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Are Household Portfolios  
Efficient? An Analysis  
Conditional on Housing



## **Are Household Portfolios Efficient? An Analysis Conditional on Housing**

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### **Abstract**

Standard tests of portfolio efficiency neglect the existence of illiquid wealth. The most important illiquid asset in household portfolios is housing: if housing stock adjustments are infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk through a hedge term and tests for portfolio efficiency of financial assets must be run conditionally upon housing wealth. We use Italian household portfolio data and time series on financial assets and housing stock returns to assess whether actual portfolios are efficient. We find that housing wealth plays a key role in determining whether portfolios chosen by home-owners are efficient.

### **Keywords**

Housing and portfolio choice, Portfolio efficiency.

### **JEL Codes**

*D91, G11*

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## **I. Introduction**

There has been an increased interest in recent years in household portfolio choice. A number of country studies have looked at the way households allocate their financial wealth across different financial instruments and found that a decreasing but still sizeable proportion of households fail to invest in the stock exchange (Guiso, Haliassos and Jappelli, 2002).

Households allocate their wealth into financial and real assets, but the portfolio allocation problem has typically been addressed empirically focusing solely on financial assets. A few studies have extended the analysis to cover other forms of household wealth, notably own business (Heaton and Lucas, 2000) and housing equity (Flavin and Yamashita, 2002, and Cocco, 2005). Both assets are illiquid, that is subject to non-negligible trading costs. These trading costs are likely to be particularly high for the housing stock for homeowners. When consumption and investment needs differ, and rental markets are imperfect (Henderson and Ioannides (1983)), short run adjustments can be all but impossible. Flavin and Yamashita (2002) stress that in this sense “demand for housing is over-determined”, and investment considerations may be of secondary importance.

In this paper we address the issue of efficiency of household portfolios when illiquid housing wealth is also considered. This issue has been investigated by Grossman and Laroque (1990) and more recently by Flavin and Nakagawa (2004). Grossman and Laroque show that the standard CAPM holds in a dynamic setting when households derive utility from just one good that is durable and illiquid (and therefore infrequently adjusted). In their model there are risky financial assets, and also a risk-free asset: given that the numeraire is the durable good, this implies that the nominal return on this asset has unit correlation with the housing return. Flavin and Nakagawa’s paper extends Grossman and Laroque’s model by allowing for the presence of two goods in the utility function. In their model, there is no correlation between housing returns and financial asset returns. Flavin and Nagakawa prove that over those periods where the housing stock is not adjusted, all households hold a single optimal portfolio of risky assets (the standard Markowitz optimal risky

portfolio), the standard CAPM holds and housing wealth affects portfolio allocations only through the relative risk aversion of individual investors. A number of recent papers have produced micro evidence on the role of housing on portfolio allocations within this framework, in which housing wealth contributes to background risk (Flavin and Yamashita, 2002, Kullman and Siegel, 2003, Yamashita, 2003, LeBlanc and Lagarenne, 2004, Cauley, Pavlov and Schwartz, 2005, and Cocco, 2005).

We extend the analysis to cover the case where returns are correlated, and show how efficient financial portfolios should be after allowance is made for the presence of a given housing stock. In these portfolios housing wealth affects the optimal shares in two distinct ways: indirectly, via risk aversion, and directly, via a hedge motive. In particular, we observe that all households will hold a single optimal portfolio of risky assets (the standard Markowitz optimal portfolio) and a hedge term covering house price risk.

On the basis of our theoretical analysis, we expect optimally chosen financial portfolios not to be mean-variance efficient in the standard sense when asset and housing returns are correlated. Also, if the housing stock is not frequently adjusted, we also expect the overall portfolios (that include financial assets and housing wealth) not to be mean-variance efficient. However, we show that optimal portfolios should be conditionally mean-variance efficient, that is mean-variance efficient when housing wealth is treated as given but stochastic. Our analysis provides the economic rationale for implementing the conditional test of mean-variance efficiency that treats housing wealth as predetermined suggested by Gourieroux and Jouneau (1999).

Our paper builds upon recent work by Flavin and Yamashita (2002), but differs from it in a number of important respects. Flavin and Yamashita characterize the efficiency frontier for house owners, when the house cannot be changed in the short run and there are non-negativity constraints on all assets. But they consider the case where financial returns are not correlated with housing returns, and therefore the main effect of housing is to change the background risk faced by investors. We instead allow for non-zero correlation, and show that even without imposing non-

negativity constraints the optimal portfolio changes, because investors who are house owners hedge housing price risk. We also formally test for the efficiency of household portfolios, and are able to show that many financial portfolios that appear inefficient when housing is neglected are instead efficient, but many others that appear efficient when the hedge term is neglected are instead inefficient.

Our paper is also closely related to Cocco (2005). Cocco numerically derives solutions of an intertemporal optimization problem that includes a risk-free asset, one risky asset, housing and human capital under borrowing and short sale restrictions. That paper is ideally suited to address the issue of limited participation in the risky assets market, but does not investigate how short-term financial portfolio decisions should be made to hedge housing risk. Not only does Cocco limit the investment set to just one risky asset, but he also assumes zero correlation between housing and financial asset returns, thus ruling out hedging motives. Our paper complements Cocco's analysis, by showing how financial portfolios should be chosen at a given point in time, when housing wealth is given, and investigating whether household portfolios are optimally chosen in the presence of housing wealth risk.

To our knowledge, our paper is the first that formally tests for the efficiency of household portfolios and investigates the role played by housing wealth in making portfolios more or less efficient. In particular, our paper shows to what extent households use financial assets to hedge the risk posed by their housing position.

In our application, we use Italian household portfolio data from the Bank of Italy Survey on Household Income and Wealth (SHIW) for 1998 and time series data on financial assets returns as well as housing stock returns to test the hypothesis that observed portfolios are efficient. We show that in our data there are significant partial correlations between financial and housing returns, and argue that similar patterns can be found in other European countries and also in the US.

The paper is organized as follows: section II presents the theory, section III discusses the test statistic and econometric issues, section IV describes the data used, sections V and VI report

efficiency test results, section VII presents results from robustness analysis and section VIII concludes.

## II. Theory

In this section we show how housing wealth can be introduced in the standard mean-variance one-period model – in the Appendix we provide conditions under which our analysis is valid in a multi-period model where housing is not just an investment good but also provides consumption services.<sup>1</sup>

We shall now derive an equation for optimal financial assets holdings in the static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem (see Mayers (1973) and Anderson and Danthine (1981) for the general case where an asset is constrained).

Let us consider a market with a risk-less asset,  $n$  unconstrained and one constrained risky assets.

Denote the first two moments of asset returns as  $\underline{m} + r_f$  (where  $\underline{m} = \begin{pmatrix} \underline{\mu} \\ \mu_H \end{pmatrix}$  and  $\underline{\mu}$  is the expected excess return) and  $\Omega$ . The variance covariance matrix for excess returns can be decomposed in four blocks, corresponding to the  $n$  unconstrained risky assets and the constrained risky asset as follows:

$$(1) \quad \Omega = \begin{bmatrix} \Sigma & \Gamma_{b_i,P} \\ \Gamma'_{b_i,P} & \sigma_P^2 \end{bmatrix}$$

Consider an investor whose portfolio allocation in the risky assets is:

$$(2) \quad Z = \begin{pmatrix} x_0 \\ h_0 \end{pmatrix}$$

where  $\underline{x}_0 \equiv \frac{X_0}{W_0}$  and  $h_0 \equiv \frac{H_0 P_0}{W_0}$

and  $(1-Z)^T \underline{1}$  in the risk-less asset ( $\underline{1}$  is an  $n+1$  vector of ones). Assume that this investor is constrained in his  $h_0$  (that is  $h_0$  is given and equal to  $\bar{h}_0$ ), but otherwise behaves according to the mean-variance model. The investor problem becomes:

$$(3) \quad \begin{cases} \min_Z Z^T \Omega Z \\ \text{s.t.} \begin{cases} Z^T \underline{m} + r_f = m^* \\ h_0 = \bar{h}_0 \end{cases} \end{cases}$$

where  $m^*$  is a given level of expected return.

The problem can be solved by defining the lagrangian:

$$(4) \quad \Lambda = (\underline{x}_0 \Sigma \underline{x}_0^T + h_0^2 \sigma_P^2 + 2h_0 \underline{x}_0 \Gamma_{bP}) - 2\gamma [\underline{x}_0 \underline{\mu} + h_0 \mu_H + r_f - m^*]$$

The first order conditions are:

$$(5) \quad \frac{\partial \Lambda}{\partial \underline{x}_0} = (2\Sigma \underline{x}_0^T + 2h_0 \Gamma_{bP}) - 2\gamma [\underline{\mu}] = 0$$

$$(6) \quad \frac{\partial \Lambda}{\partial \gamma} = \underline{x}_0 \underline{\mu} + h_0 \mu_H + r_f - m^* = 0$$

The solution is:

$$(7) \quad \underline{x}_0 = \gamma \Sigma^{-1} \underline{\mu} - h_0 \Sigma^{-1} \Gamma_{bP}$$

where  $\gamma$  is the Lagrange multiplier of the constraint on the expected return, which has the standard relative risk aversion interpretation (Samuelson (1970)).

This result means that investors have to be efficient with respect to the risky financial assets and choose the efficient Markowitz portfolio according to their risk aversion (see Markowitz (1992)). However, they also use the risky financial assets to hedge their exposure on the constrained asset. If  $\Gamma_{bP}=0$  the hedge term vanishes and portfolio choice can be separated between financial and real assets.

### III. Econometric Issues

In section II we have seen that the notion of efficiency of household portfolios depends on the assumption we make on the nature of housing investment. If housing investment is costless, then the efficient frontier should be computed using all financial assets returns as well as the return on

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<sup>1</sup> The Appendix is available at <http://depts.washington.edu/jfqa/>

housing<sup>2</sup>. If transaction costs affect housing investment, then the analysis differs according to the correlation between housing and financial returns. If this correlation is zero, household portfolios will be mean-variance efficient in the usual sense (i.e.: with respect to the standard financial assets frontier). If this correlation is instead non-zero, household portfolios will be mean-variance efficient once we condition on the value of the housing stock, as shown in equation (7).

In this section we show how we can test for the efficiency of the observed household portfolios in all cases discussed above. In order to do this, we use time series data on asset returns for a period prior to the survey to estimate the mean variance frontier, taking into account the theoretical assumptions of rational expectations and normal return distributions. In particular, we use weighted sample means and covariances in order to estimate expected excess returns and risk (i.e. the first two unconditional moments). The weights are a declining function of the time distance from the end of the sample period.<sup>3</sup>

In the vast literature on efficient portfolios, only a few papers incorporate real estate as an asset. Goetzmann and Ibbotson (1990) and Goetzmann (1993) use regression estimates of real estate price appreciation, and Ross and Zisler (1991) calculate returns from real estate investment trust funds, to characterize the risk and return to the real estate investment. Flavin and Yamashita (2002) use data from the 1968-1992 waves of the Panel Study of Income Dynamics that contain records on the owner's estimated value of the house and compute rates of return from regional real estate price data.

Mean-variance efficiency is usually assessed on the basis of a graphical comparison. However, Jobson and Korkie (1982, 1989) and Gibbons, Ross and Shanken (1989) have proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding

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<sup>2</sup> Housing can be neglected if its return is spanned by financial assets.

<sup>3</sup> Another way is to consider the first two conditional moments from a time series model of the returns data that allows for time-varying conditional heteroskedasticity, as in Blake (1996). This modeling framework requires long time series.



efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return-standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater.

Gourieroux and Jouneau (1999) derive efficiency tests for the conditional or constrained case, i.e. for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as:

$$(8) \quad S_1 = \underline{\mu}^T \Sigma^{-1} \underline{\mu}$$

The Sharpe ratio for the observed (constrained) portfolio made of the first  $n$  (financial) assets is defined in this notation as:

$$(9) \quad S_1(Z) = \frac{[\underline{\mu}^T v_1]^2}{v_1^T \Sigma v_1}$$

where  $v_1^T = \underline{x}_0^T + h_0 \Sigma^{-1} \Gamma_{bp}$  (see equation 7), that is the actual risky financial asset portfolio after eliminating the hedge term.

When all asset returns are normally distributed, Gourieroux and Jouneau show that the Wald statistic:

$$(10) \quad \xi_1 = T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \frac{Z^T \Omega Z}{v_1^T \Sigma v_1}}$$

is distributed as a  $\chi^2(n-1)$  under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier.<sup>4</sup>

Gourieroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting  $v_1 = Z$ . The test statistic becomes

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<sup>4</sup> For the sake of simplicity we do not stress in our notation that the test statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

$$(11) \quad \xi_e = T \frac{\hat{S} - \hat{S}(Z)}{1 + \hat{S}(Z)}$$

where  $\hat{S} = \underline{m}^T \Omega^{-1} \underline{m}$  and  $\hat{S}(Z) = \frac{[\underline{m}^T Z]^2}{Z^T \Omega Z}$ .

$\xi_e$  is distributed as a  $\chi^2(n)$  under the null hypothesis that mean and standard deviation of the observed portfolio lie on the efficient frontier. In this special case, this test is asymptotically equivalent to the test derived by Jobson and Korkie (1982, 1989) and Gibbons, Ross and Shanken (1989).

The intuition behind the conditional (constrained) test is the following. The standard test for portfolio efficiency is based on (the square of) the Sharpe ratio. The Sharpe ratio is in fact the same along the whole efficient frontier (with the exception of the intercept), that is along the capital market line. This test breaks down when one asset is taken as given, because the efficient frontier in the mean-variance space corresponding to all assets is no longer a line, rather a curve. However, equation (7) implies that we can go back to the standard case when the analysis is conducted conditioning on a particular asset, once the hedge term component is subtracted from the observed portfolio. That is, a Sharpe ratio can be used to test for efficiency in the mean variance space corresponding to the “unconstrained” assets, after allowance has been made for the presence of the same hedge term in all efficient portfolios.

It is worth stressing that the test statistic is based on the square of the Sharpe ratio, thus portfolios with Sharpe ratios of the same magnitude but opposite sign are treated in the same way. In our empirical application of the constrained case we shall treat as inefficient those portfolios that have a negative excess return.<sup>5</sup>

In our empirical analysis, we compute efficiency test statistics (either  $\xi_e$  or  $\xi_l$ ) for each household in our sample. In particular, we compute the standard test ( $\xi_e$ ) twice: once for the

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<sup>5</sup> We do this after checking that portfolios with the same standard deviation and excess returns just above zero are indeed counted as inefficient by the formal test.

financial portfolio (as in standard practice), and once for the whole portfolio (inclusive of housing). In this latter case, we also compute the constrained test ( $\xi_I$ ).

We use the computed test statistics in two different ways. First, we show what proportions of household portfolios fail the efficiency tests for a range of possible test sizes (from .10 to .01)<sup>6</sup>. Second, we regress the computed test statistic ( $\xi_I$ ) on household characteristics, income and housing wealth, as a way to investigate possible causes for inefficient portfolio allocations.

#### **IV. The data.**

To show the implications of our theoretical analysis we use data on Italian asset returns and household portfolios. Italy provides a good test case to study the effect of housing on portfolios because home ownership is wide spread and household stock market participation is relatively low but has much increased in recent years. As we shall see, in Italy housing returns unambiguously correlate with financial returns, thus providing the need for a hedge term in house owners portfolios. Finally, an attractive feature of Italy for our purposes is that pension wealth, whose amount is typically not recorded in survey data, is still almost entirely provided by the public pay-as-you-go social security system and is therefore both out of individual investors' control and not directly related to the financial markets performance.

Italian households traditionally have held poorly diversified financial portfolios (Guiso and Jappelli, 2002). In the 1980s and even more in the 1990s, though, the stock exchange has grown considerably and mutual funds have become a commonly held financial instrument. Household financial accounts reveal that the aggregate financial portfolio share in stocks and funds amounted to 16.15% in 1985, 20.69% in 1995 and rose to an unprecedented 46.95% in 1998. This growth in the equity market paralleled the sharp decrease in importance of bank accounts and short-term government debt in household portfolios. These aggregate statistics are uninformative on the participation issue, though. To this end, an analysis of survey data is required. The most widely

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<sup>6</sup> Throughout the paper, we use the term “test size” to denote the probability of type-I error (probability of rejecting the null hypothesis when the null is true). This is sometimes known as significance level.

used Italian survey data, the Bank of Italy-run Survey on Household Income and Wealth (SHIW), shows direct or indirect participation in equity markets (broadly defined to include life insurance, private pensions and own business) to have increased from 26.43% in 1985 to 38.19% in 1995 and to 48.24% in 1998. For comparison, the percentage of homeowners in the same sample hovered around 63-65% over the period. Finally, the share of financial to total wealth in SHIW was 11.7% in 1991 and rose to 14.59% in 1998 – housing wealth accounted for a 68.91% of total wealth in 1991 and fell slightly to 65.81% in 1998 (50.11% to 48.84% if we consider the principal residence only).

These summary statistics clearly show that household financial portfolios have changed a great deal over the years, and that a key role in total household wealth is played by real estate. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios. A financial portfolio may deviate from the mean variance frontier for financial assets simply as a result of its covariance properties with the return on housing equity. This is a relevant issue whether housing wealth is treated as liquid or instead as an illiquid asset.

In our application we use household portfolio data for 1998 and asset return data for the period 1989-1998. The 1998 SHIW wave contains detailed information on asset holdings of 7115 households as of 31.12.1998, as well as self assessed value of their housing stock (both principal residence and other real estate) and actual or imputed rent for each dwelling. For each household we also know the region of residence and a number of demographic characteristics (that are used to characterize departures from efficiency). The survey does not over sample the very rich, and it therefore captures about a third of total household financial wealth. It does cover a relatively large number of assets, including individual pension funds: these are still remarkably unimportant in Italy, though, partly because of inadequate tax incentives. Occupation pension schemes are also

relatively minor, even though recent reforms of the Italian Social Security system (particularly the Dini reform of 1995) imply that they should become wide-spread.<sup>7</sup>

Asset return data cover five major assets: short term government bonds (3-month BOT), medium term government bonds, long-term government bonds (BTP), corporate bonds and equity (the MSCI Italy stock index)<sup>8</sup>. We treat the short-term bond as risk free, and assume that this is the relevant return on bank deposits, once account is taken of non-pecuniary benefits. For medium term, long term and corporate bonds we derive the holding period returns by standard methods. In particular, for medium term we use the RENDISTAT index (the index of the medium term government bonds yields) and we determine the holding period return by assuming a duration of two years. For corporate bonds we use the RENDIOBB index (the index of Italian corporate bond yields) and assume a duration of three years. For long term bonds we use the estimated term structure of interest rates and determine the holding period returns of an equally weighted portfolio based on two assets with a duration of three years and five years. We checked the quality of this estimation by regressing our monthly returns determined with this procedure on those of the MSCI Italian bond index (that are only available since December 1993) and found that the fit is almost perfect ( $R^2$  is equal to 99.62%).

We express all returns net of withholding tax, on the assumption that for most investors other tax distortions are relatively minor (financial asset income is currently subject to a 12.5% withholding tax. Housing is taxed on the basis of its ratable value, while dividends on stocks directly held and actual rental income is taxed at the marginal income tax rate).

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<sup>7</sup> Further information on the survey is provided in Guiso and Jappelli, (2002) and D'Alessio and Faiella (2000). Information on the Italian pension system and its recent reforms is presented in Brugiavini and Fornero (2001).

<sup>8</sup> We take returns on the Italian stock exchange because, as we shall see later, direct investment in foreign assets is rare in our data. We have checked that our results are robust to assuming that roughly half of indirect investment in stocks is held in the MSCI world market index, in line with aggregate statistics on Italian mutual funds portfolios in 1998.

To evaluate the efficiency of households' portfolio we need to determine the expected return and the expected variance covariance matrix of the assets. Given long, stationary series we could simply compute the corresponding sample moments of the assets excess returns. However, this approach is unlikely to work in our case: our sample period is 1989-98 (and cannot be extended because some assets did not exist prior to the mid 1980's), and in the decade we consider we observe a long convergence process of Italian interest rates to German interest rates that accelerated dramatically in the few years before the introduction of the Euro on 1<sup>st</sup> January 1999.

Estimation error is of particular concern for first moments and calls for use of prior information in estimation (see for instance Merton, 1980, and Jorion, 1985). In our case, we should estimate the first moments by a Bayesian method that exploits prior information on convergence of particularly long-term government bond rates to its German equivalent, and possibly a multivariate GARCH for the second moments. Unfortunately, we do not have enough data points to perform sophisticated estimation exercises. In fact, housing returns are available at a semiannual frequency, and we are therefore forced to use at most twenty-one data points. However, we can exploit prior information on convergence by using a simple Weighted Least Squares procedure, where the raw return series data are down weighted more the farther away they are from December 1998. More precisely, we construct the weights to be a geometrically declining function of the lag operator multiplied by  $\alpha$  (where  $\alpha$  is set to 0.8). The weights are then multiplied by a constant so that the expected returns on long term government bills are in line with the actual returns of the German Treasury bond in 1998-9. The weighted series are used to compute sample first and second moments<sup>9</sup>.

In Table 1 we show the first and second moment of the excess returns data we use. These are expressed as percentage semi-annual rates of return net of the time-varying risk-free rate: for the risk-free rate we report only the January 1999 six month Italian Treasury bill interest rate.

[INSERT Table 1 ABOUT HERE]

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<sup>9</sup> A similar procedure for second-order moments is often used in the financial industry (see RiskMetrics, 1999) and can be shown to be equivalent to particular GARCH models (Phelan, 1995).

We see that stocks have higher expected return and higher variance than all other risky financial assets. Correlation coefficients between bonds are quite high (they range between .84 and .97) – correlation coefficients of stocks and bonds are much lower (between .38 and .64). Most correlation coefficients are significantly different from zero at the 1% level.

This picture is however largely incomplete. We know that two households out of three own real estate, and we argued that this type of investment is highly illiquid. It is therefore of great interest for us to compute first and second moments of the housing stock. To this end we use province-level semiannual price data (source: Consulente Immobiliare<sup>10</sup>) covering the whole 1989-98 period. We compute the return on housing by assuming that rent minus maintenance costs is a fixed proportion,  $\kappa$ , of the house price. We set  $\kappa=.025$  (5% on an annual basis), as in Flavin and Yamashita (2002). It is worth stressing that the choice of  $\kappa$  is immaterial in the analysis of the constrained case, as long as  $\kappa$  is a fixed number (see equation (10)), but is important in the case where housing is treated as unconstrained, given that it affects its expected return directly.

Finally, we aggregate housing returns to the macro-region level (provincial resident population numbers were used to generate weights). This way we generate average return data for the North West (NW), North East (NE), Centre (CE) and South (SO).<sup>11</sup> The first and second moments are then determined using the Weighted Least Squares procedure described above.

Table 2 reveals that expected excess returns on housing are highest in the North East and in the South and lowest in Central Italy (they range between 0.73% and 0.61% on a semiannual basis). They are close to returns on bonds, but are much lower than returns on stocks. Housing excess return standard deviations range between 0.46% and 0.78%, and are therefore much lower than on stocks, but comparable to Government and Corporate Bonds. Of interest to us is the negative

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<sup>10</sup> Index constructed using repeat sales of houses.

<sup>11</sup> This is a standard split. The North West includes the three large industrial cities of Milan, Turin and Genoa; the North East includes many middle-sizes cities and towns, such as Bologna, Venice, Verona, Trieste; the Centre (that includes the capital city, Rome, and many medium-sized town such as Florence,

correlation between housing return and most financial asset returns. This is also found in the raw series that is in the series that are not weighted in the way described above.

[INSERT Table 2 ABOUT HERE]

The issue arises of whether these correlations are negligible. Some of the simple correlation coefficients are significantly different from zero (for the NW and CE regions). But simple correlation is not the relevant concept for our analysis: partial correlations are important in a multiple asset setting. The simplest way to assess the relevance of partial correlations is to estimate the coefficients of the hedge term in equation (17) that is to estimate the beta hedge ratio  $\Sigma^{-1}\Gamma_{bp}$ . This can be done by running the regression of housing returns on financial asset returns, as suggested by De Roon, Eichholtz and Koedijk (2002). In our case we use WLS instead of OLS for internal consistency, but stress that OLS point estimates are similar<sup>12</sup>. Parameter estimates and their standard errors are summarized in Table 3.

[INSERT Table 3 ABOUT HERE]

We see that in all regions there is at least one non-zero parameter at the 5% significance level and the slope coefficients are jointly significantly different from zero at the 5% level or lower (the p-value of the F-test is reported at the bottom of the table, together with the  $R^2$ ). The region where this test is least significant is the South (with a p-value of 2.5%).

On the basis of this evidence, we conclude that housing returns present significant correlations with financial asset returns in Italy, and that this provides the basis for introducing a hedge term in household portfolios of house-owners.<sup>13</sup>

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Perugia and Ancona). In our classification the South, that is largely rural, despite the presence of important cities such as Naples and Bari, also includes the two large islands, Sicily and Sardinia.

<sup>12</sup> Significant coefficients retain their signs, but their magnitude and standard errors are inflated.

<sup>13</sup> In our analysis we assume that the relevant stock return is purely domestic. However, according to Bank of Italy aggregate statistics Italian equity mutual funds invested 52% in foreign stocks, 48% in the domestic stock exchange in the fourth quarter of 1998. Unfortunately, we do not know how household indirect equity holdings were split between domestic and foreign stocks, or across countries, but can run a robustness check.



In Tables 4 and 5 we report the percentage participation for each asset and liability recorded in SHIW98 and the corresponding aggregate portfolio share. For instance, we see that almost 75% of the sampled households have a bank current (i.e. checking) account, and that the 27.24% of all financial wealth is held in such accounts. We also show in the last column of Table 4 where each asset is classified, given that we use asset returns data at a much coarser aggregation level. So the first seven assets (cash, various deposits, and repos) are all classified as risk-free. Of particular interest is the relatively low direct stock market participation (7.42% hold listed shares; 1.58% shares in unlisted companies).

[INSERT Table 4 ABOUT HERE]

[INSERT Table 5 ABOUT HERE]

However, 10.86% of all households have mutual funds, and these holdings we classify partly as stocks and partly as bonds. Of great interest to us is the high proportion of households who own some housing stock (almost 70%) and the magnitude of this type of investment (that accounts for 85% of total wealth, see Table 5). Liabilities are relatively wide-spread (10.41% households report mortgage; 12.33% other forms of consumer debt), but their quantitative importance is relatively minor.

In Table 6, we treat mortgages as negative holdings of long-term bonds (the only long term bonds available are on government debt, BTP) and other debt as negative holdings of corporate

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Given that direct stock holdings by SHIW households were mostly domestic, we assume a 50-50 domestic-foreign split in household portfolios, and take as stock return the average of the Italian stock exchange return and the MSCI world stock index return (in local currency). The simple correlation between the domestic return and this mixed stock return is .88; compared to the domestic return, the mixed return has a lower first moment (2.08% rather than 2.29%), but also a lower standard deviation (5.22% rather than 7.49%) resulting in a larger Sharpe ratio. The key regressions of housing returns on financial asset returns produce results quite similar to those shown in Table 3: the coefficients on the stock return lie in the (-.07,-.14) interval for NW, NE and CE, and are all significant. For SO, we find a significant, positive coefficient of .07. The rest of our analysis is largely unaffected. We thank the referee for suggesting this check to us.

bonds (other debt typically has medium term maturity like corporate bonds). On this basis we re-classify our households in 4 mutually exclusive groups. We then show how this classification changes according to macro region. We see that the highest proportion of risk-free asset portfolios (30.1%) is found in SO, the lowest in CE (24.2%). The combination of risk-free and housing assets is highest in SO (49.4%), lowest in NW (33.8%). The combination of risk-free and risky financial assets (included debts) is most common in NE (5.6%), whereas the presence of all three assets is most common in NE (36.2%) and least common in SO (only 18.4%).

[INSERT Table 6 ABOUT HERE]

## V. Estimation and test results: standard analysis.

First we show the mean variance frontier for financial assets alone, using the returns information described in Section IV. We follow the literature and neglect both housing wealth and mortgages and debts. Given that the latter are mostly incurred to purchase housing stock, this is the most natural course of action when analyzing purely financial decisions.

[INSERT Figure 1 ABOUT HERE]

In the upper panel of Figure 1 we show the risky financial assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line). Individual assets are also displayed there: to the far right we have stocks (+ sign), to the extreme left of the risky frontier we find corporate bonds (denoted by a \*). In the lower panel we show where individual portfolios lie. Notice that households who have a financial portfolio are 5920 in total: 76.92% of these only have the risk-free asset while 23.08% also have risky assets.

The tangency of the upper portion of the broken line and the risky financial assets financial frontier defines the market portfolio. In Table 7, first column, we report its weights: the mean-variance efficient portfolio is made of long positions in BTP (long-term government bonds) MTG bonds and stocks, and short position in Corporate bonds<sup>14</sup>.

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<sup>14</sup> In our conditional analysis we treat mortgages as negative corporate bond positions - a negative equilibrium value of corporate bonds is thus possible. To avoid this result, one could aggregate MTG bonds

[INSERT Table 7 ABOUT HERE]

We run the formal efficiency test (described in Section 3) on observed household portfolios. The test statistic is computed for all valid observations (households whose wealth is not entirely in the risk-free asset) and the percentages of non-rejections are computed at different values of the test size (from 10% to 1%). In this case we can compute the test statistic for 1366 households who have at least one risky asset. We find that 612 such households hold efficient portfolios when the test size is set at 10%. When the test size is set at 5% or 1% all 1366 portfolios are considered efficient.

It is perhaps surprising that all portfolios are considered efficient when the test is run at the 5% or 1% levels. This probably reflects three different facts:

- a) most households do not invest in stocks, in line with the tangency portfolio;
- b) returns on bonds are highly correlated – optimization errors on their shares do not result in major efficiency losses;
- c) the efficient frontier is estimated using a small number of observations, and is therefore estimated with limited precision.

All this suggests that the test may have relatively low power, and the appropriate test size should be chosen at a conservative 10%.

Markedly different conclusions on the efficiency of household portfolios are reached if the investment set is extended to housing, and housing is treated as any other asset (that is, it is treated as unconstrained). We find that at any size of the test, there are very few efficient portfolios.

To understand why this happens, we show in Figures 2 and 3 the mean variance frontier for financial assets and housing for two macro regions (given that we know where the households live and house prices differ by region, we compute the relevant statistics for each macro region). We

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and Corporate bonds, given their similar duration and the high correlation of their returns, to obtain a tangency portfolio that has all positive weights (.1747, .8032, .0221). Our key empirical results do not change much if we follow this route.

now treat outstanding mortgages as negative holdings of long-term bonds (BTP) and debts as negative holdings of medium term (corporate) bonds.

[INSERT Figure 2 ABOUT HERE]

In the upper panel of Figure 2 we show the risky assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line) for households living in North Western Italy (NW). Individual assets are also displayed there: to the far right we still have stocks (+ sign), to the extreme left of the risky frontier we find MTG bond (denoted by a star) and corporate bonds (denoted by a \*). Just above corporate bonds is housing (denoted by a square). Even though corporate bonds seem to be a dominated asset, we know from Tables 1 and 2 that its standard deviation is actually lower than the standard deviation on the house. Also, its highly positive correlation with MTG bonds, BTP and stocks gives its short position some insurance value. This is borne out by the mean-variance efficient portfolio weights: as shown in the second column of Table 7, the optimal portfolio weight for housing in the NW region is 60% (and the BTP weight falls relatively to the purely financial portfolio shown in the first column). This high wealth percentage in housing is of course largely explained by our assumption that housing rental rate is as high as 5% in real terms.

In the lower panel we show where individual portfolios lie. We can see graphically that fewer portfolios lie close to the efficient frontier than could be seen in Figure 1. We do not show graphs for NE and CE, because the tangency portfolios are similar to the NW (see the third and fourth column in Table 7) and the actual graphs are quite similar. For Southern Italy (SO) the picture is quite different (see Figure 3): the housing expected return is quite large and partial correlations are not in line with the rest of the country (see Table 3). As a result the optimal portfolio has an extremely large weight on housing (160%). In this case, some observed portfolios appear to be close to the efficient frontier, so a formal test is required.

[INSERT Figure 3 ABOUT HERE]

When we run the formal efficiency test for the country as a whole, though, we find that only a handful of observations are efficient: 1 at 10%, 11 at the 5% and 17 at 1% level.

## **VI. Estimation and test results: financial assets conditioning on housing.**

Our results so far can be summarized as follows:

- When we consider only financial assets, household portfolios are mostly (76%) made of just the risk free asset. Of the diversified portfolios, at most 45% are mean-variance efficient at the 10% level.
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). However, only a tiny fraction diversified household portfolios are now found to be efficient.

We have already argued (see Section II) that the illiquid nature of housing should be taken into account. If consumers hold a large fraction of their wealth in housing for reasons other than investment (because rental markets are imperfect, due to information asymmetry, as argued by Henderson and Ioannides, 1983), and do not trade frequently because of high pecuniary and non-pecuniary costs (Flavin and Nakagawa, 2004), then we should investigate their portfolio efficiency conditional on housing. It is in fact possible (and plausible) that their financial decisions are partly dictated by the need to hedge some of the risks connected with their illiquid housing investment.

For each household who has non-zero housing wealth we can compute a specific conditional efficiency frontier that treats housing as constrained (for those without housing the frontier displayed in Figure 1 still applies). It's worth stressing that in the constrained case the risk-free portfolio cannot be attained, except trivially (zero housing). This explains why the efficient frontiers we display in Figure 4 are not broken lines, contrary to what we have in Section V. We display the unconstrained and a few constrained frontiers, corresponding to a random sub-sample of homeowners whose actual portfolio is also shown (marked with a plus sign).

Let us consider Figure 4 in detail. This depicts the unconditional frontier with housing for the North West: the presence of a risk-free asset makes it a broken line. We also show two constrained frontiers for the same region, corresponding to two different shares of housing to total wealth (the frontier marked 1 has 18% of wealth into housing; the frontier marked 2 has 47% of total wealth into housing. They correspond to two observed portfolios, displayed as points  $x_1$  and  $x_2$ ). These frontiers lie entirely to the right of the unconstrained frontier (apart from a tangency point, corresponding to the case where the housing portfolio share is at its optimal value). They do not touch the vertical axis, because a risk free position cannot be achieved with positive housing wealth, given the correlations shown in Table 2.

[INSERT Figure 4 ABOUT HERE]

We can now compute the test statistic for the conditional portfolios,  $\zeta_l$  (defined in equation (10), Section III), and calculate for how many portfolios the test fails to reject the null hypothesis of mean variance efficiency. The test is not defined in the case of portfolios made entirely of risk-free assets (it is a ratio of zero to zero), and is identical to the standard test of Section V for portfolios consisting of just financial assets.

In the case of portfolios consisting of risk free plus housing, the test statistic takes the same value within the same region by construction. Also, in our application the expected return for all these portfolios (net of the hedge term) within regions is negative. Therefore, all portfolios with housing but no financial assets are inefficient.

When we consider housing as a constraint, we classify a much smaller number of households in the risk-free portfolios category: 1567 instead of 4554. In fact, of those without risky financial assets, house-owners without a mortgage are now classified in the risk-free + house category (2499 households in all), house-owners with a mortgage or debt (491) could be classified in the last category (risk free+ house+ financial assets), because the mortgage is treated as a negative position on long term government bonds, but we keep them separate in our analysis because all such

portfolios turn out to be inefficient for all test sizes. Therefore, the diversified portfolios we consider conditional on housing are 1140.

Table 8 reports efficiency results for the 1363 households who have diversified portfolios<sup>15</sup> (223 with have a well-diversified financial portfolio, but no housing, and 1140 well-diversified financial portfolio and housing). We see that the test fails to reject efficiency in 261 cases (19%) at the 10% significance level, and this number rises to 595 (44%) at the 5% level and 901 (66%) at the 1% level. Not surprisingly, we find that conditioning on housing many more portfolios are efficient than treating housing as an unconstrained asset (as stressed graphically in Flavin and Yamashita, 2002).

[INSERT Table 8 ABOUT HERE]

If we look at the group of households who have a well-diversified financial portfolio (but no housing), we find that 46.64% of these portfolios are efficient when the test is conducted at the 10% significance level. It's worth stressing that the households that fall in this category are just 223.

In the more interesting case, where the household holds both housing and risky financial assets (1140 observations), we find that 157 cases are efficient at the 10% significance levels (14%). When we run the test at the 5% significance level, we find that 33% of these households hold efficient portfolios (372 in all, see Table 8). This number rises to 678 (59% of the group) at the 1% level.

The efficiency test results displayed in Table 8 suggest that a non-negligible proportion of house owners hold portfolios that are not far from their conditional (or constrained) mean variance frontier. This is in stark contrast to the case where housing is treated as a freely-chosen asset (the unconditional test discussed in section V).

Let us now consider the 1140 fully diversified portfolios (risk free, risky financial assets and housing). In Table 9 we cross tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion (at the 10% level for both test statistics):

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<sup>15</sup> Compared to Section V, we have dropped three observations because of a missing value on the house value

[INSERT Table 9 ABOUT HERE]

We find that as many as 437 portfolios are classified as efficient when housing is neglected, but inefficient when it is considered. This suggests that hedging opportunities are not fully exploited. This is partly compensated by the presence of 86 portfolios for which the reverse holds. This could be evidence that these households use financial assets to hedge housing risk, but could also reveal that housing has diversification properties (for house owners, financial risks are relatively small compared to total wealth). Given the high correlations found (see Table 3) and the very large weight attached to housing wealth, the failure to exploit hedging opportunities outweighs the benefits from diversification, and the number of conditionally efficient portfolios (157) is smaller than the number of efficient financial portfolios (508). Similar conclusions can be drawn when the chosen test size for the conditional test is 5% (and kept at 10% for the financial test).

It's worth stressing that the estimated coefficients in Table 3 are the relevant indicators of the way hedging should be performed. For instance, in three regions out of four, more should be invested in MTG bonds compared to the mean-variance efficient portfolio weights displayed in Table 7 (the exception is the SO).

In Table 10 we display efficiency results by region (for two different test sizes: 10% in Panel A and 5% in Panel B): we see that the highest proportion of efficient portfolios obtains in the NW. This is particularly true for the conditional analysis – apparently NW households are the best at hedging housing risk. Purely financial portfolios are instead most often efficient in the SO.

[INSERT Table 10 ABOUT HERE]

The question arises of what makes a household more likely to hold an efficient portfolio. To address it we run a regression of the test statistic ( $\xi_I$ ) on observable household characteristics such as age, education and employment position of the head, region, household income and housing wealth (see Pelizzon and Weber, 2006, for further details). Our key findings are that for high or low income levels inefficiency is lower, residence in the NW also has a negative effect on inefficiency, whereas a larger home value has a strong, positive impact.



## VII. Discussion of empirical results and extensions

An important issue that arises when housing is included in the asset mix is how to account for the liability every household has – to live somewhere. This issue has been stressed in a number of papers (Sinai and Souleles, 2005, Banks et al, 2004, Yao and Zhang, 2005), who point out that housing is a hedge against increases in the price of housing services. It is clear that the risk posed by price increases of housing services is the more important; the less easy it is to substitute out of housing into other goods and services. An extreme example where this substitution cannot take place at all is the case where housing consumption is already at its physical minimum.

In the framework we propose in this paper, we can account for housing needs in a relatively simple way. We define a minimum physical threshold for the main residence as  $\overline{H}$  and estimate it in our data. We then take the observed price per squared meter as given and include in wealth only the difference between the current housing wealth and its minimum multiplied by that price. However, when the reported price is much higher than the local average, we replace it with a large, but more sensible value (on the assumption that the household could buy at that lower price if they moved into the smallest possible residence within the same area).

We define  $(P_H H - \overline{P_H} \overline{H})$  as net housing wealth where  $P_H H$  is the declared house value,  $\overline{H}$  is the minimum house size for a given family size and  $\overline{P_H}$  is the relevant alternative house price within the current area of residence. We take the sample first percentile of squared meters for all possible family sizes as the minimum house size (this gives us: 20 square meters for a single, 35 for a couple, 40 for couple with one or two children, 46 for larger families). These values are in line with housing regulations (a single room must be at least 9 square meters in Italy).<sup>16</sup> As for prices, for each household we take the observed price per-square meter, except in those few cases where reported house prices are at the upper end of the distribution (top percentile), where we set them to 6m lire per square meter (roughly 3000 euros).

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<sup>16</sup> A square meter is roughly equivalent to 10 square feet.

The resulting net housing wealth variable has an average value of almost 143,000 euros, whereas the original home value is 216,000. Our procedure suggests that about a third of housing wealth should be disregarded when deciding the financial portfolio allocation, because of the housing liability discussed so far.

[INSERT Table 11 ABOUT HERE]

When we can compare the results of Table 11 to those in Table 8, we see that taking the housing liability into account makes some 6%-10% more fully diversified portfolios conditionally efficient, depending on the chosen size of the test.<sup>17</sup>

[INSERT Table 12 ABOUT HERE]

Table 12, which compares directly to Table 9, shows that the fraction of portfolios that are efficient according to both financial and conditional tests raises substantially (from 6.2% to 9.9%). This is due to a reduction in the number of portfolios that are efficient financially, inefficient conditionally.

The definition of net housing wealth as the difference between the existing main residence and a minimal residence, that meets exogenously defined housing needs, is attractive, but fails to capture preference heterogeneity. Households with a strong preference for housing may consider a much higher minimum threshold for housing services than households with a weaker preference for housing. A better approximation of housing needs may then be as a given proportion of housing services currently enjoyed.

We thus consider an alternative way to account for the notion that only a part of the main residence is perceived as wealth. We assume that households are not willing to reduce their housing consumption below a given fraction,  $x$ , of their existing consumption. This has an effect on the way they consider their main residence, but no effect on other real estate. We thus define net housing wealth,  $nhw$ , as:

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<sup>17</sup> We lose thirty observations because total wealth becomes negative, five of which have fully diversified portfolios. This explains why we have 1135 households in Table 15, as opposed to 1140 in Table 8.

$$nhw = (1 - x) * \text{main residence} + \text{other real estate} - \text{housing debt} .$$

We see that  $x$  times the main residence is the minimum threshold below which a household is not willing to go – for this reason this is not counted as wealth.

Total wealth is the sum of financial wealth and net housing wealth. If  $x = 0$  we are in the case considered in Section VI, if  $x = 1$  total wealth is financial wealth (as in the first part of Section V), for those households who have neither other real estate, nor housing-related debt. However, even if  $x=1$ , total wealth does not coincide with financial wealth for households who have other real estate or housing-related debt.<sup>18</sup>

We then check to what extent conditional efficiency coincides with financial efficiency as a function of  $x$ . Figure 5 shows the results for the sample of 1140 households who have both housing and risky financial assets. The lower curve represents the fraction of portfolios that are conditionally efficient out of all portfolios that are financially efficient (508 observations). The upper curve represents the fraction of portfolios that are conditionally inefficient out of all portfolios that are financially inefficient (632 observations). As expected, these two fractions increase in  $x$  that is conditional efficiency tends to coincide with financial efficiency when the less the main residence is counted as wealth. As explained above, we apply this  $x$  correction to the main residence only – this explains why these proportions in Figure 10 do not reach unity even when we subtract 100% of the main residence value from housing wealth. In fact, 174 out of 508 (34%) households whose portfolios are financially efficient have other real estate, 244 out of 632 (39%) of households whose portfolios are financially inefficient have other real estate.

[INSERT Figure 5 ABOUT HERE]

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<sup>18</sup> For all values of  $x$ , we make total wealth coincide with financial wealth also for those households whose net housing wealth is negative. In this case in fact we replace **nhw** with zero: this replacement occurs in relatively few cases in our sample, because housing debt is a relatively minor item (we set **nhw** to zero in less than 50 cases for  $x < 90\%$ , 65 cases when  $x = 90\%$  and 119 cases when  $x = 100\%$ )

We see that the proportion of portfolios that are efficient on both counts steadily increases: there are only 71 (14%) such portfolios when  $x=0$  (see the first main diagonal entry in Table 9), and as many as 389 (77%) when  $x=1$ . This suggests that households who neglect real estate in their portfolio choice (while achieving financial efficiency) may do so for good reasons – because they do not consider most of their main residence disposable (part of wealth).

The picture for financially inefficient portfolios is different: the fraction of inefficient portfolios on both counts declines in  $x$  from 546 (86%) when  $x = 0$  (see Table 9, second main diagonal entry) down to 515 (81%) when  $x=70\%$ , then increases to 586 (93%). This suggests that some of these households may hedge housing risk, particularly if they consider about a third of the total main residence value as wealth, but at least four out of five hold inefficient portfolios irrespective of the hedging motive.

Another issue worth considering is the effect of differential underreporting. We know from D’Alessio and Faiella (2000) that SHIW98 underestimates financial wealth by a wide margin (it accounts only for a third of aggregate household financial wealth), whereas housing wealth is in line with aggregate statistics. The reasons why financial wealth falls short of aggregate statistics can be non-response among the rich and under-reporting among those who do respond. To assess whether the latter has an important impact on our test, we take the extreme case where differential non-response is not an issue, multiply all financial wealth holdings by a factor of three and re-run the test.

Table 13 displays efficiency test results in this hypothetical case, where all households report the same fraction of their financial wealth. If we compare these results with those in Table 8, we see that more fully diversified portfolios are counted as efficient (for instance: 321 instead of 157 at the 10% level). This increase is in line with expectations (the hedge motive is relatively less important if housing wealth has a lower portfolio weight) and is quite sizeable (now 28.16% of fully diversified portfolios are efficient, rather than 13.78%).

[INSERT Table 13 ABOUT HERE]

One final issue is worth careful consideration. First and second moments of financial asset returns have been estimated using relatively accurate asset price data. Housing returns are instead based on averages of local house price data that are more likely to be affected by sampling variability. In Pelizzon and Weber (2006) we show how our analysis carries through to the case where housing returns are measured with error.

### **VIII. Conclusions**

In this paper we have argued that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk.

We have shown that, if financial assets' and housing returns are correlated, the intertemporal expected utility model subject to transaction costs in housing investment implies that financial decisions are affected by the need to hedge some of the risks connected with the existing illiquid housing position. In particular, the investors' optimal strategy is to choose the standard Markowitz portfolio according to their risk aversion and use the risky financial assets to hedge their expositions on the constrained asset (this last decision is independent of their risk aversion). This hedging motive disappears in the case of zero correlation between housing return and financial returns, in which case housing price risk only affects the investor's degree of risk aversion (Flavin and Nakagawa (2004)).

We have also shown that the optimal investment in risky financial assets is equal to the one derived in a static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem. Gourieroux and Jouneau (1999) have proposed an efficiency test for analyzing the performance of a portfolio of risky assets (in a mean-variance framework) when some constraints exist on a part of the portfolio. We are then able to claim that this test can be applied for a more general test of portfolio efficiency.

In our application, we have used Italian household portfolio data and time series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another, unconstrained asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework.

Our empirical results support the view that the presence of illiquid wealth plays an important role in determining whether portfolios chosen by home-owners are efficient.

Our results can be summarized as follows:

- When we consider only financial assets, three portfolios out of four are made of just the risk free asset. Of the diversified portfolios, a large fraction (45%) is mean-variance efficient;
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). But very few diversified household portfolios are found to be efficient when housing is treated as unconstrained.
- When we calculate the efficiency test conditional on housing we find that one in seven of fully diversified portfolios (that include the risk-free asset, housing and risky assets) are mean-variance efficient. We also find that these are largely not the same households whose financial portfolios were considered efficient.
- An important issue that arises when housing is included in the asset mix is how to account for the liability households have to live somewhere. In our robustness analysis, we propose two alternative ways to do this. First, we define net housing wealth as the difference between the home value and the value of the smallest property a household could move to in the same area. On average, our net housing wealth variable is worth around two thirds of gross housing wealth. We show that taking the housing liability into account this way increases the number of conditionally efficient fully diversified portfolios by a half. Second, we assume that housing needs are a given fraction of the housing services currently enjoyed. Net housing wealth is the

difference between total housing wealth and the fraction required to meet housing needs. As this fraction approaches unity, we find that an increasing proportion of financially efficient portfolios are also conditionally efficient. Financially inefficient portfolios, instead, are more often conditionally efficient when this fraction rises to 70%, then more often inefficient.

In summary, compared to the efficiency results relating to portfolios consisting solely of financial assets such as stocks, bonds and a risk-free asset, the introduction of housing and mortgage alters the risk and return trade-off in a direction which pushes very few household portfolios to be efficient. This is not the case, once the illiquid nature of housing investment is taken into account, but there is strong evidence that hedging opportunities are not fully exploited even by those Italian households who hold well-diversified portfolios. This widespread failure to hedge house price risk has important implications for portfolio management.

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**Table 1: Sample First and Second Moments of Asset Excess Returns (1989-98)**

	<b>BOT</b>	<b>BTP</b>	<b>MTG-Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>Expected return %</b>	1.3169	0.8021	0.427	0.4495	2.2932
<b>Standard Deviation %</b>		1.2223	0.6469	0.7809	7.4875
<b>CORRELATION</b>					
<b>BTP</b>		0.965**	0.842**	0.379	
<b>MTG-bonds</b>			0.871**	0.383	
<b>Corporate Bonds</b>				0.635**	

The upper panel of Table 1 shows the descriptive statistics for the semiannual weighted excess returns of the four risky assets used in the analysis, as well as the risk-free rate that refers to 1<sup>st</sup> January 1999. The lower panel reports correlation coefficients of the same excess returns; \*\* indicates significant at 1% level.

**Table 2: Expected Excess Returns and Correlation Matrix of Housing (1989-98)**

	<b>NW</b>	<b>NE</b>	<b>CE</b>	<b>SO</b>
<b>Expected excess return %</b>	0.6143	0.7108	0.6517	0.7303
<b>Standard deviation %</b>	0.7816	0.4607	0.5439	0.4986
<b>Correlations</b>				
<b>BTP</b>	0.018	-0.140	-0.237	-0.274
<b>MTG-bonds</b>	-0.0752	-0.246	-0.355	-0.142
<b>Corporate bonds</b>	-0.150	-0.086	-0.524*	-0.245
<b>Stocks</b>	-0.671**	-0.270	-0.675**	0.031

The upper panel of Table 2 shows the descriptive statistics for the semiannual weighted excess returns of housing for the four macro regions. The lower panel reports correlation coefficients of the same excess returns with financial asset excess returns; \* indicates significant at 5% level \*\* indicates significant at 1% level.

**Table 3: Regression of Excess Return on Housing on Financial Assets Excess Returns**

<b>Variable</b>	<b>NW</b>	<b>NE</b>	<b>CE</b>	<b>SO</b>
<b>Constant</b>	-0.00141 (.00107)	-0.0003 (.00095)	-0.00041 (.00095)	-0.00037 (.00106)
<b>r<sub>BTP</sub></b>	0.928275 (.28242)	0.559817 (.24867)	0.714788 (.25063)	-0.82673 (.27920)
<b>r<sub>MTG</sub></b>	-2.38735 (.60929)	-1.88857 (.53646)	-1.29275 (.54069)	2.014983 (.60233)
<b>r<sub>BONDS</sub></b>	1.080841 (.3165)	0.848318 (.27867)	-0.11126 (.28087)	-0.73687 (.31289)
<b>r<sub>STOCKS</sub></b>	-0.12004 (.01756)	-0.04496 (.015464)	-0.04314 (.01559)	0.0354 (.01736)
<b>p-value</b>	0.000034	0.014788	0.001414	0.02506
<b>R<sup>2</sup></b>	0.784422	0.518884	0.649422	0.482323

Table 3 shows estimation results from the regression of each macro region housing excess return on financial asset returns. The p-value refers to the F-test of joint significance of all slope parameters; **R<sup>2</sup>** is the unadjusted coefficient of determination. Standard errors are in parentheses. Number of observations = 21.

**Table 4: Participation Decision - Individual Financial and Real Assets**

<b>Asset</b>	<b>Participation</b>	<b>Broad Asset</b>
Cash	100%	Risk-free
Bank Current Account Deposits	74.94%	Risk-free
Bank Savings Deposits (Registered)	19.31%	Risk-free
Bank Savings Deposits (Bearer)	10.90%	Risk-free
Certificates of deposit	3.68%	Risk-free
Repos	0.94%	Risk-free
Post Office Current Accounts and Deposit Books	11.43%	Risk-free
Post Office Savings Certificates	6.55%	Long-Term
BOT (Italian T-bills)	9.67%	Risk-free
CCT (Italian T-certificates)	4.74%	Risk-free
BTP (Italian T-bonds)	2.70%	Long-Term
CTZ (Italian zero-coupon)	0.78%	Medium-Term
Other Italian Government Debt (CTEs, CTOs, etc.)	0.31%	Medium-Term
Corporate Bonds	5.55%	Bonds
Mutual Funds	10.86%	Bonds (1/2) Stocks (1/2)
Shares of listed companies	7.42%	Stocks
<i>of which:</i> of privatized companies	4.30%	Stocks
Shares of unlisted companies	1.58%	Stocks
Shares of limited liability companies	0.53%	Stocks
Shares of partnerships	0.15%	Stocks
Managed Savings (by banks)	2.03%	Bonds (1/2) Stocks (1/2)
Managed Savings (by other financial intermediaries)	0.5%	Bonds (1/2) Stocks (1/2)
Managed Savings by Trust Companies	0.06%	Bonds (1/2) Stocks (1/2)
Foreign bonds and government securities	0.52%	Bonds (1/2) Stocks (1/2)
Foreign Stocks and Shares	0.46%	Bonds (1/2) Stocks (1/2)
Other foreign assets	0.05%	Bonds (1/2) Stocks (1/2)
Loans to co-operatives	1.67%	Stocks
House	69.76%	House
Mortgage	10.41%	Long Term (neg. position)
Debt	12.33%	Bonds (neg. position)

Table 4 shows the proportion of households reporting positive holdings of each asset recorded in SHIW98, as well as the way each asset is classified for the purpose of our efficiency analysis.

**Table 5: Portfolio Share - Individual Financial and Real Assets**

<b>Asset</b>	<b>Portfolio share (financial wealth)</b>	<b>Portfolio share (Total wealth)</b>
Cash	2.13%	0.31%
Bank Current Account Deposits	27.24%	2.86%
Bank Savings Deposits (Registered)	4.94%	1.00%
Bank Savings Deposits (Bearer)	2.75%	0.48%
Certificates of deposit	2.52%	0.50%
Repos	1.19%	0.25%
Post Office Current Accounts and Deposit Books	2.54%	0.38%
Post Office Savings Certificates	2.00%	0.31%
BOT (Italian T-bills)	7.64%	1.22%
CCT (Italian T-certificates)	3.92%	0.58%
BTP (Italian T-bonds)	2.14%	0.37%
CTZ (Italian zero-coupon)	0.31%	0.06%
Other Italian Government Debt (CTEs, CTOs, etc.)	0.34%	0.04%
Corporate Bonds	4.92%	0.74%
Mutual Funds	13.99%	2.25%
Shares of listed companies	5.90%	0.99%
<i>of which:</i> of privatized companies	1.86%	0.29%
Shares of unlisted companies	0.77%	0.12%
Shares of limited liability companies	2.19%	0.26%
Shares of partnerships	1.30%	0.16%
Managed Savings (by banks)	6.62%	1.23%
Managed Savings (by other financial intermediaries)	1.53%	0.31%
Managed Savings by Trust Companies	0.04%	0.01%
Foreign bonds and government securities	0.25%	0.08%
Foreign Stocks and Shares	0.14%	0.03%
Other foreign assets	0.00%	0.00%
Loans to co-operatives	0.80%	0.14%
House		85.05%
Mortgage		-2.07%
Debt		-0.54%

Table 5 shows the aggregate portfolio shares for each asset recorded in the SHIW98. They are defined relative to financial wealth or total wealth (the sum of financial wealth, housing wealth net of mortgage and debt).

**Table 6 – Classification by Region.**

	Total		NW		NE		CE		SO	
	N°	%	N°	%	N°	%	N°	%	N°	%
<b>Risk-free asset</b>	1567	26.47%	385	27.13%	217	20.43%	291	24.25%	674	30.10%
<b>Risk-free asset + housing</b>	2499	42.21%	479	33.76%	402	37.85%	511	42.58%	1107	49.44%
<b>Risk-free + risky assets</b>	223	3.77%	78	5.50%	59	5.56%	41	3.42%	45	2.01%
<b>Risk-free + risky assets + housing</b>	1631	27.55%	477	33.62%	384	36.16%	357	29.75%	413	18.45%
<b>Total assets</b>	<b>5920</b>	<b>100%</b>	<b>1419</b>	<b>100%</b>	<b>1062</b>	<b>100%</b>	<b>1200</b>	<b>100%</b>	<b>2239</b>	<b>100%</b>

Table 6 shows the number and proportion of households holding various combinations of assets in each macro region and in the country as a whole (total). Mortgages and debt are treated as negative positions in risky assets.

**Table 7: Tangency Portfolio Weights**

	Financial Assets	Financial assets and housing			
		NW	NE	CE	SO
<b>BTP</b>	0.2923	-0.5462	-0.3324	-0.3829	1.4222
<b>MTG-bonds</b>	0.8932	1.5346	1.21856	0.7724	-3.099
<b>Corporate bonds</b>	-0.2030	-0.6615	-0.5252	0.0476	1.1269
<b>Stocks</b>	0.0175	0.0735	0.02842	0.0243	-0.052
<b>House</b>	--	0.5995	0.61071	0.5386	1.6027

Table 7 reports the tangency portfolio weights for the case when wealth is made of financial assets alone (column 1), and when wealth includes housing and debt (columns 2, 3, 4 and 5). Different tangency portfolios are computed for the four macro regions (NW, NE, CE and SO).

**Table 8. Efficient Portfolios Conditional on Housing**

Test size		10%		5%		1%	
Portfolios	Tot. N.	N	%	N	%	N	%
<b>Risk-free + Risky fin. Ass.</b>	<b>223</b>	104	46.64%	223	100.00%	223	100.00%
<b>Risk-free + Risky fin. ass + Housing</b>	<b>1140</b>	157	13.78%	372	32.63%	678	59.40%
<b>Total</b>	<b>1363</b>	<b>261</b>	<b>19.15%</b>	<b>595</b>	<b>43.65%</b>	<b>901</b>	<b>66.10%</b>

Table 8 shows the numbers and proportions of efficient portfolios conditional on housing, for three different test sizes. The sample is restricted to households who have at least one risky financial asset.

**Table 9. How Diversified Portfolios are Classified: a Comparison**

Test size = 10%	Efficient (Financial)	Inefficient (Financial)	Total
Efficient (conditional)	71	86	157
Inefficient (conditional)	437	546	983
<b>Total</b>	508	632	1140

Table 9 shows the numbers of portfolios with at least one risky financial asset or liability, according to the way they are classified by the efficiency tests run at the 10% level.

**Table 10. Efficient Portfolios Conditional on Housing by Region**

Panel A: Test size = 10%								
	NW		NE		Centre		South	
Portfolios	N	%	N	%	N	%	N	%
Risk-free + Risky fin. Ass.	30	38.46%	23	38.98%	17	41.46%	34	75.56%
Risk-free + Risky fin. Ass + house	112	29.71%	11	3.86%	15	5.86%	19	8.56%
<b>Total</b>	<b>142</b>	<b>31.21%</b>	<b>34</b>	<b>9.88%</b>	<b>32</b>	<b>10.77%</b>	<b>53</b>	<b>19.85%</b>
Panel B: Test size = 5%								
	NW		NE		Centre		South	
Portfolios	N	%	N	%	N	%	N	%
Risk-free + Risky fin. Ass.	78	100.00%	59	100.00%	41	100.00%	45	100.00%
Risk-free + Risky fin. Ass + house	169	44.83%	83	29.12%	73	28.52%	47	21.17%
<b>Total</b>	<b>247</b>	<b>54.29%</b>	<b>142</b>	<b>41.28%</b>	<b>114</b>	<b>38.38%</b>	<b>92</b>	<b>34.46%</b>

Table 10 shows the numbers and proportions of efficient portfolios conditional on housing by macro region. Panel A reports results for a 10% test size, Panel B for a 5% test size. The sample is restricted to households who have at least one risky financial asset.

**Table 11. Efficient Portfolios Conditional on Net Housing Wealth**

Test size		10%		5%		1%	
Portfolios	Tot. N	N	%	N	%	N	%
Risk-free + Risky fin. Ass.	223	104	46.64%	223	100.00%	223	100.00%
Risk-free + Risky fin. ass + House	1135	216	19.03%	479	42.20%	756	66.61%
<b>Total</b>	<b>1358</b>	<b>320</b>	<b>23.56%</b>	<b>702</b>	<b>51.69%</b>	<b>979</b>	<b>72.09%</b>

Table 11 shows the numbers and proportions of efficient portfolios conditional on housing, for three different test sizes. The sample is restricted to households who have at least one risky financial asset. Housing wealth is defined net of exogenously determined housing needs.

**Table 12. How Diversified Portfolios are Classified – Net Housing Wealth**

<b>Test size = 10%</b>	<b>Efficient (Financial)</b>	<b>Inefficient (Financial)</b>	<b>Total</b>
<b>Efficient (conditional)</b>	<b>112</b>	104	216
<b>Inefficient (conditional)</b>	393	<b>526</b>	919
<b>Total</b>	<b>505</b>	<b>630</b>	<b>1135</b>

Table 12 shows the numbers of portfolios with at least one risky financial asset, according to the way they are classified by the efficiency tests run at the 10% level. Housing wealth is defined net of exogenously determined housing needs.

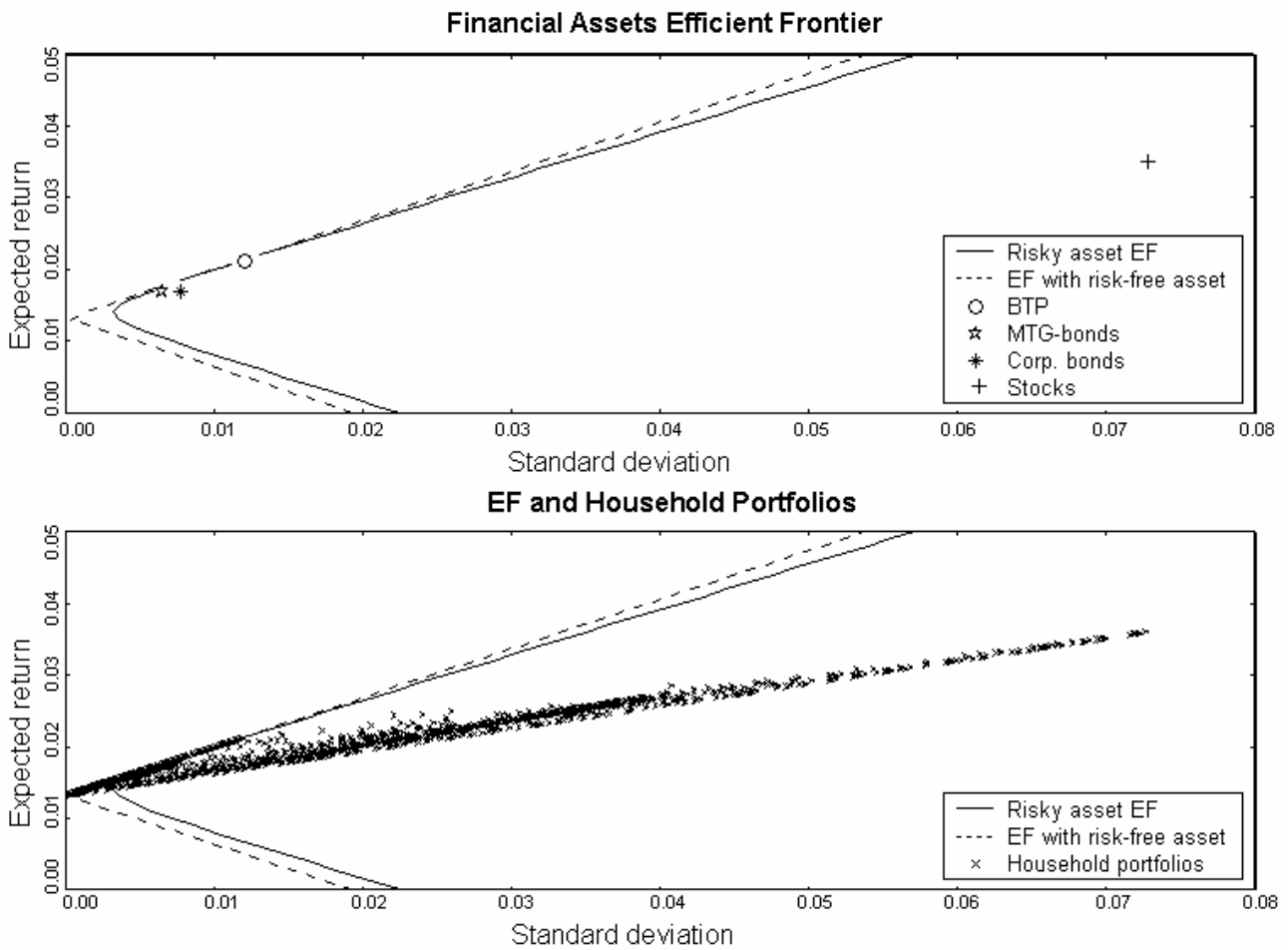
**Table 13. Efficient Portfolios Conditional on Housing Corrected for Under-Reporting**

<b>Test size</b>		<b>10%</b>		<b>5%</b>		<b>1%</b>	
<b>Portfolios</b>	<b>Tot. N</b>	<b>N</b>	<b>%</b>	<b>N</b>	<b>%</b>	<b>N</b>	<b>%</b>
<b>Risk-free + Risky fin. Ass.</b>	<b>223</b>	104	46.64%	223	100.00%	223	100.00%
<b>Risk-free + Risky fin. ass + House</b>	<b>1140</b>	321	28.16%	682	59.82%	895	78.51%
<b>Total</b>	<b>1363</b>	<b>425</b>	<b>31.18%</b>	<b>905</b>	<b>66.40%</b>	<b>1118</b>	<b>82.02%</b>

Table 13 shows the numbers and proportions of efficient portfolios conditional on housing, for three different test sizes. The sample is restricted to households who have at least one risky financial asset. Financial wealth is corrected for under-reporting.

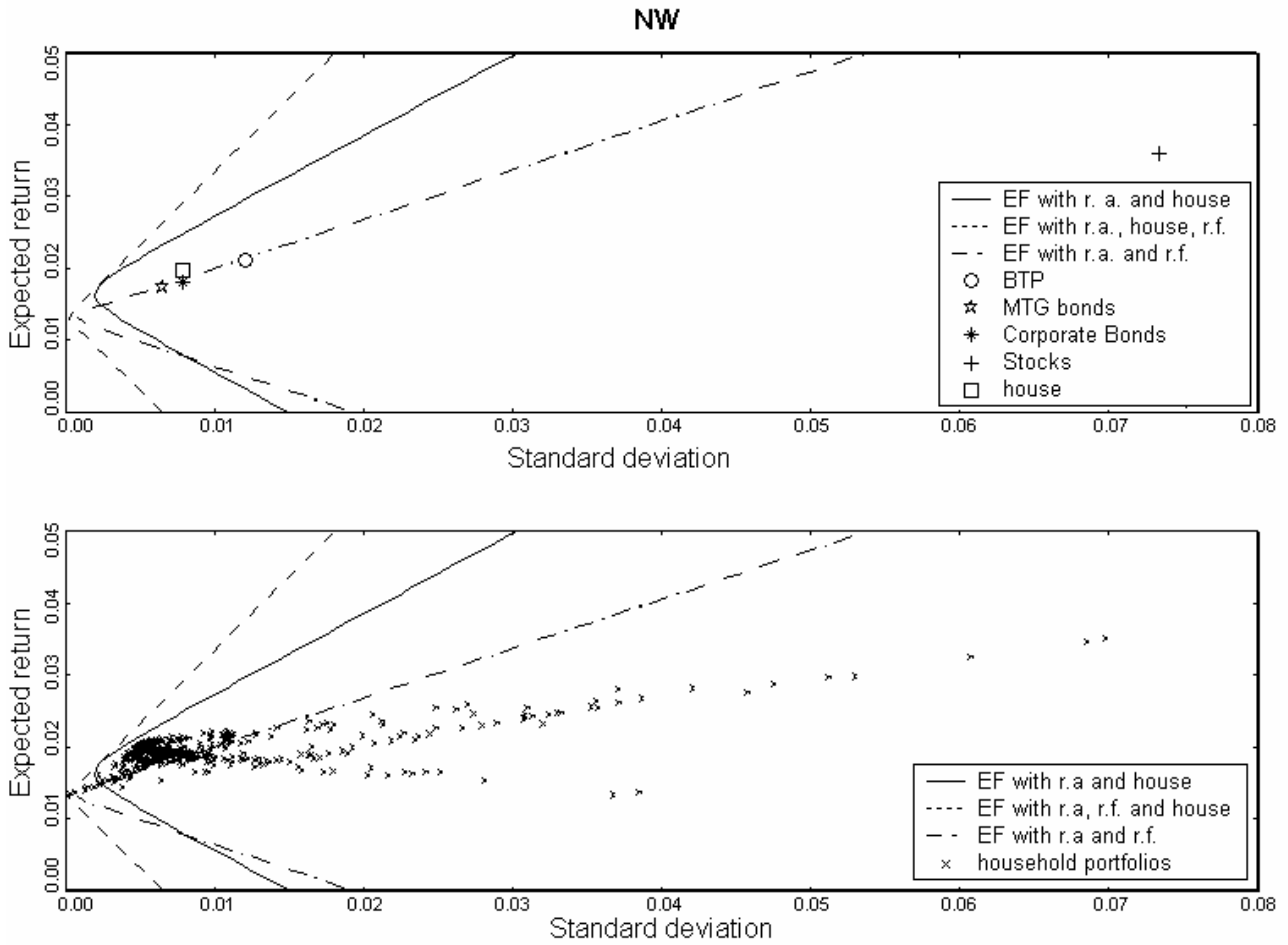


**Figure 1 – Financial Asset Efficient Frontier.**



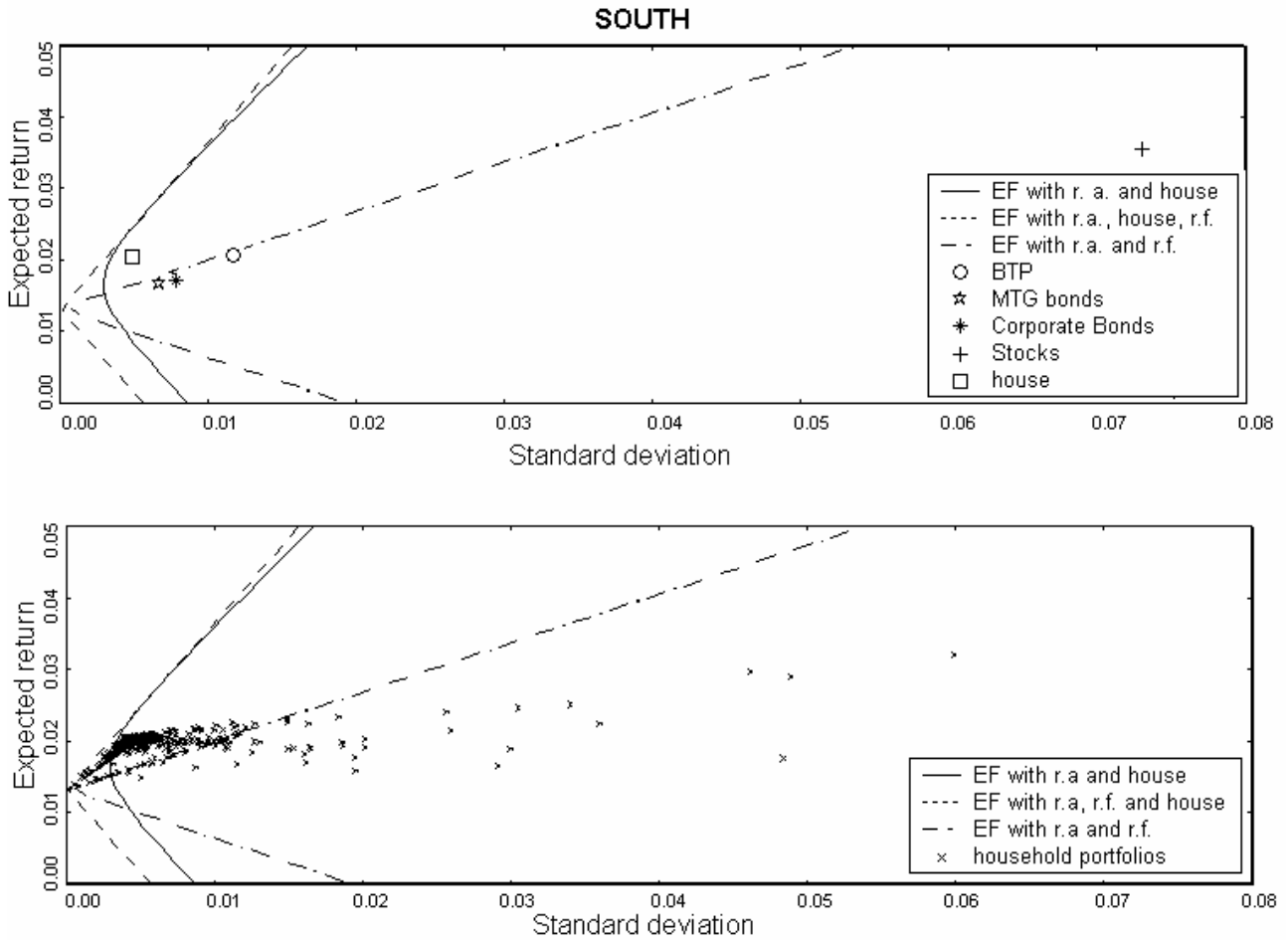
The upper portion of Figure 1 shows the four individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky financial assets efficient frontier and the efficient frontier with the risk-free asset. The lower portion shows, on the same scale, the efficient frontiers as above and the observed household portfolios.

**Figure 2 – NW Efficient Frontiers with Housing**



The upper portion of Figure 2 shows the five individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky assets efficient frontier and the efficient frontier with the risk-free asset. The corresponding efficiency frontiers without housing (see Figure 1) are also shown for comparison. The lower portion shows, on the same scale, the efficient frontiers as above and the observed household portfolios. All these refer to the North West macro area (NW).

**Figure 3 – SO Efficient Frontiers with Housing**



The upper portion of Figure 3 shows the five individual asset expected returns (y-axis) and standard deviations (x-axis) as well as the risky assets efficient frontier and the efficient frontier with the risk-free asset. The corresponding efficiency frontiers without housing (see Figure 1) are also shown for comparison. The lower portion shows, on the same scale, the efficient frontiers as above and the observed household portfolios. All these refer to the South macro area (SO).

**Figure 4 – NW efficient frontiers conditional on housing**

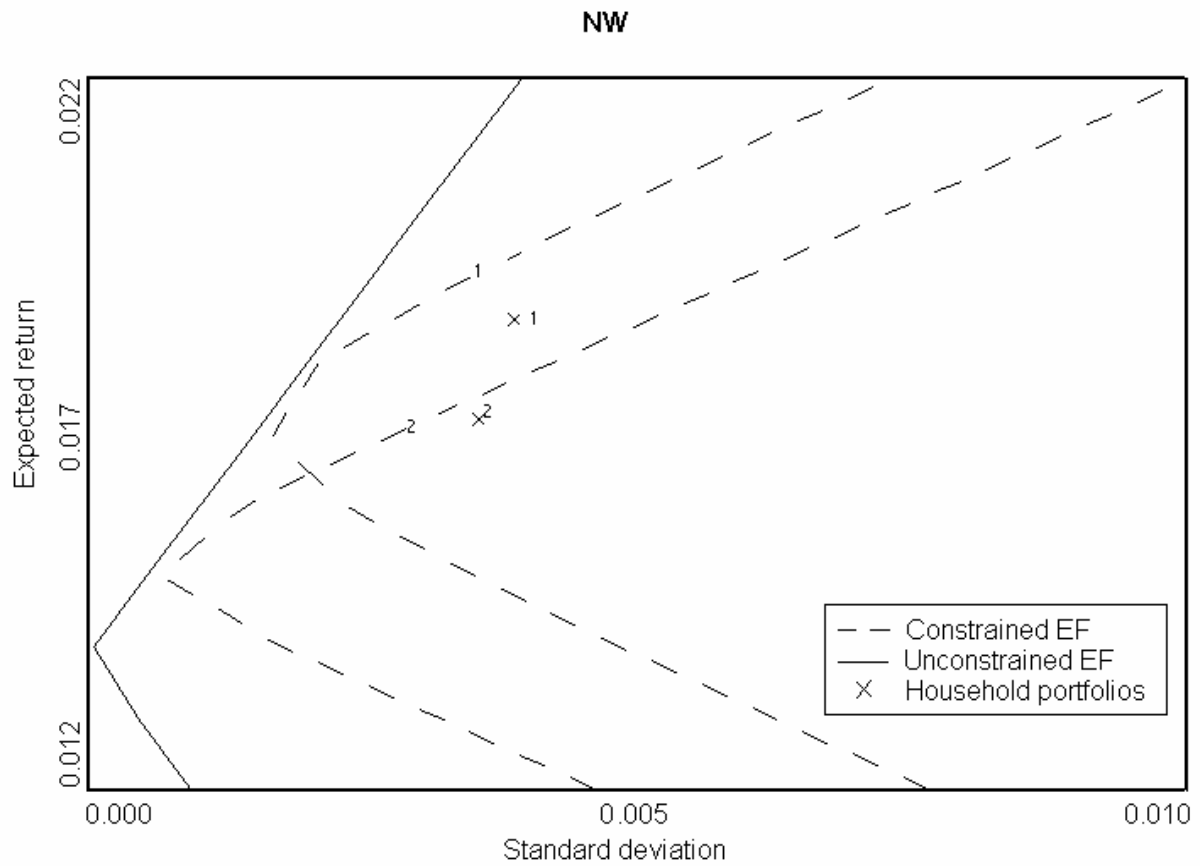


Figure 4 shows the unconstrained efficient frontier for the NW where housing is included in the investment set (solid broken line). It also shows two household portfolios expected returns (y-axis) and standard deviations (x-axis), and the corresponding constrained efficient frontiers (dashed curves).

**Figure 5 – Sensitivity of Efficiency to Changes in Housing Wealth Definition**

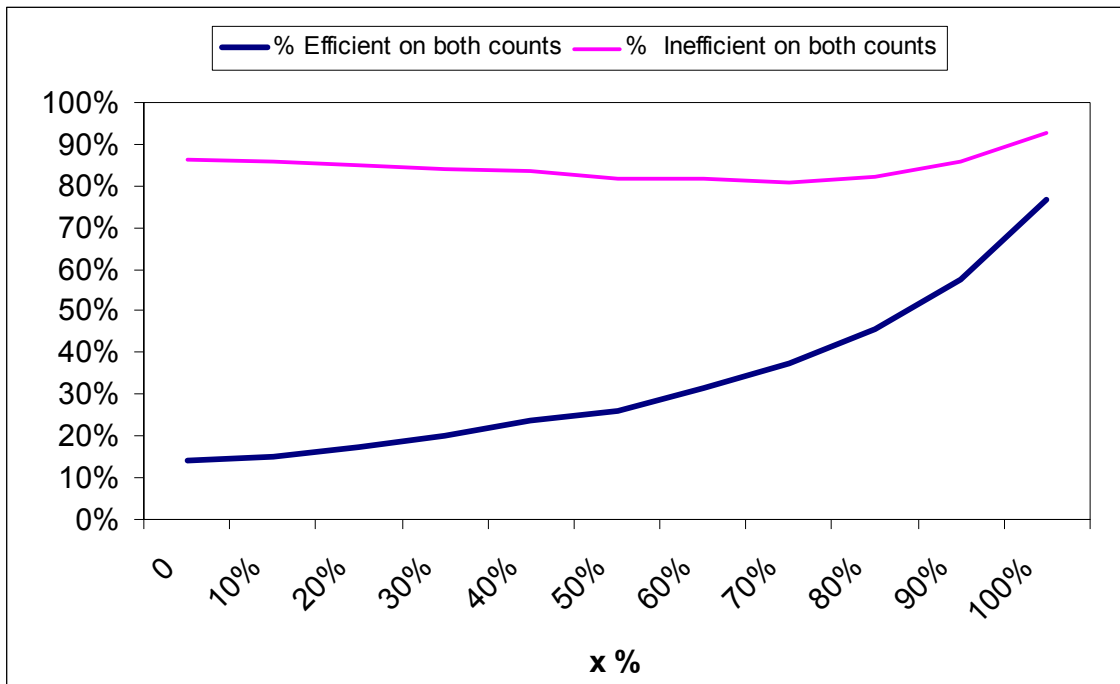


Figure 5 shows on the x-axis the percentage minimum threshold of the main residence below which a household is not willing to go. On the y-axis it shows the percentage of portfolios that are conditionally efficient out of all portfolios that are financially efficient (solid line) and the percentage of portfolios that are conditionally inefficient out of all portfolios that are financially inefficient (thin line).

## APPENDIX – Derivation of equation (7) in a multi-period context.

In this appendix we build on Flavin and Nakagawa's (2004) analysis of the dynamic optimization problem with housing, and use the same notation for comparison's sake. We show that the dynamic optimization problem produces the same asset allocation rule as a static problem that treats housing wealth as given.

Flavin and Nakagawa generalize Grossman and Laroque's model by making current utility a function of both a durable good, a house (H), and a non-durable good (C). The non-durable good is infinitely divisible and costlessly adjustable. As in Grossman and Laroque, the durable good is instead subject to an adjustment cost proportional to its value and is therefore adjusted infrequently. This generalization is of great relevance for the analysis of portfolio choice, because it allows us to consider explicitly the relationship between the real rate of return on housing investment and the real rates of return on financial assets.

The household maximizes expected lifetime utility:

$$(A1) \quad U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t) dt$$

For analytical simplicity, the house is not subject to physical depreciation.<sup>19</sup> Using the non-durable good as numeraire, define:

$$(A2) \quad P_t = \text{house price (per square meter) in the household's market}^{20}.$$

Assume that wealth is held only in the form of financial assets and housing. The household can invest in a risk-less asset and in any of  $n$  risky financial assets. Holdings of the financial assets can be adjusted with zero transaction cost.

---

<sup>19</sup> Damgaard, Fuglsbjerg and Munk (2003) have developed a model similar to ours, by deriving the numerical solution for the case with non-zero depreciation. Depreciation implies that the target value for housing is above the mid-point of the  $(s, S)$  interval. We prefer to subtract maintenance costs from the return on housing, assuming that maintenance restores the housing stock to its previous state.

<sup>20</sup> Unlike Flavin and Nakagawa, we do not consider a separate price process for the next house to be bought

Thus wealth is given by:

$$(A3) \quad W_t = P_t H_t + B_t + \underline{X}_t \underline{\ell}$$

where  $\underline{X}_t = (1 \times n)$  vector of amounts (expressed in terms of the non-durable good) held of the risky assets and  $\underline{\ell} = (n \times 1)$  vector of ones.  $B_t$  is the amount held in the form of the riskless asset. All financial assets, including the riskless asset, may be held in positive or negative amounts.<sup>21</sup>

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let  $b_{it}$  = the value (per share) of the  $i$ -th risky asset, and assume that asset prices follow an  $n$ -dimensional Brownian motion process:

$$(A4) \quad db_{it} = b_{it}((\mu_i + r_f)dt + d\omega_{it})$$

The vector  $\underline{\omega}_{Ft} \equiv (\omega_{1t}, \omega_{2t}, \dots, \omega_{nt})$  follows an  $n$ -dimensional Brownian motion with zero drift and with instantaneous covariance matrix  $\Sigma$ , the corresponding vector ( $n \times 1$ ) of expected excess returns on risky financial assets is  $\underline{\mu} \equiv (\mu_1, \mu_2, \dots, \mu_n)$ , and  $r_f$  is the risk-less rate. The  $i$ -th element of  $\underline{X}_t$  in equation (3) is given by  $X_{it} \equiv N_{it} b_{it}$  where  $N_{it}$  is the number of shares held of asset  $i$ . Since asset prices,  $b_{it}$ , are taken as exogenous, the household determines  $X_{it}$  by its choice of  $N_{it}$ <sup>22</sup>.

House prices also follow a Brownian motion:

$$(A5) \quad dP_t = P_t((\mu_H + r_f)dt + d\omega_{Ht})$$

where  $\omega_{Ht}$  is a Brownian motion with zero drift and instantaneous variance  $\sigma_p^2$ .

Stacking equations (4) and (5), define the  $((n+1) \times 1)$  vector  $d\underline{\omega}_t$ :

---

<sup>21</sup> This model does not deal with labor income or borrowing restrictions, that are instead considered in the model developed by Cocco (2005).

<sup>22</sup> We follow Flavin, and use  $X_{it}$  rather than  $N_{it}$  as the choice variable representing the portfolio decision.

$$(A6) \quad d\underline{\omega}_t = \begin{bmatrix} d\omega_{lt} \\ \vdots \\ d\omega_{mt} \\ d\omega_{Ht} \end{bmatrix}$$

which has instantaneous  $((n+1) \times (n+1))$  covariance matrix  $\Omega$ :

$$(A7) \quad \Omega = \begin{bmatrix} \Sigma & \Gamma_{b_i, P} \\ \Gamma_{b_i, P}' & \sigma_P^2 \end{bmatrix}$$

where:

$$(A8) \quad \Gamma_{bP} = \begin{bmatrix} \sigma_{b1P} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{bnP} \end{bmatrix}$$

Note that we depart from Flavin and Nakagawa here, in that we do not assume the covariance matrix  $\Omega$  to be block diagonal. This is the substantial difference between our models, that generates qualitatively different results.

We shall show that, under the assumptions listed in this Appendix, the optimal holding of risky financial assets, is given by:

$$(A9) \quad \underline{X}_0^T = \begin{bmatrix} -\frac{\partial \mathcal{V}}{\partial W} \\ \frac{\partial \mathcal{W}}{\partial^2 V} \\ \frac{\partial \mathcal{W}^2}{\partial \mathcal{W}^2} \end{bmatrix} \Sigma^{-1} \underline{\mu} - P_0 H_0 \Sigma^{-1} \Gamma_{bP}$$

In equation (A9), the expression in square brackets is the reciprocal of the coefficient of absolute risk aversion:

$$(A10) \quad ARA \equiv -\frac{\frac{\partial^2 V}{\partial \mathcal{W}_t^2}}{\frac{\partial \mathcal{V}}{\partial \mathcal{W}_t}} > 0$$



It is worth pointing out that risk aversion affects the first term on the RHS of equation (A9) but not the second term, that bears the interpretation of a hedge portfolio<sup>23</sup>. In Flavin and Nakagawa's analysis this second term disappears, because they assume  $\Gamma_{bp} = 0$ , and therefore can prove that the traditional CAPM holds.

Suppose that at time  $t=0$ , the household decides that it is not optimal to change the housing stock immediately. During a time interval  $(0,s)$  when the possibility of such change is negligible, wealth evolves according to:

$$(A11) \quad dW_t = \left[ P_t H_0 (\mu_H + r_f) + \underline{X}_t (\underline{\mu} + r_f) + r_f B_t - C_t \right] dt + \underline{X}_t d\omega_{Ft} + P_t H_0 d\omega_{Ht}$$

or, rewriting in order to eliminate the term representing risk-free bonds,

$$(A12) \quad dW_t = \left[ r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t \right] dt + \underline{X}_t d\omega_{Ft} + P_t H_0 d\omega_{Ht}$$

Let  $V(H, W, P)$  denote the supremum of household expected utility be twice continuously differentiable, conditional on the current values of the state variables  $(H, W, P)$ . Bellman's principle of optimality can be stated as:

$$(A13) \quad V(H_0, W_0, P_0) = \sup_{\{\underline{X}_t\}, \{C_t\}} E \left[ \int_0^s e^{-\delta t} u(H_0, C_t) dt + e^{-\delta s} V(H_0, W_s, P_s) \right]$$

subject to the budget constraint (A12) and the process for house prices (A5). The term inside the brackets intuitively represents the sum of the rewards on the interval  $(0,s)$  and the maximized expected value by proceeding optimally on the interval  $(s,\infty)$  with the system started at time  $s$  in state  $(H_0, W_s, P_s)$ <sup>24</sup>.

Subtracting  $V(H_0, W_0, P_0)$ , dividing by  $s$  and taking the limit as  $s \rightarrow 0$  gives:

$$(A14) \quad 0 = \lim_{s \rightarrow 0} \sup_{\{\underline{X}_t\}, \{C_t\}} E \left[ \frac{1}{s} \int_0^s e^{-\delta t} u(H_0, C_t) dt + \frac{1}{s} (e^{-\delta s} V(H_0, W_s, P_s) - V(H_0, W_0, P_0)) \right]$$

<sup>23</sup> This term is different from the classical Merton hedge term that accounts for shifts in the investment opportunity set.

<sup>24</sup> We assume that the transversality condition holds such that  $V(H_0, W_s, P_s)$  is bounded.

Evaluating the integral and using Ito's lemma, equation (A14) can be rewritten as:

$$(A15) \quad 0 = \sup_{\underline{X}_0, C_0} \left\{ u(H_0, C_0) - \delta V(H_0, W_0, P_0, P'_0) + \frac{\partial V}{\partial W} (r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) \right. \\ \left. + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} (\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2 + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}) + \frac{1}{2} \frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 \right. \\ \left. + \frac{\partial^2 V}{\partial W \partial P} (P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}) \right\}$$

that is:

$$(A16) \quad 0 = \sup_{C_0} \left\{ u(H_0, C_0) - C_0 \frac{\partial V}{\partial W} \right\} - \delta V(H_0, W_0, P_0, P'_0) + \frac{\partial V}{\partial W} (r_f W_0 + P_0 H_0 \mu_H) \\ + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2} \frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P} (P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}) + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 \\ + \sup_{\underline{X}_0} \left\{ \frac{\partial V}{\partial W} \underline{X}_0 \underline{\mu} + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} (\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}) \right\}$$

Non-durable consumption satisfies the standard first order condition:

$$(A17) \quad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}$$

The vector of holdings of risky financial assets,  $\underline{X}_0$ , is chosen according to:

$$(A18) \quad 0 = \text{constant} + \frac{\partial V}{\partial W} (r_f W_0 + P_0 H_0 \mu_H - C_0) + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 \\ + \sup_{\underline{X}_0} \left\{ \frac{\partial V}{\partial W} \underline{X}_0 \underline{\mu} + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} (\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}) + \frac{\partial^2 V}{\partial W \partial P} (P_0 \underline{X}_0 \Gamma_{bP}) \right\}$$

Assuming that  $\frac{\partial^2 V}{\partial W \partial P} = 0$  we can derive the optimal holding of risky financial assets as:

$$(A19) \quad \underline{X}_0^T = \begin{bmatrix} -\frac{\partial V}{\partial W} \\ \frac{\partial^2 V}{\partial W^2} \end{bmatrix} \Sigma^{-1} \underline{\mu} - P_0 H_0 \Sigma^{-1} \Gamma_{bP}$$

and the amount held of the risk-less asset is:

$$(A20) \quad B_0 = W_0 - P_0 H_0 - \underline{X}_0 \underline{\ell}$$

Equation (A19) is the same as equation (7), if both members are divided by total initial wealth,  $W_0$ .

The assumption that  $\frac{\partial^2 V}{\partial W \partial P} = 0$  is justified under two sets of circumstances:

- a) if the utility function does not depend on housing, as pointed out by Damgaard, Fuglsbjerg and Munk (2003)
- b) if the utility function is additive in housing and non-durable consumption.

While condition a) rules out a consumption role for housing, condition b) provides a useful benchmark for the analysis.

### **Further Reference**

Damgaard, A., B. Fuglsbjerg and C. Munk. "Optimal Consumption and Investment Strategies with a Perishable and an Indivisible Durable Consumption Good." *Journal of Economic Dynamics and Control*, 28 (2003), 209-253.