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VOTING ON PENSIONS: A SURVEY

G. de Walque [(*)]

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(*) NBB, Research Department, (e-mail: gregory.dewalque@nbb.be).

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Abstract

The paper presents a non-exhaustive survey of the literature designed to explain emergence, size and political sustainability of pay-as-you-go pension systems. It proposes a simple framework of analysis (a small open two overlapping generation economy model), around which some variants are displayed. Dictatorship of the median voter is assumed. The text is organized to answer the following questions: (i) do political equilibria with PAYG pension schemes exist, (ii) why do they emerge, (iii) what are the conditions for the participation constraint of the pension game to be verified, and finally (iv) what is the size of the pension system chosen by the median voter and how is this size influenced by an exogenous (e.g. demographic) shock.

JEL classification: D72; D91; H55.
Keywords : public pensions; voting.

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1 Introduction

In the decade following the second World War, most developed countries set up unfunded pension systems. The main particularity of such ascending transfer devices is that they provide retired people with resources that are directly levied on the wages of the workers. By contributing to the financing of the system, workers become its creditors, allowing them to expect pension benefits when they retire. In this sense, pay-as-you-go pension schemes could be compared with public debt. However, the main feature of this kind of old age consumption financing is that the pension benefits of the workers are determined by the contributions paid by the next generations of workers rather than by the contributions they pay themselves in their careers. In this respect, three parameters are essential to evaluate the return of the investment a worker makes by participating in the pension scheme: first, the size of his investment in the pension scheme as defined by the tax rate paid on his labour income in his career, second, the tax rate paid by the contributor to the system on his retirement, and finally the economy's growth rate determined by the demographic and the technological growth rates.[1]

In the following pages we will adopt a positive viewpoint in order to explain first the emergence of such public transfers and second their size. From this angle, the contributions paid to or received from the transfer system are at the core of a political debate, while the demographic factor is considered as exogenously given. The problem under analysis has been examined in an expanding literature which has already been exhaustively surveyed by Breyer (1994a), Boadway and Wildasin (1989b) and more recently by Galasso and Profeta (2002). The present paper does certainly not pretend to be exhaustive. Instead it aims at developing a simplified, unified, pedagogical framework within which to present, to compare and to comment upon some of the various models designed to explain the redistribution of resources among generations. We will develop the reasoning behind each of the models presented within an analytical model, allowing simplifications with respect to some technical points developed in the literature.

[1] Of course the length of the period during which agents are entitled to pension benefits is also crucial. This is determined both by the effective retirement age and the longevity. Nevertheless we do not introduce this parameter in the present survey. This is dealt with in papers by Conde-Ruiz and Galasso (2000) and Casamatta (2002).

The paper is organised as follows. Section 2 presents the simplified setup we adopt as a tool of analysis and the variants of which we will examine. Throughout the exercise, the assumption of a two-period overlapping generations small open economy is upheld, with agents enjoying perfect foresight. Agents are supposed to have self-oriented preferences and homogeneous productive skills. We set a borrowing constraint holding that bankers refuse to lend money to a worker on the basis of ample pension benefits expectations.[2] The political debate is assumed to take place through a majority voting procedure.

Within this setup section 3 addresses the essential question of the existence of political equilibria with unfunded pensions. However, the possibility of this being the case is not sufficient to explain the effective emergence of such a device. This is examined in section 4. Both these sections use the tools of the non-cooperative game theory and they are based on the assumption that every generation has an incentive to participate in the pension scheme. Section 5 analyses the conditions for this assumption to be realised. Finally, section 6 addresses the question of the size of ascending public transfers. This section is based on the assumption that the contribution rate which workers are asked to determine through a majority voting procedure will stand forever. Of course, alternatives to the one-dimension majority vote have also been developed in the literature: section 6 concludes with a discussion of majority voting on two dimensions while section 7 intends to give some indication of the outcomes obtained under various types of non-majority political modelling. Section 8 draws the conclusions.

2 The reference setup and its variants

As presented in the introductory section, we choose for reference throughout the following pages the simplest model possible to analyze the question of intergenerational financial public transfers. We consider a small open economy

[2] In the basic setup, this assumption precludes the adoption of irrealistically large pensions transfers. Breyer (1994b) and Persson and Tabellini (2000) display that this assumption is no longer required if labor supply is endogenously determined. Subsection 5.1 also reveals that the non-borrowing constraint is never binding in a 2 OLG closed economy with a non-linear production function .

with one consumption good. Agents are assumed identical but for their age. As a consequence, they may be fully identified by their date of birth, say $t \in I\!N$. Let us study the case where agents live for two periods. In the first period of life, they are of the "young" type. They supply work inelastically and produce a quantity w_t which may also be interpreted as their labour income. Within the considered economy, technological growth is assumed exogenous with a constant growth factor denoted g, so that

$$w_{t+1} = gw_t, \ \forall t \in I\!N \tag{1}$$

In the second period of life, agents are retired and of the "old" type. Let us denote by c_t^1 the consumption of an agent young in t while c_{t+1}^2 represents the consumption of this agent when old. Each agent gives birth to m children so that young agents are m times as numerous as the old ones, m being assumed exogenous and constant over time. Finally, agents behave as selfishly as the traditional homo economicus.

In this oversimplified model we consider two devices allowing to smooth consumption over the life-cycle: a storage technology and public intergenerational transfers. We suppose the storage technology yields an exogenous and constant return factor $R \in I\!R_+$. Within the two overlapping generation setting, each generation t cohabits successively with generation $t-1$ to which it possibly pays a transfer $\tau_t w_t$ $(0 \leq \tau_t \leq 1)$ and generation $t+1$ from which it possibly receives a transfer $m\tau_{t+1}w_{t+1}$ $(0 \leq \tau_{t+1} \leq 1)$. Agents decide to store or not in their private capacity while participation to a public transfer system is mandatory whenever one is established.

Let us represent the utility of an agent over his life-cycle by a function $U(c_t^1, c_{t+1}^2)$ satisfying the following assumptions:

H1:	continuity	**H4:**	$U_1'(0, c_{t+1}^2) = U_2'(c_t^1, 0) = +\infty$
H2:	$U_1' > 0$ and $U_2' > 0$	**H5:**	strictly quasi-concavity
H3:	twice differentiability	**H6:**	homotheticity

For a given pair $\{\tau_t, \tau_{t+1}\}$, an agent who is young at t solves the following program

$$\text{Max}_{c_t^1, c_{t+1}^2} \ U(c_t^1, c_{t+1}^2) \tag{2}$$

$$\text{s.t.} \begin{cases} w_t \cdot (1 - \tau_t) = c_t^1 + s_t \\ Rs_t + m \cdot \tau_{t+1} \cdot w_{t+1} = c_{t+1}^2 \\ s_t \geq 0 \end{cases}$$

where s_t is the amount an agent born in period t stores for old age consumption. The solution of this problem yields indirect utility $V_t(\tau_t, \tau_{t+1})$. This problem is illustrated in figure 1, with the bold lines representing respectively the budget and the non-negative saving constraint.

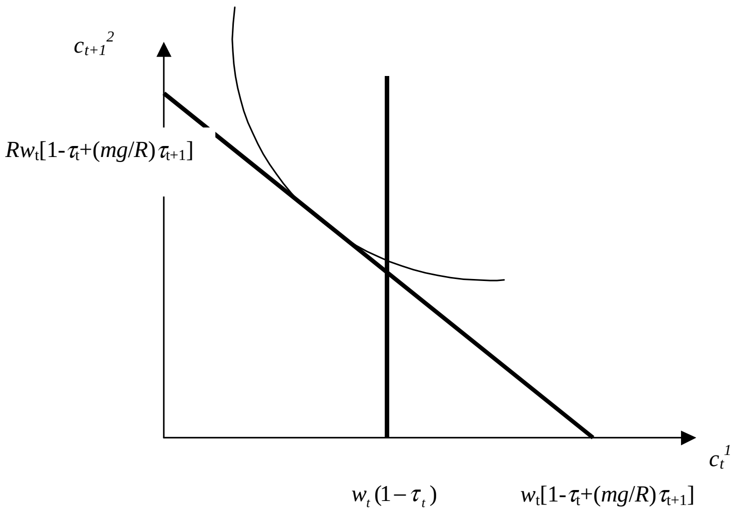

Figure 1: Decision-making in the reference setup

The problem is trivial as long as τ_t and τ_{t+1} are fixed. However, the present survey is built on the assumption that public transfers are not exogenously determined. Instead, they are at the core of a political debate among agents living contemporaneously in the economy. We have thus to aggregate individual preferences. Throughout the following pages we shall rely on the majority voting procedure. In the basic model described so far, agents differ only with respect to age. Throughout the paper we assume that population increases ($m > 1$) so that young voters outnumber the old ones and the median age voter is young and active on the labor market.

Though extremely simple, this model allows to raise in sections 3 and 4 the main difficulty of the problem under analysis: the decisive generation at time t has only the power to set the level of the transfer it wants to pay and it lacks decision power with respect to the pension it would like to receive.

In section 5 we will extend this framework in two directions, on the basis of simple examples. We will first consider a closed economy so that intergenerational transfers crowd out productive capital. Second, we introduce ascending altruism. The virtue of both these extensions is to show that, in the two overlapping generation setting, intergenerational public transfers may be politically sustainable even while the economy is dynamically efficient.

In the sixth section, which studies how the size of intergenerational transfers is decided through majority voting, we extend the basic setup first, by considering more than two overlapping generations and second, by assuming that agents differ not only in their age, but also in their productive capacity.

3 Political equilibria with positive transfers

As argued in the introduction, the existence of a pay-as-you-go pension scheme implies that the workers who finance the pensions of their retired predecessors are creditors of the public pension system managed by the government. It is a common assumption in the literature concerning intergenerational public transfers that this debt is guaranteed by the government. However, when addressing transfers from the political point of view, public decisions are assumed to be taken by the voters. It could actually be the case that a majority of voters decide to repudiate (totally or partially) the pension debt. A pension system is said politically sustainable if, at each considered period, a majority of the living voters has an incentive to participate to the pension system and subsequently not to repudiate it.[3]

At this first stage of analysis, we consider that agents do not vote directly on their preferred public transfer policy. Instead, it is assumed for simplicity that some public transfer path is proposed and agents decide wether to accept or reject it through their vote. Let us first adopt the following definition:

Definition 1 *A public transfer proposal from period* 0 *through* T *(with* $T \in \mathbb{N}$*) is a sequence* $(\tau_t^p)_{t=0}^T$ *with* $\tau_t^p \in [0,1]$ *for every* $t \in [0,T]$

The superscript p stands for "proposal". We assume that at each period

[3] This definition is voluntarily simplified. Indeed, a pension system which will be repudiated with certainty may nevertheless be politically sustainable if the horizon at which it will be abandoned is unknown to the living agents. Boldrin and Rustichini (2000) show that agents may still be interested in participating to such a pension system as long as their expected return is sufficiently large.

$0 \leq t \leq T$, a majority vote is organized with two alternatives: either accept the public transfer proposal $\left(\tau_{t+h}^{p}\right)_{h=0}^{T-t}$ (an alternative denoted 1) or reject it (an alternative denoted 0). If the outcome of the vote at some period t is 1 (acceptation), generation t pays a participation fee of $\tau_t^p w_t$, and it receives the pension benefit $m\tau_{t+1}^p w_{t+1}$ if and only if 1 is the outcome of the period $t+1$ vote. If the outcome of the vote at some period t is 0 (rejection), generation t pays no participation fee. Of course, in the two overlapping generation setting with $m > 1$, the decisive voter belongs to the young generation.

This problem structure is reminiscent of sequential games. Each generation is a player, and it plays only once. Players are indexed by their date of birth $t \in [0, T]$. Actions available to each player are $v_t \in \{0, 1\}$ and a history of the game at period t is $h_t = (v_i)_{i=0}^{t-1}$. A strategy for player t is a map

$$\sigma_t \left(h_t, \left(\tau_{t+h}^{p}\right)_{h=0}^{T-t} \right) \longmapsto v_t \in \{0, 1\}$$

Finally, the pay-off of each player t is $V_t\left(\tau_t, \tau_{t+1}\right)$ where τ_t is the participation rate actually paid by player t, i.e.

$$\tau_t = v_t \cdot \tau_t^p$$

Let us adopt a second definition:

Definition 2 *A political equilibrium is a sequence $(\tau_t)_{t=0}^{T}$ such that there exists a sequence of strategies $(\sigma_t)_{t=0}^{T}$ for the pension game which is a subgame perfect equilibrium and such that $(\tau_t)_{t=0}^{T}$ is the sequence of effective contribution rates associated with the equilibrium history.*

We may now state the following

Proposition 1 *The realization of the participation constraint*

$$V_t\left(\tau_t^p, \tau_{t+1}^p\right) > V_t(0, 0) \quad \forall t \in [0, T] \tag{3}$$

is a necessary condition for strategy $\sigma_0\left(h_0, (\tau_t^p)_{t=0}^{T}\right) = 1$ to belong to a subgame perfect equilibrium sequence of strategies.

Proof. If inequality (3) does not hold, then,

$$V_t(0, \tau_t^p) > V_t(0, 0) > V_t(\tau_t^p, \tau_{t+1}^p) > V_t(\tau_t^p, 0)$$

and $\sigma_t = 1$ is a dominated strategy. By backwards induction $\sigma_i = 1$ is also a dominated strategy for every $i < t$. ■

Note that the participation constraint (3) cannot be fulfilled if T is finite since $V_T(\tau_T^p, 0) < V_T(0,0)$. We may also state the following :

Proposition 2 (Boldrin and Rustichini, 2000) *The set of political equilibrium sequences includes all and only all the sequences $(\tau_t)_{t=0}^{\infty}$ such that, for any $l \in I\!N$,*

$$\begin{aligned} \tau_t &= 0 \text{ if } t \leq l \\ &= \tau_t^p \text{ if } t > l \end{aligned}$$

Proof. cf. technical appendix. ■

We conclude that, in the perfect information setting, from the moment every generation has an incentive to participate to the pension system, political equilibria with public transfers exist. Furthermore, the pension scheme may become active after l periods without transfers. Once the public transfers system is in place, it must be forever for public transfers to be a political equilibrium outcome. However, the question of why public transfers emerge at some period l is left unanswered and is the subject of next section.

4 Trigger strategies and the emergence of public transfers

Having in mind the profile of a political equilibrium (see Proposition 2), we may address the next question. Given a sequence $(\tau_t^p)_{t=0}^{\infty}$, why would some player l suddenly decide to vote 1 while all his predecessors in the game played 0? What is the condition for strategy $\sigma_l = 1$ to become a dominant strategy?

The choice of σ_t depends on the expectation generation t has concerning σ_{t+1}. However, as long as generation t believes σ_{t+1} fixes v_{t+1} independently of v_t, the unique Nash equilibrium is non-cooperation. Indeed,

$$\begin{aligned} &\text{for } v_{t+1} = 0: \quad V_t(\tau_t^p, 0) < V(0,0) \quad \forall \tau_t^p \in [0,1] \\ &\text{for } v_{t+1} = 1: \quad V_t\left(\tau_t^p, \tau_{t+1}^p\right) < V\left(0, \tau_{t+1}^p\right) \quad \forall \tau_t^p, \tau_{t+1}^p \in [0,1] \end{aligned}$$

This is a typical prisoner dilemma game. However, it is well-known that, if a non-cooperative game between two players is infinitely repeated and if strategies are allowed to be reaction functions, cooperation may occur at every stage of a subgame-perfect outcome even though the only Nash equilibrium of the stage game is non-cooperation (Folk theorem).

The framework is a bit different here since, even though the game is infinitely repeated, the players are different at each period.[4] However, trigger strategies may also be defined in this context. Let us consider a strategy $\sigma_t^* \left(h_t, (\tau_{t+h}^p)_{h=0}^{\infty} \right)$ such that, if the participation constraint (3) is satisfied from period $z \leq t$ on:

$$\begin{array}{ll} v_t = 1 & \text{if } v_{t-1} = 1 \text{ or } t - z = 0 \ \ \forall t - z \geq 0 \\ v_t = 0 & \text{if } v_{t-1} = 0 \text{ and either } v_{t-2} = 1 \text{ or } t - z = 1 \ \ \forall t - z \geq 1 \\ v_t = 1 & \text{if } v_{t-1} = 0,\ v_{t-2} = 0 \text{ and either } v_{t-3} = 1 \text{ or } t - z = 2 \ \ \forall t - z \geq 2 \\ v_t = 0 & \text{if } v_{t-1} = 0,\ v_{t-2} = 0,\ v_{t-3} = 0 \text{ and either } v_{t-4} = 1 \text{ or } t - z = 3 \ \ \forall t - z \geq 3 \\ \ldots & \ldots \end{array}$$

Such a strategy simply states that a transfer chain should be implemented from the moment it is Pareto improving and then never be interrupted . If the previous player deviates from this rule, he must be punished and receive no pension. The punishing generation must be rewarded and receive a pension even though it did not contribute to the PAYG scheme. This strategy has been generalized by Hammond (1975) as follows

$$\sigma_t^* \left(h_t, (\tau_{t+h}^p)_{h=0}^{\infty} \right) = \left\{ \begin{array}{l} 0 \text{ either if } t \leq z \text{ or if } t > z \text{ and } t - J(h_t) - 1 \text{ is odd} \\ 1 \text{ if } t > z \text{ and } t - J(h_t) - 1 \text{ is even (zero included)} \end{array} \right. \tag{4}$$

where

$$J(h_t) = \left\{ \begin{array}{l} \max \{ j \,|(v_j \in h_t) = 1 \} \\ 0 \text{ if } \ (v_j \in h_t) = 0 \end{array} \right.$$

If player t believes the next player will apply the σ^* strategy, playing σ^* himself garantees him either $V_t(0, \tau_{t+1})$ or $V_t(\tau_t, \tau_{t+1})$ while if he deviates, his pay-off is only $V(0,0) < V_t(\tau_t, \tau_{t+1}) < V_t(0, \tau_{t+1})$ and σ^* is then a subgame perfect equilibrium.

Proposition 3 (Hammond, 1975) *Consider that the participation constraint (3) is verified from period z on. It is sufficient that, from period*

[4] For papers dealing with the "folk theorem" in overlapping generation setting, see Salant (1991), Smith (1992) and Kandori (1992).

$l > z$ on, players believe their successors will apply a strategy of the type of σ^* defined in (4) for the set of political equilibrium sequences described in Proposition 2 to reduce to $\tau_t = \tau_t^p \quad \forall t \geq l$.

5 When is the participation constraint verified?

In Propositions 1, 2 and 3 the realization of the participation constraint (3) is crucial. Let us circumscribe, in our reference framework, the set of pairs $\left\{\tau_t^p, \tau_{t+1}^p\right\}$ such that this inequality is always verified. We define $\mathcal{I}(\tau_t)$ the value of τ_{t+1} such that, given $\tau_t \in [0,1]$,

$$V_t(\tau_t, \mathcal{I}(\tau_t)) = V_t(0,0) \tag{5}$$

We may then state the next Proposition, which is illustrated in figure 2.

Proposition 4 *Let us consider the reference small open two overlapping generation economy described in section 2. Then*

- *a (unique) fixed point $\bar{\tau}$ such that $0 < \bar{\tau} = \mathcal{I}(\bar{\tau}) < 1$ exists if and only if $mg > R$*
- *for a given public transfers proposal $(\tau_t^p)_{t=0}^{\infty}$, the participation constraint (3) is verified $\forall t \in I\!N$ if and only if, $\forall t \in I\!N$:*

$$\tau_t^p \in [0, \bar{\tau}] \tag{6}$$

$$\tau_{t+1}^p \in [\mathcal{I}(\tau_t^p), \bar{\tau}] \tag{7}$$

Proof. cf. technical appendix. ■

The $\mathcal{I}(\tau_t^p)$ curve is linear as long as the non negative saving constraint is loose. As soon as it becomes binding, the second derivative becomes positive so that τ_{t+1}^p compensates the intertemporal inefficiency in the consumption allocation caused by a marginal increase in τ_t^p.

The necessary and sufficient condition for the existence of the fixed point $\bar{\tau}$ is obvious from figure 1: if $R > mg$, $\mathcal{I}(\tau_t^p)$ is above the 45 degree line for all $\tau_t^p \in [0,1]$. Figure 1 helps also to illustrate the second point of Proposition 4: if $\tau_{t+1}^p > \bar{\tau}$, it should be the case that $\tau_{t+2}^p > \mathcal{I}(\tau_{t+1}^p)$ for player $t+2$ to participate to the transfer scheme and so on for the $t+2+j$ players (with $j \in I\!N$), requiring that within finite time τ^p becomes larger than unity. Since the borrowing constraint $s_t \geq 0$ is imposed, this is inadmissible.

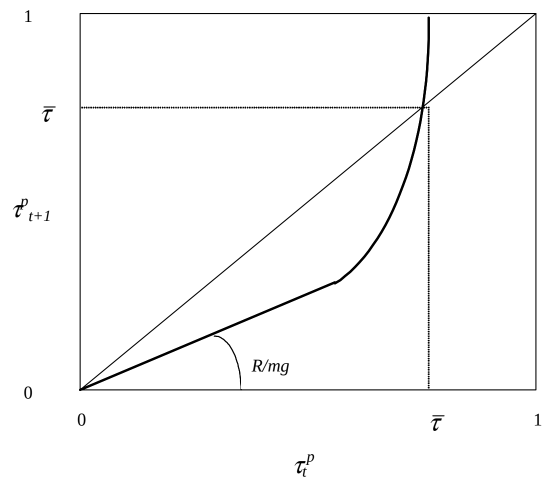

Figure 2: Illustration of Proposition 4: the $\mathcal{I}(\tau_t^p)$ curve

The latter Proposition displays the crucial role played by the $\frac{R}{mg}$ ratio, sometimes named the Aaron variable (from Aaron, 1966). If this ratio is smaller than unity, the human wealth in present value within the economy is finite if and only if the time horizon is finite. The combination of infinite horizon with $R/mg < 1$ induces non convergence of human wealth and there exists a possibility for the economy to capture resources from the far future at no cost for any generation by rolling (pay-as-you-go pension) debt over forever. In the absence of intergenerational public transfers, such an economy is Pareto inefficient and any political equilibrium $(\tau_t)_{t=0}^{\infty} \neq (0)_{t=0}^{\infty}$ is Pareto improving.

This result may appear a bit disappointing since the condition for the existence of a fixed point -which precondition the realization of the participation constraint at every period t- is similar to the condition justifying ascending public transfers obtained in the traditional normative analysis (cf. Samuelson, 1958 and Diamond, 1965). It thus seems that the positive approach adopted leads to a somewhat tautological outcome: a public unfunded pension policy can be politically sustainable only if it is Pareto efficient. Furthermore, even though the debate cannot be considered as closed, there is a large body of literature tending to prove that developed economies are dynamically efficient.[5] If this is right, our positive approach is unable to explain the observed existence of (large) unfunded pension systems. Fortunately, we may prove that the median voter's participation constraint may be satisfied in dynamically efficient economies under some slight modifications of the reference setup.

The first change we apply to the model is to consider a closed economy instead of a small open economy. Another way explored in the literature is to introduce ascending altruism. Note that similar results could also be obtained by considering more than two periods of life or by introducing heterogeneous individual productive ability. In this section we will focus on the first two amendments of the reference setup cited above, while the other ones will be studied in the next section, the purpose of which is to explain the size of the unfunded pension system.

5.1 An example with non-linear production function and crowding out

We intend to prove that, if the prices of the production factors are endogenously determined, dynamic inefficiency is not a necessary condition anymore for a political equilibrium sequence different from $(0)_{t=0}^{\infty}$ to emerge. As a corollary, a political equilibrium sequence with positive intergenerational public transfers is not necessarily Pareto improving.

The proof is easily obtained by introducing public transfers within the famous loglinear utility - Cobb Douglas production function example of Diamond (1965). Let us consider the following neoclassical production function with two factors, capital (K) and labor (L) and Harrod neutral technological progress

$$F(K_t, A_tL_t) = K_t^{\alpha}(A_tL_t)^{1-\alpha} \quad \text{with } 0 < \alpha < 1$$

Adopting the usual definitions

$$\frac{K_t}{A_tL_t} \equiv k_t \quad \text{and} \quad \frac{1}{A_tL_t}F(K_t, L_t) \equiv f(k_t)$$

[5] Homburg (1991) for example proves that, adding land to the traditional factors of the neoclassical production function, dynamic inefficiency is ruled out. Abel et al. (1989) consider an extension of Diamond (1965) to allow for uncertainty. They display that their criterion for dynamic efficiency to hold seems to be verified from 1929 up to now. However, their analysis cannot assess wether this observation is made at steady state or not.

the production function may be expressed in terms of productive units

$$f(k_t) = k_t^\alpha$$

Necessary conditions for profit maximizing are:

$$R_t = \alpha k_t^{\alpha-1} \quad \text{and} \quad w_t = (1-\alpha)k_t^\alpha \tag{8}$$

Assuming loglinear utility, indirect utility $V(\tau_t, \tau_{t+1})$ is such that

$$\begin{aligned} V_t(\tau_t, \tau_{t+1}) \equiv \text{Max}_{s_t} \ & \log\left(w_t \cdot (1-\tau_t) - s_t\right) + \\ & + \delta \log\left(R_{t+1}s_t + m \cdot \tau_{t+1} \cdot w_{t+1}\right) \end{aligned} \tag{9}$$

with $0 < \delta < 1$, the subjective discount factor. The equivalent of Proposition 4 is

Proposition 4' *Let us consider an overlapping generation closed economy with Cobb-Douglas production function and loglinear utility and characterized by parameters m, $g \in I\!R_+$ and α, $\delta \in]0,1[$. In this setting,*

- *a (unique) fixed point $\bar{\tau}$ such that $0 < \bar{\tau} = \mathcal{I}(\bar{\tau}) < 1$ exists if and only if the factor share for capital is sufficiently small with respect to (a monotonically increasing function of) the utility discount factor*
- *for a given public transfers proposition $(\tau_t^p)_{t=0}^\infty$, the participation constraint (3) is verified at each $t \in I\!N$ if and only if, $\forall t \in I\!N$, $\tau_t^p \in [0, \bar{\tau}]$ and $\tau_{t+1}^p \in [\mathcal{I}(\tau_t^p), \bar{\tau}]$*

Proof. cf. technical appendix ■

In order to help explain the first point made in the Proposition, the reader may note that, for δ approaching zero, the monotonically increasing function of δ (which is displayed in the appendix) is approximately equal to 0.667, while for δ approaching unity, it is equal to unity. This indicates that the necessary and sufficient condition for the fixed point to exist is not difficult to satisfy.

By accepting to pay a participation fee of $\tau_t w_t$, generation t is entitled to pension benefits $m\tau_{t+1}w_{t+1}$. It thus saves less for two reasons: reduced disposable wage and existence of a public pension. As shown in (8), reduced saving has a twofold effect: it increases the savings returns but reduces the wage bill of the next generation and consequently the return of the pay-as-you-go pension system. As long as the first effect is larger than the second, inequality (3) holds.

The main statement in Proposition 4' is that, contrarily to what we observed in the fixed factor price setting, given public transfers proposition $(\tau_t^p)_{t=0}^{\infty}$, the existence of a political equilibrium different from $(0)_{t=0}^{\infty}$ is completely independent of the mg/R ratio .[6] This example has been inspired from Mahieu and Rottier (2000) and Diamond (1965). The literature emphasizing the crowding out effect of unfunded pension schemes on the capital stock accumulation has been developed by Hu (1982), Coaley and Soares (1999) and Boldrin and Rustichini (2000) to name a few.

A numerical example: Let us illustrate Proposition 4' with a numerical example, choosing the following set of parameters: $\alpha = 0.3$, $mg = 1.2$, $\delta = 0.4$. For these parameters values, the economy is dynamically efficient[7] and the first point of Proposition 4' is satisfied. The evolution of wages and consumptions is displayed in Figure 3 and Figure 4 plots the corresponding life-cycle indirect utility. It is supposed that, at period 11, a PAYG system is introduced with τ constant and equal to 0.2. It first induces a decrease of the net wage and the gross wage begins to decrease only one period later, because of the crowding out effect of the pension system. The first period of life consumption follows the net wage and the second period of life consumption increases sharply in period 11 since generation 10 had not anticipated the "free lunch" coming from the introduction of the pension system. The latter induces a lower incentive to save, which rises the interest rate and explain that the new equilibrium level for the second period of life consumption remains high. This results in a larger life-cycle utility for generations born in period 10, 11 and 12 (Figure 4). However, all the ensuing generations are less well off. The figures also show the symmetric effect of renegading the PAYG pension system (in period 21): agents born in period 21 stop to contribute to the pension system so that the net wage increases. However, this is not translated into a higher first period of life consumption since they save to their second period of life. Furthermore, the gross wage remains low since the capital stock is still at its low equilibrium level. The second period of life

[6] This is actually a very convenient property of the loglinear utility-Cobb Douglas production function setting.

[7] Indeed, one may verify that $\frac{\alpha}{1-\alpha} > \frac{\delta}{1+\delta}$ which in the Cobb-Douglas production function-loglinear utility function case is the corollary of $R > mg$.

consumption falls down in period 21 due to the unanticipated loss of pension benefits for agents born in period 20. This fall goes on in period 22 because of the capital increase. The net effect for the agents is better displayed in terms of life-cycle utility: generations 20, 21 and 22 are suffering from the brutal change and all the ensuing generations are better off. The virtue of this example is to illustrate that, for a given public transfers proposition $(\tau_t^p)_{t=0}^{\infty}$ such that $\tau_t^p = 0.2\ \forall t$, the introduction of the PAYG system will be beneficial for the introducing generations but detrimental for their descendants. However, as shown for generation 21, none of these generations has an incentive to dismantle it, even though it would improve all the future agents welfare. We may then conclude that, in this example, $\sigma_t = 1$ always belong to a subgame perfect equilibrium sequence of strategies, even though it is not Pareto improving.

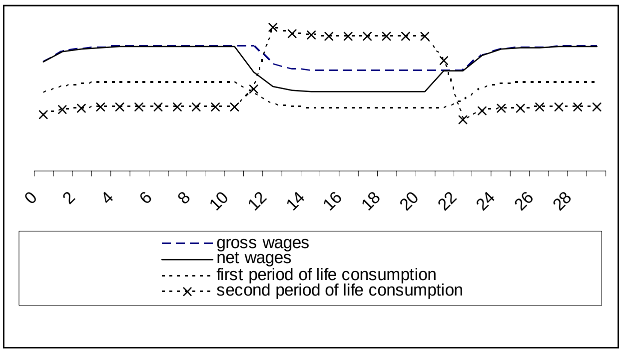

Figure 3: Wages and consumptions in the numerical example

5.2 An example with ascending altruism

Let us come back to the small open economy setting. The reference setup is now modified to introduce the concern of the young generation for the old one. It is furthermore assumed that utility is additively separable with respect to the consumption of both periods of life and of the retired parent.[8] As expressed by Veall (1986), an agent endowed with ascending altruistic

[8] Actually, altruistic feeling may take two different forms: either the concern of the altruistic agent is the utility the targeted agent derive from his consumption or the gift is motivated by the sole "joy-of-giving". Under joy-of-giving altruism, the gift is always positive and its size is completely independent from the resources of the recipient.

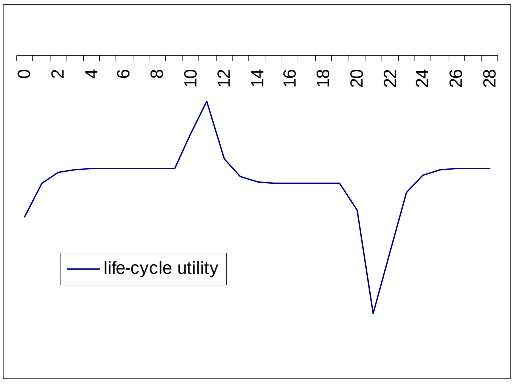

Figure 4: Life-cycle utility in the numerical example

feeling receives a positive externality from his parent second period of life consumption.[9] Under this way to model altruism, gifts are not necessarily operative. However we will display that agents may behave strategically in order to extract as much as possible from their children.

The voluntary transfers we will now focus on are actually *private* transfers. We will first derive through the example below a necessary and sufficient condition for private transfers to occur. Then, we will display that ascending *private* transfers lead to a Pareto sub-optimal equilibrium. In this case, ascending *public* transfers (under the form of a pay-as-you-go social security for example) are Pareto improving, irrespective of the R/mg ratio.

An agent born at time t seeks to maximize the same objective as in (2) except that a positive externality for the parent consumption is added. Adopting a loglinear utility, the problem faced by an agent born in t is the following:

$$\mathrm{Max}_{s_t, \mathrm{G}_t} \;\; \log c_t^1 + \delta \log c_{t+1}^2 + \frac{\gamma}{m} \log c_t^2 \tag{10}$$

$$\text{s.t.} \quad \begin{cases} w_t \cdot (1 - \mathrm{G}_t) = c_t^1 + s_t \\ R s_t + m \cdot \mathrm{G}_{t+1} \cdot w_{t+1} = c_{t+1}^2 \\ R s_{t-1} + m \cdot \mathrm{G}_t \cdot w_t = c_t^2 \\ s_t \geq 0, \;\; \mathrm{G}_t \geq 0 \end{cases}$$

[9] It is essential for the argument presented in this subsection that an agent is concerned only by the instantaneous utility his parent obtains during his retirement period. Should it take into account his parent whole life-cycle utility, the argument presented in this literature would disappear.

where $w_t \mathrm{G}_t$ is the gift he wants to give to his parent so that the latter can afford consumption c_t^2.[10] The reason to impose $\mathrm{G}_t \geq 0$ is to forbid involuntary descending transfers. We furthermore assume

$$1 > \delta > \gamma/m > 0 \tag{11}$$

Because of the non-negative saving constraint, the young generation can only capture future wage resources from the next generation (under the form of a gift). It also benefits from the old generation savings. The overall budget constraint at time t may be written as

$$Y_t = \frac{R}{m} s_{t-1} + w_t \left(1 + \frac{mg}{R} \cdot \mathrm{G}_{t+1}\right) = c_t^1 + \frac{c_t^2}{m} + \frac{c_{t+1}^2}{R} \tag{12}$$

while the optimal consumptions from the player t viewpoint are

$$c_t^{1*} = \frac{Y_t}{1 + \delta + \frac{\gamma}{m}}; \quad c_t^{2*} = \gamma \cdot c_t^{1*}; \quad c_{t+1}^{2*} = \delta R \cdot c_t^{1*} \tag{13}$$

It is obvious from (12) and (13) that the saving behavior of a generation influences the next generation gift policy: the less one saves, the larger the gift received in second period of life. Anticipating that player $t+1$ may have an incentive to undersave, player t may find it worthwhile to undersave himself and be the leader of the Stackelberg game. If he does, he will actually adopt a saving behavior such that player $t+1$ should allocate his entire wage resource w_{t+1} between c_{t+1}^1 and c_{t+1}^2 so that $s_{t+1} = 0$.

Let us assume that until period t an equilibrium without gift was prevailing so that $\frac{s_{t-1}}{w_{t-1}} = \frac{\delta}{1+\delta}$. We are then looking for the necessary and sufficient condition for generation t to deviate from an equilibrium without private transfers, i.e. to undersave in order to attract positive gifts. We may then state:

Proposition 5 *Given R, m and $g \in I\!R_+$ with $mg < R$, δ and $\frac{\gamma}{m} \in]0,1[$ with $\delta > \frac{\gamma}{m}$ and $\frac{s_{t-1}}{w_{t-1}} = \frac{\delta}{1+\delta}$, player t will chose the saving ratio $0 \leq \frac{s_t}{w_t} < \frac{\delta}{1+\delta}$ if and only if*

$$\frac{mg}{R} > \Phi(\delta, \gamma/m) \tag{14}$$

$$\textit{with } \Phi(\delta, 0) = \delta, \;\; \Phi_1'(\delta, \gamma/m) > 0, \; \Phi_2'(\delta, \gamma/m) < 0 \tag{15}$$

[10] It is implicitly assumed that siblings solve exactly the same maximization program and that they cooperate in the parent gift provision.

If this condition holds, private ascending transfers are strictly positive and the economy converges towards an equilibrium such that

$$\frac{s}{w} = 0; \text{G} = \frac{\gamma/m}{1+\gamma/m}$$

Proof. see technical appendix. ■

Note that

$$\Phi(\delta, \gamma/m) = \frac{\delta}{1+\gamma/m} \cdot \frac{\delta + \sqrt{(2+\delta)^2 + 4(1+\delta)\gamma/m}}{2(1+\delta)}$$

From the characteristics of Φ displayed in (15), we may conclude that condition (14) is not difficult to satisfy even for $mg < R$. From the moment (14) is respected, the distribution of consumption over the life-cycle is suboptimal. Indeed, deriving from program (10) the optimal stationary gift rate within an economy with zero saving, we obtain

$$\text{G} = \frac{\delta + \gamma/m}{1+\delta+\gamma/m} > \frac{\gamma/m}{1+\gamma/m} \tag{16}$$

From this inequality it appears that in the equilibrium with positive private transfers, agents overconsume in first period of life in order to attract future positive gifts. Every generation $t+i$ (with $i \in \mathbb{N}$) behaves so that $s_{t+i+1} = 0$. However, if the saving of generation $t+i-1$ is also zero, they must entirely rely on gifts for their second period of life consumption. The only generation to benefit from the situation is generation t, the ultimate Stackelberg leader, who takes profit first of the positive saving of his parent, second of his own saving and third of the private transfer from his children. All the ensuing generations are trapped in a suboptimal equilibrium and would like to transfer resources from their first to their second period of life. At the individual level, the only way to operate this transfer is saving. However, the ensuing reduction in gift would make the saving agent even worse off. The other way to proceed is to increase the ascending transfer. We are then back to a commitment problem similar to this studied in sections 3 and 4 above. At the equilibrium with strictly positive ascending transfers, every generation would be better off by adding a complementary transfer $\frac{\delta+\gamma/m}{1+\delta+\gamma/m} - \frac{\gamma/m}{1+\gamma/m}$ to the private one if it were sure to receive the same in old age. Public ascending transfers may emerge then as a Pareto optimal contract between living and future generations even while the economy is dynamically efficient. Verbon (1988) enlarges this result to the case of non strategic behavior.

6 The size of the public transfers system

Let us come back to the reference setup of section 2 (small open economy without altruism) and assume $mg > R$ so that there is a case for public transfers. So far, agents were not endowed with the power to choose the size of the transfer system itself since the sequence $(\tau_t^p)_{t=0}^{\infty}$ was assumed exogenously given. Let us drop this assumption and enlarge the set of actions in order to allow the sequence $(\tau_t^p)_{t=0}^{\infty}$ to be shaped as time evolves.[11]

Let us consider the same game as in section 3 above except for the action set of player t. Let us assume that it consists of two components: first accept or refuse the proposition τ_t^p of the previous player and second, make a proposition about the contribution rate to be paid by the next player $\tau_{t+1}^p \in [0,1]$. Let us pick some $\hat{\tau} \in [0,\bar{\tau}[$ and consider the particular strategy $\hat{\sigma}_t$ inspired from Sjoblom (1985):

- player t accepts τ_t^p if and only if $\tau_t^p \leq \check{\tau}$ such that $V_{t-1}(\tau_{t-1},\check{\tau}) = V_{t-1}(\hat{\tau},\hat{\tau})$
- player t proposes τ_{t+1}^p such that $V_t\left(\tau_t,\tau_{t+1}^p\right) = V_t(\hat{\tau},\hat{\tau})$

If player t believes all players $t+k$ ($k \in I\!N$) will adopt strategy $\hat{\sigma}$, strategy $\hat{\sigma}$ will be a dominant strategy for him. Under strategy $\hat{\sigma}$, the transfer system gives each player some preselected utility level $V(\hat{\tau},\hat{\tau})$ and the transfer rate converges from below to $\hat{\tau} \in [0,\bar{\tau}[$ which is reached within finite time.

Through this example, Sjoblom simply shows that the (stationary) size of the transfer system may virtually be any $\hat{\tau} \in [0,\bar{\tau}[$. This outcome may not be very helpful to answer the question: "What is the democratically chosen size of the public transfer system?" Nevertheless, it can be interesting to hold it in mind in order to counterbalance some of the results obtained below.

Since the seminal paper of Browning (1975), most of the models dealing with the question of the size of ascending public transfers are restricted to a stationary transfer rate. Under this assumption, the size of the transfer system is identified with the stationary transfer rate $\hat{\tau}$ and one examines the choice of agents alive in period t when asked to choose the $\hat{\tau}$ which maximizes their life-time utility.

[11] This is actually the way Boldrin and Rustichini (2000) play their generation game.

6.1 Voting for a stationary transfer rate $\hat{\tau}$

Let us suppose a referendum is organized at period t to fix the size of the transfer system, $\hat{\tau}$. Of course, agents in their second period of life in period t prefer

$$\hat{\tau}_2 = \arg\max_{\hat{\tau}}(Rs_{t-1} + m\hat{\tau}w_t) = 1$$

If $\tau_t = \tau_{t+1}$ the return of the transfer system is exactly equal to mg and if $mg > R$, young agents prefer to "save" through the transfer device rather than via the storage technology. Denoting $\left\{c_t^{1**}, c_{t+1}^{2**}\right\}$ the outcome of

$$Max_{c_t^1, c_{t+1}^2} U\left(c_t^1, c_{t+1}^2\right) \quad \text{s.t. } w_t = c_t^1 + \frac{c_{t+1}^2}{mg}$$

we may establish that agents in their first period of life in period t prefer

$$\hat{\tau}_1 = \arg\max_{\hat{\tau}} V\left(\hat{\tau}, \hat{\tau}\right) = \frac{w - c_t^{1**}}{w} < \bar{\tau} \tag{17}$$

If the population increases ($m > 1$), the median voter is young and active and the Condorcet winner is $\hat{\tau}^* = \hat{\tau}_1$. Under assumption **H6**, we may furthermore state that

$$\hat{\tau}^* = \arg\max_{\hat{\tau}} \sum_{i=0}^{\infty} V_{t+i}\left(\hat{\tau}, \hat{\tau}\right) \tag{18}$$

Remark 1 *The optimal outcome of the majority vote proceeds from three combined elements: (i) fixed factors prices, (ii) the two overlapping generation setting and (iii) the assumption of homogeneous abilities.*

Because of these three elements, the median voter is young, represents perfectly the preference of the young generation and in the absence of general equilibrium effects, the maximization of his life-cycle utility leads to the maximization of the life-cycle utility of all the ensuing generations. Let us now consider three generalizations: (i) if the prices of the production factors are endogenously decided, decentralized decision concerning saving and public transfer by the young generation are not necessarily optimal, (ii) if there are more than two generations coexisting at each period, pensioners obtain the political support of the workers close to retirement which distort the outcome of the majority vote towards larger ascending transfers and (iii) in the case of heterogeneous abilities the vote of the young workers is not homogeneous anymore and again it gives more strenght to the retired agents' viewpoint. This is what we intend to display throughout the next three subsections.

6.2 A quick incursion in the endogenous factor price setting

Let us come back to the loglinear utility - Cobb Douglas production function example developed in subsection 5.1 above. In this example, $\hat{\tau}_1$, the stationary transfer rate preferred by the young generation, is independent of the capital stock[12] and thus of the R/mg ratio. We can thus say that, if the first point of Proposition 4' is verified,

$$\hat{\tau}_1 = \arg\max_{\hat{\tau}} V_t\left(\hat{\tau}, \hat{\tau}\right) \ \forall t$$

However, this is not sufficient for (18) to hold since, if the contribution rate is fixed equal to the Condorcet winner $\hat{\tau}^* = \hat{\tau}_1 \ \forall t \in I\!N$, the economy converges towards a steady path such that

$$R(\hat{\tau}^*) = \alpha \frac{1 + \delta + \frac{1-\alpha}{\alpha}\hat{\tau}^*}{\delta(1-\alpha)(1-\hat{\tau}^*)} > \alpha \frac{1+\delta}{\delta(1-\alpha)} = R(0)$$

and there is no a priori reason for this steady path to be closer to the golden path (characterized by $R = mg$) than the no-transfer steady path. This will in particular never be true if $R(0) > mg$.

6.3 Discussion of the n OLG setting

6.3.1 No storage technology

If we are back to the fixed factor price setting, the optimal outcome of the democratic decision concerning $\hat{\tau}$ may also disappear if we increase the number of contemporaneous generations from 2 to n (with $n > 2$ and possibly infinite if we consider a model with continuous time). Let us keep the discrete time modelling, so that agents live now for n periods (with n, an integer). Agents of age $i \geq 1$ are active on the labor market if $i < p$ and retired if $i \geq p$ (with $p < n$). In this economy the median age voter belongs to generation i_{med} with i_{med} the smallest i such that $\sum_{j=1}^{i} m^{i-j} \geq \frac{1}{2}\sum_{j=1}^{n} m^{n-j}$.

Proposition 6 (Browning, 1975) *In a n overlapping generation small open economy, if there is no technology to transfer resources from $i < p$ to $i \geq p$ periods of life but a PAYG pension scheme,*

$$\frac{\partial \hat{\tau}_i}{\partial i} > 0 \ \ \forall i \in [1, p[\quad \textit{and} \quad \hat{\tau}_i = 1 \ \ \forall i \in [p, n] \qquad (19)$$

[12] This is true for the same reason that, in this example, the private savings to wage ratio s/w is independent of capital stock.

The reason for the inequality displayed in (19) is evident: the closer an agent is to the age of pension, the less he internalizes the cost of a higher $\hat{\tau}$. From (19) we infer that the outcome of a majority voting over the size of the intergenerational public transfer system is dictated by the median age voter at $\hat{\tau}_{i_{med}}$. For parameters m and n such that $i_{med} > 1$, the median age voter is not at the very beginning of his life-cycle anymore and consumption will not be spread optimally over the life-cycle.

The usual interpretation of this outcome is that elderly people derive political power from the irreversible passing of time and the absence of political representation of unborn generations. The uncomfortable conclusion to be drawn from the Browning's analysis is that early retirement policies and increased longevity, by respectively lowering p and increasing n, contribute to reinforce the bias towards large pensions and suboptimal distribution of consumption over the life-cycle.

6.3.2 Storage technology and intertemporal inconsistency

When voting for a stationary transfer rate $\hat{\tau}$, agents believe the outcome of the vote in period t will drive participation to and benefits from the pension system until they die. This belief is rational

- either if the referendum over the size of the intergenerational transfers system is organized sufficiently infrequently (or equivalently with an "entrenched interests" interpretation)
- or if voters in t forecast that the Condorcet winner of a majority voting election organized in period $t+1$ is the same as the Condorcet winner in t.

Noteworthy, in the 2 OLG economy *with* storage technology as well as in the n OLG economy *without* storage technology, if majority voting is organized at each period, the Condorcet winner will actually remain unchanged.[13] In other words, if a majority voting procedure is organized at each period, in the above mentioned settings, no intertemporal inconsistency occurs.

[13] This remark is also valid in the 2 OLG closed economy as long as $\hat{\tau}_1 = \arg\max_{\hat{\tau}} V_t(\hat{\tau}, \hat{\tau})$ is time independent (which in particular is the case in the loglinear utility - Cobb Douglas production function example).

Boadway and Wildasin (1989a) show that this property does generally not extend to the n OLG economy *with* storage technology.[14] Because of the possibility to smooth consumption across active periods of the life-cycle, agents are not only heterogeneous with respect to age anymore; they also differ with respect to the size of the amount they saved in the storage technology. For a given generation i, we denote s_t^i the amount of deposits in the storage technology at period t, and $\hat{\tau}_0$ the current transfer tax. Assuming additive separability of the life-time utility function, if agent i believes $\hat{\tau}_0$ will hold forever, he faces the following problem

$$Max \sum_{j=i}^{n} \delta^{j-i} u(c_j) \tag{20}$$

$$\text{s.t.} \quad s_j^i = \begin{cases} w_j(1-\hat{\tau}_0) + Rs_{j-1}^i - c_j \text{ if } j < p \\ w_j \hat{\tau}_0 \sum_{h=1}^{p-1} m^{n-h} + Rs_{j-1}^i - c_j \text{ if } j \geq p \end{cases}$$

$$s_j^i \geq 0 \quad \forall i \leq j \leq n$$

and s_j^i is decreasing in $\hat{\tau}_0$ for agents younger than p. If at period t a vote is organized to replace $\hat{\tau}_0$ by the Condorcet winner of the election, preference of an agent of age $i < p$ with respect to the contribution rate is single-peaked and the peak is obtained for some $\hat{\tau}_i(s_t^i(\hat{\tau}_0))$ with

$$\frac{\partial \hat{\tau}_i(s_t^i(\hat{\tau}_0))}{\partial i} > 0 \tag{21}$$

$$\text{and } \frac{\partial \hat{\tau}_i(s_t^i(\hat{\tau}_0))}{\partial s_t^i(\hat{\tau}_0)} > 0 \tag{22}$$

The reason for the sign of inequality (21) is simply that, the closer an agent is to the age of pension, the more an increase in $\hat{\tau}$ enhances his life-cycle budget. However, for $i < p$, if $\hat{\tau}$ is too large, the agent may be trapped in the borrowing constraint from some period $i + h$ on.[15] For a liquidity constrained agent, there exists a trade-off between the increase of the life-cycle budget and the cost of the constraint. However, the larger s_t^i, the larger

[14] By simply considering a storage technology (i.e. non negative saving constraint) we limit ourselves to the "earnings and pension benefits constraints" case studied by Boadway and Wildasin (1989a). These authors actually extend their analysis to the "pensions benefits constraint" case.

[15] It is important to note that saying at period t that an agent i is trapped in the borrowing constraint does not imply that $s_t^i = 0$. It simply means that, there exists a period $t \leq z < n$such that $s_z^i = 0$.

the $\hat{\tau}$ for which the constraint becomes binding, which explains inequality (22).

Remark 2 *In a n OLG small open economy, if $mg \geq R$, the wealth effect from an increase in $\hat{\tau}$ is positive for every generation i. If $mg < R$, there exists some age $\hat{\imath}$ such that, for every $i > \hat{\imath}$, this effect is positive. By opposition to the 2 OLG setting, it can be the case that $\hat{\imath} < p$ with the consequence that $mg > R$ is not a necessary condition for the median age voter to opt for public transfers.*

We observe from (21) and (22) that savings influence an agent's vote, which in turn will influence his saving behavior. As a consequence, the Condorcet winner at period t (let us denote it $\hat{\tau}_t^*$) will probably be defeated in $t+l$ if an election is (unexpectedly) held l periods later. Let us denote τ_∞ the unique stationary transfer rate such that, if it is the Condorcet winner of an election, it will remain the Condorcet winner forever. The question of interest is wether convergence towards τ_∞ will occur or not. Two cases must be distinguished:

c1 if $\tau_t = \tau_\infty \forall t$, the median voter
is liquidity constrained and $s^{i_{med}} = 0$

c2 if $\tau_t = \tau_\infty \forall t$, the median voter
is liquidity constrained and $s^{i_{med}} > 0$

Boadway and Wildasin (1989a) assume that a vote on $\hat{\tau}$ is held at each period but that agents do not anticipate it and vote as if the outcome of the vote should remain unchanged forever. The next proposition summarizes the main results obtained by the authors under this extremely strong assumption:

Proposition 7 (Boadway and Wildasin, 1989a) *In a n OLG fixed prices economy, given parameters m, g and $R \in I\!R_+$, p and $n \in I\!N$ with $p < n$, considering an additively separable life-cycle utility with strictly concave instantaneous utility, if **c1** holds, for $\hat{\tau}_0 = 0$, $\hat{\tau}_t^*$ jumps to τ_∞ within at most one period of overshooting. If **c2** holds, for a given initial $\hat{\tau}_0 \in [0, \tau_\infty[$, assuming $\hat{\tau}_t^* = (\hat{\tau}_{i_{med}})_t$ $\forall t$, the Condorcet winner will first overshoot then undershoot and cycle around τ_∞. If $\left|\frac{\partial \hat{\tau}_{i_{med}}}{\partial \hat{\tau}_0}\right| < 1$ holds uniformly, the sequence of Condorcet winner converges towards τ_∞. Noteworthy, while $\hat{\tau}_{i_{med}}$ is for sure the Condorcet winner in the overshooting sequences of the cycle,* *there is no a priori reason for $\hat{\tau}_{i_{med}}$ to be also the Condorcet winner in the undershooting sequences of the cycle.*

Proposition 7 displays the limits of the assumption that agents vote as if the outcome of the vote should hold forever. Indeed, it is only if **c1** is verified that the transfer system reaches its equilibrium size within a finite and perfectly determined number of periods (two more precisely). If **c2** is fulfilled, convergence towards τ_∞ is far from being guaranteed. Of course, as in the economy without storage technology, the equilibrium size τ_∞ is too large with respect to the preference of generation of age 1.

6.4 Heterogeneous abilities

So far we considered that agents of the same age were endowed with the same human capital and consequently the same labor income. Under this assumption, there is no case for *intra*generational transfers. However, it is well known that in most countries having adopted a pay-as-you-go pension system, the latter does not only transfer resources across generations but also within generations. In order to take this feature into account, Tabellini (1991, 2000) introduced explicitly heterogeneity of the agents along a second dimension, viz. abilities. In this new setting, two parameters are required to describe the transfer scheme: the level of the contribution rate and the redistributiveness of the benefit rule. The benefit rule may be either Bismarckian, in which case it is purely contributory and not redistributive at all, or Beveridgean, which means that it pays the same benefit to every pensioner, independently of the contribution they paid when active on the labor market. Empirically, one observes that most countries have adopted an intermediate degree of redistribution (cf. Johnson (1998)).

Following Casamatta *et al.* (2000) we denote by $0 \leq \beta \leq 1$ the Bismarckian parameter which fixes the degree of intragenerational redistribution.[16] When $\beta = 1$ (resp. $\beta = 0$), the system is purely Bismarckian (resp. Beveridgean). We represent by e_i^y the ability characterizing young agent i, with e^y a continuous variable distributed over the support $\left[e_-^y, e_+^y\right]$ and characterized by $\bar{e}^y$ and $e_{med.}^y$, respectively the mean and the median of the distribution. We also define e^o, the ability characterizing an old agent,

[16] The implicit assumption is that parameter β is chosen at the constitutional stage and once decided is not renegotiated. For an analysis of the choice of β, see Casamatta (1999).

with $e_i^o = 0 \ \ \forall i$. Let us finally denote e^P a variable with density function $f(e^P) = f(e^y) + f(e^o)$ and median $e^P_{med.} < e^y_{med.}$.

In order to ease the following reasoning, let us complete the list of assumptions **H1** through **H6** posed on the life-cycle utility function $U(c_t^1, c_{t+1}^2)$ with

H7: additive separability of the life-cycle utility function

H8: constant elasticity of substitution σ

For a given size of the transfer system $\hat{\tau}$, the problem faced by an agent i young in period t may be written as

$$\text{Max}_{c^1_{i,t}, c^2_{i,t+1}} \ u(c^1_{i,t}) + \delta u(c^2_{i,t+1}) \tag{23}$$

$$\text{s.t.} \begin{cases} w_t \cdot e_i^y \cdot (1-\hat{\tau}) = c^1_{i,t} + s_{i,t} \\ R s_{i,t} + m\hat{\tau} w_{t+1} e_i^y \cdot \Theta(e_i^y) = c^2_{i,t+1} \\ s_{i,t} \geq 0 \end{cases}$$

where

$$\Theta(e_i^y) = \beta + (1-\beta)\frac{\bar{e}^y}{e_i^y}$$

and with $\delta \in]0,1[$, the subjective discount factor. For an agent of ability e_i^y, the return of an asset invested in the storage technology is R while in the public transfer system, the return is $mg\Theta(e_i^y)$ which is decreasing in e_i^y. In particular,

$$mg\Theta(e_i^y) \geq R \Leftrightarrow e_i^y \leq \frac{\bar{e}^y(1-\beta)}{(R/mg) - \beta} \tag{24}$$

We may conclude that in the case of a vote on the size of the transfer system, voters will be divided in three groups:

- pensioners who pick up $\hat{\tau}_2 = 1$
- young agents endowed with $e_i^y < \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}$, who vote for $\hat{\tau}^{\inf}_{1,i} \in]0,1[$
- young agents endowed with $e_i^y > \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}$, who choose $\hat{\tau}^{\sup}_1 = 0$

This statement illustrates an essential feature of this kind of model: the old generation takes advantage of the heterogeneity of the young generation. In this setting, retired agents' political power is enhanced with respect to the homogeneous abilities setting where their power mainly came from the natural asymmetry that every young agent will become old and not the other way round.

Remark 3 *A particular consequence of the introduction of heterogeneous abilities is that, in the 2 OLG small open economy, dynamic inefficiency is no longer a necessary condition for young voters to prefer $\hat{\tau} > 0$.*

It is obvious from the discussion above that the focus will be on the group of young voters endowed with an ability $e_i^y \in \left[e_-^y; \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}\right]$. As said above, these voters strictly prefer to "save" through the public transfer system rather than in the storage technology, which amounts to decrease the relative price of second period of life consumption from $\frac{1}{R}$ to $\frac{1}{mg\Theta(e_i^y)}$. Their preferred "saving rate" in the public transfer system is

$$\hat{\tau}_{1,i}^{\inf} = \arg\max_{\hat{\tau}} U\left(w_t e_i^y(1-\hat{\tau}), m\hat{\tau}w_{t+1}e_i^y \cdot \Theta(e_i^y)\right)$$

In an economy where the government fixes unilaterally $\hat{\tau} = 0$, agents choose the ratio $\frac{s^*}{w}$ which maximizes $V(0,0)$. Under assumptions **H7** and **H8**, it is well known that the inequalities presented in the left column of table 1 hold true. Translated in our story where agents prefer $s = 0$ and choose $\hat{\tau}_{1,i}^{\inf}$ which maximizes $V(\hat{\tau}, \hat{\tau})$, private savings become PAYG savings with a return factor $mg\Theta(e_i^y)$ and the the right column of table 1 displays the inequalities corresponding to those in the left one.

Table 1: savings, return factors and the elasticity of substitution

$\frac{\partial s^*/w}{\partial R} \begin{cases} > 0 \text{ if } \sigma > 1 \\ = 0 \text{ if } \sigma = 1 \\ < 0 \text{ if } \sigma < 1 \end{cases}$	$\frac{\partial \hat{\tau}_{1i}^{\inf}}{\partial e_i^y} \begin{cases} < 0 \text{ if } \sigma > 1 \text{ (substituability)} \\ = 0 \text{ if } \sigma = 1 \\ > 0 \text{ if } \sigma < 1 \text{ (complementarity)} \end{cases}$

From this table we conclude that, if $\sigma \geq 1$, the Condorcet winner is strictly positive and smaller than unity if and only if $e_{med.}^P \in \left[e_-^y; \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}\right]$. In this case, retirees and (poor) young agents endowed with $e_i^y \in \left[e_-^y; e_{med.}^P\right[$ strictly prefer a contribution rate larger than the Condorcet winner while (richer) young agents endowed with $e_j^y \in \left]e_{med.}^P; \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}\right]$ and $e_k^y \in \left]\frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}; e_+^y\right]$ oppose prefer a movement in the other direction. This kind of result is quite common in the literature.[17] Figure 5 gives an illustration of this case.

If $\sigma < 1$, coalitions at work change and the result of the vote is no longer dictated by the young agent endowed with $e_{med.}^P$ but by a young voter characterized by an ability of $\hat{e}$ where $\hat{e}$ is such that pensioners and (middle class) young agents endowed with $e_i^y \in \left[\hat{e}; \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}\right]$ represent a half of the living population. More formally, $\hat{e}$ is such that

$$\int_{\hat{e}}^{\frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}} f(e^y)de^y = \frac{m-1}{2m}$$

[17] See for example Mahieu and Rottier (1999) and Tabellini (2000).

Of course, if $\int_{\frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}}^{e_+^y} f(e^y)de^y > \frac{m-1}{2m}$, such a $\hat{e}$ does not exists and the winning coalition opposes public transfers. In other words, workers at the ends of the e^y distribution vote against workers in the middle of this distribution, which remind us of the equilibrium of Epple and Romano (1996). This is illustrated by figure 6.

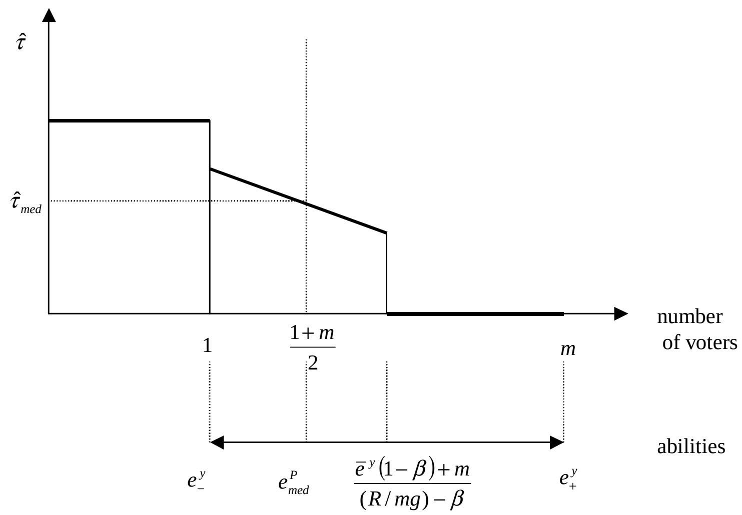

Figure 5: Illustration of the $\sigma > 1$ case.

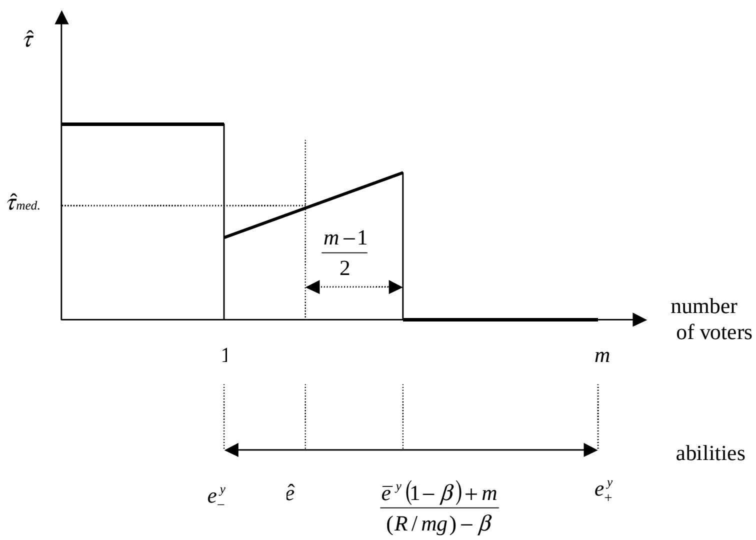

Figure 6: Illustration of the $\sigma < 1$ case.

Observing that, for $mg < R$

$$\frac{\partial \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}}{\partial \beta} < 0 \text{ and } \frac{\partial \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}}{\partial mg} < 0 \tag{25}$$

and

$$\begin{cases} \frac{\partial \hat{\tau}^{\inf}_{1,i}}{\partial \beta} > 0 & \text{if } \sigma < 1 \\ \frac{\partial \hat{\tau}^{\inf}_{1,i}}{\partial \beta} \leq 0 & \text{if } \sigma \geq 1 \end{cases} \quad \text{and} \quad \begin{cases} \frac{\partial \hat{\tau}^{\inf}_{1,i}}{\partial mg} < 0 & \text{if } \sigma < 1 \\ \frac{\partial \hat{\tau}^{\inf}_{1,i}}{\partial mg} \geq 0 & \text{if } \sigma \geq 1 \end{cases} \tag{26}$$

the following proposition summarizes this discussion:

Proposition 8 (Casamatta et al., 2000) *Let us consider a small open two overlapping generation economy, given parameters m, g, $R \in I\!R_+$ such that $mg < R$. Let us also consider that agents' life-cycle utility function $U(c_t^1, c_{t+1}^2)$ satisfies H1 through H8. Then,*

- *if $\sigma \geq 1$, the median voter is a young worker endowed with an ability $e^P_{med.}$, whatever the value of β. However, the ability of the median voter $e^P_{med.}$ is increasing in mg. For $\sigma > 1$ (resp. $= 1$), the Condorcet winner is continuously decreasing in β and may be either increasing or decreasing in mg (resp. independent of β and mg) as long as inequality $\frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta} > e^P_{med.}$ holds.*

- *if $\sigma < 1$, the ability of the median voter $\hat{e}$ is decreasing in β and can be either decreasing or increasing in mg while the Condorcet winner may be either increasing or decreasing in β and in mg.*

Proof. see Casamatta et al. (2000). The proof is directly deduced from Table 1 as well as from (25) and (26). ■

It is interesting to note that, contrarily to the conclusion of subsection 6.3, and in opposition to Tabellini (2000),[18] it may be the case that a reduction of the economy growth rate induces a decrease of the size of the

[18] Note that Tabellini (2000) does not impose the vested rights assumption and replaces it by bilateral weak altruism. Altruism is weak in the sense that private transfers do not occur in any direction. In the absence of vested rights, the rule linking benefits to contributions is broken. However, young voters endowed with low values of e^y favor a relatively high contribution to the unfunded social security system since the costs they

transfer system.[19] This conclusion may hold for an elasticity of substitution larger or smaller than unity. This outcome reconciles our political economy approach with the observation that, faced with the demography crisis, many governments are implementing reforms of the pension system that are less favourable to the elders.

Johnson (1998) displays some evidence that there is a negative correlation between the intragenerational redistributiveness of unfunded pension systems and the size of the intergenerational transfer as a percentage of GDP (cf. table 1 of Casammatta et al. (2000)). Within this modelling, this stylized fact is met for sure if $\sigma \geq 1$, while under some circumstances, it may be verified for $\sigma < 1$.

Empirical evidence concerning the relationship between the size τ of the unfunded pension system and the labor income inequality is rather inconclusive. Lindert (1996) and Perotti (1996) find no effect. Tabellini (2000) presents cross countries evidence that PAYG pension schemes are more generous the larger a Gini index of inequality. However Persson and Tabellini (2000) admits that such inequality measure is probably not consistent across countries, casting doubts to the positive correlation. The conclusions we can derive from the modelling exercise displayed in this subsection are also somewhat mixed. Suppose there is a large proportion of young agents endowed with $e_i < \frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}$. In particular, if $\sigma > 1$ (resp. $\sigma < 1$), this model predicts that τ will be larger if within this subpart of the distribution, the density function $f(e^y)$ is larger for values of e^y close to $\underline{e}^y_-$ (resp. close to $\frac{\bar{e}^y(1-\beta)}{(R/mg)-\beta}$).

In this section, the redistributive role of pension *within* generations is essential to explain the political sustainability of this intergenerational transfer scheme. One may wonder why, in most existing economies, both types of redistribution are implemented through one policy instrument instead of two distinct ones. Conde Ruiz and Galasso (1999) address this question by adding to the economy described so far a second welfare instrument which redistributes income among workers. Agents are asked to vote on both types of transfers. Old voters are indifferent with respect to redistri-

incur is lower than the benefits received by their parents. Note also that in this setting, because of their altruistic feeling towards their children, old voters are not unilaterally in favor of a large social security: their attitude will depend on their children labor revenue. However, they will always favor a larger unfunded pension than their direct descendants.

[19] It is interesting to note that Meijdam and Verbon (1996) obtain a similar conclusion, although in a very different setting.

bution among workers,[20] while poorly endowed workers are in favour of it and richly endowed workers oppose it. In order to avoid Condorcet cycles which usually occur in two-dimensions voting process, the authors adopt a structure-induced equilibrium à la Shepsle (1979). In equilibrium, the two types of instrument are used. Everything happens as if the old generation buys the votes of the poorly endowed workers on the "pension" dimension by giving them political support to obtain labor income redistribution among workers.[21] Casamatta (2000) obtains a very similar result in an economy where the old generation derives political power both from its unity of opinion regarding the pension question and the heterogeneity of workers with respect to both dimensions of the vote.

7 Alternative modelling of the political process and other extensions

As announced, this section is intended to give some basic informations on various points that have not been reviewed in the previous pages. In particular, so far, we focused on the majority voting political process while the literature has considered other types of public decision taking. We also ignored the uncertainty dimension of the problem. Finally, we have not considered the consequences of introducing descending altruism within the picture. Let us quickly lift a corner of the veil covering these topics.

Alternative political processes: Instead of just considering majority voting, one may also consider that the minority is given some extra political weight through *veto power*. In an economy similar to our reference setup (section 2 above), Azariadis and Galasso (2002) prove that adding veto power reduces the set of political equilibrium transfer sequences in a voting game of the kind studied by Sjoblöm (1985, see section 6 supra). Cyclical sequences as well as dynamic inefficient transfer sequences are shown to be excluded from this reduced set. In an economy with weak bilateral altruism (i.e. with no operative private transfers), Hansson and Stuart (1989) show that veto power is essential to obtain Pareto efficient public transfers (larger

[20] This is of course only true only if taxes are not distortionary. Conde Ruiz and Galasso (1999) considers distortionary taxation and old agents oppose redistribution of income among the young agents.

[21] In the presence of distortionary income taxation, pensioners face a trade off since income redistribution among workers is costly to them.

than the private gift considered in sub-section 5.2 supra).

Leaving the majority voting political setting, some authors (e.g. Verbon (1988), Verbon and Verhoeven (1991,1992) or Arthus and Legros (1997)) consider a government able to impose policies to citizens, which may be influenced by different *pressure groups*. Such a government objective function could then be written as

$$W_t = \phi \cdot U_t^y(c_t^1, c_{t+1}^2) + (1-\phi) \cdot U^o(c_t^2) \tag{27}$$

where U_t^y (resp. U_t^o) represents the utility of an agent young (resp. old) in t . Parameter ϕ represents the political weight of the young generation. For simplicity this parameter is assumed to be time invariant. Assuming loglinear utilities, the maximization program of the government is actually similar to (10) if the "private gift rate" is renamed "pension tax rate".[22] Then following Verbon (1988), young agents may behave strategically on behalf of their future political influence when old. Verbon and Verhoeven (1992) consider that both the young and the government play Nash when choosing respectively the saving and the pension tax rate and conclude that a unique equilibrium emerges with a PAYG pension scheme and zero savings if the political weight of the elders is sufficiently large.

Grossman and Helpman (1998) extend the analysis by considering short-lived governments that cannot precommit to future pension transfers. They focus on stationary equilibria so that each successive government chooses at equilibrium the same tax function. In this setting, the outcome of the game depends mainly of the political weight parameter ϕ. In a second step, Grossman and Helpman introduce the possibility for individuals of both groups to form lobbies. The political support function (27) is then modified so that governments also value the pressure groups contributions. Lobbies seek then to influence political decisions about intergenerational transfers by offering policy contingent campaign contributions. It is shown that the old group, when the only one to lobby, will expropriate totally the young group. The same does not hold for young agents since they anticipate that such an extreme policy will be detrimental to them in the next period. When both groups compete, the government takes advantage of the political competition and captures all the revenues.

[22] Replacing the parameter γ/m in (10) by $\frac{1-\phi}{\phi}$, there is no reason to impose $\delta > \frac{1-\phi}{\phi}$. In the private framework, condition $\delta > \gamma/m$ was intended to reflect a "charity begins at home" principle.

A closely related analysis has been undertaken by Mulligan and Sala-i-Martin (1999). They don't make use of a political support function, but assume that both groups compete directly through a pressure function with the level of political activities chosen as arguments.[23] This paper considers that political activity is time-intensive and requires leisure time.[24] As usual in collective action, individuals are willing to benefit from the group activity without contributing themselves. In order to prevent free-riding, the group promotes a distortionary policy inducing individual behavior to match with the group interest. The distortionary policy in the present case is aimed at reducing individual labor supply (or increasing leisure and consequently political activity) via labour income taxes. Since elderly people are assumed to be endowed with a lower labor productivity, it is easier for the old group to induce its members to retire and spend all their time in political activity. This argument adds to the classical "young will get old" one to explain the old group success in obtaining transfers from the young. It furthermore offers an explanation of why pensioners are inactive instead of taking it as a prerequisite as other models do.

Uncertainty is absent in most of the contributions to the topic. For Hu (1982), young agents consider their future pension benefits as uncertain since they will be decided by the median age voter when they are retired. This influences their private saving behavior. Boldrin and Rustichini (2000) introduce uncertainty on the demographic growth factor, which is a major determinant of the pension return. They show that this element of risk may soften the participation constraint (3).

Descending altruism à la Ramsey is the form of altruism most familiar to economists. In such models, it is usually assumed that negative bequest are prohibited. Cuckierman and Meltzer (1989) wrote a pioneering paper on the topic and showed that in the fixed factor prices setting, it is the most liquidity constrained dynasty that decides the size of the ascending transfer. This is because for this size of transfer, this constraint is loose for all the other dynasties and Ricardian equivalence holds. When selfish intermissions

[23] What matters in this kind of competition is simply the efforts each group invests in political activity. The pressure function is symmetrical in both arguments so that the winner is simply the more politically active group. The government displays absolutely no preference for one group or the other.

[24] Becker and Mulligan (1998) present a similar analysis where political activity is measured in cash, i.e. campaign contributions, etc.

occur within dynasties (cf. de Walque and Gevers, 2001), agents' preference are single-peaked with respect to public ascending transfer policy. This leads to a unique stable political equilibrium.

8 Conclusion

The traditional normative analysis of intergenerational public transfers states that such transfers are justified if and only if they are Pareto improving. The positive approach in the simplified reference setup presented in section 2 leads to a somewhat similar result, since, within this scope, ascending public transfers are only politically sustainable if they are Pareto improving. Within the narrow setup of section 2, this requires that the economy growth rate is larger than the savings return (dynamic inefficiency).

However, allowing for ascending altruism, Veall (1986) has shown that there is a case for Pareto improving pension programmes, even if the economy meets the efficiency requirement. Switching from a small open economy to an economy with endogenous prices for the production factors, Hu (1982), Cooley and Soares (1999) and Boldrin and Rustichini (2000) have shown that there are politically sustainable transfers that are not Pareto improving. This may explain why a pay-as-you-go pension system survives even though it is not beneficial to every generation. The present generation might even suffer, should it no longer apply, while it would have been better off had the pension system never been implemented.

The political power of the pensioners is also often invoked to explain the large transfer systems observed in developed countries. This political power derives from the fact that every worker is a future beneficiary of the pension system. The closer he is to the retirement age, the more he supports generous pension transfers. Along those lines, pre-retirement policies and increased longevity are two elements favouring an extension of the pay-as-you-go system. Browning (1975) has developed this in a simple setting without storage technology but Boadway and Wildasin (1989a) have shown how unwieldy this analysis becomes as soon as this is introduced. In particular, the conditions for intertemporal consistency to be met are rather strict.

If heterogeneous productive skills are introduced, retired people benefit from divisions among the workers. Because most pension systems redistribute resources both across and within generations, workers are divided as to the system size they prefer. In this case, even while the median voter is still part of the younger generation, the political weight of the pensioners forces the Condorcet winner to be larger than if they were prevented from voting. It is noteworthy that, contrary to the observation of Browning (1975), this type of model predicts that a permanent fall of the economy growth rate may induce a reduction of the size of the transfer system.

This discussion shows that the basic framework used and the variations considered already allowed a better understanding of the questions as to the existence and the size of pension schemes. However, the results obtained in the variants considered do not necessarily go in the same direction. In order to increase the predictive power of this positive approach a lot of work has still to be done in order to build a more complex model meeting reality more closely. We should then abandon the analytical approach and carry out computed simulations.

Nevertheless, when conducting a simple static comparative exercise, most of the models presented above come to the same basic conclusion: in the event of a sharp drop of the economic growth rate (i.e. the combination of demographic and technological growth rates), there will still be broad political support for unfunded pension systems, which may prove to be even broader. This observation contrasts sharply with the normative view holding that this fall of economic growth should result in a reduction of ascending intergenerational transfer programmes.

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9 Technical appendix

Proof of Proposition 2

A strategy sequence $(\sigma_t)_{t=0}^{\infty}$ is a subgame equilibrium if and only if $\forall t$

$$V_t\left(\sigma_t(h_t)\cdot\tau_t^p, \sigma_{t+1}(h_t, \sigma_t(h_t))\cdot\tau_{t+1}^p\right) > V_t\left(\bar{\sigma}_t(h_t)\cdot\tau_t^p, \sigma_{t+1}(h_t, \bar{\sigma}_t(h_t))\cdot\tau_{t+1}^p\right) \tag{A.1}$$

where

$$\bar{\sigma}_t = \begin{cases} 1 & \text{if} \quad \sigma_t = 0 \\ 0 & \text{if} \quad \sigma_t = 1 \end{cases}$$

If the participation constraint (3) is not respected, $\sigma = 1$ is a dominated strategy. Let us then consider the case where inequality (3) is verified from some period on and let us arbitrarily index this period $t = 0$. Then, for every $t \in I\!N$

$$V_t(0, \tau_{t+1}^p) > V_t(\tau_t^p, \tau_{t+1}^p) > V_t(0,0) > V_t(\tau_t^p, 0) \tag{A.2}$$

is always true. We may then verify that inequality (A.1) holds if and only if

- if $\sigma_t(h_t) = 0$, then either $\sigma_{t+1}(h_t, 0) = 1$ or $\sigma_{t+1}(h_t, 1) = 0$
- if $\sigma_t(h_t) = 1$, then $\sigma_{t+1}(h_t, 0) = 0$ and $\sigma_{t+1}(h_t, 1) = 1$

Q.E.D.

Proof of Proposition 4

Under assumptions **H2** and **H3**, for $\tau_t = \tau_{t+1} = 0$, the amount stored for old days consumption will always be positive. We denote $\left\{c_t^{1*}, c_{t+1}^{2*}\right\}$ the pairs yielding indirect utility $V_t(0, 0)$. Under assumptions **H4** and **H5**, we ensure uniqueness of this optimal pair. Indirect utility $V_t(\tau_t, \mathcal{I}(\tau_t))$ may be reached by consuming $\left\{c_t^{1*}, c_{t+1}^{2*}\right\}$ as long as the non-negative saving constraint is not binding. In this case, $\mathcal{I}(\tau_t)$ is such that

$$\mathcal{I}(\tau_t) = \tau_t \cdot \frac{R}{mg} \quad \text{for } 0 \leq \tau_t \leq \frac{w_t - c_t^{1*}}{w_t} \tag{A.3}$$

For any $\tau_t \in \left]\frac{w_t - c_t^{1*}}{w_t}, 1\right]$, the pair $\left\{c_t^{1*}, c_{t+1}^{2*}\right\}$ is out of reach of an agent born in t and the only way for him to enjoy the same life-cycle utility as yielded by the pair $\left\{c_t^{1*}, c_{t+1}^{2*}\right\}$ with $c_t^1 < c_t^{1*}$ is to increase the life-cycle budget above w_t. This requires $\mathcal{I}(\tau_t) > \tau_t \frac{R}{mg}$. From assumptions **H3**, **H4** and **H5** we made upon utility function $U(c_t^1, c_{t+1}^2)$, we may furthermore state that, for $\tau_t \in \left]\frac{w_t - c_t^{1*}}{w_t}, 1\right]$,

$$\lim_{\tau_t \to \left(\frac{w_t - c_t^{1*}}{w_t}\right)^+} \frac{\partial \mathcal{I}(\tau_t)}{\partial \tau_t} > \frac{R}{mg} \tag{A.4}$$

$$\lim_{\tau_t \to 1} \frac{\partial \mathcal{I}(\tau_t)}{\partial \tau_t} = +\infty \tag{A.5}$$

$$\frac{\partial^2 \mathcal{I}(\tau_t)}{\partial \tau_t^2} > 0 \tag{A.6}$$

Since we posed that the utility function U fulfills assumption **H6**, the ratios $\frac{c_t^{1*}}{c_{t+1}^{2*}}$ and $\frac{w_t - c_t^{1*}}{w_t}$ are time independent and the indifference curve $\mathcal{I}(\tau_t)$ will have exactly the same shape in the (τ_t, τ_{t+1}) plane for every $t \in I\!N$. Finally, if $\tau_{t+1}^p > \bar{\tau}$, it should be the case that $\tau_{t+2}^p > \mathcal{I}(\tau_{t+1}^p)$ for player $t + 2$ to participate to the transfer scheme and so on for the $t + 2 + j$ players (with $j \in I\!N$). However, this requires that within finite time τ^p becomes larger than unity. From the moment the borrowing constraint $s_t \geq 0$ is imposed this is of course inadmissible.

Q.E.D.

Proof of Proposition 4'

From the FOC of (9) we obtain[25]

$$s_t = \frac{1}{1+\delta}\left[\delta w_t(1-\tau_t) - \frac{m}{R_{t+1}}\tau_{t+1}w_{t+1}\right] \tag{A.7}$$

If capital completely depreciates in one period, then

$$k_{t+1} = \frac{s_t}{mg}$$

In a competitive economy with agents enjoying perfect foresight, we derive from (A.7) the dynamics of capital stock

$$k_{t+1} = \frac{k_t^\alpha}{mg}\left[\frac{\delta(1-\alpha)(1-\tau_t)}{1+\delta+\frac{1-\alpha}{\alpha}\tau_{t+1}}\right] \tag{A.8}$$

From expressions (9), (A.7) and (A.8) we may compute that

$$V_t(\tau_t, \tau_{t+1}) \geq V_t(0,0) \Leftrightarrow \tau_t \leq f(\tau_{t+1})$$

with

$$f(\tau_{t+1}) = 1 - \exp\left[\frac{1+\delta}{\alpha\delta-1}\log\left(1+\frac{1-\alpha}{\alpha}\tau_{t+1}\right) + \frac{1+\alpha\delta}{\alpha\delta-1}\log\left(1+\frac{1-\alpha}{\alpha(1+\delta)}\tau_{t+1}\right)\right]$$

Since $f(\tau_{t+1})$ is increasing and concave, we obtain

$$\mathcal{I}(\tau_t) =_{def.} f^{-1}(\tau_t)$$

and we observe that this expression is monotonically increasing and convex in τ_t. A (unique) fixed point $\bar{\tau}$ such that $\mathcal{I}(\bar{\tau}) = \bar{\tau}$ exists if and only if

$$\lim_{\tau_{t+1}\to 0} \frac{\partial f(\tau_{t+1})}{\partial \tau_{t+1}} > 1$$

$$\Leftrightarrow \alpha < \frac{2+(1+\delta)^2 - \sqrt{9+12\delta+2\delta^2-4\delta^3-3\delta^4}}{2\delta^2}$$

Q.E.D.

Proof of Proposition 5

[25] We do not need to specifiy a non-negative saving constraint in this case. Indeed, if $s_t = 0$, agents young in t are doomed to a zero consumption in $t+1$ since w_{t+1} would be zero. Given assumption **H3**, we can be sure that $s_t > 0$ holds.

Let us consider the case where player t has adopted a saving policy such that player $t+1$ saves nothing. The problem faced by player $t+1$ reduces then to

$$\text{Max}_{s_t,\mathrm{G}_t} \ \log c^1_{t+1} + \frac{\gamma}{m} \log c^2_{t+1}$$
$$\text{s.t.} \ \begin{cases} w_{t+1} \cdot (1 - \mathrm{G}_{t+1}) = c^1_{t+1} \\ R s_t + m \cdot \mathrm{G}_{t+1} \cdot w_{t+1} = c^2_{t+1} \\ s_t \geq 0, \ \ \mathrm{G}_t \geq 0 \end{cases}$$

From this maximization program we may compute that the following relationship between G_{t+1} and s_t holds :

$$\mathrm{G}_{t+1} = \max\left\{0; \frac{\gamma/m}{1+\gamma/m} - \frac{R}{mg(1+\gamma/m)} \frac{s_t}{w_t}\right\} \tag{A.9}$$

Of course, we are interested in the case undersaving leads to a gift $\mathrm{G}_{t+1} > 0$. Substituting (A.9) within (10), and maximizing with respect to G_t and s_t one obtains respectively :

$$\mathrm{G}_t = \max\left\{0; \frac{1}{1+\gamma/m}\left(\frac{\gamma}{m} - \frac{\gamma}{m}\frac{s_t}{w_t} - \frac{R}{mg}\frac{s_{t-1}}{w_{t-1}}\right)\right\} \tag{A.10}$$

$$\frac{s_t}{w_t} = \max\left\{0; \frac{\delta}{1+\delta}(1 - \mathrm{G}_t) - \frac{mg}{R}\frac{1}{1+\delta}\right\} \tag{A.11}$$

Combining the non-negative gift constraint $\mathrm{G}_t \geq 0$ with the borrowing constraint $s_t \geq 0$, four different cases may logically be considered :

(I) $\frac{s_t}{w_t} > 0$ and $\mathrm{G}_t > 0$
(II) $\frac{s_t}{w_t} > 0$ and $\mathrm{G}_t = 0$
(III) $\frac{s_t}{w_t} = 0$ and $\mathrm{G}_t > 0$
(IV) $\frac{s_t}{w_t} = 0$ and $\mathrm{G}_t = 0$

First of all, it may be proved that

Lemma 1: *Given R, m and $g \in I\!R_+$, δ and $\frac{\gamma}{m} \in \,]0,1[$ with $\delta > \frac{\gamma}{m}$, $\frac{mg}{R} > \frac{\delta}{1+\gamma/m}$ is a necessary condition for $s_{t+1} = 0$ when player t is the first one to play Stackelberg. If $s_t = 0$ this condition becomes necessary and sufficient.*

Proof. Whatever the saving and giving policy of player t, his objective as leader of the Stackelberg game is to yield the saving and giving behavior (*III*) at period $t+1$. From expressions (A.10) and (A.11) it may be established that $\text{G}_{t+1} > 0$ and $s_{t+1} = 0$ if and only if the following conditions are simultaneously met:

$$\begin{cases} \frac{s_t}{w_t} < \frac{\gamma}{m}\frac{mg}{R} \\ \frac{s_t}{w_t} \leq \frac{mg}{R\delta}\left[\frac{mg}{R}(1+\gamma/m) - \delta\right] \end{cases}$$

Because of the borrowing constraint $s_t \geq 0, \forall t$, conditions for case (*III*) to hold at $t+1$ cannot be simultaneously fulfilled if $\frac{\delta}{1+\gamma/m} > \frac{mg}{R}$. This proves that $\frac{mg}{R} > \frac{\delta}{1+\gamma/m}$ is a necessary condition for case (*III*) to emerge. However, if ever $s_t = 0$ both conditions for case (*III*) are fulfilled for sure so that $s_{t+1+j} = 0, \forall j \geq 0$. ■

From this Lemma, we observe that case (*III*) represents an equilibrium: from the moment the strategic behavior of a generation has lead the saving rate to fall down to zero, it will be a suboptimal strategy for any ensuing generation to save a strictly positive amount.

We may now enter the proof of Proposition 5.

Given δ, γ/m, and $\frac{mg}{R}$ and given that $\frac{s_{t-1}}{w_{t-1}} = \frac{\delta}{1+\delta}$ we want to know what are the conditions for case (*J*) (with *J*=*I*, *II*, *III*, *IV*) at generation t to lead to case (*III*) at generation $t+1$. From expressions (A.10) and (A.11), we may derive that :

- case (*I*) emerges at generation t if and only if

$$\frac{mg}{R\delta}\left[\frac{mg}{R}\left(1+\frac{\gamma}{m}\right) - \delta\right] < \frac{\delta}{1+\delta} < \frac{\gamma}{m}\frac{mg}{R}\frac{1+\frac{mg}{R}}{1+\delta}$$

  The first inequality may be rewritten as

$$\frac{mg}{R} > \frac{1}{2}\sqrt{\frac{1}{\gamma/m}}\left[\sqrt{\gamma/m+4\delta} - \sqrt{\gamma/m}\right] =_{def.} \Gamma(\delta,\gamma/m)$$

  and the second as

$$\frac{mg}{R} < \frac{\delta}{2(1+\gamma/m)}\left[1+\sqrt{\frac{\delta+5+4\gamma/m}{1+\delta}}\right] =_{def.} \text{B}(\delta,\gamma/m)$$

  Furthermore, case (*I*) at period t leads to case (*III*) at period $t+1$ if and only if simultaneously

$$\begin{cases} \frac{s_t}{w_t} < \frac{mg}{R\delta}\left[\frac{mg}{R}\left(1+\frac{\gamma}{m}\right) - \delta\right] & \text{(condition for } s_{t+1} = 0) \\ \frac{s_t}{w_t} < \frac{\gamma}{m}\frac{mg}{R} & \text{(condition for } \text{G}_{t+1} > 0\text{)} \end{cases}$$

These conditions may respectively be rewritten as

$$\begin{cases} \frac{mg}{R} > \frac{\delta}{2[\delta\gamma/m+(1+\gamma/m)^2]}\left[1+\frac{\sqrt{5+\delta+4\gamma/m(2+\delta+\gamma/m)}}{\sqrt{1+\delta}}\right] =_{def.} \Delta(\delta,\gamma/m) \\ 0 < \left(\delta - \frac{mg}{R}(1+\gamma/m)\right)\left[\frac{1}{1+\delta+\gamma/m}+\frac{mg/R}{\delta}\right]+\frac{\delta^2/(1+\delta)}{\frac{mg}{R}(1+\delta+\gamma/m)} =_{def.} \mathrm{E}(\delta,\gamma/m) > 0 \end{cases}$$

In short, case (I) emerges at generation t and leads to case (III) at generation $t+1$ if and only if simultaneously

$$\begin{cases} \Gamma(\delta,\gamma/m) < \frac{mg}{R} < \mathrm{B}(\delta,\gamma/m) \\ \frac{mg}{R} > \Delta(\delta,\gamma/m) \\ \mathrm{E}(\delta,\gamma/m) > 0 \end{cases}$$

Let us adopt the same methodology for the three next cases. We then obtain that

- case (II) emerges at generation t and leads to case (III) at generation $t+1$ if and only if simultaneously

$$\begin{cases} \Phi(\delta,\gamma/m) < \frac{mg}{R} < \Gamma(\delta,\gamma/m) \\ \frac{mg}{R} < \delta \end{cases}$$

- case (III) emerges at generation t and leads to case (III) at generation $t+1$ if and only if

$$\begin{cases} \frac{mg}{R} \geq \mathrm{B}(\delta,\gamma/m) \\ \frac{mg}{R} > \mathrm{A}(\delta,\gamma/m) \end{cases}$$

- case (IV) emerges at generation t and leads to case (III) at generation $t+1$ if and only if

$$\delta \leq \frac{mg}{R} \leq \mathrm{A}(\delta,\gamma/m)$$

where

$$
\begin{aligned}
&\mathrm{A}(\delta,\gamma/m) = \frac{\delta}{\frac{\gamma}{m}(1+\delta)}; \mathrm{B}(\delta,\gamma/m) = \frac{\delta}{2(1+\gamma/m)}\left[1+\sqrt{\frac{\delta+5+4\gamma/m}{1+\delta}}\right]\\
&\Gamma(\delta,\gamma/m) = \frac{1}{2}\sqrt{\frac{1}{\gamma/m}}\left[\sqrt{\gamma/m+4\delta}-\sqrt{\gamma/m}\right];\\
&\Delta(\delta,\gamma/m) = \frac{\delta}{2\left[\delta\gamma/m+(1+\gamma/m)^2\right]}\left[1+\frac{\sqrt{5+\delta+4\gamma/m(2+\delta+\gamma/m)}}{\sqrt{1+\delta}}\right];\\
&\mathrm{E}(\delta,\gamma/m) = \left(\delta-\frac{mg}{R}(1+\gamma/m)\right)\left[\frac{1}{1+\delta+\gamma/m}+\frac{mg/R}{\delta}\right]+\frac{\delta^2/(1+\delta)}{\frac{mg}{R}(1+\delta+\gamma/m)};\\
&\Phi(\delta,\gamma/m) = \frac{\delta}{2(1+\delta)(1+\gamma/m)}\left[\delta+\sqrt{(2+\delta)^2+4(1+\delta)\gamma/m}\right]
\end{aligned}
$$

It may be shown that, for any given $\delta = \overline{\delta} \in [0,1]$, the functions $\mathrm{A}(\overline{\delta},\gamma/m)$, $\mathrm{B}(\overline{\delta},\gamma/m)$ and $\Gamma(\overline{\delta},\gamma/m)$ cross at the point $(\overline{\delta}, \frac{\gamma}{m} = \frac{1}{1+\overline{\delta}})$. At the left of this point, one may verify that

$$
\begin{cases}
\mathrm{A}(\overline{\delta},\gamma/m) > \Gamma(\overline{\delta},\gamma/m) > \mathrm{B}(\overline{\delta},\gamma/m) > \overline{\delta} > \Phi(\overline{\delta},\gamma/m) > \frac{\overline{\delta}}{1+\gamma/m}\\
\Delta(\overline{\delta},\gamma/m) > \mathrm{B}(\overline{\delta},\gamma/m)
\end{cases}
$$

One observes that for $\gamma/m \leq \frac{1}{1+\overline{\delta}}$, generation t will never choose to save and give positive amounts. It will decide to save nothing and make a strictly positive gift to the previous generation if and only if $qg > \mathrm{A}(\overline{\delta},\gamma/m)$. As long as $\delta \leq \frac{mg}{R} \leq \mathrm{A}(\delta,\gamma/m)$, generation t neither saves nor gives. If $\Phi(\overline{\delta},\gamma/m) < \frac{mg}{R} < \delta$ generation t saves but gives nothing. It directly follows that, whatever the saving and giving policy adopted by generation t, $\frac{mg}{R} > \Phi(\overline{\delta},\gamma/m)$ is a necessary and sufficient condition for $s_{t+1} = 0$ when $\gamma/m \leq \frac{1}{1+\overline{\delta}}$. Note also that $\overline{\delta} \leq \frac{1}{2}\left(\sqrt{5}-1\right)$ implies $\frac{1}{1+\overline{\delta}} \leq \overline{\delta}$ and given the constraint $\delta > \gamma/m$ the domain of interest is for sure at the left of $\frac{1}{1+\overline{\delta}}$.

If $\overline{\delta} > \frac{1}{2}\left(\sqrt{5}-1\right)$, it is worthwhile to study what happens at the right of $\frac{1}{1+\overline{\delta}}$. One may verify that

$$
\begin{cases}
\overline{\delta} > \mathrm{B}(\overline{\delta},\gamma/m) > \Gamma(\overline{\delta},\gamma/m) > \mathrm{A}(\overline{\delta},\gamma/m) > \frac{\overline{\delta}}{1+\gamma/m}\\
\Gamma(\overline{\delta},\gamma/m) > \Phi(\overline{\delta},\gamma/m) > \frac{\overline{\delta}}{1+\gamma/m}\\
\Gamma(\overline{\delta},\gamma/m) > \Delta(\overline{\delta},\gamma/m)\\
\frac{mg}{R} > \Gamma(\overline{\delta},\gamma/m) \Rightarrow \mathrm{E} > 0
\end{cases}
$$

As a consequence, if $\frac{1}{1+\overline{\delta}} < \gamma/m < \overline{\delta}$, generation t will never save and give anything because the altruistic factor is too large. If $\frac{mg}{R} > \mathrm{B}(\overline{\delta},\gamma/m)$

generation t save nothing and make a positive gift[26], while for $\Gamma(\delta, \gamma/m) < \frac{mg}{R} < \mathrm{B}(\delta, \gamma/m)$, generation t saves and gives strictly positive amounts. When $\Phi(\delta, \gamma/m) < \frac{mg}{R} < \Gamma(\delta, \gamma/m)$, generation t saves but gives nothing. Once again, it appears that $\frac{mg}{R} > \Phi(\overline{\delta}, \gamma/m)$ is a necessary and sufficient condition for $s_{t+1} = 0$, whatever the saving and giving policy adopted by generation t.

Q.E.D.

[26] This is actually the result derived by Verbon (1988). We observe that this author implicitely assumed $\gamma/m > 1/(1+\delta)$.

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