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On the Welfare Properties of the Lucas and Romer Endogenous Growth Models

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Abstract

We present results from quantitative exercises using the Lucas and Romer endogenous growth models, from which we calculate welfare losses from the distortions presented in the Romer model. Moreover, comparing the models to data, we show that an economy governed by the Romer model would attain a higher welfare than one governed by the Lucas model, contrary to what can be interpreted from some previous theoretical contributions.

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1 Introduction

Nearly twenty years ago Lucas (1988) and Romer (1990) made two of the main seminal contributions that established endogenous growth theory. As usual, some of the following articles studied the properties of the steady-state and convergence of both models.

Barro and Sala-i-Martin (1995) and Faig (1995) made a dynamic analysis that intend to show that the Lucas model steady-state is saddle-path stable. Asada, Semmler, and Novak (1998) have shown that locally the steady-state of the optimal growth path is a saddle-point. The market equilibrium of the Lucas model is also analysed by Benhabib and Perli (1994), showing that indeterminacy can arise under very plausible parameters.

Benhabib, Perli, and Xie (1994) show that in the Romer model, indeterminacies can arise under complementarity in the intermediate inputs. Arnold (2000a) shows that the steady-state is locally saddle-path stable. Additionally, Arnold (2000b) demonstrates that for any initial endowments of capital and knowledge, an unique path converging to the steady-state exists. This result is valid globally as the author exploits a notational adjustment that converts the analysis done to study the Lucas' model, also applicable to the Romer's model. This article also showed that an optimal subsidy scheme implies that the decentralized equilibrium of the Romer model is equivalent to the optimal equilibrium.

In this paper, we pursue two goals. First, we derive the welfare losses for the market economy of the Romer model relative to the optimal growth framework of the Lucas model. This is useful to determine how much the economy losses when it stands without the optimal subsidy scheme described in Arnold (2000b). Second, we argue that when we compare the models to data, a decentralized Romer economy is better than a Lucas economy in terms of welfare, contrary to what may be interpreted by the analysis of the Arnold (2000b) article. We present these results taking the transition paths into account.

The next section makes a brief presentation of the models, describing the growth problem for both. Section 3 presents and compares the convergence trajectories of both models, calculates welfare and welfare losses, and discusses the adherence to reality. The fourth section concludes.

2 The Model

This section recapitulates the Arnold (2000b) comparison between the Lucas (1988) and the Romer (1990) models of endogenous growth.

2.1 Setup of the Models

The optimal-growth problem for an economy with human capital accumulation as in the Lucas (1988) model is:

$$\max_{C,u} \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}-1}{1-\sigma} dt$$
$$s.t.: \dot{K} = K^\alpha (uH)^{1-\alpha} - C \tag{1}$$
$$\dot{H} = B(1-u)H$$

where C is consumption, K is physical capital, H is human capital, u is the share of human capital employed in the final good production ($Y = K^\alpha (uH)^{1-\alpha}$); B is the productivity in the human capital sector, ρ is the intertemporal discount factor, and σ governs the elasticity of substitution.

The transitional dynamics of the problem is described by a system of three differential equations, derived by Barro and Sala-i-Martin (1995), Faig (1995), and also presented in Arnold (2000b), as follows:

$$g_z = (1-\alpha)\left(\frac{B}{\alpha} - z\right) \tag{2}$$
$$g_\chi = \chi - \frac{\rho}{\sigma} - \left(1 - \frac{\alpha}{\sigma}\right) z \tag{3}$$
$$g_u = \frac{B}{\alpha} - \chi - B\,(1-u) \tag{4}$$

where g_y is the growth rate of a given variable y, $z = (uH/K)^{1-\alpha}$ and $\chi = C/K$.

The optimal growth problem for the Romer model can be formalized as follows:

$$\max_{C,L_Y} \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}-1}{1-\sigma} dt$$
$$s.t.: \dot{K} = K^\alpha (AL_Y)^{1-\alpha} - C \tag{5}$$
$$\dot{A} = \delta A(L - L_Y)$$

where A is technology, L is labor and L_Y is labor allocated to the final good sector. If $u = L_Y/L$; $H = AL$ and $B = \delta L$, the problem described by (5) would be converted on the problem described by (1).

The decentralized equilibrium is described in Arnold (2000b:223-224) as well as the differential equations that describe the transition to the steady-state. Without considering any subsidy scheme in the decentralized equilibrium, the transition is described by a system of three differential equations as follows:

$$g_z = (1-\alpha)\,(Bu - \alpha z) \tag{6}$$
$$g_\chi = \chi - \frac{\rho}{\sigma} - \left(1 - \frac{\alpha^2}{\sigma}\right) z \tag{7}$$
$$g_u = (Bu - \alpha z) + z - \chi - B\,(1-u) \tag{8}$$

In the following section, we calculate the growth path of the Lucas economy, integrating equations (2) to (4) and of the Romer economy, integrating equations (6) to (8). We perform this exercise using backward integration as in Brunner and Strulik (2002). We begin arbitrarily close to the steady-state and we backward integrate equations that describe the evolution of z, χ, and u. The integration proceeds until the allocation of resources to the final good u reaches 1.

3 Convergence and Welfare

Arnold (2000b) showed that the optimal path of the Romer model could be analysed through the optimal path of the Lucas model, through mere notational re-arrangements. Thus we simulate the transition path of both models and compare their behaviour. We consider the following values for parameters: $\alpha = 0.36$; $\sigma = 2$; $\rho = 0.02; B = 0.15$. These values are quite standard in the literature. However, parameter B may deserve further discussion, which we do in the following subsection.

Considering the same parameter values in both models is crucial for the comparison. This implies a higher growth rate in the optimal Lucas problem (6.50%) than in the decentralized equilibrium of the Romer model (1.44%). The evolution of the output growth rate (g_Y) and of the share of human capital allocated to knowledge production (human capital in the Lucas model and ideas in the Romer model) - u_Y - are presented below:

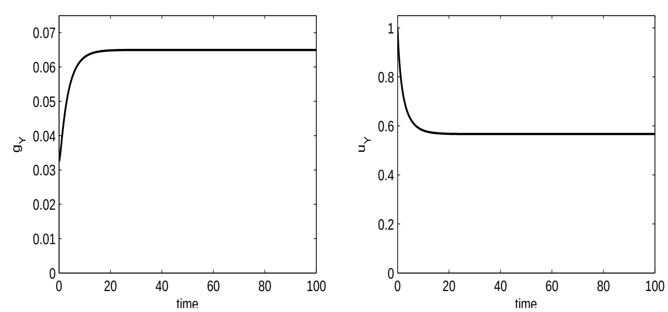

Figure 1: Transition Paths in the Lucas Economy

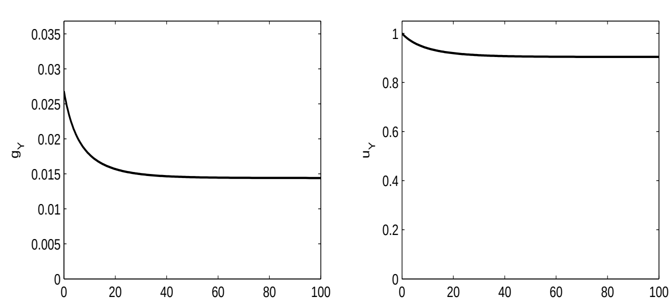

Figure 2: Transition Paths in the Romer Economy

The main difference to note from the figures is that the growth rate is increasing in the Lucas model and decreasing in the Romer model. This happens because, due to the markup, the Romer economy allocates less resources to R&D than the Lucas economy to the human capital accumulation. Initially, the share of resources allocated to the growth-source activity increases slowly in the Romer economy, which, together with the decreasing growth rate of physical capital, implies a decreasing output growth rate. The opposite occurs in the Lucas economy. This economy sharply re-allocates resources from the final good into the human capital accumulation sector, which implies an increasing output growth rate. As it is evident from the figures, the Lucas economy has a higher growth rate and a lower share of human capital in the final good production than the Romer economy. These results will have significant impact in terms of welfare as will be shown below.

To calculate welfare, we have computed a measure of utility: according to the objective function $\int_0^\infty e^{-\rho t}\frac{C^{1-\sigma}-1}{1-\sigma}$ in (1) and (5), utility is calculated as $U = U^{tr} + \frac{\frac{C_{ss}^{1-\sigma}-1}{1-\sigma}+\frac{g_C}{\rho}}{\rho-(1-\sigma)g_C}$, where U^{tr} is the utility level calculated through the transition and is calculated using the trapezoid rule. It represents the welfare attained until the last period of the transition path. The second term of the right hand side calculates utility after the economy attains the steady-state, where C_{ss} is the level of consumption at the steady-state. We note that as utility is a discounted value, it is independent of the number of years of transition. Thus, we stress that utility is directly calculated from its equation, taking the transition path into account. As a normalization, we set $C(0) = 1$. The welfare in the Lucas economy is 81.233 and the welfare in the Romer decentralized equibrium is 72.727. The welfare gained throughout transition is $U\_tr = 31.239$ in the Lucas economy against $U\_tr = 22.794$ in the Romer economy. The optimal subsidy scheme presented by Arnold (2000b), if implemented from the beginning, will achieve a welfare gain of 8.506.

3.1 Taking the Models to Data

The interpretation of the Lucas model as the optimal problem of the Romer decentralized equilibrium, due to Arnold (2000b), may indicate that the welfare attained by means of a human capital accumulation economy must be higher than the welfare attained by the innovating economy described by Romer (1990). However, this is an idea that we want to confront. When we take both models to data, there are crucial differences between their reasonable parameters and thus, between their predictions towards welfare. A simple exercise is to consider a (World) economy with a reasonable growth rate of 2% (see e.g. Maddison, 2001) and derive the implications of both models in terms of welfare. The expression of the steady-state output growth rate in the Lucas model is $g_Y^{ss} = \frac{B-\rho}{\sigma}$, which implies $B^{Lucas} = 0.06$, and $u_Y^{ss\_L} = 0.667$. However, in the Romer model $g_Y^{ss} = \frac{\alpha B-\rho}{\sigma+\alpha}$, which implies $B^{Romer} = 0.186666$, and $u_Y^{ss\_R} = 0.893$. These values are realistic as the corresponding share of resources allocated to learning is higher that the share allocated to R&D, as it is in reality. When calculating welfare we now see that an innovating economy always implies a higher welfare than a human capital economy. This happens because the Romer economy allocated more resources to the final good that is provided for consumption. This implies a higher rate of growth of consumption throughout transitional dynamics in the Romer economy than in the Lucas economy, eventhough the steady-state growth rate is equal between both economies. This result is in fact obtained through the analysis of transitional dynamics, which is crucial for the analysis of welfare. Figure 3 and 4 show the evolution of the growth rate of consumption in both economies.

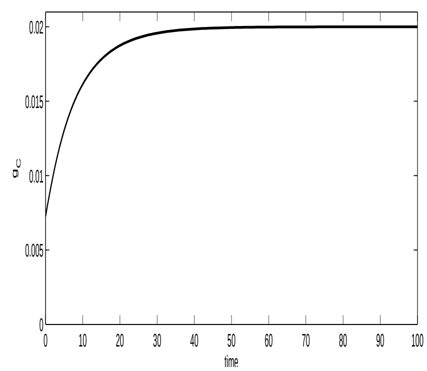

Figure 3: Growth Rate of Consumption in the Lucas Economy - Matching with data

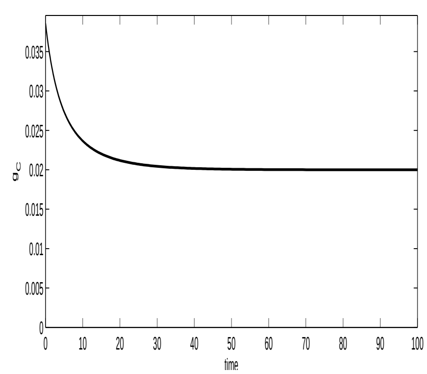

Figure 4: Growth Rate of Consumption in the Romer Economy - Matching with data

Table 1 shows welfare for different output growth rates, comparing the results of a human capital economy (Lucas) and an innovating economy (Romer) (when both imply the same output growth rate). The table shows that for reasonable values for the growth rate of the economy, it is always true that the innovative economy provides higher welfare than the human capital economy.

Table 1 - Welfare in Lucas and in Romer Model

g_Y	0.5%	1.0%	1.5%	2.0%	2.5%
Lucas Economy	55.22	63.60	68.51	71.99	74.58
Romer Economy	59.89	68.23	73.23	76.75	79.36

4 Conclusion

In this article, we depart from the comparison between the well-know endogenous growth models of Lucas (1988) and Romer (1990) provided by Arnold (2000b) to present quantitative welfare results from those models. Arnold (2000b) showed that with a simple rearrangement of parameter definition, the Romer optimal problem is similar to the Lucas optimal problem. Additionally, he showed that a subsidy scheme can internalize all the market distortions in the Romer model.

As the interpretation of the Lucas model as the optimal solution of the Romer decentralized equilibrium can lead to the conclusion that a human capital driven economy leads to a higher well-being than an ideas driven economy, we clarify this issue. In fact, when taking both models to data, for an economy with the same economic growth rate, an innovating Romer economy leaves economic agents always better-off than an human capital driven economy as in Lucas (1988).

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