1
Behavior in All-Pay Auctions with Incomplete Information

Charles Noussair and Jonathon Silver[*]

Behavior in All-Pay Auctions with Incomplete Information

Charles Noussair and Jonathon Silver

Abstract:

This paper analyzes the behavior of single-unit all-pay auctions within the independent private values environment in the laboratory. We study revenue, individual bidding behavior, and efficiency, in relation to theoretical benchmarks and to a similar study of winner-pay first-price sealed-bid auctions. We conclude that the all-pay auction yields significantly higher revenue than both the risk- neutral Bayesian equilibrium and the winner-pay auction. Bidders' decisions move closer to equilibrium levels over time in the auction.

Keywords: All-pay, auction, experiment

1. Introduction

Many examples of competition exist with the property that multiple players exert effort or expend resources in an attempt to gain a benefit, and the losers' effort or expenditure goes uncompensated. Students vying for grades in a class with a curve, lobbyists attempting to gain a favor from politicians, or rival companies battling to release a new innovative good on the market are just a few instances of this type of interaction. An auction, in which all players pay the amount of their bids, but the person or the firm that bids the highest wins the prize, is a simple and natural way to model such competition. Bidders' expenditure in the auction can be interpreted as a monetary cost or a non-monetary cost of effort.

The theoretical analysis of auctions, beginning with Vickrey (1961) is one of the richest and most highly developed research areas of applied game theory. The main focus has been on

winner-pay auctions, where only the player(s) who obtain units are required to make payments. However, in the past decade, economists have begun to study auctions where bidders forfeit their bids even if they do not obtain an item (see for example Baye et al. (1993) or Krishna and Morgan, (1997)). In a single-prize all-pay auction, each player submits a nonrefundable bid, but only the highest bidder receives the prize. This logic can represent many types of winner-take-all contest, such as a patent race, political lobbying for a government concession, or some forms of academic competition.

All experimental studies of all-pay auctions of which we are aware have found that participants tend to bid more aggressively than in Nash equilibrium, implying overdissipation of the rent available for sale in the auction (Potters et al., 1998; Davis and Reilly, 1998; Gneezy and Smorodinsky 1999; Barut et al., 2002). This tendency to overbid is not specific to all-pay auctions; results from studies of first-price winner-pay auctions (Coppinger et al., 1980; Cox et al., 1982; Harrison, 1989; Kagel et al., 1987; and Kagel and Levin, 1993), have indicated overbidding relative to equilibrium levels as well. Most previous experimental studies of all-pay auctions (Potters et al., 1998; Davis and Reilly, 1998; Gneezy and Smorodinsky, 1999) have studied environments with complete information, in which all bidders' valuations of the unit(s) for sale are common knowledge,[1] while Barut et al. (2002) consider a multiple-unit all-pay auction under incomplete information. The experiment reported here focuses on the properties of single-unit all-pay auctions in an environment with incomplete information.

We find that bidders with low valuations for the object tend to bid close to, though usually below, equilibrium predictions. However, bidding higher than equilibrium levels is common for bidders with high valuations. Many participants bid as if they do not want to commit substantial amounts of money unless they have a high probability of winning the auction. These patterns of behavior are consistent with the presence of risk aversion (Fibich et al., 2004), as well as analogous to the pattern observed by Barut et al. (2002), who studied behavior in multiple-unit all-pay auctions under incomplete information. It also corresponds to the phenomenon observed

in recent experimental work on effort in organizations by Mueller and Schotter (2003), in which high-ability workers exert greater than optimal effort, and low-ability workers drop out of the competition. Our revenue results are also consistent with the previous literature, in that the auction yields higher revenue than in equilibrium. Furthermore, we conjecture that the all-pay auction generates higher revenue than a winner-pay auction under similar parameters. A dynamic pattern of behavior is evident as the game is repeated. Many bidders suffer considerable losses in the first few periods of their session, but quickly make the adjustment to bid lower than they had bid previously, albeit still above equilibrium levels, for the rest of the auctions in the session.

Section two contains a derivation of an equilibrium for the auction in the environment we study. Section three describes the protocol of the experiment, section four describes the results, and section five is a brief summary and conclusion.

2. Theoretical Predictions

In this section we calculate a Bayesian equilibrium to the auction game we study. The equilibrium is monotonic, in that all players' bids are strictly increasing in own valuations. We also assume symmetry of bidding strategies so that every bidder uses the same strategy. The assumption of symmetry is imposed because (a) the symmetric equilibrium is simple to calculate and (b) there is reason to believe that it is the most plausible equilibrium that might emerge in the experiment; our game is symmetric and simultaneous with no obvious means to coordinate on an asymmetric equilibrium.

2.1. Environment and Auction Rules

The environment is the independent private values framework as introduced by Vickrey (1961). Suppose that n bidders, indexed by i, each receive a valuation v_i for a good to be sold in an auction. Each bidder i draws v_i from a uniform distribution $F(v)$ on the interval $[0, \bar{v}]$, that is

common to all bidders. Let b_i denote i's bid and let B_i define i's overall strategy as a function of his type so that $b_i = B_i(v_i)$. We require that $b_i \geq 0$. Each bidder knows the number of other participants, his own valuation, that there is only one unit for sale in each period, and that all bidders draw their valuations from the same common uniform distribution. However, bidders do not know what values other bidders have for the unit being sold. All of the above is common knowledge.

The rules of the all-pay auction game are the following. After drawing his own valuation, each bidder simultaneously submits a bid. Each bidder pays the amount of his bid, regardless of whether or not he obtains the unit. If player i does not submit the highest bid among the n players, he incurs a loss of $-b_i$. If i submits the highest bid among the players he receives the unit, earns value v_i from the unit and pays b_i, and thus his total payoff is equal to v_i - b_i.

2.2. Derivation of Equilibrium

The expected payoff to bidder i is denoted by

(1) $E\pi_i = v_i * P(highest\ bid) - b_i$,

where P(*highest bid*) equals the probability that b_i is the highest bid in the auction and therefore player i wins the unit for sale. Consider a symmetric strictly monotonic Bayesian equilibrium bidding function in this game, where players bid higher as their valuations increase, and use a common bidding strategy.[2] Since an individual's equilibrium bidding function, denoted as $B_i(v_i)$, is strictly monotonic, it is invertible. Since all bidders use the same strategy in a symmetric equilibrium, $B_i(v_i) = B(v)$ for all i. We denote the inverse of $B(v)$ as $V(b)$.

Since agents use a common monotonic bidding strategy, the probability that individual i submits the highest bid equals the probability that i has the highest valuation. This equals the probability that all other bidders have lower valuations. This can be calculated directly from the distribution of valuations. Substituting into equation (1), we have:

(2) $E\pi_i = v_i[F(V(b_i))]^{N-1} - b_i$,

where N is the number of bidders. In our experiment, where $N = 6$ and the distribution of

valuations is uniform on[3] $[0,1000]$ so that $F(V(b_i)) = V(b_i)/1000$, and $P(highest\ bid) =$

$[F(V(b_i))]^5$, player i's objective function becomes:

(3) $E\pi_i = v_i\left(\dfrac{V(b_i)}{1000}\right)^5 - b_i$

A necessary condition for equilibrium is that bidder i chooses b_i to maximize equation

(3). The first-order condition is:

(4) $\dfrac{dE\pi_i}{db_i} = 5v_i\left(\dfrac{V(b_i)}{1000}\right)^4 * \dfrac{V'(b_i)}{1000} - 1 = 0$

Since the function $V(b_i)$ is symmetric, we can substitute $V(b_i) = v_i$, and obtain:

(6) $\left(\dfrac{5v_i}{1000}\right)^5 * V'(b_i) = 1$.

Because $V(b_i)$ is the inverse of $B(v_i)$, $V'(b_i)$ must then equal $\dfrac{1}{B'(v_i)}$. Therefore,

(7) $B'(v_i) = \left(\dfrac{5v_i}{1000}\right)^5$.

Integrating $B'(v_i)$, we solve for $B(v_i)$ and obtain a symmetric, strictly monotonic equilibrium

bidding function, given by (8).

(8) $B(v_i) = \int_0^{v_i} \dfrac{5(v_i)^5}{1000^5} dv_i * \left(\dfrac{v_i}{1000}\right)^6 + C$

It remains to specify an initial condition. The obvious condition is that $B(0)=0$ because it is a

dominated strategy to bid an amount greater than one's valuation and bids are constrained to be

non-negative. Thus, we set $C = 0$ in equation (8), and the following expression satisfies the

necessary conditions for equilibrium.

(9) $$B(v_i) = \frac{2500}{3} * \left(\frac{v_i}{1000}\right)^6$$

It can be readily verified that the second order conditions for a maximum are satisfied. Figure 1 illustrates the equilibrium bidding function. On the horizontal axis are the valuations v_i and on the vertical axis are the bids b_i. Although $b_i > 0$ for all $v_i > 0$ the function does not reach a value of $b_i = 1$ until $v_i = 326$. The maximum possible bid consistent with equilibrium behavior is $B(1000) = 833.\overline{33}$.

[Figure 1: About Here]

3. Procedures of the Experiment

We conducted five experimental sessions, each consisting of one practice period and 25 periods that counted toward subjects' earnings. In each session, six subjects were given the role of bidders in an auction to purchase a fictitious object. There was one object auctioned in each period to the six bidders. In each period, bidders were given independently drawn valuations for obtaining the object for sale that period. The valuations were denominated in terms of an experimental currency and were integers drawn from the discrete uniform distribution on the [1, 1000] interval. The conversion rate of units of experimental currency to $US was 250 units/$1.00. New valuations were drawn each period so that an individual's valuation typically differed from period to period. Valuations were independent across bidders within a period as well for individuals over time.

All five sessions were conducted at Emory University using undergraduate students recruited from economics courses. No individual participated in more than one session during the study. All auctions were conducted manually rather than being computerized. Every subject received a $20 participation fee, announced at the beginning of each session, and this money

could be used to pay off any losses incurred during the series of auctions. In each period, bidders simultaneously submitted bids, in terms of experimental currency, for the single object available for sale during the period. The bidder with the highest bid won the item and received earnings equal to his valuation for the object minus his bid. Every other subject incurred a loss equal to the amount of his bid.[4] Table 1 shows the average, maximum, and minimum earnings among subjects for each of the five sessions.

[Table 1: About Here]

There was no communication allowed between subjects during the experiment. The information available to subjects at any point during a session was the following. Each participant had the information specified in the independent private values framework described above. Additionally, each subject knew the history of all bids from earlier auctions in the session, which was displayed on the blackboard for the entire session. Bidder identifiers and valuations were not displayed so it was not possible to associate a bid with a particular player or valuation. Players did not know the valuations that they would receive in future periods. When they turned in their record sheet to the experimenter at the end of each period, the experimenter wrote down their valuation for the next period in the appropriate column. The instructions for the experiment and a sample record sheet, used for recording players' bids and earnings, are given in the appendix[5].

4. Results

4.1. Revenue, Efficiency, and Comparison with Winner-Pay Auctions

We first consider the aggregate measures of outcomes that are generally of the greatest interest in the study of auctions: the revenue to the auctioneer and the overall surplus to the two parties. We compare observed revenue in our experiment to the Bayesian equilibrium level and

to previous data from winner-pay first-price sealed bid auctions reported by Cox et al. (1982, 1988). They investigate the first-price auction within an identical environment to ours: an independent private values information structure, six bidders, and a uniform distribution of valuations. Our observed revenue is higher than both benchmarks, the equilibrium level and the winner-pay first-price auction level.[6] Table 2 shows a comparison of the observed revenue with the Bayesian equilibrium revenue for the actual realizations of valuations for the period. In four out the five sessions, the average observed revenue considerably exceeded the equilibrium level.

[Table 2: About Here]

The table shows that 86 of 125 periods yield higher revenue than the equilibrium level. Average revenue per period is 1055, which is 47.7% higher than the predicted revenue of 714, and even higher than the maximum possible valuation for the object (1000). A t-test, under the conservative assumption that each session is the unit of observation, rejects the hypothesis that the average revenue in the session is less than or equal to the Bayesian equilibrium revenue at the five percent level ($t = 3.515$, $p < .01$, $n = 5$). This result is analogous to those obtained in previous studies of all-pay auctions in other environments (Davis and Reilly, 1998; Potters et al., 1999; Gneezy and Smorodinsky, 1999; Barut et al., 2002) that have shown that revenue is significantly greater than equilibrium levels in all-pay auctions. In conjunction with previous studies, our results suggest that more aggressive bidding than under non-cooperative game theoretic models may be a general feature of all-pay auction games.

It appears that revenue in the all-pay auction, at least for our parameters, is greater than under a winner-pay first-price auction. We compare the average revenue obtained here with that from the first-price sealed bid auction reported in Cox et al. (1982, 1988), using a pooled-variance t test.[7] Each period in each study is used as an observation. The Cox et al. data is appropriate as a basis for comparison because it consists of auctions in which six bidders bid for one item, the

bidders valuations were independently drawn from a uniform distribution and were private

information, so that the structure of the environment was identical to ours. We reject the

hypothesis that average revenue is equal under the two auction types at a five percent level of

significance ($t = 2.65$, $p < .05$), in favor of the alternative that the all-pay auction generates higher

revenue than the winner-pay auction.

Efficient allocations occur when the bidder with the highest valuation also submits the

highest bid, and therefore wins the item. A natural measure of efficiency is the ratio of the

valuation of the winning bidder to the highest valuation any bidder holds (Plott and Smith, 1978).

An efficiency level equal to 1 implies that the highest-valued bidder purchased the item. The data

for efficiency, also listed in Table 2, shows that in 62.4% of periods, an efficient allocation

occurred. Average efficiency equaled 89.1%. While this is clearly higher than would result from a

random allocation of the unit or a flawed institution, it is lower than typically obtained in winner-

pay first-price auctions. For the data that Cox et al. (1982, 1988) report for six bidders and the

same parametric structure we employ here, an average allocative efficiency of 98.26% was

obtained.[8]

4.2. Individual Bidding Behavior

Figures 2a-2e display the individual bid data from the five sessions that comprised the

experiment. The bids of each of the six subjects in a session are shown with a different symbol,

and the smooth line represents the Bayesian equilibrium bidding function calculated in section

two. The graphs show a rather similar relationship between bids and valuations in different

sessions. For lower valuations, a large majority of bids is at or close to zero, while high

valuations are generally accompanied by higher than equilibrium bids. However, there is

considerable heterogeneity of behavior between subjects.

[Figures 2a – 2e: About Here]

Although only 39% (293 out of 750) of all bids exceeded the equilibrium level, excess bidding was most evident for valuations within a particular range. Zero bidding is common; bidders with valuations of 500 or below bid zero 67.6% of the time. The percentage of subjects submitting zero bids declines steadily as valuations increase, rather than abruptly decreasing as would occur if bidders were using a common pure strategy.

For valuations below 800, the majority of bids is below the equilibrium level. The percentage exceeding equilibrium is fairly constant over the range from 0 to 700, above which it increases considerably. For valuations above 800, the majority of bids exceeds the Bayesian equilibrium level. The bids that exceed the equilibrium levels for valuations greater than 800 did so to an extent sufficient to more than offset the revenue loss relative to equilibrium from the lower than predicted bids observed for low valuations.

While the high bidding of those with valuations above 800 accounts for the greater than predicted revenue, the heterogeneity of behavior across individuals reveals the source of the inefficiency. Although behavior is reasonably common across individuals when they have low valuations, the variability of bids submitted by players with valuations above 700 suggests that inefficient allocations may be occurring, in that bidders with the highest values often fail to obtain the unit for sale. This pattern is particularly evident in session four. Closer inspection of figure 2d, which displays the data for this session, reveals a tendency on the part of bidders 3 and 6 to pursue a different type of strategy from the other four bidders.[9] Their bids are roughly a linear function of their valuations, while the other four bidders use the more typical strategy of zero or near-zero bidding for valuations below a threshold and higher than equilibrium bidding for higher valuations. This pattern of lower than equilibrium bidding for low and higher than equilibrium bidding for high valuations is consistent with the multiple-unit all-pay auction data reported in Barut et al. (2002) and with results from Mueller and Schotter's (2003) study of effort in organizations.

The aggregate pattern of individual behavior in our data is broadly consistent with the presence of risk aversion. Fibich et al. (2004) show, using perturbation analysis, that in an all-pay auction with independent private values and a small degree of risk aversion on the part of all bidders, the following patterns hold. (1) Buyers with low values bid lower than they would under risk-neutrality, (2) Bidders with high values bid higher than if they were risk neutral, and (3) buyers' expected utilities are lower than they would be under a first-price winner-pay auction. All of these patterns are observed here. Low valued players typically bid zero, consistent with (1), while those with high values bid higher than risk-neutral equilibrium levels, as (2) specifies. Expected earnings are negative, and therefore lower than in first-price auctions, which is in accordance with (3). However, because there are strong behavioral patterns in other types of auctions that are inconsistent with risk-aversion (see for example Kagel and Levin, 1993, or Kagel, 1995), other explanations for the patterns observed here may well be at least as important as risk aversion.[10]

We next consider whether or not participants change their behavior as they become more experienced with the auction process. Bidders begin with no prior experience, and their decisions might be expected to quickly improve over time. Figure 3a shows the average value of $|b^{ij}_t - b^{ij*}_t|$ the absolute difference between the player i's bid in period t of session j and the equilibrium bid for player i in the period. Figure 3b illustrates the analogous average value of $b^{ij}_t - b^{ij*}_t$, the average bias of i's bid. While the absolute difference is a measure of dispersion, bias is a measure of the extent of average over-or underbidding relative to equilibrium, with a positive value indicating a bias toward overbidding. The figures show the data by period, pooled across bidders and sessions. They illustrate that at the beginning of the sessions the average bid greatly exceeds the equilibrium level, but rapidly declines over the first five periods. However, the bias in bids is fairly constant after period 5, and remains positive, indicating long-term bidding in excess of the equilibrium level. Bidders appear to learn quite early the consequences of severe overbidding. This causes the rapid decline in bids relative to equilibrium predictions during the first few

periods. However, it appears that the feedback from moderate overbidding is not sufficiently powerful to lead agents to lower their bids to the equilibrium level, at least over the time horizon that we are able to observe in our experiment. The incidence of zero bidding does not show a systematic pattern over time, with 51.3%, 50%, 44%, 39%, and 52.7% of bids equaling zero in periods 1-5, 6-10, 11-15, 16-20, and 21-25, respectively.

[Figures 3a and 3b: About Here]

To consider the convergence process of bids relative to equilibrium levels over time, we conduct a regression with the following specification, first employed in Noussair et al. (1995).

$$b^{ij}_t - b^{ij}_t{}^* = \beta_{11}D_1(1/t) + \ldots + \beta_{15}D_5(1/t) + \beta_2((t\text{-}1)/t) + u_{it}, \qquad (10)$$

where j indexes the session, the D_j are dummy variables that take on a value of 1 for session j and zero for other sessions, and t represents the market period. Notice that in the first period of session one, (D_1/t) equals 1 but all of the (D_j/t) terms for $j \neq 1$, as well as the $(t\text{-}1)/t$ term, equal zero. Therefore, the β_{1j} coefficients can be interpreted as indicating the origin of the dependent variable at the beginning of session j. As $t \to \infty$, the D_j/t terms approach zero while the $(t\text{-}1)/t$ term approaches one so that the β_2 coefficient indicates the value that the dependent variable asymptotically approaches. The specification allows heterogeneity early on between the individual sessions, but assumes convergence to a common asymptote in all sessions. It accords well with many types of experiments, which exhibit more between-session variability early than late in the experimental sessions.

The results of the estimation are shown in table 3. The estimated coefficients of the model with the Absolute Difference as the dependent variable are given in the second column,

and those for the Bias are in the third column. The standard errors are given in parentheses. The data in the table lead to two main observations. The first is that all of the β terms are significantly positive. The fact that β_2 is positive for the Bias equation indicates that even if the declining trend in bids is extrapolated into the infinite future, the average bid would remain significantly greater than the Bayesian equilibrium level, confirming the visual impression conveyed in figures 3a and 3b. Average bids are converging to a level above the equilibrium. The second observation is that all five of the β_{1j} terms are greater than β_2, which is consistent with a trend of declining bids relative to equilibrium predictions. Thus while the average bid does not converge to the equilibrium level, it is moving in its direction.

[Table 3: About Here]

5. Conclusion

The revenue from the sealed bid first price all-pay auction that we have studied here exceeds the Bayesian equilibrium revenue in the independent private values environment. This result accords with studies of the auction in other environments. Furthermore, the all-pay auction appears to generate higher revenue than the more widely studied and used first-price winner-pay auction. This result does not extend to the multi-unit case, for which Barut et al (2002) find that in multi-unit generalizations of the auctions studied here, there is no significant revenue difference between all-pay and winner-pay variants. While we obtain the result that revenue in the all-pay auction exceeds the winner-pay auction by comparing our data with those of a previous study that may have used somewhat different procedures, the fact that average revenue in the all-pay auction exceeds the highest possible valuation gives us confidence that the finding is not due to any methodological differences. It is implausible that a first-price winner-pay auction would yield average revenue greater than even the expected highest valuation, let alone the highest possible

valuation, because of the transparently dominated nature of bidding more than one's valuation in such an auction.

At the individual level, we observe extensive use of a dichotomous bidding strategy. For relatively low valuations, bids of zero are common, and for high valuations, bids that exceed equilibrium levels are typical. Thus in this form of competition, agents appear to exert either a great deal or very little effort, where effort is represented in our experiment by a monetary commitment. This pattern, aggressive bidding on the part of those with high valuations and passive bidding by those with low valuations, is consistent with risk aversion on the part of bidders, although of course the existence of the pattern does not prove that risk aversion is the cause. Furthermore, subjects are heterogeneous with regard to the valuation at which they tend to change their approach from non-competitive to competitive. This heterogeneity, in cases where the highest-valued bidder behaves non-competitively, can create inefficient allocations. Indeed, we observe more inefficiency here than is typically the case in winner-pay auctions.

References

Anderson, S., J. Goeree, and C. Holt, 1998. Rent Seeking with Bounded Rationality: An Analysis of the All-Pay Auction. *The Journal of Political Economy* 106, 828-853.

Barut, Y., D. Kovenock, and C. Noussair, 2002. A Comparison of Multiple-Unit All-Pay and Winner-Pay Auctions Under Incomplete Information. *International Economic Review* 43, 675-707.

Baye, M., D. Kovenock, C. de Vries, 1993. Rigging the Lobbying Process: An Application of the All-Pay Auction. *The American Economic Review* 83, 289-94.

Coppinger V., V. Smith and J. Titus, 1980. Incentives and Behavior in English, Dutch, and Sealed Bid Auctions. *Economic Inquiry* 18, 1-22.

Cox, J., B. Roberson, and V. Smith, 1982. Theory and Behavior of Single-Object Auctions. In: V.L. Smith, ed., *Research in Experimental Economics.* JAI Press, Greenwich, CT, USA, pp. 537-79.

---, V. Smith, and J. Walker, 1988. Theory and Individual Behavior of First-Price Auctions. *Journal of Risk and Uncertainty* 1, 61-100.

Davis, D., and R. Reilly, 1998. Do Too Many Cooks Always Spoil the Stew? An Experimental Analysis of Rent-Seeking and the Role of a Strategic Buyer. *Public Choice* 95, 89-115

Fibich, G., A. Gavious, and A. Sela, 2004. All-Pay Auctions with Weakly Risk Averse Buyers. Tel Aviv University working paper.

Gneezy, U., and R. Smorodinsky, 1999. All-Pay Auctions—An Experimental Study," Discussion Paper 99.01. University of Haifa.

Harrison, G., 1989. Theory and Misbehavior of First-Price Auctions. *American Economic Review* 79. 749-762.

Kagel, J., 1995. Auctions: A Survey of Experimental Research. In J. Kagel and A. Roth eds, <u>The Handbook of Experimental Economics</u>. Princeton University Press, Princeton, NJ, USA, pp. 501-585.

Kagel, J., and D. Levin, 1993. Independent Private Value Auctions: Bidder Behavior in First, Second and Third Price Auctions with Varying Numbers of Bidders. *Economic Journal* 103, 868-879

Kagel, J., R. Harstad and D. Levin, 1987. Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study. *Econometrica* 55. 1275-1304.

Krishna V. 2002. <u>Auction Theory</u>, Academic Press, Elsevier Science, San Diego, CA, USA..

Krishna, V., and J. Morgan, 1997. An Analysis of the War of Attrition and the All-Pay Auction. *Journal of Economic Theory*, 72, 343-362.

Mueller, W., and A. Schotter, 2003. Workaholics and Drop Outs in Optimal Organizations. discussion paper 2003-43, Center for Economic Research, Tilburg University.

Plott, C., and V. Smith, 1978. An Experimental Examination of Two Exchange Institutions. *Review of Economic Studies* 45, 133-153.

Potters, J., C. de Vries, and F. van Winden, 1998. An Experimental Examination of Rational Rent-Seeking. *European Journal of Political Economy* 14. 783-800.

Vickrey, W., 1961. Counterspeculation, Auctions, and Competitive Sealed Tenders. *Journal of Finance* 16. 8-37.

Appendix I: Instructions for the Experiment

General Instructions

This is an experiment in the economics of market decision-making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money, which will be paid to you in cash at the end of the experiment. The experiment will be broken up into a series of 25 periods in which you will be bidding in a series of auctions for units of a good called X. You will be given a Record Sheet to keep track of results. You are not to reveal the results on your Record Sheet to any other participant.

In each period, you will see a number on your Record Sheet in the column labeled "Redemption Value." This number is your value for the product, and indicates the amount of experimental currency you will receive if you obtain a unit of X in the auction that period. Your Redemption Value is chosen randomly before the experiment and is equally likely to be any integer between 1 and 1000. You will receive a new randomly chosen redemption value in each period, and your redemption value will typically be different than the one of every other player. Other players' redemption values are independent of your redemption value, that is, each other player's number is still equally likely to be any number between 1 and 1000, no matter what your number happens to be.

You can obtain units of X by participating in the market process which is described below.

The Auction Process

Each period, you will be grouped with five other participants. There will be only one unit sold to each group each period.

During each period, you may submit a bid for one unit of the commodity by filling out the column entitled "Bid" for the appropriate period on the Record Sheet. After all participants have submitted their bids, the highest bid in each group will be accepted and that bidder will

receive the unit of X awarded. If there is a tie for the highest bid, the unit is randomly assigned to

the tied buyers by a coin flip.

In this auction, you pay the amount of your bid regardless of whether or not you receive a

unit of X. A practice round will initially be conducted to make sure that all participants

understand the auction process.

Determining Your Earnings

Please refer to your record sheet to determine your earnings. You must record your earnings on

your record sheet at the end of each period. In column 4, labeled Value of Units Received, enter

your redemption value if you made the highest bid for the period and 0 if you did not. Subtract

column 3 from column 4, and these are your earnings for the period, which are to be entered in

column 5.

Example: Someone with a redemption value of 750 bids 500 in an auction and has the

highest bid. This person would enter *750* into the Value of Units Received column and *250*, 750-

500, into the Earnings for Period column on the record sheet. Another person with a redemption

value of 700 bids 400 in the same auction. This person would enter *0* into Value of Units

Received and *–400*, 0-400, into Earnings for Period.

At the conclusion of the experiment, add of all of the earnings for each period and enter

the total at the bottom of the record sheet. The currency used in this market is "francs," and the

total earnings will be converted into dollars at a rate of 250 francs/dollar. This value will be

added (or subtracted if negative) to the $20 participation fee and will be paid out at the conclusion

of the experiment.

Appendix II: Sample Record Sheet

Record Sheet

ID#___A_____

Period	Redemption Value	Bid	Value of Units Received	Earnings for Period
Practice	500			
1	577			
2	47			
3	157			
4	381			
5	823			
6	717			
7	321			
8	614			
9	994			
10	881			
11	34			
12	216			
13	928			
14	641			
15	160			
16	249			
17	883			
18	546			
19	432			
20	572			
21	139			
22	631			
23	301			
24	465			
25	385			
Total Earnings for the Auction is:				

Table 1: Earnings of Individual Subjects in the Experiment

Session Number	Average Earnings	Maximum Earnings	Minimum Earnings	Number of Subjects Earnings Less Than Participation Fee
1	$20.75	$27.80	$13.50	2/6
2	$15.45	$19.98	$12.00	6/6
3	$15.01	$17.60	$11.99	6/6
4	$9.44	$20.66	-$2.24	5/6
5	$15.00	$19.21	$5.87	6/6

Table 2[*]: Average Revenue and Efficiency, All Sessions

Session	Average Revenue – BE	# Periods Revenue>BE	Average Efficiency	Efficiency = 1
1	-3.62	10/25	.906	15/25
2	386.90	20/25	.937	17/25
3	354.27	18/25	.873	16/25
4	597.23	20/25	.822	11/25
5	366.23	18/25	.919	19/25
Total	340.20	86/125=68.9%	.891	78/125=62.4%

---

[*] In Table 2, Average Revenue – BE equals the average difference between the actual revenue and the Bayesian equilibrium revenue per period during each session. Revenue>BE indicates the number of periods in each session that the total revenue exceeds the Bayesian equilibrium revenue for the actual valuations bidders held in the period.

Table 3: Estimates of Convergence Model for Absolute Difference and Bias Relative to Equilibrium

Independent Variable	Dep. Var. = Absolute Difference $\lvert b^{ij}_t - b^{ij}_t{}^* \rvert$	Dep. Var = Bias $b^{ij}_t - b^{ij}_t{}^*$
D_1/t	279.30 (64.37)	211.37 (76.40)
D_2/t	108.26 (49.52)	157.89 (60.84)
D_3/t	193.38 (57.96)	183.82 (63.08)
D_4/t	417.14 (109.11)	412.33 (116.26)
D_5/t	175.97 (64.25)	219.69 (69.99)
$(t\text{-}1)/t$	103.47 (7.87)	24.22 (9.20)

Figure 1: Equilibrium Bidding Function

Figure 2a: Bids and Valuations, Session 1
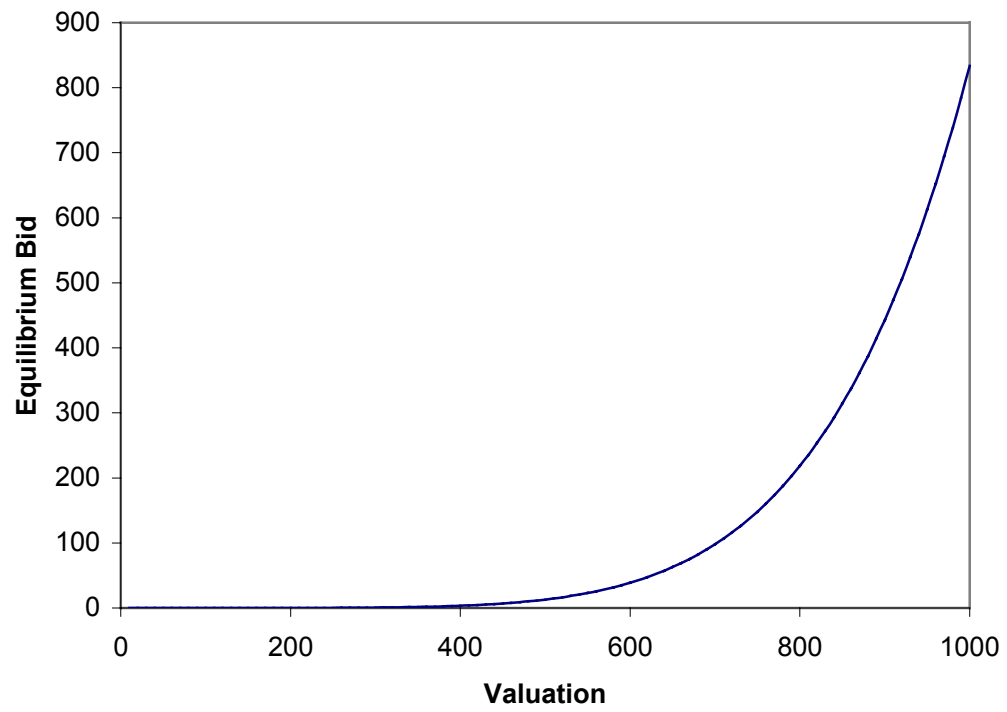
Figure 2b: Bids and Valuations, Session 2
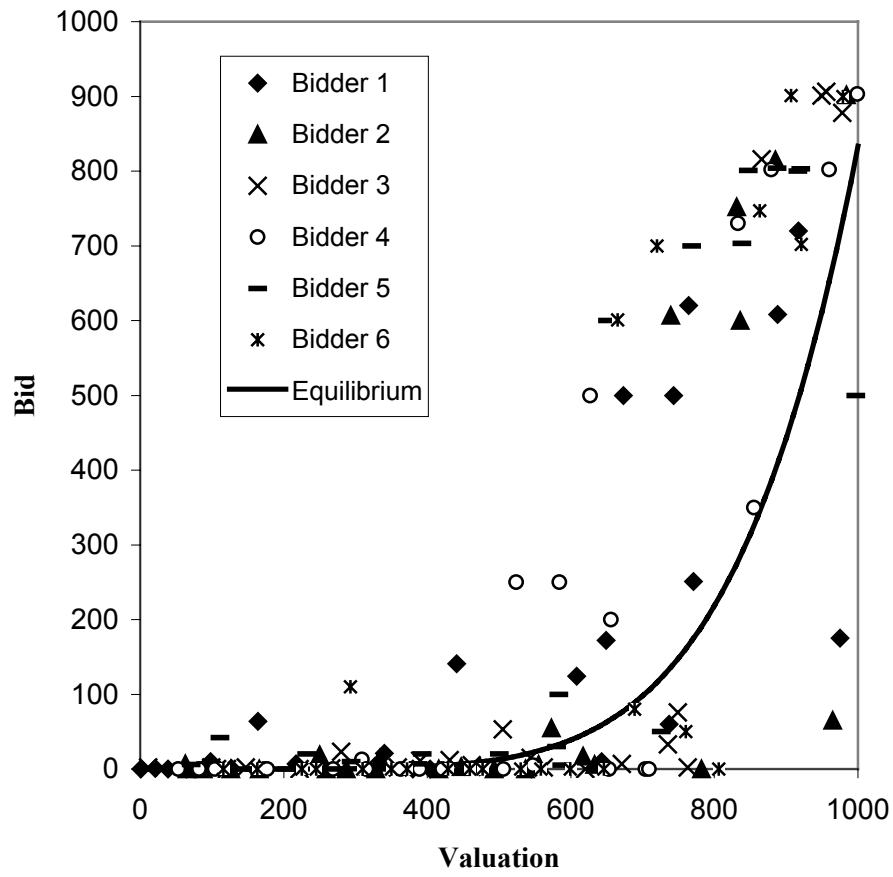
Figure 2c: Bids and Valuations, Session 3
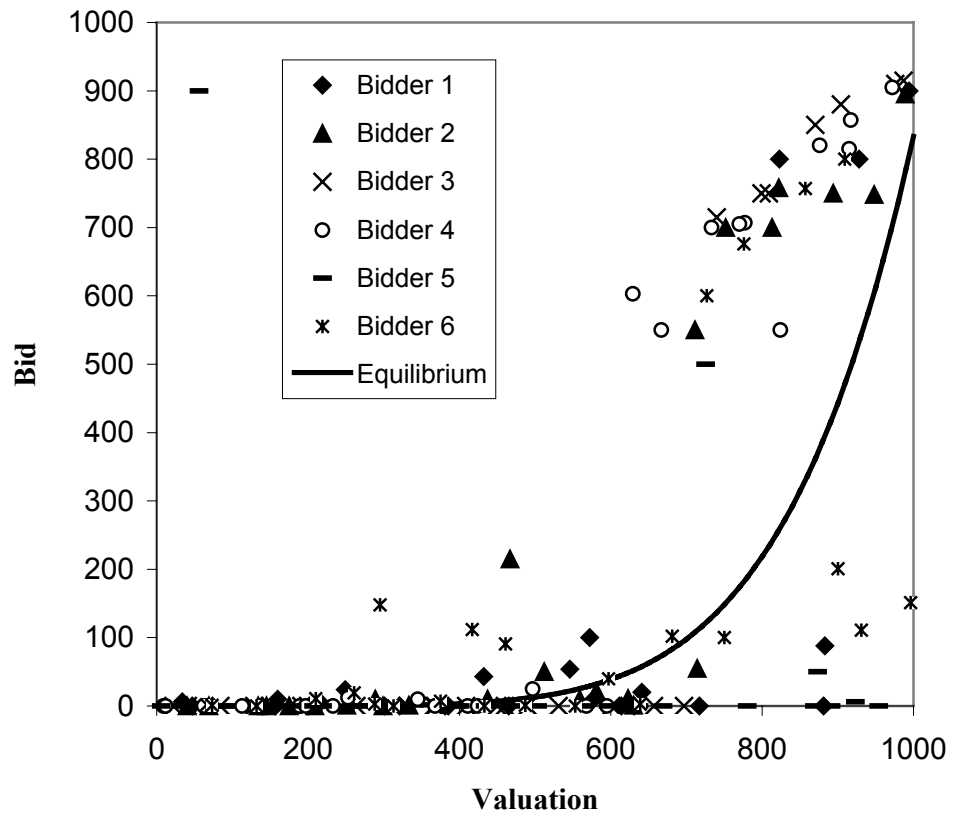
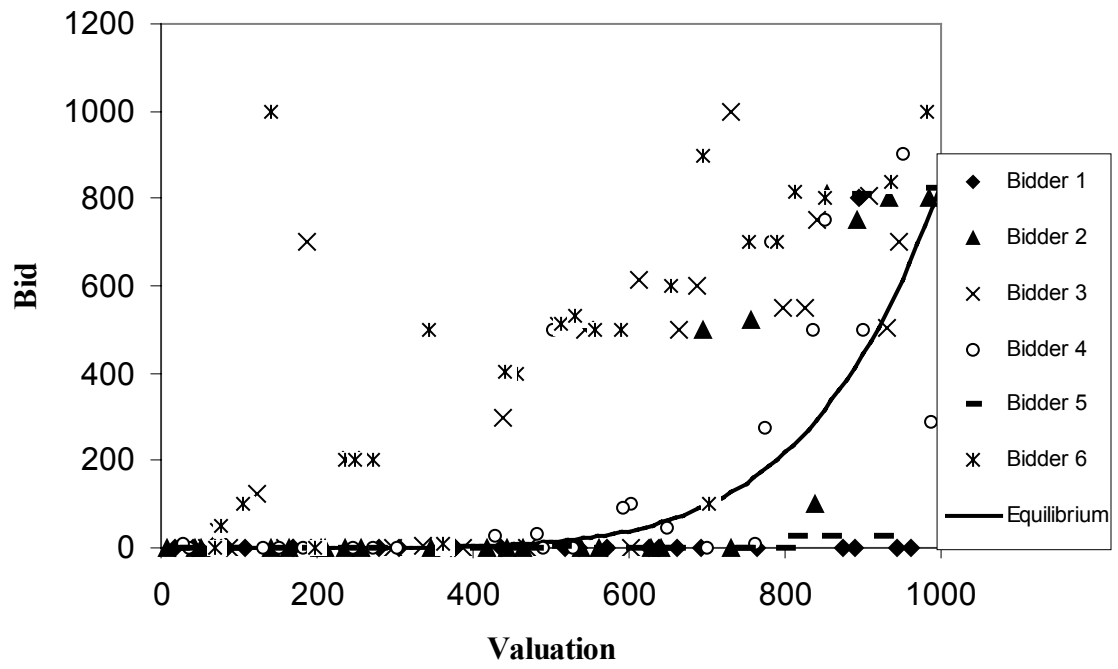**Figure 2d: Bids and Valuations, Session 4**
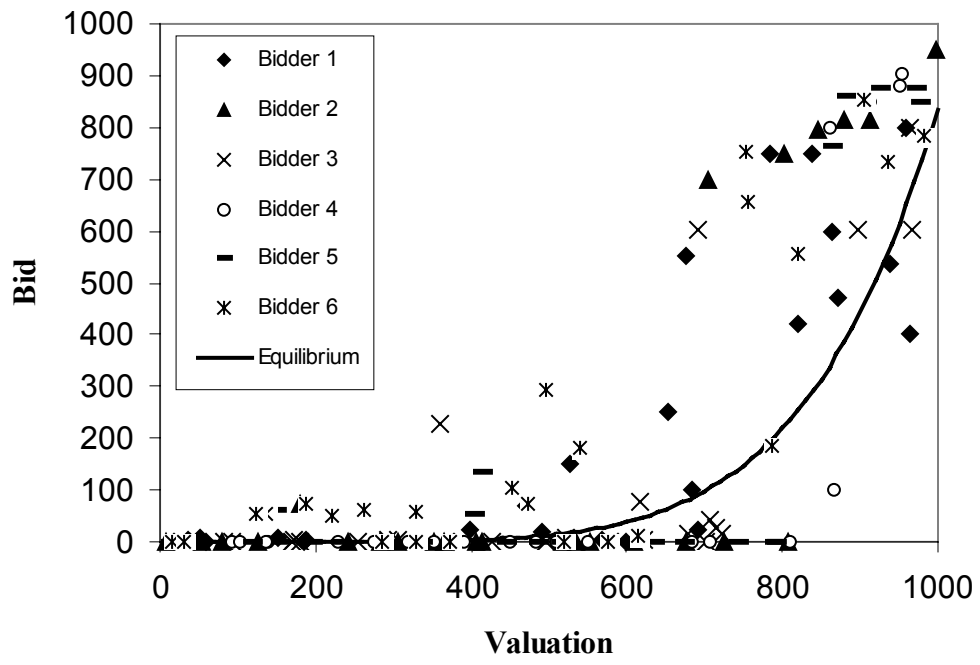
Figure 2e: Bids and Valuations, Session 5

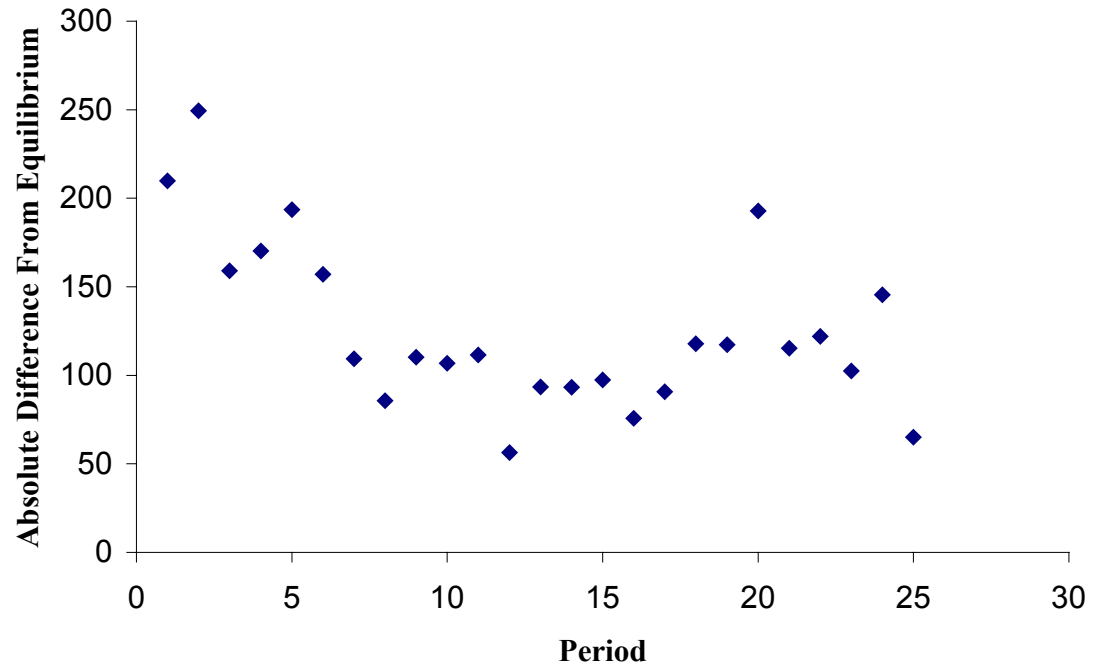**Figure 3a: Average Absolute Difference Between Observed and Equilibrium Bid, By Period**
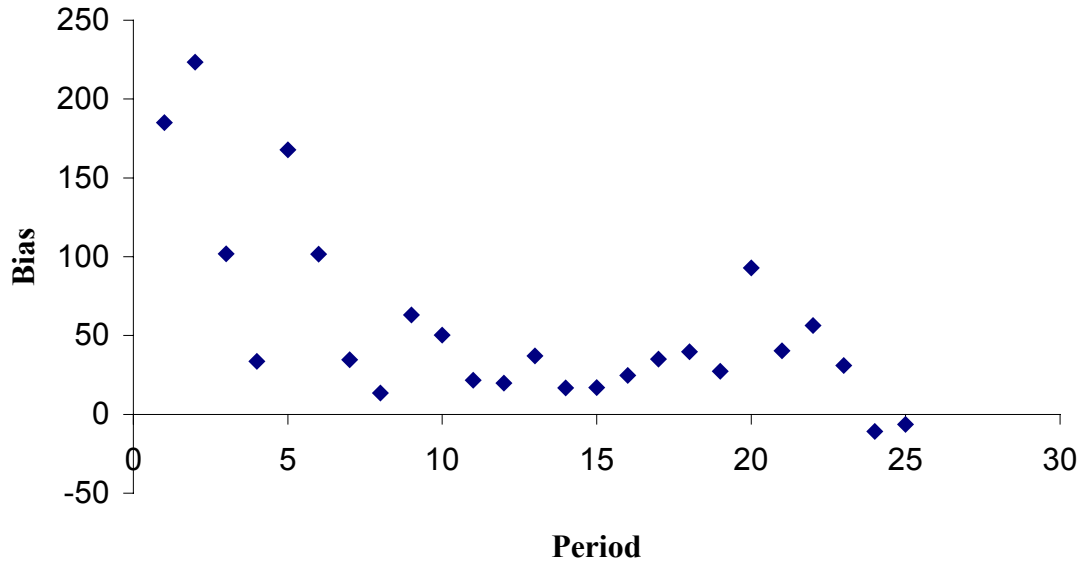
**Figure 3b: Average Bias of Observed Bid Relative to
Equilibrium, By Period**

ENDNOTES

1. Anderson et al. (1998) show that in the case of complete information, overdissipation is consistent with a logit equilibrium, in which agents may commit "errors" by choosing actions that do not have the highest expected payoffs, but the probability of choosing a particular action is increasing in its expected payoff.

2. See Krishna and Morgan (1997) or Krishna (2002) for a derivation of the Bayesian equilibrium for more general cases.

3. Although we use a discrete uniform distribution in the experiment, we derive the equilibrium here assuming a continuous distribution. The main analytical difference is that if the distribution is discrete, the possibility of a tie for the highest bid must be considered, while with a continuous distribution of types and strictly monotonic strategies, the probability of a tie is zero. With 1000 different valuations (each integer in the 1-1000 interval) and a uniform distribution, as in our experiment, the probability of a tie for the highest valuation and therefore for highest bid is extremely small.

4. If a subject had overall net losses for the auction, the amount of the loss would be deducted from the $20 participation fee while positive earnings would be added to the fee. In principle, a player could lose more than the initial twenty dollars. This only occurred for one person in the entire study. An individual in Session 4 lost $22.24 in the auction for overall losses of $2.24 for the session. In this case, the individual did not receive any money for his participation in the experiment. Since cumulative earnings are not calculated after each period, the subject appeared to be unaware that he had sustained more than the maximum allowable losses. His cumulative earnings became negative in period 23.

5. The sheet in the appendix contains the valuation of the individual for all 25 periods. However, during the experiment itself, individuals did not observe the valuations for

future periods. They knew only their valuations for the previous and current period at any time.

6. While the procedures may have differed slightly between our study and Cox et al. (1988) it is clear that revenue is higher here than it would be in a first-price winner-pay auction. All studies of the first-price winner pay auction have reported that revenue is below the highest valuation held among the bidders, although higher than the Bayesian equilibrium assuming risk neutrality. Here average revenue exceeds the highest valuation held among the bidders. In the winner pay auction, the highest bidder must use a dominated strategy for revenue to be greater than the highest valuation, while this is not the case for the all-pay auction we study here.

7. Valuations in the experiment of Cox et al. (1988) were drawn from a uniform distribution on [0, 16.9] in contrast to our interval of [0,1000]. We normalized our data for comparison by multiplying all bids in our study by 0.0169, and then calculating the mean and variance of the resulting transformed bids.

8. These very high efficiency results that Cox et al. obtain are typical of first-price winner-pay auctions. See for example, Kagel et al., 1987, who report efficiency levels of 98% to 99.5% in private value first-price winner-pay auctions.

9. Bidder six was the only person from the entire experiment to lose more money than the twenty-dollar participation fee.

10. A explanation of the deviations from the risk-neutral Bayesian equilibrium based on the existence of a utility of winning the auction cannot be a complete explanation for the pattern we observe. If a utility of winning exists, then bids would exceed Bayesian equilibrium levels for both low and high valuations. The pattern of underbidding on the part of players with low valuations seems inconsistent with the existence of a utility of winning the auction.