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Bankruptcy: Is It Enough to Forgive or Must we Also Forget?

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Abstract
In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed. We model this provision and determine conditions under which it is optimal. We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We show that forgetting a default makes incentives worse, ex-ante, because it reduces the punishment for failure. However, following a default it is generally good to forget, because by improving an entrepreneur's reputation, forgetting increases the incentive to exert effort to preserve this reputation. Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore information available to him.

Keywords
Bankruptcy, Information, Incentives, Fresh Start
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I Introduction

In studying the "fresh start" provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed; for example, in the United States information about a bankruptcy cannot be reported after 10 years.

In this paper we model this provision and determine conditions under which it is optimal. We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. In this setup, reputation effects encourage agents to exert high effort; however, it is typically the case that reputation is not efficacious until agents have accumulated a sufficiently good credit history to make default unattractive.

We show that in our model forgetting a default makes incentives worse, ex-ante, because it reduces the punishment from failure. On the other hand, following a default forgetting may be beneficial, because by improving an entrepreneur's reputation, it increases the incentive to exert effort to preserve this reputation. Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then welfare is higher in the presence of an appropriate amount of forgetting, that is, by limiting the information used by lenders on borrowers' credit history. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore the information available to him.

The focus of this paper is the effect on investment of laws governing bankruptcy.[1] We thus have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable. Data from the 1993 National Survey of Small Business Finance (NSSBF) suggests that the majority of small businesses do indeed finance themselves with some sort of personal loan or guarantee; see also Berger and Udell (1995). These entrepreneurs are also three times as likely to file for personal bankruptcy as their counterparts in the general population — see Sullivan, Warren, and Westbrook (1989). In such a setting we are then naturally led to explore the incentive effects of these laws. This seems to be where the greatest economic impact should be found, and indeed, incentives featured prominently in the policy debate surrounding the adoption of

[1] It should also be noted that similar restrictions exist in other contexts, such as motor vehicle records (Lemaire, 1985) and juvenile delinquency records (Funk, 1996).

these laws (discussed below). An alternative approach, however, would be to study the risk-sharing and redistributive impact of these laws on consumers (see Chatterjee, Corbae and Rios-Rull, 2007).

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to ten years, after which it must be removed from the records made available to lenders.[2] Similar provisions exist in many other countries — in the European Union the median time that a bankruptcy can stay on records is even shorter — only six years (see Jentzsch, 2006 and Miller, 2003 for a summary of international regulations). And this has also recently become a topic of renewed interest, as many developing countries seek to set up credit bureaus of their own (see International Finance Corporation, 2006).

Musto (2004) finds that these restrictions are binding. He shows that for those bankrupts who are more creditworthy, access to credit increases significantly when the bankruptcy "flag" is dropped from their credit file (after 10 years).[3] He also finds that those individuals who obtain new credit are subsequently likelier than average to default on this new credit; he interprets this as evidence that these laws are suboptimal. Our model will be consistent with his findings, although we will argue that it need not be evidence of inefficiency.

In the Congressional debate surrounding the adoption of the FCRA (U.S. House, 1970 and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults (or other negative information): (1) limited computer storage capacity, (2) old information might be less reliable or salient, and (3) if information was not erased the stigmatized individual would not obtain a "fresh start" and so would be unable to continue as a productive member of society. On the other hand, the arguments raised against forgetting this information were (1) it forces honest borrowers to subsidize the dishonest ones, (2) it discourages borrowers from repaying their debts by reducing the penalty of failure,[4] (3) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (4) that it increases the chance of costly fraud or other crimes by making it harder to identify (and exclude) seriously bad risks.[5] We will show that

[2]Other derogatory information can be reported for a maximum of seven years; see Hunt (2006) for a discussion of the history and regulation of consumer credit bureaus in the United States.

[3]For the less creditworthy bankrupts, dropping the flag has little impact, because they have many other derogatory indicators in their file.

[4]See also Staten (1993), who ties the increase in bankruptcy rates to the increasing availability of post-bankruptcy credit.

[5]Some of these arguments have also appeared in the criminology literature, particularly

our model captures many of these arguments, and will use it to study the tradeoffs between the positive and negative effects of forgetting.

The paper is organized as follows. We first briefly discuss the relevant literature. In section II we present the model and the choice problems of the entrepreneurs and lenders. In the following section the equilibrium notion considered (Markov Perfect Equilibrium) is defined. Its existence is then established and properties of the equilibrium strategies are characterized. The efficiency of equilibrium is also discussed. In section IV we study the effects of the specification of the forgetting policy on the properties of the equilibrium, in particular welfare. We derive conditions under which forgetting a default can be socially optimal — and relate them to the empirical evidence and the policy debate surrounding the adoption of the FCRA. Section V provides examples that illustrate these results. Finally, we consider an extension of the model in section VI. Section VII concludes, and the proofs are in the Appendix.

Previous Literature

Our basic model is one of reputation and incentives, like those of Diamond (1989), Mailath and Samuelson (2001), and Fishman and Rob (2005). In these models, principals and agents interact repeatedly under conditions of both adverse selection and moral hazard. The equilibrium in our model shares many similarities with the ones in these papers; in particular, agents build reputations over time, so that strategies are characterized by a period of low effort followed by one in which risky agents exert high effort. There are nevertheless some differences between our model and theirs — in both the setting (experience goods for Mailath and Samuelson, 2001 and Fishman and Rob, 2005, credit markets for Diamond, 1989 and our paper) and in the structure the structure of markets and information (in our model a competitive lending sector makes offers to the agents; the offer adopted by a particular agent is not observed by future lenders).

Our key contribution is to consider the impact of government policies that regulate the dissemination of information on borrowers' past history. Such information plays a key role in determining borrowers' reputations, and hence their incentives. In particular, we use our model to study the effects of these policies on the dynamics of borrowers' incentives and lenders' financing decisions.

The possible benefits for borrowers' incentives of limiting the informa-

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in the case of juvenile delinquency records; see Funk (1996).

tion available have also been explored by Padilla and Pagano (2000) and Vercammen (1995) in different settings. Padilla and Pagano study the effect of information sharing by lenders in a two-period model. They show that while information sharing can improve welfare, it may be optimal to restrict the type of information shared. In particular, if information about an agent's type is too precise, then a borrower's effort choice will have no effect on his reputation; this eliminates the incentive to exert effort. This effect is also important in our model. Vercammen presents an example in a dynamic setting that suggests that the optimal policy might involve restricting the memory of a credit bureau. In his example, however, the benefit comes from forgetting past *successes*, since if an agent has experienced many successes — and so is believed to be good with high probability — then his incentives to exert effort will be weak, because a failure in the current period will have little impact on his reputation.[6]

Another possible benefit of restricting the availability of information on agents' past behavior is that it may restrict the punishments a principal can impose on an agent. This effect has been explored by Crémer (1995), who shows that using an inefficient monitoring technology can sometimes improve incentives because by monitoring less efficiently a principal ensures that he will have less information about a default and can thereby commit to punish more severely (because he will not have enough information to renegotiate punishments). By contrast, in our model forgetting facilitates financing after failures, thereby making punishments *weaker*. In a model of Chapter 11 negotiations, Bernhardt and Nosal (2004) show that it may be optimal for a court to ignore all information and make liquidation decisions arbitrarily, since this encourages the parties to reach a better settlement on their own.

On the other hand, the positive effects that a credit bureau can have through increasing the information publicly available on borrowers' histories has also been widely discussed. One noteworthy paper that focuses on lenders' incentives to voluntarily share information is Pagano and Jappelli (1993). More recently, Brown and Zehnder (2006) construct a laboratory experiment in which they show that introducing a credit bureau into an lending game with anonymous short-term contracts can improve

[6]In this sense his basic model resembles Holmstrom (1982). We believe, however, that a model such as ours, in which incentives are worst at the beginning of an agent's life (rather than at the end) is better-suited as a description of a consumer credit market. It is well known, for example that older borrowers, as well as those with longer credit histories, are less likely to default (see Elul and Subramanian, 2002, and Fair Isaac and Co., 2003).

borrowers' incentives to repay. They also show that this can substitute for the incentive effects of long-term contracts. de Janvry, McIntosh and Zadoulet (2006) study the introduction of a credit bureau in a microfinance market in Guatemala. They first show that the introduction of a credit bureau allows lenders to screen borrowers more effectively, thereby reducing default rates. They identify this as a reduction in adverse selection. They then split their population of borrowers into two; only half of the borrowers are then actually informed as to the existence and workings of the bureau. They find that these informed borrowers are likelier to repay, i.e. that awareness of the bureau leads to reduced moral hazard.

II The Model

Consider an economy made up of a continuum (of unit mass) of risk-neutral *entrepreneurs*, who live forever and discount the future at the rate $\beta \leq 1$. In each period $t = 0, 1, \ldots$ an entrepreneur receives a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure). Output is non-storable, so entrepreneurs must seek external financing in each period. In addition, there is limited liability, so if a project fails in the current period, then the entrepreneur is not required to make payments out of any future income.

We assume that there are two types of entrepreneurs. There is a set of measure $p_0 \in (0, 1)$ of *riskless* agents, whose projects always succeed (i.e., their return is R with probability one),[7] and a set of *risky* agents, with measure $1 - p_0$, for whom the project may fail with some positive probability. The returns on the risky agents' projects are independently and identically distributed among them. The success probability of a risky agent depends on his effort choice. He may choose to exert high effort (h), at a cost $c(h) \equiv c > 0$ (in units of the consumption good), in which case the success probability will be $\pi_h \in (0, 1)$. Alternatively, he may choose to exert low effort. Low effort (l) is costless ($c(l) = 0$), but the success probability under low effort is only $\pi_l \in (0, \pi_h)$.

We assume:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

[7] We discuss the role that this assumption plays in remark 5 below.

In addition, we require the cost of effort c to be sufficiently high, which will imply that entrepreneurs face a nontrivial incentive problem. More specifically, as we will see, this condition implies that high effort cannot be implemented in the absence of reputational incentives (e.g. in a static framework) when the entrepreneur is known for certain to be risky.

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Finally, we introduce one further parameter restriction, requiring that π_h and π_l not be too far apart. This condition will be used only in some parts of the analysis below (and then only to ensure that a stronger notion of equilibrium obtains):

Assumption 3. $\pi_h^2 \leq \pi_l$

In addition to entrepreneurs, there are lenders, who provide external funding to entrepreneurs in the loan market. More specifically, we assume that in each period there are N lenders (where N should be thought as large) who compete among themselves on the terms of the contracts offered to borrowers with the objective of maximizing expected profits. Each lender lives only a single period, and is replaced by a new lender in the following period. Since lenders live only a single period, they cannot write long-term contracts. This is seems consistent with actual practice in U.S. unsecured credit markets, where borrowers often switch between lenders.

A contract is then simply described by the interest rate r at which an entrepreneur is offered one unit of financing at the beginning of a period (if the entrepreneur is not offered financing in this period then we set $r = \emptyset$). If the project succeeds, the entrepreneur makes the required interest payment r to the lender. On the other hand, if the project fails, the entrepreneur is unable to make any payment and we assume that the debt that was incurred is forgiven - or discharged. Since borrowers have no funds to repay lenders other than the proceeds from their project in this period, with no loss of generality r can be taken to lie in $[0, R] \cup \emptyset$.

We assume that an entrepreneur's type, as well as the effort he undertakes, is his private information. The loan market is hence characterized both by the presence of adverse selection and moral hazard. Since under Assumption 1 it is only profitable to lend to a risky agent if he exerts high effort, this creates an incentive problem: a risky entrepreneur may in fact prefer to exert low effort even though the total surplus in that case is lower (indeed negative).

At the same time, in a dynamic framework such as the one we consider, the history of past financing decisions and past outcomes of the projects of an agent may convey some information regarding the agent's type. Since lenders do not live beyond the current period, we assume that there is a *credit bureau* that records this information in every period and makes it available to future lenders.

Let σ_t^i denote the *credit history* of agent $i \in [0, 1]$ at date t, describing for each previous period $\tau < t$ whether the agent's project was funded and if so, whether it succeeded or failed. Hence, denoting by S a success, F a failure, and $\emptyset$ the event where the project is not funded (either because the agent is not offered financing or because he does not accept any offers), σ_t^i is given by a sequence of elements out of $\{S, F, \emptyset\} : \sigma_t^i \in \Sigma_t \equiv \{S, F, \emptyset\}^t$.

We show below that only pooling equilibria can exist in this economy;[8] that is, lenders are unable to separate borrowers by offering a menu of contracts to entrepreneurs with the same credit history. Note, however, that they may optimally choose to differentiate the terms of the contracts offered on the basis of entrepreneurs' credit histories. Hence, without loss of generality we can focus our attention on the case where a lender offers a single contract $r(\sigma_t)$ to borrowers with a given credit history σ_t. We let $\mathcal{C}(\sigma_t)$ denote the set of contracts offered at date t by the N lenders to entrepreneurs with credit history σ_t, and let $\mathbb{C}_t \equiv \cup_{t,\sigma_t \in \Sigma_t} \mathcal{C}(\sigma_t)$ be the set of contracts offered by lenders for any possible history up to date t.

We assume that while lenders present at date t know $\mathbb{C}_t$, i.e., the set of contracts which were *offered* to borrowers in the past, they do not know the particular contracts which were *chosen* by an *individual* borrower. This in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts available is in fact provided by databases such as Comperemedia®.

As discussed earlier, the focus of our paper is the effect of restrictions on the transmission of credit bureau records. We model the *forgetting policy* in this economy as follows: when an entrepreneur's project fails, with probability q the credit bureau ignores the failure and updates the entrepreneur's record as if his project succeeded in that period.[9] That is, S now represents either a success or a failure that is forgotten, and F represents a failure that has not been forgotten. The parameter $q \in [0, 1]$ then describes the forgetting policy in the economy. Note that we take q as being fixed over time, which is in line with existing laws.

We adopt this representation of forgetting to make the analysis simpler,

[8]To be precise, we focus on Markov Perfect Equilibria, and show that these must be pooling.

[9]A similar approach is also taken by Padilla and Pagano (2000).

though it is somewhat different from existing institutions. As discussed above, in the United States a personal bankruptcy filing may be reported for up to 10 years, after which it must be removed from records made available to lenders. We intend to argue however that the effects on borrowers' incentives and access to credit are similar; in particular, that the consequences of higher values of q are analogous to those of allowing for a shorter period until negative information is forgotten.[10]

The timeline of a single period is then as follows. Each entrepreneur must obtain a loan of 1 unit in the market in order to undertake his project. Lenders simultaneously post the rate at which they are willing to lend in this period to an agent with a given credit history, and do this for all possible credit histories at that date. If an entrepreneur is offered financing, and if he chooses to be financed, he undertakes the project (funds lent cannot be diverted to consumption), and if he is risky he also chooses his effort level. The outcome of the project is then realized: if the project succeeds the entrepreneur uses the revenue R of the project to make the required payment r to the lender, while if the project fails the entrepreneur defaults and makes no payment (since his default is forgiven). Note that — purely for convenience — we assume that entrepreneurs repay at the end of the same period in which they borrow.

The credit history of the entrepreneur is then updated by adding a $\emptyset$ to the sequence if the project was not financed and, if it was financed, S if the project succeeded in the period or, with probability q, if it failed, and an F otherwise. This timeline is illustrated in the following figure for the case of high and low effort (when $q = 0$).

Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer, each entrepreneur then freely chooses the best contract among the ones he is offered,[11] and so on for every t.

Since the updated credit history may affect the contracts the agent will get in the future, and hence his future expected utility, and since for a risky agent such history is, at least partly, affected by his current effort choice, this will affect the agent's incentives to choose high versus low effort. In particular, the agent may care for having a good credit history (i.e., a good

[10] This is indeed exactly so for the polar cases of $q = 0$, which implies that a failure is remembered forever, and $q = 1$, which is equivalent to forgetting immediately, i.e., not keeping any record of failures.

[11] We assume therefore that entrepreneurs are unable to commit to any future choice of contract.

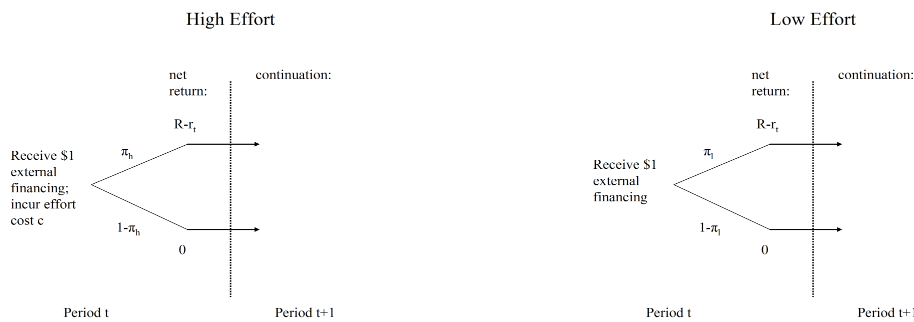

Figure 1: Timeline: $q = 0$

reputation), as this might improve his future funding prospects, and this may strengthen the agent's incentives with respect to the case of a static contracting problem. Under assumption 2, as we will show in what follows, incentives may be sufficiently poor that we need reputational effects to elicit high effort (and as a result financing cannot take place at all nodes).

To summarize, a lender's strategy consists in the choice of the contracts to offer to entrepreneurs at any given date, for any possible credit history. The strategy of an entrepreneur then describes, in every period and for every possible credit history, the choice of the contract among the ones he is offered and, if the entrepreneur is risky, also his choice of effort. To evaluate the expected profit of a loan offered to an entrepreneur with credit history σ_t an important role is played by the lender's belief, $p(\sigma_t)$, that the entrepreneur is a risky type. At the initial date such belief is given by the prior probability p_0; the belief is then updated over time on the basis of the knowledge of the credit history σ_t as well as of the contracts $\mathbb{C}_t$ offered up to such date, and of the entrepreneurs' borrowing and effort strategies. We will often refer to $p(\sigma_t)$ as an entrepreneur's *credit score*.[12]

[12]We will sometimes drop the reference to the borrower's credit history and refer simply to p.

III Equilibrium

A Markov Perfect Equilibrium

In what follows we will focus on *Markov Perfect Equilibria* (MPE) in which players' strategies depend on past events only through *credit scores* — i.e., through the belief p, entertained by lenders, that an entrepreneur with a given credit history is of the safe type. A key appeal of such equilibria is not only that players' strategies are simpler, but also that they resemble actual practice in consumer credit markets, where many lending decisions are conditioned on credit scores, most notably the "FICO score" developed by Fair Isaac and Company. In addition, we will discuss below the differences between MPE and other equilibria and argue that the latter exhibit properties of players' behavior that we consider less plausible.[13]

We can now describe players' strategies more formally for the Markov Perfect Equilibria that we consider. Let $r^n(p) \in [0, R] \cup \emptyset$ denote the strategy of lender n, i.e., the contract offered (or, when $r = \emptyset$, the fact that no contract is offered) to entrepreneurs with credit score p; then let $\mathcal{C}(\cdot)$ denote the collection of the strategies of all lenders.

The strategy of an entrepreneur, whatever his type, consists in the choice, for every credit score p he may have, and given that he is offered a set of contracts $\mathcal{C}'$, whether or not to accept any of the loan contracts offered, and if so, which one to accept. In addition, a risky entrepreneur has to choose the effort level he exerts. We will allow for mixed strategies with regard to effort and hence denote the effort level by $e \in [0, 1]$, where e signifies the probability with which the entrepreneur exerts high effort.[14] Thus $e = 1$ corresponds to a pure strategy of high (h) effort, $e = 0$ to a pure strategy of low (l) effort, and $e = 1/2$ corresponds to mixing between high and low effort with equal probability. The entrepreneur's choices are based on the evaluation of both his immediate payoff, which depends on the level of the interest rate on the contracts presently offered to him and his effort choice, as well as on his anticipation of the contracts he will be offered in the future conditional on the outcome of his project, which in turn depend on the lenders' strategies and on how they update their beliefs concerning the agent's type in light of the outcome of the current project.

In particular, we will establish the existence and analyze the properties

[13]Restricting attention to MPE to rule out implausible equilibria is common in the analysis of reputation games; see Mailath and Samuelson (2001), for example.

[14]This is the only form of mixed strategies that we allow; we demonstrate below that mixing only occurs for at most a single period along the equilibrium path.

of *symmetric, sequential MPE*, where all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategies. Let $p^S(p, \mathcal{C}')$ specify how lenders update their beliefs in case of success (or forgotten failure) of the project of a borrower with credit score p and facing current contracts $\mathcal{C}'$. Analogously, $p^F(p, \mathcal{C}')$ denotes the updated belief in case of a failure (which is not forgotten) and $p^{\emptyset}(p, \mathcal{C}')$ when the entrepreneur is not financed.[15] The updated beliefs will be computed according to Bayes' rule whenever possible; when this is not possible they will be required to be *consistent* in the Sequential Perfect Equilibrium sense.

Observation 1. Since only risky agents can fail, we must have $p^F(p, \mathcal{C}') = 0$ for any p and $\mathcal{C}'$.

Furthermore, when entrepreneurs are not offered any loan, $\mathcal{C}' = \emptyset$, and hence are not financed, it is immediate to verify that beliefs remain unchanged: $p^{\emptyset}(p, \emptyset) = p$ for all p.

We are now ready to write the formal choice problem for the entrepreneurs. Each period they have to choose which loan of the loan contracts they are offered to accept, if any, and in that case their effort level. Let $v^r(p, \mathcal{C}')$ denote the maximal discounted expected utility that a risky entrepreneur with credit score p, facing a set of contracts $\mathcal{C}'$, can obtain, given the lenders' updating rules $p^S(\cdot), p^F(\cdot), p^{\emptyset}(\cdot)$ and their strategies $\mathcal{C}(\cdot)$, determining future offers of contracts (to simplify the notation we do not make the dependence of v^r on these terms explicit). Observe that $v^r(\cdot)$ is recursively defined as the solution to the following problem:

$$v^r(p, \mathcal{C}') =$$

$$\max_{e \in [0,1], r \in \mathcal{C}' \cup \emptyset} \begin{cases} (e\pi_h + (1-e)\pi_l)(R - r) - ec \\ +\beta[e(\pi_h + (1-\pi_h)q) + (1-e)(\pi_l + (1-\pi_l)q)]v^r(p^S, \mathcal{C}(p^S)) \\ +\beta[(e(1-\pi_h) + (1-e)(1-\pi_l)][1-q]v^r(0, \mathcal{C}(0)), \quad \text{if} \quad r \neq \emptyset; \\ \\ \beta v^r(p^{\emptyset}, \mathcal{C}(p^{\emptyset})), \ \text{if } r = \emptyset. \end{cases} \tag{1}$$

Note that in writing this expression we have used the fact that, by observation 1, $p^F(\cdot) = 0$. Let us denote the solution of problem (1) by $e^r(p, \mathcal{C}'), r^r(p, \mathcal{C}')$, which describes the risky entrepreneur's strategy as p and $\mathcal{C}'$ vary.

[15]We will sometimes omit the arguments and write simply $p^S, p^F, p^{\emptyset}$.

Analogously, letting $v^s(p, \mathcal{C}')$ be the maximal discounted expected utility for a safe entrepreneur, we have:

$$v^s(p, \mathcal{C}') = \max_{r \in \mathcal{C}' \cup \emptyset} \begin{cases} R - r + \beta v^s(p^S, \mathcal{C}(p^S)) \text{ if } \ r \neq \emptyset; \\ \beta v^s(p^\emptyset, \mathcal{C}(p^\emptyset)), \text{ if } r = \emptyset. \end{cases} \tag{2}$$

The solution to this problem is denoted by $r^s(p, \mathcal{C}')$; once again this describes the safe entrepreneur's strategy as p and $\mathcal{C}'$ vary.

Since lenders cannot observe the specific contract chosen by an individual borrower in any given period, but only whether or not he was financed, and if so the outcome of his project, we have:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all $p, \mathcal{C}'$ we have $r^j(p, \mathcal{C}') \in \min(\mathcal{C}') \cup \emptyset$, for $j = s, r$.

Next, we determine the expected profits for an arbitrary lender n from a loan with interest rate r offered to a unit mass of entrepreneurs with credit score p, given the entrepreneurs' strategies, $r^s(\cdot), r^r(\cdot)$, and $e^r(\cdot)$, and the contracts $\mathcal{C}^{-n}$ offered by the *other* lenders. If r is strictly lower than any rate offered by the other lenders, and if both types of entrepreneurs choose to accept financing, the expected profits of n (per unit mass of entrepreneurs with this score) are $(p + (1 - p)\pi_h)r$ if the risky entrepreneurs exert high effort and $(p + (1 - p)\pi_l)r$ if they exert low effort (and a convex combination of the two when risky entrepreneurs mix over effort levels). When r is still lower than any rate offered by the other lenders, but only the safe entrepreneurs accept financing, expected profits are given by the first term of the above expressions, pr, whereas if only the risky entrepreneurs accept, they are $(1-p)\pi_h r$ or $(1-p)\pi_l r$ depending on the effort strategy of the risky borrower. In the same situations, if the rate is equal to the lowest one offered by other lenders, i.e., $r = \min \mathcal{C}^{-n}$, the market is equally shared among all lenders offering the minimal rate and hence the above expressions have to be divided by the number of lenders offering the minimal rate. Finally, if lender n offers no loan contract, or a rate above the lowest one offered by the other lenders ($r > \min \mathcal{C}^{-n}$), or if all borrowers reject financing, then expected profits are 0. Thus we have:

$$\Pi(r, p, \mathcal{C}^{-n}, r^S(\cdot), r^r(\cdot), e^r(\cdot)) =$$

$$\begin{cases} \{p + (1-p)\left[e(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e(p, \mathcal{C}^{-n} \cup r))\pi_l\right]\} r / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}), \text{ and } r^S(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset \\ (1-p)\{e(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e(p, \mathcal{C}^{-n} \cup r))\pi_l\} r / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}) \text{ and } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset \\ pr / [1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ \quad \text{if } r \leq \min(\mathcal{C}^{-n}), r^s(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset \\ 0, \text{ if either } r > \min(\mathcal{C}^{-n}), \text{ or } r = \emptyset, \text{ or } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset. \end{cases} \tag{3}$$

Notice that in the above expression (3) for lenders' profits we used observation 2 that entrepreneurs never choose a rate above the lowest interest rate offered in $\mathcal{C}^{-n} \cup r$.

Since a lender lives only a single period, his objective is simply to choose r so as to maximize his expected profits given by (3). Given our focus on symmetric MPE, we can denote the solution simply by $r(p)$.

We are now ready to give a formal definition of a MPE:

Definition 1. A symmetric, sequential Markov Perfect Equilibrium is a collection of strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $p(\cdot)$, such that:

- Lenders maximize profits, given $r^s(\cdot), r^r(\cdot), e^r(\cdot)$: for every p, $r = r(p)$ maximizes (3), when $\mathcal{C}^{-n} = r(p)$;
- Entrepreneurs' strategies are sequentially rational. That is,
  - for all $p, \mathcal{C}'$, $(r^r(p, \mathcal{C}'), e^r(p, \mathcal{C}'))$ solves (1) when $\mathcal{C}(p) = r(p)$.
  - for all $p, \mathcal{C}'$, $r^s(p, \mathcal{C}')$ solves (2) when $\mathcal{C}(p) = r(p)$.
- Beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.

Observe that along the equilibrium path, strategies and beliefs can be written solely as functions of the credit score p, i.e., $r(p), r^r(p), r^S(p), \mathcal{C}(p)$ and $\{p^S(p), p^F(p), p^{\emptyset}(p)\}$. Similarly, entrepreneurs' discounted expected utility can be written as $v^s(p), v^r(p)$.

It will also be useful to have the notation $r_{zp}(p, e)$ to denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p, when risky entrepreneurs exert effort e, and all agents accept financing at this rate. That is,

$$r_{zp}(p, e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \tag{4}$$

B Existence and Characterization of Equilibrium

The following proposition establishes that a Markov Perfect Equilibrium exists, and characterizes its properties. The proof is constructive, and we show in what follows that the equilibrium we construct is the most efficient MPE.

Proposition 1. *Under assumptions 1-3, a (symmetric, sequential) Markov Perfect Equilibrium always exists with the following properties:*

i. *Entrepreneurs never refuse financing and always take the contract with the lowest interest rate offered to them:* $r^s(p, \mathcal{C}') = r^r(p, \mathcal{C}') = \min(\mathcal{C}')$, *whenever* $\mathcal{C}' \neq \emptyset$.

ii. *Lenders never offer financing to entrepreneurs known to be risky with probability 1:* $r(0) = \emptyset$, *and so* $v^r(0) = 0$.

iii. $p^{\emptyset}(p, \mathcal{C}') = p$ *whenever* $\mathcal{C}' \neq \emptyset$. *That is, if a borrower refuses financing, which only happens off-the-equilibrium path, a consistent belief for lenders is that the probability remains unchanged at p. On the other hand, lenders' beliefs after financing and success are always updated via Bayes' rule as follows:*

$$p^S(p, \mathcal{C}') = \frac{p}{p + (1-p)[e^r(p, \mathcal{C}')(\pi_h + (1-\pi_h)q) + (1 - e^r(p, \mathcal{C}'))(\pi_l + (1-\pi_l)q)]},$$

for all $p, \mathcal{C}' \neq \emptyset$.

Furthermore, along the equilibrium path, the value functions $v^r(p)$ *and* $v^s(p)$ *are weakly increasing and the players' strategies are as follows:*

a. *if* $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h - \pi_l}$ *then:*

- *when* $p \geq p_{NF} \equiv \frac{1-\pi_l R}{(1-\pi_l)R}$, *the agent will be financed at the rate* $r(p) = r_{zp}(p, 0)$, *and if risky exerts low effort* ($e^r(p) = 0$).
- *when* $p < p_{NF}$ *the agent is not financed* ($r(p) = \emptyset$).

b. *if* $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h - \pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ *then there exists* $0 < p_l \leq p_m \leq p_h < 1$ *such that:*

- *if* $p \geq p_h$ *then the agent is financed at* $r(p) = r_{zp}(p, 1)$ *and if risky exerts high effort;*

- *if* $p \in [p_m, p_h)$*, risky agents mix over high and low effort with probability* $e^r(p) > 0$*, increasing in* p*, and a loan is offered at the rate* $r(p) = r_{zp}(p, e^r(p))$*;*

- *if* $p \in [p_l, p_m)$ *the entrepreneurs are financed at the rate* $r(p) = r_{zp}(p, 0)$ *and if risky exert low effort;*

- *if* $p < p_l$ *there is no financing (*$r(p) = \emptyset$*).*

c. *if* $\frac{c}{\pi_h - \pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l + (1-\pi_l)q)}$ *then there is financing at the rate* $r(p) = r_{zp}(p, 1)$*, for all* $p > 0$*, and risky agents exert high effort.*

When c is high (region a.), then incentives are weak. As a result the risky entrepreneurs exert low effort whenever they are financed. Nevertheless, financing can still obtain as long as p is not too low, since lenders are able to recoup their losses on lending to the risky agents from the safe entrepreneurs who are also in the pool of agents who have not yet failed, since the latter never default. By contrast, when c is low (region c.) then incentives are strong enough that the risky entrepeneurs exert high effort for all $p > 0$. This makes financing profitable for all $p > 0$. Finally, for intermediate values of c (region b.), high effort is implemented for high values of p. The reason is that when there are sufficiently many safe types in the pool, interest rates (both current and future) are low, which improves incentives. By contrast, for low values of p this subsidy from the safe types to the risky will be insufficient to make high effort incentive compatible; when p is particularly low this will also imply that financing is unprofitable for lenders.

Figure 2 illustrates the equilibrium outcomes as p varies for the case $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h - \pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$. Note that the low-effort and mixing regions may be empty, while the high-effort and no-financing regions must always exist for this case. That is, we must have $0 < p_l \leq p_m \leq p_h < 1$.

We first establish property ii. of Proposition 1, that entrepreneurs who are known to be risky are never financed, and show that this is actually a general property of Markov equilibria. The basic intuition is that once an entrepreneur is known to be risky, his continuation utility in a Markov Perfect Equilibrium must be the same regardless of the outcome of his project, which makes it impossible to provide him with incentives to exert high effort.

Lemma 1. *Under assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when* $p = 0$*: i.e.,* $r(0) = \emptyset$ *and hence* $v^r(0) = 0$*.*

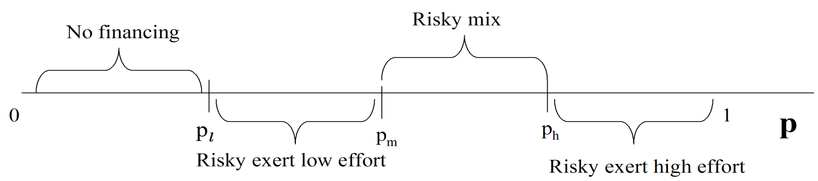

Figure 2: MPE when $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$

The proof of this result, and all subsequent ones, can be found in the Appendix.

Combining the lemma with observation 1 we get:

Corollary 1. *If an agent fails and this failure is not forgotten, he can no longer obtain financing in any MPE.*

Remark 1. As this corollary makes clear, when an entrepreneur fails in our set-up he is identified as risky and in that case can no longer obtain financing (since he would always exert low effort). In practice, although it is indeed the case that those borrowers with a bankruptcy in their credit record do find it considerably more difficult to obtain credit, both Staten (1993) and Musto (2004) point out that some post-bankruptcy credit is in fact available.

The assumption that only risky agents can fail obviously simplifies the analysis. Without this assumption, the posterior following a failure would be above $p = 0$ and so could result in continued financing. This is discussed further below, where we provide an example that has this property that failure can result in continued financing; we show that the effect of forgetting is nevertheless qualitatively similar to that obtained here.

Lemma 1 then implies that all Markov Perfect Equilibria must be a pooling equilibrium. The reason is that risky entrepreneurs will not be able to obtain financing if separated. Recall that we referred to this property earlier, when specifying the equilibrium strategies.

Lemma 2. *Any (symmetric, sequential) Markov Perfect Equilibrium must be a pooling equilibrium.*

The rest of the proof of Proposition 1 (in the Appendix) establishes the remaining properties (i. and iii.) of the MPE, and the specific characteristics of the equilibrium we construct for the parameter regions a.,b., and c.

Finally, we establish that the equilibrium characterized in Proposition 1 is also the MPE that maximizes welfare. The welfare criterion we consider in this paper, given the transferability of agents' utility implied by risk-neutrality, is the total surplus generated by entrepreneurs' projects that are financed, or, equivalently, the sum of the discounted expected utilities of all agents in the economy, including lenders.

Proposition 2. *The equilibrium constructed in Proposition 1 is the most efficient MPE.*

To prove the result, we first show that the construction of the equilibrium in Proposition 1 guarantees that the equilibrium implements the highest possible effort at any p. To be able to conclude that the total surplus is maximized, it remains then to show that any other equilibrium that does not offer financing at a node where financing is granted in the MPE we construct will result in lower level of total surplus.

Remark 2. To properly understand the differences between the MPE as defined and other Perfect Bayesian equilibria of the game considered, it is important to observe that the Markov property of players' strategies only binds at nodes where the entrepreneur is not financed. This is because when the agent is financed the updated belief in case of success will always be higher than the prior one. Hence p never hits the same value twice, so that on this path the Markov restriction is not binding. Where it is binding is at the nodes where the agent is denied financing, i.e., when p equals zero after a failure (or in cases a. and b. of the Proposition, when p_0 is sufficiently low). There the Markov property prescribes that lenders' behavior has to remain the same in the following period as well, since p remains unchanged, and so on, so that entrepreneurs are always denied financing.

By contrast, at non-Markov equilibria, lenders' strategies may entail financing when p first hits 0, i.e. after a first failure, as well as at any successor node as long as the agent succeeds, and denial of financing after a second failure and forever after. The threat of exclusion after two failures could be enough to induce high effort and hence to make financing profitable for lenders. The fact that these strategies imply that the entrepreneur is treated differently at nodes with $p = 0$ requires some coordination among lenders. Such an equilibrium thus seems somewhat fragile, being open to the possibility of breakdowns in coordination, or to renegotiation, which is not the case for the MPE we consider. Moreover, while such non-Markov equilibria have some similarities with the MPE with forgetting, in that a risky entrepreneur who fails may obtain additional periods of financing, they only exist for a limited set of parameter values — when c is low and lies in region c. of Proposition 1, so that incentives are sufficiently strong. By contrast, with forgetting financing with high effort obtains also for intermediate values of c (lying in region b.) This is because forgetting a failure in our setup entails pooling the risky types with the safe entrepreneurs anew, so that financing is granted at a lower interest rate than if their type had been revealed, and this improves their incentives (see also Proposition 4 below).

Remark 3. It is also useful to compare the MPE we consider with the equilibria we would obtain with long-term contracts. When entrepreneurs can write long-term contracts with lenders, the non-negativity condition for lenders' expected profits need no longer hold period-by-period but only intertemporally. This feature will be used in the design of the contracts which obtain in equilibrium, so as to minimize the cross-subsidy from safe to risky entrepreneurs, by postponing any net revenue from the projects financed as far into the future as possible. This will lead to rather extreme and unrealistic contracts, where interest payments are equal to R in the initial periods, and subsequently zero. The benefit for safe entrepreneurs is that fewer risky entrepreneurs will survive to share in the future surplus. We conjecture that such an equilibrium, while preferred by the safe entrepreneurs, will be less efficient (total surplus will be lower) than that considered in Proposition 1 because of the negative effect that postponing payments has on incentives.

If the Markov property is also relaxed, a separating equilibrium may obtain, since postponing payments can make the safe entrepreneurs' contract sufficiently unattractive to risky entrepreneurs. This requires that the risky entrepreneurs be able to obtain some financing if separated; as discussed above, this can only occur in region c.

IV Optimal Forgetting

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy. That is when, in the equilibrium characterized in Proposition 1, $q > 0$ dominates $q = 0$. The welfare criterion we use is again the total surplus.

What are the effects of the forgetting policy on the equilibrium properties? When we are in regions a. and c. of Proposition 1, q has no effect on the surplus generated in equilibrium by financing to safe entrepreneurs. This follows because, within each region, the set of nodes for which the safe agents are financed does not depend on q: in region c. there is financing for all $p > 0$, and in region a. there is financing for $p > p_{\text{NF}}$, where recall that p_{NF} does not depend on q. So in these cases the only effect of q is on the surplus generated by financing to risky entrepreneurs.

In this regard, a first implication of raising q is that the probability that a risky entrepreneur will be excluded from financing decreases: the failure of his project implies in fact exclusion only with probability $1 - q$. Recall that under Assumption 1 an extra period of financing to a risky entrepreneur makes a strictly positive contribution to the social surplus, given by $G \equiv \pi_h R - 1 - c > 0$, when he exerts high effort and a strictly negative one, $B \equiv \pi_l R - 1 < 0$, when he exerts low effort.

But the increase in q has another effect which needs to be taken into account: since exclusion after a project's failure is less likely, the incentives to exert high effort will be weaker.

In region a. (in which low effort is always exerted when financing takes place), the weakening of incentives manifests itself in the fact that the lower bound of this region, $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, is decreasing in q, so that this region expands when q is increased. Analogously, the upper bound of parameter region c. (where high effort is always exerted), $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, also decreases in q, so that this region becomes smaller when q is bigger.

Let $q(p_0)$ denote the welfare maximizing level of q (which clearly depends on the proportion p_0 of risky types in the population, as the equilibrium depends on it). From the above discussion the properties of the optimal forgetting policy when the parameters of the economy are in region a. or c. of Proposition 1 immediately follow:

Proposition 3. *The welfare maximizing forgetting policy respectively for high and low values of c is as follows:*

1. *If $\frac{c}{\pi_h-\pi_l} \geq \frac{R-1}{1-\beta\pi_l}$, no forgetting is optimal for all p_0: $q(p_0) = 0$.*

2. *If $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, for any $p_0 > 0$ some degree of forgetting is optimal: $q(p_0) > 0$.*

Thus in region c., when incentives are strong and high effort is implemented everywhere, some positive level of forgetting is optimal.

We now turn our attention to region b., the intermediate values of c, where the level of effort varies along the equilibrium path (switching at some point from low to high). The weakening of incentives due to forgetting now manifests itself not only in the change of the boundaries of this region, which again altogether shifts to the left as q increases, but also in the change of the points in the equilibrium paths where the switch from low to high effort takes place. Such switching points are identified by the levels of $p_h(q), p_m(q)$, and $p_l(q)$ introduced in Proposition 1, i.e. of lenders' posterior beliefs at which in equilibrium risky entrepreneurs, respectively, switch to high effort, to mixing or start getting financed, where their dependence on q is now emphasized. In what follows we will restrict attention to prior probabilities $p_0 > p_{\mathrm{NF}}$, in which case there is financing in the initial period regardless of the level of q; this will allow us to ignore any possible effect of q on $p_l(q)$.

Since, as we said, an extra period of financing with high effort makes a positive contribution to the social surplus, while one with low effort makes a negative contribution, an evaluation of the overall welfare effect of increasing q is now more complicated as we have to determine the expected number of extra periods of financing with low and with high effort. Notice first that when $p_0 > p_h(0)$ no switching takes place, as high effort is always exerted by a risky entrepreneur when financed. Hence an analogous argument to that used to prove case 2. of Proposition 3 establishes that the socially optimal level of q is above 0 in this case.

On the other hand, when $p_0 \leq p_h(0)$ raising q above 0 does not necessarily increase welfare: we face in fact a tradeoff between the positive effect, when high effort is exerted (i.e., when along an equilibrium path $p > p_h(0)$), of the lower probability of exclusion we have with $q > 0$ and its negative effect when low effort is exerted (when $p < p_h(0)$). There are two facets to this negative effect. First, as discussed above, when $q > 0$ a failure results in exclusion with only probability $1 - q$, whereas with probability q the agent is financed again in the future, in which case he may exert low effort again. In addition, raising q will "slow down the updating". That is, $p^S(p)$ will be closer to p, and thus a longer string of successes will be required until the risky entrepreneurs exert high effort. We will show in what follows that the first, positive, effect prevails over the second, negative, one when (i) agents are sufficiently patient (β close to 1), (ii) $|B|$ is sufficiently small relative to G, and (iii) p_0 is sufficiently close to $p_h(0)$, since the positive effect follows the negative one on the equilibrium path.

In addition, we must also take into account that raising q may increase

$p_h(q)$ as well, since the fact that failures are less costly can weaken incentives.[16] When β is close to 1, however, we are able to show that $p_h(q)$ does not grow too much, because the positive effect of raising q on the continuation utility in case of success is larger, thereby mitigating the negative effect on incentives from the weaker punishment after failure that we have with $q > 0$.

Proposition 4. *For intermediate values of c,* $\frac{R-1/\pi_h}{1-\beta\pi_l} \leq \frac{c}{\pi_h-\pi_l} < \frac{R-1}{1-\beta\pi_l}$, *the optimal policy might also exhibit forgetting. More precisely:*

1. *If* $p_0 > p_h(0)$, *welfare is always maximized at* $q(p_0) > 0$.
2. *If* $p_0 \in [p_{NF}, p_h(0)]$ *and* $-\frac{B}{G} < \frac{p_0(1-p_h(0))(1-\pi_l)}{p_h(0)((1-\pi_h)(1+(1-p_0)\pi_h))+\pi_h^2-p_0(1-\pi_h+\pi_h^2)}$, *then for* β *sufficiently close to 1 we also have* $q(p_0) > 0$.

While the condition in case 2. is stated in terms of $p_h(0)$, which is an endogenous variable, it is possible to show that it is not vacuous (this is also evident from the examples in the next section).[17]

Remark 4. While the above results demonstrate that it is possible to achieve an improvement in ex-ante welfare by forgetting past failures, it is useful to distinguish the impact of forgetting across the two types of entrepreneurs. It is easy to see that — if forgetting leads to an improvement in social welfare — the risky entrepreneurs must gain, since the improvement arises precisely because rather than being excluded from financing after failing, with some probability they are permitted to re-enter the pool of agents who receive financing. By contrast, forgetting generally hurts the safe types, since it slows down the updating, and the lower is p, the higher the interest rate paid. The only way in which forgetting might possibly benefit the safe types is if it were to decrease the high-effort cutoff $p_h(q)$, since the interest rate will be lower when the risky entrepreneurs exert high effort. We will see that this is *not* the case for the examples presented in section V below; so in those cases forgetting, while socially optimal, hurts the safe types.

Remark 5. As we discussed above, the social benefit of forgetting failures arises from the additional periods of financing under high effort which it permits. In light of this, we can also understand the importance of our

[16] This, however, may not always be the case, since a higher value of q also increases the continuation utility upon success.

[17] In particular, let $\pi_l \to 1/R$, so that $B \to 0$. If we hold c and R fixed, then it is not hard to show that $p_h(0)$ will be bounded away from 0, so that the condition will be satisfied.

assumption that the risky entrepreneur can fail even when he exerts high effort, i.e., that $\pi_h < 1$. When this is not the case and we have $\pi_h = 1$ (as, for example, in Diamond, 1989) then high effort ensures success, and there is no benefit from forgetting a failure, since such failures only result from low effort.

Discussion — Empirical Evidence and Policy Implications

We now discuss the empirical and policy implications of our results. We begin with an appraisal of the policy debate surrounding the adoption of the FCRA in the light of our theoretical findings. We subsequently discuss Musto (2004)'s empirical results. Finally, we point out that forgetting cannot be the outcome of the choice of individual lenders, but only of a regulatory intervention by the government.

Our model captures many of the key arguments made in that policy debate, which we summarized in the Introduction. Notice first that the main argument put forward in favor of forgetting — that it allows agents to obtain a true fresh start and hence to continue being productive members of society — also applies to our model, where the main positive effect on welfare of forgetting is that it gives risky entrepreneurs who fail access to new financing, and this increases aggregate surplus if they exert high effort.[18] On the other hand, if they exert low effort, then this additional financing will lead to a decrease in surplus; moreover, forgetting also weakens the risky entrepreneurs' incentives, and this may cause an additional decrease in surplus. These capture two of the arguments that were made against forgetting — that it increases the chance of costly fraud or other crimes by making it harder to identify bad risks, and that it hurts incentives. Our model allows us to identify when the first, positive effect is present, and, furthermore, to analyze the tradeoff between the positive and the negative effects so as to determine the conditions under which forgetting is socially optimal.

Another argument that was advanced against forgetting is that it might lead to a tightening of lending standards. This is also captured by our model. By making incentives worse, the introduction of forgetting can shift the equilibrium from region c., for which there is financing for all prior beliefs $p_0 > 0$, to region b., where agents are only financed when p_0 is sufficiently high. Just as suggested in the policy debate, the cohorts who are thus ex-

[18] In addition, two other arguments were also made in favor of forgetting — that old information may be less relevant, and limited storage space — which do not have a role in our model.

cluded from financing by such a policy are precisely those with a higher percentage of risky types (also see example 3. in the next section). Regarding the remaining argument made against forgetting — that it forces good agents to subsidize bad ones — observe that in our model forgetting indeed increases the subsidy from the safe to the risky types, by extending the time it takes for lenders to learn the type of an entrepreneur. However, this has no adverse effect on welfare and may actually increase it, as the subsidy improves the risky entrepreneurs' incentives while the surplus generated by the safe agents' projects is unaffected, because only the risky types face an incentive problem in our model.[19]

We now turn to the question of whether our results are consistent with the empirical evidence in Musto (2004), and whether we concur with his assessment that removing this flag is suboptimal. Recall that Musto (2004) finds that those who receive credit after their default is forgotten are likelier to default in the future and that their credit quality (as measured by their FICO score) declines over time. These are also implications of our model. Only the risky entrepreneurs ever default; when they are reinserted into the pool of entrepreneurs who are financed they are always more likely than the average to default in the future, regardless of the effort they exert. Moreover, the average posterior for those agents who have been reinserted into the pool eventually converges to zero, since in each future period they are at risk of experiencing a failure that is not forgotten. Nevertheless, Propositions 3 and 4 show that forgetting can still be optimal. The reason is that while these agents are indeed riskier than average, their projects can still generate positive surplus when they are pooled anew and hence financed, which would not be the case were they to be separated and excluded from financing.

Finally, while we have shown that forgetting past defaults can be welfare improving, this would never arise in equilibrium as the outcome of the choice of lenders. As shown in Lemma 1, there cannot exist any Markov Perfect Equilibrium in which agents who are known to be risky (and hence for whom the belief is $p = 0$, i.e. after a failure) obtain financing. Thus the introduction of forgetting can only be the result of a regulatory intervention by the government on the information disclosure rules of the credit bureau.

[19] On the other hand, if both types were subject to moral hazard, this effect might lead to a reduction in social welfare.

V Examples

In this section we present a few examples to illustrate the results of the previous sections. Let $R = 3, \pi_h = 0.5$, and $\pi_l = 0.32$. With regard to the remaining parameters, c, β and p_0, we consider some alternative specifications, which allow us to obtain the different types of equilibria described in Proposition 1. Note that for assumptions 1 and 2 to be satisfied, the effort cost c must lie in the interval $(0.18, 0.5)$.

1. First suppose that $c = 0.48$ and $\beta = 0.75$; these values fall into region a. of Proposition 1, for which at an MPE the risky entrepreneurs exert low effort whenever financed (regardless of q). So from Proposition 3 the optimal level of forgetting is given by $q(p_0) = 0$ for all levels of p_0. The reason is, first, that the level of q does not affect the region in which the agents receive financing, which is always $p \geq p_{\mathrm{NF}} = 0.0196$. An increase in q merely leads to more rounds of financing with low effort, and thus to a reduction in total surplus. In figure 3 we plot the values of the total surplus $\mathcal{W}(q, p_0)$ as a function of the prior belief p_0, for different levels of the forgetting policy q. Observe that surplus is decreasing in q, and also that $\mathcal{W}(q, p_0) = 0$ for $p_0 < p_{\mathrm{NF}} = 0.0196$.

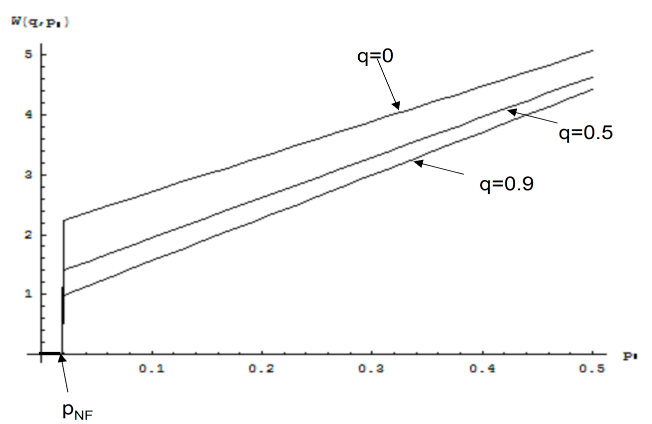

Figure 3: Region a.: $q(p_0) = 0$

2. Let $c = 0.4$ and $\beta = 0.975$. For these values we are in region b. of Proposition 1, for which high effort is implemented when $p \geq p_h(q)$. The threshold $p_h(0)$ above which high effort is exerted when $q = 0$ can be computed from equation (9) in the Appendix, which yields: $p_h(0) = 0.241$.

   When $p_0 > p_h(0) = 0.241$, from Proposition 4 we know that $q(p_0) > 0$ is optimal, because forgetting failures increases the rounds of financing to risky entrepreneurs and in these new rounds they always exert high effort. On the other hand, when $p_0 \in [p_{\rm NF}, p_h(0)) = [0.0196, 0.241)$ this may not necessarily be the case, because for $p < p_h(0)$ low effort is exerted. However, for the parameters in this example B/G satisfies the condition stated in 2. of Proposition 4 whenever $p_0 > 0.205$. Thus some degree of forgetting will be optimal for β sufficiently close to 1; we will verify that this is indeed the case, for instance, when $\beta = 0.975$. The reason is that for these parameters the increase in surplus $G = \pi_h R - 1 - c = 0.1$ from a project undertaken with high effort is high, relative to the decrease in surplus $B = -0.04$ from a project undertaken with low effort and so, for agents who are sufficiently patient the additional periods of high effort provided by forgetting outweigh the cost of the extra periods of low effort at the start of the game.

   For example, consider $p_0 = 0.206$. When $q = 0$, we have $p^S(p, 0) = 0.448$, and so low effort is exerted for the first round of financing along the equilibrium path, and high effort forever after, as long as the projects succeed. [20] However, when $q > 0$, more rounds of financing with low effort may be needed before risky entrepreneurs begin to exert high effort, both because the updating is slower and because $p_h(q)$ is higher. For example, with $q = 0.735$ three periods of financing with low effort followed by success of the project are needed until the posterior exceeds $p_h(0.735) = 0.322$. We now compare welfare levels for different specification of the forgetting policy. In figure 4 we plot the value of the total surplus $\mathcal{W}(q, 0.206)$ as a function of q, when $p_0 = 0.206$. From this figure one can see that the optimum obtains at $q(0.206) = 0.77$, in which case $\mathcal{W}(0.77, 0.206) = 16.648$.

   We also plot in figure 5 the optimal level of the forgetting policy[21] as

[20] The risky entrepreneurs do not use a mixed effort-strategy along the equilibrium path for any of the values of p_0 any q considered in this example.

[21] We discretize the domains of p_0 and q. For each point in the grid we compute $p_h(q)$ and then the welfare $\mathcal{W}(q, p_0)$. We assign $q(p_0)$ to be the value of q that maximizes this surplus, given p_0.

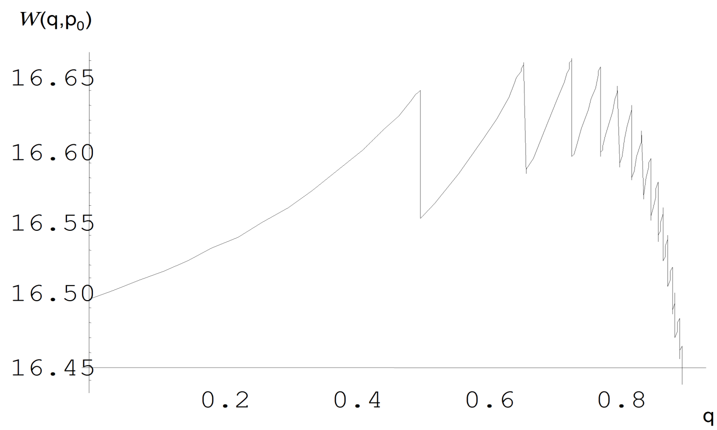

Figure 4: Region b.: social welfare as a function of q ($p_0 = 0.206$)

we vary the prior probability p_0.[22]

3. Consider next $c = 0.26$ and $\beta = 0.975$. We are now in region c. of Proposition 1, for which high effort is exerted for all $p > 0$, as long as $q \leq 0.359$ (i.e., as long as q is sufficiently low that we remain in region c.), forgetting provides additional opportunities for projects to be undertaken with high effort, and so is clearly efficient. Hence, as we can see in figure 6, we have $\mathcal{W}(0.359, p) > \mathcal{W}(0, p)$ for all $p > 0$.

   For higher values of q we move into region b.; it is then no longer the case that the risky entrepreneurs exert high effort for all p.[23] As long as $p_0 > p_h(q)$, however, high effort is still undertaken on all projects which are financed and so raising q continues to improve welfare, as it increases the rounds of financing to risky agents. For example, with $q = 0.975$ this is the case for all $p_0 > p_h(0.975) = 0.1139$.

   By contrast, for very low values of p_0 there may be no financing when we raise q. The reason is that there is no feasible interest rate which

[22] Although the condition in 2. of Proposition 4 is violated for $p_0 \leq 0.205$, we can nevertheless still have $q(p_0) > 0$, since the condition is sufficient but not necessary.

[23] For the values of c and β under consideration we are never in region a., no matter how high q.

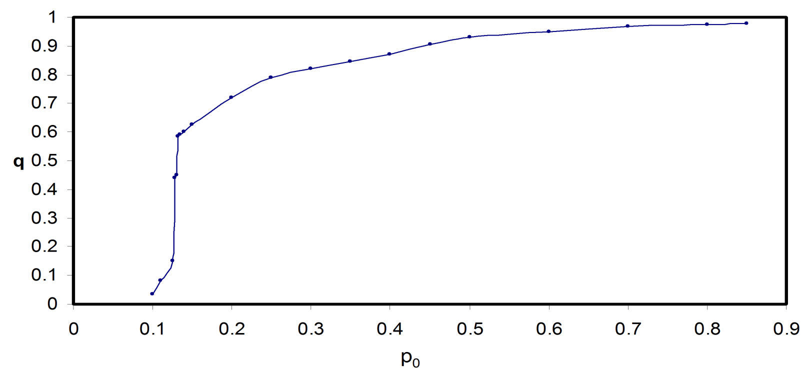

Figure 5: Region b.: welfare-maximizing value of q

would allow lenders to break even for these values of p_0; the lenders make losses on the risky entrepeneurs because they exert low effort, and there are too few safe entrepreneurs from which to recoup these losses. Thus raising q too much can lead to a tightening in lending standards, as discussed in the previous section. For example, when $q = 0.975$, then there is no financing for $p_0 < p_l(0.975) = p_{\mathrm{NF}} = 0.0196$; clearly for these values of p_0 a lower value of q must be optimal.

Finally, for $p_0 \in (p_{\mathrm{NF}}, p_h(q)]$ we face the same tradeoff discussed in example 2. above. A higher q leads to more rounds of financing where both low and high effort are exerted. When p_0 is sufficiently high, the time spent in the low effort region will be relatively short, and thus increasing the level q of forgetting above 0.359 may still increase surplus. As we see in figure 6, when $p_0 > 0.066$ we have $\mathcal{W}(0.975, p_0) > \mathcal{W}(0.359, p_0)$. On the other hand, when p_0 is low the cost of additional rounds of financing with low effort dominates, in which case welfare is higher for lower values of q.

4. Finally, consider $\beta = 0.8$, $c = 0.48$ and a slightly lower value for $\pi_l = 0.3$. While these parameters are in region b., as in example 2 above, the contribution G to total surplus of a project undertaken

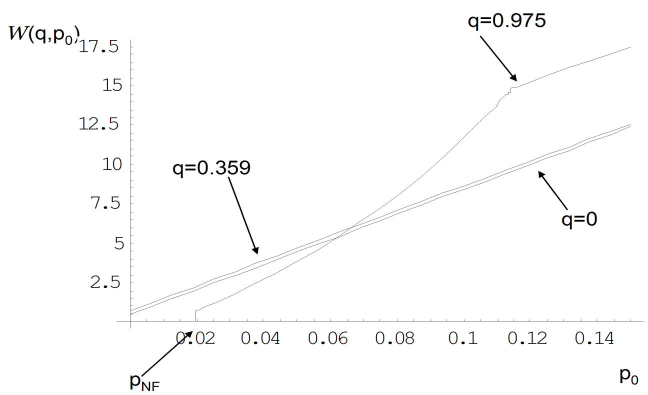

Figure 6: Region c.: $q(p_0) \geq 0.359$ for all p_0

with high effort is now much lower and agents are less patient. As a consequence, the condition stated in 2. of Proposition 4 is now violated for all $p_0 < p_h(0) = 0.628659$, and we find that, for p_0 sufficiently low, welfare is decreasing in q as the cost of a less frequent exclusion in the low effort region dominates the benefit in the high effort region. This is illustrated in figure 7 for the case $p_0 = 0.2$.

VI Extension — Both Types can Fail

We extend the model to allow both projects of risky and safe types to fail, with some positive probability. In this case an agent who fails can no longer be identified for certain as being a risky type and we will show that, as a consequence, he may be able to obtain additional periods of financing even without forgetting. We present an example where we show that this is indeed the case; in other respects, the features of the equilibria are similar to the ones found above and, moreover, we find that some amount of forgetting still increases welfare.

Let $\pi \in (\pi_h, 1]$ denote the probability that the project of a safe entrepreneur fails. Consider the following parameter values: $R = 3, \pi_h =$

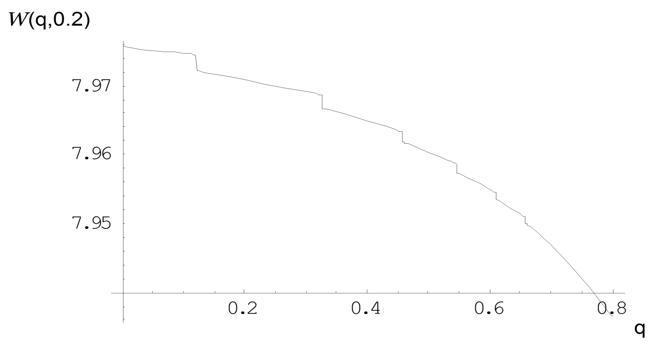

Figure 7: Region b.: condition in Proposition 4 violated ($p_0 = 0.2$)

$0.5, \pi_l = 0.32, \beta = 0.975, c = 0.35$. When $\pi = 1$ (the projects of safe types always succeed) these parameters fall in region b. of Proposition 1, where (for $q = 0$) high effort is exerted for all $p \geq p_h(0) = 0.113$, and agents are financed for all $p \geq p_{\mathrm{NF}} = 0.0196$. The situation is thus analogous to the one of Example 2 in the previous section; by similar computations we obtain that, when the prior belief is $p_0 = 0.10$, the optimal forgetting policy is $q = 0.80$.

Next suppose projects of safe entrepreneurs only succeed with probability $\pi = 0.99$. We find that equilibrium strategies exhibit, in most respects, analogous properties to those in Proposition 1 (i.e. when $\pi = 1$). To construct a Markov Perfect Equilibrium, we must however follow a different procedure which relies on numerical methods. We discretize the domain of p and, for each pair of candidates values for $p_l \geq p_{\mathrm{NF}}$ and $p_h < 1$, we compute the value function for the risky entrepreneurs, using value function iteration. We then determine whether these values are indeed associated with an equilibrium by verifying whether any possible deviation is profitable. For the borrowers, we must show that high effort is incentive compatible in the region $[p_h, 1)$, and that low effort is incentive compatible in $[p_l, p_h)$. For the lenders, it suffices to show that there are no profitable deviations in the regions where low effort is exerted or in that where no financing is granted, since deviating when high effort is exerted cannot be profitable for a lender. In particular, as in the body of the paper, we must associate a consistent belief with any deviation, and show that this belief makes the deviation unprofitable. For all the admissible values of p_h and p_l we found in this way we select then the lowest ones.

When $q = 0$ we find an MPE where high effort is exerted as long as $p \geq p_h(0) = 0.1065$, entrepreneurs are financed for $p \geq p_l = p_{\text{NF}} = 0.0199$. So long as $p < 0.58$, we have $p^F(p) < p_l$ and so a single failure still results in exclusion; however, for higher values of p an agent *will* be able to obtain financing following a failure. Comparing these values with those found for the case $\pi = 1$, we see that the region of prior beliefs for which high effort is exerted is larger, thus the fact that an entrepreneur gets funded also after a failure when p is sufficiently high appears to strengthen the incentives of risky entrepreneurs to exert high effort. The fact that the failure of a project does not necessarily lead to exclusion has two effects on incentives. First, it weakens them when p is high since in that case the punishment for failure is less harsh. In addition, however, the fact that the agent may be financed following a failure in the future raises his continuation utility upon success no matter what is the current level of p, which has a positive impact on incentives. For relatively low values of p, which are the ones where the threshold $p_h(0)$ of the high effort region lies, the latter is the relevant effect.

Along the same lines, we also construct MPE for other, positive values of q. We then compute the surplus function $\mathcal{W}(q, p_0)$. In figure 8 we have plotted the improvement in total surplus (relative to its level when $q = 0$) for various values of q: we see that when $p_0 = 0.1$ total surplus is maximal when $q = 0.80$.

An interesting feature of this extension is that forgetting may now also increase the surplus generated by the projects of safe entrepreneurs who are financed. Recall that, when $\pi = 1$ this surplus was either unaffected or decreased by the introduction of forgetting. On the other hand now, since safe entrepreneurs are also at risk of failing and hence of being excluded, forgetting may benefit them by increase the likelihood that their projects will be financed in the future.

VII Conclusion

We have analyzed a model in which entrepreneurs must repeatedly seek external funds in the market to finance a sequence of risky projects under

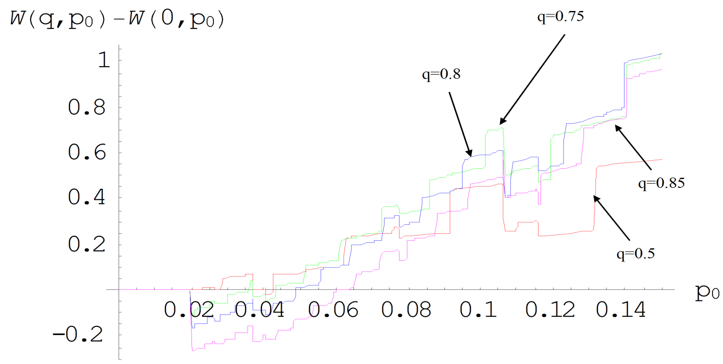

Figure 8: Both types can fail: forgetting still optimal

conditions of both adverse selection and moral hazard. We are interested in determining whether the introduction of some degree of "forgetting", i.e. of some restriction on the information available to lenders on a borrower's past defaults, may be welfare improving in this economy. Forgetting a default makes incentives worse, ex-ante, because it reduces the punishment for failure. However, following a default it is generally good to forget, because it makes the separation between safer and riskier types more difficult, and pooling the two types of agents reduces the cost of borrowing for the riskier ones, which makes exerting high effort to preserve their (undeservedly good) reputation more attractive. The determination of the optimal level of forgetting trades off these effects.

Our key result is that if agents are sufficiently patient, low effort is not too inefficient, the cost of high effort is not too high and the fraction of riskier types in the population is not too large, the introduction of an appropriate level of forgetting — that is, of some limitation on lenders's access to the information on borrowers' past credit history — is welfare improving. We also show that the introduction of forgetting must be the outcome of some regulatory intervention — no lender would willingly agree to forget or to ignore the information which is available to him.

As noted in the Introduction, there are some cross-country differences in the laws governing the memory of the credit reporting system; in general, European countries tend to allow defaults to be forgotten more quickly. It would be interesting to study how such differences are related to differences in the economic environments in such countries, in line with our findings on the relation between the optimal level of forgetting and the features of the underlying economy.

A stark feature of our model is the fact that in equilibrium a borrower's default leads to his permanent exclusion from any future loan. This is due to our assumption that only risky agents can fail. If also the safer agents could default, exclusion might no longer follow after the first failure, although experiencing sufficiently many failures would eventually preclude further financing. That case is less tractable. However we conjecture, also on the basis of the analysis of some examples (see section VI) that the qualitative nature of our results would not be that different — and that under similar conditions forgetting would continue to be an optimal policy.

VIII Appendix — Proofs

A Lemma 1 — No financing when risky

If $p = 0$, since only risky agents fail, we must have $p^S(p, C'^F(p, C') = 0$ whatever C', i.e., the agent will be known to be risky in the future as well.

Furthermore, under assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., if the interest rate r which is offered must be such that

$$\pi_h(R-r) - c + (\pi_h + (1-\pi_h)q)\beta v^r(p^S(p)) + (1-\pi_h)(1-q)\beta v^r(p^F(p)) \geq$$

$$\pi_l(R-r) + (\pi_l + (1-\pi_l)q)\beta v^r(p^S(p)) + (1-\pi_l)(1-q)\beta v^r(p^F(p)),$$

or

$$\frac{c}{\pi_h - \pi_l} \leq R - r + \beta(1-q)[v^r(p^S(p)) - v^r(p^F(p))]. \tag{5}$$

But since, as argued, when $p = 0$ we have $p^S(p) = p^F(p) = 0$, given the Markov property of the equilibrium we have $v^r(p^S(p)) = v^r(p^F(p))$, and so

(5) reduces to the static incentive compatibility constraint:

$$c \leq (\pi_h - \pi_l)\,[R - r]$$

By assumption 2, this can only be satisfied if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$.■

B Lemma 2 — All MPE are pooling

Suppose this is not the case; consider a candidate separating equilibrium. Let r^s denote the contracts offered to the safe types and r^r those offered to the risky in such an equilibrium. From lemma 1 we know that in a separating MPE the risky types cannot be financed, i.e. we must have $r^r = \emptyset$ for all nodes along the equilibrium path, and so they receive $v^r = 0$. Hence for the risky entrepreneurs not to pretend to be safe, we must have either $r^s = R$ or $r^s = \emptyset$ in every node. But if $r^s = \emptyset$ the equilibrium would in fact be pooling, contrary to the stated claim. We now argue that $r^s = R$ cannot be an equilibrium strategy for lenders, because each lender would have an incentive to undercut and offer $R - \epsilon$.

Consider in particular the second period of the game (i.e. the node following a success in the initial period). In this period a lender can deviate and offer $R - \epsilon$ (for ϵ small) to the safe entrepreneurs (only); such a deviation would clearly be profitable, thus overturning the proposed equilibrium. Note that this offer can be made to the safe agents alone because the credit history of a safe agent differs from that of a risky one by virtue of the fact that only the safe agents are financed in the initial period in the proposed equilibrium.■

C Proposition 1 — Characterization of the Equilibrium

To complete the proof of Proposition 1, we establish the remaining properties of the MPE, i. and iii., and the specific features of this equilibrium for the parameter regions a., b., and c.

We begin by verifying property iii. of Proposition 1. The second part of Property iii., that lenders' use Bayes' Rule to update their beliefs when an entrepreneur does not refuse financing, is an immediate consequence of the specification of the equilibrium strategies. When a borrower refuses financing, however, then Bayes' Rule cannot be applied. The first part of Property iii. requires us to show that a consistent belief for lenders is to keep their beliefs unchanged if an entrepreneur refuses financing. To see this, simply let both safe and risky borrowers refuse financing at some node with probability $\varepsilon > 0$, and let $\varepsilon \to 0$. Consistency of the above belief can then be readily verified using Bayes' Rule.

Given property iii., we have the following result, which will be used in the proof to verify property i. in each region:

Lemma 3. *When property iii. of Proposition 1 holds, as long as $v^s(p)$ and $v^r(p)$ are weakly increasing then it is never optimal for an entrepreneur to refuse financing.*

Proof. Consider first a safe entrepreneur with credit score p.

Let $\mathcal{C}' \neq \emptyset$ denote the set of contracts offered, either on or off-the-equilibrium path, and let r' be the lowest rate in this set. Recall that $r' \in [0, R]$ and $p^S(p, \mathcal{C}')$ describes lenders' beliefs when the project is financed and the agent succeeds. From the second part of property iii., these beliefs are such that $p^S(p, \mathcal{C}') > p$.

So if the entrepreneur accepts r' his expected discounted utility will be $R - r' + \beta v^s(p^S(p, \mathcal{C}'))$. Conversely, if he deviates and instead refuses financing, his posterior is unchanged and so his expected discounted utility will be $\beta v^s(p)$ from property iii. By the weak monotonicity of $v^s(\cdot)$, $v^s(p^S(p, \mathcal{C}')) \geq \beta v^s(p)$ because $p^S(p, \mathcal{C}') > p$ and so, since $r' \leq R$, accepting financing must be (weakly) better.

The same argument applies to risky entrepreneurs. ∎

We now verify property i., as well as the characterization of the equilibrium strategies in the various regions.

a. We show first that when $\frac{c}{\pi_h - \pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l + (1-\pi_l)q)}$ an MPE exists where, as long as $p \geq p_{\text{NF}}$, entrepreneurs are always financed, $e^r(p) = 0$, $r(p) = r_{zp}(p, 0)$ and $p^S(p) = \frac{p}{p+(1-p)(\pi_l+(1-\pi_l)q)}$.

   To show that such strategies constitute an MPE in this case, we need to demonstrate that (a-i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (a-ii) that $r_{zp}(p, 0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., that the contract is admissible; and (a-iii) that there are no profitable deviations for any player.

   a-i. Given the above strategies and beliefs, from (1) we get:

$$v^r(p) = \pi_l(R - r_{zp}(p, 0)) + (\pi_l + (1 - \pi_l)q)\beta v^r(p^S(p)). \tag{6}$$

By analogy with (5) above, for low effort to be incentive compatible we need to show that:

$$\frac{c}{\pi_h - \pi_l} \geq R - r(p,0) + \beta(1-q)v^r(p^S(p)), \tag{7}$$

since from lemma 1 we necessarily have $v^r(p^F(p)) = v^r(0) = 0$.

Since $r_{zp}(p,0) > r_{zp}(1,0) = 1$ for all $p < 1$,

$$v^r(p) < \frac{\pi_l(R-1)}{1-\beta(\pi_l + (1-\pi_l)q)}$$

for all $p < 1$, where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period (until there is a failure which is not forgotten) at $r = 1$. So for any $p \in (p_{\text{NF}}, 1)$, we have

$$R - r_{zp}(p,0) + \beta(1-q)v^r(p^S(p)) \quad < R - 1 + \beta(1-q)\tfrac{\pi_l(R-1)}{1-\beta(\pi_l+(1-\pi_l)q)}$$
$$= \tfrac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \tfrac{c}{\pi_h - \pi_l}$$

where the last inequality follows from the condition on c that defines this case. This then verifies (7).

a-ii. Note that $r_{zp}(p,0) \leq R$ if and only if $\frac{1}{p+(1-p)\pi_l} \leq R$, or equivalently $p \geq p_{\text{NF}}$.

a-iii. Finally, we show that there can be no profitable deviation from this equilibrium.

First consider possible deviations by a borrower. We have shown above that at the contract $r(p)$ the borrower never wants to switch from low to high effort; also from observation 2 it follows that borrowers would never choose a rate above $r(p)$ when $r(p)$ is also offered. So it only remains to consider a deviation consisting of refusing an offer of financing.

Since $r(p) \equiv r_{zp}(p,0)$ is strictly decreasing and $p^S(p)$ is increasing, it is immediate from (6) that the discounted expected utility $v^r(p)$ is weakly increasing in p in this case. The same argument also applies to the safe agents' expected discounted utility $v^s(p)$. On the basis of Lemma 3, this implies that refusing financing is never profitable for any borrower, which establishes property i.

Next consider a deviation by a lender. Since $r(p) = r_{zp}(p,0)$, lenders always break even when they offer financing at $r(p)$, and so they would not be able to increase their profits by refusing to offer financing when $p \geq p_{\text{NF}}$.

Consider then the alternative deviation consisting in the offer of a different contract, with interest rate r'. Without loss of generality we can restrict attention to deviations in which $r' < r(p)$, if $r(p) \neq \emptyset$ (since otherwise entrepreneurs would not accept the new offer) and $r' > 1$ (since otherwise the deviation would not be profitable for the lender).

Let the new set of contracts (which includes the deviation r') be $\mathcal{C}'$. Observe that by property iii., $p^S(p, \mathcal{C}') < 1$ whenever $p < 1$. But then by the same argument as in a-i. above we can show that since $r' > 1$ the optimal effort choice for risky entrepreneurs is to exert low effort, i.e., $e(p, \mathcal{C}') = 0$. When $p \geq p_{\text{NF}}$, since $r' < r(p) = r_{zp}(p, 0)$, this makes the deviation unprofitable. Alternatively when $p < p_{\text{NF}}$, since $r(p) = \emptyset$, for the deviation to be profitable under low effort we would need $r' > r_{zp}(p, 0)$; however, when $p < p_{\text{NF}}$ this implies $r' > R$, i.e., that the contract is not admissible.

b. Next, we show that for intermediate values of c, $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0, 1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (b-i) p_h, (b-ii) p_m and (b-iii) p_l, showing that the effort choices specified above for the risky entrepreneurs is optimal. We then show (b-iv) that there are no profitable deviations.

b-i. We first determine the lower bound p_h on the high effort region. Let $\tilde{p}^S(p, e) \equiv \frac{p}{p+(1-p)[e(\pi_h+(1-\pi_h)q)+(1-e)(\pi_l+(1-\pi_l)q)]}$; this is the posterior belief, following a success, that an entrepreneur is risky, when the prior belief is $p \in (0, 1)$ and the effort undertaken if risky is e. That is, $\tilde{p}^S(p, e)$ is calculated via Bayes' Rule as in property iii. of the Proposition, but assuming that the risky agents' effort is e. Also, let $\tilde{v}^r(p, 1)$ denote the discounted expected utility for a risky entrepreneur with credit score p when he is financed in every period until experiencing a failure that is not forgotten, he exerts high effort ($e = 1$), beliefs are updated according to $\tilde{p}^S(p, 1)$ and the interest rate is $r_{zp}(p', 1)$ for all $p' \geq p$. Then $\tilde{v}^r(p, 1)$ satisfies the following equation:

$$\tilde{v}^r(p,1) = \pi_h(R - r_{zp}(p,1)) - c + \beta(\pi_h + (1-\pi_h)q)\tilde{v}^r(\tilde{p}^S(p,1),1). \tag{8}$$

Note that while $\tilde{v}^r(p,1)$ and $\tilde{p}^S(p,e)$ are well defined for all $p \in (0,1)$, they coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ only for, respectively, $p \geq p_h$ and $e = e^r(p)$.

We then define p_h as the value of p that satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) \tag{9}$$

Observe that, since $\tilde{p}^S(p,1)$ is strictly increasing in p, and $r_{zp}(p,1)$ is strictly decreasing, $\tilde{v}^r(p,1)$ is strictly increasing in p. Thus the term on the right-hand side of (9) is increasing in p, and so (9) has at most one solution.

We now show that a solution $p_h \in (0,1)$ to (9) always exists. Since $\tilde{p}^S(p,1)$ and $r_{zp}(p,1)$ are both continuous for all $p \in (0,1)$, $\tilde{v}^r(p,1)$ is also continuous. As $p \to 1^-$, $r_{zp}(p,1) \to 1$ and $\tilde{p}^S(p,1) \to 1$, and so $\tilde{v}^r(p,1) \to \frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)}$. Thus as $p \to 1^-$, we have

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) \to R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

For the values of c in the region under consideration it is easy to verify[24] that $R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)} > \frac{c}{\pi_h-\pi_l}$, and so as $p \to 1^-$, we have

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) > \frac{c}{\pi_h - \pi_l}.$$

Conversely, as $p \to 0^+$ it is immediate to see that (since $\tilde{p}^S(p,1) \to 0$ and $r_{zp}(p,1) \to 1/\pi_h$),

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) \to R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))}.$$

[24] Suppose this were not the case, so that $R - 1 + \beta(1-q)\frac{\pi_h(R-1)-c}{1-\beta(\pi_h+(1-\pi_h)q)} \leq \frac{c}{\pi_h-\pi_l}$. If we multiply both sides of this inequality by $(\pi_h - \pi_l)(1 - \beta(\pi_h + (1-\pi_h)q))$ and then simplify, this becomes $\frac{c}{\pi_h-\pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1\pi_l)q)}$, contradicting the lower bound on c that defines case b.

It is also not difficult[25] to show that under the condition on c defining case b.,

$$R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))} < \frac{c}{\pi_h-\pi_l},$$

and so as $p \to 0^+$,

$$R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1) < \frac{c}{\pi_h-\pi_l}.$$

Thus by the continuity and monotonicity of $\tilde{v}^r(\cdot,1)$, there must be a unique solution $p_h \in (0,1)$ to (9). It is then immediate to see, given the monotonicity of the term on the right-hand side of (9), that for all $p \geq p_h$ the incentive compatibility constraint for high effort (5) is satisfied.

b-ii. Next, we find p_m, the lower bound of the region where risky entrepreneurs mix over high and low effort, and establish the properties of the equilibrium in this mixing region.

For mixing to be an equilibrium strategy at p, risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$R - r_{zp}(p,e) + \beta(1-q)v^r(\tilde{p}^S(p,e)) = \frac{c}{\pi_h-\pi_l} \tag{10}$$

for some $e \in [0,1]$. Now, let $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ denote the preimage of p_h of the map $\tilde{p}^S(p,1)$, i.e., $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),1\right) = p_h$.[26] We define p_m to be the lowest value of $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ for which a solution of (10) can be found for some e. Observe that by the construction of p_h, $e=1$ is a solution to (10) when $p=p_h$, and so $p_m \leq p_h$.

Now, for any $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ we have $\tilde{p}^S(p,e) \geq p_h$ for all e (since $\tilde{p}^S(p,e)$ is increasing in p and decreasing in e[27]). Thus

[25] The argument is analogous to that given earlier. Suppose this is not the case, so that $R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))} \geq \frac{c}{\pi_h-\pi_l}$. Then multiplying both sides of this inequality by $(\pi_h-\pi_l)(1-\beta(\pi_h+(1-\pi_h)q))$ and simplifying, we obtain $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \geq \frac{c}{\pi_h-\pi_l}$, contradicting the lower bound on c that defines this case.

[26] That is, the posterior belief of lenders, after observing a success, is equal to p_h when the prior belief was $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ and the entrepreneur exerts high effort if risky.

[27] This property can be easily verified from the expression of $\tilde{p}^S(p,e)$ and can be understood as follows: for any given p, the lower the probability e that the risky entrepreneurs exert high effort, the stronger is success a signal that the entrepreneur is a safe type.

for any $p \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, using our results from b-i., we have $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e. By the continuity of $\tilde{v}^r(p,1)$ and $r_{zp}(p,e)$ it follows that the minimum value p_m must exist.

We can also show that $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, which will imply that there is only a single period of mixing along the equilibrium path. To see this, note that by assumption 3, we have $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right) \leq \tilde{p}^S(p_h,1)$ for any e.[28] That is, no matter what effort level risky entrepreneurs exert when lenders' prior belief is $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$, the posterior belief of lenders following a success will be lower than when their prior belief is p_h (in which case entrepreneurs exert high effort). Therefore, since $r_{zp}(\left(\tilde{p}^S\right)^{-1}(p_h,1),e) > r_{zp}(p_h,1)$ for any e and $\tilde{v}^r(p,1)$ is strictly increasing in p, we must have

$$\begin{aligned} & R - r_{zp}\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right) + \beta(1-q)\tilde{v}^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right),1) \\ < \quad & R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) = \frac{c}{\pi_h - \pi_l} \end{aligned}$$

for any e. From our previous findings we also have $\tilde{v}^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right),1) = v^r(\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),e\right))$. We conclude therefore that (10) has no solution for e at $\left(\tilde{p}^S\right)^{-1}(p_h,1)$ and so we must have $p_m > \left(\tilde{p}^S\right)^{-1}(p_h,1)$.

To conclude this part, it remains to establish that for all $p \in [p_m, p_h]$ we can indeed find a value of e satisfying (10), and moreover that such value is strictly increasing with respect to p.

Suppose a solution to (10) with respect to e exists for some $p \in [p_m, p_h]$; since we can always take $p = p_m$, this is always possible.

[28] Even without assumption 3, it would still be true that we have only a single period of mixing along the equilibrium path, although the proof would be longer. Hence the characterization would remain essentially unchanged. To see this, suppose it were not the case and that we mixed both at p and its successor $p^S(p)$. Since, as shown below in the text, $e(p) < e(p^S(p))$, we have $r(p) > r(p^S(p))$, so for mixing to be incentive compatible, i.e., for equation (10) to hold both at p and $p^S(p)$, we would need $v^r(p^S(p)) > v^r(p^S(p^S(p)))$. But this is impossible, since from (13) we have $v^r(p) = v^r(p^S(p)) = v^r(p_h)$, whereas $v^r(p^S(p^S(p)) \geq v^r(p^S(p))$ since $p^S(p^S(p)) \geq p^S(p)$.

As we discuss below in footnote 30, assumption 3 is only strictly necessary in order to be able to associate an equilibrium response consistent with Bayes' Rule to any deviation.

Let $e(p)$ denote this solution (if there is more than one solution, we pick the highest one):

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p, e(p)) + \beta(1-q)v^r(\tilde{p}^S(p, e(p))).$$

To prove the claim it suffices to show that for all $p' \in (p, p_h)$ a solution $e(p')$ of (10) also exists, and $e(p') > e(p)$.

Having established above that $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, since $p \geq p_m$ we have $\tilde{p}^S(p, e(p)) > p_h$ and, for all $p' > p$, $\tilde{p}^S(p', e') > p_h$ whatever e' is. Hence $v^r(\tilde{p}^S(p, e(p))) = \tilde{v}^r(\tilde{p}^S(p, e(p)), 1)$ and $v^r(\tilde{p}^S(p', e')) = \tilde{v}^r(\tilde{p}^S(p', e'), 1)$ for all e'. Since, as we showed, $\tilde{v}^r(\cdot, 1)$, as well as $\tilde{p}^S(\cdot, e)$, are strictly increasing while $r_{zp}(\cdot, e)$ is strictly decreasing (whatever e is), when $p' > p$ we must have

$$\frac{c}{\pi_h - \pi_l} < R - r_{zp}(p', e(p)) + \beta(1-q)v^r(\tilde{p}^S(p', e(p))).$$

By the same properties, since $p' < p_h$ we also have

$$\frac{c}{\pi_h - \pi_l} > R - r_{zp}(p', 1) + \beta(1-q)v^r(\tilde{p}^S(p', 1)).$$

Hence, by the continuity of $\tilde{v}^r(\cdot, 1)$ there must be a solution $e' \in (e(p), 1)$ to (10) at p'.

b-iii. We still have to determine p_l, the lower bound on the financing region.

If $p_m \geq p_{\text{NF}}$, set $p_l = p_{\text{NF}}$. By construction, $r_{zp}(p, 0) \leq R$ for all $p \geq p_{\text{NF}}$; hence the contract $r_{zp}(p, e^r(p))$, with $e^r(p) = 0$ for $p \in [p_l, p_m)$, $e^r(p) = e(p)$ for $p \in [p_m, p_h)$ and $e^r(p) = 1$ for $p \geq p_h$ is admissible for all $p \geq p_{\text{NF}}$.

Alternatively, if $p_m < p_{\text{NF}}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e(p))$ is admissible (i.e., not greater than R). Note that since $r_{zp}(p, e)$ is decreasing in e, we have $r_{zp}(p, e(p)) \leq r_{zp}(p, 0)$ for all $p \in [p_m, p_{\text{NF}}]$, so this will imply that $p_l \leq p_{\text{NF}}$. In this case we also redefine p_m, with some abuse of notation, to be equal to p_l; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{\text{NF}}$, which implies that $r_{zp}(p, 0) > R$ for $p < p_l$. Furthermore, $p_l \leq p_m$, with p_m as defined in the preceding paragraphs.

It remains thus to show that $e^r(p) = 0$ for $p \in [p_l, p_m)$, i.e., that low effort is optimal in this region. We prove this in what follows, together with the property that $v^r(p)$ and $v^s(p)$ are (weakly) increasing for all p, which will also be used in part c-iv. of the proof.

Solving the recursive expression (1) for $v^r(p^S(p))$ and substituting into the different expressions of the IC constraint for the three regions of values of p, we obtain:[29]

$$v^r(p) \geq \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if } e^r(p) = 1; \tag{11}$$

$$v^r(p) \leq \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if } e^r(p) = 0; \tag{12}$$

$$v^r(p) = \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R-r)\frac{q}{1-q}, \text{ if (10) holds (mixing).} \tag{13}$$

As established in b-i. above, when $p \geq p_h$ we have $e^r(p) = 1$, in which case $v^r(p) = \tilde{v}^r(p, 1)$, which we have shown is strictly increasing. From (13) we also find that $v^r(p)$ is constant for all $p \in [p_m, p_h]$ and hence, using (11), that it is weakly increasing for all $p \geq p_m$.

To prove that $e^r(p) = 0$ for $p \in [p_l, p_m)$ it suffices to consider the case $p_l = p_{\text{NF}}$ (when $p_l < p_{\text{NF}}$, we showed above that $p_l = p_m$). First consider $p \in \Big[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h, 1)], p_m\Big)$. We will show that the contract $r_{zp}(p, 0)$, together with lenders' beliefs in case of success $p^S(p) = \tilde{p}^S(p, 0)$, satisfy the IC constraint for low effort. Suppose this were not true, i.e.,

$$R - r_{zp}(p, 0) + \beta(1-q)v^r(\tilde{p}^S(p, 0)) > \frac{c}{\pi_h - \pi_l}.$$

We will prove this leads to a contradiction. Note that for p in the above interval $p \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, hence $\tilde{p}^S(p, 1) \geq p_h$ and

[29] When $e^r(p) = 1$ then (1) reduces to $v^r(p) = \pi_h(R - r(p)) - c + \beta(\pi_h + (1-\pi_h)q)v^r(p^S(p))$, and hence we get $\beta(1-q)v^r(p^S(p)) = \frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q}$. The expression of the IC constraint for high effort (5) can then be rewritten as follows: $\beta(1-q)v^r(p^S(p)) \geq \frac{c}{\pi_h-\pi_l} - (R-r(p))$. Substituting for $v^r(p^S(p))$ from the above equation, yields $\frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q} \geq \frac{c}{\pi_h-\pi_l} - (R-r(p))$, or $(v^r(p) + c - \pi_h(R-r(p)))(1-q) \geq \left(\frac{c}{\pi_h-\pi_l} - (R-r(p))\right)(\pi_h + (1-\pi_h)q)$. Simplifying, we get (11). The other expressions are similarly obtained.

$v^r(\tilde{p}^S(p,1)) = \tilde{v}^r(\tilde{p}^S(p,1),1)$. Also note that $p < p_h$, and so we have $\tilde{p}^S(p,1) < \tilde{p}^S(p_h,1)$. Thus from the monotonicity of $r_{zp}(\cdot,1)$ and $\tilde{v}^r(\cdot,1)$, we have

$$\begin{aligned}&R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) = R - r_{zp}(p,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p,1),1)\\ &< R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) = \tfrac{c}{\pi_h-\pi_l},\end{aligned}$$

where the latter equality follows from the construction of p_h. Since $\tilde{p}^S(p,e) \geq \tilde{p}^S(p,1)$ for all e, so that we also have $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$, and $\tilde{v}^r(\cdot,1)$ is continuous, the two inequalities above imply that there must be a solution $\tilde{e}$ to (10) at p, which contradicts the construction of p_m as minimal in $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_h\right]$. So we conclude that $e^r(p) = 0$ for $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_m\right)$. By the argument in a. this also implies that $v^r(p)$ is strictly increasing in $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)], p_m\right)$ and, using (11) - (13) above, that it is increasing for all $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(p_h,1)]$. If $p_l \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ we are done. Otherwise, we extend the result by induction. It is convenient to use here the shorthand $\tilde{p}^{S^{-1}}$ to denote the term $\left(\tilde{p}^S\right)^{-1}(p_h,1)]$. We will now demonstrate that (i) $e^r(p) = 0$ for $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)], \tilde{p}^{S^{-1}}\right)$ and (ii) $v^r(p)$ is increasing for $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)]$. Consider $p \in \left[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}},0)], \tilde{p}^{S^{-1}}\right)$. To show (i), first note that since $p < \tilde{p}^{S^{-1}} < p_m$, we must have $p^S(p) = \tilde{p}^S(p,0)$, since lenders update via Bayes' Rule and the equilibrium prescribes low effort ($e^r(p) = 0$) in this region. So

$$R - r(p) + \beta(1-q)v^r(p^S(p)) = R - r_{zp}(p,0) + \beta(1-q)v^r(\tilde{p}^S(p,0)).$$

In addition, since $p < \tilde{p}^{S^{-1}} = \left(\tilde{p}^S\right)^{-1}(p_h,1)$, by assumption 3 we must also have $p^S(p) = \tilde{p}^S(p,0) \leq \tilde{p}^S(p_h,1) = p^S(p_h)$. Thus $v^r(p^S(p)) \leq v^r(p^S(p_h))$, since $v^r(\cdot)$ is increasing above $\tilde{p}^{S^{-1}}$, as shown in the previous paragraph, and $p^S(p) \geq \tilde{p}^{S^{-1}}$. Then using the fact that $r(p) = r_{zp}(p,0) > r_{zp}(p,1) > r_{zp}(p_h,1) = r(p_h)$, since $r_{zp}(\cdot,\cdot)$ is strictly decreasing, yields:

$$R - r_{zp}(p,0) + \beta(1-q)v^r(\tilde{p}^S(p,0)) < R - r(p_h) + \beta(1-q)v^r(p^S(p_h)) = \frac{c}{\pi_h - \pi_l},$$

with the latter equality following from the construction of p_h. Thus $e^r(p) = 0$, i.e., low effort is IC at p. The same argument as above can then be used to establish that $v^r(p)$ is increasing for $p \geq \max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 0)]$.

Now, if $p_l \geq \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 1)$ we are done. Otherwise, iterate the same argument over the interval $\Big[\max[p_l, \left(\tilde{p}^S\right)^{-1}(\left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 1), 0)], \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 1)\Big)$ and continue doing so until we reach p_l.

Finally, to show that $v^s(p)$ is increasing, simply note that $e^r(p)$ is (weakly) increasing, and so $r(p) = r_{zp}(p, e^r(p))$ is decreasing. The result then follows from the definition of $v^s(p)$ in (2).

b-iv. Finally, we show that there are no profitable deviations.

First consider the possibility of deviations by entrepreneurs, in particular, the rejection of an offer of financing (as argued in observation 2 this is the only deviation we need to consider). We showed in part b-iii. above that $v^r(p)$ and $v^s(p)$ are (weakly) increasing, so we can use lemma 3 to verify property i., i.e., that it is never optimal for entrepreneurs to refuse financing.

Next consider deviations by lenders. By the same argument as in a-iii., refusing to finance an agent when $p \geq p_l$ is not profitable for lenders. So when $p \geq p_l$ it suffices to consider deviations consisting in the offer of a contract $r' < r(p) \equiv r_{zp}(p, e(p))$. By contrast, when $p < p_l$ and there is no financing in equilibrium, the deviations consist of the offer of a contract $r' \leq R$. Without loss of generality we can also restrict our attention to the region $p < p_h$, since there can be no profitable deviations when the contract offered in equilibrium supports high effort (as in that case $r(p) = r_{zp}(p, 1) \leq r_{zp}(p, e)$ for all e).

In the statement of the Proposition we did not describe the risky entrepreneurs' effort strategy $e^r(p, r')$ off the equilibrium path. We do so here and show that $e^r(p, r')$ renders any possible deviation r' described in the previous paragraph unprofitable.

First consider the case $p \in \Big(\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h\Big)$. Now if

$$R - r' + \beta(1 - q)v^r(\tilde{p}^S(p, 0)) \leq \frac{c}{\pi_h - \pi_l}, \tag{14}$$

then $e^r(p, e') = 0$ is an optimal effort choice of entrepreneurs when they are offered the rate r' and lenders' belief is that they exert low effort. If in addition $p \geq p_l$ then $r' < r(p) \leq r_{zp}(p, 0)$ and so the deviation is unprofitable. If $p < p_l$, from c-iii. above we know that $r_{zp}(p,0) > R$, while the admissibility of the contract requires $r' \leq R$, so that $r' < r_{zp}(p,0)$, i.e., the deviation is unprofitable in this case as well.

Alternatively, suppose the reverse inequality to (14) holds. Then since $\tilde{p}^S(p,e)$ is decreasing with respect to e and $v^r(\cdot)$ is weakly increasing, we either have[30]

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) \geq \frac{c}{\pi_h - \pi_l}, \text{ for } e' = 1 \tag{15}$$

or

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) = \frac{c}{\pi_h - \pi_l} \text{ for some } e' \in (0,1). \tag{16}$$

so that the optimal effort choice of risky entrepreneurs when offered rate r' is $e^r(p,r') = e'$. Suppose $r' > r_{zp}(p,e')$; we will prove in what follows that this implies a contradiction, thus establishing that $r' \leq r_{zp}(p,e')$, i.e., that again the deviation to r' is unprofitable.

When $e' = 1$, $r' > r_{zp}(p,e') = r_{zp}(p,1)$ together with (15) imply $R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) \geq \frac{c}{\pi_h - \pi_l}$. But since, as we argued, $v^r(\cdot)$ is increasing and $r_{zp}(\cdot,1)$ strictly decreasing, this would imply that $R - r_{zp}(p_h,1) + \beta(1-q)v^r(\tilde{p}^S(p_h,1)) > \frac{c}{\pi_h - \pi_l}$, contradicting the construction of p_h in (9).

So consider instead $e' < 1$. Then from $r' > r_{zp}(p,e')$ and equation (16) we get

$$R - r_{zp}(p,e') + \beta(1-q)v^r(\tilde{p}^S(p,e')) > \frac{c}{\pi_h - \pi_l}.$$

Recall that $p \in \left(\left(\tilde{p}^S\right)^{-1}(p_h,1), p_h\right)$, so that $v^r(\tilde{p}^S(p,1)) = \tilde{v}^r(\tilde{p}^S(p,1),1)$ and, from the definition of p_h,

$$R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) < \frac{c}{\pi_h - \pi_l}.$$

[30] By assumption 3, when $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$, we have $v^r(\tilde{p}^S(p,\tilde{e})) = \tilde{v}^r(\tilde{p}^S(p,\tilde{e}),1)$, and so $v^r(\cdot)$ is continuous in $\tilde{e}$.

This is the only point in the proof where assumption 3 proves strictly necessary. Without this assumption we could still prove the existence of an MPE, but the off-equilibrium path beliefs would not necessarily be consistent everywhere with Bayes' Rule and hence we would not be able to claim that our equilibrium is also a sequential MPE.

Since, as we argued, $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e, by the continuity of $\tilde{v}^r(p,1)$ it follows that there must be a solution $\tilde{e} \in (e',1)$ to (10). If $p < p_m$ the existence of such a solution contradicts the construction of p_m as the minimal value of p for which a solution $e(p)$ to (10) with $r_{zp}(p,e(p)) \leq R$ exists in the region $p \in [(\tilde{p}^S)^{-1}(p_h,1), p_h]$, since $r_{zp}(p,\tilde{e}) < r_{zp}(p,e') < r' < r(p)$. Alternatively, consider $p \geq p_m$. If $\tilde{e} > e(p)$ this contradicts the construction of $e(p)$ as the highest solution of (10) at p; if $\tilde{e} \leq e(p)$, this implies $e' < e(p)$, and thus $r' > r_{zp}(p,e') > r_{zp}(p,e(p))$, another contradiction.

We have thus shown that $r' \leq r_{zp}(p,e')$ when $p \in \left((\tilde{p}^S)^{-1}(p_h,1), p_h\right)$, so that the deviation r' will not be profitable given the risky entrepreneurs' optimal response $e^r(p,r') = e'$.

It remains only to consider the case $p \in \left(0, (\tilde{p}^S)^{-1}(p_h,1)\right)$. We restrict attention to deviations $r' > r_{zp}(p,1)$; this is without loss of generality, since if this were not the case the deviation could never be profitable, regardless of the risky entrepreneurs' effort choice. We can show that in this case $e' = 0$ is an equilibrium response to r' for the risky borrowers. To see this, note that since $r' > r_{zp}(p,1)$ and $p < (\tilde{p}^S)^{-1}(p_h,1)$, by assumption 3 it must be that $\tilde{p}^S(p,0) \leq \tilde{p}^S(p_h,1)$, and so $v^r(\tilde{p}^S(p,0)) \leq v^r(\tilde{p}^S(p_h,1))$ by the monotonicity of $v^r(\cdot)$ (established in c-iii. above). Using this property and the fact that $r' > r_{zp}(p,1)$ and $r_{zp}(p,1) > r_{zp}(p_h,1)$ (by the monotonicity of $r_{zp}(\cdot,1)$), yields

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,0)) < R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,0))$$

$$< R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p_h,1)) = \frac{c}{\pi_h - \pi_l},$$

with the latter equality following from the definition of p_h. This establishes that $e' = 0$ is indeed the risky entrepreneurs' optimal effort choice when offered r'.

We now argue that this renders the deviation unprofitable. First, if $p \geq p_l$ in equilibrium there is financing and so the offer of r' will only be accepted if $r' < r(p)$. But $r(p) = r_{zp}(p,0)$ in this region, which means that the deviation will be unprofitable. Alternatively, if $p \in (0,p_l)$, then when r' is admissible ($r' \leq R$) we necessarily have $r' < r_{zp}(p,0)$ since, as shown above, $p_l \leq p_{NF}$, which again makes the deviation unprofitable.

c. Finally, we show that for values of c such that $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ an MPE exists where, for all $p > 0$ entrepreneurs are always financed, $e^r(p) = 1$, $r(p) = r_{zp}(p, 1)$ and $p^S(p) = \frac{p}{p+(1-p)(\pi_h+(1-\pi_h)q)}$.

As above, to show that such strategies constitute an MPE for the above values of c, we have to show that (c-i) risky entrepreneurs indeed prefer to exert high rather than low effort for all $p > 0$ and (c-ii) there are no profitable deviations for any player. Note that by assumption 1 $r_{zp}(p, 1) \leq R$ for all $p > 0$, so $r(p) = r_{zp}(p, 1)$ is always admissible.

c-i. To show that high effort is IC for all $p > 0$, given the above strategies, we need to show that

$$\frac{c}{\pi_h - \pi_l} \leq R - r(p) + \beta(1-q)v^r(p^S(p)) \tag{17}$$

for any $p > 0$.

We first argue that, for any $p > 0$, a lower bound for $v^r(p)$ is given by $\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))}$, which is the present discounted utility for a risky entrepreneur financed in every period (until a failure that is not forgotten) at $r = 1/\pi_h$ and exerting high effort. This follows immediately from the fact that $v^r(p)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p) = r_{zp}(p, 1) < 1/\pi_h$ for all $p > 0$.

Thus since $p^S(p) > 0$ for all $p > 0$, we have

$$R - r(p) + \beta(1-q)v^r(p^S(p)) > R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + (1-\pi_h)q)}.$$

So to verify (17) it suffices to show that

$$R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + (1-\pi_h)q)} \geq \frac{c}{\pi_h - \pi_l}.$$

But this follows immediately from the argument given in the proof of b-i. above (see footnote 25, noting that region c. encompasses precisely those values of c *below* the lower bound defining region b.).

c-ii. Now we show that there can be no profitable deviations.

First, note that lenders cannot profitably deviate. To see this, simply observe that, given property i., for any $r' < r(p) = r_{zp}(p, 1)$ a lender would make negative profits, regardless of the risky entrepreneurs' effort choices, and hence r' cannot be a profitable deviation.

Next consider possible deviations by a borrower. In light of observation 2 we can limit our attention to deviations in which an entrepreneur refuses financing. Since high effort is exerted for all $p > 0$, $r(p)$ is strictly decreasing in p and so from (1) and (2) it is easy to see that $v^r(p)$ and $v^s(p)$ are (strictly) increasing in p. Thus from lemma 3 above, property i. must hold in this case as well, i.e., refusing financing must be unprofitable.■

D Efficiency of Equilibrium

For simplicity we restrict attention to the case $q = 0$ (no forgetting); the argument for general values of q is exactly the same. We begin by showing that, as constructed in Proposition 1, our equilibrium maximizes $e^r(p)$, the effort exerted by the risky entrepreneurs for any p. This will play an important role in the proof.

Lemma 4. *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$ and utility $v^r(p)$ across all symmetric sequential MPE.*

Proof. We focus on the first statement — that concerning the effort level $e^r(p)$; the result on $v^r(p)$ will follow as a consequence. Suppose that this is not the case and that there exists some other equilibrium that implements higher effort at some p'. Let $e^r(p), p^S(p)$, and $v^r(p)$ denote the effort, updating function, and risky-entrepreneur utility, respectively, for the equilibrium of Proposition 1, and let $\overline{e}^r(p), \overline{p}^S(p), \overline{v}^r(p)$ denote the same for this other equilibrium.

First note that it is immediate that we cannot have $p' \geq p_h$. Moreover, since the equilibrium of Proposition 1 gives all of the surplus to the borrowers, it must also be the case that $v^r(p)$ must also be maximal for all $p \geq p_h$, i.e. $v^r(p) \geq \bar{v}^r(p)$ for all $p \geq p_h$. Also recall that $v^r(p)$ was shown above to be weakly increasing. We now proceed by induction.

Having established that $v^r(p)$ is maximal for all $p \geq p^*$ (where we begin the induction with $p^* = p_h$), we consider $p^{**} \equiv \left(\tilde{p}^S\right)^{-1}(p_h, 1)$, the pre-image of p^* under high effort. We first show that we also cannot have $p' \in [p^{**}, p^*)$. Were this not the case, and we had $\bar{e}^r(p') > e^r(p')$ for some p', then in order to satisfy incentive compatibility the continuation utility in the other equilibrium would need to be higher, i.e. $\bar{v}^S(\bar{p}^S(p)) > v^r(p^S(p))$.[31] But if $\bar{e}^r(p) > e^r(p)$ then we necessarily have $\bar{p}^S(p) < p^S(p)$, and so from the monotonicity of $v^r(\cdot)$, we have $v^r(\bar{p}^S(p)) \leq v^r(p^S(p))$. But since $\bar{p}^S(p) > p^S(p) \geq p^*$, we must have $\bar{v}^S \leq v^r(p^S(p))$, as $v^r(p)$ has been established to be maximal for $p \geq p^*$, a contradiction.

Thus we must have $e^r(p) \geq \bar{e}^r(p)$, and hence $v^r(p) \geq \bar{v}^r(p)$ for all $p \geq p^{**}$. We then set $p^* = p^{**}$ and continue the induction, thereby establishing the desired result for arbitrary p. ∎

The following corollary is also immediate, since for lenders to break even when $p < p_l$ would require a higher level of effort than our equilibrium, which we have shown is impossible.

Corollary 2. *No MPE equilibrium can implement financing when $p < p_l$.*

We now turn to demonstrating that our equilibrium is the most efficient MPE. From corollary 2, we can restrict attention to $p_0 \geq p_l$, without loss of generality.

As mentioned above, given an ex-ante probability that an entrepreneur is the safe type of p_0, then welfare is given by the expected total surplus accruing from the agents' projects. We can define this sequentially. Let p_n denote the posterior probability that an agent who has not failed thus far is safe in period n. Recall that we have shown that in any MPE, risky agents who fail must be excluded thereafter. So define b_n to be the *measure* of risky agents who have not failed through period n; note that $b_0 = 1 - p_0$, and, more generally, $b_n = p_0(1/p_n - 1)$. Also let $e_n = e^r(p_n)$ denote the risky entrepreneurs' equilibrium effort choice in period n.

Since the equilibrium constructed in Proposition 1 implements financing for all $p \geq p_0$, the surplus generated in period n is given by:

$$\mathcal{W}_n = p_0(R-1) + b_n[e_n(\pi_h R - 1 - c) + (1 - e_n)(\pi_l R - 1);$$

where we have made use of the fact that the safe agents never fail and that there is financing in all subsequent periods. Observe that $\mathcal{W}_n \geq 0$ for all n, since otherwise lenders could not break even in this period; this is a general property of all MPE.

[31] More precisely, if we are in the mixing region (in which the risky agents are indifferent between high and low effort) then the other equilibrium could implement higher effort even when $\bar{v}^S(\bar{p}^S(p)) = v^r(p^S(p))$. But recall that we chose effort in the mixing region to be maximal in constructing the equilibrium in Proposition 1.

The total ex-ante surplus $\mathcal{W}$ is then given by the discounted sum of the $\mathcal{W}_n$:

$$\mathcal{W} = \sum_{n=0}^{\infty} \beta^n \mathcal{W}_n.$$

Now consider some other equilibrium that implements a higher level of ex-ante surplus at p_0. Let $\overline{\mathcal{W}}, \overline{\mathcal{W}}_n, \overline{p}_n, \overline{b}_n, \overline{e}_n$ denote the analogous quantities for this other equilibrium (where $\overline{\mathcal{W}}_n = 0$ if there is no financing in period n). We will show that it is not possible for this other equilibrium to generate a higher ex-ante surplus.

First suppose that $p_0 < p_m$, where p_m is the mixing cutoff in our equilibrium (recall that if there is no mixing in our equilibrium then $p_m \equiv p_h$ and then this case is empty). Consider those values of n such that $p_n < p_m$. Now, from lemma 4, any equilibrium will either implement low effort for period n, or else it cannot offer financing in period n. First note that this implies that we can restrict attention to equilibria which offer financing for all such n. Otherwise, by the Markov Property, any equilibrium which does not offer financing in period n could not offer financing in any subsequent period, thus implying that $\overline{\mathcal{W}}_{n^*} = 0$ for all $n^* \geq n$, and hence a lower total discounted surplus. Thus all equilibria we consider implement low effort when $\bar{p}_n < p_m$, implying that $p_n = \overline{p}_n$ for these values of n.

Next, consider n such that $p_n \in [p_m, p_h)$, the mixing region.[32] From above we also have $p_n = \overline{p}_n$ for this n. As above, we can restrict attention to equilibria offering financing in this period, without loss of generality. So by Proposition 4, we must have $e_n \geq \bar{e}_n$, and hence $\mathcal{W}_n \geq \overline{\mathcal{W}}_n$.

Note, however, this implies that $p_{n+1} \leq \overline{p}_{n+1}$ (see footnote 27), and hence $b_{n+1} \geq \bar{b}_{n+1}$. That is, there will be (weakly) more risky agents remaining in the pool in the equilibrium of Proposition 1.

The rest of the argument is now immediate. Since the equilibrium of Proposition 1 implements high effort from the risky entrepreneurs in period $n+1$ and all subsequent periods, and since we have just shown that there are more risky entrepreneurs in period $n+1$, this must clearly dominate any other equilibrium.

If $p_0 \in [p_m, p_h)$ or $p_0 \geq p_h$ then the argument is identical, omitting the irrelevant cases.■

[32] Recall from the proof of Proposition 1 that this can be the case for at most one period.

E Proposition 3 – Optimal Forgetting (regions a. and c.)

Consider case 1. When $\frac{c}{\pi_h-\pi_l} \geq \frac{R-1}{1-\pi_l\beta}$, since $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q, the condition defining region a.
in Proposition 1 is satisfied for all q. At the MPE there is financing only when $p_0 \geq p_{\mathrm{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q.

Hence if $p_0 \geq p_{\mathrm{NF}}$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is $\frac{B}{1-(\pi_l+(1-\pi_l)q)\beta}$, , which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\mathrm{NF}}$, such surplus is zero for all q, and so $q = 0$ is also (weakly) optimal.

Consider now case 2. Again notice that $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q. Thus when $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, the condition defining region c. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\beta\pi_l)}{\beta\left((R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\pi_l)\right)} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is

$$\frac{G}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.[33] ■

F Proposition 4 – Optimal Forgetting (region b.)

For case 1 ($p_0 > p_h(0)$) the proof is an immediate corollary of the second case of Proposition 3.

Consider then case 2. Since $p_0 \geq p_{\mathrm{NF}}$, the agents will always be financed at the initial date, irrespective of q. Thus, by the argument given above, it suffices to show that we can increase the surplus generated by the risky entrepreneurs' projects. Letting $\mathcal{W}^r(q, p_0)$ denote the surplus from the risky agents' projects, when the forgetting policy is q and the prior probability of being safe is p_0, we will show that under the conditions stated in the Proposition, we can find some $\bar{q} > 0$ such that $\mathcal{W}^r(\bar{q}, p_0) > \mathcal{W}^r(0, p_0)$.

We proceed as follows. For any $q > 0$ we first find a threshold $\tilde{p}_h(q)$ for $p_h(q)$, relative to $p_h(0)$, such that *if* $p_h(q) < \tilde{p}_h(q)$ then this surplus is

[33]The optimal value of q could be higher than q^*, which would push us out of region c., into region b.

higher at q than at 0. We then show that the parameter restrictions stated in the Proposition ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(q)$.

Let $n(q, p_0)$ denote the number of successes (or forgotten failures), starting from the prior p_0, until the risky entrepreneurs first exert high effort, when the forgetting policy is q. Then the following upper and lower bounds for the surplus generated by lending to risky entrepreneurs can be shown to hold:[34]

$$\mathcal{W}^r(0, p_0) \leq \frac{B(1 - (\pi_l\beta)^{n(0,p_0)-1})}{1 - \pi_l\beta} + \frac{G(\pi_l\beta)^{n(0,p_0)-1}}{1 - \pi_h\beta} \tag{18}$$

and

$$\mathcal{W}^r(q, p_0) \geq \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}. \tag{19}$$

So to show that $\mathcal{W}^r(q, p_0) > \mathcal{W}^r(0, p_0)$, it suffices to show that we can find $q > 0$ such that

$$\frac{B(1 - (\pi_l\beta)^{n(0,p_0)-1})}{1 - \pi_l\beta} + \frac{G(\pi_l\beta)^{n(0,p_0)-1}}{1 - \pi_h\beta} < \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

Letting $\beta \to 1$ and simplifying, the above expression reduces to:

$$\frac{\frac{B}{G}}{1 - \pi_l} + \frac{\pi_l^{n(0,p_0)-1}}{(1 - \pi_l)(1 - \pi_h)}\left[(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)\right]$$
$$< \frac{\frac{B}{G}}{(1 - \pi_l)(1 - q)} + \frac{(\pi_l + (1 - \pi_l)q)^{n(q,p_0)}}{(1 - q)(1 - \pi_l)(1 - \pi_h)}\left[(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)\right],$$

since $1 - (\pi_l + (1 - \pi_l)q) = (1 - \pi_l)(1 - q)$ and $1 - (\pi_h + (1 - \pi_h)q) = (1 - \pi_h)(1 - q)$, or, equivalently, to

$$\pi_l^{n(0,p_0)-1}(1 - q) - \frac{\frac{B}{G}(1 - \pi_h)q}{(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)} < (\pi_l + (1 - \pi_l)q)^{n(q,p_0)} \tag{20}$$

[34]When there is no mixing in equilibrium (i.e. $p_m(q) = p_h(q)$), $\mathcal{W}^r$ is simply equal to the discounted expect surplus generated by consecutive successes of the project (the first $n(q, p_0)$ of which with low effort, the remainder with high effort):

$$\mathcal{W}^r(q, p_0) = \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

With mixing in equilibrium, the exact expression of $\mathcal{W}^r$ depends on the equilibrium level of effort exerted in the mixing region. However, since there can be at most only a single period of mixing in equilibrium, an upper and lower bound for such utility is given by (18) and (19), independent of the mixing probability.

It will be useful to rewrite (20) in terms of a condition on $p_h(q)$ and $p_h(0)$. To this end, notice that $p_h(q)$ and $n(q,p_0)$ are related by the following expression: $n(q,p_0)$ is the smallest integer for which[35]

$$\frac{p_0}{p_0+(1-p_0)[\pi_l+(1-\pi_l)q]^{n(q,p_0)}} \geq p_h(q), \tag{21}$$

so that $\pi_l^{n(0,p_0)} \leq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(0)}-1\right)$ and $(\pi_l+(1-\pi_l)q)^{n(q,p_0)-1} \geq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(q)}-1\right)$. Thus to satisfy (20) it suffices to show that:

$$\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)\frac{p_0}{p_h(q)}\left(\frac{1-p_h(q)}{1-p_0}\right).$$

Simplifying, we obtain the following sufficient condition for q to implement a welfare improvement as $\beta \to 1$:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{p_0(\pi_l+(1-\pi_l)q)}{p_0(\pi_l+(1-\pi_l)q)+(1-p_0)\left[\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}\right]}. \tag{22}$$

We now show that the condition on B/G stated in the Proposition ensures that we can find $\bar{q}>0$ such that $p_h(\bar{q})$ satisfies (22) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for the level of $p_h(q)$.

For intermediate values of c, lying in the region where type b. equilibria obtain when $q=0$, $p_h(0)$ belongs to $(0,1)$ and satisfies equation (9) above. It is then easy to see from the definition of this region in Proposition 1 that, when β is sufficiently close to 1, c will remain in the same region for any $q>0$.[36] So for β close to 1, $p_h(q)$ also lies in $(0,1)$ and satisfies an expression analogous to (9):

$$\frac{c}{\pi_h-\pi_l} = R - r_{zp}(p_h(q),1)+\beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q), \tag{23}$$

[35] When there is no mixing in equilibrium, i.e., $p_m(q)=p_h(q)$, the validity of this expression follows immediately from the definition of $p_h(q)$ and $n(q,p_o)$. The fact that it also holds with mixing can be seen by noticing that in such case the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p,e(p)) \leq \tilde{p}^S(p,0)$. Hence $n(q,p_0)$ will be greater or equal than the term satisfying (21). But $n(q,p_0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

[36] For β close to 1, the boundaries of the region are approximately equal to $\frac{(R-1/\pi_h)}{1-\pi_l}$ and $\frac{(R-1)}{1-\pi_l}$, both independent of q.

where, similarly to (9), $\tilde{v}^r(p,1;q)$ denotes the discounted expected utility of a risky entrepreneur with credit score p, when he exerts high effort for all $p' > p$ and the contracts offered are $r_{zp}(p,1)$, highlighting the dependence of the utility on the forgetting policy q. From (23) and (9) we obtain then:

$$-r_{zp}(p_h(0),1)+\beta\tilde{v}^r(\tilde{p}^S(p_h(0),1),1;0) = -r_{zp}(p_h(q),1)+\beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q). \tag{24}$$

By a similar argument to that in the proof of parts a. and c. of Proposition 1, a (strict) upper bound for $\tilde{v}^r(\tilde{p}_h^S(p_h(0),1),1;0)$ is given by the utility of being financed in every period at the constant rate $r=1$ until a failure occurs, while exerting high effort, i.e., by $\frac{\pi_h(R-1)-c}{1-\beta\pi_h}$. Conversely, when the forgetting policy is q, a (strict) lower bound for $\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q)$ is given by $\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\beta(\pi_h+(1-\pi_h)q)}$, that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q),1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (24) this implies that:

$$-r_{zp}(p_h(0),1)+\beta\frac{\pi_h(R-1)-c}{1-\beta\pi_h} > -r_{zp}(p_h(q),1)+\beta(1-q)\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

When $\beta \to 1$, the above inequality becomes

$$-r_{zp}(p_h(0),1)+\frac{\pi_h(R-1)-c}{1-\pi_h} > -r_{zp}(p_h(q),1)+\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\pi_h},$$

or, simplifying,

$$r_{zp}(p_h(q),1) > (1-\pi_h)r_{zp}(p_h(0),1)+\pi_h.$$

Using then the definition of $r_{zp}(\cdot,\cdot)$ in (4), the previous expression can be rewritten as follows:

$$\frac{1}{p_h(q)+(1-p_h(q))\pi_h} > (1-\pi_h)\frac{1}{p_h(0)+(1-p_h(0))\pi_h}+\pi_h,$$

or

$$\begin{aligned} p_h(0)+(1-p_h(0))\pi_h &> (1-\pi_h)[p_h(q)+(1-p_h(q))\pi_h]+\pi_h[p_h(q)+(1-p_h(q))\pi_h][p_h(0)+(1-p_h(0))\pi_h] \\ &= [p_h(q)+(1-p_h(q))\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0)]\right], \end{aligned} \tag{25}$$

which is in turn equivalent to:

$$p_h(0)(1-\pi_h)+\pi_h > [p_h(q)(1-\pi_h)+\pi_h]\left[1-\pi_h(1-\pi_h)(1-p_h(0)]\right],$$

i.e.,

$$\frac{p_h(0)(1-\pi_h)+\pi_h}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} > [p_h(q)(1-\pi_h)+\pi_h].$$

The above inequality implies that when β is close to 1 the following upper bound on the level of $p_h(q)$ must hold, for all q:

$$p_h(q) < \bar{p}_h \equiv \frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]}. \tag{26}$$

Finally, note that for q close to 1, $\tilde{p}_h(q)$ is approximately equal to $p_0 \frac{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}{p_0(1-\pi_l)-\frac{B}{G}(1-\pi_h)}$. Hence, under the condition on B/G stated in the Proposition we have that

$$\frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} < p_0 \frac{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}{p_0(1-\pi_l)-\frac{B}{G}(1-\pi_h)},$$

or equivalently that, for q close to 1 we have $\bar{p}_h < \tilde{p}_h(q)$.

Thus on the basis of the previous discussion we can conclude that there exists $\bar{q}$ yielding a welfare improvement over $q = 0$. ■

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