The Role of Functional Form in Estimating Elasticities of Housing Expenditures*

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I. Introduction

In a typically formulated housing expenditure function, observations on housing expenditures (H_i) are stochastically related to observations on income (Y_i) and price (P_i) according to the log-linear model

$$\log H_i = \beta_0 + \beta_Y \log Y_i + \beta_P \log P_i + U_i \tag{1}$$

where U_i denotes a disturbance term having standard properties and where the β_j ($j = 0, Y, P$) are constant across i. While there have been discussions in the literature regarding such matters as: (1) methods of measuring H_i, Y_i and P_i [5; 10; 11]; (2) problems of aggregation and the relative merits of micro and metro data [12; 17; 19; 20] (3) simultaneous equations bias and the neglect of the supply side [18; 23] and (4) the specification errors created by omitting the price term and other variables [7; 15; 16; 24], virtually all estimates have been constructed from a variant of this model. The log-linear specification is convenient because it allows the income and price elasticities of housing expenditure (β_Y and β_P) to be estimated by a direct application of ordinary least squares (OLS). However, as will be argued momentarily, equation (1) may have a questionable basis in theory for two reasons. First, measures of transport costs are frequently excluded as regressors in housing expenditure functions even though such measures are theoretically relevant variables.[1] Second, and this point will be stressed in the discussion to follow, income and price elasticities are likely to vary across observation units. As a result, existing estimates of these elasticities must be interpreted as complicated weighted averages of their underlying and observation specific counterparts. Both of these problems are of potential importance because accurate estimates of these income and price elasticities of expenditure are useful not only in developing housing policy but also in studying issues involving suburbanization and property taxation.

* Thanks are due to H. M. Kaufman, R. D., Rowe, D. N. Steinnes and an anonymous referee for constructive comments on a previous version of this manuscript. In addition, the first named author wishes to acknowledge research support from the John S. Bugas endowment in economics at the University of Wyoming. Any remaining errors, however, are our own responsibility.

1. Two exceptions are Kau and Lee [8] and Straszheim [20].

The purpose of this paper is to provide evidence, both theoretical and empirical, on the appropriate functional form of a housing expenditure equation. Section II uses Muth's model of housing choice as a point of departure. This model demonstrates the role of transport costs in determining housing expenditure levels and shows that constant income and price elasticities of housing expenditure across observation units are unlikely. Then, in Section III, new estimates of a housing expenditure function that are obtained from the generalized functional form approach [1] are discussed. These estimates, which are obtained from metro data of the type used by DeLeeuw [5] and Vaughn [23], indicate that income and price elasticities should be expected to vary across SMSAs. Consequently, a search for *the* income and price elasticity of housing expenditure may be misdirected. The empirical results also suggest that the omission of the transport cost variable may not be damaging to housing expenditure estimates constructed from metro data. In those regressions where it was used, the estimated coefficient on this variable was never significant at even the 25% level. Finally, some conclusions and implications are presented in Section IV.

II. The Model

The assumption of a log-linear housing demand function has little basis in theory. In general, log-linear demand functions are not derivable from any standard utility maximization framework [3, 81–6]. Demand elasticities are usually rather involved functions of the variables and parameters of the utility maximization problem. To illustrate this point, consider Muth's [14] model of housing choice. The Muth model was chosen as the vehicle for this analysis largely because it has proven to be a useful point of departure in discussions not only of housing demand but also of urban form and land use patterns. In this model, all consumers maximize the common utility function:

$$U = U(Q,X) \tag{2}$$

subject to the income constraint

$$Y = P(K) \cdot Q + X + T(K,Y) \tag{3}$$

where Q denotes quantity of housing services, X denotes quantity of all other goods, T denotes transport costs to and from work, and K denotes the distance of the consumer's residence from his workplace (which is assumed to be located in the central business district (CBD)). In addition, the price of X has been normalized to unity, the price per unit of housing is specified to be a function of distance from the CBD while journey-to-work costs are specified to vary with both distance traveled and income.

Assuming that both first and second order conditions for a constrained maximization of the Lagrangian

$$L = U(Q,X) - \lambda[Y - P(K) \cdot Q - X - T(K,Y)] \tag{4}$$

are satisfied [14, 38], expressions for the housing expenditure function and for the income and price elasticities of housing expenditure may be easily obtained. Since in this model, individuals are price-takers and their income endowments are fixed, the housing expenditure function at a particular location is

$$H = P \cdot Q = H(P,Y,T) \tag{5}$$

and the income and price elasticities of housing expenditure are

$$\epsilon_Y = (\partial H/\partial Y)(Y/H) = (\partial Q/\partial Y)(Y/Q) \quad (6)$$

and

$$\epsilon_P = (\partial H/\partial P)(P/H) = 1 + (\partial Q/\partial P)(P/Q) \quad (7)$$

where $P = P(K)$, $T = T(K, Y)$ and where $\partial Q/\partial P$ is interpreted as the change in the quantity of housing services consumed at a particular location, given a small parallel shift in the price gradient. Further, from the comparative statics properties of (4)

$$\partial Q/\partial Y = \{(1 - T_Y)\lambda(T_{KK} + QP_{KK})[U_{XX}P(K) - U_{XQ}] - \lambda^2 P_K T_{KY}\}/D \quad (8)$$

and

$$\partial Q/\partial P = \{-U_{XX}P(K)Q\lambda(T_{KK} + QP_{KK}) + U_{QX}Q + \lambda\}/D \quad (9)$$

where

$$D = \lambda(T_{KK} + QP_{KK})[U_{XX}P^2(K) + U_{QQ} - 2U_{XQ}P_K] + \lambda^2 P_K^2 . \quad (10)$$

Substituting (8) and (9) into (6) and (7) yields the conclusion that ϵ_Y and ϵ_P are each somewhat involved functions of the variables and the parameters of the maximization problem. Therefore, since these variables take on different values both across individuals within an SMSA or across the median (or average) individual in different SMSAs, the housing expenditure function in (5) is unlikely to have the constant elasticity form of equation (1).

Although this exercise has illustrated the general misspecification of assuming a constant elasticity housing expenditure function, it has not provided concrete suggestions as to the proper functional form. As is the case with many other microeconomic models of consumer choice, the Muth model indicates only the relevant explanatory variables. Without assuming a specific form for the utility function, little more can be said regarding the form of the housing expenditure equation. For this reason, the search for the most appropriate functional form becomes an empirical question.

III. Expenditure Elasticities From a Flexible Functional Form

In this section, some results derived from estimating a general form of Equation (5) are reported. The equation to be estimated is

$$H_i^{(\lambda_0)} = \alpha_0 + \alpha_1 Y_i^{(\lambda_1)} + \alpha_2 P_i^{(\lambda_2)} + \alpha_3 T_i^{(\lambda_3)} + U_i \quad (11)$$

where

$$Z^{(\lambda)} = \begin{cases} (Z^\lambda - 1)/\lambda & \lambda \neq 0 \\ \ln Z & \lambda = 0 \end{cases} \quad (12)$$

Equation (11) is the most general specification permitted within the class of Box and Cox [1] transformations and is employed for three reasons. First, it permits the parameters λ_i, $i = 0, 1, 2, 3$ which determine the form of the housing expenditure function to be estimated from sample information rather than imposing them on an ad hoc basis. Second, the appropriateness of the log-linear functional form can be subjected to a statistical test since, by L'Hopital's rule, the log-linear form is just a special case of equation (11) with $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = 0$. Third, if the log-linear form proves to be inappropriate (as turns out to be the case), observation specific income and price elasticities can be easily computed. These elasticities are of some interest as metro or SMSA median data are used in the analysis.[2]

Estimates of equation (11) were constructed using a maximum likelihood procedure. In particular, under the assumption that the U_i are normally, independently, and identically distributed with zero mean and variance σ^2 for all i, estimates were obtained by maximizing the log-likelihood function

$$L = \text{Constant} + \log J - (n/2) \log \sigma^2 - (1/2\sigma^2)\Sigma_i \{H_i^{(\lambda_0)} - \alpha_0 - \alpha_1 Y_i^{(\lambda_1)} - \alpha_2 P_i^{(\lambda_2)} - \alpha_3 T_i^{(\lambda_3)}\}^2 \quad (13)$$

where $\log J = (\lambda_0 - 1)\Sigma_i \log H_i$ and where J is the determinant of the Jacobian of the transformation of the U_i to the H_i. Partially differentiating (13) with respect to the α_i, $i = 0, 1, 2, 3$, and σ^2 yields solutions for the maximum likelihood estimates of these parameters in terms of the λ_i. Since the λ_i are unknown, a direct search for the optimal values of these parameters was conducted in order to find the combination $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\lambda}_0, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3$ and $\hat{\sigma}^2$ that maximized the concentrated likelihood function

$$L_C = \text{Constant} + \log J - (n/2) \log \hat{\sigma}^2 . \quad (14)$$

More specifically, the λ_i were varied at intervals of 0.5 from 8.0 to −8.0 in order to locate approximate maximum likelihood estimates for all parameters. In the neighborhood of these approximations, the λ_i were first varied at intervals of 0.1 and then at intervals of 0.01 in order to obtain the results to be presented. Values of L_C were printed for each iteration of each equation estimated so that the maximum value of this statistic could be determined directly, thus reducing the possibility of reporting estimates pertaining to local, rather than global, optima.[3]

Estimates of equation (11) were constructed for both homeowners and renters disaggregated by family size using data on 37 SMSAs. The income and housing data are reported in the U.S. Department of Commerce publication *Metropolitan Housing Characteristics* [22], while the remaining data are taken from the Bureau of Labor Statistics Bulletin No. 1735 entitled, *Handbook of Labor Statistics* [21]. These data are similar to those employed by Vaughn [23] in his study of simultaneity problems in the housing demand function. The variables H, Y, P, and T are measured as:

HH = SMSA specific median value of an owner-occupied unit in 1970 for families of size 1, 2, 3, 4, 5, 6 persons and for the aggregate of all family sizes deflated by a 1971 SMSA specific index of homeowner costs for a four person family.

HR = SMSA specific median gross expenditure for rent in 1970 for families of size 1, 2,

2. A more complete treatment of the methodology underlying the Box and Cox transformations is contained in Zarembka [25] and Box and Cox [1].

3. Obviously, this procedure would fail to find the maximum value of L_C if: (1) there was a "large enough" spike in the likelihood function falling between the trial values of the λ_i or (2) this maximum occurred outside the search region. While either of these cases is possible, both appear remote since, for all equations estimated, the values of L_C were well-behaved when relatively small changes were made in the λ_i. In addition, a few trials at values beyond the range over which the concentrated search was carried out supported the view that the likelihood function is unimodal.

3, 4, 5, 6 persons and for the aggregate of all family sizes deflated by a 1971 SMSA specific index of renter costs for a four person family.

YH = SMSA specific median income of homeowners for families of size 1, 2, 3–4, 5, 6 and for the aggregate of all family sizes deflated by a 1971 SMSA specific index of non-housing costs.

YR = SMSA specific median income of renters for families of size 1, 2, 3–4, 5, 6 and for the aggregate of all family sizes deflated by a 1971 SMSA specific index of non-housing costs.

PH = SMSA specific index of homeowner costs for a family of size four in 1970 deflated by 1971 SMSA specific index of non-housing costs.

PR = SMSA specific index of renter costs for a family of size four in 1970 delfated by a 1971 SMSA specific index of non-housing costs.

T = SMSA specific index of transport costs for a family of size four in 1970 deflated by a 1971 SMSA specific index of non-housing costs.

Four features of these variable definitions deserve further comment. First, the grouped median income data serve as a proxy for permanent income on the assumption that the individual transitory components wash out. However, as Vaughn observed, the census income data for both renters and homeowners are aggregated for the three and four person family size categories. As a result, the same income measure was used as an explanatory variable in both the three and four person family size regression equations. Second, the price indices were developed for the BLS typical family of four. Consequently, these price variables may be mismeasured in the regression equations for families of other sizes. Third, the transport cost variable measures total transport costs, rather than only those connected with the journey to work. However, in Muth's model, these two measures would be the same since all travel other than work travel is assumed away. Fourth, given the variable definitions in Muth's model, the regressors were deflated using an index of non-housing prices rather than an index of prices for all goods.[4]

In any case, the estimates of the homeowner equations are presented in Table I, while the estimates of the renter equations are presented in Table II. To interpret these results, recall that these estimates were obtained by a search procedure that involved systematically varying the λ_j parameters. Consequently, the estimates presented refer to the iteration that had the highest concentrated likelihood value. Also, estimates of the λ_j together with asymptotic confidence intervals for each of these estimates are provided in both tables. To see how these confidence intervals were obtained consider the interval for λ_0. Kendall and Stuart [9] have shown that under general conditions the statistic $-2[L_{max}(\lambda_0^0) - L_{max}(\hat{\lambda}_0)]$ is distributed as χ^2 with one degree of freedom where $L_{max}(\hat{\lambda}_0)$ denotes the value of the unconditional maximum of the likelihood function and where $L_{max}(\lambda_0^0)$ denotes the value of the conditional maximum of the likelihood function with λ_0 constrained to λ_0^0 (all other parameter estimates unconstrained). Hence, a $100(1 - \alpha)$ percent confidence interval for λ_0 can be obtained by finding that value of λ_0^0 on either side of $\hat{\lambda}_0$ such that the relation

$$L_{max}(\hat{\lambda}_0) - L_{max}(\hat{\lambda}_0^0) = 1/2\chi_1^2(\alpha) \qquad (15)$$

is satisfied. An analogous procedure was used to derive the confidence intervals for the estimates of the remaining λ_j parameters in the two tables.

4. This point is discussed in greater detail in Lee [11, 300].

Table I. Maximum Likelihood Estimates of the Demand for Housing (Owner)

Equation[a]	λ_0[b]	λ_1[b]	λ_2[b]	λ_3[b]	Constant[c]	Coefficient on YH[c]	Coefficient on PH[c]	Coefficient on T[c]
Aggregate	1.02 (0.6/2.0)	1.50 (0.8/2.2)	−2.20 (−1.7/−2.6)	0.5 (−2.5/3.0)	.118+02 (.259+02)	.246+00 (.317−01)	−.818+02 (.343+02)	.753+01 (.858+02)
Aggregate	1.02 (0.6/2.0)	1.50 (0.8/2.2)	−2.20 (−1.7/−2.6)	--- ---	.153+02 (.213+02)	.220+00 (.494−01)	−.773+02 (.343.02)	--- ---
Six	1.10 (0.02/2.0)	1.70 (0.2/2.3)	−1.61 (−1.1/−2.6)	--- ---	−.248+02 (.478+02)	.140+00 (.201−01)	−.990+02 (.446+02)	--- ---
Five	1.15 (0.5/1.85)	1.02 (0.25/2.0)	−3.22 (−2.5/−3.6)	--- ---	−.158+03 (.412+02)	.278+01 (.332+00)	−.664+02 (.366+02)	--- ---
Four	1.10 (0.4/1.5)	1.00 (9.25/2.1)	−3.06 (−2.5/−3.5)	--- ---	−.158+03 (.456+02)	.278+01 (.382+00)	−.656+02 (.207+02)	--- ---
Three	1.10 (0.5/1.5)	1.00 (0.15/2.0)	−2.23 (−1.6/−2.7)	--- ---	−.137+03 (.437+02)	.260+01 (.331+00)	−.711+02 (.218+02)	--- ---
Two	1.08 (0.5/1.5)	1.00 (0.13/1.7)	−2.50 (−2.0/−2.9)	--- ---	−.302+02 (.422+02)	.222+01 (.311+00)	−.617+02 (.207+02)	--- ---
One	0.81 (0.12/1.5)	0.80 (0.13/1.5)	−2.81 (−3.3/−2.3)	--- ---	.584+02 (.123+02)	.281+01 (.470+00)	−.381+02 (.155+02)	--- ---

[a] Family size indicated as equation name
[b] Confidence bounds in parentheses are 95% intervals
[c] Standard errors are in parentheses

Table II. Maximum Likelihood Estimates of the Demand for Housing (Renter)

Equation[a]	λ_0[b]	λ_1[b]	λ_2[b]	λ_3[b]	Constant[c]	Coefficient on YR[c]	Coefficient on PR[c]	Coefficient on T[c]
Aggregate	0.12 (-.12/.81)	2.6 (2.4/4.1)	-4.4 (-3.6/-5.1)	0.5 (-2.5/3.0)	.397-02 (.195-01)	.562-06 (.232-06)	-.795+00 (.182+00)	.641-01 (.764+00
Aggregate	0.12 (-.12/.81)	2.8 (2.4/4.1)	-4.4 (-3.6/-5.1)	--- ---	.426-02 (.561-01)	.548-05 (.192-06)	-.764+00 (.166+00)	--- ---
Six	0.50 (0.1/1.1)	2.0 (0.9/2.8)	-5.0 (-4.1/-5.4)	--- ---	-.437+00 (.718-01)	.193-03 (.211-04)	-.351+00 (.107+00)	--- ---
Five	0.12 (0.04/1.02)	4.42 (3.6/5.0)	-1.4 (-0.6/-2.1)	--- ---	.466-01 (.509-01)	.329-08 (.354-09)	-.498+00 (.134+00)	--- ---
Four	0.42 (0.01/1.1)	4.05 (3.2/5.0)	-1.5 (-0.3/-2.0)	--- ---	.148+00 (.722-01)	.105-07 (.605-08)	-.609+00 (.199+00)	--- ---
Three	0.46 (0.01/1.2)	4.02 (3.1/4.8)	-3.0 (-2.3/-4.0)	--- ---	.875-01 (.645-01)	.127-07 (.539-08)	-.658+00 (.175+00)	--- ---
Two	0.50 (-.02/0.9)	4.00 (3.1/4.8)	-3.5 (-2.6/-4.0)	--- ---	.858-01 (.648-01)	.146-07 (.467-08)	-.804+00 (.186+00)	--- ---
One	0.10 (-0.2/0.8)	0.10 (-0.2/1.0)	-4.5 (-3.1/-5.1)	--- ---	-.182+01 (.232+00)	.397+00 (.520-01)	-.910+00 (.116+00)	--- ---

[a] Family size indicated as equation name
[b] Confidence bounds in parentheses are 95% intervals
[c] Standard errors are in parentheses

Both Tables I and II report that the aggregate homeowner and renter equations were estimated initially by including transport costs as a regressor. However, since the standard errors exceeded the estimated coefficients on this variable, both equations were re-run with T excluded. As indicated in the tables, this change produced only minor adjustments in the estimated coefficients on the remaining explanatory variables and (somewhat surprisingly) no change at all in the estimated values for the transformation parameters. Because the transport cost variable entered the two aggregate equations so insignificantly, it was not included as a regressor in any of the disaggregated family size equations.

In comparing the seven equations in each table where T was excluded, the functional form parameters do not appear to vary as much across family size categories within either the homeowner or renter groups as they do between these groups. Estimates of λ_1 and λ_2 in the renter equations tend to lie above their counterparts in the homeowner equations while the opposite situation prevails for the estimates of λ_0. In addition, the coefficients on income and price in all 14 equations have the expected sign and are significantly different from zero at the one percent level of significance. But perhaps most importantly, none of the confidence intervals about the estimated λ_j in the homeowner equations bracket the value zero, while the same statement holds for the renter equations in all except three cases.[5] This result tends to suggest that, taken separately, the housing expenditure, income, and price variables seldom appropriately enter the metro housing expenditure function with a logarithmic transformation.

The constant elasticity estimates produced by the log-linear functional form are presented in Table III for each of the 14 equations where T was excluded. These estimates are consistent with the results of previous studies using metro data in at least three respects. First, the income elasticities in the homeowner equations are generally higher and the price elasticities generally lower (in absolute value) than for the corresponding renter equations. Second, the income elasticity estimates for homeowners are generally larger than unity. Third, and finally, the income and price elasticity estimates for the three and four person family size categories are quite close to the results obtained by Vaughn [23] who, as previously indicated, used essentially the same data set. In particular, Vaughn aggregated the three and four person homeowner categories, used observations on 32 rather than 37 SMSAs and obtained $\hat{\beta}_Y = 1.88$, $\hat{\beta}_P = -.33$ for homeowners and $\hat{\beta}_Y = .32$ and $\hat{\beta}_P = -.48$ for renters.

As previously indicated, the generalized functional form estimates can be compared with the log-linear estimates of the housing expenditure equations on the basis of "goodness of fit" criteria. Since both types of estimates were obtained from members of the same family of parametric functions, a likelihood ratio test is used for this purpose. Again, this test is based on the fact that, under general conditions, the quantity $-2\,(L^0_{max} - L_{max})$ is χ^2 distributed with three degrees of freedom where: (1) L_{max} now corresponds to the unconditional likelihood value obtained by estimating one of the 14 housing expenditure equations using the generalized functional form approach; and (2) L^0_{max} corresponds to the likelihood value obtained from the log-linear specification of the same equation; that is, the value obtained by setting $\lambda_0 = \lambda_1 = \lambda_2 = 0.0$. The results of this pairwise comparison are presented in Table IV. These comparisons indicate that the null hypothesis of no difference in the "goodness of fit" in the log-linear specification and the "goodness of fit" in the generalized functional form specification would be rejected for all 14 equations at less than the 0.1 percent level of signif-

5. These three cases are: (1) the estimate of λ_0 in the family size = 2 equation, (2) the estimate of λ_0 in the family size = 1 equation, and (3) the estimate of λ_1 in the family size = 1 equation.

Table III. Estimates of Log-Linear Housing Expenditure Functions[a]

Family Size	Owner			Renter		
		Coefficient on			Coefficient on	
	Constant	YH	PH	Constant	YR	PR
Aggregate	-1.135 (0.182)	1.353 (0.182)	-0.458 (0.137)	-1.049 (0.582)	0.726 (0.162)	-0.653 (0.147)
Six	-2.766 (1.230)	1.650 (-.252)	-0.351 (0.147)	-4.499 (0.530)	1.067 (0.126)	-0.322 (0.103)
Five	-3.057 (1.106)	1.720 (0.227)	-0.335 (0.133)	-3.657 (0.678)	0.843 (0.154)	-0.469 (0.116)
Four	-2.476 (0.974)	1.613 (0.201)	-0.484 (0.126)	-0.360 (1.002)	0.368 (0.230)	-0.505 (0.163)
Three	-2.759 (1.016)	1.659 (0.204)	-0.405 (0.129)	-1.688 (0.936)	0.421 (0.213)	-0.553 (0.158)
Two	-0.018 (0.735)	1.144 (0.161)	-0.380 (0.147)	-1.153 (0.668)	0.315 (0.154)	-0.682 (0.169)
One	2.778 (0.486)	0.637 (0.138)	-0.310 (0.211)	-2.079 (0.524)	0.556 (0.141)	-0.892 (0.127)

[a] Standard errors in parentheses

Table IV. Pairwise Comparisons Between Log-Linear and Generalized Functional Form Estimates

Family Size	Owner		Renter	
	$\chi^2(3)$	Probability of a Larger Value than $\chi^2(3)$	$\chi^2(3)$	Probability of a Larger Value than $\chi^2(3)$
Aggregate	83.34	.001	72.81	.001
Six	67.17	.001	26.77	.001
Five	58.28	.001	64.26	.001
Four	91.64	.001	47.16	.001
Three	87.26	.001	42.24	.001
Two	16.51	.001	25.62	.001
One	22.82	.001	18.21	.001

icance. Obviously, this conclusion reinforces the earlier finding that the variables in a correctly specified metro housing expenditure function do not enter with a logarithmic transformation.

Because of the superiority of the generalized functional form over the log-linear form of the housing expenditure function, neither the income nor the price elasticity of housing expenditure appears to be constant across 37 SMSAs in the sample. To illustrate their range of variation as well as their numerical magnitude, SMSA specific income and price elasticities have been constructed from the estimates of the aggregate family size homeowner and renter equations (where T was excluded), given in Tables I and II. These estimates, which are presented in Table V, indicate that for homeowners, estimates of ϵ_Y range from 1.00 in Los Angeles-Long Beach to 1.65 in Indianapolis, while estimates of ϵ_P vary between −0.25 for Washington, D. C. and −0.62 for Baltimore. On the other hand, for renters, the highest estimate of ϵ_Y was Washington, D. C.'s 1.38 and the lowest estimate was Cleveland's 0.15, while the highest and lowest estimates of ϵ_P were −0.17 and −1.36 for San Francisco-Oakland and Baton Rouge.[6]

Finally, Table VI shows that the log-linear estimates of the income and price elasticities always lie within the range of values defined by the minimum and maximum obtained from the generalized functional form approach. In particular, the entries in this table indicate the number of SMSA specific elasticity estimates (always less than 37) that exceeded their corresponding log-linear estimates in absolute value. Intuitively, this result should not be unexpected since the log-linear estimates must be weighted averages of their underlying and observation specific counterparts where the weights are determined by the $\hat{\alpha}_i$ and the $\hat{\lambda}_i$. Unfortunately, there do not appear to be any more definitive statements that can be made about the relation between the two sets of elasticity estimates which would hold using any metro data set. This point is evidenced by comparing the homeowner columns with the renter columns in Table VI. While for the renter equation, the log-linear estimates of both ϵ_Y and ϵ_P tended to fall near the lower end of the range of the SMSA specific values, in the case of the homeowner equations no distinct pattern is present. In any case, there does not seem to be a marked tendency for the log-linear estimates to lie near the median of the SMSA specific estimates.

V. Implications and Conclusions

As this paper has suggested, a correctly specified housing expenditure function may not yield income and price elasticities that are constant across observation units. More specifically, the Muth [14] model of housing choice was used as a vehicle to show that these elasticities are likely to be functions of (among other things) housing price levels, real income levels, and transport costs. To the extent that these factors vary across observation units, income and price elasticities of housing expenditure will also vary. In addition, the extent of variation in these elasticities was investigated by applying the generalized Box-Cox [1] regression model to metro data of the type used by DeLeeuw [5] and Vaughn [23]. These estimates indicate

6. SMSA specific estimates of ϵ_Y and ϵ_P have also been constructed from the remaining 12 equations reported in Tables I and II. To conserve space, these estimates are not presented here; however, they exhibit similar ranges in variation to those presented in Table V. Interested readers may obtain these additional results from either of the authors on request.

Table V. SMSA Specific Income and Price Elasticities for the Aggregate Family Size Categories of the Owner and Renter Groups

	Owner		Renter	
SMSA	Income Elasticity	Price Elasticity	Income Elasticity	Price Elasticity
Boston	1.40	-.26	.44	-.62
Buffalo	1.25	-.40	.52	-.65
Hartford	1.28	-.27	.87	-.47
Lancaster	1.09	-.49	.71	-.71
Philadelphia	1.42	-.52	.56	-1.29
Pittsburgh	1.13	-.50	.59	-1.28
Portland	1.15	-.44	.33	-.65
Cedar Rapids	1.25	-.37	.73	-.65
Champaign-Urbana	1.28	-.37	.40	-.24
Chicago	1.25	-.28	.95	-.45
Cincinnati	1.29	-.38	.57	-1.29
Cleveland	1.29	-.27	.15	-1.18
Dayton	1.23	-.40	1.27	-.82
Detroit	1.35	-.37	.90	-.88
Greenbay	1.22	-.40	.75	-.85
Indianapolis	1.65	-.44	.82	-.71
Kansas City	1.26	-.48	.64	-.85
Milwaukee	1.33	-.30	.91	-.74
Minnesota-St. Paul	1.11	-.31	.62	-.73
St. Louis	1.33	-.44	.63	-1.14
Witchita	1.39	-.52	.62	-.75
Atlanta	1.07	-.40	.94	-.91
Austin	1.01	-.49	.46	-1.16
Baltimore	1.19	-.62	.80	-.38
Baton Rouge	1.02	-.42	.44	-1.36
Dallas	1.29	-.43	1.01	-.71
Durham	1.03	-.48	.35	-.50
Houston	1.30	-.55	1.01	-1.21
Nashville	1.16	-.44	.53	-.91
Orlando	1.02	-.47	.50	-.31
Washington, D.C.	1.29	-.25	1.38	-.52
Bakersfield	1.13	-.51	.61	-.75
Denver	1.17	-.39	.54	-.94
Los Angeles-Long Beach	1.00	-.40	.68	-.48
San Diego	1.07	-.29	.55	-.62
San Francisco-Oakland	1.07	-.26	.62	-.17
Seattle-Everett	1.28	-.30	.73	-.54

that: (1) the hypothesis of constant income and price elasticities across SMSAs should be rejected at very small significance levels; and (2) even though theoretically relevant, transport costs are not an important determinant of housing expenditure. An obvious implication of the first result (which must be tempered with the recognition that no micro data were used in the analysis) is that the almost exclusive reliance on the log-linear functional form in previous empirical investigations of housing expenditure patterns may be rather misplaced.

Furthermore, since the outcome of discussions on topics such as urban population distributions [14, 70–83; 13, 78–89] and property taxation [6, 327–30] often depend upon the

Table VI. Number of SMSA Specific Estimates of ϵ_Y and ϵ_P Exceeding Log-Linear Estimates in Absolute Value

Family Size	Owner		Renter	
	ε_Y	ε_P	ε_Y	ε_P
Aggregate	4	10	14	21
Six	21	15	28	22
Five	20	24	30	25
Four	24	18	25	26
Three	21	18	25	26
Two	18	21	25	26
One	5	10	35	25

values assumed for the income and price elasticities of housing expenditure, the behavior of these elasticities in the face of income and price changes may be an important piece of information. One way to obtain such information would be to examine the behavior of $\epsilon_Y{}^i$ and $\epsilon_P{}^i$ when either Y or P is changed and the other is held constant using the estimates from the previously reported housing expenditure functions. In particular, expressions from which $\epsilon_Y{}^i$ and $\epsilon_P{}^i$ may be computed are given in equations (16) and (17)

$$\hat{\epsilon}_Y{}^i = [\hat{\alpha}_1 Y_i^{\hat{\lambda}}1]/[H_i^{\hat{\lambda}}0] \tag{16}$$

$$\hat{\epsilon}_P{}^i = [\hat{\alpha}_2 P_i^{\hat{\lambda}}2]/[H_i^{\hat{\lambda}}0] \tag{17}$$

while values for the estimated parameters in these equations may be taken from Tables I and II. Tables VII and VIII, then provide evidence on the behavior of these elasticity estimates for the aggregate homeowner and renter equations in the case where transport costs were not included as an explanatory variable. For example, Table VII shows the calculations for $\hat{\epsilon}_{YH}{}^i$ and $\hat{\epsilon}_{PH}{}^i$ for three values of both YH_i and PH_i. The three values correspond to the sample minimum, average, and maximum observations on these variables. Table VIII presents corresponding calculations for the aggregate renter equation. Also, the figures reported in both tables were obtained using the approximation

$$H_i^{\hat{\lambda}_0} \approx \hat{\lambda}_0\{\hat{\alpha}_0 + \hat{\alpha}_1\ Y_i^{(\hat{\lambda}_1)} + \hat{\alpha}_2 P_i^{(\hat{\lambda}_2)}\} + 1 \tag{18}$$

Since it is not clear what value H_i "should" take on when Y_i and P_i are arbitrarily varied over their range of observations in the sample. In any case, Tables VII and VIII demonstrate that the shape of the housing expenditure functions for renters and owners are quite similar at least in qualitative terms. That is, in each equation, both $\hat{\epsilon}_Y{}^i$ and $\hat{\epsilon}_P{}^i$ tend to increase with increases in income and price. However, even though these results provide an indication of how the housing expenditure elasticities vary with changes in income and price, it would be

Table VII. Behavior of Income and Price Elasticities in the Aggregate Owner Equation[a]

$\hat{\varepsilon}^{i}_{YH}$

	PH minimum = .747	PH average = .962	PH maximum = 1.305
YH minimum = 98.92	1.105	1.298	1.465
YH average = 113.26	1.158	1.327	1.466
YH maximum = 147.57	1.245	1.371	1.467

$\hat{\varepsilon}^{i}_{PH}$

	PH minimum = .747	PH average = .962	PH maximum = 1.305
YH minimum = 98.92	-.750	-.504	-.291
YH average = 113.26	-.641	-.420	-.238
YH maximum 147.57	-.463	-.292	-.160

[a] Income figures are stated in hundreds of dollars while the base figure for the price index is unity.

Table VIII. Behavior of Income and Price Elasticities in the Aggregate Renter Equation[a]

	$\hat{\varepsilon}^i_{YR}$		
	PH minimum = .844	PH average = .988	PH maximum = 1.358
YH minimum = 44.55	.220	.225	.229
YH average = 66.73	.669	.684	.695
YH maximum = 82.52	1.186	1.210	1.230

	$\hat{\varepsilon}^i_{PR}$		
	PH minimum = .844	PH average = .988	PH maximum = 1.358
YH minimum = 44.55	-1.560	-.797	-.120
YH average = 66.73	-1.529	-.781	-.196
YH maximum = 82.52	-1.494	-.763	-.191

[a] Income figures are stated in hundreds of dollars while the base figure for the price index is unity.

premature to derive any substantive policy implications. In fact, until the variations in these expenditure elasticities have been studied in alternative data sets, there is really no way to tell whether a consistent pattern will emerge. Also, the possibility of aggregation bias in the metro data used in this study could have resulted in spurious associations between the elasticity estimates and the income and price variables. Therefore, an obvious next step for future research on this topic might be to construct some estimates of a generally specified, rather than a log-linear, housing expenditure function using micro data.

References

1. Box, G. E. P. and D. R. Cox, "An Analysis of Transformations." *Journal of the Royal Statistical Society*, Series B, Part 2, 1964, 211–43.
2. Boyes, W. J. and S. D. Gerking, "A Liquidity Trap in the Foreign Exchange Market: The Case of Mexico." *International Economic Review*, October, 1978, 777–85.
3. Bridge, J. L. *Applied Econometrics*. Amsterdam: North-Holland, 1971.
4. Carliner, G., "Income Elasticity of Housing Demand." *Review of Economics and Statistics*, November, 1973, 528–32.
5. DeLeeuw, F., "The Demand for Housing: A Review of Cross-Section Evidence." *Review of Economics and Statistics*, February, 1971, 1–10.
6. Heilbrun, J. *Urban Economics and Public Policy*. New York: St. Martin's Press, 1974.
7. Kartman, A. E., "New Evidence on the Demand for Housing." *Southern Economic Journal*, April, 1972, 525–30.
8. Kau, J. B. and C. F. Lee, "Functional Form, Density Gradient and Price Elasticity of Demand for Housing." *Urban Studies*, 1976, 193–200.
9. Kendall, M. B. and A. Stuart. *The Advanced Theory of Statistics*, Volume II. London: Charles Griffin and Co., Ltd., 1961.
10. Lee, T. H., "Housing and Permanent Income: Tests Based on a Three-Year Reinterview Survey." *Review of Economics and Statistics*, November, 1968, 480–90.
11. Lee, T. H. and C. M. Kong, "Elasticities of Housing Demand." *Southern Economic Journal*, October, 1977, 298–305.
12. Maisel, S. J., J. B. Burnham, and J. S. Austin, "The Demand for Housing: A Comment." *Review of Economics and Statistics*, November, 1971, 410–413.
13. Mills, E. S. *Urban Economics*. Glenview, Illinois: Scott, Foresman and Co., 1972.
14. Muth, R. *Cities and Housing*. Chicago: University of Chicago, 1969.
15. Polinsky, A. M., "The Demand for Housing: A Study in Specification and Grouping." *Econometrica*, March, 1977, 447–61.
16. Polinsky, A. M. and D. T. Ellwood, "An Empirical Reconciliation of Micro and Grouped Estimates of the Demand for Housing." *Review of Economics and Statistics*, forthcoming.
17. Roistacher, E. A., "Short-Run Housing Responses to Changes in Income." *American Economic Review*, February, 1977, 381–86.
18. Smith, B. A., "The Supply of Urban Housing." *Quarterly Journal of Economics*, August, 1976, 389–405.
19. Smith, B. A. and J. M. Campbell, Jr., "Aggregation Bias and the Demand for Housing." *International Economic Review*, June, 1978, 495–505.
20. Straszheim, M. R., "Estimation of the Demand for Urban Housing Services from Household Interview Data." *Review of Economics and Statistics*, February, 1964, 82–9.
21. U.S. Bureau of Labor Statistics, "Three Budgets for an Urban Family of Four Persons, 1971," Bulletin No. 1735, U.S. Government Printing Office, 1972.
22. U.S. Bureau of Census, Census of Housing: 1970, "Metropolitan Housing Characteristics," Final Report HC(2): Various volumes, U.S. Government Printing Office, 1973.
23. Vaughn, G., "Sources of Downward Bias in Estimating the Demand Income Elasticity for Urban Housing." *Journal of Urban Economics*, January, 1976, 45–56.
24. Winger, A., "Housing and Income." *Western Economic Journal*, June, 1968, 226–32.
25. Zarembka, P. "Transformation of Variables in Econometrics," in *Frontiers in Econometrics*, edited by P. Aarembka. New York: Academic Press, 1974.