Demand Relationships Among Juice Beverages: A Differential Demand System Approach

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Abstract

Nielsen ScanTrack data were used to study how income and prices influenced consumer juice beverage demand in the United States during the period from 1988-89 through 1991-92. Alternative differential demand models combining the features of the Rotterdam model and the almost ideal demand system (AIDS) were tested. Results indicate the CBS type demand responses describe consumer behavior better than the other specifications for this particular data set.

Key Words: differential demand systems, juice beverages

In the last several decades consumer demand analysis has moved in the direction of the system-wide approach. There are now numerous algebraic specifications of demand systems, including the linear and quadratic expenditure systems, the Rotterdam model, translog models, the almost ideal demand system, and Working's model. Two demand systems which have become popular in agricultural economics are the Rotterdam model and the almost ideal demand system (AIDS) (e.g., Seale et al.; Lee et al. 1992; and Alston and Chalfant). However, the assumptions used to parameterize these two models have different implications. For example, the marginal expenditure share and the Slutsky terms are assumed constants in the Rotterdam model, while they are assumed functions of budget shares in the AIDS.

Economic theory does not provide criteria to choose *ex ante* between these two models; instead, researchers usually rely on statistical inferences. When the competing models are nested, the likelihood ratio test, Wald's test, or the Lagrangian multiplier test (Amemiya, p. 142) can be used to choose a model which best represents the data used. However, when the models are not nested, one needs an alternative testing procedure for the competing alternatives. Deaton (1978) applied a non-nested test to compare models with the same dependent variables, but his test is not suitable to compare the Rotterdam model and the AIDS, because these two models do not have the same dependent variables. Barten (1990) demonstrates that the Rotterdam model and the AIDS are special cases of a general demand model, so that nested tests can be carried out to determine whether the Rotterdam model, the AIDS, or other hybrids of these two models can best be used to explain the data. Lee et al. used Barten's nested testing procedure to choose among the Rotterdam model and a hybrid Rotterdam-Working's model. Alston and Chalfant have also developed a test for choosing between the Rotterdam model and the

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AIDS;[1] however, the Alston and Chalfant test can only be applied to pair-wise comparisons, which is less powerful than the one proposed by Barten (1990).

Although income and price elasticities vary over time, analysts frequently focus on demand elasticities calculated at sample means. Recent research has examined demand elasticities over time (Flood et al.; Seale and Theil). For example, using Japanese time series data from 1951 through 1972, Flood et al. show that the behavior of the income elasticity estimates for food is quite different under the translog and Working's models. Results from the translog model indicate that, over the time period studied, the income elasticity for food increased from about 0.4 to more than 0.7, whereas the Working's model yielded an almost equally large decline. Given that the income elasticity of demand for a good is a measure of its luxury character, one should question whether the large increase in the elasticity for food implied by the translog model is realistic. The elasticity values implied by Working's model seem more satisfactory.

In the present study, four differential demand systems examined by Barten (1990) (the Rotterdam model, a differential version of the almost ideal demand system (AIDS), and two mixed models, the CBS and NBR systems[2]) are fit to weekly retail scanner data on U.S. juice beverage consumption. This is an improvement over previous analyses on U.S. juice consumption in several important ways: the four models above are all consistent with economic theory and restrictions such as adding-up, unlike the double-log model previously used in juice beverage studies (Brown and Lee, 1986; Lee); it extends the analysis beyond the use of a single demand system such as the Rotterdam model (Lee; Brown and Lee, 1992) to four other competing models; and it is one of the first studies on juice consumption to utilize weekly retail scanner data. Additionally, the paper demonstrates the importance of functional form in terms of analytical results, elasticity measures, and in testing theoretical restrictions such as homogeneity and symmetry. It also demonstrates a statistical method to choose among several competing demand systems that goes beyond the pair-wise comparisons suggested by Alston and Chalfant and that does not suffer from adding-up and parameter identification problems.

The paper is arranged as follows. The next section introduces four competing demand systems derived using the differential approach (Barten, 1964, 1967, 1968; Theil, 1965) and the Deaton and Muellbauer model. A hybrid demand model developed by Barten (1990), which encompasses these four competing demand models, will be used to analyze the demand for juice beverages in the United States. From the estimated parameters, expenditure and price elasticities are calculated and reported. These elasticities will be used to demonstrate the differences these alternative models make in empirical work. Finally, conclusions from the study are summarized.

Four Differential Demand Systems

The Rotterdam model, due to Barten (1964) and Theil (1965), takes the form (with time subscripts omitted for convenience)

$$w_i \, d\log q_i = \theta_i \, d\log Q + \textstyle\sum_j \pi_{ij} \, d\log p_j, \quad i = 1,\ldots,n, \tag{1}$$

where w_i represents the average value or budget share for commodity i, p_i and q_i are the price and quantity of good i, respectively, $d\log p_i$ and $d\log q_i$ represent dp_i/p_i and dq_i/q_i, respectively, and $d\log Q$ is an index number (Divisia volume index) for the change in real income and can be written as

$$d\log Q = \textstyle\sum_i w_i d\log q_i. \tag{2}$$

The demand parameters, θ_i and π_{ij}, are given by

$$\theta_i = p_i(\partial q_i/\partial m), \tag{3}$$

$$\pi_{ij} = (p_i p_j/m)s_{ij},$$

$$s_{ij} = \partial q_i/\partial p_j + q_j \partial q_i/\partial m,$$

where m is total outlay or the expenditure, and s_{ij} is the (i,j)[th] element of the Slutsky substitution matrix. The parameter θ_i is thus the marginal budget share for commodity i, and π_{ij} is a compensated price effect. The constraints of demand theory can be directly applied to the parameters of the Rotterdam model, in particular,

Adding-up $\sum_i \theta_i = 1, \quad \sum_i \pi_{ij} = 0;$ (4)

Homogeneity $\sum_j \pi_{ij} = 0;$ (5)

and

Slutsky Symmetry $\pi_{ij} = \pi_{ji}.$ (6)

The Rotterdam model is a particular parameterization of equations (1) where the demand parameters, θ_i's and π_{ij}'s, are assumed to be constant. However, there is no strong *a priori* reason that the θ_i's and π_{ij}'s should be held constant. An alternative parameterization is based on Working's Engel model,

$$w_i = \alpha_i + \beta_i \log m, \quad i = 1,...,n. \tag{7}$$

As the sum of the budget shares is unity, it follows from (7) that $\sum\alpha_i = 1$ and $\sum\beta_i = 0$. To derive the marginal shares implied by Working's model, one multiplies (7) by m and then differentiates with respect to m, which results in

$$\partial(p_i q_i)/\partial m = \alpha_i + \beta_i(1 + \log m), \text{ or} \tag{8}$$

$$\theta_i = w_i + \beta_i.$$

Hence, under Working's model the *i*th marginal share differs from the corresponding budget share by β_i; as the budget share is not constant with respect to income, neither is the associated marginal share.

By replacing θ_i in (1) with (8) and rearranging terms, one obtains

$$w_i d\log q_i = (\beta_i + w_i)\, d\log Q + \sum_j \pi_{ij}\, d\log p_j, \tag{9}$$

where β_i and π_{ij} are constant coefficients (Keller and van Driel; Theil and Clements, 1987). Equation (9) will be referred to as the CBS model following Keller and van Driel.

The AIDS model, which can be viewed as an extension of Working's model by allowing price effects, is specified as

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(m/P); \tag{10}$$

where $\log P$ is a price index implicitly defined by

$$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \tfrac{1}{2} \sum_k \sum_l \gamma_{kl} \log p_k \log p_l.$$

The adding up restriction requires that

$$\sum_i \alpha_i = 1, \quad \sum_i \beta_i = 0, \quad \sum_i \gamma_{ij} = 0;$$

homogeneity is satisfied if and only if

$$\sum_j \gamma_{ij} = 0;$$

and symmetry is satisfied provided that

$$\gamma_{ij} = \gamma_{ji}.$$

The AIDS can also be expressed in differential form (Deaton and Muellbauer; Barten, 1990). Specifically, approximating $\log P$ by Stone's price index and the logarithmic change in Stone's price index by the Divisia price index, $\sum_i w_i d\log p_i$, one can obtain the differential AIDS specification

$$dw_i = \beta_i\, d\log Q + \sum_j \gamma_{ij}\, d\log p_j. \tag{11}$$

As shown by Barten (1990),

$$\beta_i = \theta_i - w_i, \text{ and}$$

$$\gamma_{ij} = \pi_{ij} + w_i \delta_{ij} - w_i w_j;$$

where δ_{ij} is the Kronecker delta, equal to unity if $i = j$ and zero otherwise. Equation (11) can be rewritten as

$$w_i d\log q_i = (\beta_i + w_i)\, d\log Q$$

$$+ \sum_j (\gamma_{ij} - w_i(\delta_{ij} - w_j))\, d\log p_j \tag{11a}$$

using the relations that $dw_i = w_i(d\log p_i + d\log q_i - d\log m)$, $d\log m = d\log P + d\log Q$, and $d\log P = \sum_i w_i d\log p_i$ is the Divisia price index. Note that the CBS system has the AIDS income coefficients β_i's and the Rotterdam price coefficients π_{ij}'s.

A fourth alternative model, the NBR model (Neves), can be derived by substituting θ_i-w_i for β_i in (11). The NBR model has the Rotterdam income coefficients and AIDS price coefficients. Specifically the NBR model is

$$dw_i + w_i\, d\log Q = \theta_i\, d\log Q + \sum_j \gamma_{ij}\, d\log p_j. \quad (12)$$

Similarly, equation (12) can be rewritten as

$$w_i d\log q_i = \theta_i\, d\log Q + \sum_j (\gamma_{ij} - w_i(\delta_{ij} - w_j))\, d\log p_j. \quad (12a)$$

The four models (equations (1), (9a), (11a), and (12a)) have the same left-hand-side variable $w_i d\log q_i$ and right-hand-side variables $d\log Q$ and $d\log p_j$, $i,j = 1,\ldots,n$. These models can be considered as four different ways to parameterize a general model: the marginal budget shares are assumed to be constants (i.e., θ_i) in the Rotterdam and NBR models but variables (i.e., $\beta_i + w_i$) in the AIDS and CBS models; while the Slutsky terms are considered to be constants (i.e., π_{ij}) in the Rotterdam and CBS models and variables (i.e., $\gamma_{ij} - w_i(\delta_{ij} - w_j)$) in the AIDS and NBR models. The CBS and the NBR models can be considered as income-response variants of the Rotterdam model and the AIDS, respectively.

Choice among Four Parameterizations

The four models presented above are not nested. However, a general model can be developed which nests all four models (Barten, 1990). Specifically, the general model is

$$w_i\, d\log q_i = (d_i + \delta_1 w_i)\, d\log Q + \sum_j (e_{ij} - \delta_2 w_i(\delta_{ij} - w_j))\, d\log p_j; \quad i = 1,\ldots,n; \quad (13)$$

where $d_i = \delta_1\beta_i + (1-\delta_1)\theta_i$, $e_{ij} = \delta_2\gamma_{ij} + (1-\delta_2)\pi_{ij}$, and δ_1 and δ_2 are two additional parameters to be estimated. Note that (13) becomes the Rotterdam model when both δ_1 and δ_2 are restricted to be zero; the CBS model when $\delta_1 = 1$ and $\delta_2 = 0$; the AIDS model when $\delta_1 = 1$ and $\delta_2 = 1$; and the NBR model when $\delta_1 = 0$ and $\delta_2 = 1$. The demand restrictions on (13) are

Adding-up $\sum_i d_i = 1 - \delta_1$ and $\sum_i e_{ij} = 0$;

Homogeneity $\sum_j e_{ij} = 0$; and

Symmetry $e_{ij} = e_{ji}$. (14)

Thus the general model is consistent with economic theory and does not suffer from adding-up or parameter identification problems.

The general model nests not only the four elementary demand systems, but also all combinations of any two of these demand systems (which can be derived by imposing different restrictions on δ_1 and δ_2). It has two parameters more than the elementary systems and is therefore somewhat more flexible. Specification (13) can also be taken as a demand system in its own right. Although w_i appears on both sides of (13) and w_j appears on the right-hand-side, for estimation one might use lagged w_i and w_j on the right-hand-side as an approximation, as similarly done by Eales and Unnevehr (1988, 1993) for the AIDS.

As equations (13) and (14) nest the various Rotterdam/AIDS specifications and demand restrictions, the likelihood ratio test (LRT, Amemiya, pp. 141-6) can be used for testing the hypotheses of homogeneity, symmetry, and for selecting a model which can best explain the data. The differential demand specifications discussed to this point have been in terms of infinitesimal changes. For application to discrete data, the specifications are approximated by replacing w_i by $(w_{it} + w_{it-1})/2$, $d\log q_i$ by $\log(q_{it}/q_{it-1})$, and $d\log p_i$ by $\log(p_{it}/p_{it-1})$, where subscript t indicates time.

Data and Results

Weekly observations on juice beverage consumption for the weeks ending December 10, 1988 through November 11, 1992 were analyzed. The data were retail scanner data from stores with sales of more than four million dollars per year collected by A. C. Nielsen Company. Basic information on seven juice beverage groups were available. These groups are orange juice, grapefruit juice, apple juice, blended juices, juice drinks, juice cocktails, and remaining juices; the expenditure shares for the seven juice beverage groups for the study period were 0.35, 0.03, 0.08, 0.05, 0.32, 0.10, and 0.08, respectively. Due to the lack of information on the prices and quantities purchased for other commodities, it is assumed that juice beverage consumption is weakly separable from other commodities; therefore, the parameter estimates presented here are conditional demand parameters. To obtain the unconditional demand parameters, one needs knowledge of how the budget is allocated among juice beverages and other commodities, which is beyond the focus of this study.

Since all five models automatically satisfy the adding-up conditions, only six equations were estimated for the seven equation systems by excluding the remaining juice equation (Barten, 1969). The models analyzed in this study were estimated by the maximum likelihood method (Barten, 1969; Bewley). The first-order autocorrelation coefficient estimates (Berndt and Savin) for all five models were statistically insignificant. The log-likelihood values and their corresponding test statistics for each of the models are presented in Table 1. The numbers in the first three columns are the log-likelihood values; the numbers in the last three columns are the log-likelihood ratio test statistics (Barten, 1969; Deaton, 1974). As shown in Table 1, both homogeneity and symmetry restrictions are rejected at $\alpha = 0.01$ level for the AIDS, the NBR, and the general models but not for the Rotterdam and CBS models. The different parameterizations of price coefficients -- the Rotterdam and the CBS models have π_{ij} as their price parameters while the AIDS and NBR models have γ_{ij} as their price parameters -- seem to determine whether these two hypotheses are rejected or not rejected. This result may be an indication that the AIDS price parameter specification is too restrictive with the current data set.

The test results show that the general model rejects the Rotterdam and the NBR models as single models. The models which are not rejected by the general model are the CBS and AIDS models, an indication that the Working's income specification fits the data better than any of the other models. Since the homogeneity and symmetry hypotheses were not rejected using the CBS model and the CBS was not rejected by the general model, the results for the CBS model are presented and discussed.[3]

There has recently been much discussion concerning potential endogeneity problems caused by the expenditure variable in conditional demand systems (Attfield, 1985 and 1991; LaFrance). Attfield indicates that the rejection of homogeneity may be an indication that $d\log Q$ is endogenously determined, and $d\log Q$ and the disturbance terms used in the conditional demand systems are not independent. Remember that homogeneity was rejected for the AIDS and NBR models, but not for the Rotterdam and CBS models. Rejection of homogeneity (see Table 1) in the former (but not the latter) models may indicate an endogeneity problem with the expenditure variable. Still to further ensure that endogeneity is not a problem for the CBS juice beverage sub-demand model, the theory of rational random behavior (Theil 1975, 1976, 1980; Theil and Clements (1978), Duffy (p. 1060)) was invoked. Theil shows that if $d\log Q$ is indeed exogenous (i.e., the disturbance term is normal with zero mean and independent of $d\log Q$) then the covariances of the disturbance terms are proportional to the Slutsky terms, or in other words,

$$\mathrm{cov}(\varepsilon_i\varepsilon_j) = \gamma\pi_{ij},$$

where γ (<0) is a factor of proportionality. Using the covariance of disturbance terms and the Slutsky terms from the CBS model, a simple regression shows that $\mathrm{cov}(\varepsilon_i\varepsilon_j) = -0.0586\ (0.1960) - 8.9244\ (1.4260)\pi_{ij}$ with $R^2 = 0.68$, where the numbers in the parentheses are standard errors of the estimates. The regression results, an insignificant intercept term and a significant negative slope term, indeed support the theory of rational random behavior as the $\mathrm{cov}(\varepsilon_i\varepsilon_j)$s are approximately proportional to the Slutsky terms. Hence, in the present case, treating the disturbance term as independent of $d\log Q$ does not seem to be troublesome.[4]

Out of the 35 parameter estimates for the CBS model, 25 are statistically different from zero at $\alpha = 0.01$ level and all own-price Slutsky terms are statistically different from zero and have expected negative signs (Table 2). Single equation R^2s and Durbin-Watson statistics are also presented in Table 2. However, single equation R^2 statistics are not necessarily good measures for the goodness of fit for these models and single equation Durbin-Watson statistic should only be used as a guide to the possible existence of first-order autocorrelation

Table 1. Log-likelihood Values for the Four Basic Models and the General Model

Model	Log-Likelihood Value			Test Statistics[a,b]		
	No Restriction	Homogeneity	Homogeneity and Symmetry	Homogeneity	Homogeneity and Symmetry	Model Specification[c]
Rotterdam	6140.46	6137.63	6122 38	5 66 (6)	36 16 (21)	42.58 (2)
CBS	6161.94	6160.41	6142.66	3 06 (6)	38.56 (21)	2.02 (2)
AIDS	6169 81	6158.71	6140.95	22 20 (6)	57.22 (21)	5 44 (2)
NBR	6147 06	6134 70	6119.61	24 72 (6)	54 90 (21)	48.12 (2)
General	6171.77	6162.21	6143 67	19.52 (6)	56 60 (21)	

[a]Table values of χ^2 are 9 21, 16.81, and 38.93 for 2, 6, and 21 degrees of freedom, respectively, at α = 0.01 level
[b]Number in parentheses are degrees of freedom for tests
[c]Models with homogeneity and symmetry constraints imposed.

Table 2 Parameter Estimates for the CBS Model

Beverage	Expenditure (β_i)	Price (π_{ij})							R^2 (DW)
		Orange Juice	Grapefruit Juice	Apple Juice	Blended Juices	Juice Drinks	Juice Cocktails	Remaining Juices	
Orange Juice	-0.0517*	-0.3074*	0 0112*	0 0319*	0 0156*	0 1354*	0.0722*	0 0412*	0.8487
	(0.0099)[a]	(0.0212)	(0.0027)	(0.0075)	(0.0050)	(0.0172)	(0.0092)	(0 0060)	(2.0636)
Grapefruit Juice	0 0002		-0 0516*	0 0019	0 0047	0 0149*	0.0209*	-0 0020	0 8186
	(0 0010)		(0 0032)	(0 0028)	(0 0024)	(0 0024)	(0 0034)	(0.0032)	(2.0380)
Apple Juice	-0.0067*			-0 1581*	0 0092	0.0658*	0 0389*	0.0104	0.8339
	(0 0029)			(0 0082)	(0 0043)	(0.0062)	(0.0074)	(0 0055)	(1 9296)
Blended Juices	0 0006				-0.0893*	0 0377*	0 0251*	-0 0081	0 7661
	(0.0020)				(0 0045)	(0 0042)	(0.0054)	(0 0051)	(1 9120)
Juice Drinks	0.0564*					-0 3879*	0 0958*	0.0384*	0 9102
	(0.0092)					(0 0175)	(0 0081)	(0.0051)	(1.8913)
Juice Cocktails	0.0059						-0 2557*	0.0027	0 8617
	(0.0036)						(0 0128)	(0.0062)	(2.1025)
Remaining Juices	-0 0047							-0 0826*	[b]
	(0.0023)							(0.0079)	

[a]Numbers in parentheses are standard errors of parameter estimates First-order autocorrelation was not evident (Berndt and Savin)
[b]Derived from the other six equations, therefore, no R^2 nor DW-statistic was calculated
*Statistically different from zero at α = 0.01 level

(Bewley). Therefore, a statistic developed by Bewley et al. was used to measure goodness of fit; this statistic is

$$R_L^2 = 1 - 1/(1 + LR/(T(n\text{-}1)),$$

where T is the number of observations, n is the number of equations in the system, and LR is twice the difference between the log likelihood of the model and the log likelihood of the same dependent variables on $d\log Q$ term only.[5] For the model in Table 2, R_L^2 = 0.53. The inappropriate nature of the single equation statistic becomes clear when it is noted that the whole model is judged to explain only half of the variation in allocation; whereas, on the basis of the single equation measure, the worst equation in the system explains 78 percent of the variation. The autocorrelation coefficient estimate

(Berndt and Savin) for the CBS model was statistically not different from zero at $\alpha = 0.01$ level ($\rho = 0.0016$ with a standard error of 0.0012); therefore, no adjustment was made for the first-order autocorrelation for the CBS model.

Demand Elasticity Estimates

The expenditure and compensated price elasticities corresponding to the CBS model (9) are

$$\eta_i = 1 + \beta_i/w_i, \text{ and} \qquad (15)$$

$$\eta_{ij} = \pi_{ij}/w_i.$$

The expression for the expenditure elasticity indicates that a good with positive (negative) β_i is a luxury (necessity). Elasticity (15) does not rule out inferior goods and allows a good to be normal over some range of income and inferior over another. If $\beta_i = 0$, the budget share will not change in response to income changes (again, with price held constant).

The expenditure parameter estimates for the CBS model, β_i, indicate that orange juice and apple juice are necessities; and that juice drinks are luxuries among the juice beverages studied. Parameter estimates for the expenditure term for grapefruit juice, blended juices, juice cocktails, and remaining juices are not statistically different from zero, an indication that these juice beverage categories had unitary expenditure elasticity and their budget shares will not change in response to total juice beverage expenditure changes.

All expenditure and compensated price elasticity estimates for the CBS model calculated at sample budget share means for the study period are presented in Table 3. Results presented in Table 3 indicate that orange juice has the lowest income and own-price elasticities among the seven juice beverages studied. In addition, the demand for orange juice was price inelastic. The high expenditure share and low demand elasticities for orange juice indicate that orange juice can be considered a staple juice among the juice beverages studied. Results also show that the own-price elasticity estimates for other juice beverages were close to two in absolute value except the ones for juice drinks and remaining juices. As expected, most cross-price elasticity estimates were small and less than half of their corresponding own-price elasticity estimates.

Expenditure and own-price elasticity estimates were also derived for the four seasons studied at the respective average weekly seasonal budget shares; the results are presented in Table 4. The expenditure elasticity estimates for orange juice for the CBS model decreased slightly from 0.86 in 1988-89 to 0.84 in 1991-92 while its own-price elasticity estimates increased over the study period in absolute value (Table 4). These results are due to the fact that the orange-juice expenditure share decreased over the study period from 0.37 in 1988-89 to 0.33 in 1991-92. Likewise, changes in the expenditure elasticities for the other beverage groups between 1988-89 and 1991-92, although relatively small, are explained by changes in the expenditure shares for the groups. Expenditure elasticity estimates from the CBS model indicate that the demands for grapefruit juice and remaining juices have become slightly more sensitive to expenditure while for apple juice, blended juices, juice drinks, and juice cocktails have become slightly less expenditure sensitive.

The demands for orange juice, grapefruit juice, and apple juice have become more price elastic over the four seasons and the demands for blended juices, juice drinks, juice cocktails, and remaining juices have become less price elastic. The result may be attributed to the fact that the expenditure shares of orange juice, grapefruit juice, and apple juice have decreased and the expenditure shares of other juice beverages have increased over the four seasons. Similar results can be derived for the cross-price elasticity estimates. For example, the cross-price elasticity estimate between juice drinks and orange juice increased from 0.3628 in 1988-89 to 0.4142 in 1991-92.

A Comparison of Models

As indicated above, the basic demand responses for the alternative models differ and are limited in important ways. For example, the Slutsky terms are assumed to be constants in the Rotterdam and CBS models but functions of budget shares in the AIDS and NBR models. The limitations of the models may lead to unexpected,

Table 3. Conditional Expenditure and Compensated Price Elasticity Estimates Calculated at Sample Means for the CBS Model

Beverage	Expenditure	Price						
		Orange Juice	Grapefruit Juice	Apple Juice	Blended Juices	Juice Drinks	Juice Cocktails	Remaining Juices
Orange Juice	0.8518	-0.8816	0.0321	0.0914	0.0448	0.3882	0.2071	0.1180
Grapefruit Juice	1.0070	0.4075	-1.8791	0.0702	0.1719	0.5411	0.7624	-0.0740
Apple Juice	0.9192	0.3832	0 0232	-1.9007	0 1103	0 7910	0.4677	0.1253
Blended Juices	1.0135	0 3477	0.1049	0 2040	-1 9852	0.8392	0.5585	-0.1791
Juice Drinks	1 1761	0 4224	0.0463	0.2053	0 1177	-1 2106	0.2989	0 1199
Juice Cocktails	1.0609	0.7503	0.2173	0.4041	0 2609	0 9950	-2.6558	0 0282
Remaining Juices	0.9403	0.5214	-0.0257	0.1321	-0 1020	0.4867	0 0344	-1.0468

Table 4. Conditional Expenditure and Compensated Own-price Elasticity Estimates by Season (CBS model)[a]

Season[a]	Orange Juice	Grapefruit Juice	Apple Juice	Blended Juices	Juice Drinks	Juice Cocktails	Remaining Juices
				Average Expenditure Share (w_i)[b]			
Average	0.3487	0.0274	0.0832	0.0450	0.3205	0.0963	0.0789
				Conditional Expenditure Elasticity Estimate[c]			
88-89	0.8615	1.0063	0 9200	1.0136	1.1886	1 0630	0.9376
89-90	0.8565	1.0071	0.9198	1.0139	1.1804	1.0632	0.9406
90-91	0.8453	1.0071	0.9197	1.0138	1.1709	1.0589	0.9420
91-92	0.8419	1.0078	0 9172	1.0127	1.1661	1.0589	0 9409
Average	0.8518	1 0070	0 9192	1.0135	1.1761	1 0609	0.9403
				Conditional Own-Price Elasticity Estimate[c]			
88-89	-0.8237	-1.6943	-1.8829	-2 0050	-1.2964	-2.7455	-1.1606
89-90	-0.8535	-1.8909	-1.8866	-2 0380	-1.2400	-2.7559	-1.1048
90-91	-0.9200	-1.9021	-1.8882	-2.0367	-1.1747	-2.5665	-1.0774
91-92	-0.9405	-2.0706	-1.9470	-1.8690	-1 1413	-2.5668	-1.0983
Average	-0.8816	-1.8791	-1.9007	-1.9852	-1.2106	-2.6558	-1 1094

[a]From December through November.
[b]Weekly average.
[c]Calculated at weekly sample average.

implausible results in empirical work. To demonstrate the impact of parameterization assumptions on the demand elasticities, expenditure and compensated own-price elasticity estimates for the Rotterdam, AIDS, NBR, and general models (calculated at sample means and two selected seasons (1988-89 and 1991-92)), are presented in Table 5. For comparison, recall that the conditional expenditure elasticity estimates presented in Table 4 did not change very much and the compensated own-price elasticity estimates for orange juice, grapefruit juice, and apple juice became more elastic while those for the remaining juice groups became less elastic over the study period.

Using general model (13) the expenditure elasticity for each commodity group (η_i) can be derived by dividing the marginal budget share by the corresponding budget share, and the compensated price elasticity estimates (η_{ij}) can be

Table 5. Expenditure and Own-price Elasticity Estimates Calculated at Sample Means and Two Selected Periods

Juice	Expenditure Elasticity			Compensated Own-Price Elasticity		
	88-89	mean	91-92	88-89	mean	91-92
			The Rotterdam Model			
Orange Juice	0.8165	0.8738	0.9322	-0.8368	-0.8956	-0.9555
Grapefruit Juice	0.8781	0.9738	1.0730	-1.6700	-1.8521	-2.0408
Apple Juice	0.9106	0.9192	0.9416	-1.9033	-1.9212	-1.9681
Blended Juices	1.0171	1.0070	0.9481	-1.9982	-1.9785	-1.8628
Juice Drinks	1.2194	1.1387	1.0736	-1.3373	-1.2488	-1.1774
Juice Cocktails	1.1351	1.0980	1.0612	-2.7676	-2.6772	-2.5874
Remaining Juices	1.0095	0.9650	0.9553	-1.1354	-1.0854	-1.0744
			The AIDS			
Orange Juice	0.8635	0.8539	0.8441	-0.8355	-0.8747	-0.9115
Grapefruit Juice	1.0077	1.0086	1.0094	-1.7704	-1.8608	-1.9538
Apple Juice	0.9183	0.9175	0.9155	-1.8702	-1.8800	-1.9055
Blended Juices	1.0159	1.0157	1.0148	-1.9999	-1.9891	-1.9258
Juice Drinks	1.1872	1.1748	1.1648	-1.2642	-1.2057	-1.1562
Juice Cocktails	1.0643	1.0622	1.0601	-2.6995	-2.6378	-2.5763
Remaining Juices	0.9322	0.9352	0.9358	-1.1197	-1.1077	-1.1050
			The NBR Model			
Orange Juice	0.8184	0.8759	0.9344	-0.8488	-0.8889	-0.9267
Grapefruit Juice	0.8795	0.9754	1.0748	-1.7462	-1.8338	-1.9241
Apple Juice	0.9090	0.9175	0.9399	-1.8908	-1.9007	-1.9267
Blended Juices	1.0193	1.0093	0.9502	-1.9928	-1.9821	-1.9192
Juice Drinks	1.2180	1.1373	1.0723	-1.3051	-1.2439	-1.1922
Juice Cocktails	1.1364	1.0993	1.0624	-2.7218	-2.6594	-2.5972
Remaining Juices	1.0042	0.9600	0.9503	-1.0935	-1.0826	-1.0802
			The General Model			
Orange Juice	0.8731	0.8470	0.8204	-0.8235	-0.8763	-0.9291
Grapefruit Juice	1.0382	1.0157	0.9923	-1.7211	-1.8818	-2.0483
Apple Juice	0.9218	0.9188	0.9108	-1.8743	-1.8899	-1.9306
Blended Juices	1.0133	1.0156	1.0290	-2.0052	-1.9878	-1.8854
Juice Drinks	1.1807	1.1849	1.1883	-1.2781	-1.2001	-1.1366
Juice Cocktails	1.0457	1.0522	1.0587	-2.7276	-2.6455	-2.5638
Remaining Juices	0.9185	0.9329	0.9360	-1.1562	-1.1151	-1.1060

derived by dividing the Slutsky parameter by the budget share for the good, that is,

Expenditure Elasticity: $\eta_i = (d_i + \delta_i w_i)/w_i$,

Compensated Price Elasticity:

$$\eta_{ij} = (e_{ij} - \delta_i w_i(\delta_{ij} - w_j))/w_i. \qquad (16)$$

Note that in (16) both expenditure and price elasticities are functions of expenditure shares, offering somewhat more flexibility. With negative d_i and positive δ_i, a good can be a luxury (inferior) for high (low) values of w_i; for positive d_i and negative δ_i the opposite can occur. If d_i and δ_i are both positive the elasticity is between $(d_i + \delta_i)$ and ∞; if they are both negative the elasticity is between

$-\infty$ and $(d_i + \delta_i)$. For price elasticities, similar flexibility exists, i.e., the sign of η_{ij} is in part dependent on the value taken by the variable w_j. With changing budget shares a pair of goods can turn from (Hicksian) complements into (Hicksian) substitutes. Of course, a negative η_{ii} can turn into a positive one, which is the undesirable aspect of flexibility.

As expected, the demand elasticity estimates derived from the general model are similar to those derived from the CBS model. Expenditure elasticity estimates for orange juice are smaller than those for other juice categories and the demand for orange juice is relatively less price sensitive than the demand for other juice categories.

The conditional expenditure elasticity estimates from the Rotterdam and NBR models demonstrate different patterns from those derived from the CBS model over the study period. For example, expenditure elasticity estimates derived from the Rotterdam-type marginal budget share (the Rotterdam and the NBR models) indicate that expenditure elasticities for orange juice and apple juice increased during the study period. The expenditure elasticities derived from the Working-type marginal budget share (the AIDS and the general model) indicate the expenditure elasticities decreased over the same period and are similar to the expenditure elasticity patterns for the CBS model (Table 4). In addition, the expenditure elasticity estimates derived from the Rotterdam-type marginal budget shares are less stable than those derived from the Working-type marginal budget shares. For the short time period studied, this instability in expenditure elasticities was unexpected.

The conditional compensated own-price elasticity estimates derived from all models indicate that the demands for orange juice, grapefruit juice, and apple juice have become more price elastic over the study period and the demands for blended juices, juice drinks, juice cocktails, and remaining juices have become less price elastic. This result is similar to the findings from the CBS model. Again, the own-price elasticities estimated from the Rotterdam-type price responses are less stable over the study period than those derived from the AIDS-type price responses. The stability of the own-price elasticities derived from the general model falls in between those derived from the Rotterdam-type and the AIDS-type responses.

Concluding Remarks

Estimating demand systems in applied research is much more common than in the past when many demand studies utilized single-equation models. Two of the more popular models of choice have been the Rotterdam and the AIDS models. In this paper, these two models as well as two hybrid (the CBS and NBR) models were fit to U.S. juice beverage data. A general model which nests the other four was also fit to the data.

Results indicate that many findings were functional-form specific. For example, homogeneity and symmetry were rejected by the AIDS model, the NBR models which has AIDS-type price terms, and the general model. Homogeneity and symmetry were not rejected by the Rotterdam model and the CBS model which has Rotterdam-type price terms.

As Deaton has discussed, rejection of such innocuous assumptions such as homogeneity and symmetry is always puzzling. Rejection of homogeneity according to Attfield may be due to the endogeneity of the expenditure variable in conditional demand systems. Based on the rejection of homogeneity by the AIDS and NBR models, one would be lead to reject the exogeneity of expenditure. However, results from the CBS model do not give the same indication. Indeed, a test of rational random behavior with the CBS model rejected the endogeneity of expenditure.

This paper also demonstrated that the CBS model fit the data better than the AIDS, NBR, or Rotterdam models. Further, the behavior of expenditure elasticity estimates over time were found to be functional-form specific. The models with Working-type income terms produce expenditure elasticity movements corresponding to predictions from economic theory; those with constant marginal share did not. Price elasticity estimates were also found to be affected by choice of functional form.

This study has demonstrated that mixed models --- the CBS model with Working's income responses and Rotterdam-type price responses and the NBR model with Rotterdam-type income responses and AIDS type price responses --- provide flexibility beyond the Rotterdam and AIDS models, but are themselves limited in the same basic manner the Rotterdam and AIDS models are. A general model suggested by Barten combines the features of the Rotterdam, CBS, AIDS, and NBR models and offers further flexibility. The mixed models and the general model are interesting for empirical work as they do not involve many additional parameters to estimate. The results of the present study suggest that the Working-type income and the Rotterdam-type price responses work better for explaining U.S. juice beverage consumption behavior. Further research based on other data sets may reveal additional insight in consumer expenditure behavior.

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Footnotes

1. The approach proposed by Alston and Chalfant does not differentiate the demand parameters θ_i, β_i, π_{ij}, and γ_{ij} used in the Rotterdam model and the AIDS; and of the six models they proposed, only Models I and III satisfy the adding-up restrictions for demand systems. When Models I and III were used in conjunction with their compound models, their proposed test is identical to the pair-wise test proposed by Barten.

2. The models were named after the institutes, the Netherlands Central Bureau of Statistics and the National Bureau of Research, where Keller and van Driel and Neves worked, respectively, when the models were developed.

3. Barten's study (1990) also found the CBS model worked better than other specifications for the Dutch expenditure data set.

4. In addition to the theory of random rational behavior, Bieri and de Janvry indicate that under the two-stage budget allocation scheme, the second-stage demand equations can be fitted independently of the first-stage expenditure functions (p. 21).

5. The naive model $w_i(d\log q_i - d\log Q) = \beta_i d\log Q + e_i$, also satisfies the adding-up condition $\sum_i \beta_i = 0$ by construction.