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*Production and financial linkages*

*in inter-firm networks:*

*structural variety, risk-sharing and resilience*

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PROVINCIA AUTONOMA DI TRENTO

# Production and financial linkages in inter-firm networks: structural variety, risk-sharing and resilience*

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October 15, 2010

**Abstract**

The paper analyzes how (production and financial) inter-firm networks can affect firms' default probabilities and observed default rates: an issue the recent crisis has brought to the front of the debate. A simple theoretical model of shock transfer is built up to investigate some stylized facts on how firm-idiosyncratic shocks tend to be allocated in the network, and how this allocation changes firms' default probability.

The model shows that the network works as a perfect "risk-pooling" mechanism, when it is both strongly connected and symmetric. But the resort to "risk-sharing" does not necessarily reduce default rates in the network, unless the shock they face is lower on average than their financial capacity. Conceived as cases of symmetric inter-firm networks, industrial districts might have a comparative disadvantage in front of "heavy" financial crises such as the current one.

*Keywords*: Firm clusters; industrial districts; interlinking transactions; resilience; systemic risk.

*JEL Classification*: R11; R12; G20.

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# 1 Introduction

The world is now experiencing the economic tail of the sub-prime financial crisis burst in 2007. The idiosyncrasy of this crisis (e.g. Shiller, 2008; Reinhart and Rogoff, 2008, 2009) and the different resilience exhibited by countries (e.g. Horta et al., 2008; Frenkel and Rapetti, 2009) have already been investigated. On the contrary, at the best of our knowledge, the resilience of the different models of production organization to the crisis has received no attention.

Nonetheless, this is a further test for the alleged superiority of the "flexible specialization" model of production (e.g. Storper and Christopherson, 1987; Hirst and Zeitlin, 1989; Storper, 1995; Herrigel, 1996; Le Heron, 2009), and of industrial districts in particular (e.g. Harrison, 1992; Guerrieri et al., 2003).

In trying to fill this gap, the paper analyzes how inter-firm (production and financial) networks can actually affect the firms' resilience to financial shocks. In particular, it focuses on the risk of default entailed by firm-specific credit constraints and investigates: (i) how the allocation of risk is affected by the structure of such networks; (ii) how this allocation changes firms' default probabilities.

The remainder of the paper is organized as follows. In Section 2, we review and combine the theoretical literature on network models of risk-sharing, contagion and financial stability, with the empirical literature on the different typologies of inter-firm networks. In Section 3, we build up a simple theoretical model to analyze the transfer mechanisms of firms' idiosyncratic financial shocks in inter-firm networks. Section 4 presents the main theoretical results of the model. Section 5 discusses their empirical implications and concludes.

# 2 Background literature and stylized facts

The scope and speed of diffusion of the recent financial crises have stimulated the analysis of the conditions under which *financial contagion* can actually arise. Diamond and Dybvig (1983) refer to phenomena of "bank run" and self-fulfilling panic in the banking system. Drawing on them, Allen and Gale (2000) analyze how interbank lending brings about domino-like effects, which could increase the risk of collapse of the whole financial system, that is "systemic risk". Iori et al. (2006), Nier et al. (2007), Gallegati et al. (2008) and Battiston et al. (2009) have recently re-examined this issue and found a possible trade-off between the mutual insurance of financial institutions and systemic risk.

Out of the financial literature, the conditions for domino effects (here called "cascading failures") to occur and produce "global cascades" have been studied by Watts (2002), Motter and Lai (2002) and Whitney (2009). Along the same research stream, diffusion and contagion in networks have been investigated by, among the others, Pastor-Satorras and Vespignani (2001, 2002), Dodds and Watts (2005) and López-Pintado (2008). Finally, recent economic studies have analyzed the efficient and stable configurations of "risk-sharing networks", whose links guarantee the nodes bilateral mutual insurance (Bramoullé and Kranton, 2007b,a; Fafchamps and Gubert, 2007; Bloch et al., 2008).

However, this literature has not yet been applied to investigate how inter-firm networks affect the resilience of firms in clusters to external shocks. This is unfortunate, as a number of empirical and theoretical studies have addressed inter-firm relations in clusters and their actual structures. Taxonomies of them (constellations, hub-and-spokes, satellite platforms and different kinds of industrial districts) have been put forward by Markusen (1996), Paniccia (1998) and Carbonara (2002). Their evolutionary patterns have been studied in an *industry life-cycle* perspective by Carbonara et al. (2002) and Albino et al. (2006, 2007) – for supply chains in industrial districts – and, more recently, by Ter Wal and Boschma (2009) – in terms of co-evolutionary processes of industries, firms and networks in clusters. Finally, the structure of ownership and non-ownership ties in industrial districts, their evolution over time and the idiosyncrasies of business groups in industrial districts have been investigated by Brioschi et al. (2002, 2004).

These inter-firm networks are mainly made up of production linkages between different typologies of firms (e.g. final producers vs. subcontractors) and heterogeneous capabilities.[1] These production linkages become extremely important in the aftermath of crises that expose firms to demand declines and credit restrictions from formal banking institutions.[2]

Indeed, inter-firm *production* relationships usually entail inter-firm *credit* relationships. On the one side, firms can obtain credit from subcontractors through payments delays: a *trade credit* which requires different contractual power between the parties (Peterson and Rajan, 1997). On the other side, the supplier may obtain credit from the buyer on the basis of an underly-

[1]Firms look for networking also in other spheres, such as innovation. For an analysis of the networks of R&D collaborations see, for instance, Orsenigo et al. (2001), Goyal and Moraga-Gonzalez (2001) and Goyal and Joshi (2003).

[2]Alessandrini et al. (2008, 2009) and Alessandrini and Zazzaro (2008) suggest that local banking systems, affecting information asymmetries between lenders and borrowers at the local level, can reduce firms' financing constraints. As a matter of fact, physical proximity, involving long-lasting relationships and in-depth cultural affinity, allows local banks to collect a greater amount of "soft" information on local borrowers, thus increasing the quality of screening and monitoring. Nonetheless, since bank decisional centers have been concentrated over the last decade in a few places, the "functional" distance between banks and local production systems is increased, thus balancing the positive effects of local closeness. Their findings show that these negative effects prevail on those positive effects associated to "operational" distance, making firms' financing constraints more binding.

ing commercial transaction, possibly by discounting the refunding from the relative payment.

As Dei Ottati (1994) argued in investigating the industrial district of Prato (Florence, Italy), in local production systems of this kind the *interlinking of credit and subcontracting* is the common mean to overcome the problems entailed by supplier-user bilateral monopolies, and to make credible mutual commitments in long-term relationships (Williamson, 1983, 1985). Indeed, in order to be implemented, this interlinking requires a minimum level of cooperation and mutual trust between firms. On the other hand, it helps reducing the emergence of opportunistic behaviors and thus tends to increase the level of social capital in turn. Hence, as much empirical literature shows – in particular, but not only, with respect to the Italian economy – the interlinking of credit and subcontracting is a relevant phenomenon when spatial proximity, face-to-face contacts, long-lasting relationships and in-depth social and cultural closeness play an important role (Cainelli, 2008).[3]

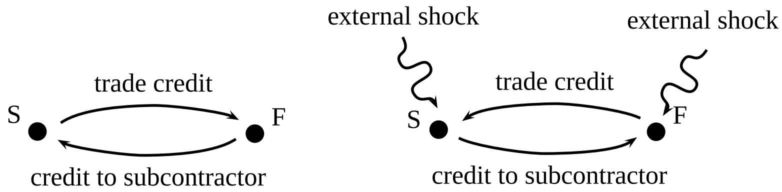
# 3 Model

Let us consider a network of $n$ firms. Assume these firms can be linked through production relationships only, in which one firm acts as supplier ($S$) of intermediate commodities for another final producer ($F$).[4] On the basis of the previous arguments (see Section 2), these production relationships could entail two possible credit relationships between $S$ and $F$: (i) trade credit, that is the credit granted by $S$ to $F$ via payments delays, whose extent depends on the relative contractual power of the parties; (ii) the credit to the subcontractor, which is granted by $F$ to $S$ in a interlinking of subcontracting and credit, as a mean to reduce opportunistic behaviors and sustain long-term relationships (Figure 1(a)).

In front of firm-specific (i.e. individual) shocks, such as, for instance, credit restrictions from banks or demand downturns, these credit channels can be conceived as possible transfer mechanisms of the shock, or part of it, between $S$ and $F$. In particular, trade credit can allow $F$ to transfer the shock to $S$. The credit granted to the subcontractor, as well as the pre-existence of a credit relationship, can instead enable $S$ to transfer the shock to $F$ (Figure 1(b)).

[3]As is well-known, the Italian manufacturing system is an emblematic example of the coexistence of all these features. On the one hand, most of the Italian manufacturing activities are concentrated within local systems of small and medium firms and industrial districts. On the other hand, in these local production systems subcontracting and trade credit relationships are very pervasive too Omiccioli (2000); Cocozza (2000).

[4]Although at the price of a certain lack of realism, the model is kept in its simplest benchmark version, in order to better show its functioning and potentiality.




(a) Credit linkages (b) Shock transfer (dual)

Figure 1: Inter-firm credit relationships and shock transfer

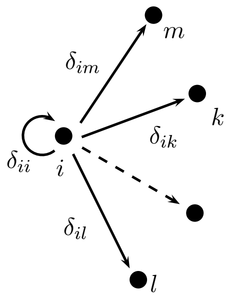



Figure 2: Shock transfer

The working of these transfer mechanisms crucially depends on the network structure. In order to study this effect, assume that each firm $i$ of the $n$ in the network is hit by an external shock $x_{i0}$. We then represent the transfer mechanisms of these idiosyncratic shocks by a *weigthed directed network* $\Gamma$, where the *valued directed edge* from firm/node $i$ to firm/node $j$ ($\delta_{ij}$) measures the share of $i$'s total idiosyncratic shock that $i$ can transfer to $j$ ($\sum_{j=1}^{n} \delta_{ij} = 1$) (Figure 2).

With this assumptions, the shock experienced by firm $i$ after one *round* ($x_{i1}$) is simply equal to:

$$x_{i1} = \sum_j \delta_{ji} x_{j0}$$

and:

$$\begin{pmatrix} x_{11} & \dots & x_{n1} \end{pmatrix} = \begin{pmatrix} x_{10} & \dots & x_{n0} \end{pmatrix} \begin{pmatrix} \delta_{11} & \dots & \delta_{1n} \\ \vdots & \ddots & \vdots \\ \delta_{n1} & \dots & \delta_{nn} \end{pmatrix}$$

or, in matrix form:

$$\mathbf{x}'_1 = \mathbf{x}'_0 \mathbf{T} \tag{1}$$

where $\mathbf{T}$ is the *adjacency matrix* of the network $\Gamma$, and $\mathbf{x}'_t$ the row vector of firm-specific shocks after $t$ rounds.

It follows that:

$$\mathbf{x}'_t = \mathbf{x}'_{t-1} \mathbf{T} = \mathbf{x}'_0 \mathbf{T}^t \tag{2}$$

$$\hat{\mathbf{x}}' = \mathbf{x}_0' \left( \lim_{t \to \infty} \mathbf{T}^t \right) \tag{3}$$

where $\hat{\mathbf{x}}'$ is the row vector of the allocation of the idiosyncratic shocks in equilibrium.

We model the initial exogenous shock $\mathbf{x}_0$ as a random vector whose generic element $x_{i0}$ is made up of of a common trend ($\mu$) and an idiosyncratic random component ($\epsilon_i$):[5]

$$x_{i0} = \mu + \epsilon_i \tag{4}$$

Moreover, each node/firm $i$ is characterized by a given *threshold* $\theta_i$, which represents its resistance to external shocks.

Assuming that $\hat{\mathbf{x}}$ exists and it is in fact unique, the default condition for firm $i$ can be:[6]

$$\hat{x}_i(\mathbf{x}_0; \mathbf{T}) > \theta_i \tag{5}$$

# 4 Results

Of the simple model above, we first search for the limit distribution of the idiosyncratic shocks in the network and, then, provided that it exists and is indeed unique, investigate its impact on firms' default probabilities. In so doing, we will analyze the change in the firms' default rate induced by the network of shock transmissions formalized by $\mathbf{T}$.

The model results are first presented here in formal terms, while their economic implications are discussed in the following section.

## 4.1 Shock transfer and risk distribution

As far as the analysis of the limit distribution of the idiosyncratic shocks in the network is concerned, it is important to note that, in spite of the fact that, for what concerns the shock transfer, our model is not probabilistic, $\mathbf{T}$ is formally a *right stochastic matrix*. Hence, in order to study the allocation of shocks in equilibrium, we can use a number of useful results from the theory of finite Markov chains.[7]

First of all, following the standard definitions, we define the network $\Gamma$ (and the related matrix $\mathbf{T}$) as *strongly connected* if each node can reach every other by a *directed path*, i.e. a sequence of distinct nodes $i_1, i_2, \ldots, i_K$ such that $T_{i_k, i_{k+1}} > 0$, for each $k \in \{1, 2, \ldots, K\}$.

[5]This idiosyncratic component can capture the individual differences in the experienced shock or in the buffer level of the internal absorption of the shock.

[6]The higher (lower) the speed of converge of the system to the equilibrium, the more (less) reasonable such condition is. We will return on the issue later.

[7]For a textbook treatment of Markov chains see, for instance, Karlin and Taylor (1975, 1981) and the references therein. Iterated iteraction matrices have been used also in studies on convergence of beliefs in networks (DeGroot, 1974; DeMarzo et al., 2003; Golub and Jackson, 2009), prestige and status (Bonacich, 1987; Rogers, 2006), and strategic games in networks with neighbours' influence (Ballester et al., 2006).

Let us say that the network $\Gamma$ (and the related matrix $\mathbf{T}$) is *aperiodic* if the greatest common divisor of the lengths of its directed cycles is 1, where a *directed cycle* is a directed path joining a node to itself, and the *length* of the cycle is the number of distinct nodes in the path.[8]

We can therefore state our first proposition.

**Proposition 1** *If the inter-firm network $\Gamma$ is strongly connected and aperiodic, the system always reaches an equilibrium in which each firm bears a definite proportion ($s_i$) of the sum of all the idiosyncratic shocks ($\sum_i x_{i0}$):*

$$\hat{\mathbf{x}}' = \mathbf{s}'(\textstyle\sum_i x_{i0})$$

*where $\mathbf{s}'$ is the left eigenvector of $\mathbf{T}$ corresponding to eigenvalue 1 whose entries sum to 1: $\mathbf{s}'(\mathbf{I} - \mathbf{T}) = \mathbf{0}$, $\sum_i s_i = 1$.*

Let us note first that the condition of aperiodicity for the network is rather weak and can be assumed as almost always satisfied in the present framework. Indeed, a sufficient condition for the network to be aperiodic is that there is at least one loop (i.e. $\delta_{ii} > 0$ for some $i$), that is at least one firm which is not able to transfer all the experienced shock in each round.

A formal proof of the proposition is provided in the Appendix. Here we present a simple illustrative example in case of a network made up of three firms – 1, 2, 3 – and with the following structure (Figure 3(a)):

$$\mathbf{T} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

In this case, we have:

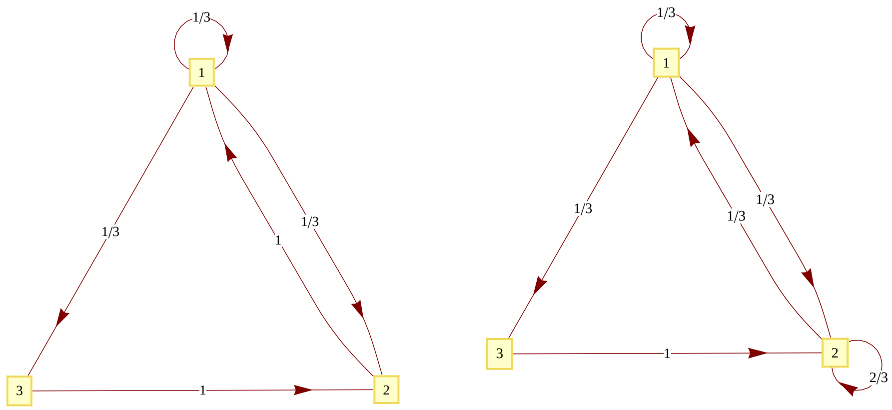
$$\mathbf{T}^{25} \approx \mathbf{T}^{26} = \ldots = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

So, although firms 2 and 3 are able to transfer all the one-round shock to the others, the system soon converges to the equilibrium and such equilibrium entails the following redistribution of the sum of all the shocks: $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$. Thus, when firms are hit by the same initial shock, the system of relations is detrimental (beneficial) for firm 1 (3).[9]

In general, thus, the proportion of the sum of all the shocks that, in equilibrium, accrues to each firm is not the same. However, given a *symmetric* network, i.e. a network whose adiacency matrix is symmetric ($\delta_{ij} = \delta_{ji}$ for each $i, j$), the following proposition holds:

[8] Strictly speaking, a cycle is not a path because the starting (and ending) node appears twice. However, apart from this minor inconsistency, the definition is correct and is made here for convenience.

[9] So, for instance, if $\mathbf{x}'_0 = (100, 100, 100)$, we have $\hat{\mathbf{x}}'_0 = (150, 100, 50)$




(a) (b)

Figure 3: Asymmetric strongly connected networks

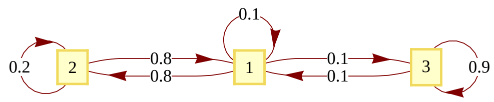



Figure 4: A symmetric strongly connected network

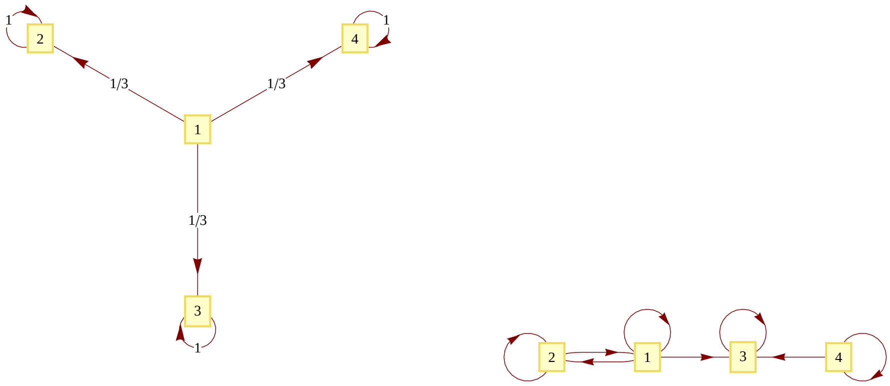
**Proposition 2** *If the network is strongly connected, aperiodic and symmetric, the risk distribution is equalitarian, i.e. each firm gets in equilibrium a common shock amounting to the average of all the shocks:*

$$\hat{x}_i = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}, \qquad \forall i \in N$$

A formal proof of the statement is given in the Appendix. Let us note here that, apart from the strong connectivity and aperiodicity, the condition for this "equalitarian" distribution to occur is not that all the linkages are equal, but only that they are perfectly reciprocated, as the example of Figure 4 illustrates. In this simple three-firm network, the equilibrium values are: $s_1 = s_2 = s_3 = \frac{1}{3}$, in spite of the fact that the structure of relations of the three firms is strongly different.[10]

As we better argue in Section 5, the symmetry condition could be retained to hold in case of industrial districts and, more in general, of local production systems characterized by a high level of social capital, sys-

[10]Indeed, at a first glance, firm 1 might look in a better position than 3, because it is able to transfer a much greater portion of its initial shock to the others (0.9 against 0.1 of firm 3).




(a) (b)

Figure 5: Shock releasers/absorbers in non strongly connected networks

tems in which the interlinking of credit and subcontracting is used to rebalance the contractual power of the parties in the supplier-user relationships (Dei Ottati, 1994).

In general, however, when the symmetry condition does not hold, the risk distribution is not equalitarian and a different portion of the total shock accrues to firms ($s_i \neq s_j$). What is important is that such portion depends on the overall structure of relations.

An example is provided in Figure 3(b), where, despite the simmetry in the reciprocal relationship, 1 and 2 get different parts of the total shock because of the differences in their relation with 3: $(s_1, s_2, s_3) = (.3, .6, .1)$.

Finally, when the network is not strongly connected, one or more nodes/firms may have zero *outdegree* or *indegree*, i.e. no outgoing or ingoing linkages.

Nodes with zero indegree and a positive outdegree are *shock releasers*: firms which in the limit are always able to transfer their shock to the others completely. This is the case showed in Figure 5(a), where firm 1 (e.g. a final firm) transfers all its shock to the others (its suppliers via trade credit).

Nodes with zero outdegree and a positive indegree can be called *shock absorbers*: they are firms that sustain the shock of the others without being able to transfer theirs. This is the case of firms 2, 3 and 4 in the previous example,[11] as well as of firm 3 in Figure 5(b).

[11] In the example, each supplier (firms 2-4) faces its idiosyncratic shock plus a fraction (1/3) of the buyer's shock.

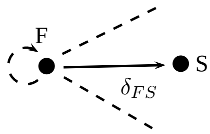



Figure 6: A supplier-buyer relation

### 4.2 Risk distribution and default probabilities

By affecting the actual allocation of idiosyncratic shocks, inter-firm networks can change firms' *default probabilities* and, via this, observed *default rates*.

In this section, we investigate only the two most paradigmatic cases, that is: i) the default probability of a supplier in a subcontracting relation without interlinking credit (Figure 6); ii) the default rate of firms in a symmetric and strongly connected network (such as that illustrated by Figure 4). In addition, we provide some insights about the more complex situations of strongly and asymmetric networks.

The section ends up with a discussion on the speed of convergence, which is of utmost importance. In fact, given our analysis of default probabilities at the equilibrium, the lower the speed at which the system reaches the equilibrium, the more unrealistic is our operational suggestion to analyze default probabilities at the equilibrium allocation.

#### 4.2.1 Default probabilities of suppliers in supplier-buyer relations

As for the default probabilities of suppliers in a supplier-buyer relation, given the idiosyncratic shock:

$$x_i = \mu + \epsilon_i$$

assuming that $\epsilon_i \sim (0, \sigma^2)$, the shock faced by the supplier is:

$$\hat{x}_S = x_{S0} + \delta_{FS} x_{F0} \sim \left( (1 + \delta_{FS})\ \mu,\ (1 + \delta_{FS}^2)\ \sigma^2 \right)$$

Thus, the shock experienced by the supplier ($\hat{x}_S$) is higher on average and it is also more volatile than its initial shock ($x_{S0}$). Clearly, this increases its default probability with respect to the one of an isolated firm facing the shock $x_i$ (Figure 7).

#### 4.2.2 Default rates in strongly connected networks

In case of firms in a symmetric and strongly connected network, from Proposition 2 it follows that $\hat{x}_i = \hat{x}_j = \bar{x}_0$, for each $i, j$. Still assuming that $\epsilon_i \sim (0, \sigma^2)$, this implies:

$$\hat{x}_i = \bar{x}_0 \overset{a}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

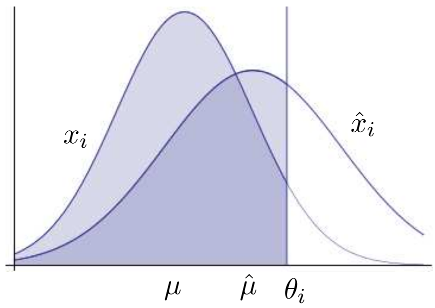



Figure 7: Default probabilities of supplier

Thus, given that each firm faces in equilibrium an average of all the shocks, its volatility is reduced. This leads us to state two further propositions.

**Proposition 3** *Assume that the idiosyncratic components of the shocks are independently distributed and the number of firms in the network sufficiently large. Then, the default probability of a firm in a symmetric and connected network is higher (lower) than that of the same firm in isolation when the expected value of the random shock is higher (lower) than its threshold.*

A formal proof of this proposition is in the Appendix. Figure 8 shows the case of a normally distributed shock for a firm $i$ with threshold $\theta_i$. The shadow area under the curves measure the probability that the shock is lower than the firms' threshold, that is the probability of survival for the firm. As it clear emerges from the Figure, this area for $x_i$ is higher (lower) than the corresponding area for $\hat{x}_i$ when the average of the shock ($\mu$) is lower (higher) than the firm's threshold ($\theta_i$).

Moving from the individual firm to the network of firms, if the threshold is heterogeneous across firms, but drawn from a common distribution, the following proposition holds:

**Proposition 4** *Assume the idiosyncratic components of the shocks are independently distributed, and so are the threshold values ($\theta_i$), and in addition the number of firms sufficiently large. Then, the average default rate in a symmetric and connected network is higher (lower) than the one for isolated firms if* $\Pr(\theta_i < \mu) > \Pr(\theta_i - x_{i0} < 0)$ *(* $\Pr(\theta_i < \mu) < \Pr(\theta_i - x_{i0} < 0)$*).*

For the simpler case of an homogeneous threshold, the previous proposition reduces to the following proposition:

**Proposition 5** *Assume the idiosyncratic components of the shocks are independently distributed, the threshold value is homogeneous across firms, and the number of firms sufficiently large. Then, the average default rate in a symmetric and connected network is higher (lower) than the one for isolated firms if the expected value of the random shock is higher (lower) than the common threshold.*

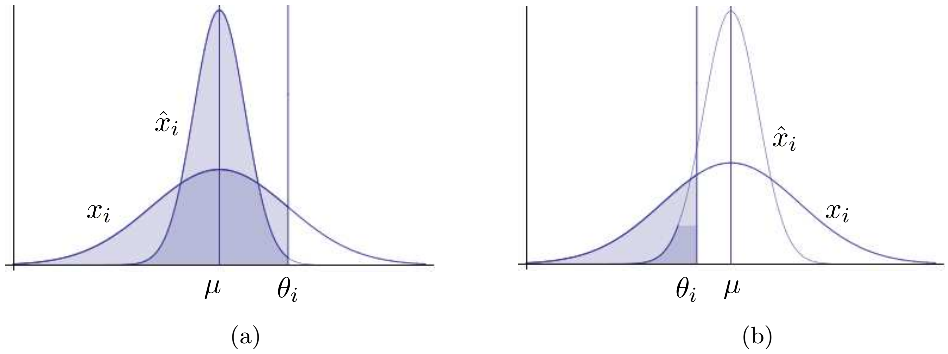



Figure 8: Default probabilities of firms in symmetric networks

Finally, in case of strongly connected asymmetric networks, the allocation of the total shock is not equal across firms. But depends on the overall structure of bilateral relations and no easy generalisation can be made. In general, if $s_i$ is the fraction of the total shock accruing to firm $i$, $i$ will face in equilibrium a shock which is asymptotically normally distributed (by the central limit theorem) with expected value:

$$E\left[\hat{x}_i\right] = E\left[s_i \sum_i x_{i0}\right] = s_i n \mu$$

and variance:

$$\mathrm{Var}\left[\hat{x}_i\right] = \mathrm{Var}\left[s_i \sum_i x_{i0}\right] = s_i^2 n \sigma^2$$

Hence, if $s_i < 1/n$, the firm in the network have to face a shock which is less volatile and lower on average than the one it faced in isolation. If instead $1/n < s_i < 1/\sqrt{n}$, the firm's shock in the network is higher on average but still less volatile. Finally, when $1/\sqrt{n} < s_i \leq 1$ the firms' shock is both higher on average and more volatile.

On the basis of the previous argument, the following proposition can be stated (and proved in the Appendix):

**Proposition 6** *Assume the idiosyncratic components of the shocks are independently distributed and the number of firms in the network sufficiently large. Then, the default probability of a firm in a strongly connected asymmetric network is higher (lower) than that of the same firm in isolation if the expected value of the random shock is higher (lower) than* $\frac{\theta_i}{s_i n}$.

Therefore, in this case, in order to assess the gains of the network, in terms of a lower default probability, we need to take into account the share of the overall shock the firm gets in equilibrium. In particular, if this share actually makes the expected value of the shock exceed the firms' threshold.

Finally, looking at the default rate for the overall network, what matters is the correlation between the equilibrium shares, as determined by the network, and the firms' thresholds. So, for instance, if the firms that in equilibrium get the higher shares of the overall shocks are those with the lowest thresholds, the default rate exhibited by these networks can be relatively high in case of low shocks. But comparatively lower in case of strong common shocks. Indeed, the system of relations makes the weakest firms, which would have died anyway, take a larger share of the total shock.

### 4.2.3 Default analysis and speed of convergence

Our default analysis strongly relies on the operational device to work out default probabilities at the limiting distribution of shocks. In fact, the slower the rate at which the system converges to the equilibrium, the less realistic is our assumption.

Hence, understanding the relationship between the structure of the firms' network and the speed of convergence is crucial. In formal terms, the question amounts to calculate how long it does take the Markov matrix $\mathbf{T}$ to approach its limit.

This is a well known issue, on which there is in fact a large literature. As reported by Golub and Jackson (2009), the convergence time is proportional to $1/\log(|\lambda 2(\mathbf{T})|)$, where $\lambda 2(\mathbf{T})$ denotes the second largest eigenvalue of $\mathbf{T}$. Therefore, a second eigenvalue close to 1 implies a very low speed of convergence.

As for the relationship between this mathematical condition and its insights for our model, a useful perspective is the one provided by the approach based on measuring *bottlenecks* (Diaconis and Stroock, 1991). The basic idea is that if there are pieces of the network connected only by narrow linkages, then convergence is slow.

## 5 Discussion and final remarks

The results of the model suggest a number of interesting interpretations, once related to the background literature and stylized facts we reviewed.

First of all, the network capacity of working as a "system" in financial terms – in which individual firms exchange their idiosyncratic shock for a certain portion of the total shock of the network – crucially depends on the structure of the network itself. In particular, the strong connectivity of the network is crucial. Should some or even only one of the firms be "isolated" from the twofold transfer mechanism we have described, the network would lose its system properties.

This can be considered the case of clusters, in which firms are linked through subcontracting relationships but with little socio-economic embeddedness. Unless the subcontractors in turn further subcontract part of their work – which is not uncommon[12] – in these chains of "atomistic" producer-user relationships, the client firms exploit their larger market power to transfer, via trade credit, their risk to the subcontractors themselves, which thus get subject to financial default exclusively and/or earlier than the former.[13]

While strongly connected networks are able to work as financial systems, on the other hand, their capacity to translate the idiosyncratic risk of each firm into an average of the risk of all the firms in the cluster is not guaranteed. In order to have such "equalitarian" risk-pooling, the inter-firm bilateral relationships need to be perfectly reciprocated. With benefit of handy sight, we could say that the district atmosphere the network is embedded in must be such to exactly compensate, or tend to compensate, the asymmetries in contractual powers which emerge from user-supplier differences in size. Quite interestingly, this result is consistent with the trade-off local studies find between contractual opportunistic behaviors, on the one side, and social capital, on the other.[14]

Interesting implications can be drawn also in terms of default probability, that is of the actual capacity of the network firms to bear a financial risk such as the current one. In the case of non strongly connected networks, when "isolated firms" are present, those firms which act as pure absorbers have been shown to be two times in troubles: not only because do they end up receiving a shock larger than the average, but also because such a shock encapsulates the variability of that faced by the other firms.

Definitively more interesting is the result for the district network, where trade credit and interlinking of credit and subcontracting coexist. District firms have been shown to be more shock-resilient than isolated ones *only* under two important conditions: in the case of symmetric relationships and, providing the average shock is lower than the threshold of the firm itself. Conversely, belonging to the district could even increase the default rate of the firm.

This is possibly the most important result of the model. Indeed, it seems to show that the industrial district model, while enabling firms to share the risk of a moderate shock, and to be actually more resilient in "normal" conditions, does not help and is actually disadvantageous in front of "heavy" financial crisis (such as possibly the current one).[15] Quite interestingly, this

[12]And actually one of the possible extensions of the model.

[13]The previous result can be used to interpret what is happening, for example, in the supply-chain network of Fiat automobile in Italy (Abatecola, 2009). Here, the small subcontractors of components are actually providing the producer with a remarkable margin of flexibility both in production and financial terms. The FIAT contractual power and the absence of a district-like environment for the supply-chain are crucial for this to occur.

[14]In the Italian region of Emilia-Romagna, for example, this has been shown to be the case of the footwear district of San Mauro Pascoli (Brioschi et al., 2004).

[15]Under a different perspective, the same result points to the production specialization of the districts, making more (less) fragile those which are specialised in sectors more (less) exposed to international competition: the different destiny of the ceramic tales district of

result is invariant with respect to the actual structure of the relationships in the district: "canonically" or not (Paniccia, 1998) does not make any difference for its financial behavior.

If in strongly connected networks, the twofold credit relationships we have envisaged are asymmetric, the implications of the model becomes more blurred, as they depend on the ratio between the share of the overall shock firms get (in equilibrium) and the network firms' threshold. Still, the insight is that in this case, the network firms actually split into two groups: the "winners", so to say, which are able to transfer to the others part of both the average and the variance of their shock; the "losers", whose default rate increases both because of a higher and a more variable shock. This is another interesting result, which recovers the relevance of the structure of local production systems in evaluating their resilience to the crisis: indeed, such a structure turns out to be more important than the bilateral relationships on which local studies usually focus.

# A Appendix

**Proof of Proposition 1** When $\mathbf{T}$ is strongly connected, it is a standard result of the theory of Markov chains that aperiodicity is a necessary and sufficient condition for $\mathbf{T}$ to be convergent (e.g. Kemeny and Snell, 1960). Moreover, when this happens $\mathbf{T}$ is also *primitive*, i.e. $\mathbf{T}^t$ has only strictly positive entries for some $t \geq 1$ (e.g. Perkins, 1961), and there is a unique (up to scale) left eigenvector $\mathbf{s}$ of $\mathbf{T}$, corresponding to the unit eigenvalue, such that for any $\mathbf{v}$:

$$\lim_{t\to\infty} \mathbf{T}^t\mathbf{v} = \mathbf{s}'\mathbf{v}.$$

Since $\mathbf{T}$ is convergent, $\mathbf{S} \equiv \lim_{t\to\infty} \mathbf{T}^t$ exists and hence:

$$\mathbf{S}\mathbf{T} = \lim_{t\to\infty} \mathbf{T}^t\,\mathbf{T} = \lim_{t\to\infty} \mathbf{T}^t = \mathbf{S}$$

where each row of $\mathbf{S}$ is equal to $\mathbf{s}'$.

It follows that:

$$\hat{\mathbf{x}}' = \mathbf{x}_0' \left(\lim_{t\to\infty} \mathbf{T}^t\right) = \mathbf{x}_0'\mathbf{S} = \mathbf{x}_0' \begin{pmatrix} \mathbf{s}' \\ \vdots \\ \mathbf{s}' \end{pmatrix} = \mathbf{s}'(\textstyle\sum_i x_{i0}). \qquad \blacksquare$$

**Proof of Proposition 2** A symmetric network implies $\mathbf{T} = \mathbf{T}'$ and therefore:

$$\mathbf{S}' = (\lim_{t\to\infty} \mathbf{T}^t)' = \lim_{t\to\infty} \mathbf{T}^t = \mathbf{S}$$

Sassuolo and of the mechanical one of Bologna in Italy, for example, can also be read in this terms.

i.e. $\mathbf{S}$ must be symmetric too ($s_{ij} = s_{ji}$). As in $\mathbf{S}$ by definition $s_{ji} = s_{ii}$, the symmetry implies $s_{ii} = s_{ij}$.

Moreover, since the sum by column of each row is one, it follows that:

$$\sum_{j=1}^{n} s_{ij} = n \; s_{ii} = 1$$

for each $i$ and all the elements of $\mathbf{S}$ are equal to $1/n$. Hence:

$$\hat{x}_i = \mathbf{x}_0' \begin{pmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{pmatrix} = \frac{\sum_i x_{i0}}{n} = \bar{x}$$

for each $i \in N$. ∎

**Proof of Proposition 3** Given that, in a connected-symmetric network $\hat{x}_i = \bar{x}_0$ and this variable is asimptotically normally distributed with variance $\sigma^2/n$ and mean $\mu$, when $n$ gets larger it converges in probability toward $\mu$. Therefore, we have:

$$\lim_{n\to\infty} \Pr(\hat{x}_i > \theta_i) = \begin{cases} 1 & \text{if } \theta_i > \mu \\ 0 & \text{if } \theta_i < \mu \end{cases}$$

On the contrary, since $\sigma^2 > 0$, there is always a $\epsilon > 0$ such that $\epsilon < \Pr(x_{i0} > \theta_i) < 1 - \epsilon$ and this probability is so strictly bound between 0 and 1. ∎

**Proof of Propositions 4** Assuming that $\theta_i$ are identically and indipendently distributed, and so are $x_{i0}$, the number of firms' defaults follows a binomial distribution with expected value: $n \Pr(\theta_i - x_{i0} < 0)$. The expected value of the default rate is therefore simply: $\Pr(\theta_i - x_{i0} < 0)$.

For firms in a symmetric-connected network, the expected value of the binomial is instead: $n \Pr(\theta_i - \bar{x} < 0)$, with an expected default rate equals to: $\Pr(\theta_i - \bar{x} < 0)$.

Given that $\bar{x} \xrightarrow{p} \mu$ we have that:

$$\lim_{n\to\infty} \Pr(\theta_i - \bar{x} < 0) = \Pr(\theta_i < \mu)$$

Hence, the expected default rate of firms when the number of firms gets large is higher (lower) in isolation than in a symmetric-connected network if $\Pr(\theta_i - x_{i0} < 0) > \Pr(\theta_i < \mu)$ ($\Pr(\theta_i - x_{i0} < 0) < \Pr(\theta_i < \mu)$). ∎

**Proof of Propositions 5** For a common threshold ($\theta_i = \theta$), we have :

$$\lim_{n\to\infty} \Pr(\bar{x} < \theta) = \begin{cases} 1 & \text{if } \theta > \mu \\ 0 & \text{if } \theta < \mu \end{cases}$$

while $\Pr(x_{i0} < \theta)$ remains stricly bound between 0 and 1. ∎

**Proof of Propositions 6** The probability of default in a strongly connected asymmetric network for firm $i$ is:

$$\Pr(\hat{x}_i > \theta_i) = \Pr(s_i \sum_i x_{i0} > \theta_i) = \Pr(s_i n \frac{\sum_i x_{i0}}{n} > \theta_i) = \Pr(\bar{x} > \frac{\theta_i}{s_i n})$$

$\bar{x}$ converges in probability toward $\mu$, therefore we have that $\Pr(\hat{x}_i > \theta_i)$ tends to 1 if $\bar{x} > \frac{\theta_i}{s_i n}$ and 0 if instead $\bar{x} < \frac{\theta_i}{s_i n}$.

On the contrary, since $\sigma^2 > 0$, there is always a $\epsilon > 0$ such that $\epsilon < \Pr(x_{i0} > \theta_i) < 1 - \epsilon$ and the probability in this case is strictly bound between 0 and 1. ∎

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