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Non-exclusivity and adverse selection: An application to the annuity market*

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Abstract
Using a common agency framework, we characterize possible equilibria when annuities contracts are not exclusive. We discuss theoretical and empirical implications of these equilibria. First, we show that at equilibrium prices are not linear.
Then we characterize an equilibrium. We provide conditions for existence and show that this equilibrium is efficient.

Keywords
Menus, Common Agency, Insurance, Annuity Markets, Adverse Selection, Efficiency

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D82, H5, J2, G1

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1 Introduction

If life time is uncertain and if agents (consumers or workers) are risk averse, then annuities are an appropriate tool. If the price is actually fair, consumers should invest all their wealth in annuity products. This intuitive and simple principle has been formalized by Yaari (1965). From a theoretical point of view the issue seems clear. But empirically, markets for annuities are really small and only few people choose to use them. For example Friedman and Warshawsky (1990) indicate that only two percent of the American elderly population own individual annuities of any sort.

Market imperfections, and especially adverse selection, are commonly use to explain what seems to be a well established stylized fact. Following the Rothschild and Stiglitz (1976)'s analysis, If consumers have better knowledge of their own survival probability than insurance compagnies, annuities cannot be fairly priced and the aggregated outcome is not Pareto-efficient.

The presence of adverse selection in the annuity markets has been empirically tested. Chiappori and Salanié (2002) note that evidences of adverse selection has been repeatedly found in the literature. Warshawsky (1988) computes the value of an annuity by using the mortality probabilities of the general population and mortality probabilities of the annuities purchasers. He interprets the difference between these two values as the cost of adverse selection and concludes that adverse selection is significant. Friedman and Warshawsky (1990) show that there is a substantial difference between the survival probability of the general population and the survival probability of the annuity purchasers which confirm the Warshawsky (1988)'s previous results.

More recently, Finkelstein and Porteba (2002) and Finkelstein and Porteba (2004) use English datas to measure the importance of adverse selection. In the first paper explores adverse selection in the voluntary and compulsory individual annuity markets in the United Kingdom. They find that annuitants are longer-lived than non-annuitants. These mortality differences are more pronounced in the voluntary than in the compulsory annuity market. They estimate that the amount of adverse selection in the compulsory market is about one half of that in the voluntary market.

In their second paper, using the same data set, they also find systematic relationships between ex post mortality and annuities' characteristics, such as the timing of payments and the possibility of payments to the annuitant's estate. However, they find no evidence of substantive mortality differences by annuity size. These results suggest that the absence of selection on one contract dimension does not preclude its presence on others.

All the latter contributions rely on the Rothschild and Stiglitz's (1976) insurance model (RS hereafter). First, this model is not explicitly a model of annuity market, and hence it does not introduce explicitly mortality probabilities or or retirement scheme. Second, the RS's model assume explicitly that insurance contracts are "exclusive". An agent cannot hold simultaneously two or more contracts. These assumptions are crucial, for the economic methodology and for the equilibrium allocation properties.

Concerning the first limit of RS, we will not introduce a more explicit model. The literature considers generally as precise enough. Exclusivity is not a good assumption when we consider annuity markets (see Filkenstein and Porteba, 2004 for example). We construct a model of insurance with adverse selection in the spirit of Rothschild and Stiglitz (1976) but we allow for non-exclusive contracts.

Non exclusivity has been studied in insurance theory. Kahn and Mookherjee (1998) study the consequences of moral hazard in this setting. But their model is quite different from our, not only because they consider another kind of asymmetric information. They adopt a very unusual timing: the agent (the insure) plays first, he receives sequentially offers from different insurance companies and selects his favorite offers. In this paper will keep the traditional timing, insurances offers simultaneously one or more contracts. The agents observe the offers and select their favorite ones.

Bisin and Guaitoli (2004) consider a model of non-exclusive insurance affected by moral hazard in a timing similar to ours. Hence it is an interesting reference point has the main difference between our paper and their is the kind of market imperfection that we consider. Their consider Moral Hazard, we consider adverse selection. Except that, our models are quite comparable.

Our first finding is that at equilibrium, competitive insurance companies do not offer "linear contracts". By linear contracts, we mean contracts such that the premium is linear function of the repayment. Technically, this first result does not depend on the restriction that we can make on communication scheme, it means that it a very general feature of the model.

Surprisingly, in the literature it is often argued that non-exclusivity leads to equlibria in which only linear contracts are offered. Chiappori (2000) claims that with non-exclusivity *agents can always linearize the schedule by buying a large number of small contracts from different insurers*. It is true if insurers are offering small contracts, which not the case at equilibrium. We show that at equilibrium insurance companies have no incentive to offer small contract and hence the equilibrium contracts are not linear.

Then, we characterize an equilibrium which has several interesting properties. It is different from the equilibrium characterized by RS. There is a cross-subsidization between the agents. High risk agent are fully insured, and the low risk agents are not fully insured as in the RS's model. Finally, we show that this equilibrium is efficient.

The paper is organized as follows. In section 2, we state the model. Then in section 3, we present our first the result, the impossibility of linear equilibrium. In section 4 we characterize and analyze an equilibrium of our model. Finally in section 5 we conclude.

2 The model

The agents have all the same preferences, represented by a vNM utility function $U(.)$. Each agent can enjoy a "short" or a "long" life, with some probabilities. We keep a very stylized model of of insurance and consider that the utility of the agent can be represented as:

$$\pi U(w-p-l+r)+(1-\pi)U(w-p), \tag{1}$$

where w is the exogenous income of the agent, p is the net price of the annuity, l is the extra cost induce by a longer life and r represents the extra annuities that the agent get if he lives long life. The probability of living longer is denoted π. The utility can also be interpreted as a traditional insurance problem, π is the probability of accident, l the losses generated by and accident, p the insurance premium and r is the repayment. We call the couple (p,r) a contract. For convenience, it will be sometime denoted C.

To simplify the notation, we will use a short notation for the utility function

$$\forall (p,r) \in \mathbb{R}^2_+,\ \forall \pi \in [0,1] \quad V((p,r),\pi) = \pi U(w-p-l+r) + (1-\pi)U(w-p)\,. \quad (2)$$

We consider markets, agents will be taken as "small" players. Hence they will play independently and they will not interact in any way. Our model can be interpreted as well as a model with a single agent.

Insurance companies are formalized as profit maximizing firms with the same constant return-to-scale production technology. Namely, an annuity contract (p,r) owns by agent having a probability π to live a long life, generates the profit:

$$p - \pi r. \quad (3)$$

We do not introduce any administrative cost. By doing this, we follow the RS's model and keep our results comparable to the RS's ones. Moreover, administrative costs do not seem to be important and cannot explain alone why annuity markets are empirically so small. Porteba (2001) give some econometric evidence showing that administrative costs can explain only a reduced share of annuities prices.[1] As we have said, we do not want to depart from traditional insurance model.

The main issue of the model is the adverse selection problem. Then, we introduced two kind of agents, differing by their probability to live longer. Some agents have a high life expectancy, their probability to live a long life is π_2 and some other have a lower life expectancy, i.e; they have a probability π_1 to live longer life (and by necessity $\pi_2 > \pi_1$). The proportion of high and low life expectancy agents are respectively n_2 and n_1, and $n_1 + n_2 = n$.

Probabilities π_2 and π_1 and proportion n_1 and n_2 are common knowledge. Agents know their life expectancy, insurance companies cannot distinguish between the two kind of agents and cannot offer contract contingent on the their life expectancy.

Insurance company cannot prevent agents to buy contracts from other companies. In that sense contracts are incomplete. Hence we consider a model of non-exclusive contracts, or in the jargon of contract theory, a common agency game.

[1] Murthi, Orszag, and Orszag (1999) and Cannon and Tonks (2003) confirm these results.

If agent are allowed to hold simultaneously several contracts, market are not anonymous. They cannot buy to contracts to the same company, and they cannot resell their contract to some other agent.

To make the problem tractable, we assume that the agents' preferences satisfy the "single crossing" condition. The restriction is critical and it is know that properties derived under this assumption are sometimes not robust. Formally:

$$\forall\, (r,p) \in \mathbb{R}_+^2, \\ \frac{\pi_2 U'(w-p-l+r)}{\pi_2 U'(w-p-l+r)+(1-\pi_2)U'(w-p)} > \frac{\pi_1 U'(w-p-l+r)}{\pi_1 U'(w-p-l+r)+(1-\pi_1)U'(w-p)}. \tag{4}$$

Graphically, in the space (r,p), if two indifference curves cross each others, they cross only once.

We can represent this property in the space (r,p).

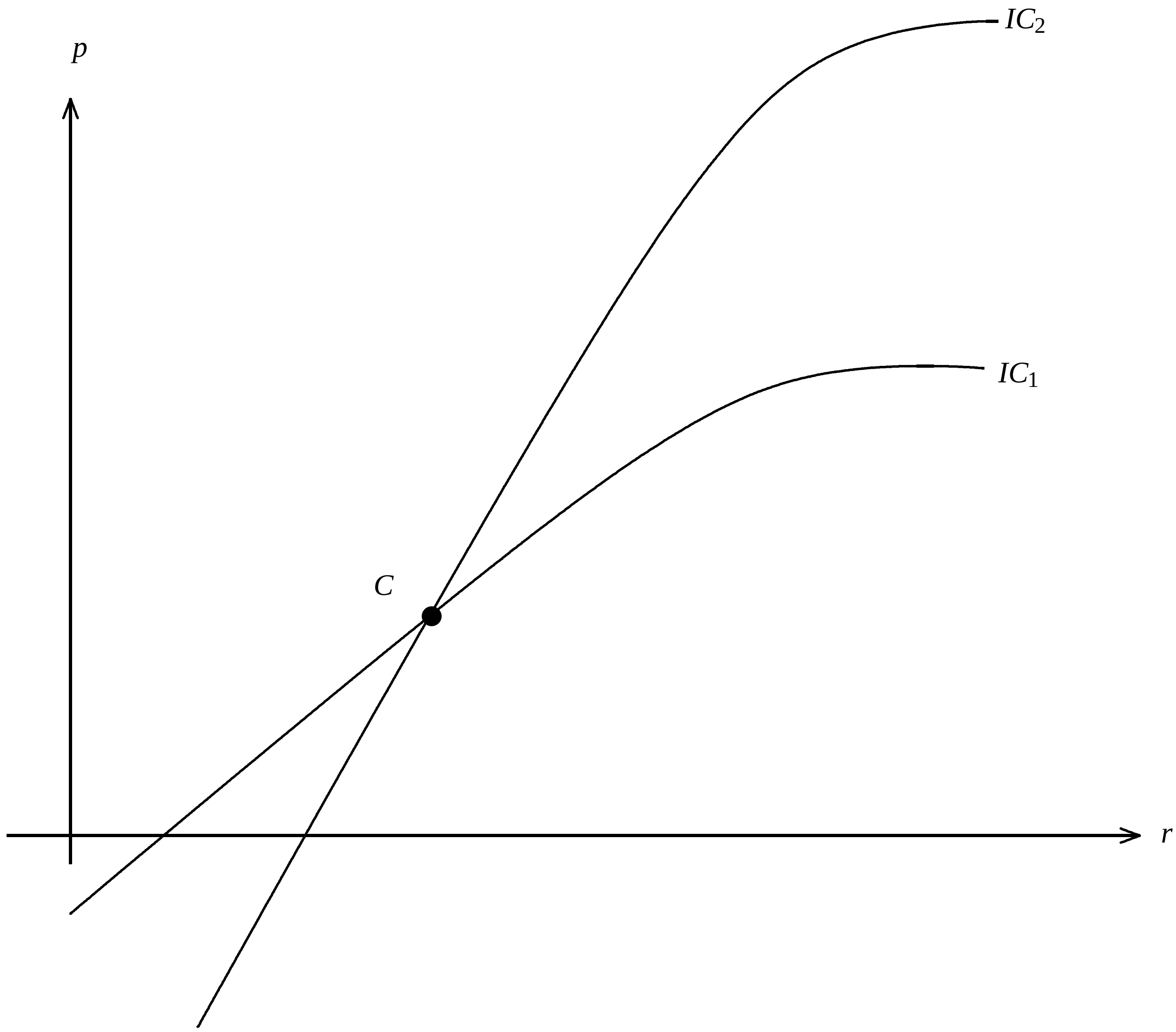


Figure 1: Single Crossing Property

In the figure 1, two indifference curves intersect at the point C. The indifference curve of the high probability agents (denoted IC_2) has always a higher derivative than the indifference curve of the low probability agents (denoted IC_1). Hence they cannot intersect twice.

We follow the literature on common agency games and we consider that insurance companies (the principals) compete through "menus" (generically denoted M in the following.) In other words, we will consider that insurance companies offer a set of contract, and the agent is allow to choose one in each of these sets. Peters (2001) and Martimort and Stole (2002) have shown that there is no loss of generality by considering these kind of mechanisms. Hence our results do not rely on any ad hoc restriction on the offers that can be made by the insurers.[2] For obvious technical reasons, we will constrain these sets M to be compact and measurable.

To be precise we give the definition of a linear equilibrium.

Definition 1 *We said that an equilibrium is linear if:*

- *The insurance companies offer the same menu,*
- *this menu is a line in the space* (p, r).

In other words, an equilibrium is linear if the equilibrium price is linear.

Finally, two benchmarks are interesting to interpret our results. The first important benchmark, is the competitive equilibrium when the information is complete. We denote the equilibrium contracts C_1^{FB} and C_2^{FB}. The both type of agents receive a fair insurance contract with respect to their probability: C_1^{FB} belongs to the line L_1, characterized by the equation $p = \pi_1 r$, the contract C_2^{FB} belong to L_2, (equation $p = \pi_2 r$). They are fully insured. The equilibrium is efficient.

[2] Let us remark that we do not allow for random mechanisms. In our model insurers cannot offer lotteries over contracts. We do not think that random mechanisms are realistic: to be implemented, insurance companies should be able to commit on random variable. Nevertheless, this restriction is not without loss of generality.

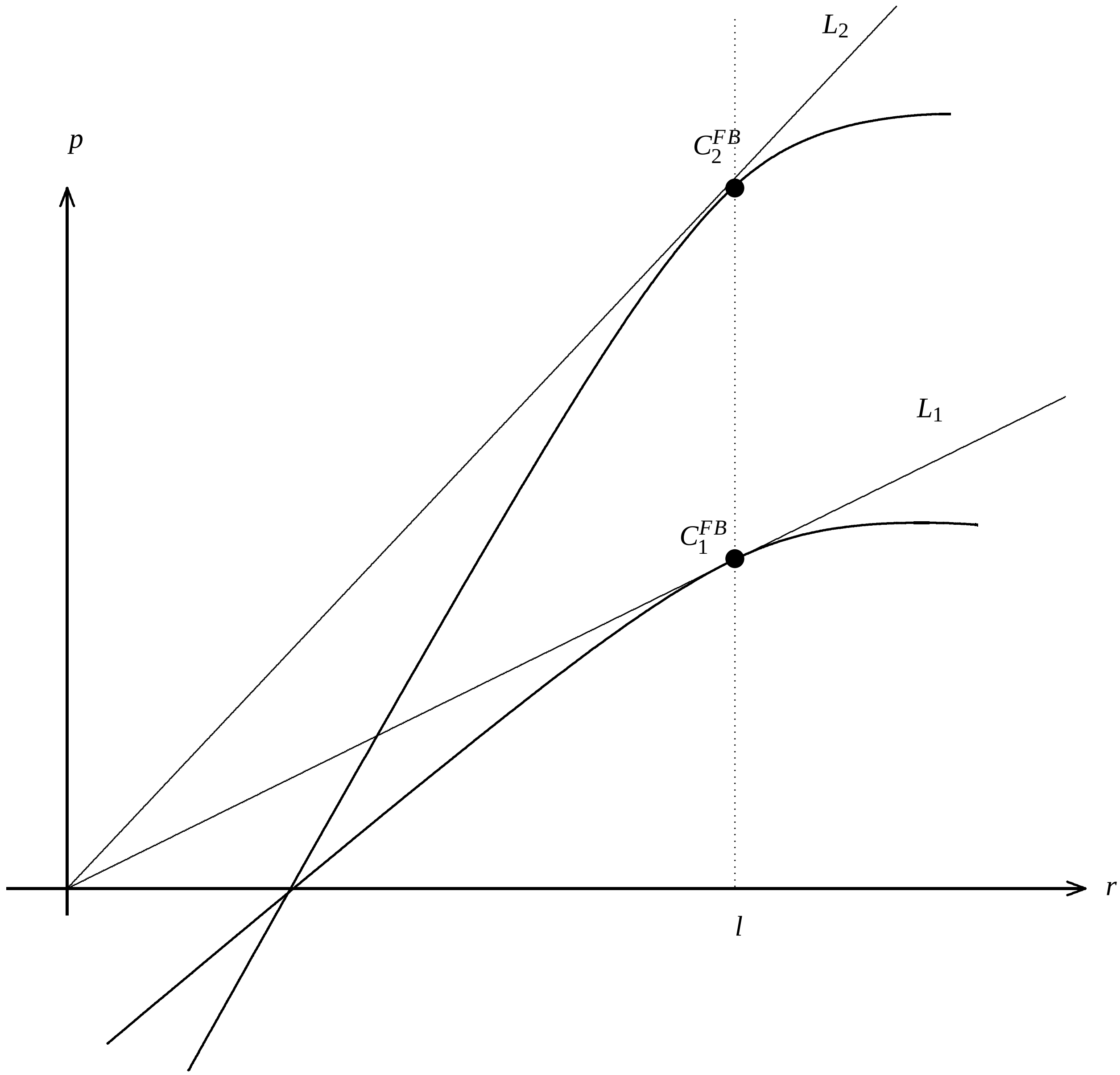


Figure 2: The competitive equilibrium under full information

The RS equilibrium can be represented graphically as follows

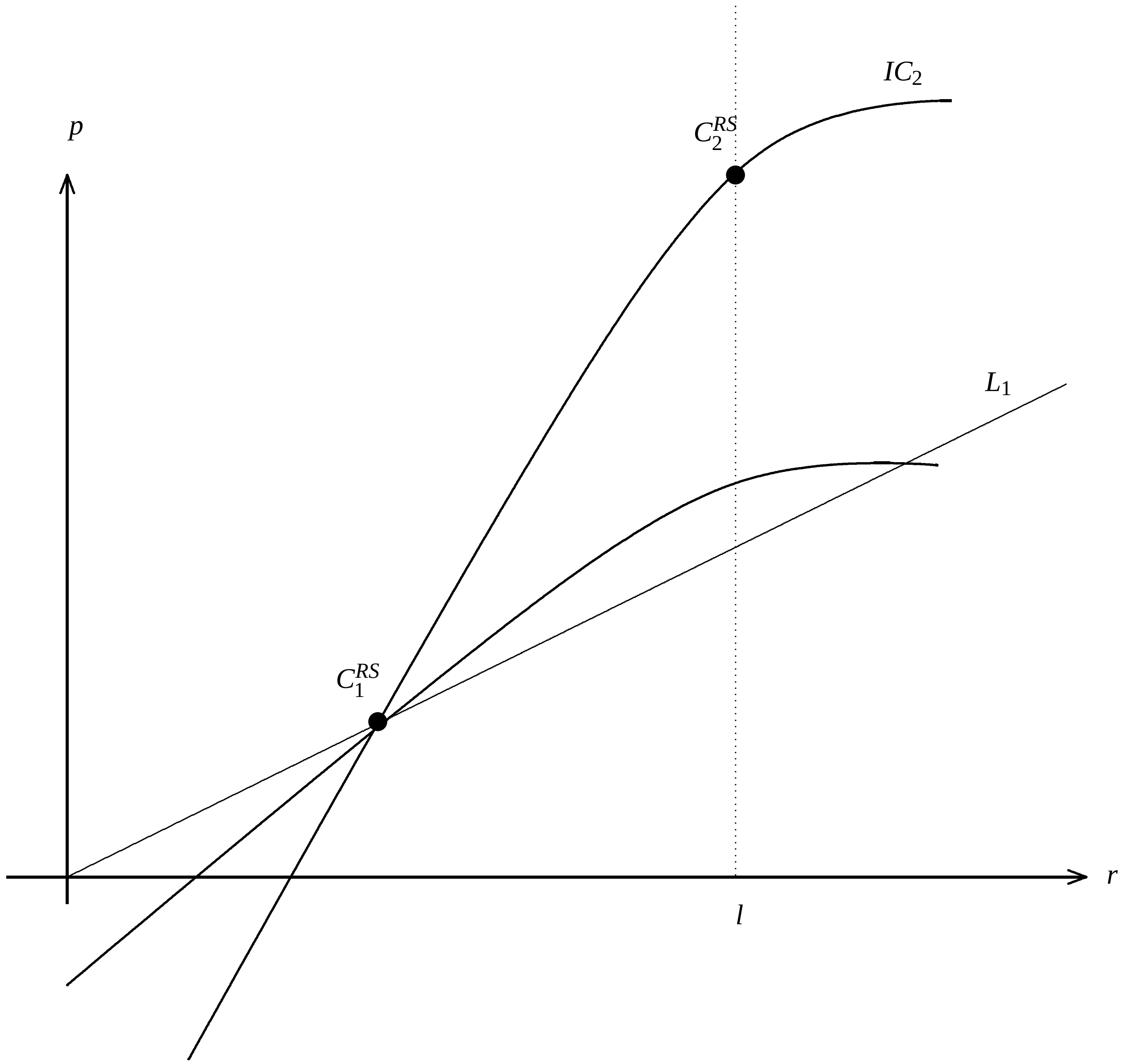


Figure 3: The Rothschild-Stiglitz equilibrium

We denote C_1^{RS} and C_2^{RS} the equilibrium contracts. The high probability agents receive their competitive outcome: $C_2^{RS} = C_2^{FB}$. Hence they fully insured. The low probability agents receive the contract c_1^{RS} which is defined has the intersection of the line L_1 and the indifference curve of the high probability agents passing through C_2^{FB} (here denoted IC_2). The low probability agent are not fully insured. The equilibrium exists if the proportion of high probability agents n_2 is large enough. This equilibrium (when it exists) is not efficient, even if we consider the constraints due to information. An imperfectly informed planner could improve the allocation.

3 Linear Prices

We consider a scenario in which n insurance companies ($n > 1$) interact by offering linear contracts. By linear contracts, we mean that they offer menus in which the premium p is proportional to the indemnity r, more precisely, menus M_α that can be defined as

$$M_\alpha = \left\{ (p, r) \in \mathbb{R}_+^2 \, | p = \alpha r, \text{ with } \alpha \in [0, 1] \right\}. \tag{5}$$

We will show that if $n-1$ firms are offering the linear contract M_α, the n^{th} has no incentive to offer that contract.

In order to do that, let us assume that at equilibrium each insurance company offers a contract such that the premium p is proportional to the indemnity r. In that case, agent with the probability of accident π, (with $\pi \in \{\pi_1, \pi_2\}$) who has chosen a contract $(\alpha r, r)$ gets the utility:

$$\pi U (w - \alpha r - l + r) + (1 - \pi) U (w - \alpha r). \tag{6}$$

Let us remark that choosing a contract for an agent, is equivalent to choose a particular r. For the agent contracting with only one company or accepting several contract from different companies is equivalent.

Let us denote the contract chosen by the agent π:

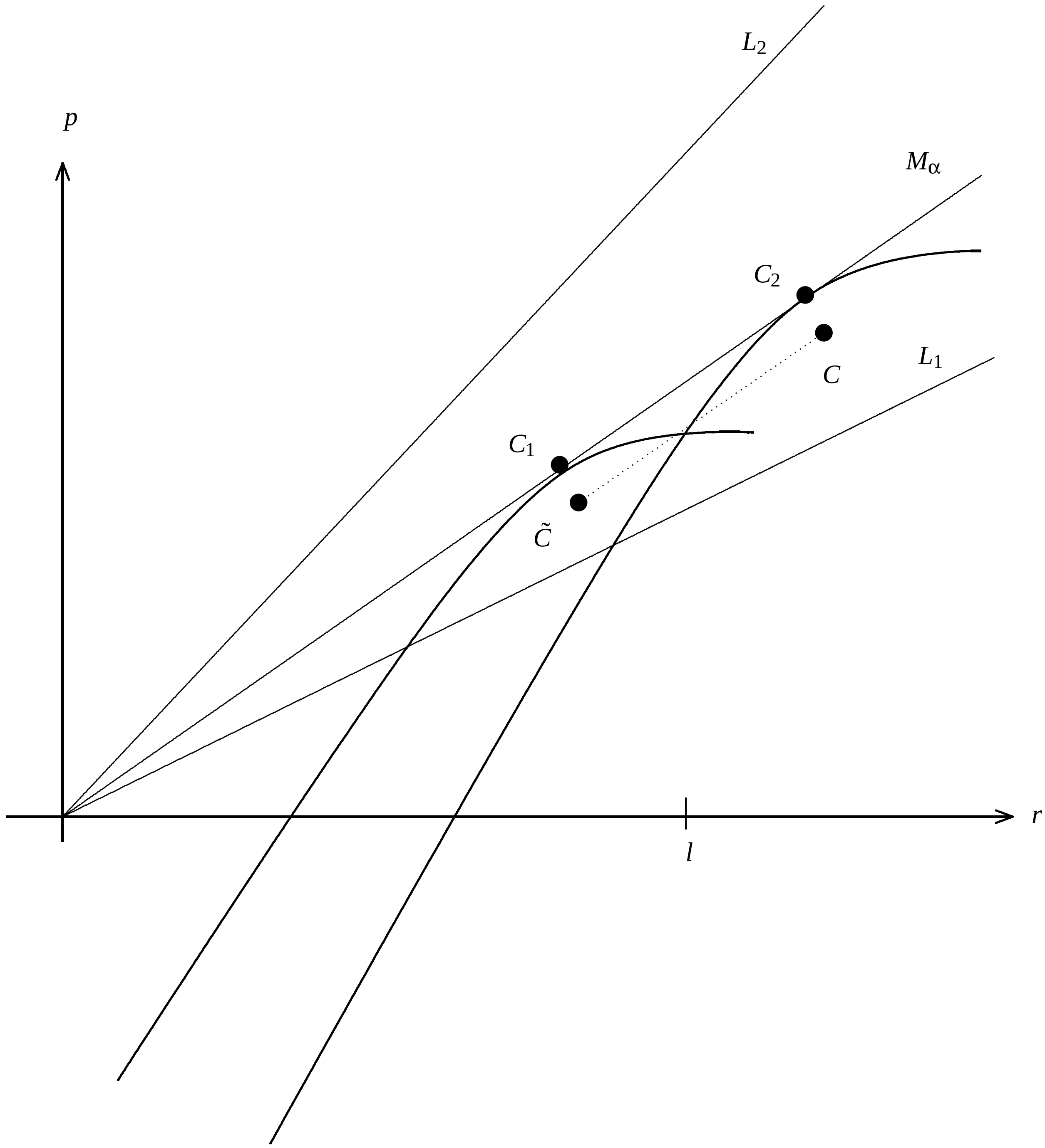
$$[r^* (\alpha, \pi); p^* (\alpha, \pi) = \alpha r^* (\alpha, \pi)] \tag{7}$$

If $\alpha \leq \pi$ then $r^* (\alpha, \pi) = l$, and if $\alpha > \pi$ then $r^* (\alpha, \pi) < l$. Moreover, for every $\alpha \in (\pi_1, \pi_2]$ we have $r^* (\alpha, \pi_1) < r^* (\alpha, \pi_2) \geq l$. To simplify the notations, we define $\Pi (\alpha)$ as

$$\Pi (\alpha) = n_1 [\alpha r^* (\alpha, \pi_1) - \pi_1 r^* (\alpha, \pi_1)] + n_2 [\alpha r^* (\alpha, \pi_2) - \pi_2 r^* (\alpha, \pi_2)]. \tag{8}$$

Lemma 1 *Let us consider a linear symmetric equilibrium in which each company offer the menu of contract M_α where $\alpha \in (0, \pi_2)$. Then it exists a contract $(\tilde{p}, \tilde{r}) \in \mathbb{R}^2_+$ such that $\tilde{p}/\tilde{r} < \alpha$ and $n_1\tilde{p} - n_1\pi_1\tilde{r} + n_2\tilde{p} - n_2\pi_2\tilde{r} > \Pi(\alpha)$.*

Graphically:



If the Menu M_α is offered, low risk agent buy the contract C_1 and high risk agents buy the contract C_2 as indicated in the graph. It would be better or a principal to constrain the high risk agents to buy the contract C_1. This can be (almost) done if he offers the single contract $\tilde{C}$. The low risk agents prefer this new contract. The high agents to. They by it and they supplement it in order to get the contact C. If $\tilde{C}$ is close enough to C_1, the deviation is profitable. This is formally stated in our first proposition.

Proposition 1 *If the single crossing condition is satisfied, at equilibrium, insurance companies are offering linear contracts.*

Let us interpret a little bit more this first proposition. It is show that if all other companies offer the same linear menu M_α, the best reply of a company is to offer something else. But it has stronger implications. Imagine that the offers made by the $(n-1)$ companies lead to an aggregate menu equal to M_α. This can happen if they are infinitely many companies that offer each a single contract which belongs to the line defined by $p = \alpha r$. Then the best reply of the remain company is to offer a contract or a set of contract which is not in the line defined by $p = \alpha r$.

Our result does not rely on the number of company n (as long as $n > 1$). Hence even if n is very large, in other words, even if the market is very competitive, we should not observe linear prices.

If the single crossing condition is not satisfied, it may exist a linear equilibrium.[3] Concerning our model, even if we do not have a formal proof, the existence of a linear equilibrium would be very surprising, locally the the same kind of argument can be applied.

4 Nonlinear Prices

In the previous section we have shown that at equilibrium menus are not linear. Hence it is legitimate to try to characterize nonlinear equilibria. One natural candidate for an equilibrium is the RS equilibrium. In fact it turns out that the RS equilibrium is not implement anymore if insurance companies cannot impose an exclusive relationship to the agent.

Proposition 2 *The RS equilibrium is never implementable under non-exclusivity.*

[3] Under exclusivity, Chassagnon and Chiappori (1997) show that insurance competition that the RS equilibrium is modified, even if some of its predictions remain valid.

The formal proof is in appendix, but the proposition can be easily illustrated by an graphic.

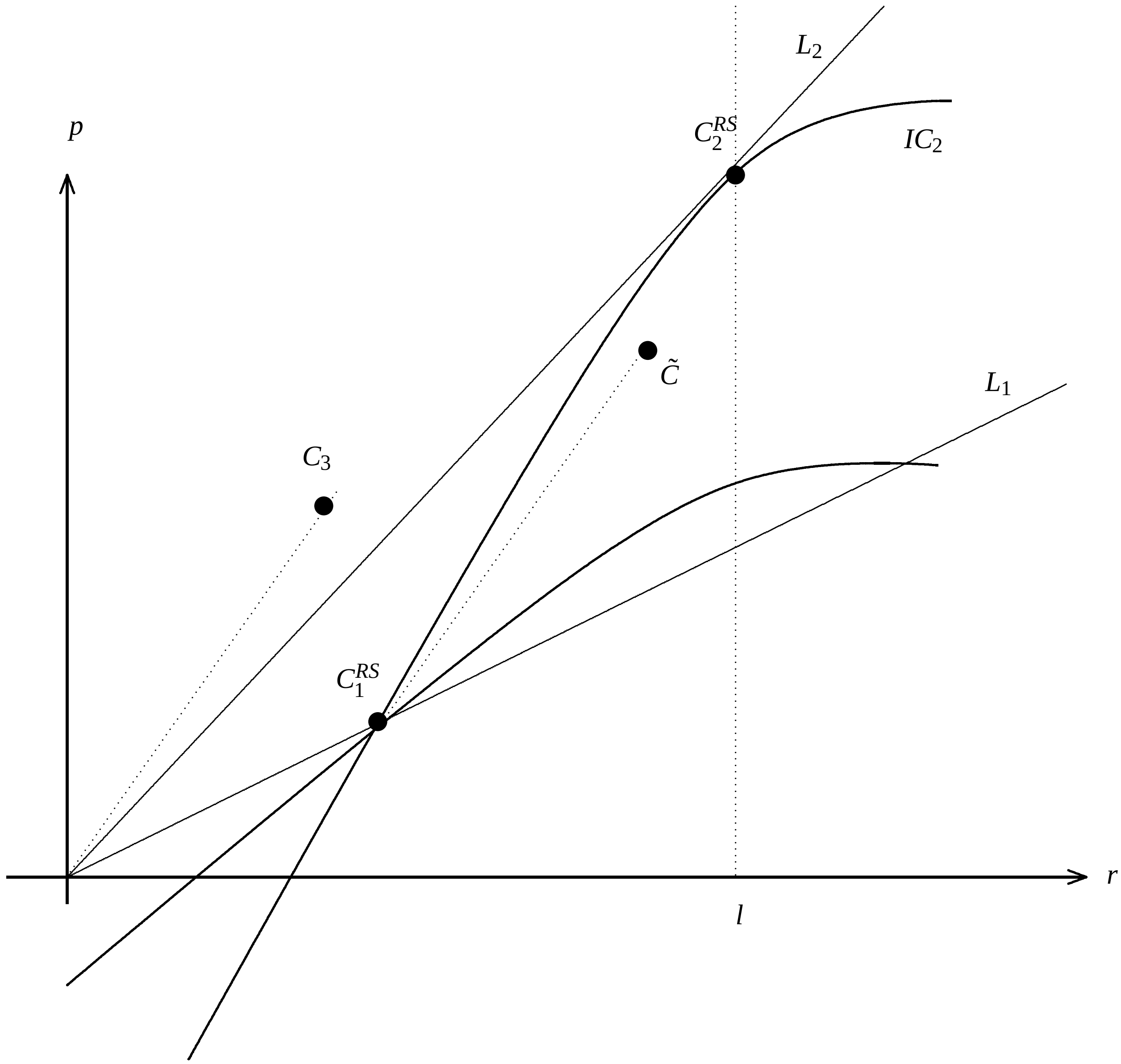


Figure 4: The Rothschild-Stiglitz equilibrium is not robust

In the example presented in the graphic, if all other insurance companies offer the RS contracts, a insurance company can offer the contract C_3. As shown in the graphic the contract C_3 generates positive profit if it bough by the high risk agents as it is above the line L_2. The graphic also shows that this contract is never bough alone by a high risk agent, it is above the curve IC_2. But the high risk agents can buy the contract C_1 and the contract C_3 together, then they reach the point $\tilde{C}$ which is strictly preferred to the point C_2^{RS}. Hence, the deviation is profitable for the deviating principal.

The RS equilibrium is not implementable because an insurance company can use the contract offered by its competitors as bases for its own offers. Roughly speaking an insurance company can ask to a high risk agent to misreport his type to an other company. He will enjoy a cheap contract, but with a low coverage. The deviating company then propose to him to supplement this contract with a small contract. The resulting contract, will remains relatively cheap, but with a acceptable coverage. Obviously, this deviation implies losses for the non-deviating companies.

Under non-exclusivity, insurance companies must prevent this kind of deviation, and hence they cannot propose anymore the RS contracts.

Let us consider the following contracts.

- First we define the contract $\tilde{C}_1 = (\tilde{p}_1, \tilde{r}_1)$ as the contract that maximizes $V(p_1, r_1, \pi_1)$ over $(p_1, r_1) \in \mathbb{R}_+^2$ under the constraint $p_1 = (n_1\pi_1 + n_2\pi_2)\, r_1$.
- Second we define the contract $\tilde{C}_2 = (\tilde{p}_2, \tilde{r}_2)$ in the following way. The coverage r_2 is such that $r_1 + r_2 = l$ and $\tilde{p}_2 / \tilde{r}_2 = \pi_2$.
- Finally, we define $\hat{C} = (\hat{p}, \hat{r})$. The contract $\hat{C}$ is such that $V(\tilde{C}_2, \pi_2) = V(\tilde{C}_1 + \hat{C}, \pi_2)$.

These contracts will be the contract offered at equilibrium, together with the linear menu $\left\{(p,r) \in \mathbb{R}_+^2 \,|p = \pi_2\, r\right\}$. We need to define few more contracts, that are convenient to characterize our equilibrium but which are not offered at equilibrium.

- The contract $\overline{C}_1 = (\overline{p}_1, \overline{r}_1)$ is defined in the following way. It is a contract that satisfied the three following conditions: $V\left(\tilde{C}_1, \pi_1\right) = V\left(\overline{C}_2, \pi_2\right)$, $\overline{p}_2 = \pi_1 \overline{r}_2$, $\tilde{r}_1 < l$.
- The contract $\overline{C}_2 = (\overline{p}_2, \overline{r}_2)$ is defined in a similar way. It is a contract that satisfied the three following conditions: $V\left(\tilde{C}_1 + \tilde{C}_2, \pi_2\right) = V\left(\overline{C}_1, \pi_2\right)$, $\overline{p}_1 = \pi_1 \overline{r}_1$, $\tilde{r}_1 < l$.

- The contract $\check{C}_1 = (\check{p}_1, \check{r}_1)$ is defined as $\check{C}_1 = \overline{C}_1 + \hat{C}$.
- Finally, $\check{C}_2 = \overline{C}_2 + \hat{C}$.

Graphically these contracts can be represented as follows

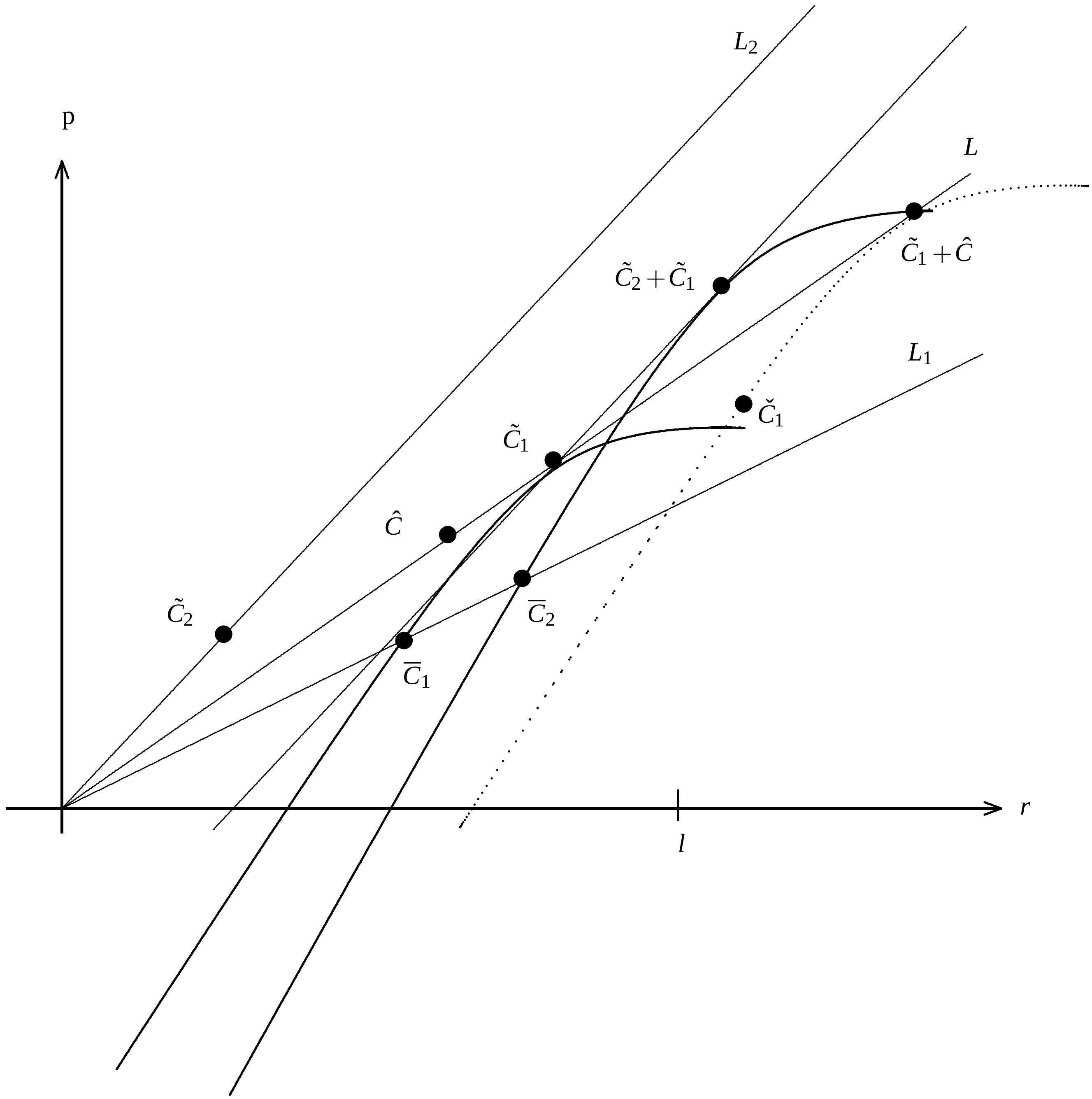


Figure 5: The new contracts

The line L is the line having the equation $p = (n_1 \pi_1 + n_2 \pi_2)\, r$.

We need conditions to characterize our equilibrium.

Condition 1 *The condition (E) is satisfied is and only if*

- $V\left(\tilde{C}_1+\tilde{C}_2,\pi_2\right) > V\left(2\tilde{C}_1,\pi_2\right)$,
- $2\hat{C} < \tilde{C}_1$,

These two conditions ensure that the high risk agents will prefer the contracts designed for them (the contract $\tilde{C}_1+\tilde{C}_2$) rather than the contract than a contract that would result from the duplication of the other existing contracts. In figure 5 the condition $2\hat{C} < \tilde{C}_1$ is not satisfied while the other condition is.

Condition 2 *The condition (E′) is satisfied is and only if for all contracts* $C=(p,r)\in\mathbb{R}_+^2$ *such that* $V(C,\pi_1)=V\left(\tilde{C}_1,\pi_1\right)$ *and* $p\geq\pi_1 r$, *we have* $V\left(\tilde{C}_1+\tilde{C}_2,\pi_2\right)\leq V\left(C+\hat{C},\pi_2\right)$.

These conditions are needed to be sure that if an insurance deviates from the contract $\tilde{C}_1$ and offer a contract $\underline{C}$ then the high risk agent will by it together with the contract $\hat{C}$. As we will see, it will make the deviation non profitable.

Proposition 3 *If the condition (E) and (E′) are satisfied: Hence the following menus constitute an equilibrium:*

- *n companies offer the menu* $\left\{\tilde{C}_1\right\}\bigcup\left\{(p,r)\in\mathbb{R}_+^2\,|p=\pi_2 r\right\}$,
- *2 companies offer the menu* $\left\{\tilde{C}_1,\hat{C}\right\}\bigcup\left\{(p,r)\in\mathbb{R}_+^2\,|p=\pi_2 r\right\}$.

The agents buy the contract $\tilde{C}_2$ *if their type is* π_2 *and the contract* $\tilde{C}_1$ *when their type is* π_1.

Before to state the conditions under which the equilibrium exists, we will make few remarks.

At equilibrium the contract $\hat{C}$ is never bought, it is a "latent contract".[4] Even if this contract is not bought, it plays a crucial role. If it is not offered, a principal can profitably deviate by offering a contract close to $\underline{C}_1$ bought by the low risk agents only. This deviation is not possible because the high risk would by this contract together with

[4]The term latent contract has been first introduced by Hellwig (1983).

the contract $\hat{C}$. Martimort and Stole (2002) analyse in detail the implications in term of direct mechanisms and menus. It is because we need "latent contracts" at equilibrium that we cannot restrict attention to direct mechanisms and that we should consider menus instead. The same remark applies for almost all contracts in the menu $\left\{(p,r) \in \mathbb{R}_+^2 \,|\, p = \pi_2 r\right\}$.

The high risk agents buy two contract. The equilibrium can obviously rewritten with a new contract $C_2 = \tilde{C}_1 + \tilde{C}_2$. The important thing is that risk agents can buy the contracts $\tilde{C}_1$ and $\tilde{C}_2$. In that interpretation of the equilibrium, we can emphasis that a kind of cross-subsidization occurs at equilibrium. The low risk agents buy the contract $\tilde{C}_1$ which is not actuarially fair given their probability π_1. They buy a relatively expensive contract. On the other hand, the high risk agents buy a relatively cheap contract, C_2 or $\tilde{C}_1 + \tilde{C}_2$. This contract is sustainable only because it is subsidized by the low risk agents. In the RS's model, the equilibrium contract are acturially fair and any kind of cross-subsidization can be sustained at equilibrium.

Insurance companies do not make profit at equilibrium. It is a surprising result compared to one obtained by Bisin and Guaitoli (2004). As we have said, the main difference between our model and their, is that we consider adverse selection and their consider moral hazard. Contrary to us, they found positive profit equilibria (for some values of their parameters). These positive profit equilibria are very strong, they are robust to free entry.[5]

The intuition behind these very different results could be that in moral hazard model a little change in the offers may induce very different behaviors. Agent who was making efforts may stop suddenly. Hence in a moral hazard setting the Bertrand's argument does not apply because a aggressive offer may change the agent's behavior.

Here, agent takes no decisions, except to buy or not contracts. Hence an insurance company can make aggressive offers provided that its offer remains incentive compatible. It turns out that the only aggressive offers that an insurance can make are the ones bought by all the agents. Hence at equilibrium insurances propose a contract that is bought by all agent and leads to zero profits.

[5]Positive profit equilibria seems to be a common feature of common agency modal of moral hazard. Parlour and Rajan (2001) in a model of strategic default show that at equilibrium profits may be strictly positive.

This interpretation should be taken carefully. Biais, Martimort, and Rochet (2000) proposed a model close to our and characterize positive profit equilibria. Their model is not a real model of insurance and it is interpreted as a model of finance. Nevertheless the reason why their results are different from ours remains unclear. An explanation could be that they consider a continuum of type and we consider only two types of agents. In any case their model and our are quite different and cannot be compared directly.

At equilibrium, high risk agents are fully insured, low risk agent are only partially covered. It justify the approach taken in applied papers. It is not a surprising result. Our model can be interpreted as the RS's model plus some extra constraints. Hence, it is natural to find that it exhibits the RS's equilibrium main property.

Finally, the number of firms do not affect the equilibrium (as long as $n \geq 1$). Hence even if n becomes very large, the equilibrium does not converge toward a linear equilibrium. Hence our model does not provide any foundation for a general equilibrium approach of incomplete information models.[6]

Proposition 4 *If π_1 and π_2 are closed enough and is n_1 is small enough, the equilibrium characterized in proposition 3 exists.*

The existence conditions are different from RS. They need a sufficiently high proportion of high risk agents. We require exactly the reverse, a sufficiently high proportion of low risk agents. Moreover they do not need any assumption one the probabilities π_1 and π_2. We need the restrictive condition π_1 and π_2 not to much different.

Hence the RS equilibrium and our equilibrium cannot be compared straightforwardly. They are likely to exist for different values of the parameters.

Nevertheless, we can state a last important property of our equilibrium.

Proposition 5 *The equilibrium characterized in proposition 3 sustains an optimal allocation.*

[6] Let us remark that Biais, Martimort, and Rochet (2000) find an allocation equilibrium which depends on n. But in their model, as in our, the limit equilibrium is not a linear equilibrium.

The equilibrium allocation is different from the full information equilibrium described in figure 2. As the utilities are not transferable, the optimal quantities are not unique. Hence we can have two different allocations that are both optimal.

The equilibrium allocation is also first best optimal. We mean that it would be an optimal allocation in a full information economy. At a first glance it sounds very surprising. It is not. The information structure, here the fact that agents know their probability of survival, constrains the set of optimal allocation. In other words some optimal are not implementable because of information issues. For example, it is clear the full information competitive equilibrium allocation is not sustainable anymore under asymmetric information. If some optimal allocations are not implementable anymore, some remain implementable, as it is the case for our equilibrium allocation. In general, optimal allocations that give a lot of surplus to the high risk agents are likely to be implementable under asymmetric information. As they enjoy an high level of welfare, high risk agents have no reason to adopt a strategic behavior.

Our interpretation of this result is the following. First, let us remark that at equilibrium the risk agents are not indifferent between their contract $\tilde{C}_1 + \tilde{C}_2$ and the contract designed for the low risk agents, $\tilde{C}_1$. High risk agent prefer $\tilde{C}_1 + \tilde{C}_2$ than $\tilde{C}_1$ and low risk agent prefer $\tilde{C}_1$ than $\tilde{C}_1 + \tilde{C}_2$. High risk agents have many way to cheat on the insurance companies. They can choose one type contract from one company an another type of contract from another company. In the RS's model they can only choose the contract designed for the low risk agents. As they have many way to misbehave, insurance companies must let top them a lot of surplus. As they get a lot of surplus at equilibrium they do not have reason to misbehave, i.e; to choose contracts not designed for them. It follows that at equilibrium information is not an issue, and hence market forces lead to an optimal allocation.

There is no standard results concerning optimality in common agency games. Bisin and Guaitoli (2004) show that equilibria characterized in their moral hazard setting are not efficient. In the other hand, Attar, Campioni, and Piaser (2006) show that the equilibria of Parlour and Rajan's (2001) common agency model are efficient. In the same manner the equilibrium characterized in Biais, Martimort, and Rochet (2000) is not efficient.

To conclude our remarks about optimality, we would emphasis that if the equilibrium is efficient, it may remain issues about redistribution. For example compared to RS allocation, high risk agents prefer the non-exclusive equilibrium, but the low risk agents prefer the RS equilibrium, and insurance companies are indifferent.[7] Then, even if it is not efficient, the RS equilibrium may be preferred to the non-exclusive outcome for redistribution motives.

5 Conclusion

The literature on annuity markets focuses on equilibria in which prices are linear. In this note, we have shown that there is no clear game theoretic foundations for that approach. More precisely, in a context of adverse selection, if insurance companies compete through nonlinear prices, there is no equilibrium with linear price or converging toward linear prices, even if the number of companies is very large.

As it appears that empirically prices seems to be linear, (see for example Finkelstein and Porteba (2004)), we could conclude that adverse selection may not be relevant phenomenon in annuity markets. It does not means that adverse selection is totally absent, but other imperfections or features should be at work. Mainly we see two potential lines of research:

Moral hazard may be a relevant aspect of annuity markets. Agents may have influence on their expected lite time. If an agent knows that it will receive a decent pension in his old days, he may change if behavior (alcohol, cigarettes) to enjoy a longer life. This is a potentially important aspect of annuity markets which has not been fully explored yet.

The positive correlation between life time and annuity could be explained by different in wealth rather than by adverse selection effect as claimed by Attanasio and Hoynes (2000). The potential importance of wealth distribution in retirement assets management has been studied,[8] but its policy implications are not been intensively studied.

[7] We should keep in mind that to make this comparison, the to equilibria must exist.

[8] For a survey see Webb (2004).

Dushi and Webb (2004)'s findings suggest that wealth distribution is very important factor that cannot be neglected.

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A Proof of Lemma 1

Let us consider the profit $\Pi(\alpha)$.

$$\Pi(\alpha) = n_1 \left[\alpha r^*(\alpha,\pi_1) - \pi_1 r^*(\alpha,\pi_1)\right] + n_2 \left[\alpha r^*(\alpha,\pi_2) - \pi_2 r^*(\alpha,\pi_2)\right]. \tag{9}$$

Let us consider that the variable $r^*(\alpha,\pi_2)$ is given. In that case, as $\alpha < \pi_2$,

$$\frac{\partial \Pi(\alpha)}{\partial r^*(\alpha,\pi_2)} < 0. \tag{10}$$

The function $\Pi(\alpha)$ is linear with respect to the variable $r^*(\alpha,\pi_2)$, hence $n_1 \alpha r^*(\alpha,\pi_1) - n_1 \pi_1 r^*(\alpha,\pi_1) + n_2 \alpha r^*(\alpha,\pi_1) - n_2 \pi_2 r^*(\alpha,\pi_1) > \Pi(\alpha)$. By continuity, it exists a contract $(\tilde{p},\tilde{r})$ as described in the statement of the lemma.

B Proof of Proposition 1

If there is a linear equilibrium characterized by $\tilde{\alpha}$.

- If $\alpha \in [0,\pi_1]$, companies make negative profits. Hence it cannot be equilibrium value.
- If $\alpha \in (\pi_1,\pi_2)$, lemma 1 implies that at least one company has a profitable deviation, the contract $\tilde{C}$. Even if the contract is bought by all agent, this contract if more profitable than the linear menu M_α.

- If $\alpha \in [\pi_2, 1]$ hence $\Pi(\alpha) > 0$, and a standard argument applies: a firm can attract all the agents by offering a infinitesimally less profitable linear menu $M_{\alpha-\varepsilon}$, where $\varepsilon > 0$ and "very small".

C Proof of Proposition 2

Proof. Let us imagine that the principals offer the RS contracts, namely they all offer the menu $\left\{\left(p_1^{RS}, r_1^{RS}\right), \left(p_2^{FB}, r_2^{FB}\right)\right\}$.

If $V(p_2^{FB}, r_2^{FB}, \pi_2) < V(2\,p_1^{RS}, 2\,r_1^{RS}, \pi_2)$, then the π_2-type agents will not by the contract $\left(p_2^{FB}, r_2^{FB}\right)$, but two $\left(p_1^{RS}, r_1^{RS}\right)$ contracts. Hence the contract $\left(p_2^{FB}, r_2^{FB}\right)$ is never bought and some insurance companies experience losses.

If not, then a insurance company can profitably deviate from $\left\{\left(p_1^{RS}, r_1^{RS}\right), \left(p_2^{FB}, r_2^{FB}\right)\right\}$. We consider the space (p, r) and the agents' indifference curves in that space. At the RS equilibrium the π_2-type agents are indifferent between the contracts $\left(p_1^{RS}, r_1^{RS}\right)$ and $\left(p_2^{FB}, r_2^{FB}\right)$. Hence, for those agents, there is a indifference passing through the points $\left(p_1^{RS}, r_1^{RS}\right)$ and $\left(p_2^{FB}, r_2^{FB}\right)$, that we denote IC_2. At the point $\left(p_1^{RS}, r_1^{RS}\right)$, the indifference curve IC_2, has derivative strictly greater than $\mathrm{p}_2^{\mathrm{FB}}/\mathrm{r}_2^{\mathrm{FB}}$. A insurance company can propose a single contract (p', r'), with p' and r' "small" but different from zero and such that $(p', r') > \mathrm{p}_2^{\mathrm{FB}}/\mathrm{r}_2^{\mathrm{FB}}$.

If p' and r' are sufficiently small, then we have

$$V(p_2^{FB}, r_2^{FB}, \pi_2) < V(p_1^{RS} + p', r_1^{RS} + r', \pi_2). \tag{11}$$

The deviation is profitable for the insurance company. Whoever buy the contract, it makes positive and it bought with probability 1 by the π_2-type agents. ∎

D Proof of Proposition 3

D.1 Singleton

Let us imagine that one insurance company deviates by just offering a menu $\left\{\left(\underline{p}, \underline{r}\right)\right\}$ and we take the notation $\underline{C} = \left(\underline{p}, \underline{r}\right)$.

First, if the contract $\underline{C}$ is bought by the π_2-type agents only, it cannot be profitable. They buy the contracts $\tilde{C}_1$ and $\tilde{C}_2$, and those contracts would induces losses it they were bought by π_2-type agents only. Hence, all contracts that are preferred to $\tilde{C}_1+\tilde{C}_2$ by the π_2-type agents induce loss for the insurance companies.

We assume that the contract $\underline{C}$ is bought by all agents. In order to be bought by the π_1-type agents, the contract $\underline{C}$ must be below the line define by the equation $p=(n_1\pi_1+n_2\pi_2)\,r$. Hence, to be profitable, it must be bought be the π_1-type agents only.

To be bought by the π_1-type agents only, the contract $\underline{C}$ must belong to the set Zdefined in the following way:

$$Z=\left\{(p,r)\in\mathbb{R}_+^2\;\middle|\;\begin{array}{l}p\geq\pi_1 r,\\ V\left((p,r),\pi_1\right)\geq V\left(\tilde{C}_1,\pi_1\right),\\ V\left((p,r),\pi_2\right)\leq V\left(\tilde{C}_2,\pi_2\right)\end{array}\right\}.$$

This set has a simple graphical representation. We denote $I\tilde{C}_1$ the indifference of the π_1-type agents passing through the contract $\tilde{C}_1$, and We denote $I\tilde{C}_2$ the indifference of the π_2-type agents passing through the contract $\tilde{C}_2$. We denote by C^* their intersection.

Under the basic assumption of our model, this intersection exists. Unambiguously we can write $C^*>\tilde{C}_1$, as the curve $I\tilde{C}_2$ is tangent to a line passing through the point $\tilde{C}_1$, the curve $I\tilde{C}_1$ and $I\tilde{C}_2$ intersect at the right of $\tilde{C}_1$. Moreover under the condition (E) we have $\overline{C}_1<\overline{C}_2$. The set Z can be seen as the curviline triangle $C^*\overline{C}_1\overline{C}_2$.

Let us remark that under our assumptions, the set Z is nonempty.

Let us consider the contracts (p,r) of Z such that $V\left((p,r),\pi_1\right)=V\left(\tilde{C}_1,\pi_1\right)$ and $r\leq\tilde{r}_1$. It is a segment of the curve $I\tilde{C}_1$, that we denote $\tilde{S}_1$.

The translation with respect to the vector $\hat{C}$ is denoted $T_{\hat{C}}$. We denote $\overline{IC}_1$ the translate of $I\tilde{C}_1$ by the translation $T_{\hat{C}}$. The translate of the segment $\tilde{S}_1$ by the translation $T_{\hat{C}}$ is the segment $overline S_1$, from $\tilde{C}_1+\hat{C}_1$ to $\check{C}_1$. We have assumed that $\overline{C}_2<\check{C}_1$.

Under our assumptions (condition (E$'$), the curve $I\tilde{C}_2$ and the segment $\tilde{S}_1$ cannot intersect two times.

From that discussion it comes that every point of the segment $\overline{S}_1$ is preferred to the point $\tilde{C}_1 + \tilde{C}_2$ by the π_2-type agents.

Now we consider the set of contract (p, r) such that:

$$\begin{aligned}
&p \geq \pi_1 r, \\
&V((p,r), \pi_1) \geq V(\tilde{C}_1, \pi_1), \\
&V((p,r), \pi_2) = V(\tilde{C}_2, \pi_2), \\
&p \leq \tilde{p}_1.
\end{aligned} \tag{12}$$

It is easy to check that this set (denote $\tilde{S}_2$) is nonempty. Using simple arguments, it can be shown that if the contract C belongs to $\tilde{S}_2$, then

$$V\left(C + \hat{C}, \pi_2\right) \geq V\left(\tilde{C}_1 + \tilde{C}_2, \pi_2\right). \tag{13}$$

We define the set $\tilde{Z}$ as

$$\tilde{Z} = \left\{ (p,r) \in \mathbb{R}_+^2 \;\middle|\; \begin{array}{l} p \geq \pi_1 r, \\ V((p,r), \pi_1) \geq V(\tilde{C}_1, \pi_1), \\ V((p,r), \pi_2) \leq V(\tilde{C}_2, \pi_2), \\ p \leq \tilde{p}_1 \end{array} \right\}.$$

Hence we can state that if the contract $\underline{C}$ belongs to the set $\tilde{Z}$, then

$$V\left(\underline{C} + \hat{C}, \pi_2\right) \geq V\left(\tilde{C}_1 + \tilde{C}_2, \pi_2\right). \tag{14}$$

We define the set $\check{Z}$ as

$$\tilde{Z} = \left\{ (p,r) \in \mathbb{R}_+^2 \;\middle|\; \begin{array}{l} p \geq \pi_1 r, \\ V((p,r), \pi_1) \geq V(\tilde{C}_1, \pi_1), \\ V((p,r), \pi_2) \leq V(\tilde{C}_2, \pi_2), \\ p \geq \tilde{p}_1 \end{array} \right\}.$$

If the contract $\underline{C}$ belongs to the set $\tilde{Z}$, then by construction it exists an $\alpha > 0$ "small enough" such that

$$V\left(\underline{C} + \alpha\tilde{C}_2, \pi_2\right) \geq V\left(\tilde{C}_1 + \tilde{C}_2, \pi_2\right). \tag{15}$$

To conclude, deviating toward a single contract $\underline{C}$ is never profitable for a principal.

D.2 Two contracts menus

Let us assume that a principal deviates by offering a menu $\{\underline{C}_1, \underline{C}_1 + \underline{C}_2\}$. To be profitable, the preceding offer must be such that

$$V(\underline{C}_1, \pi_1) \geq V\left(\tilde{C}_1, \pi_1\right), \tag{16}$$

$$V(\underline{C}_1 + \underline{C}_2, \pi_2) \geq V\left(\tilde{C}_2, \pi_2\right), \tag{17}$$

$$V(\underline{C}_1 + \underline{C}_2, \pi_2) > V\left(\underline{C}_1 + \hat{C}, \pi_2\right). \tag{18}$$

It implies that the contract $\underline{C}_1$ is below the line L. Moreover, the contract $\underline{C}_2$ must be the line L_2. It implies that the offer $\{\underline{C}_1, \underline{C}_1 + \underline{C}_2\}$ cannot be profitable.

Any deviation involving two contracts that can be written as a menu $\{\underline{C}_1, \underline{C}_1 + \underline{C}_2\}$ are not profitable. Using the same kind or argument, one can see that there is no profitable deviation by using menus of two contracts.

D.3 General Menus

Let us consider a menu M (compact subset of $\mathbb{R}^2_+$). As agent are "small", the do not interact (the game is similar to a game with one agent only), we can neglect mixed strategies. We assume that π_1-type agents choose the contract $\underline{C}_1$ and the π_2-type agents choose the contract $\underline{C}_2$. Hence rather than offer the menu M, the deviating principal could equivalently offer the menu $\{\underline{C}_1, \underline{C}_2\}$. As we have seen, these deviations are not profitable.

E Proof of Proposition 4

First let us imagine that $n = n_1$ and $\pi_1 < \pi_2$. Using continuity, we can state that that if π_1 are π_2 closed enough, $\hat{C}$ is as small as we want. Moreover if $n = n_1$, then $\tilde{C}_1 = \overline{C}_1$ and $\check{C}_1 = \hat{C} + \tilde{C}_1$. Hence by construction we have $\overline{C}_1 < \overline{C}_2 < \check{C}_1$. Hence conditions (E) and

(E′) are satisfied. Apply a simple argument of continuity, we can argue that it remains true if n_1 close enough to n.

F Proof of Proposition 5

If one want to increase π_2-type agents utility, it can only be done by decreasing the profit of the insurance company. This due to the indifference curve passing through $\tilde{C}_2 + \tilde{C}_1$ has derivative equal to π_2 at the point $\tilde{C}_2 + \tilde{C}_1$.

In order to get positive or zero profits, insurance companies must charge the π_1-type agents, which decrease their utility.

The same argument applies to every other players (companies and π_1-type agents).

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