, F T, h}} \frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}} \frac{\bar{P}_{V}^{F T, h}}{\bar{P}_{Z}^{F T, h}} \frac{\bar{P}_{Z}^{F T, h}}{\bar{P}_{h}^{C T, h}} \frac{\bar{P}_{h}^{C T, h}}{\bar{P}^{C, h}} \\
& =\frac{\eta_{V T, F T, h}^{h}}{\eta_{V T, F T, h}^{U T}} \frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}} \overline{\bar{P}}_{V}^{F T, h}  \tag{101}\\
\bar{P}_{Z}^{F T, h} & \bar{P}_{h}^{C T, h}
\end{align*} \frac{\overline{P^{C T h}}}{\bar{P}^{C T, h}} .
$$

To find $\frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}}$, we replace its denominator by the steady state of the dual CES price aggregator of the intermediate inputs from Plasmans et al. (2006a), eqs. (60-61) for $m=F T:{ }^{37}$

$$
\bar{P}_{V}^{m, h} \equiv\left[\begin{array}{c}
\nu_{m 1, h}\left(\bar{P}_{h}^{V T, m, h}\right)^{1-\chi_{m, h}}+\nu_{m 2, h}\left(\bar{P}_{h}^{V N, m}\right)^{1-\chi_{m, h}}+\nu_{m 3, h}\left(\bar{P}_{U}^{V T, m, h}\right)^{1-\chi_{m, h}} \\
+\left(1-\nu_{m 1, h}-\nu_{m 2, h}-\nu_{m 3, h}\right)\left(\bar{P}_{W}^{V T, m, h}\right)^{1-\chi_{m, h}}
\end{array}\right]^{\frac{1}{1-\chi_{m, h}}},
$$

or

$$
\begin{align*}
\frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}} & =\frac{\bar{P}_{h}^{V T, F T, h}}{\left[\begin{array}{c}
\nu_{F T 1, h}\left(\bar{P}_{h}^{V T, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 2, h}\left(\bar{P}_{h}^{V N, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 3, h}\left(\bar{P}_{U}^{V T, F T, h}\right)^{1-\chi_{F T, h}} \\
+\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\bar{P}_{W}^{V T, F T, h}\right)^{1-\chi_{F T, h}}
\end{array}\right]^{\frac{1}{1-\chi_{F T, h}}}},
\end{align*}
$$

[^18]Defining:

$$
\overline{\mathcal{R}}_{h}^{V} \equiv \frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{h}^{V N, F T, h}}, \bar{\tau}_{U}^{V T, h} \equiv \frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}_{h}^{V T, F T, h}} \text { and } \bar{\tau}_{W}^{V T, h} \equiv \frac{\bar{P}_{W}^{X, V T, F T, h}}{\bar{P}_{h}^{V T, F T, h}}
$$

the price ratio (102) can be written as:

$$
\frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}}=\frac{1}{\left[\begin{array}{c}
\nu_{F T 1, h}+\nu_{F T 2, h}\left(\frac{1}{\overline{\mathcal{R}}_{h}^{V}}\right)^{1-\chi_{F T, h}}+\nu_{F T 3, h}\left(\frac{\bar{\tau}_{U}^{V T, h}}{\eta_{V T, F T, U}^{h}}\right)^{1-\chi_{F T, h}}  \tag{103}\\
+\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\frac{\bar{\tau}_{W}^{V T, h}}{\eta_{V T, F T, W}^{h}}\right)^{1-\chi_{F T, h}}
\end{array}\right]^{\frac{1}{1-\chi_{F T, h}}}} \equiv \Theta_{h, 1}^{V T, F T}
$$

To look for $\frac{\bar{P}_{V}^{F T, h}}{\bar{P}_{Z}^{F T, h}}$, we replace the denominator by the disaggregation of $\bar{P}_{Z}^{F T, h}$ (see eq. (50) in Plasmans et al. (2006a) or the dual function to production function (10) in this paper), and we define the relative prices of inputs in the production of tradable (non-tradable) final goods as $\Lambda_{h, 2}^{F T} \equiv \frac{\bar{W}^{F T, h}}{\bar{P}_{V}^{F T, h}}\left(\Lambda_{h, 2}^{F N} \equiv \frac{\bar{W}^{F N, h}}{\bar{P}_{V}^{F N, h}}\right)$ to obtain:

$$
\begin{align*}
\frac{\bar{P}_{V}^{F T, h}}{\bar{P}_{Z}^{F T, h}} & =\frac{\bar{P}_{V}^{F T, h}}{\left[v_{L F T, h}\left(\bar{W}^{F T, h}\right)^{1-\gamma_{F T, h}}+\left(1-v_{L F T, h}\right)\left(\bar{P}_{V}^{F T, h}\right)^{1-\gamma_{F T, h}}\right]^{\frac{1}{1-\gamma_{F T, h}}}} \\
& =\frac{1}{\left[v_{L F T, h}\left(\Lambda_{h, 2}^{F T}\right)^{1-\gamma_{F T, h}}+\left(1-v_{L F T, h}\right)\right]^{\frac{1}{1-\gamma_{F T, h}}}} \equiv \Theta_{h, 2}^{F T} \tag{104}
\end{align*}
$$

Finally, to find $\frac{P_{Z}^{F T, h}}{P^{C, h}}$, we use the marginal cost expression (79) for the domestic tradable final goods sector at the steady state:

$$
\begin{equation*}
\bar{P}_{h}^{C T, h}=\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1} \bar{P}_{Z}^{F T, h} \Rightarrow \frac{\bar{P}_{Z}^{F T, h}}{\bar{P}_{h}^{C T, h}}=\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1}\right)^{-1} \tag{105}
\end{equation*}
$$

So, replacing (103), (104), (105) and (89) in (101), we obtain:

$$
\begin{equation*}
\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}}=\frac{\eta_{V T, F T, h}^{h}}{\eta_{V T, F T, h}^{U}} \Theta_{h, 1}^{V T, F T} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1}\right)^{-1} \Theta_{h, 1}^{C T} \tag{106}
\end{equation*}
$$

The other real export prices follow a similar reasoning:

$$
\begin{align*}
\frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}} & =\frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}_{h}^{V T, F N, h}} \frac{\bar{P}_{h}^{V T, F N, h}}{\bar{P}_{V}^{F N}} \frac{\bar{P}_{V}^{F N}}{\bar{P}_{Z}^{F N}} \frac{\bar{P}_{Z}^{F N}}{\bar{P}_{h}^{C N, h}} \frac{\bar{P}_{h}^{C N, h}}{\bar{P}^{C, h}} \\
& =\frac{\eta_{V T, F N, h}^{h}}{\eta_{V T, F N, h}^{U}} \Theta_{h, 1}^{F N} \Theta_{h, 2}^{F N}\left(\frac{\theta_{C N, h}^{h}}{\left(\theta_{C N, h}^{h}-1\right)} \frac{1}{\varpi_{F N, h}}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\bar{w}_{F N, h}}-1}\right)^{-1} \Theta_{h, 1}^{C N} \tag{107}
\end{align*}
$$

$$
\begin{align*}
\frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}^{C, h}} & =\frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}_{h}^{V T, F T, h}} \frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T}} \frac{\bar{P}_{V}^{F T}}{\bar{P}_{Z}^{F T}} \frac{\bar{P}_{Z}^{F T}}{\bar{P}_{h}^{C, h}} \frac{\bar{P}_{h}^{C, h}}{\bar{P}^{C, h}} \\
& =\frac{\eta_{V T, F T, h}^{h}}{\eta_{V T, F T, h}^{W}} \Theta_{h, 1}^{V T, F T} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1}\right)^{-1} \Theta_{h, 1}^{C T}=\frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}_{h}^{X, V T, F T, U}} \frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} \\
& =\frac{\eta_{V T, F T, h}^{U}}{\eta_{V T, F T, h}^{W}} \frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} \text { and } \tag{108}
\end{align*}
$$

$$
\begin{equation*}
\frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}^{C, h}}=\frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}_{h}^{X, V T, F N, U}} \frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}}=\frac{\eta_{V T, F N, h}^{U}}{\eta_{V T, F N, h}^{W}} \frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}}, \tag{109}
\end{equation*}
$$

where we used the following definitions and identities:

$$
\begin{align*}
\Theta_{h, 1}^{F N} & \equiv \frac{\bar{P}_{h}^{V T, F N, h}}{\bar{P}_{V}^{F N, h}}=\frac{1}{\left[\begin{array}{c}
\nu_{F N 1, h}+\nu_{F N 2, h}\left(\frac{1}{\mathcal{R}_{h}^{V}}\right)^{1-\chi_{F N, h}}+\nu_{F N 3, h}\left(\eta_{V T, F T, U}^{h} \bar{\tau}_{U}^{V T, h}\right)^{1-\chi_{F N, h}} \\
+\left(1-\nu_{F N 1, h}-\nu_{F N 2, h}-\nu_{F N 3, h}\right)\left(\eta_{V T, F T, W}^{h} \bar{\tau}_{W}^{V T, h}\right)^{1-\chi_{F N, h}}
\end{array}\right]^{\frac{1}{1-\chi_{F N, h}}}} .  \tag{110}\\
\Theta_{h, 2}^{F N} & \equiv \frac{\bar{P}_{V}^{F N}}{\bar{P}_{Z}^{F N}}=\frac{1}{\left[v_{L F N, h}\left(\Lambda_{h, 2}^{F N}\right)^{1-\gamma_{F N, h}}+\left(1-v_{L F N, h}\right)\right]^{\frac{1}{1-\gamma_{F N, h}}}},  \tag{111}\\
\frac{\bar{P}_{Z}^{F N}}{\bar{P}_{h}^{C N, h}} & =\left(\frac{\theta_{C N, h}^{h}}{\left(\theta_{C N, h}^{h}-1\right)} \frac{1}{\varpi_{F N, h}}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\varpi_{F N, h}}-1}\right)^{-1} \text { and }  \tag{112}\\
\frac{\bar{P}^{C N N}}{\bar{P}^{C, h}} & =\left(\left[\alpha_{T, h}\left(\overline{\mathcal{R}}_{h}\right)^{1-\eta_{C, h}}+1-\alpha_{T, h}\right]^{\frac{1}{1-\eta_{C, h}}}\right)^{-1} \equiv \Theta_{h, 1}^{C N} . \tag{113}
\end{align*}
$$

Next, we look for the following import prices:

$$
\frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}^{C, h}}, \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}^{C, h}}, \frac{\bar{S} \bar{P}_{W}^{X, V T, F T, h}}{\bar{P}^{C, h}} \text { and } \frac{\bar{S} \bar{P}_{W}^{X, V T, F N, h}}{\bar{P}^{C, h}} .
$$

Noting from equations.(98-100) that

$$
\frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}_{h}^{X, V T, F T, U}}=\frac{\eta_{V T, F T, h}^{U}}{\eta_{V T, F T, h}^{h} \bar{\tau}_{U}^{V T, h}},
$$

the price $\frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}^{C, h}}$ is expressed as the following product of relative prices:

$$
\begin{equation*}
\frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}^{C, h}}=\frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} \frac{\bar{P}_{U}^{X, V T, F T, h}}{\bar{P}_{h}^{X, V T, F T, U}}=\frac{\eta_{V T, F T, h}^{U}}{\mathrm{y}_{V T, F T, h}^{h} \bar{\tau}_{U}^{V T, h}} \frac{\bar{P}_{h}^{X, V T, F T, U}}{\bar{P}^{C, h}} . \tag{114}
\end{equation*}
$$

Similarly:

$$
\begin{align*}
& \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}^{C, h}}=\frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}} \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}_{h}^{X, V T, F N, U}}=\frac{\mathrm{\eta}_{V T, F N, h}^{U} \mathrm{y}_{V T, F T, U}^{h}}{\mathrm{y}_{V T, F N, h} \mathrm{y}_{V T, F N, U}^{h} \bar{\tau}_{U}^{V T, h}} \frac{\bar{P}_{h}^{X, V T, F N, U}}{\bar{P}^{C, h}}  \tag{115}\\
& \frac{\bar{S}^{\prime} \bar{P}_{W}^{X, V T, F T, h}}{\bar{P}^{C, h}}=\frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}^{C, h}} \frac{\bar{P}_{W}^{X, V T, F T, h}}{\bar{P}_{h}^{X, V T, F T, W}}=\frac{\eta_{V T, F T, h}^{W}}{\mathrm{y}_{V T, F T, h}^{h} \bar{\tau}_{W}^{V T, h}} \frac{\bar{P}_{h}^{X, V T, F T, W}}{\bar{P}^{C, h}} \tag{116}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{\bar{S} \bar{P}_{W}^{X, V T, F N, h}}{\bar{P}^{C, h}}=\frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}^{C, h}} \frac{\bar{P}_{W}^{X, V T, F N, h}}{\bar{P}_{h}^{X, V T, F N, W}}=\frac{\eta_{V T, F N, h}^{W} \mathrm{\eta}_{V T, F T, W}^{h}}{\mathrm{y}_{V T, F N, h}^{h} \mathrm{\eta}_{V T, F N, W}^{h} \bar{\tau}_{W}^{V T, h}} \frac{\bar{P}_{h}^{X, V T, F N, W}}{\bar{P}^{C, h}} . \tag{117}
\end{equation*}
$$

The terms of trade of the final goods that consider the effect of the intermediate goods structure can be expressed in terms of their marginal costs:

$$
\bar{\tau}_{U}^{h} \equiv \frac{\bar{P}_{U}^{C T, h}}{\bar{P}_{h}^{C T, h}}=\frac{\left(\frac{\theta_{C T, U}^{h}}{\left(\theta_{C T, U}^{h}-1\right)}\right)^{2}}{\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)}} \frac{\frac{1}{\varpi_{F T, U}}\left[\bar{Y}_{U}^{F T}\right]^{\frac{1}{\varpi_{F T, U}}-1}}{\frac{1}{\varpi_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\varpi_{F T, h}}-1}} \frac{P_{Z}^{F T, U}}{P_{Z}^{F T, h}},
$$

where

$$
\frac{P_{Z}^{F T, U}}{P_{Z}^{F T, h}}=\frac{\bar{P}_{V}^{F T, h}}{P_{Z}^{F T, h}} \frac{\bar{P}_{V}^{F T, U}}{\bar{P}_{V}^{F T, h}} \frac{P_{Z}^{F T, U}}{\bar{P}_{V}^{F T, U}}=\frac{\Theta_{h, 2}^{F T}}{\Theta_{U, 2}^{F T}} \frac{\bar{P}_{V}^{F T, U}}{\bar{P}_{V}^{F T, h}}
$$

and
$\frac{\bar{P}_{V}^{F T, U}}{\bar{P}_{V}^{F T, h}}=\frac{\left[\begin{array}{c}\nu_{F T 1, U}\left(\bar{P}_{U}^{V T, F T, U}\right)^{1-\chi_{F T, U}}+\nu_{F T 2, U}\left(\bar{P}_{U}^{V N, F T, U}\right)^{1-\chi_{F T, h}}+\nu_{F T 3, U}\left(\bar{P}_{h}^{V T, F T, U}\right)^{1-\chi_{F T, U}} \\ +\left(1-\nu_{F T 1, U}-\nu_{F T 2, U}-\nu_{F T 3, U}\right)\left(\bar{P}_{W}^{V T, F T, U}\right)^{1-\chi_{F T, U}}\end{array}\right]^{\frac{1}{\chi_{F T, U}-1}}}{\left[\begin{array}{c}\nu_{F T 1, h}\left(\bar{P}_{h}^{V T, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 2, h}\left(\bar{P}_{h}^{V N, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 3, h}\left(\bar{P}_{U}^{V N, F T, h}\right)^{1-\chi_{F T, h}} \\ +\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\bar{P}_{W}^{V N, F T, h}\right)^{1-\chi_{F T, h}}\end{array}\right]^{\frac{1}{\chi_{F T, h-1}}}}$

A similar expression can be derived for $\bar{\tau}_{U}^{h}$ and the terms-of-trades for the RoMU and the RoW:
where

$$
\frac{P_{Z}^{F T, W}}{P_{Z}^{F T, h}}=\frac{\bar{P}_{V}^{F T, h}}{P_{Z}^{F T, h}} \frac{\bar{P}_{V}^{F T, W}}{\bar{P}_{V}^{F T, h}} \frac{P_{Z}^{F T, W}}{\bar{P}_{V}^{F T, W}}=\frac{\Theta_{h, 2}^{F T}}{\Theta_{W, 2}^{F T}} \frac{\bar{P}_{V}^{F T, W}}{\bar{P}_{V}^{F T, h}}
$$

and

$$
\begin{aligned}
& \frac{\bar{P}_{V}^{F T, W}}{\bar{P}_{V}^{F T, h}}=\frac{\left[\begin{array}{c}
\nu_{F T 1, W}\left(\bar{P}_{W}^{V T, F T, W}\right)^{1-\chi_{F T, W}}+\nu_{F T 2, W}\left(\bar{P}_{W}^{V N, F T, W}\right)^{1-\chi_{F T, W}}+\nu_{F T 3, W}\left(\bar{P}_{h}^{V T, F T, W}\right)^{1-\chi_{F T, W}} \\
+\left(1-\nu_{F T 1, U}-\nu_{F T 2, U}-\nu_{F T 3, U}\right)\left(\bar{P}_{W}^{V T, F T, U}\right)^{1-\chi_{F T, U}}
\end{array}\right]^{\frac{1}{\chi_{F T, U}-1}}}{\left[\begin{array}{c}
\nu_{F T 1, h}\left(\bar{P}_{h}^{V T, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 2, h}\left(\bar{P}_{h}^{V N, F T, h}\right)^{1-\chi_{F T, h}}+\nu_{F T 3, h}\left(\bar{P}_{U}^{V N, F T, h}\right)^{1-\chi_{F T, h}} \\
+\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\bar{P}_{W}^{V N, F T, h}\right)^{1-\chi_{F T, h}}
\end{array}\right]^{\frac{1}{\chi_{F T, h-1}}}}
\end{aligned}
$$

## B. 3 Steady state of final consumption

Using the steady state price ratios, we are able to derive the steady state formulas for real aggregate consumption demands (see equations (12) and (14) in Plasmans et al. (2006a)):

$$
\left.\begin{array}{c}
\bar{C}_{h}^{T, h}=n_{h}^{h}\left(\frac{\bar{P}_{h}^{C T, h}}{\bar{P}^{C T, h}}\right)^{-\eta_{h}} \bar{C}^{T, h}=n_{h}^{h}\left(\Lambda_{h, 1}\right)^{-\eta_{h}} \bar{C}^{T, h} \equiv n_{h}^{h} \Xi_{1}^{h} \bar{C}^{T, h} \\
\bar{C}_{U}^{T, h}=\left(n_{U}^{h}-n_{h}^{h}\right)\left(\frac{\bar{P}_{U}^{C T, h}}{\bar{P}^{C T, h}}\right)^{-\eta_{h}} C^{T, h}=\left(n_{U}^{h}-n_{h}^{h}\right)\left(\frac{\bar{P}_{U}^{C T, h}}{\bar{P}_{h}^{C T, h}} \bar{P}_{h}^{C T, h} \bar{P}^{C T, h}\right.
\end{array}\right)^{-\eta_{h}} C^{T, h}, ~\left(n_{U}^{h}-n_{h}^{h}\right)\left(\bar{\tau}_{U}^{h} \Lambda_{h, 1}\right)^{-\eta_{h}} C^{T, h} \equiv\left(n_{U}^{h}-n_{h}^{h}\right) \Xi_{2}^{h} \bar{C}^{T, h} .
$$

and

$$
\left.\begin{array}{rl}
\bar{C}_{W}^{T, h} & =\left(1-n_{U}^{h}\right)\left(\frac{\bar{P}_{W}^{C T, h}}{\bar{P}^{C T, h}}\right)^{-\eta_{h}} C^{T, h}=\left(1-n_{U}^{h}\right)\left(\frac{\bar{P}_{W}^{C T, h}}{\bar{P}_{h}^{C T, h}} \bar{P}_{h}^{C T, h} \bar{P}^{C T, h}\right.
\end{array}\right)^{-\eta_{h}} C^{T, h}
$$

where we used (88) and terms-of-trade (85). Moreover:

$$
\begin{aligned}
\bar{C}^{T, h} & =\alpha_{T}^{h}\left(\frac{\bar{P}^{C T, h}}{\bar{P}^{C, h}}\right)^{-\eta_{C, h}} \bar{C}^{h} \\
& =\alpha_{T}^{h}\left(\frac{\bar{P}^{C T, h}}{\left[\alpha_{T, h}\left(\bar{P}^{C T, h}\right)^{1-\eta_{C, h}}+\left(1-\alpha_{T, h}\right)\left(\bar{P}_{h}^{C N}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}}\right)^{-\eta_{C, h}} \bar{C}^{h} \\
& =\alpha_{T}^{h}\left(\frac{1}{\left[\alpha_{T, h}+\left(1-\alpha_{T, h}\right)\left(\frac{1}{\overline{\mathcal{R}}_{h}}\right)^{1-\eta_{C, h}}\right]^{\frac{1}{1-\eta_{C, h}}}}\right)^{\bar{\eta}_{C, h}} \bar{C}^{h} \\
& \equiv \alpha_{T}^{h}\left(\Theta_{3, h}\right)^{-\eta_{C, h}} \bar{C}^{h} \equiv \alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h}
\end{aligned}
$$

and finally:
$\bar{C}_{h}^{N}=\left(1-\alpha_{T}^{h}\right)\left(\frac{\bar{P}_{h}^{C N}}{\bar{P}^{C, h}}\right)^{-\eta_{C, h}} \bar{C}^{h}=\left(1-\alpha_{T}^{h}\right)\left(\frac{\bar{P}_{h}^{C N}}{\bar{P}^{C T, h}} \frac{\bar{P}^{C T, h}}{\bar{P}^{C, h}}\right)^{-\eta_{C, h}} \bar{C}^{h}=\left(1-\alpha_{T}^{h}\right)\left(\frac{\Theta_{3, h}}{\overline{\mathcal{R}}_{h}}\right)^{-\eta_{C, h}} \bar{C}^{h} \equiv\left(1-\alpha_{T}^{h}\right) \Xi_{4}^{h} \bar{C}^{h}$,
where we used $\overline{\mathcal{R}}_{h} \equiv \frac{\bar{P}^{C T, h}}{\bar{P}_{h}^{C N}}$.
Summarizing, the steady state formulas of real consumption demands in terms of total consumption are:

$$
\begin{align*}
\bar{C}_{h}^{T, h} & =n_{h}^{h} \Xi_{1}^{h} \bar{C}^{T, h}  \tag{119}\\
\bar{C}_{U}^{T, h} & =\left(n_{U}^{h}-n_{h}^{h}\right) \Xi_{2}^{h} \bar{C}^{T, h}  \tag{120}\\
\bar{C}_{W}^{T, h} & =\left(1-n_{U}^{h}\right) \Xi_{3}^{h} \bar{C}^{T, h}  \tag{121}\\
\bar{C}_{h}^{N} & =\left(1-\alpha_{T}^{h}\right) \Xi_{4}^{h} \bar{C}^{h} \text { and }  \tag{122}\\
\bar{C}^{T, h} & =\alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h} \tag{123}
\end{align*}
$$

Similar derivations can be made for foreign economies yielding for $U$ :

$$
\begin{align*}
\bar{C}_{U}^{T, U} & =\left(n_{U}^{U}-n_{h}^{U}\right) \Xi_{1}^{U} \bar{C}^{T, U}  \tag{124}\\
\bar{C}_{h}^{T, U} & =n_{h}^{U} \Xi_{2}^{U} \bar{C}^{T, U}  \tag{125}\\
\bar{C}_{W}^{T, U} & =\left(1-n_{U}^{U}\right) \Xi_{3}^{U} \bar{C}^{T, U}  \tag{126}\\
\bar{C}_{U}^{N} & =\left(1-\alpha_{T}^{U}\right) \Xi_{4}^{U} \bar{C}^{U} \text { and }  \tag{127}\\
\bar{C}^{T, U} & =\alpha_{T}^{U} \Xi_{5}^{U} \bar{C}^{U} \tag{128}
\end{align*}
$$

and for W :

$$
\begin{align*}
\bar{C}_{W}^{T, W} & =\left(1-n_{U}^{W}\right) \Xi_{1}^{W} \bar{C}^{T, W}  \tag{129}\\
\bar{C}_{h}^{T, W} & =n_{h}^{W} \Xi_{2}^{W} \bar{C}^{T, W}  \tag{130}\\
\bar{C}_{U}^{T, W} & =\left(n_{U}^{W}-n_{h}^{W}\right) \Xi_{3}^{W} \bar{C}^{T, W}  \tag{131}\\
\bar{C}_{W}^{N} & =\left(1-\alpha_{T}^{W}\right) \Xi_{5}^{W} \bar{C}^{W} \text { and }  \tag{132}\\
\bar{C}^{T, W} & =\alpha_{T}^{W} \Xi_{5}^{W} \bar{C}^{W} \tag{133}
\end{align*}
$$

Using the above formulas we can rewrite total production in tradable final goods sectors as:

$$
\bar{Y}_{h}^{F T}=\bar{C}_{h}^{T, h}+\bar{C}_{h}^{T, U}+\bar{C}_{h}^{T, W}=n_{h}^{h} \Xi_{1}^{h} \alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h}+n_{h}^{U} \Xi_{2}^{U} \alpha_{T}^{U} \Xi_{5}^{U} \bar{C}^{U}+n_{h}^{W} \Xi_{2}^{W} \alpha_{T}^{W} \Xi_{5}^{W} \bar{C}^{W}
$$

and

$$
\begin{aligned}
\bar{Y}_{U}^{F T} & =\bar{C}_{U}^{T, U}+\bar{C}_{U}^{T, h}+\bar{C}_{U}^{T, W} \\
& =\left(n_{U}^{U}-n_{h}^{U}\right) \Xi_{1}^{U} \alpha_{T}^{U} \Xi_{5}^{U} \bar{C}^{U}+\left(n_{U}^{h}-n_{h}^{h}\right) \Xi_{2}^{h} \alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h}+\left(n_{U}^{W}-n_{h}^{W}\right) \Xi_{3}^{W} \alpha_{T}^{W} \Xi_{5}^{W} \bar{C}^{W}
\end{aligned}
$$

A similar expression is obtained for the RoW.

## B. 4 Steady state of the intermediate goods net exports

We can rewrite demands for intermediate inputs in the production of tradable and non-tradable final goods from Plasmans et al. (2006a), subsection 5.4 in the steady state as:

$$
\begin{aligned}
\bar{V}^{F T, h} & =\left(1-v_{L F T, h}\right)\left(\frac{\bar{P}_{V}^{F T, h}}{\bar{P}_{Z}^{F T, h}}\right)^{-\gamma_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\omega_{F T, h}}}=\left(1-v_{L F T, h}\right)\left(\Theta_{h, 2}^{F T}\right)^{-\gamma_{F T, h}}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\omega_{F T, h}}} \text { and } \\
\bar{V}_{h}^{F N} & =\left(1-v_{L F N, h}\right)\left(\frac{\bar{P}_{V}^{F N, h}}{\bar{P}_{Z}^{F N, h}}\right)^{-\gamma_{F N, h}}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\sigma_{F N, h}}}=\left(1-v_{L F N, h}\right)\left(\Theta_{h, 2}^{F N}\right)^{-\gamma_{F N, h}}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\bar{\sigma}_{F N, h}}}
\end{aligned}
$$

Defining terms of trade for $U$ and $W$ as:

$$
\begin{aligned}
& \bar{\tau}_{1}^{V T, U} \equiv \frac{\bar{P}_{h}^{V T, F T, U}}{\bar{P}_{U}^{V T, F T, U}} \text { and } \bar{\tau}_{2}^{V T, U} \equiv \frac{\bar{P}_{W}^{V T, F T, U}}{\bar{P}_{U}^{V T, F T, U}} \\
& \bar{\tau}_{1}^{V T, W} \equiv \frac{\bar{P}_{h}^{V T, F T, W}}{\bar{P}_{W}^{V T, F T, W}} \text { and } \bar{\tau}_{W}^{V T, h} \equiv \frac{\bar{P}_{U}^{V T, F T, W}}{\bar{P}_{W}^{V T, F T, W}}
\end{aligned}
$$

we can rewrite demands for particular intermediate goods inputs in the steady states as:

$$
\begin{aligned}
& \bar{V}_{h}^{T, F T, h}=\nu_{F T 1, h}\left(\frac{\bar{P}_{h}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}}\right)^{-\chi_{F T, h}} V^{F T, h}=\nu_{F T 1}\left(\Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}} \bar{V}^{F T, h}, \\
& \bar{V}_{h}^{N, F T}=\nu_{F T 2, h}\left(\frac{\bar{P}_{h}^{V N, F T, h}}{\bar{P}_{V}^{F T, h}}\right)^{-\chi_{F T, h}} \bar{V}^{F T, h}=\nu_{F T 2}\left(\overline{\mathcal{R}}_{h}^{V} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}} \bar{V}^{F T, h}, \\
& \bar{V}_{U}^{T, F T, h}=\nu_{F T 3, h}\left(\frac{\bar{P}_{U}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}}\right)^{-\chi_{F T, h}} V^{F T, h}=\nu_{F T 3, h}\left(\bar{\tau}_{1}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}} V^{F T, h}, \\
& \bar{V}_{W}^{T, F T, h}=\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\frac{\bar{P}_{W}^{V T, F T, h}}{\bar{P}_{V}^{F T, h}}\right)^{-\chi_{F T, h}} V^{F T, h} \\
& =\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\bar{\tau}_{2}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}} V^{F T, h} \text {, } \\
& \bar{V}_{h}^{T, F N, h}=\nu_{F N 1, h}\left(\frac{\bar{P}_{h}^{V T, F N, h}}{\bar{P}_{V}^{F N, h}}\right)^{-\chi_{F N, h}} V^{F N, h}=\nu_{F N 1}\left(\Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}} \bar{V}^{F N, h} \\
& \bar{V}_{h}^{N, F N}=\nu_{F N 2, h}\left(\frac{\bar{P}_{h}^{V N, F N, h}}{\bar{P}_{V}^{F N, h}}\right)^{-\chi_{F N, h}} \bar{V}^{F N, h}=\nu_{F N 2}\left(\overline{\mathcal{R}}_{h}^{V} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}} \bar{V}^{F N, h} \\
& \bar{V}_{U}^{T, F N, h}=\nu_{F N 3, h}\left(\frac{\bar{P}_{U}^{V T, F N, h}}{\bar{P}_{V}^{F N, h}}\right)^{-\chi_{F N, h}} V^{F N, h}=\nu_{F N 3, h}\left(\bar{\tau}_{1}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}} V^{F N, h} \\
& \bar{V}_{W}^{T, F N, h}=\left(1-\nu_{F N 1, h}-\nu_{F N 2, h}-\nu_{F N 3, h}\right)\left(\frac{\bar{P}_{W}^{V T, F N, h}}{\bar{P}_{V}^{F N, h}}\right)^{-\chi_{F N, h}} V^{F N, h} \\
& =\left(1-\nu_{F N 1, h}-\nu_{F N 2, h}-\nu_{F N 3, h}\right)\left(\bar{\tau}_{2}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}} V^{F N, h} \\
& \bar{V}_{h}^{T, F T, U}=\nu_{F T 3, U}\left(\frac{\bar{P}_{h}^{V T, F T, U}}{\bar{P}_{V}^{F T, U}}\right)^{-\chi_{F T, U}} V^{F T, U}=\nu_{F T 3, U}\left(\bar{\tau}_{1}^{V T, U} \Theta_{U, 1}^{V T}\right)^{-\chi_{F T, U}} V^{F T, U} \\
& \bar{V}_{W}^{T, F T, U}=\left(1-\nu_{F T 1, U}-\nu_{F T 2, U}-\nu_{F T 3, U}\right)\left(\frac{\bar{P}_{W}^{V T, F T, U}}{\bar{P}_{V}^{F T, U}}\right)^{-\chi_{F T, U}} V^{F T, U} \\
& =\left(1-\nu_{F T 1, U}-\nu_{F T 2, U}-\nu_{F T 3, U}\right)\left(\bar{\tau}_{2}^{V T, U} \Theta_{U, 1}^{V T}\right)^{-\chi_{F T, U}} V^{F T, U} \\
& \bar{V}_{h}^{T, F T, W}=\nu_{F T 3, W}\left(\frac{\bar{P}_{h}^{V T, F T, W}}{\bar{P}_{V}^{F T, W}}\right)^{-\chi_{F T, W}} V^{F T, W}=\nu_{F T 3, W}\left(\bar{\tau}_{1}^{V T, W} \Theta_{W, 1}^{V T}\right)^{-\chi_{F T, W}} V^{F T, W} \text { and } \\
& \bar{V}_{U}^{T, F T, W}=\left(1-\nu_{F T 1, W}-\nu_{F T 2, W}-\nu_{F T 3, W}\right)\left(\frac{\bar{P}_{U}^{V T, F T, W}}{\bar{P}_{V}^{F T, W}}\right)^{-\chi_{F T, W}} V^{F T, W} \\
& =\left(1-\nu_{F T 1, W}-\nu_{F T 2, W}-\nu_{F T 3, W}\right)\left(\bar{\tau}_{2}^{V T, W} \Theta_{W, 1}^{V T}\right)^{-\chi_{F T, W}} V^{F T, W},
\end{aligned}
$$

where we used steady state price ratios derived in the previous subsections.

## B. 5 Combining results

Replacing the obtained steady state formulas into (75) and (76), we obtain:

$$
\begin{align*}
\frac{\overline{N X} X_{h}^{F T}}{\bar{P}^{C, h}} n x_{h, t}^{F T} & =\frac{\mathfrak{\eta}_{C T, h}^{h}}{\eta_{C T, h}^{U}} \Theta_{h, 1}^{V T, F T}\left[n_{h}^{U} \Xi_{2}^{U} \alpha_{T}^{U} \Xi_{5}^{U} \bar{C}^{U}\right]\left(p_{h, t}^{X, C T, U}+c_{h, t}^{T, U}-p_{t+1}^{C, h}\right) \\
& +\frac{\mathfrak{\eta}_{C T, h}^{h}}{\eta_{C T, h}^{W}} \Theta_{h, 1}^{V T, F T}\left[n_{h}^{W} \Xi_{2}^{W} \alpha_{T}^{W} \Xi_{5}^{W} \bar{C}^{W}\right]\left(p_{h, t}^{X, C T, W}+c_{h, t}^{T, W}-p_{t+1}^{C, h}\right) \\
& -\eta_{C T, U}^{h} \bar{\tau}_{U}^{h} \Theta_{h, 1}^{V T, F T}\left[\left(n_{U}^{h}-n_{h}^{h}\right) \Xi_{2}^{h} \alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h}\right]\left(p_{U, t}^{X, C T, h}+c_{U, t}^{T, h}-p_{t+1}^{C, h}\right) \\
& -\eta_{C T, W}^{h} \bar{\tau}_{W}^{h} \Theta_{h, 1}^{V T, F T}\left[\left(1-n_{U}^{h}\right) \Xi_{3}^{h} \alpha_{T}^{h} \Xi_{5}^{h} \bar{C}^{h}\right]\left(s_{t}+p_{W, t}^{X, C T, h}+c_{W, t}^{T, h}-p_{t+1}^{C, h}\right) \tag{134}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\overline{N X}_{h}^{V T}}{\bar{P}^{C, h}} n x_{h, t}^{V T} & =\Delta_{h, 1}\left[\bar{Y}_{U}^{F T}\right]^{\frac{1}{\sigma_{F T, U}}}\left(p_{h, t}^{X, V T, F T, U}+v_{h, t}^{T, F T, U}-p_{t+1}^{C, h}\right) \\
& +\Delta_{h, 2}\left[\bar{Y}_{U}^{F N}\right]^{\frac{1}{\omega_{F N, U}}}\left(p_{h, t}^{X, V T, F N, U}+v_{h, t}^{T, F N, U}-p_{t+1}^{C, h}\right) \\
& +\Delta_{h, 3}\left[\bar{Y}_{W}^{F T}\right]^{\frac{1}{\omega_{F} T, W}}\left(p_{h, t}^{X, V T, F T, W}+v_{h, t}^{T, F T, W}-p_{t+1}^{C, h}\right) \\
& +\Delta_{h, 4}\left[\bar{Y}_{W}^{F N}\right]^{\frac{1}{\omega_{F N, W}}}\left(p_{h, t}^{X, V T, F N, W}+v_{h, t}^{T, F N, W}-p_{t+1}^{C, h}\right) \\
& -\Delta_{h, 5}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\omega_{F} T, h}}\left(p_{U, t}^{X, V T, F T, h}+v_{U, t}^{T, F T, h}-p_{t+1}^{C, h}\right) \\
& -\Delta_{h, 6}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\omega_{F N, h}}}\left(p_{U, t}^{X, V T, F N, h}+v_{U, t}^{T, F N, h}-p_{t+1}^{C, h}\right) \\
& -\Delta_{h, 7}\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\omega_{F N, h}}}\left(s_{t}+p_{W, t}^{X, V T, F N, h} v_{W, t}^{T, F N, h}-p_{t+1}^{C, h}\right) \\
& -\Delta_{h, 8}\left[\bar{Y}_{h}^{F T}\right]^{\frac{1}{\omega_{F T, h}}}\left(s_{t}+p_{W, t}^{X, V T, F T, h}+v_{W, t}^{T, F T, h}-p_{t+1}^{C, h}\right), \tag{135}
\end{align*}
$$

where

$$
\begin{align*}
& \Delta_{h, 1} \equiv \frac{\eta_{V T, F T, h}^{U}}{\eta_{V T, F T, U}^{U}} \Theta_{h, 1}^{V T, F T} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{C}_{h}^{T}\right]^{\frac{1}{\omega_{F T, h}}-1}\right)^{-1} \\
& \Theta_{h, 1}^{C T} \nu_{F T 3, U}\left(\bar{\tau}_{1}^{V T, U} \Theta_{U, 1}^{V T}\right)^{-\chi_{F T, U}}\left(1-v_{L F T, U}\right)\left(\Theta_{U, 2}^{F T}\right)^{-\gamma_{F T, U}}, \\
& \Delta_{h, 2} \equiv \frac{\eta_{V T, F N, h}^{U}}{\eta_{V T, F N, U}^{U}} \Theta_{h, 1}^{V N} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C N, h}^{h}}{\left(\theta_{C N, h}^{h}-1\right)} \frac{1}{\varpi_{F N, h}}\left[\bar{C}_{h}^{N}\right]^{\frac{1}{\omega_{F N, h}}-1}\right)^{-1} \\
& \Theta_{h, 1}^{C T} \nu_{F N 3, U}\left(\bar{\tau}_{1}^{V T, U} \Theta_{U, 1}^{V T}\right)^{-\chi_{F N, U}}\left(1-v_{L F N, U}\right)\left(\Theta_{U, 2}^{F N}\right)^{-\gamma_{F N, U}}, \\
& \Delta_{h, 3} \equiv \frac{\eta_{V T, F T, h}^{W}}{\eta_{V T, F T, W}^{W}} \Theta_{h, 1}^{V T, F T} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{C}_{h}^{T}\right]^{\frac{1}{\omega_{F T, h}}-1}\right)^{-1} \\
& \Theta_{h, 1}^{C T} \nu_{F T 3, W}\left(\bar{\tau}_{1}^{V T, W} \Theta_{W, 1}^{V T}\right)^{-\chi_{F T, W}}\left(1-v_{L F T, W}\right)\left(\Theta_{W, 2}^{F T}\right)^{-\gamma_{F T, W}} \text {, } \\
& \Delta_{h, 4} \equiv \frac{\eta_{V T, F N, h}^{W}}{\eta_{V T, F N, W}^{W}} \Theta_{h, 1}^{V N} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C N, h}^{h}}{\left(\theta_{C N, h}^{h}-1\right)} \frac{1}{\varpi_{F N, h}}\left[\bar{C}_{h}^{N}\right]^{\frac{1}{\omega_{F N, h}}-1}\right)^{-1} \\
& \Theta_{h, 1}^{C T} \nu_{F N 3, W}\left(\bar{\tau}_{1}^{V T, W} \Theta_{W, 1}^{V T}\right)^{-\chi_{F N, W}}\left(1-v_{L F N, W}\right)\left(\Theta_{W, 2}^{F N}\right)^{-\gamma_{F N, W}}, \\
& \Delta_{h, 5} \equiv \frac{\left(\mathrm{y}_{V T, F T, U}^{h}\right)^{2}}{\mathrm{\eta}_{V T, F T, U}^{h}} \bar{\tau}_{1}^{V T, h} \Theta_{h, 1}^{F N} \Theta_{h, 2}^{F T}\left(\frac{\theta_{C T, h}^{h}}{\left(\theta_{C T, h}^{h}-1\right)} \frac{1}{\varpi_{F T, h}}\left[\bar{C}_{h}^{T}\right]^{\frac{1}{\bar{\omega}_{F T, h}}-1}\right)^{-1} \\
& \Theta_{h, 1}^{V T, F T} \nu_{F T 3, h}\left(\bar{\tau}_{1}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}}\left(1-v_{L F T, h}\right)\left(\Theta_{h, 2}^{F T}\right)^{-\gamma_{F T, h}}, \\
& \Delta_{h, 6} \equiv \frac{\bar{P}_{U}^{X, V T, F N, h}}{\bar{P}^{C, h}} \nu_{F N 3, h}\left(\bar{\tau}_{1}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}}\left(1-v_{L F N, h}\right)\left(\Theta_{h, 2}^{F N}\right)^{-\gamma_{F N, h}}, \\
& \Delta_{h, 7} \equiv \frac{\bar{S} \bar{P}_{W}^{X, V T, F N, h}}{\bar{P}^{C, h}}\left(1-\nu_{F N 1, h}-\nu_{F N 2, h}-\nu_{F N 3, h}\right)\left(\bar{\tau}_{2}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F N, h}}\left(1-v_{L F N, h}\right)\left(\Theta_{h, 2}^{F N}\right)^{-\gamma_{F N, h}} \text { and } \\
& \Delta_{h, 8} \equiv \frac{\bar{S} \bar{P}_{W}^{X, V T, F T, h}}{\bar{P}^{C, h}}\left(1-\nu_{F T 1, h}-\nu_{F T 2, h}-\nu_{F T 3, h}\right)\left(\bar{\tau}_{2}^{V T, h} \Theta_{h, 1}^{V T, F T}\right)^{-\chi_{F T, h}}\left(1-v_{L F T, h}\right)\left(\Theta_{h, 2}^{F T}\right)^{-\gamma_{F T, h}} . \tag{136}
\end{align*}
$$

## C Steady state of the complete model

We will approximate all the variables in the world economy around their symmetric steady state. The inflation rate is zero, and there are no productivity shocks in any country ( $\Omega_{h}^{m}=\Omega_{W}^{m}=\Omega_{W}^{m}=1$ ) for $m=F T, F N, V T, V N$ and other exogenous variables are zero.

Using the consumer's FOCs w.r.t. labor (4) in the steady state:

$$
\begin{equation*}
\frac{\varrho_{L m, h}}{\varrho_{L m, h}-1}\left(\bar{L}_{h}\right)^{\phi_{h}}=\bar{\Gamma}^{h}\left(1-\tau_{h}\right) \bar{W}_{h}^{m} \text { for } m=F T, F N, V T, V N, \tag{137}
\end{equation*}
$$

we get: $\left(\frac{\varrho_{L m, h}}{\varrho_{L m, h}-1}\right)\left(\frac{\varrho_{L F T, h}}{\varrho_{L F T, h}-1}\right)^{-1}=\bar{W}_{m}^{m} \bar{W}_{h}^{h T}$ for $m=F N, V T, V N$. Assuming that firms are efficient in the steady state, we invert all sector production functions. Domestic tradable and non-tradable final goods are $\bar{Z}_{h}^{F T}=$ $\frac{1}{\Omega_{F T, h}}\left(\bar{Y}_{h}^{F T}\right)^{\frac{1}{\bar{\omega}_{F T, h}}}$ and $\bar{Z}_{h}^{F N}=\frac{1}{\Omega_{F N, h}}\left(\bar{Y}_{h}^{F N}\right)^{\frac{1}{\bar{W}_{F N, h}}}$, while for domestic tradable and non-tradable intermediate goods we have $\bar{Z}_{h}^{V T}=\bar{L}_{h}^{V T}=\frac{1}{\Omega_{V T, h}}=\left(\bar{Y}_{h}^{V T}\right)^{\frac{1}{\bar{\omega}_{V T, h}}}$ and $\bar{Z}_{h}^{V N}=\bar{L}_{h}^{V N}=\frac{1}{\Omega_{V N, h}}\left(\bar{Y}_{h}^{V N}\right)^{\frac{1}{\overline{W_{V N, h}}}}$. Final goods production functions (10) can be inverted in the second level for labor input as:

$$
\bar{L}_{h}^{F T}=\frac{\left[\frac{1}{\bar{\Omega}_{F T, h}}\left(\bar{Y}_{h}^{F T}\right)^{\frac{1}{\bar{w}_{F T, h}}}\right]-\left[\left(1-v_{L F T, h}\right)^{\frac{1}{\gamma_{F T, h}-1}} \bar{V}^{F T, h}\right]}{v_{L F T, h}^{\gamma_{F T, h-1}}} \text { and } \bar{L}_{h}^{F N}=\frac{\left[\frac{1}{\bar{\Omega}_{F N, h}}\left(\bar{Y}_{h}^{F N}\right)^{\frac{1}{\bar{w}_{F N, h}}}\right]-\left[\left(1-v_{L F N, h}\right)^{\frac{1}{\gamma_{F N, h}-1} \bar{V}^{F N, h}}\right]}{v_{L F N, h}^{\frac{1}{\gamma_{F N}, h-1}}} .
$$

Using formula (70) from Plasmans et al. (2006a): $\overline{M C}{ }_{h}^{F N} \equiv \frac{1}{\varpi_{F N, h}} \bar{P}_{Z}^{F N, h} \frac{\left[\bar{Y}_{h}^{F N}\right]^{\frac{1}{\bar{\sigma}_{F N, h}}-1}}{\Omega_{F N, h}}=\frac{\left(\theta_{C N, h}^{h}-1\right)}{\theta_{C N, h}^{h}} \bar{P}_{h}^{C N}$, we get: $\bar{W}_{h}^{m}=$

$$
\begin{align*}
& \frac{\bar{\Gamma}^{h}\left(1-\tau_{h}\right)}{\left(\frac{\varrho_{L m, h}}{\varrho_{L m, h}-1}\right)} \tag{138}
\end{align*}
$$

for $m=F T, F N, V T, V N$.
Along with equations implied by (138) and log-linearizing around a steady state, where $\bar{L}_{h}^{m}=\bar{L}_{U}^{m}=\bar{L}_{W}^{m}$ for $m=F T, F N, V T, V N$, we obtain $\bar{C}^{h}=\bar{C}^{U}=\bar{C}^{W}$.

Prices in the steady state and equilibrium conditions yield the full set of equations, which define the steady state values of the endogenous variables.

## D Model in the log-linearized form

The aggregate model to be presented in this Appendix is a log-linearized version of the NOEM model for three economies with a monetary union between two of them.

The following notation is used: the country of origin is denoted in the subscript whereas the country of destination (consumption) is in the superscript.

The order of appearance of the equations corresponds to the theoretical specifications of the model in Plasmans et al. (2006a).

For sake of brevity, we present the log-linearized equations for the home country solely. The complete model can be directly obtained on simple request from the authors.

Total labor:

$$
\begin{equation*}
0 \simeq-\bar{L}_{h, t} l_{h, t}+\bar{L}_{h, t}^{F T} l_{h, t}^{F T}+\bar{L}_{h, t}^{F N} l_{h, t}^{F T}+\bar{L}_{h, t}^{V T} l_{h, t}^{V T}+\bar{L}_{h, t}^{V N} l_{h, t}^{V T} \tag{139}
\end{equation*}
$$

Total labor income:

$$
\begin{align*}
0 & \simeq-\bar{W}_{h, t} \bar{L}_{h, t} w_{h, t}-\bar{W}_{h, t} \bar{L}_{h, t} l_{h, t}+\bar{W}_{h, t}^{F T} \bar{L}_{h, t}^{F T} w_{h, t}^{F T}+\bar{W}_{h, t}^{F T} \bar{L}_{h, t}^{F T} l_{h, t}^{F T}+\bar{W}_{h, t}^{F N} \bar{L}_{h, t}^{F N} w_{h, t}^{F N}+  \tag{140}\\
& +\bar{W}_{h, t}^{F N} \bar{L}_{h, t}^{F N} l_{h, t}^{F N}+\bar{W}_{h, t}^{V T} \bar{L}_{h, t}^{V T} w_{h, t}^{V T}+\bar{W}_{h, t}^{V T} \bar{L}_{h, t}^{V T} l_{h, t}^{V T}+\bar{W}_{h, t}^{V N} \bar{L}_{h, t}^{V N} w_{h, t}^{V N}+\bar{W}_{h, t}^{V N} \bar{L}_{h, t}^{V N} l_{h, t}^{V N}
\end{align*}
$$

Consumption demands:

$$
\begin{align*}
0 & \simeq-c_{h, t}^{T, h}+c_{t}^{T, h}-\eta_{h}\left(p_{h, t}^{C T, h}-p_{t}^{C T, h}\right)  \tag{141}\\
0 & \simeq-c_{U, t}^{T, h}+c_{t}^{T, h}-\eta_{h}\left(p_{U, t}^{C T, h}-p_{t}^{C T, h}\right)  \tag{142}\\
0 & \simeq-c_{W, t}^{T, h}+c_{t}^{T, h}-\eta_{h}\left(p_{W, t}^{C T, h}-p_{t}^{C T, h}\right)  \tag{143}\\
0 & \simeq-c_{h, t}^{N}+c_{t}^{h}-\eta_{c, h}\left(p_{h, t}^{C N}-p_{t}^{C, h}\right)  \tag{144}\\
0 & \simeq-c_{t}^{T, h}+c_{t}^{h}-\eta_{c, h}\left(p_{t}^{C T, h}-p_{t}^{C, h}\right)
\end{align*}
$$

Price aggregations:

$$
\begin{align*}
& 0 \simeq-p_{t}^{C, h}+\alpha_{T, h} p_{t}^{C T, h}+\left(1-\alpha_{T, h}\right) p_{h, t}^{C N} \text { and }  \tag{145}\\
& 0 \simeq-p_{t}^{C T, h}+n_{h}^{h}\left(p_{h, t}^{C T, h}\right)+\left(n_{U}^{h}-n_{h}^{h}\right) p_{U, t}^{C T, h}+\left(1-n_{h}^{h}-\left(n_{U}^{h}-n_{h}^{h}\right)\right) p_{W, t}^{C T, h} \tag{146}
\end{align*}
$$

Consumption Euler FOC:

$$
\begin{align*}
& \left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right) \\
& \simeq\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) E_{t}\left(c_{t+1}^{h}\right)-\kappa_{h}\left(1-\sigma_{h}\right) c_{t}^{h}-\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right) \lambda_{h}^{1-\sigma_{h}}\right) E_{t}\left(c_{t+2}^{h}\right) \\
& +r_{t ; t+1}-E_{t}\left(\Delta p_{t+1}^{C, h}\right) \tag{147}
\end{align*}
$$

FOCs for labor supply:

$$
\begin{align*}
& \phi_{h} l_{h, t}-\left[\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right)\right]-w_{t}^{F T, h}+p_{t}^{h} \simeq 0  \tag{148}\\
& \phi_{h} l_{E, t}-\left[\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right)\right]-w_{t}^{F N, h}+p_{t}^{h} \simeq 0  \tag{149}\\
& \phi_{h} l_{E, t}-\left[\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right)\right]-w_{t}^{V T, h}+p_{t}^{h} \simeq 0  \tag{150}\\
& \phi_{h} l_{E, t}-\left[\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right)\right]-w_{t}^{V N, h}+p_{t}^{h} \simeq 0 \tag{151}
\end{align*}
$$

FOC for money demand:

$$
\begin{gather*}
\frac{1}{\chi_{h}} E_{t}\left(m_{t+1}^{h}\right)-p_{t}^{C, h}=-\frac{\gamma_{t}-\beta E_{t}\left(\gamma_{t+1}\right)}{1-\beta_{h}} \text { with }  \tag{152}\\
\gamma_{t}=\left[\left(-\sigma_{h}+\beta_{h} \kappa_{h}\left(\kappa_{h}\left(1-\sigma_{h}\right)+1\right)\right) c_{t}^{h}-\kappa_{h}\left(1-\sigma_{h}\right) c_{t-1}^{h}-\beta_{h} \kappa_{h}\left(1-\sigma_{h}\right) E_{t}\left(c_{t+1}^{h}\right)\right] \tag{153}
\end{gather*}
$$

Production functions for final goods (in the neighborhood of $\gamma_{m, h}=1$ ):

$$
\begin{gather*}
0 \simeq-y_{h, t}^{F T}+\varpi_{F T, h} \omega_{F T, h, t}+\varpi_{F T, h} v_{F T, h} l_{h, t}^{F T}+\varpi_{F T, h}\left(1-v_{F T, h}\right) v_{t}^{F T, h},  \tag{154}\\
0 \simeq-y_{h, t}^{F N}+\varpi_{F N, h} \omega_{F N, h, t}+\varpi_{F N, h} v_{F N, h} l_{h, t}^{F N}+\varpi_{F N, h}\left(1-v_{F N, h}\right) v_{t}^{F N, h} \tag{155}
\end{gather*}
$$

Production functions for intermediate goods:

$$
\begin{align*}
& 0 \simeq-y_{h, t}^{V T}+\varpi_{V T, h} \omega_{V T, h, t}+\varpi_{V T, h} l_{h, t}^{V T},  \tag{156}\\
& 0 \simeq-y_{h, t}^{V N}+\varpi_{V N, h} \omega_{V N, h, t}+\varpi_{V N, h} l_{h, t}^{V N} \tag{157}
\end{align*}
$$

Prices for inputs:

$$
\begin{gather*}
0 \simeq-p_{V, t}^{F T, h}+\nu_{F T 1, h} p_{h, t}^{V T, F T, h}+\nu_{F T 2, h} p_{h, t}^{V N, F T, h}+\nu_{F T 3, h} p_{U, t}^{V T, F T, h}+\left(1-v_{F T 1, h}-v_{F T 2, h}-\nu_{F T 3, h}\right) p_{W, t}^{V T, F T, h},  \tag{158}\\
0 \simeq-p_{Z, t}^{F T, h}+v_{F T, h} w_{t}^{F T, h}+\left(1-v_{F T, h}\right) p_{V, t}^{F T, h},  \tag{159}\\
0 \simeq-p_{V, t}^{F N, h}+\nu_{F N 1, h} p_{h, t}^{V T, F N, h}+\nu_{F N 2, h} p_{h, t}^{V N, F N, h}+\nu_{F N 3, h} p_{U, t}^{V T, F N, h}+\left(1-v_{F N 1, h}-v_{F N 2, h}-\nu_{F N 3, h}\right) p_{W, t}^{V T, F N, h} \tag{160}
\end{gather*}
$$

$$
\begin{equation*}
0 \simeq-p_{Z, t}^{F N, h}+v_{F N, h} w_{t}^{F N, h}+\left(1-v_{F N, h}\right) p_{V, t}^{F N, h} \tag{161}
\end{equation*}
$$

Demand for inputs from all the sectors $\left(\gamma_{m}=1\right)\left(l_{h, t}^{F T}\right.$ and $l_{h, t}^{V N}$ give us the same information as their production functions):

$$
\begin{gather*}
v_{h, t}^{F T} \simeq-\gamma_{V T, F T}^{h}\left(p_{V, h, t}^{F T}-p_{Z, h, t}^{F T}\right)+\frac{1}{\varpi_{F T}^{h}} y_{h, t}^{F T}-\omega_{F T, h, t,}  \tag{162}\\
v_{h, t}^{T, F T} \simeq-\chi_{T, F T}^{h}\left(p_{h, t}^{V T, F T}-p_{V, h, t}^{F T}\right)+v_{h, t}^{F T},  \tag{163}\\
v_{h, t}^{N, F T} \simeq-\chi_{N, F T}^{h}\left(p_{h, t}^{V N, F T}-p_{V, h, t}^{F T}\right)+v_{h, t}^{F T},  \tag{164}\\
v_{W, t}^{h, T, F T} \simeq-\chi_{M, F T}^{W}\left(s_{t}+p_{W, t}^{h, V T, F T}-p_{V, h, t}^{F T}\right)+v_{h, t}^{F T},  \tag{165}\\
v_{U, t}^{h, T, F T} \simeq-\chi_{M, F T}^{U}\left(p_{U, t}^{h, V T, F T}-p_{V, h, t}^{F T}\right)+v_{h, t}^{F T},  \tag{166}\\
v_{h, t}^{F N} \simeq-\gamma_{V T, F N}^{h}\left(p_{V, h, t}^{F N}-p_{Z, h, t}^{F N}\right)+\frac{1}{\varpi_{F N}^{h}} y_{h, t}^{F N}-\omega_{F N, t}^{h},  \tag{167}\\
v_{h, t}^{T, F N} \simeq-\chi_{T, F N}^{h}\left(p_{h, t}^{V T, F N}-p_{V, h, t}^{F N}\right)+v_{h, t}^{F N}  \tag{168}\\
v_{h, t}^{N, F N} \simeq-\chi_{N, F N}^{h}\left(p_{h, t}^{V N, F N}-p_{V, h, t}^{F N}\right)+v_{h, t}^{F N},  \tag{169}\\
v_{W, t}^{h, T, F N} \simeq-\chi_{M, F N}^{W}\left(s_{t}+p_{W, t}^{h, V T, F N}-p_{V, h, t}^{F N}\right)+v_{h, t}^{F N}  \tag{170}\\
v_{U, t}^{h, T, F N} \simeq-\chi_{M, F N}^{U}\left(p_{U, t}^{h, V T, F N}-p_{V, h, t}^{F N}\right)+v_{h, t}^{F N} \tag{171}
\end{gather*}
$$

Exchange rate equilibrium conditions:

$$
\begin{equation*}
E_{t}\left(\Delta s_{t+1}\right) \simeq r_{t, t+1}-r_{t, t+1}^{W}-\delta b_{W, t+1}^{U(R)} \tag{172}
\end{equation*}
$$

Equilibrium conditions:

$$
\begin{align*}
& 0 \simeq-\bar{Y}_{h, t}^{F T} y_{h, t}^{F T}+\bar{C}_{h, t}^{T, h} c_{h, t}^{T, h}+\bar{C}_{h, t}^{T, U} c_{h, t}^{T, U}+\bar{C}_{h, t}^{T, W} c_{h, t}^{T, W}  \tag{173}\\
& 0 \simeq-y_{h, t}^{F N}+c_{h, t}^{N}  \tag{174}\\
& 0 \simeq-\bar{Y}_{h, t}^{V T} y_{h, t}^{V T}+\bar{V}_{h, t}^{F T, h} v_{h, t}^{F T, h}+\bar{V}_{h, t}^{F T, U} v_{h, t}^{F T, U}+\bar{V}_{h, t}^{F T, W} v_{h, t}^{F T, W}  \tag{175}\\
& 0 \simeq-y_{h, t}^{V N}+v_{h, t}^{F N, h} \tag{176}
\end{align*}
$$

Marginal costs:

$$
\begin{align*}
& 0 \simeq-m c_{h, t}^{F T}+p_{Z, t}^{F T, h}+\frac{1-\varpi_{F T, h}}{\varpi_{F T, h}^{F T}} y_{h, t}^{F}-\omega_{F T, h, t}  \tag{177}\\
& 0 \simeq-m c_{h, t}^{F N}+p_{Z, t}^{F N, h}+\frac{1-\varpi_{F N, h}}{\varpi_{F N, h}} y_{h, t}^{F N}-\omega_{F N, h, t}  \tag{178}\\
& 0 \simeq-m c_{h, t}^{V T}+w_{Z, t}^{V T, h}+\frac{1-\varpi_{V T, h}}{\varpi_{V T, h}} y_{h, t}^{V T}-\omega_{V T, h, t}  \tag{179}\\
& 0  \tag{180}\\
& 0-m c_{h, t}^{V N}+w_{h, t}^{V N}+\frac{1-\varpi_{V N, h}}{\varpi_{V N, h}} y_{h, t}^{V N}-\omega_{V N, h, t}
\end{align*}
$$

Marginal costs for home import companies:

$$
\begin{gather*}
0 \simeq-m c_{U, t}^{M F, h}+p_{U, t}^{X, C T, h},  \tag{181}\\
0 \simeq-m c_{W, t}^{M F, h}+p_{W, t}^{X, C T, h}-s_{t},  \tag{182}\\
0 \simeq-m c_{U, t}^{M V, F T, h}+p_{U, t}^{X, V T, F T, h},  \tag{183}\\
0 \simeq-m c_{W, t}^{M V, F T, h}+p_{W, t}^{X, V T, F T, h}-s_{t},  \tag{184}\\
0 \simeq-m c_{U, t}^{M V, F N, h}+p_{U, t}^{X, V T, F N, h},  \tag{185}\\
0 \simeq-m c_{W, t}^{M V, F N, h}+p_{W, t}^{X, V T, F N, h}-s_{t} . \tag{186}
\end{gather*}
$$

New prices in consumption goods sectors:

$$
\begin{align*}
& 0 \simeq-p_{h, t}^{C T, h}+\varphi_{C T, h}^{h} p_{h, t-1}^{C T, h}+\left(1-\varphi_{C T, h}^{h}\right) \breve{p}_{h, t}^{C T, h}  \tag{187}\\
& 0 \simeq-p_{h, t}^{C N}+\varphi_{C N, h}^{h} p_{h, t-1}^{C N}+\left(1-\varphi_{C N, h}^{h}\right) \breve{p}_{h, t}^{C N}  \tag{188}\\
& 0 \simeq-p_{h, t}^{X, C T, U}+\varphi_{C T, h}^{U} p_{h, t-1}^{X, C T, U}+\left(1-\varphi_{C T, h}^{U}\right) \breve{p}_{h, t}^{X, C T, U}  \tag{189}\\
& 0 \simeq-p_{h, t}^{X, C T, W}+\varphi_{C T, h}^{W} p_{h, t-1}^{X, C T, W}+\left(1-\varphi_{C T, h}^{W}\right) \breve{p}_{h, t}^{X, C T, W} . \tag{190}
\end{align*}
$$

New prices for final goods home importers:

$$
\begin{align*}
& 0 \simeq-p_{U, t}^{C T, h}+\varphi_{C T, U}^{h} p_{U, t-1}^{C T, h}+\left(1-\varphi_{C T, U}^{h}\right) \breve{p}_{U, t}^{X, C T, h}  \tag{191}\\
& 0 \simeq-p_{W, t}^{C T, h}+\varphi_{C T, W}^{h} p_{W, t-1}^{C T, h}+\left(1-\varphi_{C T, W}^{h}\right) \breve{p}_{W, t}^{X, C T, h} . \tag{192}
\end{align*}
$$

New prices in home intermediate goods sectors:

$$
\begin{gather*}
0 \simeq-p_{h, t}^{V T, F T, h}+\varphi_{V T, F T, h}^{h} p_{h, t-1}^{V T, F T, h}+\left(1-\varphi_{V T, F T, h}^{h}\right) \breve{p}_{h, t}^{V T, F T, h},  \tag{193}\\
0 \simeq-p_{h, t}^{V T, F N, h}+\varphi_{V T, F N, h}^{h} p_{h, t-1}^{V T, F N, h}+\left(1-\varphi_{V T, F N, h}^{h}\right) \breve{p}_{h, t}^{V T, F N, h},  \tag{194}\\
0 \simeq-p_{h, t}^{V N, F T, h}+\varphi_{V N, F T, h}^{h} p_{h, t-1}^{V N, F T, h}+\left(1-\varphi_{V N, F T, h}^{h}\right) \breve{p}_{h, t}^{V N, F T, h},  \tag{195}\\
0 \simeq-p_{h, t}^{V N, F N, h}+\varphi_{V N, F T, h}^{h} p_{h, t-1}^{V N, F N, h}+\left(1-\varphi_{V N, F T, h}^{h}\right) \breve{p}_{h, t}^{V N, F N, h},  \tag{196}\\
0 \simeq-p_{h, t}^{X, V T, F T, U}+\varphi_{X, V T, F T, h}^{U} p_{h, t-1}^{X, V T, F T, U}+\left(1-\varphi_{X, V T, F T, h}^{U}\right) \breve{p}_{h, t}^{X, V T, F T, U},  \tag{197}\\
0 \simeq-p_{h, t}^{X, V T, F N, U}+\varphi_{X, V T, F N, h}^{U} p_{h, t-1}^{X, V T, F N, U}+\left(1-\varphi_{X, V T, F N, h}^{U}\right) \breve{p}_{h, t}^{X, V T, F N, U}  \tag{198}\\
 \tag{199}\\
0 \simeq-p_{h, t}^{X, V T, F T, W}+\varphi_{X, V T, F T, h}^{W} p_{h, t-1}^{X, V T, F T, W}+\left(1-\varphi_{X, V T, F T, h}^{W}\right) \breve{p}_{h, t}^{X, V T, F T, W},  \tag{200}\\
0 \simeq-p_{h, t}^{X, V T, F N, W}+\varphi_{X, V T, F N, h}^{W} p_{h, t-1}^{X, V T, F N, W}+\left(1-\varphi_{X, V T, F N, h}^{W}\right) \breve{p}_{h, t}^{X, V T, F N, W} .
\end{gather*}
$$

New prices in intermediate goods home import sectors:

$$
\begin{align*}
& 0 \simeq-p_{U, t}^{V T, F T, h}+\varphi_{V T, F T, U}^{h} p_{U, t-1}^{V T, F T, h}+\left(1-\varphi_{V T, F T, U}^{h}\right) \breve{p}_{U, t}^{V T, F T, h}  \tag{201}\\
& 0 \simeq-p_{U, t}^{V T, F N, h}+\varphi_{V T, F N, U}^{h} p_{U, t-1}^{V T, F N, h}+\left(1-\varphi_{V T, F N, U}^{h}\right) \breve{p}_{U, t}^{V T, F N, h} \tag{202}
\end{align*}
$$

$$
\begin{align*}
& 0 \simeq-p_{W, t}^{V T, F T, W}+\varphi_{V T, F T, W}^{W} p_{W, t-1}^{V T, F T, W}+\left(1-\varphi_{V T, F T, W}^{W}\right) \breve{p}_{W, t}^{V T, F T, W}  \tag{203}\\
& 0 \simeq-p_{W, t}^{V T, F N, W}+\varphi_{V T, F N, W}^{W} p_{W, t-1}^{V T, F N, W}+\left(1-\varphi_{V T, F N, W}^{W}\right) \breve{p}_{W, t}^{V T, F N, W} \tag{204}
\end{align*}
$$

Optimal Calvo prices in home final goods sectors:

$$
\begin{gather*}
0 \simeq-\breve{p}_{h, t}^{C T, h}+\left(1-\varphi_{C T, h}^{h} h_{h}\right) m c_{h, t}^{F T}+\varphi_{C T, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{C T, h}\right)  \tag{205}\\
0 \simeq-\breve{p}_{h, t}^{C N}+\left(1-\varphi_{C N, h}^{h} h_{h}\right) m c_{h, t}^{F N}+\varphi_{C N, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{C N}\right),  \tag{206}\\
0 \simeq-\breve{p}_{h, t}^{X, C T, U}+\left(1-\varphi_{X, C T, h}^{U} h_{h}\right) m c_{h, t}^{F T}+\varphi_{X, C T}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, C T, U}\right),  \tag{207}\\
0 \simeq-\breve{p}_{h, t}^{X, C T, W}+\left(1-\varphi_{X, C T, h}^{W} h_{h}\right) m c_{h, t}^{F T}+\varphi_{X, C T}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, C T, W}\right) . \tag{208}
\end{gather*}
$$

Optimal Calvo prices in home import sectors of final goods:

$$
\begin{align*}
0 & \simeq-\breve{p}_{U, t}^{C T, h}+\left(1-\varphi_{C T, U}^{h} h_{h}\right) m c_{U, t}^{M F, h}+\varphi_{C T, U}^{h} h_{h} E_{t}\left(\breve{p}_{U, t+1}^{C T, h}\right),  \tag{209}\\
0 & \simeq-\breve{p}_{W, t}^{C T, h}+\left(1-\varphi_{C T, W}^{h} h_{h}\right) m c_{W, t}^{M F, h}+\varphi_{C T, W}^{h} h_{h} E_{t}\left(\breve{p}_{W, t+1}^{C T, h}\right) . \tag{210}
\end{align*}
$$

Optimal Calvo prices in home intermediate goods sectors:

$$
\begin{gather*}
0 \simeq-\breve{p}_{h, t}^{V T, F T, h}+\left(1-\varphi_{V T, F T, h}^{h} h_{h}\right) m c_{h, t}^{V T, F T, h}+\varphi_{V T, F T, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{V T, F T, h}\right)  \tag{211}\\
0 \simeq-\breve{p}_{h, t}^{V T, F N, h}+\left(1-\varphi_{V T, F N, h}^{h} h_{h}\right) m c_{h, t}^{V T, F N, h}+\varphi_{V T, F N, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{V T, F N, h}\right),  \tag{212}\\
0 \simeq-\breve{p}_{h, t}^{V N, F T, h}+\left(1-\varphi_{V N, F T, h}^{h} h_{h}\right) m c_{h, t}^{V N}+\varphi_{V N, F T, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{V N, F T, h}\right),  \tag{213}\\
0 \simeq-\breve{p}_{h, t}^{V N, F N, h}+\left(1-\varphi_{V N, F N, h}^{h} h_{h}\right) m c_{h, t}^{V N}+\varphi_{V N, F N, h}^{h} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{V N, F N, h}\right)  \tag{214}\\
0 \simeq-\breve{p}_{h, t}^{X, V T, F T, U}+\left(1-\varphi_{X, V T, F T, h}^{U} h_{h}\right) m c_{h, t}^{V T}+\varphi_{X, V T, F T, h}^{U} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, V T, F T, U}\right)  \tag{215}\\
0 \simeq-\breve{p}_{h, t}^{X, V T, F N, U}+\left(1-\varphi_{X, V T, F N, h}^{U} h_{h}\right) m c_{h, t}^{V T}+\varphi_{X, V T, F N, h}^{U} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, V T, F N, U}\right)  \tag{216}\\
0 \simeq-\breve{p}_{h, t}^{X, V T, F T, W}+\left(1-\varphi_{X, V T, F T, h}^{W} h_{h}\right) m c_{h, t}^{V T}+\varphi_{X, V T, F T, h}^{W} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, V T, F T, W}\right)  \tag{217}\\
0 \simeq-\breve{p}_{h, t}^{X, V T, F N, W}+\left(1-\varphi_{X, V T, F N, h}^{W} h_{h}\right) m c_{h, t}^{V T}+\varphi_{X, V T, F N, h}^{W} h_{h} E_{t}\left(\breve{p}_{h, t+1}^{X, V T, F N, W}\right) . \tag{218}
\end{gather*}
$$

Optimal Calvo prices in home import sectors of intermediate goods:

$$
\begin{align*}
& 0 \simeq-\breve{p}_{U, t}^{V T, F T, h}+\left(1-\varphi_{V T, F T, U}^{h} h_{h}\right) m c_{U, t}^{V T, F T, h}+\varphi_{V T, F T, U}^{h} h_{h} E_{t}\left(\breve{p}_{U, t+1}^{V T, F T, h}\right)  \tag{219}\\
& 0 \simeq-\breve{p}_{U, t}^{V T, F N, h}+\left(1-\varphi_{V T, F N, U}^{h} h_{h}\right) m c_{U, t}^{V T, F N, h}+\varphi_{V T, F N, U}^{h} h_{h} E_{t}\left(\breve{p}_{U, t+1}^{V T, F N, h}\right),  \tag{220}\\
& 0 \simeq-\breve{p}_{W, t}^{V T, F T, h}+\left(1-\varphi_{V T, F T, W}^{h} h_{h}\right) m c_{W, t}^{V T, F T, h}+\varphi_{V T, F T, W}^{h} h_{h} E_{t}\left(\breve{p}_{W, t+1}^{V T, F T, h}\right)  \tag{221}\\
& 0 \simeq-\breve{p}_{W, t}^{V T, F N, h}+\left(1-\varphi_{V T, F N, W}^{h} h_{h}\right) m c_{W, t}^{V T, F N, h}+\varphi_{V T, F N, W}^{h} h_{h} E_{t}\left(\breve{p}_{W, t+1}^{V T, F N, h}\right) . \tag{222}
\end{align*}
$$

DGPs for exogenous variables are
(i) monetary policy rule: either equation (32), (33) or (34),
(ii.a) final tradables technology:

$$
\begin{equation*}
\Omega_{h, t}^{F T}=\rho_{\Omega_{h}}^{F T} \Omega_{h, t-1}^{F T}+\eta_{h, t}^{\Omega^{F T}} \tag{223}
\end{equation*}
$$

(ii.b) final non-tradables technology:

$$
\begin{equation*}
\Omega_{h, t}^{F N}=\rho_{\Omega_{h}}^{F N} \Omega_{h, t-1}^{F N}+\eta_{h, t}^{\Omega^{F N}}, \tag{224}
\end{equation*}
$$

(ii.c) intermediate tradables technology:

$$
\begin{equation*}
\Omega_{h, t}^{V T}=\rho_{\Omega_{h}}^{V T} \Omega_{h, t-1}^{V T}+\eta_{h, t}^{\Omega} \tag{225}
\end{equation*}
$$

(ii.d) intermediate non-tradables technology:

$$
\begin{equation*}
\Omega_{h, t}^{V N}=\rho_{\Omega_{h}}^{V N} \Omega_{h, t-1}^{V N}+\eta_{h, t}^{\Omega^{V N}} \tag{226}
\end{equation*}
$$

(iii) exchange rate shock:

$$
\begin{equation*}
\xi_{h, t}^{s}=\rho_{s}^{h} \xi_{h, t-1}^{s}+\eta_{h, t}^{s} \tag{227}
\end{equation*}
$$

Disturbances $\varepsilon_{1 t}, \eta_{h, t}^{\Omega^{F T}}, \eta_{h, t}^{\Omega^{F N}}, \eta_{h, t}^{\Omega^{V T}}, \eta_{h, t}^{\Omega^{V N}}, \eta_{h, t}^{s}$ are white noise with standard deviations $\sigma_{\varepsilon_{1 t}}, \sigma_{h \eta^{\Omega F_{h}^{F T}}}, \sigma_{h \eta^{\Omega F_{h} N}}, \sigma_{h \eta^{\Omega V_{h} T}}$, $\sigma_{h \eta_{h}^{V N}}, \sigma_{\eta_{h, t}^{s}}$, respectively.

## E Numerical Simulations

## E. 1 Technological shocks in tradable final goods sectors

Figure 1 IRFs, positive technological shock in the $F T$ sector (consumption side).









Figure 2 IRFs, positive technological shock in the $F T$ sector (production side).








Figure 3 IRFs, positive technological shock in the $F T$ sector (exchange rate and policies).


## E. 2 Technological shocks in tradable final goods sectors

Figure 4 Technological shock in intermediate goods sectors (consumption side)
















Figure 5 Technological shock in intermediate goods sectors (production side).











Figure 6 Technological shock in intermediate goods sectors (exchange rate and policies).


## E. 3 Exchange rate shock

Figure 7 IRFs, exchange rate shock (consumption side)











Figure 8 IRFs, exchange rate shock (production side).











Figure 9 IRFs, exchange rate shock (exchange rate and policies).


## E. 4 Policy loosening shock

Figure 10 IRFs, monetary policy loosening (consumption side)















Figure 11 IRFs, monetary policy loosening (production side).


Figure 12 IRFs, monetary policy loosening (exchange rate and policies).


## E. 5 A preference shock to consumption

Figure 13 IRFs, consumption shock (consumption side)











Figure 14 IRFs, consumption shock (production side).


wagenfr1

wagenfinT1








Figure 15 IRFs, consumption shock (exchange rate and policies).


## F Data Appendix

The economic structure of all the countries is taken from a few records of input-output (I-O) matrices for The Netherlands, the RoEMU and the US (RoW). We compute the intermediate tradables and non-tradables over total
intermediates and w.r.t. GDP. Moreover, we aggregate final demand faced by that of each industry into tradable and non-tradable consumption over GDP. We interpolate the shares in the periods in which I-O tables are not available. Then, we recover the levels of the series applying the obtained shares to the GDP values at 2000 prices from the OECD statistical compendium.

We divided tradable and non-tradable goods industries as follows. Tradable goods industries comprise: Agriculture, forestry and fish; Raw materials; Food and stimulants industry; Textile and leather industry; Paper industry, publishers and printers; Petrol industry; Chemical industry; Rubber and synthetic material industry; Metal industry; Machine-industry; Electrical industry; Transport equipment industry; Remaining industry; Energy and water supply business; Transport, raise and communication. On the other hand, non-tradable goods industries are: Construction industry; Trading and repairs; Hotel and restaurant services; Construction industry; Financial institutions; Rent of and trade in real estate; Business services and renting of movables; Authorities/Government; Health and social work services; Services n.e.c.; Goods and services n. e. c.; Corrections in connection with wages in kind. The following subsections give more details regarding each country.

## F. 1 The small open economy: The Netherlands

Sectoral data from 1969 to 2002 is taken from I-O tables on the Central Bureau of Statistics (CBS) website. From 1969 to 2002 I-O tables are computed in current prices and from 1988 onwards, they are also computed at constant prices.

In order to construct our variables we classified the 26 sectors of the I-O symmetric matrices into $F T$ and $F N$ and intermediates $V T$ and $V N .{ }^{38}$ Since the Netherlands have yearly I-O tables, we checked the stability of technical coefficients of tradable and non-tradable goods per industry.

## F. 2 Rest of the EMU

To specify the production structure of the Rest of the EMU, we consider I-O tables for Denmark, France, Germany, Italy from the OECD I-O tables (edition 1995), both in constant and in current prices. For the rest of the EMU, we have available data for the following countries: (i) Italy: I-O tables for 1985, 1992; (ii) France: I-O tables for 1972, 1977, 1980, 1985, 1990, 1995; (iii) Germany: I-O table for 1978, 1986, 1990, 1995; and (iv) Spain: I-O table for 1995.

We consider that France and Germany have a representative economic structure for the twelve countries comprising the EMU. Production of intermediates is taken from the STAN database and all other data from the OECD statistical Compendium.

## F. 3 Rest of the World

To specify the production structure of the US, we consider I-O tables for the years 1972, 1977, 1982, 1985, 1990 and 1997. Source: OECD I-O Tables. Total intermediate production is taken from the STAN database.

[^19]
## F. 4 Shares of the Netherlands, the RoEMU and the US in terms of Euros.

Figure 16 Computed sizes of the Netherlands, the RoEMU and the US


## G Calibration and Estimation

Our calibrated parameters and assumed prior densities are in Table 1.
Table 1 Calibrated parameters and prior densities

| Parameters | Our prior dens | ies |  | Levin | $t$ al.(2005) | Orteg | \&Rebei(2006) | Ratto | al. (20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type[bounds] | Mean (SD) | Based on | Type | Mean (SD) | Type | Mean (SD) | Type | Mean |
| $\alpha_{T}^{h}, \alpha_{T}^{U}, \alpha_{T}^{W}$ | B [0.45;0.55] | 0.5(0.05) | O\&R06 |  |  | B | $0.5(0.05)$ |  |  |
| $b^{h}, b^{U}, b^{W}$ |  | 0.99 | L06 |  |  |  |  |  |  |
| $\beta^{h}, \beta^{U}, \beta^{W}$ |  | 0.99 | L06 |  |  |  |  |  |  |
| $\chi^{h}, \chi^{U}, \chi^{W}$ |  | 2/3 |  |  |  |  |  |  |  |
| $\chi_{F N, h}^{U}, \chi_{F N, h}^{W}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F N, U}^{h}, \chi_{F N, U}^{W}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F N, W}^{h}, \chi_{F N, W}^{U}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F T, h}^{U}, \chi_{F T, h}^{W}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F T, U}^{h}, \chi_{F T, U}^{W}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F T, W}^{h}, \chi_{F T, W}^{U}$ |  | 7 | AD\&M |  |  |  |  |  |  |
| $\chi_{F N, h}^{N}, \chi_{F N, U}^{N}, \chi_{F N, W}^{N}$ |  | 5 |  |  |  |  |  |  |  |
| $\chi_{F T, h}^{N}, \chi_{F T, U}^{N}, \chi_{F T, W}^{N}$ |  | 5 |  |  |  |  |  |  |  |
| $\chi_{F N, h}^{T}, \chi_{F N, U}^{T}, \chi_{F N W}^{T}$ |  | 5 |  |  |  |  |  |  |  |
| $\chi_{F}^{F N, h}$, $\chi_{F}^{F} N, U, \chi_{F}^{F N, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\chi_{F T, h}^{T}, \chi_{F T, U}^{T}, \chi_{F T, W}^{T}$ |  | 5 |  |  |  |  |  |  |  |
| $\eta_{C, h}, \eta_{C, U}, \eta_{C, W}$ |  | 3 |  |  |  |  |  |  |  |
| $\eta_{h, h}, \eta_{h, U}, \eta_{h, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\gamma_{F N, h}, \gamma_{F N, U}, \gamma_{F N, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\gamma_{F T, h}, \gamma_{F T, U}, \gamma_{F T, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\gamma_{V N, h}, \gamma_{V N, U}, \gamma_{V N, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\gamma_{V T, h}, \gamma_{V T, U}, \gamma_{V T, W}$ |  | 5 |  |  |  |  |  |  |  |
| $\kappa_{h}, \kappa_{U}, \kappa_{W}$ | N [0.15;0.8] | 0.5(0.125) |  |  |  |  |  |  |  |
| $\phi_{h}, \phi_{U}, \phi_{W}$ | B [0.16;0.6] | 0.3(0.1) | Mo00 | N | 1.2(0.5) |  |  | B | 0.45 (0. |
| $\sigma_{h}, \sigma_{U}, \sigma_{W}$ | N $[1.4 ; 3]$ | $2(0.5)$ | O\&R06 | N | $2(0.5)$ |  |  |  |  |
| $\rho_{A_{N T}, h}, \rho_{A_{N T}, U}, \rho_{A_{N T}, W}$ | B [0.70;0.99] | 0.85(0.1) | O\&R06 | B | $0.5(0.25)$ | B | $0.85(0.1)$ | B | 0.5 (0.2 |
| $\rho_{A_{T}, h}, \rho_{A_{T}, U}, \rho_{A_{T}, W}$ | B [0.70;0.99] | $0.85(0.1)$ | O\&R06 | B | $0.5(0.25)$ | B | $0.85(0.1)$ | B | 0.5 (0.2 |
|  | B [0.70;0.99] | 0.85(0.1) | O\&R06 |  |  |  |  | B | 0.5 (0.2 |
| $\rho_{A_{V T}, h}, \rho_{A_{V T}, U}, \rho_{A_{V T}, W}$ | B [0.70;0.99] | 0.85(0.1) | O\&R06 |  |  |  |  | B | 0.5 (0.2 |


| $\rho_{i, U}, \rho_{i, W}$ | B [0.6;0.9] | 0.75(0.05) | Mo00 | N | 1(0.15) | B | 0.85(0.1) | B | 0.8(0.1) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\vartheta_{1, U}, \vartheta_{1, W}$ | IG [0.02;2.5] | 1.5(0.1) | T93 |  |  | IG | 1.5(0.2) | B | 1.25 (0.1) |
| $\vartheta_{2, U}, \vartheta_{2, W}$ | B [0.02;0.3] | 0.25(0.1) | T93 |  |  | N | 0.2(0.1) |  |  |
| $\vartheta_{3, U}, \vartheta_{3, W}$ | B [0.02;0.3] | 0.25 (0.1) | T93 |  |  |  |  |  |  |
| $\vartheta_{4, U}, \vartheta_{4, W}$ | B [0.02;0.3] | 0.25(0.1) | T93 |  |  |  |  |  |  |
| $\vartheta_{5, U}, \vartheta_{5, W}$ | B [0.02;0.3] | 0.25(0.1) | T93 |  |  |  |  |  |  |
| $v_{L F T, h}, v_{L F T, U}, v_{L F T, W}$ |  | 0.36 | O\&R06 |  |  | B | 0.36(0.05) |  |  |
| $v_{L F N, h}, v_{L F N, U}, v_{L F N, W}$ |  | 0.36 | O\&R06 |  |  | B | 0.34(0.05) |  |  |
| $\varphi_{n_{F T}, h}, \varphi_{n_{F T}, U}$ | B [0.35;0.8] | 0.5(0.2) |  |  |  | B | 0.67(0.05) | B | 0.5(0.2) |
| $\varphi_{n_{F T}, W}$ | B [0.15;0.8] | $0.375(0.1)$ |  | B | 0.375(0.1) |  |  |  |  |
| $\varphi_{n_{F N}, h}, \varphi_{n_{F N}, U}$ | B [0.35;0.8] | 0.5(0.2) |  |  |  | B | 0.67(0.05) | B | 0.5(0.2) |
| $\varphi_{n_{F N}, W}$ | B [0.15;0.8] | $0.375(0.1)$ |  | B | 0.375(0.1) |  |  |  |  |
| $\varphi_{n_{V T}, h}, \varphi_{n_{V T}, U}, \varphi_{n_{V T}, W}$ | B [0.15;0.8] | 0.375(0.1) |  |  |  |  |  |  |  |
| $\varphi_{n_{V N}, h}, \varphi_{n_{V N}, U}, \varphi_{n_{V N}, W}$ | B [0.15;0.8] | $0.375(0.1)$ |  |  |  |  |  |  |  |
| $\varphi_{n_{W T}, h}, \varphi_{n_{W T}, U}$ | В [0.35;0.8] | 0.5(0.2) |  |  |  | B | 0.67(0.05) | B | 0.5(0.2) |
| $\varphi_{n_{W T}, W}$ | В [0.15;0.8] | 0.375(0.1) |  | B | 0.375(0.1) |  |  |  |  |
| $\varphi_{n_{W N}, h}, \varphi_{n_{W N}, U}$ | В [0.35;0.8] | 0.5(0.2) |  |  |  | B | 0.67(0.05) | B | 0.5(0.2) |
| $\varphi_{n_{W N}, W}$ | B [0.15;0.8] | $0.375(0.1)$ |  | B | 0.375(0.1) |  |  |  |  |
| $\varpi_{F T, h}, \varpi_{F T, U}, \varpi_{F T, W}$ |  | 1 |  |  |  |  |  |  |  |
| $\varpi_{F N, h}, \varpi_{F N, U}, \varpi_{F N, W}$ |  | 1 |  |  |  |  |  |  |  |
| $\varpi_{V T, h}, \varpi_{V T, U}, \varpi_{V T, W}$ |  | 1 |  |  |  |  |  |  |  |
| $\varpi_{V N, h}, \varpi_{V N, U}, \varpi_{V N, W}$ |  | 1 |  |  |  |  |  |  |  |
| $\omega_{F T, h}, \omega_{F T, U}, \omega_{F T, W}$ | IG [0.001;2] | 0.02(2) | R05 | IG | 0.6(0.6) | IG | 1.5(2) | B | 0.1(2) |
| $\omega_{F N, h}, \omega_{F N, U}, \omega_{F N, W}$ | IG [0.001;2] | 0.02(2) | R05 | IG | 0.6(0.6) | IG | 1.5(2) | B | 0.1(2) |
| $\omega_{V T, h}, \omega_{V T, U}, \omega_{V T, W}$ | IG [0.001;2] | 0.02(2) | R05 |  |  |  |  |  |  |
| $\omega_{V N, h}, \omega_{V N, U}, \omega_{V N, W}$ | IG [0.001;2] | 0.02(2) | R05 |  |  |  |  |  |  |

Note: O\&R06 refers to Ortega and Rebei (2006), L06 to Levin et al. (2006), D05 to Dellas (2005), T93 to Taylor (1993), R05 to Ratto et al. (2005b), and Mo00 to Monacelli (2000). The probability density types beta, normal and inverted gamma are abbreviated as B, N and IG, respectively.

The calibration used by Erceg et al. (2005) assumes an annualized interest rate of 3\%, a risk aversion parameter of 2 , a habit formation parameter $\theta=0.7$ but measured in the form $C_{t}-\theta C_{t-1}$, the Frisch elasticity of labor of $1 / 5 .{ }^{39}$ Wage and price markups are calibrated at 0.2 , a Calvo probability parameter of 0.75 , and an implied annual inflation of $4 \%$.

Table 2 Results from posterior maximization Rule (II) model

[^20]| Parameters | Prior | Prior mean | Post. mean | n ${ }^{\text {Std. error }}$ | t-statistic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{T}^{h}$ | beta | 0.5 | 0.5571 | 12.6094 | 0.2135 |
| $\phi_{h}$ | beta | 0.3 | 0.3925 | $5 \quad 1.6311$ | 0.2406 |
| $\sigma_{h}$ | norm | 2 | 2.0047 | $7 \quad 0.4471$ | 4.4836 |
| $\eta_{c, h}$ | invg | 3 | 3.0071 | $1 \quad 7.2554$ | 0.4145 |
| $\eta_{h}^{h}$ | invg | 5 | 4.9997 | $7 \quad 149.1864$ | 0.0335 |
| $\kappa_{h}$ | norm | 0.5 | 0.602 | 20.018 | 33.4016 |
| $\varphi_{h}$ | beta | 0.5 | 0.5896 | $6 \quad 0.0081$ | 72.9986 |
| $\varphi_{W, h}$ | beta | 0.5 | 0.473 | $3 \quad 0.006$ | 78.2503 |
| $\varphi_{V, h}$ | beta | 0.375 | 0.3469 | $9 \quad 0.0023$ | 148.2899 |
| $\varphi_{U}$ | beta | 0.5 | 0.5409 | $9 \quad 0.0003$ | 1654.0799 |
| $\varphi_{W, U}$ | beta | 0.5 | 0.4437 | $7 \quad 0.0012$ | 359.4676 |
| $\varphi_{V, U}$ | beta | 0.375 | 0.3921 | $1 \quad 0.0016$ | 249.5846 |
| $\varphi_{W}$ | beta | 0.375 | 0.3448 | $8 \quad 0.0001$ | 2750.2192 |
| $\varphi_{W, W}$ | beta | 0.375 | 0.3609 | $9 \quad 0.0006$ | 640.233 |
| $\varphi_{V, W}$ | beta | 0.375 | 0.3855 | 50.0001 | 3471.5673 |
| $\vartheta_{4, U}^{I I}$ | beta | 0.7 | 0.6971 | $1 \quad 0.0004$ | 1973.2117 |
| $\vartheta_{4, W}^{I I}$ | beta | 0.7 | 0.7066 | $6 \quad 0.0017$ | 412.5538 |
| $\vartheta_{1, U}^{I I}$ | invg | 0.35 | 0.318 | $8 \quad 0.0001$ | 5978.2498 |
| $\vartheta_{1, W}^{I I}$ | invg | 0.35 | 0.3394 | $4 \quad 0.0004$ | 904.2846 |
| $\vartheta_{2, U}^{I I}$ | norm | 0.15 | 0.0307 | $7 \quad 0$ | 42681.0467 |
| $\vartheta_{2, W}^{I I}$ | norm | 0.15 | 0.0779 | 9 | 131921.962 |
| $\vartheta_{3, U}^{I I}$ | norm | 0.15 | 0.2077 | 7 0 | 212634.735 |
| $\vartheta_{3, W}^{I I}$ | norm | 0.15 | 0.2141 | $1 \quad 0$ | 577892.341 |
| SD of shocks | Prior | Prior mean | Mode | SD | statistic |
| $\omega_{F T, h}$ | invg | 0.15 | 0.5859 | 306.3331 | 0.0019 |
| $\omega_{F N, h}$ | invg | 0.15 | 0.4848 | 261.8096 | 0.0019 |
| $\omega_{V T, h}$ | invg | 0.15 | 0.5363 | 1415.3387 | 0.0004 |
| $\omega_{V N, h}$ | invg | 0.15 | 0.4866 | 13330.9041 | 0 |
| $\omega_{F T, U}$ | invg | 0.15 | 0.4816 | 3312.9687 | 0.0001 |
| $\omega_{F N, U}$ | invg | 0.15 | 0.4942 | 122.0622 | 0.004 |
| $\omega_{V T, U}$ | invg | 0.15 | 0.5031 | 453.3898 | 0.0011 |
| $\omega_{V N, U}$ | invg | 0.15 | 0.4989 | 185.0591 | 0.0027 |
| $\omega_{F T, W}$ | invg | 0.15 | 0.4916 | 824.2392 | 0.0006 |
| $\omega_{F N, W}$ | invg | 0.15 | 0.503 | 24.3822 | 0.0206 |
| $\omega_{V T, W}$ | invg | 0.15 | 0.6369 | 2.6335 | 0.2418 |
| $\omega_{V N, W}$ | invg | 0.15 | 0.4966 | 3.9399 | 0.126 |

## References

Abel, A.B. (1990), "Asset Prices under Habit Formation and Catching up with the Joneses", American Economic Review Paper and Proceedings, Vol. 80, pp. 38-42.

Abel, A.B. (1999), "Risk Premia and Term Premia in General Equilibrium", Journal of Monetary Economics, Vol. 43, pp. 3-33.

Ambler, S., A. Dib and N. Rebei (2004), "Nominal Rigidities and Exchange Rate Pass-Through in a Structural Model of a Small Open Economy", Working Paper 2003-29, Bank of Canada.

Ambler, S., A. Dib and N. Rebei (2004), "Optimal Taylor Rules in an Estimated Model of a Small Open Economy", Working Paper 2004-36, Bank of Canada.

Ascari, G. (2004), "Staggered Prices and Trend Inflation: Some Nuisances", Review of Economic Dynamics, Academic Press for the Society for Economic Dynamics, Vol. 7 No. 3, pp. 642-667.

Blanchard, O. and J. Kahn (1980), The solution of linear difference models under rational expectations, Econometrica, 48, pp. 1305-1312.

Benigno, P. (2001), "Price Stability with Imperfect Financial Integration", CEPR Discussion Papers No. 2854, C.E.P.R. Discussion Papers.

Benigno, P. (2004), "Optimal monetary policy in a currency area," Journal of International Economics, Vol. 63, No. 2, pp. 293-320.

Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework", Journal of Monetary Economics, Vol. 12, pp. 383-398.

Canzoneri, M., R. Cumby and B. Diba (2005), "How Do Monetary and Fiscal Policy Interact in the European Monetary Union?", NBER Working Papers No. 11055.

Choi, W.G. and Y. Jung (2003), "Optimal Monetary Policy in a Small Open Economy with Habit Formation and Nominal Rigidities", IMF Working Paper WP/03/5.

Christiano, L., M. Eichenbaum, and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy, Vol. 113, pp. 1-45.

Clarida, R., J. Galí and M. Gertler (1998), "Monetary policy rules and macroeconomic stability: Evidence and some theory" NBER Working Paper No. 6442.

Clarida, R., J. Galí and M. Gertler (2002), "A simple Framework for International Monetary Policy Analysis", Journal of Monetary Economics, Vol. 49, pp. 879-904.

Collard, F. and H. Dellas (2004), "Sticky Prices versus Sticky Information: A Horse Race", mimeo, CNRSGREMAQ, Toulouse, France (http://fabcol.free.fr/pdf/horse_race.pdf).

Dellas, H. (2005), "Monetary Policy in Open Economies", mimeo, Department of Economics, University of Bern, 23 pages (http://www.vwi.unibe.ch/content/e2071/e2077/e2949/e2951/e2954/diba_eng.pdf).

Erceg, C., L. Guerrieri, and C. Gust (2005), "SIGMA: A New Open Economy Model for Policy Analysis", Board of Governors of the Federal Reserve System International Finance Discussion Papers 835.

Erceg, C., D. Henderson, and A. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts." Journal of Monetary Economics, Vol. 46, pp. 281-313.

Galí, J., Gertler M., and J. Lopez-Salido (2003), "Markups, Gaps, and the Welfare Costs of Business Fluctuations", Discussion Economic WP, Universitat Pompeu Fabra, 46 pages.

Galí, J. and T. Monacelli (2002), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", NBER Working Paper No. 8905.

Huang, K.X.D. and Z. Liu (2005), "Sellers' local currency pricing or buyers' local currency pricing: does it matter for international welfare analysis?", Journal of Economic Dynamics and Control, In Press, Corrected Proof, Available online 18 July 2005.

Juillard M. (2005), DYNARE Manual, Version 3.0, Mimeo. http://www.cepremap.cnrs.fr/juillard/mambo/download/manual/ind
Jung, Y. (2004), "Catching Up with the Joneses in a Sticky Price Model", Journal of Money, Credit, and Banking, Vol. 36, No. 1, pp. 73-93.

Kim, J. (2000), "Constructing and Estimating a Realistic Optimizing Model of Monetary Policy", Journal of Monetary Economics, Vol. 45 pp 329-359.

Kozicki, S. and P. Tinsley (2002), "Dynamic specifications in optimizing trend-deviation macro models", Journal of Economic Dynamics 8 Control, Vol. 26, pp. 1585-1611.

Kydland, F. and E. Prescott (1977), "Rules rather than discretion: The inconsistency ofoptimal plans", Journal of Political Economy, Vol. 85, pp. 473-490.

Lane, P. (2001), "The New Open Economy Macroeconomics: A Survey", Journal of International Economics, Vol. 54(2), pp. 235-266.

Levin, A., A. Onatski, J. Williams and N. Williams (2005), "Monetary Policy Under Uncertaintly in Micro-Founded Macroeconomic Models" in M. Gertler and K. Rogoff (eds.), NBER Macroeconomics Annual, MIT Press, pp. 229-269.

Leith, C. and J. Malley (2003), "Estimated Open Economy New Keynesian Phillips Curves for the G7", CESifo Working Paper No. 834.

Lindé, J., M. Nessén and U. Söderström (2004), "Monetary Policy in an Estimated Open-Economy Model with Imperfect Pass-Through", Sveriges Riksbank Working Paper Series No. 167.

Lubik T. and F. Schorfheide (2005), "A Bayesian Look at the New Open Economy Macroeconomics", in M. Gertler and K. Rogoff (eds.), NBER Macroeconomics Annual, MIT Press.

Lucas Jr., R. (2003), "Models of Business Cycle", Basil Blakwell, New York.
Lucas Jr., R. (2003), "Macroeconomic Priorities", American Economic Review, Vol. 93(1), pp. 1-14.
Malik, H.A. (2005), "Monetary-Exchange Rate Policy and Current Account Dynamics", mimeo, Department of Economics, Lakehead University, Canada.

Mankiw G. and R. Reis (2002), "What Measure of Inflation Should a Central Bank Target?", NBER Working Papers No. 9375.

McCallum, B.T. (1979), "A Monetary Policy Ineffectiveness Result in a Model with a Predetermined Price Level". Economics Letters, Vol. 3 pp. 1-4.

McCallum, B.T. (1994), "A Semi-Classical Model of Price Level Adjustment". Carnegie-Rochester Conference Series on Public Policy No. 41 pp. 51-84.

Mc Callum, B.T. (1997), "Comment on Rotemberg and Woodford",.in B. S. Bernanke and J. J. Rotemberg (eds.), NBER Macroeconomics Annual, The MIT Press, Cambridge, MA, pp. 355-360.

Monacelli, T. (2004), "Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy", Journal of International Economics, Vol. 62, pp. 191-217.

Natalucci, F.M. and F. Ravenna (2002), "The Road to Adopt the Euro: Monetary Policy and Exchange Rate Regimes in EU Candidate Countries", International Finance Discussion Papers, No 741, Board of Governors of the Federal Reserve System.

Plasmans, J., G. Di Bartolomeo, B. Merlevede, and Bas van Aarle (2004), "Monetary policy regimes with hybrid output gaps and inflation rates with an application to EU-accession countries" in S. Kesenne and C. Reyns (eds.), Kwantitatief bekeken - Liber Amicorum Prof. dr. Robert Van Straelen", pp. 95-136, Garant, AntwerpenApeldoorn.

Onatski A. and N. Williams (2003), "Modeling model uncertaintly", Journal of the European Economic Association, Vol. 1, pp. 1087-1122.

Ortega, E. and N. Rebei (2006), "The Welfare Implications of Inflation versus Price-Level Targeting in a Two-Sector Small Open Economy", Bank of Canada Working Paper 2006-12.

Plasmans, J., T. Michalak and J. Fornero (2006a), "A microfounded sectoral model for open economies", Discussion Paper, University of Antwerp, 56 pages.

Plasmans, J., T. Michalak and J. Fornero (2006b), "Exchange Rate Pass-Through and Monetary Policy in a Sectoral Two Country Small Open Economy Model", Discussion Paper, University of Antwerp, 41 pages.

Paustian, M. (2004), "Welfare Effects of Monetary Policy Rules in aModel with Nominal Rigidities and Credit Market Frictions", Working Paper, Bonn University, 26 pages.

Pytlarczyk, E. (2005), "An estimated DSGE model for the German economy within the euro area", Discussion Paper Series 1: Economic Studies 2005, No. 33, Deutsche Bundesbank.

Pierdzioch, C. (2004), "Financial Market Integration and Business Cycle Volatility in a Monetary Union", Scottish Journal of Political Economy, Vol. 51(3), pp. 422-442.

Ratto, M., W. Roeger, J. in't Veld and R. Girardi (2005a), "An estimated new keynesian dynamic stochastic general equilibrium of the Euro area", European Economy, Economic Papers No. 220, European Comission.

Ratto, M., W. Roeger, J. in't Veld and R. Girardi (2005b), "An estimated open-economy model for the EURO area" Mimeo, June, 67 pages.

Ravina, H. (2005), "Habit Persistence and Keeping Up with the Joneses:Evidence from Micro Data", New York University Discussion Paper, 47 pages.

Selaive J. and V. Tuesta (2006), "The Consumption-Real Exchange Rate Anomaly: Non-traded Goods, Incomplete Markets and Distribution Services", Working Paper No. 359,Central Bank of Chile.

Schmitt-Grohé, S. and M. Uribe, (2003), "Closing Small Open Economy Models", Journal of International Economics, Vol. 61, pp. 163-185.

Smets, F. and R. Wouters (2003), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area". Journal of the European Economic Association, Vol. 1, pp. 1123-1175.

Smets, F. and R. Wouters (2004), "Comparing Shocks and Frictions in US and Euro Area Business Cycles: A Bayesian DSGE Approach". NBB Working Paper No. 61, October.

Taylor, J.B. (1993), "Discretion versus policy rules in practice", Carnegie-Rochester Conference Series on Public Policy, Vol. 39, pp. 195-214.

Taylor, J.B. (1999), "Introduction" in J.B. Taylor (ed.), Monetary Policy Rules, National Bureau of Economic Research Studies in Business Cycles.

Wolman, A.(1999), "Does state-dependent pricing imply coordination failure?", Working Paper 99-05, Federal Reserve Bank of Richmond.

Woodford, M. (2003), Interest and Prices, Princeton University Press.

## NATIONAL BANK OF BELGIUM - WORKING PAPERS SERIES

1. "Model-based inflation forecasts and monetary policy rules" by M. Dombrecht and R. Wouters, Research Series, February 2000.
2. "The use of robust estimators as measures of core inflation" by L. Aucremanne, Research Series, February 2000.
3. "Performances économiques des Etats-Unis dans les années nonante" by A. Nyssens, P. Butzen, P. Bisciari, Document Series, March 2000.
4. "A model with explicit expectations for Belgium" by P. Jeanfils, Research Series, March 2000.
5. "Growth in an open economy: some recent developments" by S. Turnovsky, Research Series, May 2000.
6. "Knowledge, technology and economic growth: an OECD perspective" by I. Visco, A. Bassanini, S. Scarpetta, Research Series, May 2000.
7. "Fiscal policy and growth in the context of European integration" by P. Masson, Research Series, May 2000.
8. "Economic growth and the labour market: Europe's challenge" by C. Wyplosz, Research Series, May 2000.
9. "The role of the exchange rate in economic growth: a euro-zone perspective" by R. MacDonald, Research Series, May 2000.
10. "Monetary union and economic growth" by J. Vickers, Research Series, May 2000.
11. "Politique monétaire et prix des actifs: le cas des Etats-Unis" by Q. Wibaut, Document Series, August 2000.
12. "The Belgian industrial confidence indicator: leading indicator of economic activity in the euro area?" by J.-J. Vanhaelen, L. Dresse, J. De Mulder, Document Series, November 2000.
13. "Le financement des entreprises par capital-risque" by C. Rigo, Document Series, February 2001.
14. "La nouvelle économie" by P. Bisciari, Document Series, March 2001.
15. "De kostprijs van bankkredieten" by A. Bruggeman and R. Wouters, Document Series, April 2001.
16. "A guided tour of the world of rational expectations models and optimal policies" by Ph. Jeanfils, Research Series, May 2001.
17. "Attractive Prices and Euro - Rounding effects on inflation" by L. Aucremanne and D. Cornille, Documents Series, November 2001.
18. "The interest rate and credit channels in Belgium: an investigation with micro-level firm data" by P. Butzen, C. Fuss and Ph. Vermeulen, Research series, December 2001.

19 "Openness, imperfect exchange rate pass-through and monetary policy" by F. Smets and R. Wouters, Research series, March 2002.
20. "Inflation, relative prices and nominal rigidities" by L. Aucremanne, G. Brys, M. Hubert, P. J. Rousseeuw and A. Struyf, Research series, April 2002.
21. "Lifting the burden: fundamental tax reform and economic growth" by D. Jorgenson, Research series, May 2002.
22. "What do we know about investment under uncertainty?" by L. Trigeorgis, Research series, May 2002.
23. "Investment, uncertainty and irreversibility: evidence from Belgian accounting data" by D. Cassimon, P.-J. Engelen, H. Meersman, M. Van Wouwe, Research series, May 2002.
24. "The impact of uncertainty on investment plans" by P. Butzen, C. Fuss, Ph. Vermeulen, Research series, May 2002.
25. "Investment, protection, ownership, and the cost of capital" by Ch. P. Himmelberg, R. G. Hubbard, I. Love, Research series, May 2002.
26. "Finance, uncertainty and investment: assessing the gains and losses of a generalised non-linear structural approach using Belgian panel data", by M. Gérard, F. Verschueren, Research series, May 2002.
27. "Capital structure, firm liquidity and growth" by R. Anderson, Research series, May 2002.
28. "Structural modelling of investment and financial constraints: where do we stand?" by J.- B. Chatelain, Research series, May 2002.
29. "Financing and investment interdependencies in unquoted Belgian companies: the role of venture capital" by S. Manigart, K. Baeyens, I. Verschueren, Research series, May 2002.
30. "Development path and capital structure of Belgian biotechnology firms" by V. Bastin, A. Corhay, G. Hübner, P.-A. Michel, Research series, May 2002.
31. "Governance as a source of managerial discipline" by J. Franks, Research series, May 2002.
32. "Financing constraints, fixed capital and R\&D investment decisions of Belgian firms" by M. Cincera, Research series, May 2002.
33. "Investment, R\&D and liquidity constraints: a corporate governance approach to the Belgian evidence" by P. Van Cayseele, Research series, May 2002.
34. "On the Origins of the Franco-German EMU Controversies" by I. Maes, Research series, July 2002.
35. "An estimated dynamic stochastic general equilibrium model of the Euro Area", by F. Smets and R. Wouters, Research series, October 2002.
36. "The labour market and fiscal impact of labour tax reductions: The case of reduction of employers' social security contributions under a wage norm regime with automatic price indexing of wages", by K. Burggraeve and Ph. Du Caju, Research series, March 2003.
37. "Scope of asymmetries in the Euro Area", by S. Ide and Ph. Moës, Document series, March 2003.
38. "De autonijverheid in België: Het belang van het toeleveringsnetwerk rond de assemblage van personenauto's", by F. Coppens and G. van Gastel, Document series, June 2003.
39. "La consommation privée en Belgique", by B. Eugène, Ph. Jeanfils and B. Robert, Document series, June 2003.
40. "The process of European monetary integration: a comparison of the Belgian and Italian approaches", by I. Maes and L. Quaglia, Research series, August 2003.
41. "Stock market valuation in the United States", by P. Bisciari, A. Durré and A. Nyssens, Document series, November 2003.
42. "Modeling the Term Structure of Interest Rates: Where Do We Stand?, by K. Maes, Research series, February 2004.
43. Interbank Exposures: An Empirical Examination of System Risk in the Belgian Banking System, by H. Degryse and G. Nguyen, Research series, March 2004.
44. "How Frequently do Prices change? Evidence Based on the Micro Data Underlying the Belgian CPI", by L. Aucremanne and E. Dhyne, Research series, April 2004.
45. "Firms' investment decisions in response to demand and price uncertainty", by C. Fuss and Ph. Vermeulen, Research series, April 2004.
46. "SMEs and Bank Lending Relationships: the Impact of Mergers", by H. Degryse, N. Masschelein and J. Mitchell, Research series, May 2004.
47. "The Determinants of Pass-Through of Market Conditions to Bank Retail Interest Rates in Belgium", by F. De Graeve, O. De Jonghe and R. Vander Vennet, Research series, May 2004.
48. "Sectoral vs. country diversification benefits and downside risk", by M. Emiris, Research series, May 2004.
49. "How does liquidity react to stress periods in a limit order market?", by H. Beltran, A. Durré and P. Giot, Research series, May 2004.
50. "Financial consolidation and liquidity: prudential regulation and/or competition policy?", by P. Van Cayseele, Research series, May 2004.
51. "Basel II and Operational Risk: Implications for risk measurement and management in the financial sector", by A. Chapelle, Y. Crama, G. Hübner and J.-P. Peters, Research series, May 2004.
52. "The Efficiency and Stability of Banks and Markets", by F. Allen, Research series, May 2004.
53. "Does Financial Liberalization Spur Growth?" by G. Bekaert, C.R. Harvey and C. Lundblad, Research series, May 2004.
54. "Regulating Financial Conglomerates", by X. Freixas, G. Lóránth, A.D. Morrison and H.S. Shin, Research series, May 2004.
55. "Liquidity and Financial Market Stability", by M. O'Hara, Research series, May 2004.
56. "Economisch belang van de Vlaamse zeehavens: verslag 2002", by F. Lagneaux, Document series, June 2004.
57. "Determinants of Euro Term Structure of Credit Spreads", by A. Van Landschoot, Research series, July 2004.
58. "Macroeconomic and Monetary Policy-Making at the European Commission, from the Rome Treaties to the Hague Summit", by I. Maes, Research series, July 2004.
59. "Liberalisation of Network Industries: Is Electricity an Exception to the Rule?", by F. Coppens and D. Vivet, Document series, September 2004.
60. "Forecasting with a Bayesian DSGE model: an application to the euro area", by F. Smets and R. Wouters, Research series, September 2004.
61. "Comparing shocks and frictions in US and Euro Area Business Cycle: a Bayesian DSGE approach", by F. Smets and R. Wouters, Research series, October 2004.
62. "Voting on Pensions: A Survey", by G. de Walque, Research series, October 2004.
63. "Asymmetric Growth and Inflation Developments in the Acceding Countries: A New Assessment", by S. Ide and P. Moës, Research series, October 2004.
64. "Importance économique du Port Autonome de Liège: rapport 2002", by F. Lagneaux, Document series, November 2004.
65. "Price-setting behaviour in Belgium: what can be learned from an ad hoc survey", by L. Aucremanne and M. Druant, Research series, March 2005.
66. "Time-dependent versus State-dependent Pricing: A Panel Data Approach to the Determinants of Belgian Consumer Price Changes", by L. Aucremanne and E. Dhyne, Research series, April 2005.
67. "Indirect effects - A formal definition and degrees of dependency as an alternative to technical coefficients", by F. Coppens, Research series, May 2005.
68. "Noname - A new quarterly model for Belgium", by Ph. Jeanfils and K. Burggraeve, Research series, May 2005.
69. "Economic importance of the Flemish maritime ports: report 2003", F. Lagneaux, Document series, May 2005.
70. "Measuring inflation persistence: a structural time series approach", M. Dossche and G. Everaert, Research series, June 2005.
71. "Financial intermediation theory and implications for the sources of value in structured finance markets", J. Mitchell, Document series, July 2005.
72. "Liquidity risk in securities settlement", J. Devriese and J. Mitchell, Research series, July 2005.
73. "An international analysis of earnings, stock prices and bond yields", A. Durré and P. Giot, Research series, September 2005.
74. "Price setting in the euro area: Some stylized facts from Individual Consumer Price Data", E. Dhyne, L. J. Álvarez, H. Le Bihan, G. Veronese, D. Dias, J. Hoffmann, N. Jonker, P. Lünnemann, F. Rumler and J. Vilmunen, Research series, September 2005.
75. "Importance économique du Port Autonome de Liège: rapport 2003", by F. Lagneaux, Document series, October 2005.
76. "The pricing behaviour of firms in the euro area: new survey evidence, by S. Fabiani, M. Druant, I. Hernando, C. Kwapil, B. Landau, C. Loupias, F. Martins, T. Mathä, R. Sabbatini, H. Stahl and A. Stokman, Research series, November 2005.
77. "Income uncertainty and aggregate consumption, by L. Pozzi, Research series, November 2005.
78. "Crédits aux particuliers - Analyse des données de la Centrale des Crédits aux Particuliers", by H. De Doncker, Document series, January 2006.
79. "Is there a difference between solicited and unsolicited bank ratings and, if so, why?" by P. Van Roy, Research series, February 2006.
80. "A generalised dynamic factor model for the Belgian economy - Useful business cycle indicators and GDP growth forecasts", by Ch. Van Nieuwenhuyze, Research series, February 2006.
81. "Réduction linéaire de cotisations patronales à la sécurité sociale et financement alternatif" by Ph. Jeanfils, L. Van Meensel, Ph. Du Caju, Y. Saks, K. Buysse and K. Van Cauter, Document series, March 2006.
82. "The patterns and determinants of price setting in the Belgian industry" by D. Cornille and M. Dossche, Research series, May 2006.
83. "A multi-factor model for the valuation and risk management of demand deposits" by H. Dewachter, M. Lyrio and K. Maes, Research series, May 2006.
84. "The single European electricity market: A long road to convergence", by F. Coppens and D. Vivet, Document series, May 2006.
85. "Firm-specific production factors in a DSGE model with Taylor price setting", by G. de Walque, F. Smets and R. Wouters, Research series, June 2006.
86. "Economic importance of the Belgian ports: Flemish maritime ports and Liège port complex - report 2004", by F. Lagneaux, Document series, June 2006.
87. "The response of firms' investment and financing to adverse cash flow shocks: the role of bank relationships", by C. Fuss and Ph. Vermeulen, Research series, July 2006.
88. "The term structure of interest rates in a DSGE model", by M. Emiris, Research series, July 2006.
89. "The production function approach to the Belgian output gap, Estimation of a Multivariate Structural Time Series Model", by Ph. Moës, Research series, September 2006.
90. "Industry Wage Differentials, Unobserved Ability, and Rent-Sharing: Evidence from Matched WorkerFirm Data, 1995-2002", by R. Plasman, F. Rycx and I. Tojerow, Research series, October 2006.
91. "The dynamics of trade and competition", by N. Chen, J. Imbs and A. Scott, Research series, October 2006.
92. "A New Keynesian Model with Unemployment", by O. Blanchard and J. Gali, Research series, October 2006.
93. "Price and Wage Setting in an Integrating Europe: Firm Level Evidence", by F. Abraham, J. Konings and S. Vanormelingen, Research series, October 2006.
94. "Simulation, estimation and welfare implications of monetary policies in a 3-country NOEM model", by J. Plasmans, T. Michalak and J. Fornero, Research series, October 2006.


[^0]:    (*) Corresponding author, Department of Economics, University of Antwerp, Prinsstraat 13, B2000 Antwerp; Phone: +32 (0)3 220 4149, Fax: +32 (0)3 2204585.
    Authors' e-mails: joseph.plasmans@ua.ac.be, tomasz.michalak@ua.ac.be and jorgealberto.fornero@ua.ac.be.

[^1]:    ${ }^{1}$ It is assumed that intermediate goods producers do not use other intermediate goods as their inputs.
    ${ }^{2}$ The relevance of intermediate goods in the production processes is nicely illustrated by annual Input-output tables of (for instance) the Dutch economy, where they account for around $25 \%$ of all inputs in many industries. See Data Appendix F for details.

[^2]:    ${ }^{3}$ Alternatively, these stochastic discount factors can also be considered as prices of one (domestic and foreign) assets paid to the consumer (owner of assets) at time $t+1$.
    ${ }^{4}$ For an exact definition of effective interest rates $\hat{r}_{U, t, t+1}^{h}$ and $\hat{r}_{W, t, t+1}^{h}$, see Section 8 in this paper. Notice also that $S_{t}$ is used to translate asset returns at the beginning and at the end of period $t$ so that, de facto, it represents the average exchange rate during period $t$; see also equation (20) in Selaive and Tuesta (2003).
    ${ }^{5}$ This is a Constant Rate of Risk Aversion (CRRA) utility function that allows for habit formation as in Kozicki and Tinsley (2002). Moreover, notice that following the international literature, we assume that the parameters in this utility function are equal over consumers.
    ${ }^{6}$ Recent micro-level studies report mixed evidence of the impact of habit formation in consumption (see Ravina (2005)), while studies conducted with aggregate data finds substantial evidence, e.g. Christiano et al.(2005), enphasize the role of habit persistence in the explanation of the hump-shaped behavior of aggregate consumption in response to a monetary policy shock. Notice that habit formation in consumption vanishes and consumption is as the usual CRRA ulitily if $\kappa_{h}=0$.
    ${ }^{7}$ Hence, shares and bonds are only used to carry over savings between periods.

[^3]:    ${ }^{8}$ Non-tradable intermediate goods include intermediate goods which are relatively too expensive to be transported, e.g. sand, water and various kinds of services.
    ${ }^{9}$ Similarly to household $i$ 's utility function (2), we assume that in sector $m$ producer $j$ 's production function parameters are the same across producers. We assume that intermediate goods include capital goods. For the treatment of capital goods as a separate input, see Plasmans et al. (2006a).

[^4]:    ${ }^{10}$ Note that our $\mathrm{AR}(1)$ process can be interpreted as highly persistent shock that shifts the trend level of a productivity growth (see, for instance, Erceg et al. (2005), where productivity shocks are split in two parts: a highly persistent (which corresponds to $\omega_{h, t}^{m}$ ) and a transitory ones (which corresponds to a shock as $\xi_{m, \omega, h, t}$ )).
    ${ }^{11}$ Notice that these import prices include the importers' markups (see later), while the export prices do not.

[^5]:    ${ }^{12}$ Some authors assume that producers do not know beforehand whether their goods will be used as a final good or as an input in production chain (see for example Leith and Malley (2002)). However, we follow the spirit of the General Equilibrium models à la Debreu where market clearing is defined for each (sub)market.
    ${ }^{13}$ For example, a Calvo price parameter equal to 0.75 implies an average duration of 4 periods.
    ${ }^{14}$ Note that the formulation below does not contain price indexation. This can be introduced by defining $\pi_{h, t}^{C T, h}(j) \equiv \frac{P_{h, t}^{C T, h}(j)}{P_{h, t-1}^{C T, h}(j)}$ so that, on average $P_{h, t}^{C T, h}=\pi_{h, t}^{C T, h} P_{h, t-1}^{C T, h}$. (see Erceg et al. (2005)).

[^6]:    ${ }^{15}$ The property that in (23) and (24) only the factors of proportionality $\digamma_{U, t+1}^{h}($.$) and \digamma_{W, t+1}^{h}($.$) (and not \hat{r}_{U, t, t+1}^{h}$ and $\left.\hat{r}_{W, t, t+1}^{h}\right)$ are consumer $i$-dependent is argued from this interpretation.
    ${ }^{16}$ For instance, another formulation of $\digamma_{t}($.$) can be found in Schmitt-Grohé and Uribe (2003) and Malik (2005)$ on the one side and in Erceg et al. (2005) on the other side.

[^7]:    ${ }^{17}$ Notice that $\mathcal{B}_{h, t}, \mathcal{B}_{U, t}$ and $\mathcal{B}_{W, t}$ include benefits from all the six sectors of the corresponding economies $m=$ $F T, F N, V T, V N, M F, M V$.

[^8]:    ${ }^{18}$ In order to have an idea of the importance of intermediate goods production, we observe from empirical inputoutput tables that intermediate goods account for a large share (in some industries more than $50 \%$ ) of the inputs utilized in the final (industry) outputs.

[^9]:    ${ }^{19}$ The reader could wonder why the CB would target also wage inflation if it already targets inflation. Concern about wage inflation could result for various reasons, among which: (i) wage inflation could lead to a wage-price spiral and (ii) it could target real wages, e.g. to secure competitiveness and contain demand-pull inflation it could try to keep real wages low.
    ${ }^{20}$ Welfare implications are robust to both definitions of welfare, conditional and unconditional, but the unconditional welfare gain is higher than the conditional one. The conditional welfare measure is found solving the Ramsey problem as e.g. in Levin et al. (2005). The Ramsey problem consists of the maximization of the consumer's utility, made by a benevolent planner subject to the structural model.

[^10]:    ${ }^{21}$ In the very near future, we would like to compare both approaches in an empirical setting using the Dynare software (see Juillard (2005)).
    ${ }^{22}$ Monthly exchange rate data is published on the Fed webside: http://research.stlouisfed.org/fred2/series/EXUSEU?\&cid=15.
    ${ }^{23}$ For a similar productivity shock in the setting with non-tradable goods but without intermediate goods, see Selaive and Tuesta (2006).

[^11]:    ${ }^{24}$ See Huang and Liu (2005) for a very good discussion of this issue.

[^12]:    ${ }^{25}$ Our results can be compared to those obtained by Ortega and Rebei (2006, p. 44) for a local (domestic) nominal interest rate shock.
    ${ }^{26}$ In fact, our results of the (4b) shock on the home country can be compared to those obtained by Erceg et al. (2005) for the foreign demand shock.

[^13]:    ${ }^{27}$ In the EMU series in levels, the null of the unit root cannot be rejected: (i) EMU interest rate (with p-value of 0.27 ); (ii) MU annualized inflation (with 0.33 of prob.); EMU output (with 0.8 of prob.). EMU wage inflation is at the edge (with 0.05 of prob.). All the series become stationary after taking first differences.

    Alternatively, we checked stationarity properties of the detrended part of all the series (using the standard Hoddrick-Prescott filter with $\lambda=1600$ ) and we found that the cyclical part is always stationary.

    Stationarity results for the RoW (USA) series are similar to those of the EMU.
    ${ }^{28}$ The evidence shown in the previous footnote suggests that, in order to estimate monetary rules using OLS, we must use either data in differences or the detrended part. We performed both possibilities.

[^14]:    ${ }^{29}$ Ratto et al. (2005b, p.14) check systematically for all parameters and determine which of them are more likely to lead to indeterminacy.
    ${ }^{30}$ Compare to Table 6 in Levin et al. (2005), where 31 parameters are assumed to have an independent prior density.

[^15]:    ${ }^{31}$ Remark that, obviously, also NOEM models with different types of monetary policy rules for the ECB and the Fed (e.g. Rule (I) for the ECB and Rule (II) for the Fed, etc.) could be considered. Although this seems very relevant from an empirical point of view, we do not perform this here to save space and time and leave it for future experiments.

    Results considering Rules (I) and (III) can be obtained from the authors by simple request.
    ${ }^{32}$ This value is somewhat smaller than the value 0.65 as assumed in Dellas (2005).
    ${ }^{33}$ Comparing with the OLS results in equation (37), we find a very similar reaction parameter for the smoothed interest rate (but not for the reaction to the CPI inflation).

[^16]:    ${ }^{34}$ Recall that stock variables are measured at the beginning of the period, so that $M_{t+1}^{h}, B_{U, t+1}^{h}$ and $B_{W, t+1}^{h}$ are decision variables in period $t$ and are defined without conditional expectation operators.
    ${ }^{35}$ See (25) for an exact definition of risk premium functions.

[^17]:    ${ }^{36}$ Note that the restrictions on the expected stochastic discount factors do not imply that the interest rates $r_{t, t+1}$ and $r_{t, t+1}^{W}$ are equal!

[^18]:    ${ }^{37}$ Which is the steady state of the dual function to the input relationship (11).

[^19]:    ${ }^{38}$ We checked the evolution of technical coefficients from 1969 to 2002, and in order to get a more stabe path of it, we aggregate similar industries (from 26 to 9 sectors). These 9 sectors are:
    1.Agriculture, forestry and fish.
    2.Raw materials; Metal industry; Machine-industry; Electrical industry; Transport equipment industry; Remaining industry; Energy and water supply business.
    3.Food and stimulants industry; Textile and leather industry; Paper industry, publishers and printers;
    4.Petrol industry; Chemical industry; Rubber and synthetic material industry.
    5.Construction industry.
    6.Trading and repairs; Hotel and restaurant services; Transport, raise and communication.
    7.Financial institutions; Rent of and trade in real estate; Business services and renting of movables.
    8.Authorities/Government; Health and social work services
    9.Services n.e.c.; Goods and services n. e. c.; Corrections in connection with wages in kind.

[^20]:    ${ }^{39}$ It is defined as the elasticity of the labor supply with respect to wage, leaving constant the marginal utility of consumption.

