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Trade, Growth and Increasing Returns to Infrastructure: The Role of the Sophisticated Monopolist

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TRADE, GROWTH AND INCREASING RETURNS TO INFRASTRUCTURE : THE ROLE OF THE SOPHISTICATED MONOPOLIST

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1. Introduction

We model an economy with two final goods, manufactures produced under IRS and food. The scale economies in manufacturing are external (therefore compatible with perfect competition) and traceable to internal economies in the provision of an infrastructural service (the third sector of the economy). We examine the equilibria of this economy under both autarky and free trade.

We thus revisit a theme with a voluminous literature, beginning with R. C. O. Matthews'(1950) vintage classic and including, among others, Panagariya (1991), Krugman (1991), and Venables (1996). Much of this – as well as our own work – concerns multiple equilibria: it overlaps the development literature on poverty traps from Rosenstein Rodan (1943) to Murphy, Schleifer and Vishny (1989). We differ from this body of work in a major, and some minor, respects. We trace the source of increasing returns to infrastructure, and our focus is on the role of the infrastructure provider's beliefs in determining the equilibrium and the fate of the economy.

Internal economies in infrastructure provision ensure that it is non-competitive . We consider a pure monopoly. The infrastructure provider is of course aware of the impact of his decisions on the price of his services, but he may or may not appreciate their impact, on demand for labor (in a market where he competes with all other industrial and agricultural producers) and wages and induced effects on demand for infrastructure itself. He may in short be a 'naïve' or a 'sophisticated' decision-maker.

We model the naïve infrastructure provider after Venables (1996). Venables portrays a producer of intermediates who derives the demand curve for his product on the assumption that his

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customers have already contracted for their purchases of other inputs, specifically labor. Similar beliefs on the part of our infrastructure provider generate an equilibrium that is unique in the closed economy. In the small open economy, on the other hand, equilibria, where they exist , will generally be multiple: at any world price, there will generally exist one at a low level with unexhausted scale economies and another at a high level where these have been exhausted.

Outcomes are different if the infrastructure producer has a broader vision of the economy and his role in it. Under free trade, if the world price is high enough to sustain an equilibrium at all, it will be unique: it will occur at a high level of industrialization where all scale economies are exhausted. An unlimited world market facilitates industrialization that fully exploits economies of scale. But, it is insufficient if infrastructure producers have a limited view of their place in the economy. Industrialization may also require what may, with some justice, be described as Schumpeterian entrepreneurs, monopolists with a panoramic vision of the economy and of their catalytic role in it.

Some less important questions regarding our model: First, why introduce infrastructure into the traditional analysis of trade and industrialization under increasing returns *a la* Matthews and Panagariya ? The traditional model never explicitly models the source of increasing returns. Real world external economies in manufacturing arise from two sources – (1) irreversible Arrovian learning processes (eg. growth through learning-by-doing of a skilled labor force or cross-fertilization of research), (2) fall in intermediate prices as supplier industries grow and realize internal economies of scale. The standard models are static with reversible scale economies: they cannot accommodate learning by doing. Nor do they have room for a third intermediate sector using resources under internal economies of scale. Thus external economies in these models appear out of nowhere like manna from heaven. This enables competitive assumptions to be applied to the whole economy and production equilibrium necessarily occurs on the production possibility curve (rather than inside it) – but it detracts severely from realism.

Some papers[1] introduce an intermediate good responsible for external economies in a final goods sector. Venables (1996) has such a tradable intermediate, Okuno-Fujiwara (1988) a non-tradable one, Ishikawa's (1992) intermediate may be either, while Harris (1995) and Kikuchi and Ichikawa (2002) discuss communication networks in this context. With *infrastructure* as the intermediate, there are some key divergences in outcomes. Despite increasing returns, our closed economy equilibrium, where it exists, is *unique*. Opening up the economy may lead to multiple equilibria if the infrastructure provider is "naïve"— but unlike Okuno-Fujiwara, these need not involve complete specialization. Unlike the literature cited, we also explicitly model the choices made by a more "forward-looking" infrastructural entrepreneur and show how opening up the economy catapults it to a (unique) equilibrium with industrialization.

Infrastructure has been modeled in two ways. Arrow–Kurz (1970) and Barro (1990) visualized it as an input in production. Martin and Rogers (1995) examined it as determining the fraction of output that evaporates before reaching the consumer; they model it as a state-supplied public good. Our focus is on infrastructural services as private goods (though not necessarily provided by the private sector); we wish to examine their part in the frequent failure of market processes to generate growth without assuming the problem away by assigning an indispensable role to the state in their provision. We revert therefore to the Arrow-Barro tradition in which infrastructural services enter the production function. The basic model is that of electricity, frequently identified in many poor countries (such as India) as the critical bottleneck on growth.

What are the distinctive implications of infrastructure in such a context? We stress two. *First*, it needs fixed investment, which occurs only if the return on this fixed cost at least matches the interest rate. Thus a low-level equilibrium without infrastructure and therefore without industry is possible, a pure agrarian economy that imports all its manufactures. *Second,* the non-tradable character of infrastructural services ensures that the scale economies they generate are localized. In models with increasing returns in the final goods industry (whether arising from direct increase in output or in the number of intermediates in use), one must postulate transport costs of tradable goods to localize scale economies in a particular country. One must also explain how differences in scales arise by postulating differences in consumption patterns between countries (due to factors like population size or Engel effects on the demand for manufactures). Though our model can accommodate transport costs on final products and Engel effects, its results are independent of these assumptions. Thus, its conclusions are not undermined by the secular decline in the share of transport costs in world prices.

2. The Model

Our model has two final products, food and manufactures with CRS production functions and one intermediate, infrastructural services, produced under IRS. Agriculture uses labor and the fixed land endowment to produce food. Infrastructural services are provided by labor operating fixed indivisible equipment under decreasing marginal cost; the equipment costs F and is imported. Manufacturing uses labor and infrastructural services under CRS. Investment in infrastructure is financed by free mobility of capital at the fixed world interest rate r. Internal economies of scale in the infrastructural activity rule out perfect competition: we assume that the industry is a natural monopoly**.** The rest of the economy is competitive. While the infrastructure monopolist is the sole supplier of his product, he must compete in the labor market with all other manufacturers and with farmers. The infrastructural service is non-tradable while food and manufactures are perfectly mobile in international trade. We eliminate distributional considerations by assuming Stone-Geary utility functions with a subsistence term for food, implying linear expenditure functions.

2.2 Infrastructure Monopoly

We assume Cobb-Douglas production functions

$$A = L_a^{\alpha} N^{1-\alpha} \tag{1}$$

for food and

$$M = L_m^{\beta} I^{1-\beta} \tag{2}$$

for manufactures where L_a and L_m are labor inputs in the two sectors, N is the fixed endowment of land and I the infrastructural service. Infrastructure requires a lumpy investment, F. The production function for the latter is iso-elastic in labor, once this investment F is made:

$$I = L_i^{\delta}, \qquad \delta > 1 \tag{3}$$

where L_i is the labor input in the infrastructural activity. Such a production function could result, for instance, from the division of labor as the output of the infrastructure service increases. We can choose the unit of land so that N = 1.

The utility function is

$$U = M_d^{\mu}(A_d - \overline{A})^{1-\mu} \tag{4}$$

where M_d and A_d are the consumptions of manufactures and food respectively implies that expenditure on manufactures is a fixed fraction μ of the surplus of income (net of interest payments rF) over subsistence $\overline{A}$:

$$pM_d = \mu(pM + A - rF - \overline{A}) \tag{5}$$

where p is the price of manufactures. Labor market equilibrium requires that the wage rate

$$w = \alpha L_a^{\alpha-1} \tag{6}$$

$$= \beta p(L_m / I)^{\beta-1} \tag{7}$$

Labor is supplied inelastically and the wage-rate adjusts flexibly to ensure full employment.

$$L_a + L_m + L_i = L \tag{8}$$

The price *q* of the infrastructural service is its marginal value product in manufacturing

$$q = (1-\beta)p(L_m / I)^{\beta} \tag{9}$$

We consider pure monopoly in the infrastructural activity. The inverse demand function for the monopolist's product is represented by (9). For the rest of this section as well as for sections 3 and 4, we assume that the monopolist behaves in a 'naïve' manner. Following Venables (1996), our infrastructure producer presumes that manufacturers commit themselves to employment contracts first; subsequently, they decide on their output and buy infrastructural services in the light of the price that clears the market for the latter. Thus, the monopolist takes the manufacturer's employment level as given when figuring out the derived demand for his product. Like Venables' upstream producers, he also ignores any effects of his decisions on the prices of inputs for which he must compete with downstream manufacturers or farmers and the larger macroeconomic effects of his decisions on income and demand. He then perceives the elasticity of this demand as $1/\beta$ so that the profit-maximizing equality of the monopolist's marginal revenue and marginal cost would require

$$q(1-\beta) = wI^{-(1-1/\delta)}/\delta \tag{10}$$

subject of course to the condition that profits are non-negative:

$$qI \geq wI^{1/\delta} + \mathrm{rF} \tag{11}.$$

A necessary condition for this is

$$q > wI^{(1-1/\delta)} \tag{12}.$$

(10) and (12) together imply the second order condition for the monopolist's maximization

$$1/\delta > 1-\beta \tag{13}.$$

Assume that (11) is satisfied, so that the fixed cost rF can be ignored, as it is in the short run.

Simple manipulations now yield

$$I = \left(\frac{\beta}{\delta(1-\beta)^2}\right)^{-\frac{\beta\delta}{\sigma}} M^{\frac{\delta}{\sigma}} \tag{14}$$

where $\sigma = \beta + (1-\beta)\delta$ is a weighted average of δ and 1 and therefore larger than unity. Manufactured output and infrastructural services are increasing functions of each other. Further, we have

$$w = \alpha[L - \lambda M^{1/\sigma}]^{\alpha-1} \tag{15}$$

where $\lambda = \{1 + \frac{\beta}{\delta(1-\beta)^2}\}\{\frac{\delta(1-\beta)^2}{\beta}\}^{\beta/\sigma}$.The wage rate rises as manufactures expand - since manufacturing diverts labor directly and indirectly (through infrastructural activity) from agriculture, thus raising the marginal productivity of labor in terms of food.

However, there are increasing returns to infrastructural activity. q, the price of the infrastructural service, is subject both to the upward pressure of rising wages and the down-thrust of economies of scale (indicated by the negative exponent of I in (10)). When industry is small, the latter dominates: infrastructure cheapens as it expands. The effect may well be reversed in an industrialized economy.

The unit cost of manufacturing in turn may increase or diminish with manufactured output according to the balance between economies of scale in the production of infrastructural inputs and wage pressures. Some cumbrous but simple algebraic manipulation yields

$$p = w/\tau M^{1-1/\sigma} \qquad (16).$$

where $\tau = \{\delta(1-\beta)^2\}^{\delta(1-\beta)/\sigma}\beta^{\beta/\sigma}$. As $M \to 0$ (and $L_a \to L$), w converges to the non-zero limit $\alpha L^{\alpha-1}$ and p to ∞, while, as $L_a \to 0$ and M to the finite maximum that this implies, w and p both tend to ∞. The inverse supply function of manufactures p = φ(M) is U-shaped, as confirmed by differentiation,

$$dp/p = \{\frac{1-\alpha}{\sigma}(\frac{L-L_a}{L_a}) - \frac{\sigma-1}{\sigma}\}dM/M \qquad (17)$$

Thus, $M\varphi'(M)/\varphi(M) \leq 0$ or $M\varphi'(M)/\varphi(M) \geq 0$ as

$$(L-L_a)/L_a \leq (\sigma-1)/(1-\alpha) \text{ or } (L-L_a)/L_a \geq (\sigma-1)/(1-\alpha) \qquad (18)$$

As the economy industrializes, the share of industrial to agricultural labor rises from 0 to ∞, ensuring a unique minimum to the supply curve of manufactures. External economies of scale continue to dominate manufacturing till the share of agriculture in the labor force dwindles to $(1-\alpha)/(\sigma-\alpha)$. Thereafter they are swamped by wage pressures.

Further, equation (17) indicates that

$$M\varphi'(M)/\varphi(M) > -(1-1/\sigma) > -1 \quad (19).$$

So much for the domestic supply of manufactures. What of the domestic demand ? Inserting the agricultural production function in (5), and substituting for agricultural labour in terms of M, we have the implicit domestic demand for manufactures as a function of domestic supply:

$$M_d = \mu[M+\{(L-\lambda M^{1/\sigma})^{\alpha} - \bar{A} - rF\}/\varphi(M)] = \theta(M) \quad (20)$$

for $p \geq \underline{p}$, the minimum supply price of manufactures and

$$M_d = \mu(L^{\alpha} - \bar{A})/p \quad (20a)$$

for $p < \underline{p}$.

3. The Closed Economy

A solution of the equation $M = M_d = \theta(M)$ is a closed economy equilibrium. Note that we define a *closed economy* or *autarky* as one that does not trade final goods though it may import capital.

Proposition 1: The closed economy equilibrium exists and is unique.

Proof: In the appendix.

This is illustrated graphically in Figs.1 and 2. The upper part of each diagram shows ψ(M), the supply price of manufactures, as a function of output M: any M determines a price ψ(M). M and ψ(M) determine domestic demand M_d. The supply curve of manufactures is a U-shaped curve in the upper part of the figure. In the lower part domestic demand for manufactures θ(M) is plotted against output . We prove that θ(M) crosses the 45° line just once – at U corresponding to P in the top quadrant, where supply and demand for manufactures are equal at the price PT. Q is the minimum of the curve ψ(M) and QN the associated minimum supply price of manufactures. ψ(M) being U-shaped, any price above QN is associated with two possible outputs and two possible demands for manufactures. At prices below QN, no manufactures are produced; demand for manufactures arises out of agricultural surplus.

However, θ(M) can intersect the 45˚ line either to the left of the minimum point of the supply curve (as in Fig. 1) or to its right (as in Fig. 2). Autarky equilibrium might occur with unexhausted economies of scale or it may occur on the rising segment of the supply curve.

Some simple substitutions show that if $\bar{A} = 0$ (there are no subsistence requirements) and r = 0, autarky equilibrium occurs when the ratio of non-agricultural to agricultural labor reaches $\frac{\mu\{\delta(1-\beta)^2+\beta\}}{\alpha(1-\mu)}$. If the subsistence term and the world interest rate are positive, the share of non-agriculture in the labor force would be smaller than this under autarky. However, nothing much changes if we dispense with the subsistence requirement with its non-homotheticity implications. Our model accommodates differential income-elasticities of demand , but is not driven by them.

Autarky equilibrium occurs with unexhausted scale economies if the ratio of industrial labour to agricultural is smaller under autarky than at the minimum of the supply curve of manufactures. A sufficient condition: $\frac{\mu\{\delta(1-\beta)^2+\beta\}}{\alpha(1-\mu)} < \frac{\sigma-1}{1-\alpha}$. The smaller is μ, the budget share of manufactures in the consumer's surplus income, and the larger is α, the elasticity of labor supply to industry, the likelier is this condition to be met. However, even if it is unfulfilled, a large subsistence requirement $\bar{A}$ or a high r (the world interest rate) or F (the lumpy infrastructural requirement) can result in an autarky equilibrium on the down-sloping segment of the supply curve of manufactures.

4. The Small Open Economy

In a small open economy, the system of equations cannot be closed by equating M_d to M, but by taking p as given by the world market. Because of the U-shape of the supply curve, our system generates multiple equilibria when opened up to trade. We maintain the assumption in this section that the infrastructure monopolist is "naïve".

Fig. 1

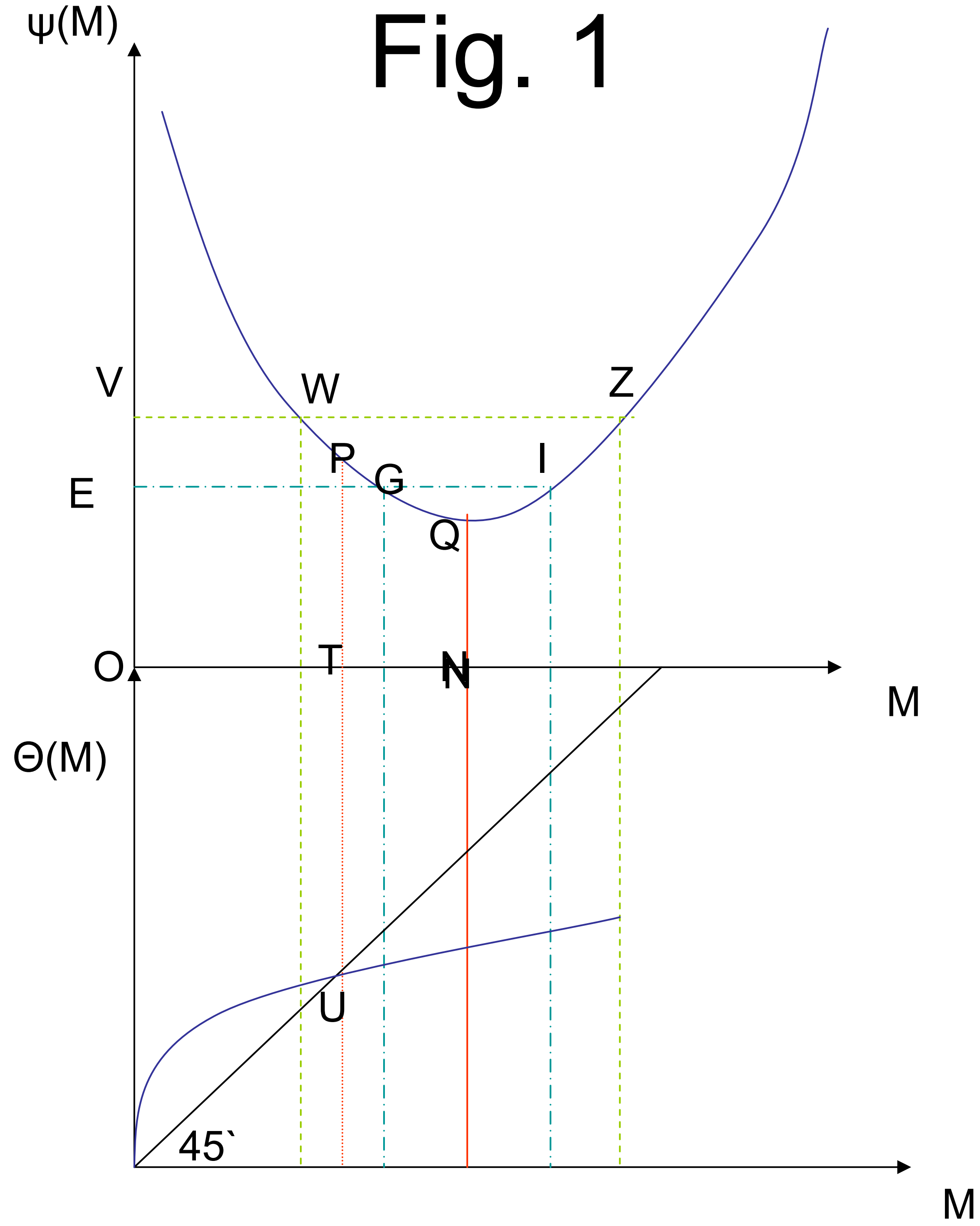

Fig. 2

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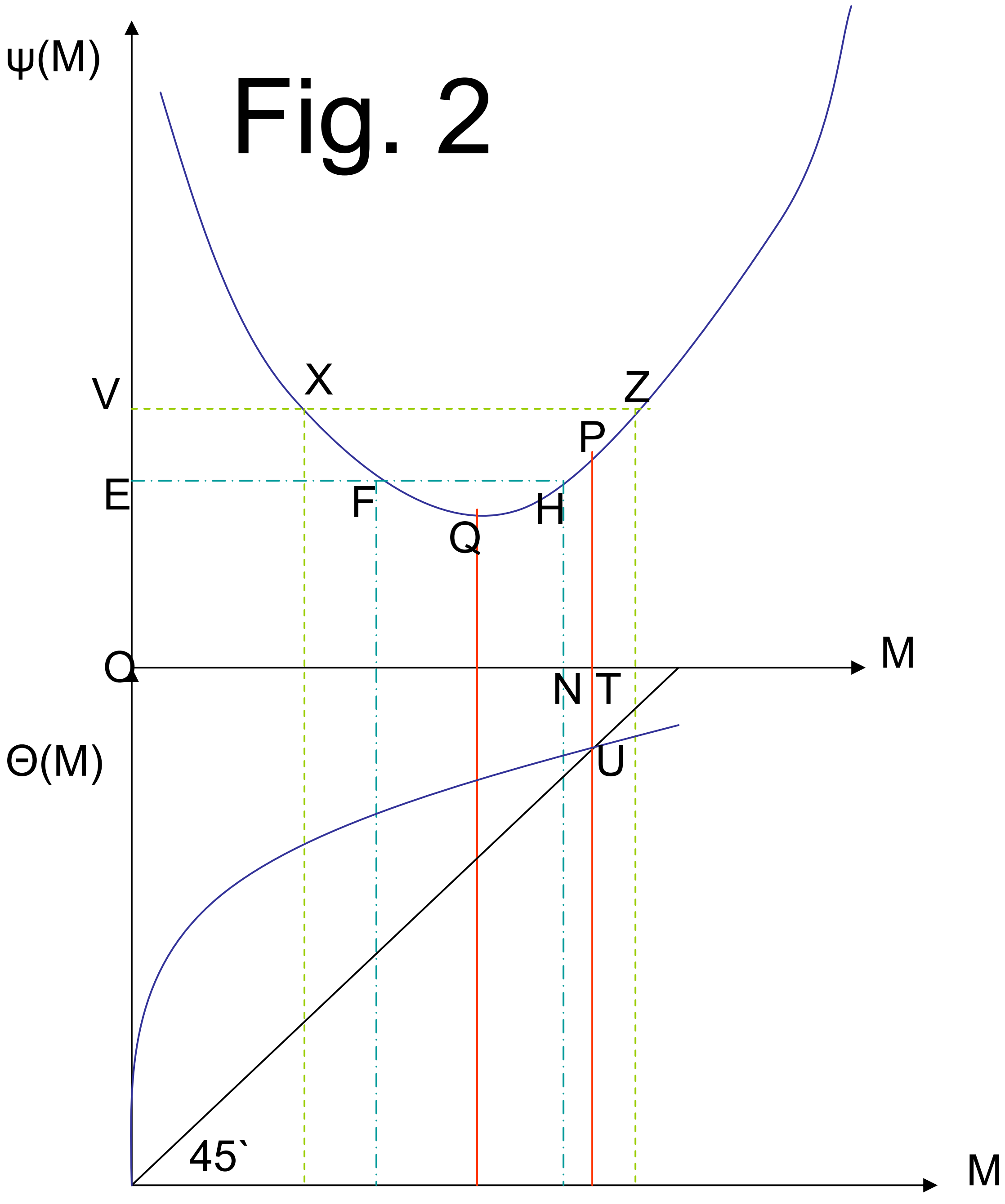


As discussed, depending on the parameters, scale economies may or may not be exhausted. Further, each of these two cases can be partitioned into two subcases according to whether the world price $\tilde{p}$ exceeds the autarky price p* or falls short.

1. In case 1, with autarky equilibrium in the decreasing return phase (Fig. 2): (a) if $\underline{p} < \tilde{p} < p^*$ (world price between the autarky price and the minimum supply price as with OE), there will be excess demand for, and imports of, manufactures; however, thanks to the U-shaped supply curve, two import equilibria will exist: OE will just cover the cost of producing either output EH or the smaller output EF; given the same price, the difference in outputs imply different incomes and therefore different domestic demand levels for manufactures; (b) if $\tilde{p} > p^*$ (as with world price OV), this may generate a large manufactured output VZ, implying exports; but it is also consistent with the smaller output VX and imports.
2. In case 2, with increasing returns in autarky equilibrium (Fig.1), (a) if $\underline{p} < \tilde{p} < p^*$ (OE in Fig. 1), this could lead to either of the outputs EG or EI (with associated domestic demands both less than domestic supply), implying manufactured exports; (b) if $\tilde{p} > p^*$ (OV in Fig. 1), this may induce output VZ with exports or the smaller output VW with imports.

Our economy therefore faces a multiplicity of possible equilibria at different levels of industrialization. If however condition (11) does not hold everywhere, several of these short run equilibria are no longer profitable in the long run. Recall that profits in the infrastructure monopoly

$$\pi = qI - wI^{1/\delta} - rF. \qquad (21).$$

Substitutions from (12) reduce this to

$$\pi = wI^{1/\delta}[1/\delta(1-\beta) - 1] - rF. \qquad (22)$$

If inequality (14) is not fulfilled, infrastructure will be unprofitable; it can never attract investment and the economy is doomed to remain permanently agricultural. If however (14) is satisfied, profit becomes an increasing function of both wages and infrastructural services. Since both increase with manufactured output, so does profit. The possibility now emerges of profits being negative at low levels of manufactured output, but positive at higher levels. With a multiplicity of short run equilibria, the ones at higher output levels could well be sustainable even if those at lower levels are not. This underlines the “big-push” flavor of our open economy multiple equilibria model.

Note: equilibria can be supported where manufactures are exported despite their autarky price exceeding the world price, if scale economies were unexhausted in autarky equilibrium : similarly, it is possible to support equilibria where manufactures are imported in spite of the world price of manufactures being higher than the autarky price, if scale economies had been exhausted in autarky.

The “naïve” equilibria described point to the multiplicity of outcomes in small open economies, so that economies with similar underlying parameters may diverge. This divergence could happen either if the infrastructure providers in the different economies were all naïve, or alternatively if some were naïve but not others. We show next how the outcome differs for a “sophisticated” infrastructure provider, yielding a unique prediction for the open as well as the closed economy.

We show below that of all the “naïve” equilibria described, that which is most profitable for the infrastructure monopolist for any set of parameters maximizes industrialization – as profits are directly related to manufactured output. However, as long as the infrastructure monopolist does not recognize his role as a “leader” there is no guarantee that co-ordination on this particular equilibrium will occur. In the next section, we will argue that a “sophisticated” Stackelberg-type – or, as we call him, Schumpeterian – monopolist will, for a wide range of parameters, choose an equilibrium with an even greater degree of industrialization than the most profitable of the “naïve” equilibria.

4.2 Profits, Wages and National Incomes in the Naïve Equilibria

We turn to a comparison of profit and wage levels in the trading equilibria and the autarky equilibrium in the naïve case. The competitive final goods industry of course converges to zero profit equilibria in all cases. Wage rates however differ - and so do profit levels in infrastructure monopoly.

Manufacturing expansion drives up labor demand and wages in terms of both final goods, as (7), (15) and (16) show. Wages in terms of both final goods will therefore be higher in (1) the industrial exporter than in the autarkic economy and (2) the autarkic economy than in the industrial importer.

As for the profit level π in the infrastructure monopoly,

$$\pi = qI - wI^{1/\delta} - rF. \tag{23}$$

Substitutions from (10) reduce this to

$$\pi = wI^{1/\delta}\,[1/\{\delta(1-\beta)\} - 1] - rF \tag{24}$$

The condition for non-negative profit (13) ensures that π will be increasing in both w and I; and since both increase with M, so will profits. Industrialization increases profits in the infrastructure monopoly.

Since however the return to land diminishes with industrialization, the change in national income or welfare is ambiguous. National income in terms of food

$$Y = A + pM \tag{25}$$

Differentiation and some manipulation yields

$$dY/dM = \sigma\tau[1 - \lambda + \lambda\{(1-\alpha)pM/wL_a\}] \tag{26}$$

$$\text{Thus } dY/dM \geq 0 \text{ iff } pM/wL_a \geq (1 - 1/\lambda)/(1-\alpha) \tag{27}$$

When manufactures account for a negligible fraction of total output, industrialization depresses national income in terms of food. However, by equation (17), manufactured output value rises with M, farm output falls - and, once the relative share of industry crosses the threshold (27), industrialization adds to national income. These results compare various "naïve" equilibria and establish that the most highly industrialized generates highest profits, wages and, after a point, national income.

5. Sophistication and the Schumpeterian Equilibrium

What if the monopolist plays the role of a full 'Schumpeterian leader' in factor, as well as product, markets? A Schumpeterian leader is defined here as a producer who is aware of and takes into account the impact of his decisions not only on the price of his product but also on all factor markets and, through that, on the rest of the economy. He believes, correctly, that other agents adjust their behavior to any given set of product and factor price-parameters: he maximizes his profits on the basis of this belief. He therefore departs from the naïve reasoning portrayed above, and does not make the assumption, attributed to him earlier in this paper, that manufacturers' employment levels are given.

The Schumpeterian monopolist maximizes

$$\Pi = p\,M - w(L_m + L_i) - rF \tag{28}$$

subject to the production functions, the full employment condition and the labor market equilibrium conditions (equations (1)-(3) and (6)-(8)). The rationale: with the rest of the economy competitive, all profit income accrues in equilibrium to the sole monopolist. The only other source of non-agrarian income is labor; so profits are the value of the gross product of this sector less the income of non-agricultural labor (as well as interest on fixed capital). Further, since the other agents in the economy respond predictably to his actions, he can in effect choose what would be for him the most profitable values as well of their control variables subject of course to the constraints we have mentioned.

In a closed economy, this exercise is performed with p set by the balance of supply and domestic demand $M = M_d$, where M_d is determined by (4) and (5) above. Then (32) can be rewritten

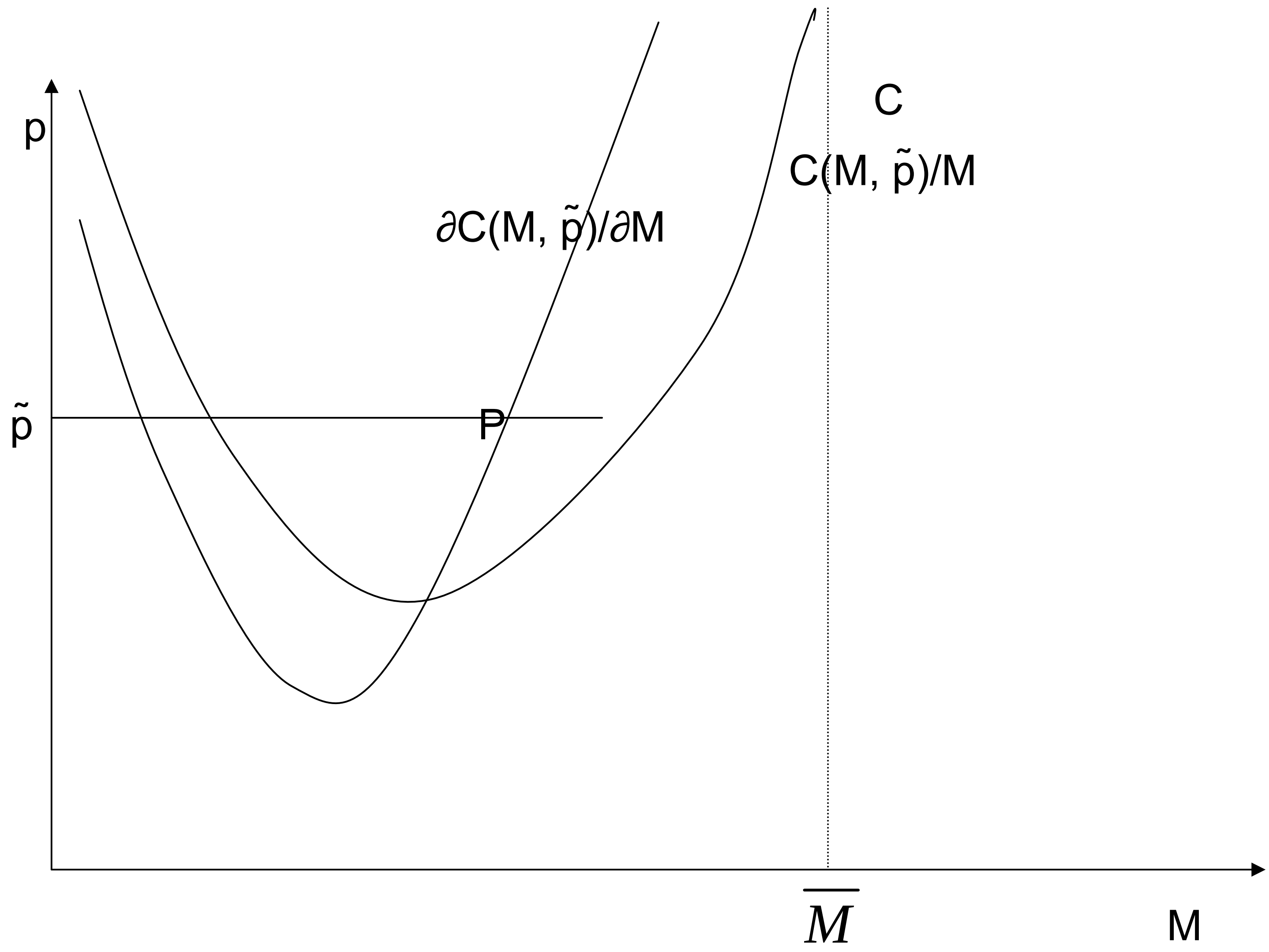
$$\Pi = p\,M_d - w(L_m + L_i) - rF$$

$$= \frac{\mu(A - rF - \overline{A})}{1-\mu} - w(L_m + L_i) - rF \tag{29}$$

(29) suggests that, under autarky, the Schumpeterian monopolist should contract output indefinitely. This shrinks non-agricultural employment ($L_m + L_i$) and wages while raising agricultural labor force, output A and surplus ($A - \bar{A}$). No positive equilibrium exists for infrastructure and manufacturing and the economy is purely agrarian. Infrastructure and manufactures cannot be sustained in a closed economy without government support.

However, the value of this last conclusion, is reduced by Gabszewics's (1972) proof that general equilibria under monopoly depends on one's choice of numeraire. A conclusion can be

Fig. 3



definitive only if robust to changes in numeraire. Our other qualitative results meet this criterion, but the last one does not: with manufactures as numeraire instead of food, Schumpeterian monopoly completely deindustrializes a closed economy for some parameter values, but for some others positive manufacturing can be sustained, though at an equilibrium level that does not exhaust scale economies.

In a small open economy, on the other hand, production and domestic demand are independent and p is exogenously set by the world market at the level $\tilde{p}$. For a given $\tilde{p}$, equations (1) – (3) and (6) – (8) enable us to solve for wages and all labor inputs as functions of manufactured output. Let $C(M, \tilde{p}) = w(L_m + L_i)$. Then, the first- and second-order conditions for a maximum of (28) are

$$\tilde{p} = C_M(M; \tilde{p}) \tag{30}$$

and

$$C_{MM}(M; \tilde{p}) > 0. \tag{31}$$

A further sufficient condition for a positive equilibrium is non-negative profit:

$$\tilde{p} > C(M; \tilde{p})/M + rF \qquad (32).$$

Now consider

$$C(M; \tilde{p})/M = w(L_m + L_i)/M$$

As $M \to 0$, $L_a \to L$, w tends to a finite constant (the marginal product of agrarian labor when all labor works in agriculture). So do the unit requirements of labor and infrastructure in manufacturing, L_m/M and I/M. However, $L_i/M = I^{1/\delta}/M$ tends to infinity as I and M tend to 0, so that in this limit $C(M; \tilde{p})/M$ goes to infinity. On the other hand, as L_a approaches 0 and M tends to the finite maximum $\overline{M}$ consistent with this, w explodes to infinity, so that in this extremity as well $C(M; \tilde{p})/M$ goes to infinity. Continuity therefore requires that between the limits represented by $L_a = L$ and $L_a = 0$, $C(M; \tilde{p})/M$ must have an odd number of minima. We prove elsewhere that it has a unique minimum – where the share of agriculture in the labor force shrinks to $(1- \alpha)/\sigma$.

Proposition 2: The "average cost" curve facing the Schumpeterian monopolist has a unique minimum.

Proof : In the appendix.

The profit maximization of the Schumpeterian monopolist then can be represented by Fig. 3. Equilibrium necessarily occurs on the rising segments of the 'average' and 'marginal cost' functions – beyond the industrialization level that exhausts all scale economies in industry. Note that the increasing returns phase for the Schumpeterian monopolist extends to a higher degree of industrialization than in the previous case. The share of agriculture in the labour force drops below $(1 - \alpha)/\sigma$, which is less than $(1-\alpha)/(\sigma - \alpha)$ – the benchmark in the previous case.

6. Public Ownership of the Infrastructure Industry

How would all this compare with the social optimum? In particular, would the profit-maximizing monopolist aim at a higher degree of industrialization than welfare-maximizing public ownership of infrastructure? We show below that the opposite is in fact the case. We make the admittedly heroic assumption that production efficiency is independent of the regime, so that the same production functions can be used in the two cases.

As is well-known, a necessary, though not sufficient, condition for social optimality is a production equilibrium that maximizes Y, the value of output at world prices, since this enables the economy to climb onto its highest consumption possibility locus. Y is affected by increased provision of I through its impact on outputs in agriculture and manufacturing.

$$dY/dI = p\,dM/dI + dA/dI.$$

Agricultural output is impaired by the withdrawal of labor into infrastructural and manufacturing employment.

$$dA/dL_a$$

$$= \alpha L_a^{\alpha - 1} = \alpha A/L_a$$

The manufacturing output effect is an average of the proportionate changes in I and manufacturing employment, weighted by their relative output shares.

$$dM/M = (qI/pM)\, dI/I + (wL_m/pM)\, dL_m/L_m.$$

Manufacturing employment, in turn, is subject to two forces: the growth in I accompanied by a proportionate rise in manufacturing labor demand (if wages were unchanged) and the rise in wages that induces a fall in the labor-infrastructure ratio.

$$dL_m/L_m = [1 - \{1/(1-\beta)\}(dw/dI)I/w]\, dI/I$$

Meanwhile, infrastructural employment rises with elasticity $1/\delta$ as I increases and agricultural employment falls (with elasticity $-1/(1-\alpha)$) as wages rise.

$$IdL_i/dI = L_i/\delta.$$

$$wdL_a/dw = - L_a/(1-\alpha)$$

Wages rise just enough to equilibriate the labor market (through an induced restriction of agricultural employment) in the face of the rise in labor demand from the infrastructure and manufacturing sectors.

$$\{L_a/(1-\alpha)\}\,(dw/dI)\, I/w = L_i/\delta + L_m[1 - \{1/(1-\beta)\}(dw/dI)I/w]$$

from which

$$dw/w = \frac{(L_i/\delta + L_m)(1-\alpha)(1-\beta)}{L_a(1-\beta) + L_m(1-\alpha)}\, dI/I$$

It is now a matter of simple substitutions to work out the changes in manufacturing and agricultural employment and therefore in manufacturing and agricultural output that follow a change in infrastructure. The impact on national income can then be worked out as follows:

$$dY/dI = \frac{L_a(1-\beta)pM - \dfrac{L_i\{\beta(1-\alpha)pM + \alpha(1-\beta)A\}}{\delta} + L_m(1-\beta)\{(1-\alpha)pM - \alpha A\}}{L_a(1-\beta) + L_m(1-\alpha)}$$

Simplification of the numerator shows that $dY/dI \geq 0$ as $L_m(1-\beta)/\beta \geq L_i/\delta$. Given the fact that $(1-\beta)/\beta = qI/wL_m$, this reduces to the condition $qI \geq wL_i/\delta$.

A necessary condition for the maximization of national income (which is a necessary condition for social optimality) is $qI = wL_i/\delta < wL_i$. The infrastructure sector will be making losses in this situation. Further, if $qI > wL_i$, so that the infrastructure industry can at least cover its variable cost, $dY/dI > 0$: social optimality will require an expansion of infrastructural output till losses are made. Obviously, therefore, any free-market) equilibrium will imply a smaller infrastructural output and a lower degree of industrialization than social welfare-maximising state ownership of infrastructure.

We repeat that this conclusion requires our implausible assumption that efficiency levels are independent of the ownership pattern.

7. Contestable Cournot Oligopoly in Infrastructure

A question of some importance relates to the sensitivity of our results to our assumption that infrastructure is provided by a monopolist. While natural monopoly is typical of many major elements of infrastructure, an alternative market specification which has been frequently used is that of Cournot oligopoly with free entry. In this section, we assume n identical infrastructure firms playing a Cournot game. With the inverse demand function of the infrastructure industry defined by (8), the marginal revenue of each Cournot firm is (1-β/n)q, while its marginal cost is $\frac{w}{\delta}(I/n)^{1/\delta-1}$ (where I/n is its output). The equation of marginal revenue and marginal cost then implies

$$\frac{\delta(1-\beta/n)}{n^{1-1/\delta}} = \frac{w}{qI^{1-1/\delta}} \tag{37}$$

$$= \frac{\beta I}{(1-\beta)L_m I^{1-1/\delta}} \tag{38}$$

from which

$$L_m = \frac{\beta I^{1/\delta} n^{1-1/\delta}}{\delta(1-\beta)(1-\beta/n)} \quad (39).$$

Labor in the infrastructure is related to I:

$$L_i = n(L_i/n) = n(I/n)^{1/\delta} = n^{1-1/\delta} I^{1/\delta} \tag{40}$$

From (39) and (40),

$$L_i + L_m = \{1 + \frac{\beta}{\delta(1-\beta)(1-\frac{\beta}{n})}\} n^{1-1/\delta} I^{1/\delta} \tag{41}$$

Now, inserting the expression (39) for L_m in the manufacturing production function,

$$M = \{\frac{\beta n^{1-1/\delta}}{\delta(1-\beta)(1-\beta/n)}\}^{\beta} I^{\sigma/\delta} \tag{42}$$

Manipulation of (41) and (42) yields

$$M = \frac{\beta^{\beta}\{\delta(1-\beta)(1-\beta/n)\}^{\delta(1-\beta)}(L_i + L_m)^{\sigma}}{\{\delta(1-\beta)(1-\beta/n)+\beta\}^{\sigma}\, n^{(\delta-1)(1-\beta)}} \tag{43}$$

n is endogenously determined. Free entry into the infrastructure industry reduces excess profits to zero. Now the marginal revenue/average revenue ratio is $(1-\beta/n)$ and the marginal cost/average cost ratio $1/\delta$: zero profits (a.r. = a.c.) imply that the former ratio is equal to the latter:

$$1-\beta/n = 1/\delta \qquad (44).$$

Thus,

$$n = \beta\delta/(\delta-1) \qquad (45).$$

Insertion of the value of n in (43) yields

$$M = (1/\beta - 1)^{\delta(1-\beta)}(1-1/\delta)^{(\delta-1)(1-\beta)}\beta(L_i + L_m)^{\sigma} \qquad (46).$$

Then, using (7), (39), (45) and (46), we derive

$$\mathrm{p} = \frac{w\beta^{(\sigma-1-\beta)/\sigma}}{(1-\beta)^{(1-\beta)/\sigma}(1-1/\delta)^{1-1/\sigma}M^{1-1/\sigma}} \tag{47}$$

(47) is identical with (18), apart from a scalar transformation. Thus qualitatively, the shapes of the supply curves of manufactures under infrastructure monopoly and free-entry infrastructure Cournot oligopoly are identical. Equations (and inequalities) (19) to (23a) hold without any change under Cournot oligopoly - and the geometric analysis above can therefore be repeated. Our conclusions regarding multiple equilibria under infrastructure monopoly are replicated where infrastructure is provided by Cournot oligopolists in a contestable market. The "naivete" assumption in the previous part of the paper is maintained for this analysis – the oligopolists do not take into account their possible impact on manufacturers' employment decisions, or on the wage.

If integer constraints are considered, the equations (45)-(47) will be replaced by inequalities setting upper and lower bounds to n, M and p. The algebra becomes infinitely more cumbrous without changing the qualitative characteristics of the system.

8. Some Implications

How do labor force size and supply elasticity to industry affect this model? Both for the naïve infrastructure producer (as inequality (20) indicates) and his sophisticated counterpart, the larger is α, the higher must industry's share in total labor force rise before exhausting increasing returns in manufacturing. α , on the other hand, is positively related to elasticity of labor demand in agriculture and therefore to elasticity of labor supply to industry (as shown by wage equation (6)). The larger the total labour force L, the larger will be total industrial employment for any given share of industry in total labor. Thus, a large volume and elasticity of labor supply increase the likelihood – where the monopolist is naïve – of an autarky equilibrium with unexhausted economies of scale in industry and of asymmetric trading equilibria. However, with both naïve and sophisticated infrastructure producers, industrial growth prospects *a la* Arthur Lewis open up for densely populated agrarian economies with highly elastic labor supply in manufacturing - if international trade provides an outlet for their manufactures. This, of course, is the story of much of East Asian growth, of the development over the past four decades of Korea, Taiwan, Thailand, Indonesia, China and now Vietnam. Unlike Lewis, however, we do not have to assume zero marginal labor product or surplus labor in agriculture: indeed, an agricultural production function near-linear in labor favors this result most strongly.

A major implication of the analysis is that the extent of the infrastructure provider's sophistication in decision-making drastically affects the equilibrium outcome and the extent of industrialization that a small open economy can achieve. We have argued that while the closed economy equilibrium is unique – itself a departure from most papers on the theme of increasing returns in trade – multiple equilibria with different patterns and magnitudes of trade are possible when the economy opens up, if the infrastructure provider is naïve. If he is sophisticated, however, unique equilibrium emerges even in an open economy. In this equilibrium, economies of scale have been exhausted so the level of industrialization chosen is obviously high. Moreover, when the infrastructure provider is sophisticated, economies of scale themselves persist for a greater degree of industrialization than when the infrastructure monopolist does not recognize his leadership role. Open economies with similar underlying parameters may diverge if infrastructure providers in some, though not necessarily all, of these countries behave in a naïve manner – due to the possibility of multiple trading equilibria with naïve infrastructure providers . With naïve behavior, it is possible that opening up an economy will lead to a lower degree of industrialization than under autarky – a deindustrialization without Dutch disease. Not so however with a sophisticated or Schumpeterian infrastructure provider: here the economy settles at a unique trading equilibrium at a high level of industrialization.

To sum up, if international trade in manufactures is opened up, rapid industrialization through the market depends on the beliefs of the infrastructure monopolist. Our model focuses sharply on the key role of the entrepreneur in development – the role dramatically expounded by Schumpeter. We provide an interpretation of the distinctive function of the Schumpeterian entrepreneur.

However, the possibility of multiple equilibria also widens the role that government may play in industrial policy. This might become important if agents are not very forward looking ('naïve'). Expansion of infrastructure, through entrepreneurial initiative – if entrepreneurs are sufficiently sophisticated in their reasoning - or otherwise because of government ownership or persuasion[2] of less adventurous entrepreneurs, could be crucial in catalyzing industrial growth and realization of scale economies. As already shown, such expansion justifies itself in terms of higher profits. The present model is static, of course, and does not depict irreversible growth. We could however lock history into the production function for manufactures by adding a multiplicative productivity parameter that grows with manufactured output through learning-by-doing. Could this be the secret of the success of East Asian governments in nudging their industrialists down the path of industrial export growth, a path that led directly to the East Asian miracle?

Appendix A

Proof of Proposition 1: We prove that one and only one equilibrium exists in the closed economy.

As $M \rightarrow 0$, $L_a \rightarrow L$ and

$$M\psi(M) = w\, M^{1/\sigma}/\tau \rightarrow \alpha\, L^{\alpha-1} M^{1/\sigma}/\tau \rightarrow 0$$

$$\text{Now } M/\theta(M) = \frac{M\psi(M)}{\mu\{M\psi(M) + L^{\alpha} - \bar{A} - rF\}}$$

$$\rightarrow \frac{M\psi(M)}{\mu\{M\psi(M) + L^{\alpha} - \overline{A} - rF\}} = 0.$$

Thus for small M, θ(M) > M.

M assumes its maximum value M_{max} when agricultural employment dwindles to zero. At this limit, θ(M) = μ(M-rF/p) < M.

Since θ(M), a continuous function, passes from values greater than M to values less than it as M increases from 0 to M_{max}, it must have a fixed point.

Now, differentiation and some manipulation show that

$$M\theta'(M) = \mu M[1 - \frac{\lambda\tau\{1-(1-\alpha)\frac{rF+\overline{A}}{A}\}}{\alpha\sigma} + \frac{\tau(1-1/\sigma)(A-rF-\overline{A})}{wM^{1/\sigma}}]$$

while θ(M) can be written as

$$\theta(M) = \mu M + \mu[\tau M^{1-1/\sigma}(A - \bar{A} - rF)/w] \quad \text{(using (18) and (23))}$$

Comparing the expressions for Mθ'(M) with that for θ(M) we see that the first term μM is common to both and the second term in θ(M) exceeds the sum of the second and third terms in Mθ'(M) :as

$$\sigma > 0,\ 1-1/\sigma < 1$$

so

$$\mu M[\frac{\tau(A-rF-\overline{A})}{wM^{\frac{1}{\sigma}}}] > \mu M[\frac{\tau(1-1/\sigma)(A-rF-\overline{A})}{wM^{1/\sigma}}]$$

$$> \mu M[\frac{\tau(1-1/\sigma)(A-rF-\overline{A})}{wM^{1/\sigma}} - \frac{\lambda\tau\{1-(1-\alpha)\frac{rF+\overline{A}}{A}\}}{\alpha\sigma}]$$

or $\qquad M\theta'(M) < \theta(M)$

$$\text{Thus, } \frac{d\ln\theta}{d\ln M} < 1.$$

If θ(M*) = M* is a closed economy equilibrium, we have, for any M > M*,

$$\int_{M^*}^{M} d\ln\theta < \int_{M^*}^{M} d\ln M$$

$$\text{Or } \ln\theta(M) - \ln\theta(M^*) < \ln M - \ln M^*$$

Or θ(M) < M for all M >M* (as θ(M*) = M*).

Similarly, we can prove that θ(M) > M for all M < M*.

Thus, the closed economy equilibrium is unique.

Proof of Proposition 2

From equations (1)-(3) and (6)-(7), we can express labor in agriculture, manufacturing and infrastructure in terms of labor in manufacturing and manufactured output. Using this in the full employment equation (8), we have

$$L_m + \left(\frac{M}{L_m^{\beta}}\right)^{\frac{1}{\delta(1-\beta)}} + \left(\frac{\alpha L_m}{\beta p M}\right)^{\frac{1}{1-\alpha}} = L \tag{A1}$$

After total differentiation and manipulation, we get

$$\frac{dL_m}{dM} = \frac{\frac{1}{1-\alpha}\left(\frac{\alpha}{\beta p}\right)^{\frac{1}{1-\alpha}}\left\{\frac{L_m}{M^{2-\alpha}}\right\}^{\frac{1}{1-\alpha}} - \frac{1}{\delta(1-\beta)}\left\{\frac{M^{1-\delta(1-\beta)}}{L_m^{\beta}}\right\}^{\frac{1}{\delta(1-\beta)}}}{1-\frac{\beta}{\delta(1-\beta)}\left\{\frac{M}{L_m^{\beta+\delta(1-\beta)}}\right\}^{\frac{1}{\delta(1-\beta)}} + \frac{1}{1-\alpha}\left(\frac{\alpha}{\beta p}\right)^{\frac{1}{1-\alpha}}\left\{\frac{L_m^{\alpha}}{M}\right\}^{\frac{1}{1-\alpha}}} \tag{A2}$$

From (2) and (3),

$$L_i^{\delta(1-\beta)} = ML_m^{-\beta} \tag{A3}$$

Totally differentiating, we have

$$\delta(1-\beta)L_i^{\delta(1-\beta)-1}\frac{dL_i}{dM} = L_m^{-\beta} - \beta ML_m^{-\beta-1}\frac{dL_m}{dM}$$

Or

$$\frac{dL_i}{dM} = \frac{L_i^{1-\delta(1-\beta)}}{\delta(1-\beta)L_m^{\beta}} - \frac{\beta ML_i^{1-\delta(1-\beta)}}{\delta(1-\beta)L_m^{1+\beta}}\frac{dL_m}{dM} \tag{A4}$$

From (A3) and (A4)

$$\frac{dL_i}{dM} = \frac{1}{\delta(1-\beta)}\left(\frac{M^{1-\delta(1-\beta)}}{L_m^{\beta}}\right)^{\frac{1}{\delta(1-\beta)}} - \frac{\beta}{\delta(1-\beta)}\left(\frac{M}{L_m^{\beta+\delta(1-\beta)}}\right)^{\frac{1}{\delta(1-\beta)}}\frac{dL_m}{dM} \tag{A5}$$

As

$$w = \frac{\beta pM}{L_m} \tag{A6}$$

We have

$$\frac{dw}{dM} = \frac{\beta p}{L_m} - \frac{\beta pM}{L_m^{2}}\frac{dL_m}{dM} \tag{A7}$$

Average cost is given by

$$\frac{C(M,p^*)}{M}=\frac{w(L_m+L_i)}{M}$$
$$=\frac{\beta p}{L_m}\{L_m+(ML_m{}^{-\beta})^{\frac{1}{\delta(1-\beta)}}\}=\beta p[1+\{\frac{M}{L_m{}^{\beta+\delta(1-\beta)}}\}^{\frac{1}{\delta(1-\beta)}}] \tag{A8}$$

[using (A3) and (A6)].

Marginal cost is given by

$$C^{'}(M,p^*)=w(\frac{dL_m}{dM}+\frac{dL_i}{dM})+\frac{dw}{dM}(L_m+L_i)$$
$$=\frac{\beta pM}{L_m}[\frac{dL_m}{dM}\{1-\frac{\beta}{\delta(1-\beta)}(\frac{M}{L_m{}^{\beta+\delta(1-\beta)}})^{\frac{1}{\delta(1-\beta)}}\}+\frac{1}{\delta(1-\beta)}(\frac{M^{1-\delta(1-\beta)}}{L_m{}^{\beta}})^{\frac{1}{\delta(1-\beta)}}] \tag{A9}$$
$$+[\frac{\beta p}{L_m}-\frac{\beta pM}{L_m{}^2}\frac{dL_m}{dM}]L_m[1+(\frac{M}{L_m{}^{\beta+\delta(1-\beta)}})^{\frac{1}{\delta(1-\beta)}}]$$

[using (A3),(A5), (A6) and (A7)].

Now at any turning point of the AC curve, we must have AC = MC. Equating (A8) and (A9), canceling common terms from both sides, and rearranging, we must have at this point,

$$\frac{\beta pM}{L_m}\frac{dL_m}{dM}[1+(\frac{M}{L_m{}^{\beta+\delta(1-\beta)}})^{\frac{1}{\delta(1-\beta)}}]$$
$$=\frac{\beta pM}{L_m}[\frac{dL_m}{dM}\{1-\frac{\beta}{\delta(1-\beta)}(\frac{M}{L_m{}^{\beta+\delta(1-\beta)}})^{\frac{1}{\delta(1-\beta)}}\}+\frac{1}{\delta(1-\beta)}(\frac{M^{1-\delta(1-\beta)}}{L_m{}^{\beta}})^{\frac{1}{\delta(1-\beta)}}]$$

Or (after further cancellation),

$$\frac{dL_m}{dM}\{\beta+\delta(1-\beta)\}(\frac{M}{L_m{}^{\beta+\delta(1-\beta)}})^{\frac{1}{\delta(1-\beta)}}=(\frac{M^{1-\delta(1-\beta)}}{L_m{}^{\beta}})^{\frac{1}{\delta(1-\beta)}}$$

From this we immediately get

$$\frac{dM}{dL_m}\frac{L_m}{M}=\beta+\delta(1-\beta)=\sigma \tag{A10}$$

(reverting to our earlier notation). This must hold at the minimum point of the AC.

Now we know, from (2) and (3), that

$$M=L_m{}^{\beta}L_i{}^{\delta(1-\beta)}$$

So, total differentiation gives us

$$dM=\frac{\beta M}{L_m}dL_m+\frac{\delta(1-\beta)M}{L_i}dL_i$$

Or

$$\frac{dM}{dL_m}\frac{L_m}{M} = \beta + \frac{\delta(1-\beta)L_m}{L_i}\frac{dL_i}{dL_m} \qquad \text{(A11)}$$

From (A10) and (A11), at the minimum point of the AC, we must have

$$\frac{dL_m}{L_m} = \frac{dL_i}{L_i} \qquad \text{(A12).}$$

Now $w = \alpha/L_a^{1-\alpha}$

So

$$dw/w = -(1-\alpha)d L_a/L_a = (1-\alpha)(dL_i + dL_m)/L_a \qquad \text{(A13)}$$

As $C = w(L_i + L_m)$,

$dC/C = dw/w + (dL_i + dL_m)/(L_i + L_m)$

$=(dL_i + dL_m)\{(1-\alpha)/L_a + 1/(L_i + L_m)\}$

$= dL_m (1 + L_i/L_m) \{(1-\alpha)/L_a + 1/(L_i + L_m)\}$ [from (A12)]

$= dL_m (L_m + L_i)/L_m\{(1-\alpha)/L_a + 1/(L_i + L_m)\}$

$= dL_m /L_m \{(1-\alpha)(L_m + L_i)/L_a + 1\}$

$= (dL_m /L_m)(1-\alpha) L/L_a$

But at this point, $dM/M = \sigma dL_m /L_m$

At a stationary point of the AC curve, C and M must be changing at the same proportional rate: $dC/C = dM/M$.

So

$$(1-\alpha) L/L_a = \sigma$$

$$\text{Or } L_a/L = (1-\alpha)/\sigma \qquad \text{(A 14)}$$

As the economy industrializes (M increases), C/M sinks to a minimum as the share of agriculture in the labor force drops to $(1-\alpha)/\sigma$ and rises thereafter to infinity. The minimum point is unique.

Appendix B : Changing the numeraire good

In the text of our paper, we have taken food as the numeraire good and used p to denote the relative price of manufactures in terms of food. Here we look at the implications of taking manufactures as the numeraire good. Let p now denote the relative price of food in terms of manufactures. Before going into details, we summarize the implications of changing the numeraire good for our results. For the naïve infrastructure monopolist, the results are *qualitatively* identical with a unique autarky equilibrium, a U-shaped supply curve with the same turning point as before, and multiple equilibria in an open economy. For the sophisticated monopolist, there is a difference in the autarky case where complete de-industrialization is no longer optimal for him : instead incomplete specialization is. However, the open economy analysis for the sophisticated monopolist yields identical results, with economies of scale persisting beyond the point where they did for a naïve monopolist (the turning point being identical to that in the text), and with a unique equilibrium at a high level of industrialization.

Equations (1)-(4) and (8) remain the same. (5) becomes

$$M_d = \mu(M + p(A - rF - \overline{A})) \tag{5'}$$

The marginal productivity equations for the wage rate w (expressed in terms of manufactures) become

$$w = \alpha p L_a^{\alpha-1} = \beta(L_m / I)^{\beta-1} \tag{6'}$$

The price q of infrastructural services is now the marginal *physical* product of infrastructure in manufacturing :

$$q = (1-\beta)(L_m / I)^{\beta} \tag{9'}$$

Equations (10)-(15) remain the same, except for the fact that rF in (13) is multiplied by p. Substitutions lead to a positive relationship between infrastructure and manufactures which continues to be expressed by equation (16).

We now attempt to find a relationship between the supply of manufactures, M and the relative price of manufactures to food, 1/p, to parallel the supply curve of manufactures we derived in the text. Using substitutions, we find

$$L_a = L - \lambda M^{1/\sigma}$$

and

$$w = \tau M^{1-1/\sigma} \tag{18'}$$

Using these along with (6') enables us to write

$$1/p = \varphi(M) = \frac{\alpha}{[L-\lambda M^{1/\sigma}]^{1-\alpha}\tau M^{1-1/\sigma}}$$

As M→0, and L_a→L, from (18') w→0, so from (6') 1/p→∞. As L_a→0 and M tends to the finite maximum that this implies, we find from (6') that w/p→∞, and from (18'), that w tends to a finite limit : hence 1/p→∞. This leads us to conjecture that φ(M) is U-shaped. Differentiation yields

$$\varphi'(M) = \frac{\alpha}{\tau}[\frac{\lambda(1-\alpha)}{\sigma}L_a^{-2+\alpha}M^{(2-2\sigma)/\sigma} - \frac{(\sigma-1)}{\sigma}L_a^{-1+\alpha}M^{(1-2\sigma)/\sigma}]$$

φ'(M)≥0 or φ'(M)<0 as $(1-\alpha)\tau M^{1/\sigma} \geq \sigma L_a$ or $(1-\alpha)\tau M^{1/\sigma} < \sigma L_a$

or as $\frac{L-L_a}{L_a} \geq \frac{\sigma-1}{1-\alpha}$ or $\frac{L-L_a}{L_a} < \frac{\sigma-1}{1-\alpha}$.

This is identical with the case in the text : the supply cuve first decreases and then increases, with economies of scale being exhausted at the same turning point as before.

From (5'), we may write the demand function for manufactures in terms of manufactured output as

$$M_d = \mu[M+\{(L-\lambda M^{1/\sigma})^{\alpha} - \bar{A} - rF\}/\varphi(M)] = \theta(M)$$

Though identical to (23) in the text, we note that φ(M) stands for 1/p here. It can be proved that this demand function, when plotted against the output of manufactures, starts at a positive intercept above the 45 degree line, and has a unique intersection (corresponding to a unique autarky equilibrium) after which it goes below the 45 degree line. The method of proof is similar to that followed in the proof of Proposition 1 in Appendix A.

As M → 0, L_a → L and

$$M\,\varphi(M) = \frac{\alpha M^{1/\sigma}}{L^{1-\alpha}\tau} \to 0$$

Just as in Appendix A, we can also show that M/θ(M) → 0 as M→ 0 and that therefore, θ(M)>M for small M (this establishes a positive vertical intercept for the demand curve as a function of M). M assumes its maximum value M_{max} when agricultural employment dwindles to zero. At this limit, θ(M) = μ(M-prF-p$\bar{A}$) < M. Since θ(M), a continuous function, passes from values greater than M to values less than it as M increases from 0 to M_{max}, it must have a fixed point.

Now we can write

$$\theta(M) = \mu M + \mu[\frac{(A-\overline{A}-rF)\tau M^{1-1/\sigma}(L-\lambda M^{1/\sigma})^{1-\alpha}}{\alpha}]$$

$$M\,\theta'(M) = \mu M[1+(1-\frac{1}{\sigma})\frac{(A-\overline{A}-rF)\tau(L-\lambda M^{1/\sigma})^{1-\alpha}}{\alpha M^{1/\sigma}} - \frac{\tau\lambda}{\alpha\sigma}\{1-(1-\alpha)\frac{\overline{A}+rF}{A}\}]$$

Comparison readily tells us that $\theta(M) > M\,\theta'(M)$ or $\frac{d\ln\theta}{d\ln M}<1$. Now we follow exactly the same steps as in the case in Appendix A to prove uniqueness of the closed economy equilibrium.

As supply and demand for manufactures behave in the same way as in our main case when the monopolist is naïve, the analysis for the open economy case remains the same : multiple equilibria may obtain with the same type of trade patterns as described in the text.

What will happen in the case of the Schumpeterian monopolist once we change the numeraire good? In a closed economy, demand and supply of manufactures are equal. The sophisticated monopolist maximizes profits which we now write in terms of the manufactured good.

$$\Pi = M_d - w(L_m + L_i) - prF$$

$$= \frac{1}{1-\mu}[\frac{\mu(A-\overline{A})-rF}{\alpha}(L-\lambda M^{1/\sigma})^{1-\alpha}\tau M^{1-1/\sigma}] - \tau M^{1-1/\sigma}\lambda M^{1/\sigma}$$

After simplification and substitutions, and using $M_d = M$, profit maximization yields the following first order condition with respect to M:

$$\frac{(1-\alpha)\lambda M^{1/\sigma}}{L-\lambda M^{1/\sigma}} = (\sigma-1) - \tau\lambda\{\sigma\alpha(1-\mu)+\mu\} = \kappa$$

Now if κ is positive, this leads to a choice of M in the closed economy equilibrium such that

$$\frac{L-L_a}{L_a} = \frac{\kappa}{1-\alpha} < \frac{\sigma-1}{1-\alpha}$$

Thus the share of the non-agricultural sector chosen in autarky by a sophisticated monopolist is smaller than the size the non-agricultural sector reaches under a naïve monopolist at the turning point (where increasing returns to scale are exhausted). This may be smaller than the industrialization level chosen in autarky with a naïve monopolist, because as shown in the text, autarky equilibrium with a naïve monopolist may well be to the right of the turning point. Moreover, if parameters are such that κ is non-positive, there is complete de-industrialization in autarky with a sophisticated monopolist. This parallels the case in the text.

When the economy is opened up, p is exogenously set by the world market at the level $\tilde{p}$. As in the text, we let $C(M, \tilde{p}) = w(L_m + L_i)$. Since we are using manufactures as the numeraire, we replace $\tilde{p}$ in condition (34) by 1 and multiply rF in condition (36) by $\tilde{p}$. Now the function $C(M; \tilde{p})/M = w(L_m + L_i)/M$ (the average cost function) may be written as

$$C(M; \tilde{p})/M = \frac{\alpha \tilde{p}}{L_a^{1-\alpha}} \lambda M^{-(1-1/\sigma)}$$

It is easy to see that this tends to infinity as $M \to 0$ and also tends to infinity as $L_a \to 0$ and M tends to a finite maximum. The average cost curve is U-shaped and we can prove that it has a unique minimum at a level of industrialization which is greater than that prevailing at the turning point in the naïve monopolist case (so that economies of scale persist to a greater degree in the sophisticated monopolist's case). This turning point is identical to that in the case where food is the numeraire good – i.e where $\frac{L - L_a}{L_a} = \frac{\sigma - 1 + \alpha}{1 - \alpha}$. Moreover, equilibrium is necessarily on the rising segment of the average cost curve for profit maximization, therefore the degree of industrialization in an open economy under a sophisticated monopolist is very high.

The proof of the unique minimum is exactly identical with the proof of Proposition 2 in Appendix A except that 1/p in (A1) is replaced by p, and p disappears from (A6),(A7), (A8) and (A9), while the definition of w used to derive (A13) is w = $\alpha p L_a^{-(1-\alpha)}$. Note that wherever p appears in this proof, it is set at $\tilde{p}$.

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[1] Harris (1992), Ichikawa (1992), Kikuchi and Ishikawa (2002), Okuno-Fujiwara (1988), Venables (1996) for instance.

[2] It is possible that the government may play a co-ordinating role persuading naïve infrastructure providers to choose the most highly industrialized of the multiple "naïve" equilibria. However, this attributes a high degree of foresight to the government.