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## Savings, expectations and technological unemployment: a generalization of assumptions for the Hicksian fixwage traverse

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## **Non-Technical Summary**

In the dynamic analysis of technological change the Neo-Austrian model of traverse introduced by Hicks is considered a major breakthrough, as it enables the investigator to analyze the impact of innovations on economic activity and aggregate employment over subsequent time periods. The Hicksian analysis rests on two important assumptions concerning savings behaviour and expectations: With respect to savings he employs the so-called Q-Assumption, stating that consumption out of profits remains unchanged compared to the original reference path. Concerning expectations he assumes static expectations, implying that current values are expected to remain unchanged.

The present paper examines some implications of the Q-Assumption and the assumption of static expectations for the adjustment path in case a more mechanized type of technological innovation is introduced. Concerning the Q-Assumption, we derive the conditions for output and employment to rise after a one-time drop as depicted in some of the literature on our subject. We identify two overlapping but independent effects on the level of the growth path stemming from changes in efficiency and savings behaviour respectively. We argue that the Q-Assumption has rightfully been criticized for implying a less-than-sensible investment behaviour that may force termination of processes due to lacking investment funds and may even prevent the new technology from being introduced in the first place. Furthermore, the occurrence of jerks in the growth rate at the end of the early phase is investigated.

With respect to expectations, we argue that the Q-Assumption does not concur with the assumption of static expectations invoked by Hicks. When invoking several savings functions consistent with static expectations, we find that even under the most favourable assumption the possibility of permanent technological unemployment due to forced truncation of processes cannot be excluded. In all other cases permanent technological unemployment is bound to emerge even if no forced truncation occurs. Additionally, the more realistic assumption of adaptive expectations is introduced. In this case, we find technological unemployment to prevail during the early phase of the traverse. Moreover, for the growth rate of the rate of starts and employment an oscillating path emerges, indicating the possibility of technologically caused business cycles independent of the economy's monetary sphere. Finally, we argue that

even under rational expectations, in the disequilibrium context of the traverse the possibility of permanent technological unemployment cannot be excluded.

We conclude that in a disequilibrium context the effects of introducing a more mechanized type of technological innovation ultimately depend on the investors' risk aversion and their expectations concerning future effective demand. If effective demand is expected to remain low, investment will be restrained and be accompanied by permanent technological unemployment. Conversely, with less risk-averse investment behaviour the adjustment process is speeded up, increasing the level of effective demand and allowing full employment to be maintained.

# Savings, Expectations and Technological Unemployment

## A Generalization of Assumptions for the Hicksian Fixwage Traverse

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April 2000

### Abstract

This paper examines the role of alternative assumptions on savings and expectations for the fixwage traverse with strong forward biased technological change. After briefly outlining the model, some peculiarities of the adjustment path under the Hicksian Q-Assumption are investigated. Subsequently, the consequences of several savings functions consistent with the assumption of static expectations are explored. With all but one of these assumptions the possibility of permanent technological unemployment emerges. Additionally, the assumption of adaptive expectations is introduced, in which case technological unemployment throughout the early phase prevails. Furthermore, oscillating growth rates emerge, indicating the possibility of technologically caused business cycles.

JEL: O31, O33

Keywords: Traverse analysis, technological unemployment, Neo-Austrian theory

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## 1 Introduction

In the dynamic analysis of structural change the Neo-Austrian model of traverse introduced by Hicks 1973 is considered a major breakthrough, as it enables the investigator to depict the effects of technological changes over subsequent time periods in a disequilibrium framework. In particular, the analysis of the fixwage path has proven a fruitful instrument for the investigation of technological unemployment, making it possible to analyze the impact of Ricardian machinery effects over a greater length of time.

Two important assumptions are invoked by Hicks for the fixwage traverse. Concerning savings he introduces the so-called “Q-Assumption”, stating that consumption during the traverse remains unaltered compared to the original steady state, while with regard to expectations, he assumes static expectations, implying that current values are expected to remain unchanged.<sup>1</sup> Although the Q-Assumption has frequently been criticized,<sup>2</sup> alternative savings functions have rarely been discussed. The most notable exception is Nardini<sup>3</sup>, who introduces the assumption that consumption out of profits depends on a wealth index taking into account the additional revenues that can be expected from all processes currently at work. With respect to the assumption of static expectations, to our knowledge no attempt has been made to explore its implications for the course of the traverse. This, however, seems a worthwhile subject of investigation, as in the disequilibrium context of the traverse decisions have to be taken under fundamental uncertainty and expectations formation is thus likely to have crucial significance for the course of the traverse.

In the present paper, some implications of the Hicksian Q-Assumption and the assumption of static expectations are investigated. After a brief outline of Hicks’ basic Neo-Austrian model in section 2, in section 3 we explore some peculiar characteristics of the traverse path resulting from the Q-Assumption under strong forward biased technological change. Specifically, the conditions for employment to rise as soon as the rate of starts increases are derived. Moreover, we identify two separate effects on the level of the growth path under

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<sup>1</sup> Cf. Hicks 1973: 56. Strictly speaking, the assumption of static expectations is introduced with respect to the expected wage level. It would, however, be implausible to assume that expectations formation differs for different variables. We thus take the assumption of static expectations as valid for all variables during the traverse.

<sup>2</sup> Cf. e.g. Solow 1974, Bernholz 1974, Malinvaud 1986.

<sup>3</sup> Cf. Nardini 1990, 1993, 1994.

the Q-Assumption, stemming from changes in technological efficiency and changes in the rate of savings. It is argued that the savings effect caused by the Q-Assumption implies an implausible savings behaviour that may cause forced truncation of profitable processes due to a lack of investment funds and may even prevent the new technology from being introduced in the first place. Finally, the occurrence of jerks in the path of the growth rate of the rate of starts at the end of the early phase is explored. In section 4 the implications of the Hicksian assumption of static expectations are investigated. After arguing that the Q-Assumption is incompatible with static expectations, we introduce several savings function which we consider consistent with the static expectations assumption. We find that even under the most favourable assumption the possibility of permanent technological unemployment due to forced truncation of processes cannot be excluded, while in all other cases it is bound to emerge even if no forced truncation occurs. Finally, the assumption of static expectations is substituted by the more realistic assumption of adaptive expectations. In this case, we find technological unemployment to prevail in the early phase of the traverse. Additionally, for the growth rate of the rate of starts and employment an oscillating path emerges, indicating the possibility of technologically caused business cycles. In section 5, we summarize the main results.

## 2 Basic Model

To start with, we briefly restate the basic Neo-Austrian model as employed by Hicks. The production process is fully vertically integrated and is represented by a series of input- and output coefficients. For convenience we stick to the Hicksian notation and denote by  $(w_t)$  the real wage, by  $(r_t)$  the real rate of profit, by  $(a_t)$  the labour input coefficient and by  $(b_t)$  the output coefficient in period  $(t)$ . The production process is assumed to last  $(d+1)$  periods  $(t=0,1,\dots,d)$ . Setting the price of final output equal to unity, net output in period  $(t)$  is thus given by  $q_t = b_t - w_t a_t$ . From the equilibrium condition for the internal rate of return  $k_t = \sum_t (b_t - w_t a_t)(1+r_t)^{-t} = 0$  we obtain the familiar equation for the efficiency curve. Denoting the rate of starts in the current period  $(T)$  by  $(x_T)$ , total employment  $(A_T)$  and output  $(B_T)$  are given by

$$A_T = \sum_{t=0}^d x_{T-t} a_t \quad \text{and} \quad B_T = \sum_{t=0}^d x_{T-t} b_t$$

respectively. With the Hicksian “simple profile”, production is characterized by the following input and output coefficients:

Period	Construction Period 0	Utilization Period 1 to d
Input	$a_0$	$a_1$
Output	$b_0=0$	$b_1=1$
Net-Output	$q_0=-wa_0$	$q_1=1-wa_1$

Table 1

The equation for the efficiency curve is thus given by

$$\frac{1}{w} = a_1 + a_0 r_d \quad \text{with} \quad r_d = \frac{r}{[1 - (1+r)^{-d}]}$$

Again following Hicks, we assign a (\*) to all variables relating to processes carried out under the “old” technique. With the simple profile, a new technology will be adopted provided that the new efficiency curve enables higher wages at a given  $(r_d)$ , that is, if  $a_1^* + a_0^* r_d > a_1 + a_0 r_d$ . ( $r_d$ ) being positive, the ratio  $w(r)/w^*(r)$  lies between the boundaries  $h = a_0^* / a_0$  (“index of saving in constructional cost”) and  $H = a_1^* / a_1$  (“index of saving in utilizational cost”). In case  $H > h$ , the technological innovation is called *forward biased*, indicating that the main savings are attained in the utilization department. If  $H > h > 1$ , savings accrue in both construction and utilization, although the savings are larger in the utilization department (*weak forward bias*). Conversely, with  $H > 1 > h$  savings are only possible in utilization, while in construction more labour input is required than under the old technique (*strong forward bias*). In this case the new technology is introduced only if savings accruing in the utilizational period are larger than the additional costs in construction, which in turn depends on the respective values of the distributional parameters ( $w$ ) and ( $r$ ).<sup>4</sup>

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<sup>4</sup> This issue is discussed in detail by Faber 1975.

### 3 Some Peculiarities of the Adjustment Process Under the Q-Assumption

We now examine the implications of the Hicksian Q-Assumption for the course of the fixwage traverse. For this purpose, we compare the actual path under the new technique with the reference situation of the original steady state. The fixwage analysis assumes the real wage to be constant, leaving the chosen technique unchanged throughout the traverse. By contrast, the quantities of output ( $B_T$ ), rate of starts ( $x_T$ ) and employment ( $A_T$ ) may vary from period to period. In order to isolate the effects of the Q-Assumption, we stick to the “simple profile” invoked by Hicks.<sup>5</sup> With regard to the rate of starts the Hicksian “Full Performance” assumption is employed, indicating that all savings are reinvested.<sup>6</sup> As in the construction period the output coefficient is zero, current output is produced solely by processes started in previous periods. Employment, however, as far as constructional employment is concerned, is co-determined by the current rate of starts, which in turn is limited by current savings due to the Full Performance assumption. With respect to savings behaviour during the traverse Hicks introduces the Q-Assumption, stating that consumption out of profits (the “take-out”), which can be shown to equal  $Q_T = B_T - wA_T$ ,<sup>7</sup> remains unchanged compared to the reference path ( $Q_T = Q_T^*$ ).<sup>8</sup> Accordingly, during the traverse all additional profits are saved and reinvested in the start of fresh processes. From  $Q_T = B_T - wA_T$  it is obvious that under the Q-Assumption the course of employment parallels the path of current output.

We assume that the new technology is introduced in period 1 and consider the case of strong forward biased technological change ( $h < 1 < H$ ) and the limiting case ( $h = 1 < H$ ). Taking as an example the values

	$x_0$	$a_0$	$a_1$	$a_0^*$	$a_1^*$	d (duration)	g (steady-state growth rate)
$h < 1$	10	0.6	0.75	0.5	0.8	5	10%
$h = 1$	10	0.5	0.75	0.5	0.8	5	10%

Table 2

<sup>5</sup> For a discussion of the Q-Assumption with more general profiles we refer to Belloc 1996.

<sup>6</sup> The “Full Performance”-Assumption is analogous to full employment of capital stock, which in the Neo-Austrian model is not observable. Cf. Hagemann 1983: 259.

<sup>7</sup> Cf. Hicks 1973: 57.

<sup>8</sup> It is assumed that no savings out of wages occur.



the following diagrams show the path of the rate of starts and employment respectively on a logarithmic scale:

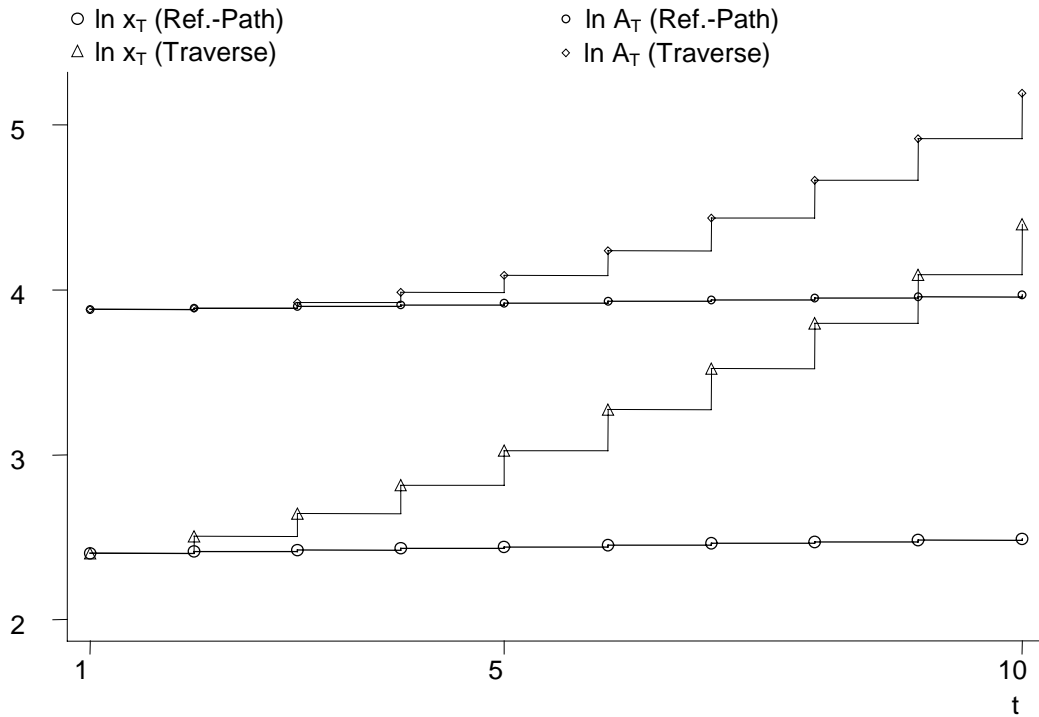


Figure 1: rate of starts and employment ( $h = 1 < H$ )

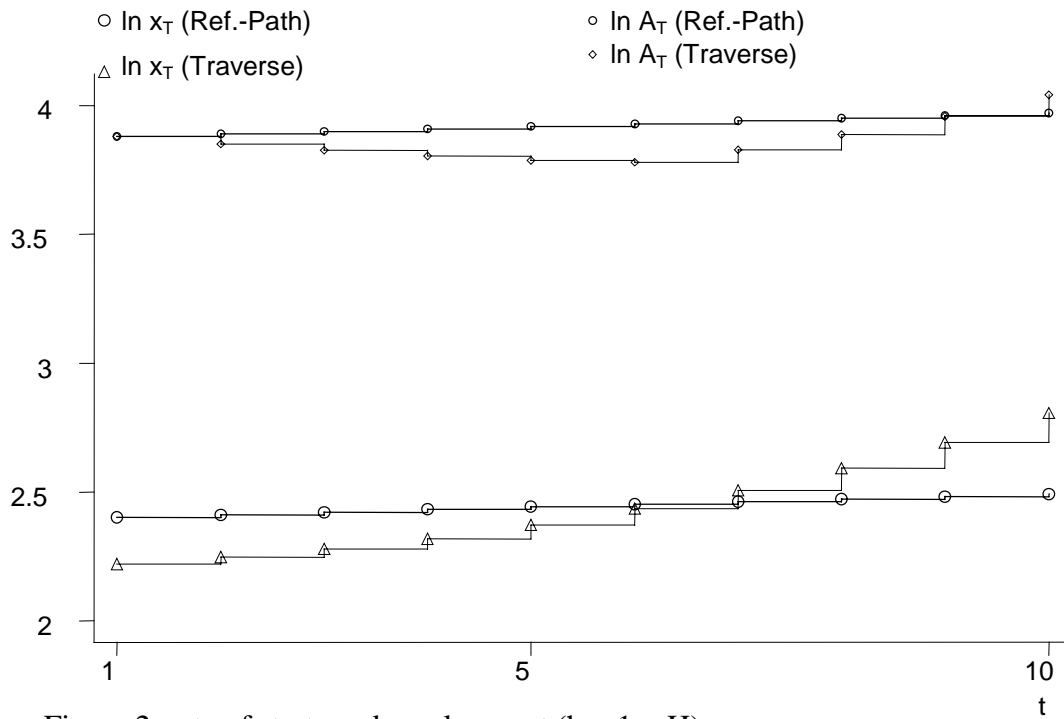


Figure 2: rate of starts and employment ( $h < 1 < H$ )

In the limiting case ( $h=1 < H$ ) the new machines require the same amount of labour in the construction department ( $h=1$ ) but less labour in the utilization department ( $H > 1$ ).<sup>9</sup> With ( $h=1$ ) we have no additional profit in period 1, leaving all quantities as yet unchanged. From period 2 onward the first new machines enter the utilization department, yielding additional profit available for investment in the start of fresh processes. Accordingly, in period 2 the rate of starts may be increased for the first time compared to the reference path and rises further in the following periods. As in the previous period no additional machines have been constructed and the number of machines in the utilization department is still unchanged, output and employment remain as yet unaffected. An increase in output and employment does not occur until period 3, by the time the additional machines constructed in period 2 enter the utilization department and increase output capacity.

By contrast, with ( $h < 1 < H$ ) more labour is required in the construction department. As the output fraction available for investment is initially unchanged, a smaller number of fresh processes may be started in period 1. Accordingly, the path of the rate of starts drops below the reference path, whereas output and employment remain unaffected for the time being. Only in period 2, when the first new machines enter the utilization department, their higher efficiency allows the rate of starts to be increased. This increase continues in the subsequent periods, eventually raising the rate of starts up to the reference level and beyond. With regard to output and employment, however, in the second period the consequences of the diminished rate of starts in period 1 become evident. As in the utilization department a smaller number of machines is available, the path of output and employment drops below the reference path. This effect is the more pronounced, the higher the labour requirement in the construction period is in comparison to the old technique. The decline of output and the displacement of workers continues until the output capacity in the utilization department has risen to the reference level. Only then an increase in output and employment will occur.

Occasionally the path of output and employment with strong forward bias is depicted as increasing from period 3 onward already.<sup>10</sup> This description, however, only holds for special cases. Due to additional profits in the utilization

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<sup>9</sup> Cf. Hicks 1973: 91ff., Hagemann 1983: 262, Mettelsiefen 1981: 101.

<sup>10</sup> Cf. e.g. Mettelsiefen 1981: 102, Hagemann 1983: 261. The authors presume, as does Hicks, the new machines to be introduced in period 0.

department the rate of starts may indeed be increased in period 3. The path of output and employment, however, initially further declines in relation to the reference path. The continuing fall is caused by the fact that as long as the rate of starts remains below the reference level, the number of old machines leaving the utilization department is still higher than the number of new machines entering it. As the output coefficient is unity for both old and new machines, total output continues to decline. Due to the wage fund argument this is also true for aggregate employment. Only when the number of machines in the utilization department and the output capacity reaches the reference level the fall of output and employment comes to an end. For this to be true the rate of starts must rise to the reference level and additionally the smaller number of new machines produced in previous periods must be compensated for.

Accordingly, output and employment may rise in period 3 compared to the reference level if and only if the number of machines entering the utilization department in period 3 exceeds the reference level by the drop of machine construction in period one. This requires the rate of starts in period 2 to satisfy  $x_2 \geq x_2^{\text{ref}} + (x_1^{\text{ref}} - x_1)$ . For this to be true the labour coefficient of the new technique in the utilization period ( $a_1$ ) must not exceed the upper limit given by

$$a_1 \leq \frac{1}{w} - \frac{a_0}{a_0^*} \left[ \frac{1}{w} + (2 + g)(a_0 - a_0^*) - a_1^* \right],$$

where ( $g$ ) denotes the original steady state growth rate. Taking the derivative with respect to ( $a_0$ ) of the right side of this inequality we may determine the location of this upper bound for different values of ( $a_0$ ) as

$$\frac{\partial}{\partial a_0} = (2 + g) \left(1 - \frac{2}{h}\right) - r_d^*$$

which due to  $h < 1$  is unambiguously negative. Consequently, if output and employment are to rise in period 3, the higher ( $a_0$ ), the smaller the labour input coefficient in the utilization department is required to be. Similarly, the derivative with respect to ( $w$ )

$$\frac{\partial}{\partial w} = \frac{1}{w^2} \left( \frac{a_0}{a_0^*} - 1 \right) = \frac{1}{w^2} \left( \frac{1}{h} - 1 \right)$$

is unambiguously positive due to  $h < 1$ , implying that the higher ( $w$ ), the higher the coefficient ( $a_1$ ) is allowed to be.

On closer examination of the traverse path under the Q-Assumption, two overlapping effects on the level of the growth path may be identified. For one thing, with a given rate of savings the level of the growth path changes in response to the different input coefficients of the new machines (efficiency effect). Additionally, however, the Q-Assumption implies a change in the savings rate for all periods in which the level of aggregate output differs from the reference path (savings effect). Specifically, a fall of ( $B_T$ ) compared to the reference path implies a decrease in the rate of savings, whereas in case ( $B_T$ ) rises compared to the reference level the converse is true. Accordingly, if ( $B_T$ ) decreases relative to the reference path, the adverse efficiency effect of  $h < 1$  is reinforced by the lower savings rate that causes an additional downward shift of the growth path. Conversely, if the level of aggregate output is higher than the reference level, the higher savings rate exerts an additional positive impact on the growth path.

The above argument makes it clear that the sensibility of the Hicksian Q-Assumption has rightfully been questioned. As keeping consumption out of profit at the reference level implies that the consumption rate is constant with respect to output on the reference path, additional profits accruing in course of the traverse are entirely saved and invested. While this assumption seems to be favourable at first glance, in the case of strong forward bias it proves to be fatal: Actually, the Q-Assumption implies that in relation to the current output level saving and investment is the lower, the more the course falls short of the reference path. Moreover, with some combinations of labour coefficients ( $a_0$ ) and ( $a_1$ ) this kind of investment behaviour may lead to the result that due to lacking investment funds no fresh processes may be started at all, or worse, old processes may have to be terminated even though they are profitable.<sup>11</sup> This may even happen in period one, preventing the new technique from being introduced in the first place. The condition for this to occur may be determined from  $Q_1 = Q_1^{\text{ref}}$  or  $B_1 - wA_1 = B_1^{\text{ref}} - wA_1^{\text{ref}}$  respectively. Collecting terms we obtain

$$w a_0 x_1 = w a_0^* x_1^{\text{ref}} \quad \text{or} \quad x_1 = \frac{a_0^*}{a_0} x_1^{\text{ref}}$$

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<sup>11</sup> This cut of the rate of starts due to a lack of resources has been stated previously by Zamagni (1984), who refers to it as the ‘‘Hayek effect’’.

Not introducing the new technique at all implies  $x_1 < 1$ . Together with  $x_1^{\text{ref}} = x_0^{\text{ref}}(1+g)$  this yields

$$\frac{a_0^*}{a_0} x_0^{\text{ref}} (1+g) < 0 \quad \text{or} \quad a_0 > a_0^* x_0^{\text{ref}} (1+g)$$

The latter inequality is true provided that starting from a moderate activity level a change from a rather labour intensive to a rather capital intensive technique occurs. Similarly, in later periods there is a danger that old processes will have to be terminated for lack of investment funds and the reference level of the rate of starts may not be reached. Output and employment in this case will remain permanently below the reference level, resulting in permanent, technologically caused capital shortage unemployment.

The fixwage path with strong forward bias exhibits an additional characteristic not discussed by Hicks: At the beginning of the late phase a one-time drop in the growth rate of the rate of starts is observed. Taking the values

$x_0$	$a_0$	$a_1$	$a_0^*$	$a_1^*$	$d$	$g$
10	0.6	0.7	0.5	0.8	4	10%

Table 3

we obtain the following diagram showing a drop of the growth rate in the path of the rate of starts in period six:

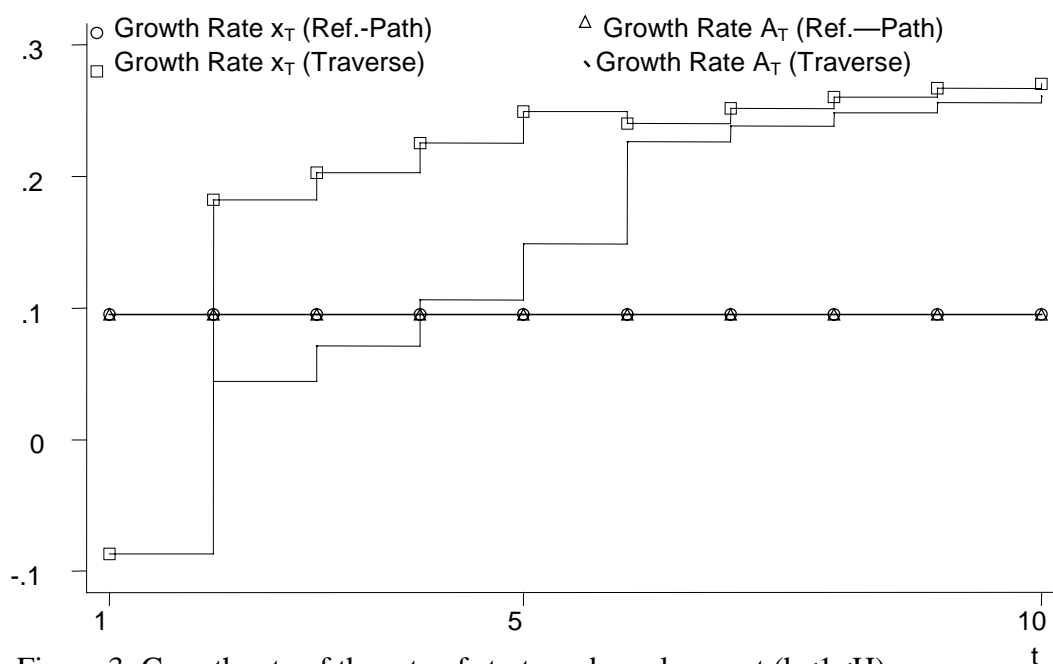


Figure 3: Growth rate of the rate of starts and employment ( $h < 1 < H$ )

To examine the cause of these jerks we denote by  $(x_{-1}^{ea})$  and  $(x_0^{ea})$  the rate of starts in the penultimate and last period of the early phase, by  $(x_1^{la})$  the rate of starts in the first period of the late phase, by  $(n_1^*)$  and  $(n_1)$  the number of old and new machines in the utilization department and by  $(d)$  utilization duration. An indication for the reason of this one-time drop in the rate of starts may be ascertained once the growth factors of the last period in the early phase and the first period of the late phase are determined. The growth factor of the last period in the early phase is given by

$$(1) \quad \frac{x_0^{ea}}{x_{-1}^{ea}} = \frac{n_{-1}^* (1 - a_1^*) + n_{-1} \cdot (1 - a_1) + [x_{-1}^{ea} \cdot (1 - a_1) - x_{-d-1}^{ref} (1 - a_1^*)] - q_{-1}^{ref} \cdot g}{n_{-1}^* (1 - a_1^*) + n_{-1} \cdot (1 - a_1) - q_{-1}^{ref}}$$

while for the growth factor of the first period in the late phase we obtain

$$(2) \quad \frac{x_1^{la}}{x_0^c} = \frac{n_0 \cdot (1 - a_1) + [(x_0^{ea} - x_{-d}^{ea}) \cdot (1 - a_1)] - q_{-1}^{ref} \cdot g^2}{n_0 \cdot (1 - a_1) - q_{-1}^{ref} \cdot g}$$

Here we can isolate a one-time effect that is related to the value of the index of savings in utilizational costs  $H = a_1^*/a_1 > 1$ , stating that with strong forward bias the input coefficient for the old machines ( $a_1^*$ ) is larger than for the new machines ( $a_1$ ). After the last period of the early phase, all of the old machines have left the utilizational department. Thus, when in the first period of the early phase new processes are introduced into utilization, they no longer replace old machines with a higher input coefficient ( $a_1^*$ ) but new machines with a lower labour requirement ( $a_1$ ). Accordingly, in the first period of the late phase for the first time no additional savings may be obtained by introducing new machines into utilization. Consequently, the growth rate of the rate of exhibits a drop compared to the last period of the early phase. More formally, for the bracket in the denominator in (1) the following inequality holds:

$$[x_{-1}^{ea} \cdot (1 - a_1) - x_{-d-1}^{ref} (1 - a_1^*)] > [x_{-1}^{ea} \cdot (1 - a_1) - x_{-d-1}^{ref} (1 - a_1)]$$

The higher input coefficient ( $a_1^*$ ) in the last period of the early phase enlarges the bracket term in (1) relative to the bracket term in (2), causing a one-time decrease of the growth rate of the rate of starts.

## 4 Expectations Formation and Technological Unemployment

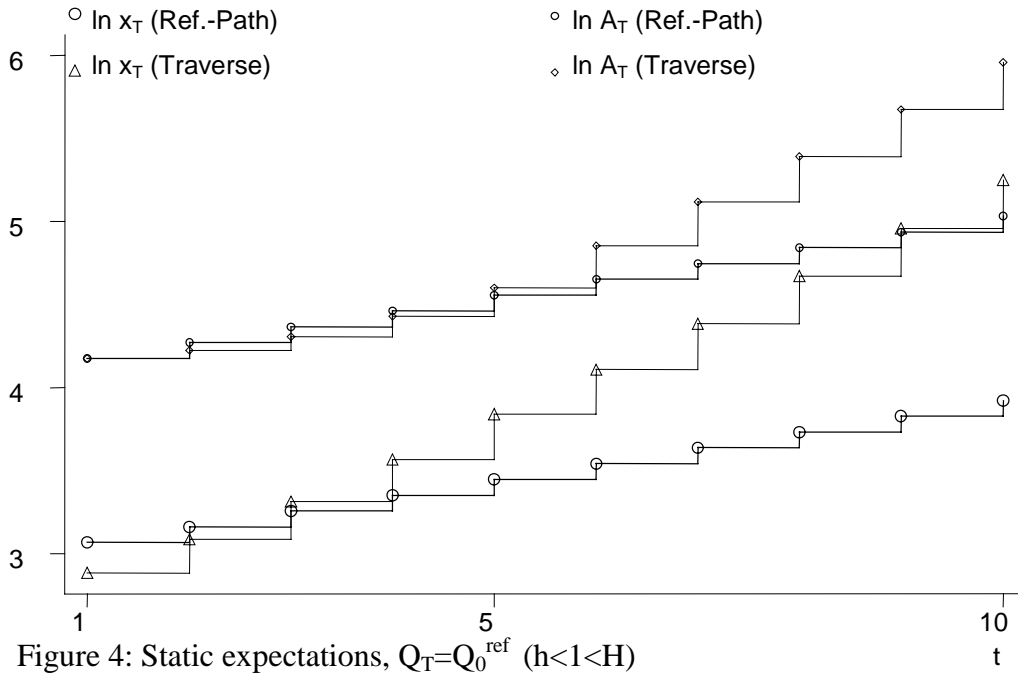
Outside steady-state equilibrium investors are confronted with fundamental uncertainty in a Keynesian sense. The subject of expectations formation is thus likely to be of crucial importance for the course of the traverse. In this respect, Hicks invokes the assumption of static expectations, that is, current values are expected to remain unchanged. On closer examination, however, the Q-Assumption employed by Hicks with respect to savings seems to be inconsistent with static expectations. Static expectations are usually understood to be based on the events of the previous period, but not on the course of a hypothetical reference path. Accordingly, from the second period onward the Q-Assumption would have to be replaced by a savings function that is oriented towards the actual course. With static expectations two alternatives may be considered:

- (a) consumption out of profits may be constant in absolute terms
- (b) the modified rate of starts in the previous period may be taken into account

With (a), the new saving assumption is given by  $Q_T = Q_0^{\text{ref}}$ . As both the Q-Assumption and the reference path imply  $Q_T = Q_T^{\text{ref}} = Q_0^{\text{ref}}(1+g)^T$ , with  $Q_T = Q_0^{\text{ref}}$  consumption is lower by the amount  $Q_T^{\text{ref}} - Q_T = Q_0^{\text{ref}}g^T$ . The rate of starts for the first period is given by <sup>12</sup>

$$x_1 = x_0^{\text{ref}} \left[ \frac{1}{wa_0} (b_1 - b_d \frac{1}{(1+g)^{d+1}}) + \frac{1}{a_0} (a_0^* + a_d^* \frac{1}{(1+g)^{d+1}} - a_1^*) \right]$$

and for subsequent periods has to be determined recursively. When calculating the paths for the rate of starts and for aggregate employment for the parameter values from Table 3, we find that the fall of the rate of starts as well as the drop-down in employment are reduced and the adjustment process is speeded up considerably:



Static expectations interpreted according to (a) thus result in a lower amount of technological unemployment compared to the Hicksian Q-Assumption. However, as with the latter, with some combinations of labour input coefficients there is the possibility of forced truncation of old processes and permanent technological unemployment due to a lack of investment funds.

With respect to (b) two possible savings assumptions are conceivable. One possibility is to determine current consumption out of profits ( $Q_T$ ) as depending on the expected rate of starts in the current period, assuming that the latter is equal to the rate of starts in the previous period ( $x_T^e=x_{T-1}$ ) due to static expectations. In this case consumption out of profits differs from the reference level by

$$Q_T^{\text{ref}} - Q_T = x_0^{\text{ref}} \left\{ \sum_{t=1}^{T-1} b_t g^t - w \sum_{t=0}^{T-1} [a_t - (1+g)^{T-t} a_t^*] \right\}.$$

The rate of starts under this assumption is given by  $x_T = x_0^{\text{ref}}$ , and is thus below the reference path for all positive ( $g$ ). Aggregate employment is determined by

<sup>12</sup> We assume that no output is produced by processes started in the current period ( $T$ ), implying that the construction phase lasts at least one period.



$$A_T = \sum_{t=0}^{T-1} a_t x_0^{\text{ref}} + \sum_{t=T}^d a_t^* x_{T-t}.$$

Considering the strong forward bias in Hicks' simple profile, the take-out in the first period is below the reference level for all  $h < 1/(1+g)$ , whereas aggregate employment is higher than the reference level due to  $a_0 < a_0^*$ . While the realized rate of starts still falls short of the reference path, it differs from the original Q-Assumption by  $x_0^{\text{ref}} [1 - (1+g)h]$ , and is thus unambiguously higher for all  $h < 1/(1+g)$ . In later periods, however, the favourable effect for employment reverses due to  $a_1 < a_1^*$  in combination with a constant rate of starts. Plotting the path of the rate of starts and employment on a logarithmic scale it is obvious that in this case the path of the rate of starts as well as employment is continuously lower than the reference path throughout the traverse, and is bound to converge to a steady state equilibrium below the original reference path.

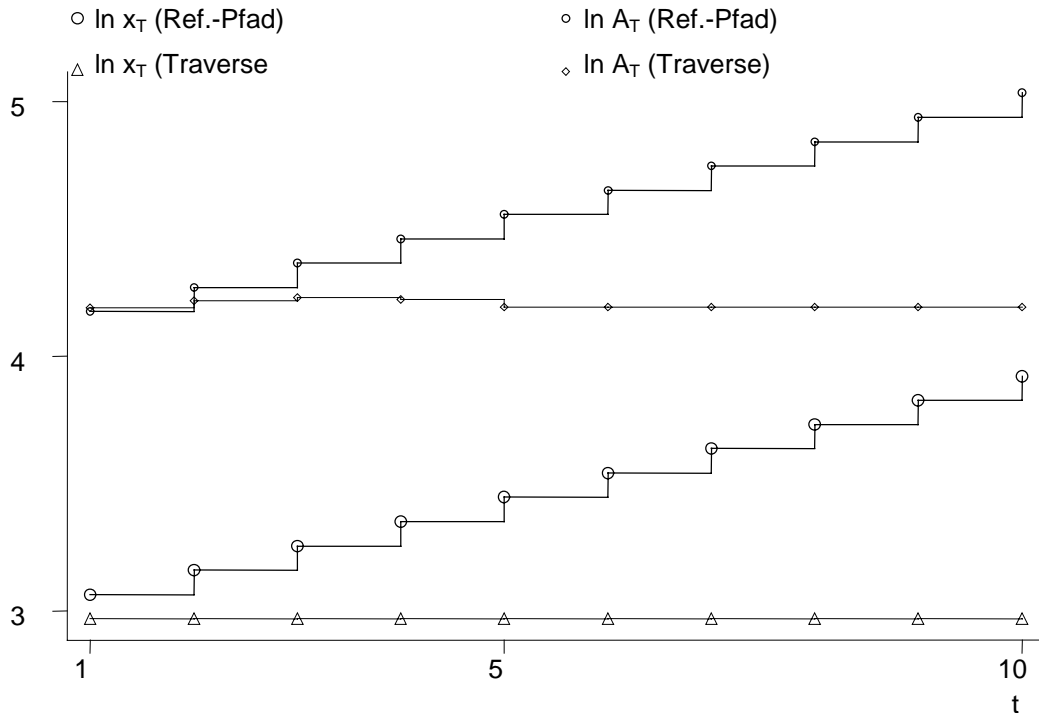


Figure 5: Static expectations,  $x_T^e = x_{T-1}$  ( $h < 1 < H$ )

The assumption of static expectations interpreted to mean  $x_T^e = x_{T-1}$  thus results in a constantly lower activity of the economy compared to the reference situation and in a high amount of technological unemployment. This is due to the fact that under  $x_T^e = x_{T-1}$  the additional profits accruing in the utilization department are almost entirely consumed by the entrepreneurs, except for the

additional investment necessary to keep up a constant rate of starts despite the higher labour input coefficient in the construction department. The implicit investment behaviour is extremely risk averse and may at most be explained by the presumption that entrepreneurs expect effective demand to at best remain constant or fall short of the previous period due to the observed technological unemployment.<sup>13</sup>

A second savings assumption consistent with static expectations would be to determine consumption out of profits depending on the rate of starts of the previous period adjusted by the steady-state growth rate ( $x_T^c = x_{T-1}(1+g)$ ). In this case, the take-out differs from the reference level by

$$Q_T^{\text{ref}} - Q_T = wx_0^{\text{ref}} \sum_{t=0}^T (1+g)^{T-t} (a_t - a_t^*).$$

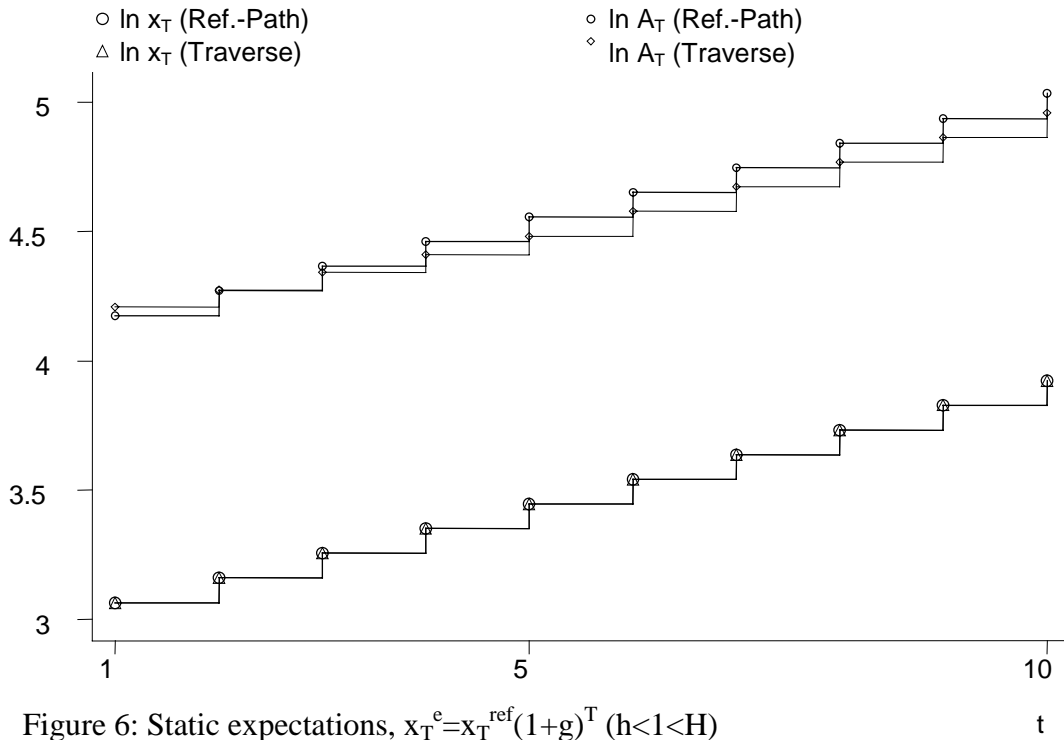
For the rate of starts we obtain  $x_T = x_0^{\text{ref}} (1+g)^T = x_T^{\text{ref}}$ . The path of the realized rates of starts is thus identical to the reference path, as the efficiency gains are exactly consumed by additional take-out. Aggregate employment is given by

$$A_T = x_0^{\text{ref}} \sum_{t=0}^{T-1} (1+g)^{T-t} a_t + \sum_{t=T}^d x_{T-t} a_t^*.$$

As the following diagram shows, the resulting path is identical to the original reference situation for the rate of starts but falls short of the reference path for employment due to the labour saving effect of the technological innovation in the utilization department:

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<sup>13</sup> The latter argument may be justified by the fact that the level of output indeed decreases in absolute terms from period 2 to the end of the early phase.



As in the previous case, static expectations interpreted to mean  $x_T^e = x_{T-1}(1+g)$  result in permanent, albeit lower, technological unemployment, even though the economy is run at the same activity level as in the original steady state situation throughout the entire traverse. The investment behaviour concurring with this version of static expectations may be based on the anticipation that effective demand under the new technology will be at the same level as with the original steady state situation.<sup>14</sup>

The assumption of static expectations has rightfully been criticized for presuming a less-than-sensible investment behaviour on the part of the entrepreneurs. We therefore introduce the more realistic assumption of adaptive expectations. In this case, the expected rate of starts is determined by  $x_T^e = x_{T-1} + (1 - \beta)(x_{T-1} - x_{T-2})$ ,  $\beta \in [0, 1]$  being the error correction factor. The rate of starts in period (T) is given by

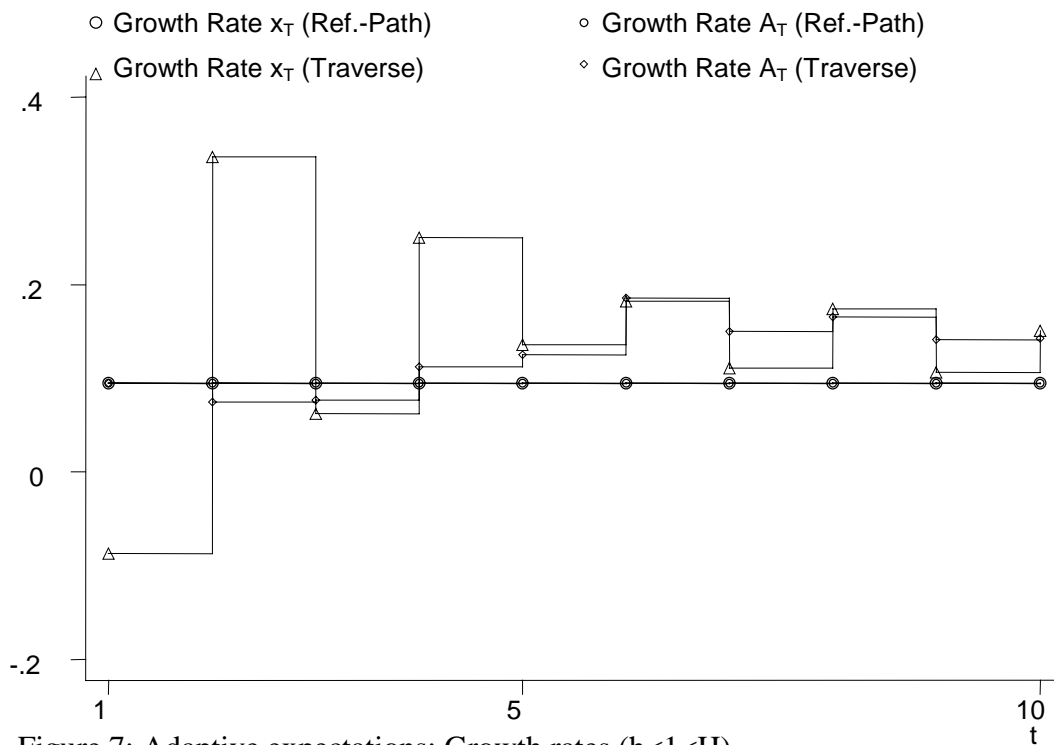
$$x_T = x_0^{\text{ref}} \left[ 1 + \sum_{t=1}^T (1 - \beta)^t - \sum_{t=1}^T \frac{(1 - \beta)^t}{(1 + g)} \right].$$

<sup>14</sup> Actually, the resulting path of aggregate output  $B_T$  coincides with the reference path.

For aggregate employment we obtain

$$A_T = x_0^{\text{ref}} \sum_{t=0}^{T-1} \left[ 1 + \sum_{t=0}^{T-1} (1-\beta)^t - \sum_{t=0}^{T-1} \frac{(1-\beta)^t}{1+g} \right] a_t + \sum_{t=T}^d x_{T-t} a_t^*.$$

Plotting the growth rate of the rate of start and employment for the example above and  $\beta=0.4$  we obtain an oscillating path for the rate of starts as well as employment:



This result indicates the possibility that technological innovations combined with adaptive expectations may give rise to business-cycle dynamics independent of the economy's monetary sphere.<sup>15</sup> The intensity of these oscillations clearly depends on the magnitude of the parameter ( $\beta$ ).

Plotting the path of the rate of starts and employment on a logarithmic scale we see that with adaptive expectations, while a low amount of technological unemployment prevails in the early phase (up to period 5), the path of employment rises above the reference level from period 6 onward.

<sup>15</sup> The possibility of cycle-trends occurring in the late phase of the traverse caused by a wealth-dependent consumption function has been noted by Nardini (1990).

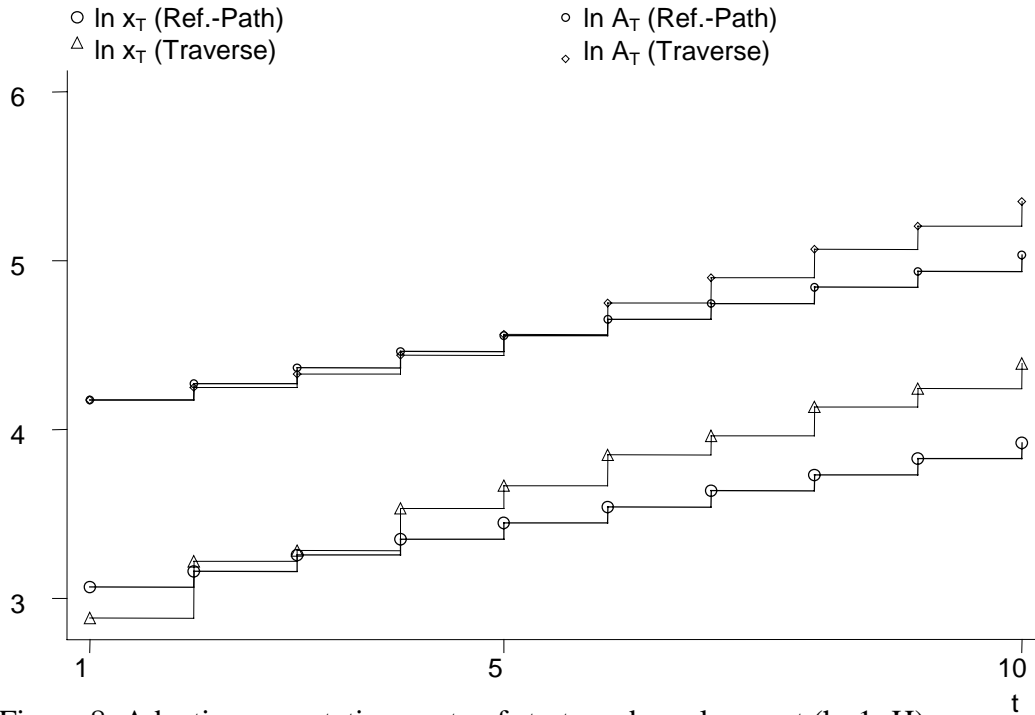


Figure 8: Adaptive expectations, rate of starts and employment ( $h < 1 < H$ )

As a subject for further investigation it might be interesting to explore the consequences of rational expectations. However, as in the disequilibrium context of the traverse fundamental uncertainty prevails, stochastic assumptions are not easily specified and the objective function to be maximized is not clear. For instance, the entrepreneur might choose to minimize the time of adjustment to the new steady state path. In this case, the rational expectations solution would clearly require  $Q_T=0$ , in which case full employment could be preserved throughout the traverse. A second objective might be to minimize the difference of the rate of starts to the reference path. In this case, the savings assumption would be identical to our case of static expectations with  $x_T^e = x_{T-1}(1 + g)$ , where the resulting path of the rate of starts equals the original reference path and permanent technological unemployment emerges. Depending on how the assumption of rational expectations is specified, in a disequilibrium context even the rational expectations assumption does not necessarily exclude the possibility of permanent technological unemployment.

## 5 Conclusion

In this paper, the role of the Hicksian Q-Assumption and alternative assumptions on expectations have been discussed. As to the Q-Assumption, we have derived the conditions for output and employment to rise after a one-time drop as depicted in some of the literature on our subject. We have identified two overlapping but independent effects on the level of the growth path that stem from changes in efficiency and savings behaviour respectively. In considering the savings effect of the Q-Assumption we have argued that this assumption has rightfully been criticized for implying a less-than-sensible investment behaviour that may force termination of processes due to lacking investment funds and may even prevent the new technology from being introduced in the first place. Furthermore, the occurrence of jerks in the growth rate of the rate of starts at the end of the early phase has been investigated.

With respect to expectations, we have argued that the Q-Assumption does not concur with the assumption of static expectations invoked by Hicks. When substituting the Q-Assumption with savings functions consistent with static expectations, we found that in all cases the possibility of permanent technological unemployment emerges. With the most favourable assumption  $Q_T = Q_0^{\text{ref}}$ , adjustment is speeded up in comparison to the Q-Assumption but permanent technological unemployment due to forced truncation of old processes cannot be excluded. With all other assumptions permanent technological unemployment is bound to emerge even if no forced truncation of processes occurs. Additionally, we have introduced the assumption of adaptive expectations. In this case, we find technological unemployment to emerge during the early phase of the traverse. Moreover, for the growth rate of the rate of starts and employment an oscillating path emerges. This indicates the possibility that, depending on expectations formation, technological innovations may cause business cycles independent of the economy's monetary sphere.

We conclude that the effects of introducing a more mechanized type of technological innovation in a disequilibrium context ultimately depend on the investors' risk aversion and their expectations concerning future effective demand. If effective demand is expected to remain low, investment will be restrained and will be accompanied by permanent technological unemployment. Both events reinforce the unfavourable development of effective demand and cause pessimistic expectations to be fulfilled. Conversely, with less risk-averse

investment behaviour the adjustment process is speeded up, increasing the level of effective demand and allowing full employment to be maintained.

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