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Keywords Productive efficiency, input-output, growth, Europe.

JEL Codes C67, O47, P51.

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1. Introduction

By utilizing the EUROSTAT input-output data base, and Italian data on the service life of capital assets, we investigate in this paper differences in productive efficiency across sixteen European countries. As productive efficiency bears upon total factor productivity, and thus on aggregate growth, in order to assess aggregate efficiency one would be tempted to simply compare real growth rates and then conclude that in economies where real growth is higher efficiency must be proportionately higher. This would be missleading, however, as productive efficiency in the aggregate economy actually reflects levels of efficiency in a range of different sectors, and also the output composition of the economy. So, in order to assess differences in productive efficiency, one has to distinguish firstly differences between countries in output composition and then differences within countries in respect of individual sectors.

As there is no common ideal output composition available, to isolate true differences in productive efficiency one has to find an alternative. A first possibility could consist of combining the matrix of the domestic multipliers of one country with the final demand of the remaining countries in the sample.

The solution adopted in this paper consists of building a dynamic input-output model and computing for each country the balanced growth rate and the balanced output composition. On this base, the actual growth rate in each country can be decomposed in two parts. The first is the balanced growth rate itself and the other is the difference between the two rates stemming from the distance separating the actual output composition from the one implying balanced growth. After briefly discussing the structure of the dynamic model and its results, we investigate how the differences existing between the output composition for balanced growth and the actual one relate to the differences between the rate of balanced growth and the actual one. In the final part of the paper we examine the influence of individual sectors on the rate of balanced growth by looking for growth-sensitive sectors i.e. whether there are some sectors which have a proportionately bigger influence and whether there are sectors where a reduction in the intermediate and capital input coefficients has relatively greater effect.

2. Productive Efficiency

One way of defining productive efficiency in a sector is by utilising the familiar concept of "production possibility frontier" i.e. the maximal amount of output that can be obtained with the available resources. According to Ten Raa, the degree by which the actual level of output is lower than the maximal one possible measures the inefficiency in that sector, from which it follows that efficiency is synonymous with proximity to that frontier (Ten Raa, 2005). An alternative way of measuring efficiency considers "total factor productivity" (Wolff, 1985). Utilising an Input-Output framework, output is measured by the value of gross production and inputs consist of labour, fixed capital and intermediate inputs. Efficiency in a given sector increases when for a given level of output, the total value of those inputs is reduced. At the aggregate level, the composition of the output matters such that aggregate efficiency reflects the sectoral composition of final demand.

In both those approaches, capital and labour are treated as primary resources even though they are quite obviously different from true natural resources. Capital goods are produced in specialized sectors of the economy. Even labour can be considered a reproducible means of production. Indeed, while natural resources do not need to be produced – although they need to be modified to be valuable to mankind – all other resources have to be produced, and this implies the use of natural resources. The technology used in the production of many capital goods is far better than it was in the past in so far as nowadays it requires less natural resources, while the production of human capital requires more of them. We are then all aware that natural or true primary resources enter in the production of the many means of production needed to obtain final outputs, whether directly or indirectly. High efficiency, in other words, is premised on low consumption of true primary resources. To be efficient, therefore, an economy must be parsimonious in the use of the means which require those resources. As goods and services used as intermediate inputs in production, and in replacing and expanding fixed capital directly or indirectly, require primary resources, it is the consumption of primary resources that must be lowered. This line of reasoning, of course, was introduced in the spirit of classical economics by Leontief (1951) in his study of the structure of the American economy over the period 1919-1939. We will adopt the same approach, and make both final consumption and gross investment endogenous in a closed input-output model, and assume that efficiency is obtained by reducing intermediate inputs in production and in both replacing and expanding fixed capital. Before applying that model, let us consider investment in capital goods which is an essential part of the data used in the analysis.

3. Fixed Investment and the Life of Capital

The recommended, and most used, method for estimating stocks of fixed, tangible and durable capital (OECD, 1992 and UNECE/Eurostat/OECD, 2004) goes by the name "perpetual inventory". In this approach, the capital stock is considered to be the sum of past investments in capital goods less scrapped or discarded capital goods. For the concept to be useful, it is necessary to make an assumption regarding the time profile of discards. To clarify the implications of this concept we use the following notation:

K_{jt} fixed capital stock in sector j and period t

I_{jt} net investment less discards in sector j and period t

d_{jt} replacement rate

t_j number of years of life or turnover time of fixed capital.

According to the concept, the capital stock existing at the end of the current period is the sum of past investments – those made in the current period included – in capital goods less discarded capital goods. Then, if in the destination sector j all capital goods are used for t_j years, the capital stock in the same sector is:

(1) $K_{jt} = (I_{jt} + d_{jt}K_{jt}) \cdot t_j$

From this definition, it is easy to derive the condition under which the number of years of life of the existing stock of capital, i.e. t_j, is constant. First note that the equation also reads:

(2) $1 - \left(\frac{I_{jt}}{K_{jt}} + d_{jt} \right) t_j = 0$,

where t_j is the turnover time at the end of period. Now observe that if during the year t there were no investment ($I_{jt} + d_{jt}K_{jt} = 0$) then the initial stock K_{jt-1} would be one year older at the end of the same year and t_j would be higher than at its beginning; while if during the same year, enough investment was made in the economy, then the average number of years of life of the existing capital, t_j, would be constant. Let us then indicate the amount of fixed investment which would be necessary to keep stable the number of years t_j of the existing stock of capital by: $I^*_{jt} + d^*_{jt}K_{jt}$. With such an amount of investment, the following condition is clearly satisfied:

(3) $\Delta t_j = 1 - \left(\frac{I^*_j}{K_j} + d^*_j \right) t_j = 0$.

If actual gross investment $I_{jt} + d_{jt}K_{jt}$ turns out to exceed $I^*_{jt} + d^*_{jt}K_{jt}$, the actual average number of years of capital decreases and vice-versa. In other, words, by accelerating or decelerating the flow of investment relative to the stock, firms can decrease or increase technical progress embodied in new capital goods.

What normally is unknown, however, is the stock of fixed capital rather than its average life and this is precisely our situation. Then the equation above could be used to derive the stock of capital from available data regarding the average life, the rate of replacement and the average amount of investment by destination sector j, as follows:

$$(4)\ K_{jt} = \frac{I_{jt}}{\frac{1}{t_j} - d_{jt}}.$$

The equation shows that the stock is a multiple of the average amount of expansion investment realized in destination sector j where the expanding factor $1/\left(t_j^{-1} - d_{jt}\right)$ is increased when firms prolong the use of the existing capital or increase the rate of replacement.

4. Productive Efficiency In a Closed Dynamic Model à la Leontief

To analyse productive efficiency in the use of intermediate and capital goods we will use the following symbols:

a_{ij} is the intermediate input i for one unit of output in sector j (the technical coefficient) for $i = 1, N$ and $j = 1, N$ where the N^{th} sector is the household sector which buys intermediate goods and services and provides labour and capital services,

a_{iN} is the input from sector i to sector N, or final consumption of products of type i of households,

a_{Nj} is the input from household sector N to sector j

e_i is the percentage of total fixed investment expenditure in the economy spent on goods produced by sector i in the given year,

c_j is the amount of replacement investment needed to produce one unity of output in sector j,

$a_{ij} + e_i c_j$ is the annual total flow of goods and services sold by sector i to sector j in order to satisfy intermediate consumption and capital consumption in sector j,

t_{ij} is the number of years of life of the capital sold by sector i and used in sector j,

$t_{ij} e_i c_j = b_{ij}$ is the unitary stock of fixed capital in use in sector j and sold by sector i,

$a_{ij} = 0$, for $i = N; j = N$ by construction (households do not hire labour)

$b_{ij} = 0$, for $i = N; j = N$ by construction (households do not produce capital goods)

A is a matrix of flow coefficients with endogenous final consumption

B is a matrix of stock coefficients with endogenous final consumption

By means of these definitions, we could rephrase the concept of efficiency. For a given level of final output in sector j, production efficiency in sector j is increased by decreasing $a_{ij} + e_i c_j$, or, equivalently, $a_{ij} + b_{ij} / t_{ij}$. Thus a sufficient condition for increased efficiency is a reduction in the combined change in the flow coefficient, the stock coefficient and the duration term, as follows:

(5) $da_{ij} + (\dot{b}_{ij} - \dot{t}_{ij}) b_{ij} / t_{ij} < 0$,

where the symbols with the dots $\dot{b}_{ij}$ and $\dot{t}_{ij}$ indicate percentage changes. This expression makes clear that it does not matter if one coefficient undergoes a positive change, provided that the other one decreases to provide a negative combined change.

It is obvious that a sufficient condition for increased aggregate efficiency is that condition (5) holds in at least one cell and that the remaining cells are constant. As some cells decrease while others increase in value, one has to consider the net result and this can be found by solving the dynamic model. A way of measuring efficiency, therefore, can be obtained by estimating the b_{ij}'s coefficients and solving the resulting dynamic model in order to find the maximum rate of uniform growth in the economy. While we claim no originality, we argue that this old approach is able to highlight the role of productive inputs reduction i.e. increased efficiency in the growth process. By defining as λ the uniform rate of economic growth, or the rate which assures that the economy grows in a balanced way, one arrives at the well known Leontief's closed dynamic model (Leontief, 1985):

(6) $x = Ax + \lambda Bx$.

To focus more closely on how a change in efficiency $da_{ij} + (\dot{b}_{ij} - \dot{t}_{ij})b_{ij} / t_{ij} \neq 0$ hinges upon the growth rate, we express the uniform growth rate λ in terms of both the model's value and technical coefficients in the Appendix.

To summarize the above discussion, we observe, firstly, that there is a practical advantage to be obtained by constructing a dynamic model based on the idea that capital stock coefficients can be obtained from data on replacement, investment and duration, as follows:

(7) $b_{ij} = t_{ij} e_i c_j$.

Secondly, we argue that productive efficiency implies a higher uniform or balanced growth rate that can be derived from what Leontief (1985) called the closed version of the dynamic model which is basically what we use in the analysis of efficiency. We argue, in particular, that productive efficiency is the highest when matrices A and B are such that the implied rate of uniform growth in the economy is maximal. The model is not entirely closed as if it were so, the matrix equation above $(I - A - \lambda B)x = 0$ would have multiple solutions. In this case, actual output could not be uniquely determined unless boundary conditions on the availability of natural resources were provided. In the section below, therefore, we will use a model with endogenous final consumption and capital formation which, being not entirely closed, is capable of having one solution only.

Provided that the model is a open one, the properties of the coefficient matrices A and B are such that there exists only one single growth rate λ which makes the equation capable of being solved by a positive vector x. As final consumption is considered as an input in the sector which produces labour and capital services, total production meets all direct and indirect needs required by the expansion of the economy at the uniform growth rate λ. To find such special positive solution, the unique making economic sense, is easy. The model can be written as a standard eigen-equation of the matrix $(I-A)^{-1}B$:

(8) $\frac{1}{\lambda}x=(I-A)^{-1}Bx$.

Let's indicate with ρ the first or maximal eigenvalue and with $\bar{x}$ the corresponding eigenvector:

(9) $\rho\bar{x}=(I-A)^{-1}B\bar{x}$.

It is well known that according to the Frobenius theorem, the largest root of $(I-A)^{-1}B$ is always simple and positive, and so are also all the elements of the associate vector $\bar{x}$ [1]. From the above it follows that the lower the maximal eigenvalue ρ, the higher the rate of balanced growth λ in the economy and thus the greater the efficiency. As we assume that changes in the single flow, and stock coefficients, hinge upon efficiency, it is much more practical to refer to the Frobenius eigenvalue rather than to the individual coefficient, although there are coefficients whose change

[1] According to the same theorem, if an arbitrary positive vector is repeatedly pre-multiplied by such a matrix, then the result tends towards the positive eigenvector $\bar{x}$. The speed of the convergence depends on the level of the second eigenvalue (λ_2) relative to that of the first (λ_1), as the closer the two values, the lower the speed of convergence. Thus, the worst possible o less efficient case is obtained when λ_1 is high and the difference $\lambda_1-\lambda_2$ large, as the available technology reaches rapidly the balanced path characterized by a low rate of balanced growth. Brody (1997) has recently investigated the relation between the two largest eigenvalues and claimed that their relation, and thus the speed of convergence, is related to the size of the coefficient matrix.

$da_{ij} + (\dot{k}_{ij} - \dot{t}_{ij})k_{ij} / t_{ij}$ have a more than average bearing on the eigenvalue[2]. Also, differences in productive efficiency that can be observed across space and over time can be ascribed to differences in those coefficients. Differences in productive efficiency, therefore, can be globally assessed also by the associate Frobenious eigenvalue ρ of matrix $(I - A)^{-1}B$. If we were interested in the overall impact, the maximal eigenvalue offers a perfect answer. A well known property of the maximal eigenvalue of $(I - A)^{-1}B$ is that the reciprocal of the eigenvalues which are different from ρ, are lower in modulus thus yielding higher growth rates. The associate eigenvectors, however, are not positive in all their elements thus the growth rate associated to the maximal eigenvalue is the lowest of all, but it is also the only one sustainable in the long run and thus the only one making economic sense. For any change in the flow and capital coefficient matrices, which occurs when $da_{ij} + (\dot{b}_{ij} - \dot{t}_{ij})b_{ij} / t_{ij} \neq 0$ in at least one case, the computation of the dominant eigenvalue immediately translates the change in the coefficients into the change in the rate of balanced growth.

5. Productive efficiency in selected European countries

Using the theoretical framework above, we have analyzed productive efficiency in 16 European countries for which EUROSTAT provides input-output tables for 30 sectors . Table 1 details the fourteen out of sixteen countries which have published the tables for 2000.

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[2] More on this in the last section.

Table 1. Countries Publishing Eurostat Input-Output Tables: 2000.

1	Austria	2000
2	Belgium	2000*
3	Czech Republic	2000
4	Denmark	2000
5	Estonia	2000
6	Finland	2000
7	Germany	2000
8	Ireland	2000
9	Italy	2000
10	Lithuania	2000
11	Norway	2001*
12	Portugal	1999*
13	Slovakia	2000
14	Slovenia	2000*
15	Spain	2000
16	Sweden	2000

In Table 1 above, "*" means that financial intermediate services indirectly measured (FISIM) are dealt with using the old methodology i.e. by adding to the standard 30 sectors a special sector that reports on the total undistributed amount of financial services. Notable omissions in Table 1 are large economies such as France, UK and the Netherlands due to a lack of data on consumption of fixed capital by sector[3]. The same problem occurred for the cases of Italy and Spain, but for these two countries we decided to use supplementary data provided by the Italian Statistical Central Office (ISTAT) to estimate the consumption of fixed capital by sector[4]. This source provides, in particular, detailed figures regarding the life-span of specific fixed-capital goods which allows us to estimate the coefficients t_{ij} as shown in Table 2.

[3] In the case of France, data is only provided on the consumption of fixed capital for several branches.

[4] ISTAT publishes data on replacement and capital stock from 1980. It also provides information regarding the duration of nine different types of capital goods for the thirty branches of economic activity. Other countries do the same (UNECE/Eurostat/OECD, 2004). This wealth of information has been utilised for all the European countries for which replacement figures are available.

Table 2. The service-life of fixed-capital goods.

	BRANCHES OF ORIGIN	DESTINATION BRANCHES (purchasers)																													
	SECTORS (suppliers)	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
01	Agriculture, hunting and forestry	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
02	Fishing	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
03	Mining and quarrying of energy producing materials	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
04	Mining and quarrying except energy producing materials	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
05	Manufacture of food products; beverages and tobacco	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
06	Manufacture of textiles and textile products	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
07	Manufacture of leather and leather products	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
08	Manufacture of wood and wood products	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
09	Manufacture of pulp, paper and paper products; publishing and printing	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
10	Manufacture of coke, refined petroleum products and nuclear fuel	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	Manufacture of chemicals, chemical products and man-made fibres	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
12	Manufacture of rubber and plastic products	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
13	Manufacture of other non-metallic mineral products	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
14	Manufacture of basic metals and fabricated metal products	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	Manufacture of machinery and equipment n.e.c. (*)	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
16	Manufacture of electrical and optical equipment (*)	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
17	Manufacture of transport equipment (*)	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
18	Manufacturing n.e.c. (*)	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
19	Electricity, gas and water supply	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	Construction (*)	51	35	35	35	36	35	35	35	35	35	35	35	35	35	35	35	35	35	40	35	65	65	50	65	79	60	57	35	56	0
21	Wholesale and retail trade; repair of motor vehicles, motorcyc., personal and household goods (*)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
22	Hotels and restaurants	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
23	Transport, storage and communication	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
24	Financial intermediation	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
25	Real estate, renting and business activities	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
26	Public administration and defence; compulsory social security	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
27	Education	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
28	Health and social work	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
29	Other community, social, personal service activities	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
30	Activities of households	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(*) Estimation based on ISTAT data source.

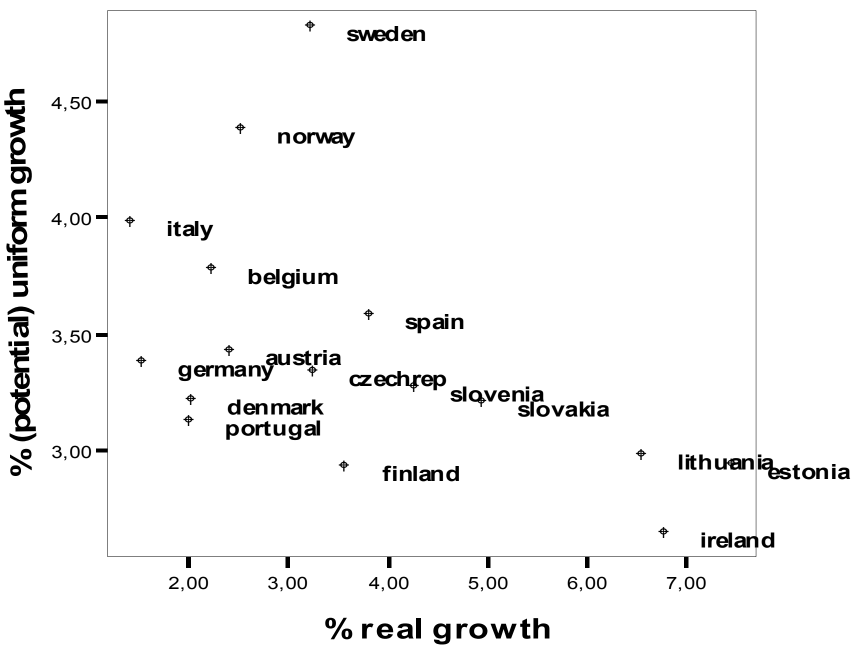
In the previous section we argued that by accelerating or decelerating the flow of investment relative to the stock, firms can tune the average number of years of their stock of capital and thus tune technical progress embodied in new capital goods. This implies that the t_{ij} 's are neither constant over time nor uniform across the different economies. This instability has obvious implications for the amount of fixed capital in the different circumstances. Obviously, the table above is not easy to estimate for each country, since related information is hardly available. Thus, we assume the service life values or simply the number of years suggested are valid for all the countries.

From the EUROSTAT tables and the data in Table 2 we have derived the matrices $(I-A)^{-1}B$ and, in turn, the corresponding dominant eigenvalues ρ. Table 3 shows the value of the dominant eigenvalue, the corresponding rate of balanced growth and the actual average rate of growth in the period 1998-2007 provided by EUROSTAT for each country. The last column in the same Table shows the differences between the actual rate of growth and the one computed from the dynamic model. Several comments are in order. First of all, it appears that the balanced growth rate is a poor predictor of actual growth and, indeed, there seems to be an inverse relation between the two rates, a relation which is more evident if the two outlier cases (Sweden and Norway) are removed from the scattergram (Figure 1). If this is done, Italy is shown to have the highest balanced growth rate but the lowest actual growth rate. Ireland has the opposite. One could further argue that the rate of actual growth (col. c) can be decomposed into the rate of balanced growth (col. b) and the residual (col. d) i.e. the difference between the actual output composition and the one corresponding to the balanced growth path. As illustrated in the same figure, actual average growth rates show a higher dispersion than that found in the uniform growth rates derived from the model. It appears that seven countries were able to grow at an average rate which is higher – in some cases markedly higher – than the balanced one.

Table 3. Maximum eigenvalues and rates of economic growth.

Country	(a) Max. Eigenvalue	(b) % Rate of balanced growth	(c) % Rate of actual growth (average 1998-07)	(d) = (c) – (b) Difference
Estonia	34,30	2,92	7,46	4,54
Ireland	37,99	2,63	6,79	4,16
Lithuania	33,78	2,96	6,56	3,60
Slovakia	31,34	3,19	4,95	1,76
Slovenia	30,73	3,25	4,29	1,04
Spain	28,08	3,56	3,82	0,26
Finland	34,39	2,91	3,59	0,68
Czech Republic	30,14	3,32	3,27	-0,05
Sweden	20,82	4,80	3,24	-1,56
Norway	22,91	4,36	2,53	-1,83
Austria	29,35	3,41	2,42	-0,99
Belgium	26,56	3,76	2,24	-1,52
Denmark	31,25	3,20	2,04	-1,16
Portugal	32,16	3,11	2,02	-1,09
Germany	29,75	3,36	1,55	-1,81
Italy	25,25	3,96	1,43	-2,53

Figure 01. Actual rates of growth and balanced growth rate.



The differences clearly require an explanation. We first observe that since the actual structure of production need not be that implied by the dominant eigenvector[5], it is necessary to verify whether the differences between the actual and the balanced growth rates are related to the differences existing between the actual production structure and that implied by the eigenvector measured with the Theil's index of similarity:

$$I_T^k = \sum_j \bar{x}_j^k \ln\left(\frac{\bar{x}_j^k}{x_j^k}\right)$$

where:

$\bar{x}_j^k$ output share of sector *j* in country *k* in the balanced growth

x_j^k output share of sector *j* in country *k* actually observed.

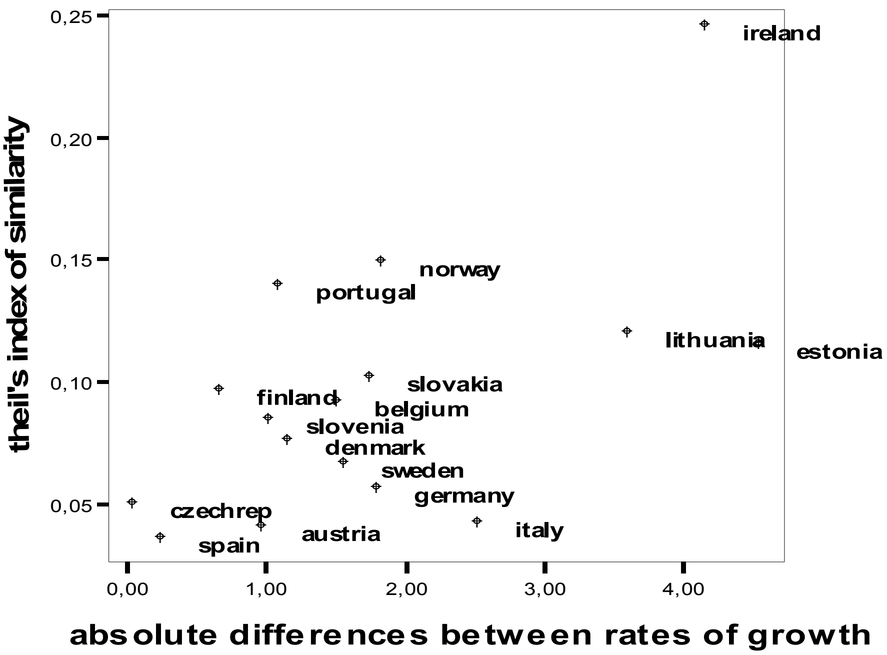
Indeed, it can be argued that growth in the economy as a whole actually reflects total factor productivity i.e. including varying efficiency in the different sectors and also the economy's output composition. The Theil's index takes a value close to zero when actual growth and balanced growth rates are equal, and increases when are not Table 4 shows that in many cases the index has a value that is rather low, and that the few exceptions are found in countries where the difference between actual and balanced growth are larger. Figure 2 shows that there is some evidence of a positive association between the two variables (the correlation coefficient is equal to 0.595 and to 0.653 if Estonia is excluded).

[5] The dominant eigenvector is the eigenvector associated with the dominant eigenvalue.

Table 04. Theil's index of similarity.

Country	Theil's Index	Difference between actual and balanced growth rates
Estonia	0,1132	4,54
Ireland	0,2435	4,16
Lithuania	0,1179	3,60
Slovakia	0,1005	1,76
Slovenia	0,0831	1,04
Finland	0,0944	0,68
Spain	0,0345	0,26
Czech Republic	0,0482	-0,05
Austria	0,0393	-0,99
Portugal	0,1375	-1,09
Denmark	0,0740	-1,16
Belgium	0,0898	-1,52
Sweden	0,0646	-1,56
Germany	0,0543	-1,81
Norway	0,1469	-1,83
Italy	0,0402	-2,53

Figure 2. Absolute differences between actual and balanced growth rate and Theil's index of similarity.



6. Principal Components of the Eigenvectors

To deepen our analysis, we have tried to ascertain whether there are sectors which, being under or overrepresented in the actual production structure, can be deemed to be more responsible than others for the difference between the actual and the balanced growth rates. This was achieved by applying principal component analysis to the output structure in the different countries. Seven components have been extracted and Table 5 displays the main results in the principal components analysis with varimax rotation over the structure of the eigenvectors of each country. The "Total" column gives the variance in the original variables accounted for by each component. The "Share of variance" column gives the percentage share of total variance accounted for by each component. The last column shows the cumulative share of variance accounted for by the single components.

Table 5. Extraction of (rotated) components.

Component	Rotated sum of squared loadings		
	Total	Share of variance	Cumulative share of variance
1	7,9	0.25	0.25
2	6,5	0.21	0.46
3	3,6	0.11	0.58
4	2,7	0.08	0.67
5	2,4	0.07	0.75
6	2,2	0,07	0.82
7	1,9	0,06	0.88

There are two dominant components, each one accounting for more than 20% of total variance. We can characterize the components according to their factorial loads in the rotated component matrix by correlating each component to the shares in the eigenvector. Table 6 shows the factorial loads for each component and sector. If the factorial load regarding component p and eigenvector's element $\bar{x}_j$ is close to 1, a country with a high $\bar{x}_j$ eigenvector's element will tend to show a high score in component p. On the contrary, if the factorial load related to component p and the eigenvector's element $\bar{x}_j$ is close to -1, a country with a high $\bar{x}_j$ eigenvector's element will tend to show a low score in factor p. Finally, if the factorial load between factor p and eigenvector's element $\bar{x}_j$ is closed to 0, the score of component p will be rather independent from such an eigenvector's element $\bar{x}_j$. Table 6 shows the factorial loads for each component and sector.

Table 6. Factorial loads by component and sector.

Sector	Component 1	2	3	4	5	6	7
1 Agriculture, hunting and forestry	0,897	0,013	-0,235	-0,231	0,021	-0,024	-0,102
2 Fishing	-0,105	-0,176	0,029	0,225	-0,132	-0,079	0,882
3 Mining and quarrying of energy producing materials	0,865	0,138	0,150	-0,196	0,103	-0,307	-0,075
4 Mining and quarrying except energy producing materials	0,002	-0,136	-0,865	-0,250	0,112	0,094	-0,079
5 Manufacture of food products; beverages and tobacco	0,875	0,081	-0,339	0,021	0,036	0,094	0,212
6 Manufacture of textiles and textile products	0,461	0,461	-0,186	0,193	0,084	0,631	-0,054
7 Manufacture of leather and leather products	0,678	0,265	-0,041	0,322	-0,149	0,396	-0,135
8 Manufacture of wood and wood products	0,627	-0,604	0,138	-0,114	-0,100	-0,202	0,222
9 Manufacture of pulp, paper and paper products; publishing and printing	-0,227	0,130	0,764	-0,290	-0,126	0,271	0,095
10 Manufacture of coke, refined petroleum products and nuclear fuel	-0,071	0,900	0,338	0,105	0,009	0,014	0,061
11 Manufacture of chemicals, chemical products and man-made fibres	0,771	0,507	0,014	-0,070	-0,019	0,204	-0,011
12 Manufacture of rubber and plastic products	0,832	0,253	-0,053	-0,181	-0,228	0,117	-0,208
13 Manufacture of other non-metallic mineral products	0,477	0,635	0,133	0,252	0,412	0,062	-0,191
14 Manufacture of basic metals and fabricated metal products	0,220	0,914	0,207	-0,060	-0,067	0,057	-0,027
15 Manufacture of machinery and equipment n.e.c.	0,187	0,584	0,388	-0,226	0,269	-0,438	-0,113
16 Manufacture of electrical and optical equipment	0,093	0,364	-0,005	-0,192	-0,856	-0,024	0,034
17 Manufacture of transport equipment	-0,290	0,372	-0,355	0,193	0,537	0,147	0,258
18 Manufacturing n.e.c.	-0,002	-0,298	-0,879	0,249	-0,031	0,054	-0,117
19 Electricity, gas and water supply	0,609	0,479	-0,100	-0,186	0,057	-0,505	-0,137
20 Construction	0,151	0,566	0,013	0,033	0,653	-0,188	-0,026
21 Wholesale and retail trade; repair of motor vehicles, personal and household goods	0,292	-0,349	0,130	0,186	0,151	0,735	-0,079
22 Hotels and restaurants	-0,224	0,005	-0,232	0,818	0,147	0,114	0,211
23 Transport, storage and communication	0,057	-0,334	0,156	-0,220	-0,648	-0,367	0,414
24 Financial intermediation	-0,582	-0,102	-0,354	0,236	-0,156	-0,132	-0,615
25 Real estate, renting and business activities	-0,869	0,026	0,011	-0,067	-0,031	-0,142	-0,111
26 Public administration and defence; compulsory social security	-0,203	-0,673	0,002	-0,529	-0,022	0,044	0,242
27 Education	-0,408	-0,612	0,295	-0,199	0,007	0,316	0,329
28 Health and social work	-0,764	-0,524	0,104	-0,178	0,127	-0,044	0,186
29 Other community, social, personal service activities	-0,459	-0,332	0,712	0,059	0,102	-0,039	-0,157
30 Activities of households	-0,085	0,060	0,054	0,888	0,209	0,186	0,009
31 Add_value & consumption	-0,381	-0,873	0,045	0,135	-0,015	0,171	-0,010

These results show that the first component is positively related to agriculture, and foodstuff manufacturing, while being in a negative relation with real estate, renting and business services. The second component is mainly positively related to the energy supplying sector and the manufacture of metal products, and negatively to the households sector. Table 7 displays the scores of the seven factors in the different countries:

Table 7. Scores of the countries for each component.

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7
Austria	-0,40800	-0,54100	0,29923	0,62587	0,08697	-0,05259	-1,16797
Belgium	-0,80683	0,63935	-0,36817	-0,41642	0,50108	0,54027	0,36967
Czech R.	0,67736	1,44057	0,00772	-0,22974	-0,32604	-1,22780	-0,61473
Denmark	-0,68824	-1,14240	1,20720	-0,36768	-0,03846	-0,38830	-0,48795
Estonia	0,53748	0,91157	0,21171	-0,21758	-3,18627	0,33487	0,53816
Finland	-0,43319	-0,51862	1,00503	-0,98044	-0,06418	0,06338	-0,37701
Germany	-0,96029	-0,18975	0,41739	0,23331	0,29765	-0,11071	-1,41917
Ireland	-0,75244	-0,84228	-3,38556	-0,14167	-0,42463	0,22129	-0,29633
Italy	0,16831	0,05981	0,31248	1,83003	-0,46248	0,97356	-0,68337
Lithuania	2,77419	-2,06893	-0,00925	-0,54684	0,15826	0,55816	-0,11512
Norway	-0,51576	-0,77084	0,30816	-0,01135	0,06309	-0,73448	2,95228
Portugal	0,34540	0,43343	-0,08164	0,46799	1,53993	0,76081	0,80110
Slovakia	1,14150	1,20419	-0,51914	-0,81424	0,93338	-2,08556	-0,37348
Slovenia	0,09310	1,49621	0,20962	-1,08697	0,76776	2,33238	0,19618
Spain	0,18059	0,33922	0,05226	2,57163	0,32339	-0,59185	0,56687
Sweden	-1,35319	-0,45053	0,33296	-0,91589	-0,16946	-0,59344	0,11086

It is interesting to see whether there is a relation between the different components and the balanced or theoretical rate of growth. As shown in Table 8, the Pearson’s correlation coefficients for the different components and the balanced growth rate are not very high, although the first component is shown to be related.

Table 8. Pearson's correlation between the principal components and the potential rate of growth.

Component 1	Pearson's correlation	-0,410
	Sig. (bilateral)	0,114
Component 2	Pearson's correlation	-0,020
	Sig. (bilateral)	0,943
Component 3	Pearson's correlation	0,313
	Sig. (bilateral)	0,237
Component 4	Pearson's correlation	0,142
	Sig. (bilateral)	0,599
Component 5	Pearson's correlation	0,106
	Sig. (bilateral)	0,696
Component 6	Pearson's correlation	-0,170
	Sig. (bilateral)	0,529
Component 7	Pearson's correlation	0,360
	Sig. (bilateral)	0,171

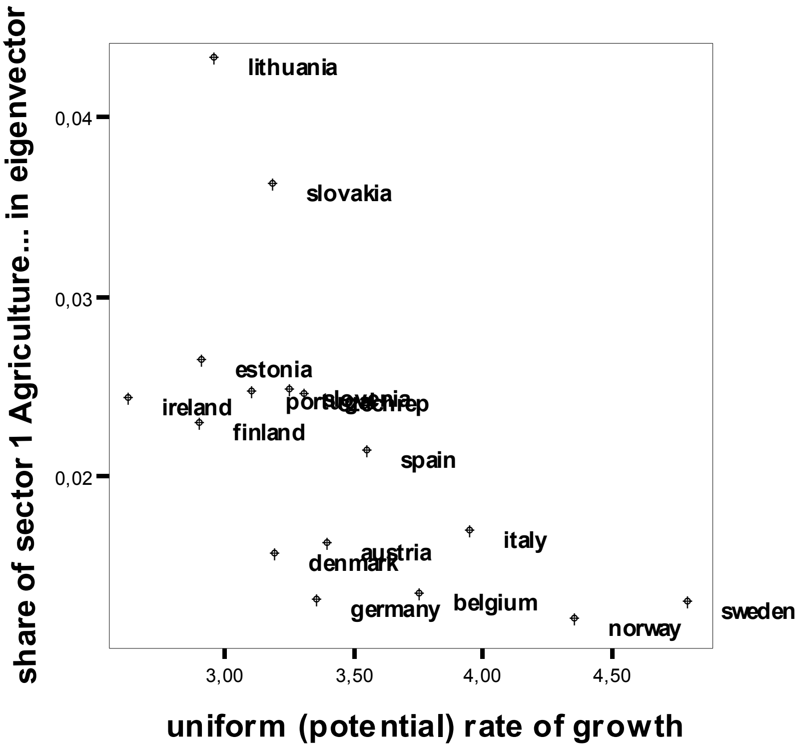
There is, however, some correlation between the balanced growth rate and the shares of two sectors having high absolute factorial loads in the first component in the eigenvector. Both *agriculture* and *real estate, renting and business services* have high absolute factorial loads in the first component (Table 6) which is nearly significant in statistical terms, as shown in the table above. Table 9 displays the results.

Table 9. Pearson's correlation between the share of sectors in eigenvector and the rate of balanced growth.

Sector	Pearson's correlation (Sig. bilateral)
1 Agriculture, hunting and forestry	-0,605 (0,013) (*)
2 Fishing	0,268 (0,315)
3 Mining and quarrying of energy producing materials	-0,182 (0,500)
4 Mining and quarrying except energy producing materials	-0,412 (0,113)
5 Manufacture of food products; beverages and tobacco	-0,382 (0,144)
6 Manufacture of textiles and textile products	-0,323 (0,223)
7 Manufacture of leather and leather products	-0,218 (0,417)
8 Manufacture of wood and wood products	-0,084 (0,758)
9 Manuf. of pulp, paper and paper products; publishing and printing	0,131 (0,628)
10 Manufacture of coke, refined petroleum products and nuclear fuel	0,178 (0,509)
11 Manufacture of chemicals, chemical products and man-made fibres	-0,303 (0,254)
12 Manufacture of rubber and plastic products	-0,487 (0,055)
13 Manufacture of other non-metallic mineral products	-0,272 (0,308)
14 Manufacture of basic metals and fabricated metal products	-0,035 (0,898)
15 Manufacture of machinery and equipment n.e.c.	0,043 (0,875)
16 Manufacture of electrical and optical equipment	-0,294 (0,269)
17 Manufacture of transport equipment	0,395 (0,130)
18 Manufacturing n.e.c.	-0,356 (0,176)
19 Electricity, gas and water supply	-0,238 (0,374)
20 Construction	-0,239 (0,372)
21 Wholesale and retail trade; repair	-0,273 (0,305)
22 Hotels and restaurants	0,135 (0,619)
23 Transport, storage and communication	0,192 (0,475)
24 Financial intermediation	-0,163 (0,546)
25 Real estate, renting and business activities	0,536 (0,032) (*)
26 Public administration and defence; compulsory social security	0,125 (0,644)
27 Education	0,278 (0,297)
28 Health and social work	0,419 (0,106)
29 Other community, social, personal service activities	0,387 (0,139)
30 Activities of households	0,210 (0,435)
31 Add_value & consumption	0,232 (0,386)

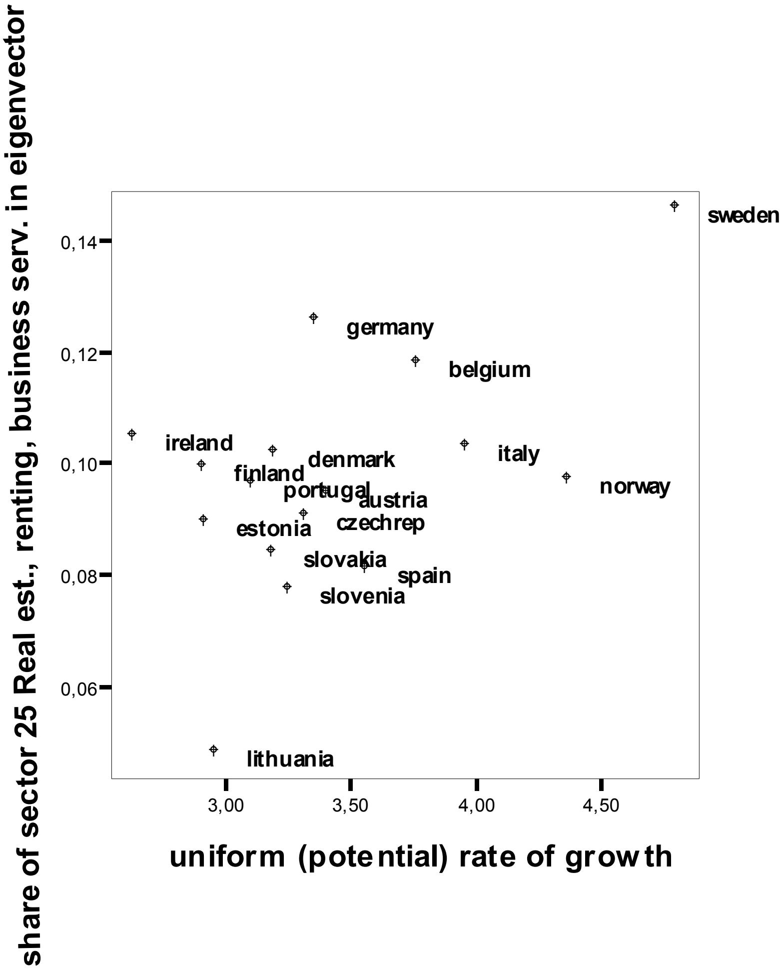
More exactly, for sector 1 (*agriculture)* the correlation is negative (Figure 3), since the higher the share of this sector in the eigenvector the lower the balanced growth rate.

Figure 3. Share of Agriculture in the eigenvector and the balanced growth rate.



On the other hand, the correlation in the case of *real estate, renting and business services* is positive i.e. the higher share of this sector in the eigenvector, the higher the balanced growth rate (Figure 4).

Figure 4. Share of sector 25 Real estate, Renting and Business Services in the eigenvector and the balanced growth rate.



We have, therefore, computed the differences between the share of both sectors in the real output, and between the shares in the eigenvector, to see whether such differences and the differences between the actual and the uniform rates of growth are correlated. Results are shown in Table 10:

Table 10. Relation between differences in shares and differences in rates of growth.

Country	Sector 1: Agriculture, hunting and forestry			Sector 25: Real estate, renting and business activities			Difference actual - uniform growth
	share in real output	share in eigenvector	difference	share in real output	share in eigenvector	difference	$\rho - g$
Austria	0,0178	0,0159	0,0019	0,1427	0,0937	0,0490	-0,9865
Belgium	0,0139	0,0131	0,0007	0,1638	0,1174	0,0463	-1,5250
Czech. R.	0,0293	0,0242	0,0051	0,1113	0,0897	0,0216	-0,0509
Denmark	0,0241	0,0153	0,0088	0,1762	0,1013	0,0750	-1,1597
Estonia	0,0426	0,0261	0,0165	0,1176	0,0887	0,0290	4,5448
Finland	0,0295	0,0226	0,0069	0,1258	0,0987	0,0272	0,6825
Germany	0,0129	0,0127	0,0002	0,1797	0,1250	0,0547	-1,8113
Ireland	0,0288	0,0240	0,0049	0,1188	0,1043	0,0146	4,1572
Italy	0,0195	0,0166	0,0029	0,1458	0,1023	0,0435	-2,5299
Lithuania	0,0502	0,0429	0,0073	0,0773	0,0475	0,0298	3,5997
Norway	0,0117	0,0117	0,0000	0,1363	0,0966	0,0397	-1,8347
Portugal	0,0280	0,0244	0,0036	0,1237	0,0958	0,0279	-1,0891
Slovakia	0,0412	0,0360	0,0052	0,1023	0,0833	0,0190	1,7592
Slovenia	0,0268	0,0244	0,0024	0,1082	0,0768	0,0314	1,0364
Spain	0,0308	0,0210	0,0098	0,1264	0,0805	0,0458	0,2590
Sweden	0,0156	0,0126	0,0030	0,2100	0,1449	0,0651	-1,5621
		correlation:	**0,65**		**correlation:**	**-0,63**	

There is evidence that over-representation in *Agriculture* widens the gap between the actual and the balanced rate of growth while over-representation in *Real estate, renting and business* services has the opposite effect. On the basis of these findings, we may conclude that countries with a large *Agriculture* sector are deemed to grow at a rate which is lower than the rate of balanced growth, while countries with a large *Real estate, renting and business* services sector tend to growth at a rate which is closer to the balanced one.

7. Sensitivity Analysis.

To identify the sectors that have the largest influence on the first eigenvector of matrix $(I-A)^{-1}B$, the rate of balanced growth in each country (and thus the indicator of efficiency which is independent from output composition), we look at the elasticity of that eigenvalue relative to the coefficients of the same matrix. Changes in the intermediate and capital input coefficients reflect the many disparate changes occurring in areas such as technical change, product composition and relative prices. As we distinguish intermediate from capital inputs, we must also consider two sets of elasticities, one for the a_{ij}'s coefficients and one for the b_{ij}'s coefficients. Overall efficiency depends on the amount of inputs that are used to produce one unit of final output in each sector of the economy, and also on the overall sector composition, but in this section we focus only on the coefficients. This analysis, therefore, completes the one we carried out earlier focused on sector composition.

A unit change in the intermediate input in a_{ij} implies a change in the Leontief inverse, in the matrix $(I-A)^{-1}B$ and thus in the first eigenvalue, while a change in the b_{ij} is directly mapped on to the first eigenvalue. It is the ensuing changes in the corresponding rate of balanced growth that we are interested in, since any reduction in the model's coefficients have a bearing on the efficiency of the system. The elasticity with regard to a single intermediate input coefficient i and in sector j in country k (i.e. a_{ij}^{k} or, simply, a_{ij}) and capital input coefficient i in sector j in country k (i.e. b_{ij}^{k} or b_{ij}) are defined, respectively, as:

$$\varepsilon_{ij}^{ka} = \frac{\Delta\rho}{\rho} \Big/ \frac{\Delta a_{ij}}{a_{ij}}, \text{ and } \varepsilon_{ij}^{kb} = \frac{\Delta\rho}{\rho} \Big/ \frac{\Delta b_{ij}}{b_{ij}}.$$

A high elasticity means that a small increase in the intermediate input coefficient a_{ij} leads, ceteris paribus, to a greater increase in the eigenvalue i.e. to a larger reduction in efficiency. The differences in the elasticities are clearly greater across sectors than across countries. This means that there are specific sectors - and particular inputs in specific sectors – which in almost all countries are more important than others in respect to their influence on the first eigenvalue. Indeed, there are particular cells in the table that have distinctly high input coefficients and thus proportionally high elasticities. The average elasticity with respect to all the inputs coefficients in sector j in country k is defined as:

$$\bar{\varepsilon}_j^{ka} = \sum_{i=1}^{N-1} \varepsilon_{ij}^{ka} \Big/ (N-1) .$$

Table 11 reports the ratios between those averages and the country average. The sectors whose input coefficients exercise the largest influence across all countries are: sector 29 (Construction), sector 25 (Real estate, renting and business activities), sector 21 (Trade), sector 23 (Transport and Communication), and sector 5 (Manufacture of foods). For particular sectors in specific countries, we observe the strong influence of sector 11 (Chemicals industry) in Lithuania, sector 16 (Manufacture of electrical and optical equipment) in Estonia, sector 19 (Electricity, gas and water supply) in Slovakia, and sector 14 (Financial intermediation) in Austria.

Table 11. Relative of the eigenvector with respect to elasticities of input coefficients (national average=100)

	Sectors	Austria	Belgium	Czech R.	Denmark	Estonia	Finland	Germany	Ireland	Italy	Lithuania	Norway	Portugal	Slovakia	Slovenia	Spain	Sweden
1	Agriculture...	70,97	52,76	84,36	87,29	99,03	100,58	57,50	103,83	50,09	173,55	46,86	78,10	136,79	82,60	68,53	45,03
2	Fishing	0,60	2,21	0,38	4,97	7,52	1,97	0,59	2,14	2,02	2,90	24,53	3,05	0,31	0,84	5,81	1,18
3	Mining (energy prod.)	37,34	42,77	64,33	4,12	43,79	45,44	62,64	0,00	17,16	148,77	11,86	236,11	82,63	31,51	39,76	44,52
4	Mining (except energy prods.)	11,01	17,60	10,44	6,20	4,90	25,89	8,75	48,47	9,26	19,16	11,22	7,41	15,41	14,15	11,81	12,21
5	Manuf. of food...	159,94	201,70	217,17	175,27	215,15	197,85	189,25	280,37	215,99	365,40	241,89	244,47	212,09	207,75	276,09	169,74
6	Manuf. of textiles...	67,62	58,94	56,33	51,59	75,13	36,77	61,74	62,76	98,04	74,68	40,63	81,87	46,87	122,15	73,85	35,75
7	Manuf. of leather...	14,48	7,89	15,41	9,07	20,42	7,11	13,30	13,03	36,52	22,45	9,09	14,05	13,15	23,28	19,93	7,25
8	Manuf. of wood...	37,39	23,78	28,98	36,10	31,81	45,41	26,15	32,62	27,82	49,57	43,46	30,00	24,73	21,05	37,54	41,54
9	Manuf. of pulp, paper...	94,65	76,79	71,64	97,54	77,88	114,76	82,76	62,98	82,04	69,18	89,59	73,65	58,61	92,57	80,22	90,86
10	Manuf. of coke, refined petr...	54,65	102,78	90,76	48,31	70,37	69,85	70,23	0,00	71,36	0,00	64,39	85,73	64,23	107,52	93,39	61,00
11	Manuf. of chemicals...	111,59	160,80	135,90	74,61	159,90	99,18	115,39	72,52	138,58	283,79	88,63	139,00	135,26	149,26	150,63	71,16
12	Manuf. of rubber and plastic...	36,83	40,56	66,77	35,23	65,57	35,95	43,80	40,02	47,39	125,68	26,59	44,52	49,76	56,78	42,80	31,55
13	Manuf. non-metallic min. prod.	41,17	42,79	53,96	40,16	32,75	36,76	48,19	22,31	54,94	59,48	31,21	58,45	51,00	52,98	58,36	25,23
14	Manuf. metal prods.	117,07	139,64	169,01	98,05	162,58	138,64	114,94	58,07	145,78	86,46	111,51	107,85	171,00	207,38	154,36	127,27
15	Manuf. of machinery n.e.c.	81,68	59,07	88,84	69,86	62,48	106,64	75,31	39,63	76,03	55,50	77,35	94,73	112,60	78,12	63,08	80,76
16	Manuf. Electrical...	97,06	98,06	169,63	105,67	459,69	150,95	119,11	158,91	86,67	72,26	102,64	83,74	95,32	98,08	91,13	144,25
17	Manuf. transport equip.	90,85	137,16	81,80	50,89	38,33	52,56	144,14	106,34	96,99	56,76	115,08	119,37	95,98	145,57	141,57	132,18
18	Manufacturing n.e.c.	44,51	38,43	36,78	32,73	27,42	28,48	38,48	95,29	46,53	52,99	37,92	39,03	31,39	23,86	47,38	36,05
19	Electricity...	90,68	54,05	178,66	46,22	120,55	49,98	52,94	54,99	77,44	116,06	26,37	89,97	292,28	59,66	81,35	40,69
20	Construction	279,16	368,25	408,35	394,79	194,10	365,55	322,26	320,82	285,48	272,76	343,19	398,05	386,63	417,80	397,74	191,10
21	Wholesale and retail trade...	255,39	288,76	142,72	262,97	206,52	293,56	225,23	170,09	376,26	224,33	237,35	249,01	187,40	256,45	187,39	183,01
22	Hotels and restaurants	106,29	70,70	42,62	66,08	52,91	77,84	66,39	130,25	117,21	30,37	103,50	107,90	51,96	73,18	142,09	61,54
23	Transport, communication	224,35	240,32	196,70	369,34	297,70	183,97	213,45	257,68	242,63	228,92	383,34	111,21	208,48	173,74	212,32	276,40
24	Financial intermediation	282,23	80,86	91,43	107,30	47,05	55,68	184,67	163,47	87,32	16,57	32,99	29,12	56,19	30,83	70,06	67,74
25	Real estate, renting, business	283,74	353,40	298,89	337,13	202,93	322,82	347,43	369,01	274,51	120,02	341,96	278,31	242,64	221,30	236,79	580,27
26	Public administration...	94,29	46,91	55,59	107,28	85,66	116,35	82,00	122,81	60,82	110,83	141,16	32,62	72,74	91,52	46,14	134,75
27	Education	21,83	18,37	21,69	59,73	38,50	59,22	31,23	39,07	26,07	31,47	48,97	28,87	12,31	30,82	20,30	79,23
28	Health and social work	102,04	95,42	41,86	115,54	39,75	93,27	100,12	132,41	67,32	55,95	89,53	73,35	37,48	62,50	65,88	126,86
29	Other services...	90,59	79,24	78,99	105,96	59,63	86,97	102,01	40,11	81,74	74,13	77,18	60,50	54,79	66,72	83,68	100,91
30	Activities of households	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	Average	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**

To make cross country comparisons in the above mentioned five sectors (29, 25, 21, 23 and 5), Table 12 shows for specific sectors the ratio between each country's average elasticities and the overall average. Through this means, we find that even though in general the ratios are rather similar across countries, there are nevertheless some countries which have higher average elasticities in sensitive secors and are thus less efficient than others in those sectors. See, for example, the cases of Estonia in sector 23 (Transport) and Sweden in sector 25 (Real Estate) (See Table 12).

Table 12. Relative elasticities of the most important input coefficients.

Sectors	5 Manufacture of food products; beverages and tobacco	20 Construction	21 Wholesale and retail trade; repair of motor vehicles, motorcyc., personal and household goods	23 Transport, storage and communication	25 Real estate, renting and business activities
Austria	59,23	71,35	92,28	77,17	78,87
Belgium	98,79	124,50	138,00	109,34	129,95
Czech. Rep	102,82	133,44	65,93	86,50	106,23
Denmark	56,55	87,91	82,78	110,69	81,65
Estonia	168,10	104,67	157,44	216,06	119,03
Finland	73,57	93,82	106,51	63,54	90,11
Germany	65,63	77,13	76,21	68,76	90,45
Ireland	158,61	125,27	93,89	135,41	156,71
Italy	91,78	83,73	156,01	95,77	87,57
Lithuania	151,22	77,91	90,59	88,01	37,29
Norway	108,13	105,88	103,52	159,18	114,75
Portugal	100,14	112,53	99,52	42,32	85,58
Slovakia	81,75	102,85	70,48	74,64	70,21
Slovenia	87,60	121,60	105,51	68,05	70,05
Spain	115,31	114,65	76,36	82,37	74,24
Sweden	80,78	62,77	84,98	122,18	207,30
Average	100,00	100,00	100,00	100,00	100,00

A capital coefficient can be decomposed into three factors according to equation (7) $b_{ij} = t_{ij} e_i c_j$, i.e. the service life, the fraction of total fixed investment supplied by the origin sector *i*, and the amount of replacement investment in the destination sector *j*. A unit change in any one of these factors implies a change in the matrix *B* and thus in the corresponding first eigenvalue of matrix $(I - A)^{-1} B$. In particular, a unit change in the coefficient regarding the capital input in the destination sector, i.e. $\Delta c_j = 1$, implies a change in the capital coefficient concerning the specific origin sector equal to $\Delta b_{ij} = t_{ij} e_i$. This change feeds upon the first eigenvector through the matrix $(I - A)^{-1} B$, and the elasticity of the corresponding first eigenvector with respect to this capital coefficient is precisely what interests us. Table 13 shows that differences among sectors are quite large, and thus that differences in the sector composition matter a great deal when included are sectors where the elasticity is high, such as Agriculture (sector 1) and Real estate (sector 25) among others.

Table 13. Relative elasticies of the eigenvector with respect to replacement investment (national average =100).

	Sectors	Austria	Belgium	Czech. R.	Denmark	Estonia	Finland	Germany	Ireland	Italy	Lithuania	Norway	Portugal	Slovakia	Slovenia	Spain	Sweden
1	Agriculture...	145,85	84,79	92,39	101,65	96,14	169,21	84,08	148,71	166,70	201,90	156,57	83,28	83,57	130,08	136,58	95,18
2	Fishing	0,18	5,27	0,25	8,39	4,62	0,97	0,83	4,83	5,13	0,25	20,63	1,40	0,07	0,58	5,10	2,49
3	Mining (energy prod.)	65,02	24,42	73,51	20,24	27,42	27,44	70,88	0,00	57,96	95,57	65,49	0,21	169,68	73,77	70,38	50,10
4	Mining (except energy prods.)	9,46	9,40	12,91	5,01	2,25	19,97	5,23	5,03	11,56	13,81	6,77	12,59	5,81	13,43	11,55	19,07
5	Manuf. of food...	66,91	61,85	53,23	34,42	53,34	41,78	42,15	51,33	63,91	114,03	49,49	64,71	51,39	89,07	58,13	74,06
6	Manuf. of textiles...	23,66	24,04	16,75	16,79	7,48	12,50	17,30	19,08	37,73	32,76	12,81	24,45	9,73	29,27	60,52	15,92
7	Manuf. of leather...	2,66	4,12	2,75	3,44	1,92	2,20	3,39	3,96	6,26	11,55	2,00	1,85	2,03	6,41	7,92	3,14
8	Manuf. of wood...	11,64	12,78	9,07	9,12	6,72	8,75	6,56	18,19	16,97	29,26	13,14	12,32	3,67	9,19	20,68	16,12
9	Manuf. of pulp, paper...	27,80	34,18	27,07	35,27	20,01	37,96	31,62	15,13	33,45	35,28	37,09	25,52	25,94	41,28	33,33	66,16
10	Manuf. of coke, refined petr...	8,14	7,23	16,90	6,48	11,48	6,45	7,69	0,00	13,94	0,00	9,51	19,67	13,75	21,34	13,60	8,97
11	Manuf. of chemicals...	35,62	42,53	52,42	27,02	22,19	27,83	31,99	16,97	73,48	21,18	42,46	61,18	70,49	79,54	83,78	41,99
12	Manuf. of rubber and plastic...	11,91	23,61	17,40	12,69	9,93	8,94	12,90	17,73	23,75	10,56	12,56	23,34	13,98	28,64	28,85	14,27
13	Manuf. non-metallic min. prod.	18,36	23,34	30,68	19,78	15,87	13,07	23,60	36,87	29,37	12,39	13,59	36,24	23,96	36,05	28,27	12,17
14	Manuf. metal prods.	36,94	46,22	60,42	30,58	19,09	25,30	30,78	26,69	78,91	30,13	32,24	44,91	18,70	73,21	91,84	49,54
15	Manuf. of machinery n.e.c.	17,25	18,45	21,19	20,14	6,82	13,99	16,83	10,08	30,30	21,23	16,53	31,28	32,32	28,12	67,52	27,36
16	Manuf. Electrical...	19,17	43,31	23,21	20,20	7,81	14,28	29,56	32,96	32,45	35,87	24,84	30,60	13,87	35,22	42,90	31,24
17	Manuf. transport equip.	13,54	20,62	16,70	11,05	10,36	10,69	26,64	13,94	45,76	26,19	20,68	37,17	11,82	24,78	35,81	40,96
18	Manufacturing n.e.c.	11,04	14,29	6,76	10,61	4,53	6,46	10,07	20,73	14,45	16,90	10,03	7,84	2,61	10,93	20,85	7,78
19	Electricity...	123,41	157,21	254,61	83,35	170,89	84,44	86,76	64,64	128,27	203,72	111,97	52,60	366,54	177,42	130,66	142,74
20	Construction	80,28	88,45	64,44	75,44	26,75	45,18	54,07	24,41	96,44	71,26	54,23	86,04	39,97	91,92	71,06	100,38
21	Wholesale and retail trade...	172,50	236,40	186,05	231,15	165,83	231,62	143,52	281,88	198,57	287,12	199,08	209,82	177,65	339,12	200,48	228,48
22	Hotels and restaurants	78,33	72,13	-3,94	43,84	34,54	25,73	29,55	48,25	78,80	21,58	83,42	61,26	19,48	101,00	76,22	52,42
23	Transport, communication	308,14	230,30	514,48	276,37	170,17	363,63	200,63	292,29	332,83	312,68	426,92	218,18	150,90	222,24	324,00	396,64
24	Financial intermediation	68,70	117,18	129,84	89,65	187,17	68,25	71,99	408,90	98,27	42,43	45,11	45,52	471,52	54,45	99,09	91,39
25	Real estate, renting, business	1175,07	1049,91	799,87	1237,61	1545,90	1311,50	1421,86	1123,02	908,49	832,02	868,93	1352,37	748,78	890,58	874,24	923,68
26	Public administration...	207,62	262,90	215,88	192,71	220,15	123,00	198,52	303,60	273,63	295,63	323,61	232,24	354,20	213,61	267,88	252,05
27	Education	61,22	97,01	149,14	107,17	81,76	90,30	76,04	0,38	27,67	101,49	111,87	86,61	39,23	51,99	27,07	92,72
28	Health and social work	64,60	74,47	47,61	71,23	20,53	64,35	96,49	10,38	41,21	61,71	108,48	44,15	42,76	51,82	38,37	55,88
29	Other services...	134,99	113,59	108,39	198,58	48,33	144,20	168,50	0,00	73,74	61,48	119,98	92,65	35,59	74,96	73,32	87,11
30	Activities of households	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	Average	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**

8. Conclusions.

By utilizing the EUROSTAT input-output data base, and Italian data on service life of capital assets, we have built a dynamic input-output model for 16 European countries. The model has been used to compute the various rates of balanced growth in the countries concerned as a starting point for analysis of national differences in productive efficiency. The assumption, indeed, has been that high productive efficiency raises total factor productivity and thus the rate of balanced growth. The actual growth rate is in our sample for the most part different from the balanced rate, and this may reflect differences between the output composition actually observed and the particular one which would be required to produce the balanced growth path. Principal component analysis of the output structure shows that two dominant components account for more than 46% of total variance in the national rates of aggregate actual growth, and there are a few sectors the presence of which – having higher absolute factorial loads in the first two components – seem to be more highly correlated with the rate of balanced growth. Among these, *Agriculture* and *Real Estate, renting and business activities* are found to be critical to growth performance in that a higher share in the economy of *Agriculture* is negatively associated with the rate of balanced growth and, respectively, a higher share of *Real Estate, renting and business activities* is positively associated. In particular, overrepresentation of *Agriculture* is associated with a larger gap between the actual and the balanced rates of growth, while overrepresentation of Real estate etc. is associated with a narrow gap between the two rates.

By computing the sensitivity of the rate of balanced growth to the levels of intermediate input coefficients and capital coefficients, we found that differences amongst sectors are larger than those amongst countries. In particular, we found that there are some sectors, notably *Agriculture* and *Real Estate*, in which intermediate and capital coefficients are critical to the rate of aggregate growth because the elasticity of the rate of balanced growth with respect to the coefficients is particularly high. To put it differently, a unit change in the coefficients in the two sectors exerts the highest impact on the balanced rate of growth. Thus a completely different exercise yields results which are convergent with those obtained from the principal component analysis. The broad conclusion that can be drawn from our investigation is that even though the sixteen European countries are rather similar from the point of view of their input-output structures, there are some countries that are less efficient, and some others that are more so, in the sectors we have called the sensitive sectors i.e. those whose size matters in establishing the gap between the actual rate of growth and the rate of balanced growth. The obvious conclusion is that overrepresentation in growth-enhancing sectors is a favourable condition while overrepresentation in growth-reducing sectors is unfavourable. From a policy point of view, it might be argued that countries which are overrepresented and inefficient in growth-reducing sectors could gain a great deal from reducing that overrepresentation or by increasing efficiency in those sectors.

Appendix.

To focus more closely on how a change in the model's coefficients hing on the growth rate, we express the uniform growth rate λ in terms of both the model's value and technical coefficients. The closed dynamic model for such an economy $(I - A - \lambda B)x = 0$ requires that $|I - A - \lambda B| = 0$, which in the simple case for two sectors only, and in expanded form reads:

$$\begin{vmatrix} 1 - a_{11} - \lambda b_{11} & -a_{12} - \lambda b_{12} \\ -a_{21} - \lambda b_{21} & 1 - a_{22} - \lambda b_{22} \end{vmatrix} = 0$$

To further simplify matters, we assume that there is minimal interaction, i.e. $a_{11} = a_{22} = 0$, and that the second sector does not produce capital goods, i.e. $b_{21} = b_{12} = 0$, thus solving, the resulting equation: $\begin{vmatrix} 1 - \lambda b_{11} & -a_{12} - \lambda b_{12} \\ -a_{21} & 1 \end{vmatrix} = 0$, or: $(1 - \lambda b_{11}) - a_{21}(a_{12} + \lambda b_{12}) = 0$, one gets: $\lambda = \dfrac{1 - a_{12}a_{21}}{b_{11} + a_{12}b_{21}}$. The expressions shows that the lower the a_{ij}'s and the b_{ij}'s, the higher the balanced growth rate; indeed, the latter increases when the amount of resources left after deduction for intermediate consumption, i.e. $(1 - a_{12}a_{21})$, increases and the amount of resources needed for making investments, i.e. $(b_{11} + a_{12}b_{21})$, decreases.

All coefficients in the model are intended to be value coefficients as they include relative prices computed over costs, factor rents included, it is then interesting to see whether changes in prices affect the growth rate. This can be done by considering the expansion of value coefficients into quantity or technical coefficients (α_{ij}) and prices (p_i) :

$$\lambda = \frac{1 - a_{12}a_{21}}{b_{11} + a_{12}b_{21}} = \frac{1 - \dfrac{p_1}{p_2}\alpha_{12}\dfrac{p_2}{p_1}\alpha_{21}}{\dfrac{p_1}{p_1}\beta_{11} + \dfrac{p_1}{p_2}\alpha_{12}\dfrac{p_2}{p_1}\beta_{21}} = \frac{1 - \alpha_{12}\alpha_{21}}{\beta_{11} + \alpha_{12}\beta_{21}}$$

The expression shows that changes in rents, and thus prices, do not alter the aggregate rate of growth and the corresponding aggregate rate of profit in the economy since the rate λ depends on the α_{ij}'s and the β_{ij}'s only. In the two sectors case, therefore, relative prices are irrelevant.

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