Correlation Hypotheses, and Contagion in the Sub-prime Crisis.

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Based on a joint work with François-Xavier Vialard, Consultant, Zeliade Systems.

Content of the talk.

- On high regimes of default correlation.
- A simple model to take randomness of correlation and systemic risk into account.
- Examples from the CDO and ABS market.
- Conclusion: importance of alternative methodologies for risk management.

Some conclusions from the subprime crisis

- Complex structured products (CDOs of ABS...) have been deviced so as to meet rating criteria. Strong model risk (too much dependence on the underlying models).
- Standard models failed to quantify default risk in CDOs of ABS. In particular, AAA credit enhancements have proved entirely mispriced.
- More broadly: General failure to capture systemic contagion and high default correlations regimes (credit quality of borrowers, monoline insurers, housing market, liquidity crunch...).

Our purpose today:

- Focus on certain shortcomings of standard pricing methods for structured finance and credit risk assessment.
- Favorite example: CDO tranches, but conclusions hold for any credit portfolio/ structured product.
- Concentrate on senior tranches which price reflects extreme risks (systemic and macroeconomic risks, liquidity, funding and counterparty risk, some operational risk...)

On Gaussian assumptions

- Recall briefly the standard methodology for large portfolios credit risk assessment (Bank exposures in Basel 2 in the advanced IRB approach, including retail investment; CDO tranches on CDS...):
- In the basic one-factor Gaussian copula model, the default of the i-th issuer is triggered by the value of a Gaussian variable:

$$G_i = \rho V + \sqrt{1-\rho^2} V_i$$

where ρ stands for the correlation of G_i to a market-wide random variable V.

On Gaussian assumptions (2)

- Shortcomings of the model are well-known for CDO tranches: different levels of correlation are associated to the various tranches (implied base correlation curves).
- Stochastic correlation or Random factor loadings (RFL) models did improve the picture (both theoretically and quantitatively):

$$X_i = \rho(V)V + \gamma V_i - m$$

  with $(V, V_1, \ldots, V_n)$ independent normal Gaussian variables, and $\rho(V) = \alpha \mathbf{1}_{V \leq \theta} + \beta \mathbf{1}_{V > \theta}$.
- Here, $\alpha > \beta$ stand for two correlation regimes (high, bearish, resp. low, bullish, since default correlation tends to increase when the economy deteriorates). The parameters m and γ are chosen so that X_i has mean 0 and variance 1 (thus m is the mean of $\rho(V)V$).

On RFL shortcomings

- But: random levels of correlation still fail to account for the price of senior tranches !
- Reason: Gaussian copula models, however sophisticated, fail to modelize correctly tails of portfolios loss distributions.
- Our conclusion is that a three parameters model fits market prices of tranches and gives a sound appreciation of risks: we add to the RFL model a systemic default intensity λ.
- The parameter measures roughly the spread of the most senior tranches of the structure and accounts for the risk premia (over the risk free interest rate) of these tranches. In particular it accounts, besides systemic risk, for risks that are mispriced in standard copula/credit risk models such as liquidity risk or counterparty risk.

A closed formula for the expected loss of the (A, B) tranche at maturity t in the large pool approximation (requiring only Normal and Bivariate Normal cumulative distributions):

$$EL_{(A,B)}(t) = (1 - \exp(-\lambda t))(B - A) + \exp(-\lambda t) \times \text{RFL term}$$

where:

$$\text{RFL term} = EL^{cond}_{(A,1)}(t) - EL^{cond}_{(B,1)}(t),$$

$$EL^{cond}_{(A,1)}(t) = (1 - R)[\Phi_2(\theta_1, \frac{C(t) + m}{\sqrt{\gamma^2 + \alpha^2}},$$

$$\frac{\alpha}{\sqrt{\gamma^2 + \alpha^2}}) - \frac{A}{1 - R}\Phi(\theta_1)] + (1 - R)[\Phi_2(\theta_2, \frac{C(t) + m}{\sqrt{\gamma^2 + \beta^2}},$$

$$\frac{\beta}{\sqrt{\gamma^2 + \beta^2}}) - \Phi_2(\theta, \frac{C(t) + m}{\sqrt{\gamma^2 + \beta^2}}, \frac{\beta}{\sqrt{\gamma^2 + \beta^2}})$$

$$-\frac{A}{1 - R}(\Phi(\theta_2) - \Phi(\theta))]\}$$

with $\theta_1 = \min(\theta, \frac{m + C(t) - \gamma\Phi^{-1}(A/(1-R))}{\alpha})$, $\theta_2 = \max(\theta, \frac{m + C(t) - \gamma\Phi^{-1}(A/(1-R))}{\beta})$, and where $C(t)$ is a default threshold calibrated on CDS spreads.

Zeliade Systems had been advocating this ERFL model since early 2006

The ERFL (enhanced random factor loadings) model combines the advantages of:

- Random Factor Loadings originally proposed by Andersen & Sidenius (2005)
- Gaussian one-factor associated with a systemic default spread model originally proposed by Minh, Thompson, Devarajan (2005)

Its parameters have a clear practitioners interpretation:

- Correlation is notoriously prone to successive regimes
- The stress regime can be interpreted as the worst year of a credit cycle
- Composite multi-name credit spreads including a systemic default risk

Implementation is easy thanks to a closed-form solution in the ”large pool” approximation. Efficient saddle-point implementation for granular portfolios allows to price 100+ names portfolios with time bucketed Greeks in a few seconds.

Calibration of the European investment grade index (September 2007)

European Investment Grade Credit Derivatives Index (iTraxx)						
Table 1	**Portfolio**	**Large Pool Model**				Proxy
21/09/2007	**Model**	**EGC**	**ERFL**	**RFL**	**Base Cor.**	rating
ITX 5 Year	**Alpha**[2]	13.9%	18.1%	31.1%		
Index 36bp	**Beta**[2]		11.4%	8.0%		
RR 40%	**Theta**		-2.2	-2.2		
	SDI[a]	20.0	14.0			
Tranches	**Mkt Price**	**Model Prices**				
0-3 (upfront)	18.7%	15.1%	19.5%	28.2%	28.6%	NR
3-6 (bp)	86.7	88.6	86.8	87.2	41.7%	BBB
6-9 (bp)	36.1	30.1	36.1	30.9	50.8%	AA
9-12 (bp)	23.2	21.7	23.2	29.7	57.6%	AAA
12-22 (bp)	14.3	20.0	15.1	16.3	73.3%	AAA

RFL and GC models do not satisfactorily calibrate both the Equity, Mezzanine and Senior tranches and explain the steep base correlation skew:

- RFL overprices (resp. GC underprices) equity risk and both do not discriminate senior and senior mezzanine

Market prices imply 3 regimes of correlations:

- Low correlation (beta=11.4%) with infrequent (theta=-2.2) higher correlation (alpha=18.1%) regimes
- Systemic default can be considered as 100% correlation

Systemic default intensity corresponds to 23% of index spread (SDI= 0.14% and RR=40%)

[a] SDI: systematic default intensity

Calibration of the European investment grade index (March 2008)

European Investment Grade Credit Derivatives Index (iTraxx)						
Table 1 bis	**Portfolio**	**Large Pool Model**				Proxy
03/06/2008	**Model**	**EGC**	**ERFL**	**RFL**	**Base Cor.**	rating
ITX 5 Year	**Alpha**[2]	31.8%	27.7%	77.8%		
Index 126bp	**Beta**[2]		11.9%	4.5%		
RR 40%	**Theta**		-1.2	-1.3		
	SDI[a]	120.1	91.8			
Tranches	**Mkt Price**	**Model Prices**				
0-3 (upfront)	42.5%	30.5%	48.8%	59.5%	47.5%	NR
3-6 (bp)	510	525.7	515.9	526.2	59.4%	BBB
6-9 (bp)	321.5	304.8	319.8	264.7	66.1%	AA
9-12 (bp)	231.5	212.1	232.0	232.1	71.1%	AAA
12-22 (bp)	126.5	151.5	132.2	217.2	84.5%	AAA

RFL overprices (resp. ECG underprices) equity risk and both poorly discrimate senior and senior mezzanine
Market implied regimes of correlations:

- Low correlation (beta=11.9%) with now more frequent (theta=-1.2) high correlation (alpha=27.7%) regime
- Systemic default intensity corresponds to 43% of index spread (SDI= 0.92% and RR=40%)

[a] SDI: systematic default intensity

Calibration of a generic European RMBS deal based on primary market statistics in the years 2004–2006

European Prime Residential Mortgage Securitization (RMBS)						
Table 2	**Portfolio**	**Large Pool Model**				Proxy
Prime RM	**Model**	**EGC**	**ERFL**	**RFL**	**Base Cor.**	rating
WAL 5 Year	**Alpha**^{2}	4.2%	8.4%	85.3%		
NIM 25bp	**Beta**^{2}		0.1%	1.5%		
RR 60%	**Theta**		-1.93	-2.13		
	SDI	30.7	27.9			
Tranches[a]	**Mkt Price**	**Model Prices**				
0-1% (upfront)	40%	37.6%	41.7%	53.5%	25.0%	NR
1-2.5% (bp)	80	81.5	79.8	92.0	49.3%	BBB
2.5-4% (bp)	40	31.1	40.3	38.9	61.0%	A
4-6% (bp)	25	30.7	28.8	38.7	70.9%	AA
6-100% (bp)	12	11	10	6.9	N/A%	AAA

Necessity of the 3 regimes:

- ”Independence” (beta=0.1% compared to 11.4%) in the low correlation regime
- Lower correlation (alpha= 8.4% compared to 18.4%) in the stress regime
- Higher influence of the systemic risk, corresponding to 45% of the pool’s spread

EGC and RFL model have poor calibration and discrimination of A and AA tranches.

[a] Attachment points include benefit of 0.4% reserve account from excess spread

Calibration of a generic European SME deal based on primary market statistics in the years 2000–2006

European SME loans securitization						
Table 3	**Portfolio**	**Large Pool Model**				Proxy
SME	**Model**	**EGC**	**ERFL**	**RFL**	**Base Cor.**	rating
WAL 4 Year	**Alpha**[2]	7.1%	8.0%	59.0%		
NIM 75bp	**Beta**[2]		0.1%	5.0%		
RR 50%	**Theta**		-1.8	-2.06		
	SDI	46.1	38.2			
Tranches[a]	**Mkt Price**	**Model Prices**				
0-4% (upfront)	35%	32.2%	35.2%	40.2%	23.5%	NR
4-6% (bp)	120	122.1	120.8	120.4	34.1%	BBB
6-8% (bp)	65	57.4	65.1	54.8	42.5%	A
8-11% (bp)	40	47.4	41.9	53.0	52.4%	AA
11-100% (bp)	18	20.1	16.7	10.7	N/A	AAA

Necessity of the 3 regimes:

- Correlation regimes close to the RMBS calibration
- Systemic risk influence similar to the IG index (25% of composite spread)

[a] Attachment points include benefit of 1.4% reserve account from excess spread

ERFL model parameters interpretation is valuable in a broader universe of ABS and multi-name credit assets

Based on pre-crisis market levels calibrations, ERFL parameters are more informative than base correlation:

- They suggest serious shortcomings of classical correlation neutral strategies
- Super senior risks captured by composite spread instead of correlations

Across credit asset classes:

- Corporate credit risk has a higher correlation than SME or Retail credit in normal and stress regime (as could be expected)
- Residential mortgage credit has a higher proportion of systemic risk in the composite spread:
  - Importance of the real estate markets
  - Leverage of households
  - Banks concentration in the segment
  - Jobs & real economy

With hindsight, the current ”sub-prime” crisis provides clear elements in its favour:

AAA and super senior risks are better captured by composite spread

- Fear contagion, liquidity squeeze, loss of confidence in origination standards or rating methodologies happen with 100% correlation (!)

Structured credit disasters such as CDOs of HEL mezzanine ABS could have been avoided

- Much higher AAA CDO-square credit enhancement from the high proportion of SDI in the composite spread
- More scrutiny would have resulted on BBB pieces of HEL ABS

Composite spread concept is equally useful for counterparty credit risk on credit derivatives or monoline credit enhancement

INTRODUCING THE CRIS PLATFORM

The future
...will require rebuilding of confidence
...reporting and disclosure will identify the winners and losers, and emphasize the long term value of risk management
...markets and regulations will require more transparency on risk and models
...requires a data, model, valuation and risk management portal whose commercial success can be ”public good” for the industry

Zeliade Systems, OTC-Conseil and JPLC, together with Dexia CLF, Microsoft France and the University of Evry are partners of such a project, called ”CRIS” recently selected by the Pôle de competitivité Finance Innovation. The project was chosen at the 5th “appel d’offre FUI”.

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