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Ca' Foscari University of Venice

Andrea Attar
Eloisa Campioni
Gwenaël Piaser
Uday Rajan

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On multiple agent models of moral hazard

Andrea Attar
IDEI, Toulouse and DSE Università di Roma, La Sapienza

Eloisa Campioni
LUISS, University of Rome

Gwenaël Piaser
University of Venice

Uday Rajan
Ross School of Business, University of Michigan

Abstract
In multiple principal, multiple agent models of moral hazard, we provide conditions under which the outcomes of equilibria in direct mechanisms are preserved when principals can offer indirect communication schemes. We discuss the role of random allocations and recommendations and relate the result to the existing literature.

Keywords
Moral Hazard, Multiple Agents, Direct Mechanism.

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Address for correspondence:
Gwenaël Piaser
Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta S.Giobbe
30121 Venezia - Italy
Phone: (++39) 041 234 91 22
Fax: (++39) 041 2349176
e-mail: piaser@gmail.com

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Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

1 Introduction

We consider multiple principal, multiple agent models of pure moral hazard. That is, there is complete information about the types of principals and agents. Principals offer allocations and agents choose non-contractible efforts, and payoffs are then realized.

Referring to recent researches on common agency games, we look for on the conditions under which pure strategy equilibria characterized by direct mechanism are robust to the introduction of richer indirect mechanisms.

We show that if principals use both stochastic allocations and private recommendations to the agents, direct mechanism equilibria remain equilibria when principals can choose more general communication schemes.

We provide two examples to show how both elements are necessary for the result. In the first example, we let a principal use recommendations but he is restricted to choose deterministically his allocation. Then it exists an indirect mechanism that gives to him a higher payoff. In the second example, we allow a principal to choose randomly among allocation without sending any recommendation to the agents. Again, the principal can improve his payoff by choosing an indirect mechanism. Finally, we discuss an example provided by Peters (2004) in detail, since it appears to contradict our theorem. In this example, Peters (2004) wanted to show that a pure strategy equilibrium characterized with direct mechanisms could be not robust to the introduction of indirect mechanisms. A critical feature for this result is that principals' strategies are restricted to be deterministic mechanisms. Allowing for stochastic mechanisms enables us to recover the robustness of equilibria in the example and the coherence with our framework.

As menus theorems do not apply in multi-principal multi-agent models, and as the methodology proposed by Pavan and Calzolari (2005) has not been yet extended to multi-principal multi-agent games, our theorem can be one step forward toward a more general characterization of equilibria in this framework.[1]

In the next section we start presenting the model.

[1]See Peters (2001) for a presentation of the menus theorems and a discussion of their extension to multi-principal multi-agent games. Han (2006) extends these theorems to a restricted class of multi-principal multi-agent games.

2 The Model

There are n principals dealing with k agents, where $n \geq 1$ and $k \geq 2$. That is, we consider a model with multiple agents. While the general model allows for multiple pricipals as well, the single principal case is a special case of some interest, and will be the focus of Sections 3 and 4.

Let Y_j be a set of deterministic allocations available to principal j, with typical element $y_j \in Y_j$. An allocation can be, for example, monetary transfers, tax rates, prices, or quantities, depending on the particular interpretation of the model. Each principal j chooses an allocation in the set $\Delta\left(Y_j\right)$, the set of lotteries that can be generated over the set of deterministic allocations Y_j.

There is complete information about agent types. However, each agent i chooses an unobservable effort $e^i \in E^i$, where E^i is a finite set. Therefore, the model is one of pure moral hazard. We denote the vector of efforts as $e = \left(e^1, e^2, ..., e^k\right) \in E = \times_{i=1}^{k} E^i$.

We use the general communication structure for principal-agent models introduced by Myerson (1982). Each principal j chooses a message space M_j^i (possibly the empty set) and a recommendation space R_j^i for each agent. We restrict M_j^i and R_j^i to be finite for each i and j. Let $R_j = \times_{i=1}^{k} R_j^i$ denote the set of recommendations principal j can make, with $M_j = \times_{i=1}^{k} M_j^i$, as before. After receiving the message array $m_j = \left(m_j^1, \cdots, m_j^k\right)$ from the agents, principal j chooses an allocation and a private recommendation to each agent.

As in Myerson (1982), his behavior is described by the choice rule $\pi_j : M_j \rightarrow \Delta\left(Y_j \times R_j\right)$. That is, principal j may choose a stochastic mechanism, which provides a lottery over allocations and recommendations for some message array m_j. When the choice rule π_j is not deterministic , we assume that the lottery over allocations is realized only after agents have chosen their efforts. However, principal j chooses a realization from the lottery over recommendations, and reports the realization r_j^i to agent i. Potentially, this allows a principal to induce a correlated equilibrium in the agents' efforts game.

With a slight abuse of notation, let $\pi_j\left(y_j, r_j \left| m_j\right.\right)$ denote the probability that allocation y_j and recommendation array r_j are chosen, given a message array m_j. A mechanism for the principal j is thus given by $\left(M_j, R_j, \pi_j\right)$. We denote by Γ_j the set of all available mechanisms to principal j. Let $\Gamma = \times_{j\in n}\Gamma_j$, with γ a generic element of Γ. Mechanisms are publicly observed, but a message from agent i to principal j, and a recommendation from principal j to agent i, are observed only by i and j. As is usual in the literature, principals commit to their mechanisms before agents send messages.

There are two stages at which agent i moves in the game. First, he sends a message array $m^i = \left(m^i_1, \ldots, m^i_n\right)$ to the principals. Then, after observing only his private recommendations $r^i = \left(r^i_1, \ldots, r^i_n\right)$, he chooses an effort $e^i \in E^i$. Given the mechanisms γ, let

$$\mu^i : \Delta\left(M^i\right) \tag{1}$$

denote the message strategy of agent i, where $M^i = \times_{j\in N} M^i_j$, and let

$$\delta^i : M^i \times R^i \to \Delta\left(E^i\right) \tag{2}$$

denote his strategy in the efforts game, where $R^i = \times_{j\in N} R^i_j$.

The time structure of the interaction is provided in Figure 1 and follows the one considered by Myerson (1982).

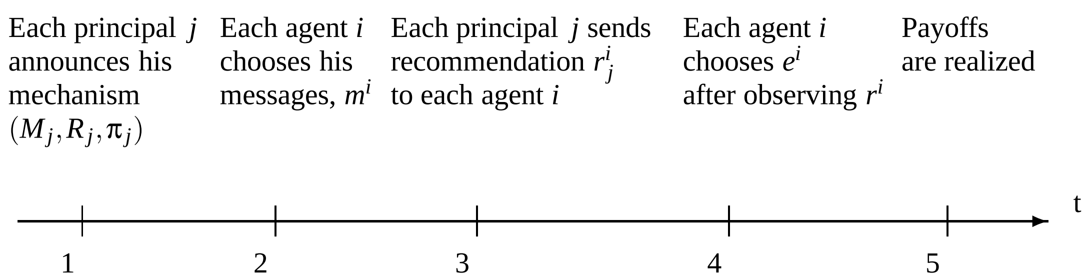

Figure 1: Timing of the generalized communication game

Agent i's payoff is given by the von Neumann–Morgenstern utility function $\overline{U}^i(y, e)$ and principal j's payoff is given by $\overline{V}_j(y, e)$. Given chosen mechanisms γ, let $\pi = \times^n_{j=1}\pi_j$ denote the strategies of the principals, $\mu = \times^k_{i=1}\mu^i$ the message reporting strategies of the agents, and $\delta = \times^k_{i=1}\delta^i$ the agents' strategies in the effort game. Also, let $U^i(\pi, \mu, \delta)$

denote the expected utility of agent i given strategies π, μ, δ, and $V^j(\pi, \mu, \delta)$ the corresponding expected utility of principal i.

In this complete information framework, a direct mechanism is defined as follows. Principals do not solicit messages from agents, and directly suggest the actions they should take. That is, $M_j^i = \emptyset$ and $R_j^i = E^i$ for every $j = 1, \ldots, n$ and for every $i = 1, \ldots, k$. Finally, $\pi_j \in \Delta\left(Y_j \times E\right)$. A mixed strategy for an agent in a direct mechanism is given by $\delta : \left(E^i\right)^n \to \Delta\left(E^i\right)$.

3 Single Principal

In the special case of a single principal, a revelation principle holds (see, e.g., Myerson (1982)). That is, any outcome (i.e., a joint distribution over allocations and efforts) that can be sustained as an equilibrium in the agents' effort game in an indirect mechanism can also be sustained as an equilibrium of the agents' effort game in an incentive compatible direct mechanism. In the direct mechanism, it is a best response for an agent to "obey" the recommendation received from the principal.

It follows that the optimal direct mechanism is optimal in the class of all communication mechanisms. That is, an equilibrium of the game depicted in Figure 1 in which the principal is restricted to choosing among direct mechanisms remains an equilibrium of the game even when the principal is allowed to choose indirect mechanisms.

In this section, we provide examples to highlight two features of the construction that are necessary to sustain the revelation principle in the single-principal case: stochastic allocations, and recommendations. Each of the two examples we consider has one principal and two agents. In each example, we show the optimal direct mechanism, and then show that, by privately communicating with one of the two agents, the principal can sustain outcomes in an indirect mechanism that are not feasible in a direct mechanism unless stochastic allocations (Example 1) and recommendations (Example 2) are allowed for.

Example 1 (Stochastic allocations):

Consider a game with one principal and two agents. The principal can choose between two allocations, and each agent chooses between two efforts levels. Following our notation, we have $Y = \{y_1, y_2\}$, $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$.

The payoffs are given by the following matrices. In each cell, the first element corresponds to the utility of the principal, and the second and third to the utilities of agents 1 and 2 respectively.

$y = y_1$

	b_1	b_2
a_1	$(1,-4,4)$	$(1,4,-4)$
a_2	$(1,4,-4)$	$(1,-4,4)$

$y = y_2$

	b_1	b_2
a_1	$(1,1,1)$	$(x,0,0)$
a_2	$(x,0,0)$	$(1,-1,-1)$

where $x > 1$.

Suppose the principal offers only deterministic allocations in any mechanism. A mechanism is then characterized by:

$$\tilde{\pi} : M \rightarrow Y \times \Delta(R),$$

where M is the message space and R the recommendation space. Hence, a direct mechanism with deterministic allocation is defined by $Y \times \Delta\left(E^1 \times E^2\right)$.

In a direct mechanism with deterministic allocations, recomemndations have no role. For each of the allocations y_1, y_2, there is a unique correlated equilibrium in the agents' effort game.

If the allocation is y_1, the payoff of the principal is trivially 1, regardless of agents' efforts. Agents play the following effort game:

$y = y_1$

	b_1	b_2
a_1	$(1, -4, 4)$	$(1, 4, -4)$
a_2	$(1, 4, -4)$	$(1, -4, 4)$

This game has a unique correlated equilibrium in which agents equally randomize between their two strategies. Hence, incentive compatibility if y_1 is chosen requires that the principal choose recommendations from the joint distribution that places a probability $\frac{1}{4}$ on each of the four effort combinations.

If the principal chooses the allocation y_2, the effort game is given by

$y = y_2$

	b_1	b_2
a_1	$(1, 1, 1)$	$(x, 0, 0)$
a_2	$(x, 0, 0)$	$(1, -1, -1)$

Each agent has a strictly dominant strategy (a_1 for agent 1 and b_1 for agent 2). Hence, there is again a unique correlated equilibrium in which the principal's payoff is still 1. Incentive compatibility if y_2 is chosen requires that $r^1 = a_1$ and $r^2 = b_1$, where r^i is the recommendation sent to agent i.

Hence, using a direct mechanism with deterministic allocations, the payoff of the principal is 1. We now show that using an indirect mechanism with deterministic allocations, he can do better. Therefore, stochastic allocations are necessary to sustain the revelation principle.

Consider the following indirect mechanism: $M^1 = \{m_1, m_2\}$, and $M^1 = R^1 = R^2 = \emptyset$, where M^i is the message space of agent i and R^i the set of recommendations that may be provided to agent i. That is, the principal communicates only with agent 1, and offers no recommendations.

The principal uses the following deterministic allocation rule $\tilde{\pi}$: if agent 1 sends message m_k, the allocation is y_k, for $k = 1, 2$.

Since agent 1 does not observe any new information (i.e., a recommendation) after sending his message, and agent 2 observes no information before choosing his effort, the following simultaneous-move game is induced between the agents:

	b_1	b_2
(m_1, a_1)	$(1, -4, 4)$	$(1, 4, -4)$
(m_1, a_2)	$(1, 4, -4)$	$(1, -4, 4)$
(m_2, a_1)	$(1, 1, 1)$	$(x, 0, 0)$
(m_2, a_2)	$(x, 0, 0)$	$(1, -1, -1)$

In the absence of recommendations, agents play a Nash equilibrium of this game. We now show that every Nash equilibrium of this game places a positive probability on the outcome $(0,0)$.

First, by inspection, we observe that there is no pure strategy equilibrium in this game. Therefore, in any Nash equilibrium, agent 2 must play both b_1 and b_2 with strictly positive probability. Further, strategy (m_2,a_2) for agent 1 is strictly dominated by (m_2,a_1).

Now, agent 1 must also be mixing in equilibrium (else agent 2 will not mix over $\{b_1,b_2\}$). Suppose that in equilibrium agent 1 mixes over only (m_1,a_1) and (m_2,a_2). Then, agent 2 must be playing each of b_1 and b_2 with probability $\frac{1}{2}$ (else agent 1 is not indifferent between his two strategies). But, against this strategy of agent 2, (m_2,a_1) is a strict best response for agent 1.

Hence, in every Nash equilibrium of this game, agent 1 must play (m_2,a_1) with positive probability, and agent 2 must play both b_1 and b_2 with positive probability. Therefore, the outcome $(0,0)$ has positive probability.

However, the outcome $(0,0)$ provides a payoff $x > 1$ to the principal. Since the payoff from every other outcome is 1, the expected payoff of the principal from any equilibrium of the indirect mechanism strictly exceeds 1. That is, the principal does strictly better with an indirect mechanism than with a direct mechanism. ■

Therefore, even with complete information and in the presence of recommendations, stochastic allocations are necessary for the revelation principle to go through. In this example, the principal uses an indirect mechanism effectively to provide agent 1 with private information about allocations, and create uncertainty about allocations for agent 2. This uncertainty, in turn, affects the strategy of agent 1 in the agents' effort game, leading to an eventual outcome that is not sustainable in a direct mechanism unless stochastic allocations are permitted.

Strausz (2003) provides an example to show that, in a setting of pure adverse selection, even with one principal and one agent, it is no longer true that any payoff implementable by a deterministic indirect mechanism can be matched with a deterministic direct mechanism. However, as Strausz shows, the principal always does weakly better with a direct mechanism, so that the optimality of direct mechanisms remains in the single-principal, single-agent setting. He provides a further example to show that a second agent with veto power may veto a direct deterministic mechanism and prefer an indirect mechanism. Our example above shows that, with pure moral hazard and two agents, the principal may strictly prefer a deterministic indirect mechanism to a deterministic direct mechanism.

Next, we provide an example to show that recommendations are necessary to sustain the revelation principle. In their absence, a principal can again do better with an indirect mechanism than with a direct mechanism.

Example 2 (Recommendations):

Again, $n = 1$ and $k = 2$. As before, $Y = \{y_1, y_2\}$, $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$.

The payoffs are given by the following matrices. In each cell, the first element corresponds to the utility of the principal, and the second and third to the utilities of agents 1 and 2 respectively.

$y = y_1$

	b_1	b_2
a_1	$(0,0,10)$	$(50,6,6)$
a_2	$(0,-10,-10)$	$(-10,0,10)$

$y = y_2$

	b_1	b_2
a_1	$(0,0,-10)$	$(-200,0,0)$
a_2	$(0,10,0)$	$(4,1,6)$

Suppose that the principal offers no recommendations (so that $R^1 = R^2 = \emptyset$), but can choose a lottery over allocations, so that $y = py_1 + (1-p)y_2$. Then, a mechanism is characterized by $\pi : M \longrightarrow \Delta(Y)$.

In a direct mechanism, the agents' effort game is as follows:

	b_1	b_2
a_1	$(0,\,0,\,20\,p-10)$	$(250p-200,\,6\,p,\,6\,p)$
a_2	$(0,\,p,\,-10\,p)$	$(4-14p,\,1-p,\,4\,p+6)$

For $p < \frac{5}{7}$, b_2 strictly dominates b_1. For $p < \frac{1}{7}$, agent 1's best response is a_2, so that the unique equilibrium is (a_2, b_2), with resultant utility for the principal $4-14p$. Thus, the principal's utility is maximized at $p = 0$ and a value of 4. For $p \in (\frac{1}{7}, \frac{5}{7})$, agent 1's best response is a_1, which results in equilibrium (a_1, b_2) and a utility of $250p - 200$ for the principal. This has a supremum at $p = \frac{5}{7}$, and a value of $-\frac{150}{7}$.

When $p = \frac{5}{7}$, agent 1's best response is a_1, and agent 2 is indifferent over b_1, b_2. The maximal utility the principal can obtain is 0, when agent 2 plays b_2. Finally, for $p > \frac{5}{7}$, the unique equilibrium of the agents' subgame is (a_1, b_1), with principal utility being 0.

Hence, the optimal allocation for the principal is y_1, with resultant equilibrium (a_2, b_2) in the agents' game, and a utility of 4 for the principal.

Now, consider the following indirect mechanism. The principal communicates with agent 1, with the message space being $M_1 = \{m_1, m_2\}$. The allocation rule, as in Example 1, is $\tilde{\pi}(m_k) = y_k$ for $k = 1, 2$.

As in example 1, a simultaneous-move game is induced between the agents, and can be represented as follows.

	b_1	b_2
(m_1, a_1)	(0, 0, 10)	(50, 6, 6)
(m_2, a_1)	(0, 0, −10)	(−200, 0, 0)
(m_1, a_2)	(0, −10, −10)	(−10, 0, 10)
(m_2, a_2)	$(0, 10, 0)$	$(4, 1, 6)$

The agents' game exhibits the following unique Nash equilibrium:

- Agent 1 mixes between (m_1, a_1) and (m_2, a_2), with probabilities $3/5$ and $2/5$.
- Agent 2 mixes between b_1 and b_2 with probabilities, $1/3$ and $2/3$.

Thus, the principal's expected payoff from the indirect mechanism is $316/15 > 4$. That is, the principal has a higher payoff from the indirect mechanism than is achievable in a direct mechanism.

Allowing for recommendations, we can resurrect the equilibrium of the indirect mechanism in a direct mechanism. A direct mechanism with recommendations in this example may be characterized as a function $\pi : Y \times E^1 \times E^2 \to [0, 1]$, where $\pi(y, a, b)$ is the probability the principal chooses allocation y and recommends effort a to agent 1 and b to agent 2.

In the equilibrium of the indirect mechanism above, the resultant distribution over allocations and efforts is $\pi(y_1, a_1, b_1) = 1/5$, $\pi(y_1, a_1, b_2) = 2/5$, $\pi(y_2, a_2, b_1) = 2/15$ and $\pi(y_2, a_2, b_2) = 4/15$. Suppose the principal plays this strategy in the direct mechanism. That is, the principals choose allocations and efforts according to $\pi(\cdot)$, and announces the resulting recommendations to the agents.

It is straightforward to check that neither agent has an incentive to deviate, so the mechanism is incentive compatible. For example, when agent 2 is told "b_2", his posterior beliefs place probability $3/5$ on (y_1, a_1) and $2/5$ on (y_2, a_2). Given these beliefs, b_2 is a (weak) best response. The principal obtains the utility $\frac{316}{15}$, as before. ■

In this example, the principal uses an indirect mechanism to communicate privately with agent 1, thereby sustaining a correlated outcome over allocations and efforts. Such correlation can be replicated in the direct mechanism only if the principal sends recommendations.

4 Multiple Principals

With multiple principals, the principals are now playing a game with each other, and their choices of mechanisms must correspond to a Nash equilibrium of this game. Further, agents choices of messages and efforts must represent continuation equilibria of the game, given the mechanisms chosen by principals and recommendations received by agents.

We first observe that, with multiple principals and stochastic mechanisms, agent's obedience of principals' recommendations is a troublesome notion. An agent may be recommended different actions by different principals. For example, if two principals are both randomizing over recommendations, since principals choose their strategies independently, there is a strictly positive probability that an agent will receive different recommendations from the principals. Which one should he obey?

Given this difficulty, we bypass the issue of agents obeying recommendations received from principals. Instead, for incentive compatibility with multiple principals, we only require that, given the strategies of principals and other agents, agents play an equilibrium of the effort game.

Though there is complete information among principals, since agents receive private recommendations from principals, agents may have private information at stage 4 (see Figure 1), when they play the effort game. Hence, in the spirit of perfect Bayesian equilibrium, we require that each agent j plays a best response following any recommendation array $r^i = \left(r_1^i, \ldots, r_n^i\right)$ he may receive.

Multiple principals introduce a new complication into the framework, since principals too must play equilibrium strategies in the mechanism design game. Since principals must choose mechanisms independently, there can be no correlation in the resultant distribution over allocations in any equilibrium of a direct mechanism. However, as Example 3 shows, such correlation can be induced in an indirect mechanism, if principals privately communicate with the same agents.

Example 3: (Multiple principals and correlated allocations)

Suppose there are two principals and two agents. The allocation sets are $Y_1 = \{c_1, c_2\}$ for principal 1 and $Y_2 = \{d_1, d_2\}$ for principal 2, and the feasible efforts are $E^1 = \{a_1\}$ for agent 1 and $E^2 = \{a_2\}$ for agent 2. Hence moral hazard plays no role.

The payoffs are given by the following matrix

	d_1	d_2	d_3
c_1	$(1,1,0,0)$	$(3,2,1,1)$	$(-1,-1,0,0)$
c_2	$(2,3,1,1)$	$(1,1,0,0)$	$(-1,-1,0,0)$
c_3	$(-1,-1,0,0)$	$(-1,-1,0,0)$	$(-1,-1,0,0)$

where in each cell the first number represents the utility of the first principal, the second, the utility of the second principal and the two last numbers indicates respectively the utlity of the first and the second agent.

Consider the following indirect mechanism. Principals 1, 2 each offer the same message space $M_j = \{m_j^1, m_j^2, m_j^3\}$ to agent 1 and 2. Let $M^2 = R^1 = R^2 = \emptyset$ (so that agent 2 is not able to send any messages, and neither agent receives any recommendations). The allocation rule used by the principals is: For each $i = 1, 2,$, principal 1 sets $\pi_1(m_1^i, m_1^1) = \pi_1(m_1^i, m_1^2) = c_i$, and if any of the two agents send the message m_1^3, he takes the decision

c_3. Principal 2 sets the same decision rule: $\pi_2\left(m_2^i, m_2^1\right) = \pi_2\left(m_2^i, m_2^1\right) = d_i$, and d_3 if one of the agents sends m_2^3.

In equilibrium, agent 1 plays each of $\left(m_1^2, m_2^1\right)$ and $\left(m_1^1, m_2^2\right)$ with probability $1/2$, and agent 2 plays $\left(m_1^1, m_2^1\right)$ with probability one. If one of the principal deviates, then the agents choose to send m_j^3 to the no-deviating principal. Hence no principal has an incentive to deviate. Let us remark that given the strategy of the other agent, such a strategy is optimal for every agent.

This strategies guarantee to the agents a payoff of 1 and to the principals a payoff of $5/2$.

The resultant distribution over allocations and efforts places a probability $1/2$ each on (c_1, d_2) and (c_2, d_1). Such a distribution is not achievable with direct mechanisms: in any game in which principals 1 and 2 play independently. If the allocations (c_1, d_2) and (c_2, d_1) are reached with positive probability, then the same must happen to (c_1, d_1) and (c_2, d_2).

Let us remark that moral hazard plays no role in our example, hence recommendations do not play any role and cannot help to create correlation between decision as in the second example.

This example illustrates a well known result in contract theory: whenever one consider multi-principal games, there may exist equilibria sustained by indirect mechanisms only. In the example agent 1 acts as a correlation device, the resulting correlation cannot be reproduced if principals play direct mechanisms only.

In the single-principal setting, every equilibrium of a direct mechanism can trivially be sustained as an equilibrium of a direct mechanism that satisfies incentive compatibility. Hence, one implication of the revelation principle is that every equilibrium of a direct mechanism remains an equilibrium when the principal is allowed to choose over indirect mechanisms instead. We show that this implication goes through with multiple principals, as long as the recommendations offered in the equilibrium of the direct mechanism are uncorrelated with the allocations.

Recall that a mechanism offered by principal j is defined by (M_j, R_j, π_j), where: $M_j = \times_{i=1}^k M_j^i$ and M_j^i is the message space for agent i to communicate with principal j,

$R_j = \times_{i=1}^{k} R_j^i$, and R_j^i is the set of recommendations principal j may make to agent i, and $\pi_j : M_j \to \Delta(Y_j \times R_j)$ is the allocation rule used by principal j.

A direct mechanism is defined by $(\emptyset, E, \pi_j)$, where $\pi_j \in \Delta(Y_j \times E)$. If the probabilities over allocations and recommendations are independent, we say the recommendations are uncorrelated with allocations.

Definition 1 *In a direct mechanism, a strategy π_j of principal j has no correlation between recommendations and allocations if there exist marginal densities $\pi_{j,y} \in \Delta(Y_j)$ and $\pi_{j,e} \in \Delta(E)$ such that $\pi_j(y,e)\,dy = \pi_{j,y}(y)\,\pi_{j,e}(e)$ for each $y \in Y_j$ and $e \in E$.*

A special case of recommendations uncorrelated with allocations is when recommendations are deterministic rather than stochastic. For example, suppose that each agent can put in a binary effort, say high or low. In addition, suppose that in equilibrium, each principal wishes that each agent choose high effort. Then, recommendations are deterministic, and regardless of allocation strategies, satisfy our definition of being uncorrelated with allocations.

In a direct mechanism, when recommendations are deterministic, the same outcomes are achieved in equilibrium as if they were publicly observed. Since obedience has not been assumed, the agents' equilibrium strategies given deterministic recommendations only need to be mutual best responses in the efforts game. That is, principals' direct mechanisms induce Nash equilibria in the agents' effort game. Therefore, the overall outcome represents a subgame-perfect equilibrium of the game in which principals design direct mechanisms.

Let $\Gamma_{\mathcal{D}}$ be the direct mechanism game among the principals. In this game, principals may choose any direct mechanism at stage 1 (see Figure 1), so the strategy choice of each principal j is restricted to $\pi_j \in \Delta(Y_j \times E)$. Let $\Gamma_{\mathcal{G}}$ be the indirect mechanism game. In this game, principals may choose indirect mechanisms as well, so that stage 2 (at which agents send messages to principals) may have a real role. Here, principal j chooses (M_j, R_j, π_j), where (with a slight abuse of notation) $\pi_j : M_j \to \Delta(Y_j \times R_j)$.

In an equilibrium of either $\Gamma_{\mathcal{D}}$ or $\Gamma_{\mathcal{G}}$, we require that (i) each principal plays a best response, given other principals' strategies and agents' strategies, and (ii) each agent i plays a best response for every recommendation array r^i he may receive, given principals' strategies and other agents' strategies.

Formally,

Theorem 1 *Suppose the direct mechanism game $\Gamma_{\mathcal{D}}$ has an equilibrium in which (i) each principal j plays π_j^* (ii) each agent j plays δ^{i*}, and (iii) for each principal j, π_j^* has no correlation between allocations and recommendations. Then, in the indirect mechanism game $\Gamma_{\mathcal{G}}$, it remains an equilibrium for each principal j to offer the mechanism $(\emptyset, E, \pi_j)$ and for each agent i to play δ^{i*}. Thus, the joint distribution over allocations and efforts that obtains in the equilibrium of the direct mechanism game remains an equilibrium outcome of the indirect mechanism game.*

Proof.

Consider the game $\Gamma_{\mathcal{G}}$. Suppose that, in this game, every principal j offers a mechanism $\left(M_j, R_j, \pi_j\right) = (\emptyset, E, \pi_j^*)$, where π_j^* is his equilibrium strategy in the direct mechanism game $\Gamma_{\mathcal{D}}$. It is immediate that $\delta^* = \times_{i=1}^{k} \delta^{i*}$ must remain a continuation equilibrium in the agents' efforts game.

Hence, we need only to show that no principal j' has an incentive to unilaterally deviate from the mechanism $\left(\emptyset, E, \pi_{j'}^*\right)$. Suppose, therefore, that some principal j' has an incentive to deviate to $\left(\tilde{M}_{j'}, \tilde{R}_{j'}, \tilde{\pi}_{j'}\right) \neq \left(\emptyset, E, \pi_j^*\right)$, while all other principals $j \neq j'$ continue to offer mechanisms $\left(\emptyset, E, \pi_j^*\right)$. Suppose the agents play $\left(\tilde{\mu}, \tilde{\delta}\right)$ in response to these mechanisms, and the mechanisms and the agents' effort strategies $\tilde{\delta}$ induce a (possibly correlated) distribution over allocations y and efforts e. Let $\tilde{\nu}(y,e)$ denote this distribution. Since agent j has an incentive to deviate to the indirect mechanism, his utility from such a deviation, $V_j(\tilde{\nu}(y,e))$ must exceed his utility from the equilibrium of the direct mechanism.

Now, every principal $j \neq j'$ is using recommendations uncorrelated with allocations. Since each agent i observes only the mechanisms and his own recommendation array $r^i = (r_1^i, \ldots, r_k^i)$, the efforts chosen must also be uncorrelated with the allocations of principals $j \neq j'$. Hence, we can write $\tilde{nu}(y,e) = \tilde{\nu}_j(y_j, e) \times \left(\times_{j \neq j'} \pi_{y,j}^*(y_j)\right)$, where $\pi_{y,j}^*(y_j)$ is the marginal distribution over the allocations of principal j, given his strategy π_j^*.

Now, it is straightforward for principal j to induce the same joint distribution in the direct mechanism game. Rather than play the strategy π_j^*, he plays the strategy $\tilde{\nu}_j$.

Since this strategy induces the same joint distribution over efforts and allocations as in the continuation equilibrium of the indirect mechanism game, it must be a best response for each agent i to obey the recommendation of principal j, and to ignore the recommendations of the others (else agent i would have a profitable deviation in the indirect mechanism game, rather than playing $\tilde{\delta}$). But if every agent i obeys the recommendation of principal j, and principal j plays $\tilde{\nu}_j(y,e)$ in the direct mechanism game, the same joint distribution over allocations and efforts is induced as in the indirect mechanism game. Hence, if principal j has a profitable deviation in the indirect mechanism game, he has a profitable deviation in the direct mechanism game as well, contradicting the assumption that $\pi^* = \times_{i=1}^n \pi_j^*$ is an equilibrium of the latter game. ■

Remarks:

1. Peters (2003), Theorem 1, shows that in a common agency (i.e., multiple-principal, single-agent) setting, pure strategy equilibria of direct mechanisms are robust to the introduction of more complex communication schemes. Our theorem above provides an analog with multiple principals and multiple agents. Note, however, that our setting is different from that of Peters (2003) due to the explicit use of recommendations.

2. Our theorem uses the same intuition as that of Myerson (1982). Suppose every principal uses recommendations uncorrelated with allocations in the direct mechanism game, and one principal now deviates to an indirect mechanism. The deviating principal may play a strategy which implies a correlation between recommendations and allocations. Nevertheless, it is incentive compatible (in the sense of being a best response to other agents' strategies) for each agent to obey the deviating principal in the indirect mechanism game. Hence, the problem of replicating an outcome from the indirect mechanism game in the direct mechanism game reduces to the same problem as with a single principal.

3. There may well be multiple equilibria in the agents' effort game. All we show is that obeying the recommendation of the deviating principal is one such equilibrium.

4. The indirect mechanism game may have other equilibria in which more than one principal offers an indirect mechanism (see Example 3).

Importantly, our theorem cannot be straightforwardly extended to games with incomplete information. The intuition is the following: even if recommendations are uncorrelated with allocations, a recommendation from principal j to agent i may communicate information about the type of some other agent i'. This may lead to a correlation between agents' efforts and principals' allocations, which is difficult for a single principal to replicate in a direct mechanism.

5 Discussion

In a recent paper, Peters (2004) provides two thought-provoking examples in a setting with two principals and two agents. His first example suggests that "In a multiple agency environment [...] pure strategy equilibria are not robust against the possibility that principals might deviate to more complex indirect mechanisms".[2] The second example shows that a "no externality" assumption (see Peters, 2003) sufficient to imply the Revelation Principle in a multi-principal, single agent context fails to do so when there are many agents.

We show in this section that his first example can be reconciled with the discussion above, if one removes the restriction to deterministic direct mechanisms. We first briefly present the example, using Peters' notation to keep the presentation as simple as possible.

Peters (2004), Example 1:

As shown in Figure 2 below, the direct mechanism game begins with principals P_1 and P_2 simultaneously choosing allocations in the space $Y_1 = Y_2 = \{A, B\}$. In Peters' framework, there are no recommendations. Thus, at the second stage, agents A_1 and A_2 observe the principals' allocation rules, and simultaneously choose a level of effort in the set $E^1 = E^2 = \{1, 2\}$. For convenience, participation constraints of the agents are ignored (it is easy to scale agent payoffs if necessary).

The game is summarized in Table 1. The principals' choices affect which cell of the larger matrix is chosen. The agents then play the 2×2 subgame in that cell. Payoffs should be interpreted in the following way: the first payoff is the payoff to principal 1, who chooses the row in the big matrix. The second payoff is the payoff to principal 2,

[2] Peters (2004, p. 184).

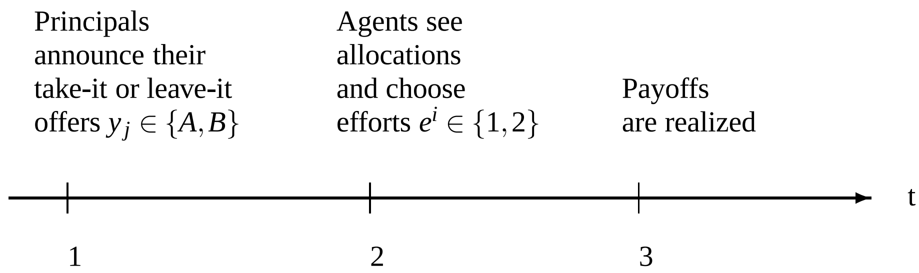

Figure 2: Peters (2004), Example 1: direct mechanism interaction

who chooses the column in the big matrix. The third payoff is to agent 1, who chooses the row in the inner matrix in each cell, and the last one is to agent 2, who chooses the column in the inner matrix in each cell.

	A	B
A	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & * & * \\ e=2 & * & * \end{array}$	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & \left(\frac{7}{8}, \frac{7}{8}, -\frac{3}{2}, \frac{5}{4}\right) & \left(\frac{7}{8}, \frac{7}{8}, \frac{5}{4}, -\frac{3}{2}\right) \\ e=2 & \left(\frac{7}{8}, \frac{7}{8}, \frac{5}{4}, -\frac{3}{2}\right) & \left(\frac{7}{8}, \frac{7}{8}, -\frac{3}{2}, \frac{5}{4}\right) \end{array}$
B	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & \left(0, 0, -\frac{3}{2}, \frac{5}{4}\right) & \left(0, 0, \frac{5}{4}, -\frac{3}{2}\right) \\ e=2 & \left(0, 0, \frac{5}{4}, -\frac{3}{2}\right) & \left(0, 0, -\frac{3}{2}, \frac{5}{4}\right) \end{array}$	$\begin{array}{ccc} & e=1 & e=2 \\ e=1 & (3, 0, -1, -1) & (1, 1, 1, 1) \\ e=2 & (1, 1, 1, 1) & (0, 3, -1, -1) \end{array}$

Table 1: Reduced form of the example in Peters (2004)

Principals' decisions are restricted to deterministic offers. That is, principals are not allowed to use lotteries over the allocations $\{A, B\}$. Given this restriction, in the direct mechanism game in Figure 2, there exist three equilibria in the direct mechanism game:

- P_1 and P_2 both play B; A_1 plays 1 and A_2 plays 2; each player gets a payoff of 1.
- P_1 and P_2 both play B; A_1 plays 2 and A_2 plays 1; each player gets a payoff of 1.
- P_1 and P_2 both play B; A_1 and A_2 randomize and play 1 with probability $1/2$; P_1 and P_2 both get a payoff of $5/4$, A_1 and A_2 get a payoff of 0.

This framework is used to show that if a principal can communicate with the agents before choosing his allocation, the first two pure strategy equilibria do not survive. More specifically, Peters considers the case where P_1 asks each agent $i = 1,2$ to send a message $m_1^i \in \{A,B\}$ and designs the following mechanism (where $\pi_1(\cdot)$ denotes his allocation strategy):

$$\pi_1(m_1^1, m_1^2) = \begin{cases} B & \text{if } m_1^1 = m_1^2, \\ A & \text{otherwise.} \end{cases}$$

Whenever the agents do not coordinate on their messages, P_1 chooses A. Since there are no recommendations, we can think about agents choosing messages and efforts simultaneously.

If principal 2 continues to play B, the deviation π_1 is profitable for principal 1. The continuation game associated with the strategies $\{\pi_1, B\}$ has multiple equilibria. In the first class of equilibria (referred to as E_1), agents both report B (alternatively, both report A) and randomize equally over efforts, inducing a payoff $5/4$ for P_1.[3] In the second equilibrium (E_2) agents randomize equally over *both* messages and actions. That is, they select each message–action pair with probability $1/4$. These behaviors induce a payoff of $17/16$ for P_1 and $-1/16$ for each agent.

Finally, the strategy profile E_3 has each agent randomizing equally over the message–action pairs $(m_1, 1)$ and $(m_2, 2)$. The corresponding payoffs are $19/16$ for P_1, $1/8$ for A_1 and $-5/4$ for A_2. However, E_3 turns out to not be an equilibrium, since the best-replies of agent 2 to the strategy of agent 1 are $(m_2, 1)$ and $(m_1, 2)$, instead of $(m_1, 1)$ and $(m_2, 2)$.

Nevertheless, the strategy π_1 is a profitable deviation in Peters' example given that, in every continuation equilibrium, the deviating principal earns a payoff strictly greater than 1. In particular, the example emphasizes that the payoffs $(1,1,1,1)$ cannot be sustained at equilibrium when P_1 is allowed to communicate with agents. ■

We now show that allowing for stochastic allocations in this example, even without introducing recommendations, recovers the result that all payoffs associated with principal 1's deviation to pi_1 can be supported using direct mechanisms.

[3] For every player, this equilibrium is payoff-equivalent to the mixed strategy equilibrium of the take-it or leave-it offer game.

To develop this argument, we first observe that, in the equilibria discussed above, principal 2 always chooses the strategy *B*. Thus, we can restrict our analysis to the second column of table 1. We can therefore focus on the optimal action of principal 1, and effectively interpret this example as a single principal, multi-agent game.

Suppose, in particular, that principal 1 randomizes equally between *A* and *B* (with principal 2 playing *B*). Agents observe the stochastic allocation, but not the realization of the randomization, before they act. Then, the agents are effectively faced with the following continuation game, where the first number in each cell is the principal's expected payoff, and the remaining two numbers are the expected payoffs to agents 1 and 2, respectively.[4]

	$e=1$	$e=2$
$e=1$	$(31/16,-5/4,1/8)$	$(15/16,9/8,-1/4)$
$e=2$	$(15/16,9/8,-1/4)$	$(7/16,-5/4,1/8)$

The agents' subgame exhibits only a mixed strategy equilibrium, with both agents equally randomizing over actions. This randomization yields the principal a payoff of $\frac{17}{16}$, and the agents a payoff $-\frac{1}{16}$.[5]

Hence, if the principal is allowed to randomize over allocations, the outcome $(1,1,1)$ cannot be supported as an equilibrium in the direct mechanism game either (i.e., this allocation results from a sub-optimal choice of principal 1, given the strategy of principal 2).

6 Conclusion

The literature on competing mechanisms with multiple agents often makes seemingly restrictive assumptions about the set of mechanisms that are feasible for principals: models focus on "take-it or leave-it" offers or direct mechanisms in which agents report information about their preferences.[6] We provide some support for this approach in a

[4] Since principal 2's strategy is fixed at *B*, his payoffs are ignored. As mentioned above, this is now equivalent to a single principal, two-agent game.

[5] If participation constraints (e.g., a reservation of utility of zero for the agents) are a factor, note that the agents payoffs in each cell of the original game can be increased by $\frac{1}{16}$ without affecting the equilibria.

[6] See for example Doğan (2004), McAfee (1993), and Prat and Rustichini (2003).

complete information scenario: equilibria defined in direct mechanisms remain equilibria in more general games in which principals can use richer communication schemes, if one correctly defines the strategy spaces. Nevertheless, as shown in the common agency literature,[7] our approach does not allow to characterize every equilibrium of multi-principal multi-agent games. If we focus on direct mechanisms, we may lose some interesting equilibria.

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[7] See Martimort and Stole (2002) for an example with complete information.

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