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Fund Managers, Career Concerns, and Asset Price Volatility*

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ABSTRACT

We propose a model of delegated portfolio management with career concerns. Investors hire fund managers to invest their capital either in risky bonds or in riskless assets. Some managers have superior information on the default probability. Looking at the past performance, investors update beliefs on their managers and make firing decisions. This leads to career concerns which affect investment decisions, generating a positive or negative "reputational premium". For example, when the default probability is high, the return on the risky bond has to be high to compensate the uninformed managers for the high risk of being fired. As the default probability changes over time, the reputational premium amplifies price volatility.

1 Introduction

The financial turmoil that began in the summer of 2007 and exploded in 2008 has fueled debate on asset price volatility and the role of financial intermediaries and their incentives. In the few years before the crisis, a number of market observers were concerned about a growing "overenthusiasm" for risky investments in debt instruments, including high-yield corporate bonds, mortgage-backed assets and emerging market bonds. Some connected these events to the role of incentives of financial intermediaries. For example, one observer comments on the bond financed leveraged buy-out boom in 2005:

> The head of one of the biggest commercial lenders in the US describes the amount of leverage on some buy-out deals as "nutty". Much of the wildest lending is being done by hedge funds awash with cash, he says. "Some funds believe they have to invest the money even if it's not a smart investment. They think the people that gave them the money expect them to invest it. But it's madness." (March 14, 2005, Financial Times)

In this paper, we propose a general equilibrium model of portfolio management where fund managers have career concerns. We show that managers' career concerns distort their investment decisions and magnify asset price volatility.

Figure 1 shows the pattern of the yield spreads of a sample of emerging market bonds, the AAA and the B-graded corporate bonds, and the BB-graded commercial mortgage-backed assets, between October 1994 and April 2010. The figure shows at least two periods in which all spreads shrunk to very low levels, close to the AAA corporate spreads: in 1996-1997 and then again from 2005 to the summer of 2007. Observers describe these periods as periods of overenthusiasm, which typically occur right before the emergence of a crisis (e.g. Kamin and von Kleist, 1999, Duffie et al., 2003). The figure also shows at least four episodes of high turbulence in which the spreads of many high-risk bonds jump up and capital tends to flow out of these markets, a phenomenon dubbed as flight-to-liquidity or flight-to-quality. Our model is able to rationalize both episodes of apparent overenthusiasm and episodes of flight-to-liquidity.

We propose a model where investors delegate their portfolio decision to risk-neutral fund managers. We assume that fund managers can invest either in risk-less assets or in risky bonds. The default probability is an iid random variable and managers differ in the degree of information about its realization. We consider an extreme environment, where some informed managers can perfectly predict the default state, while other uninformed managers only know the realization of the default probability. The core of our model builds on the
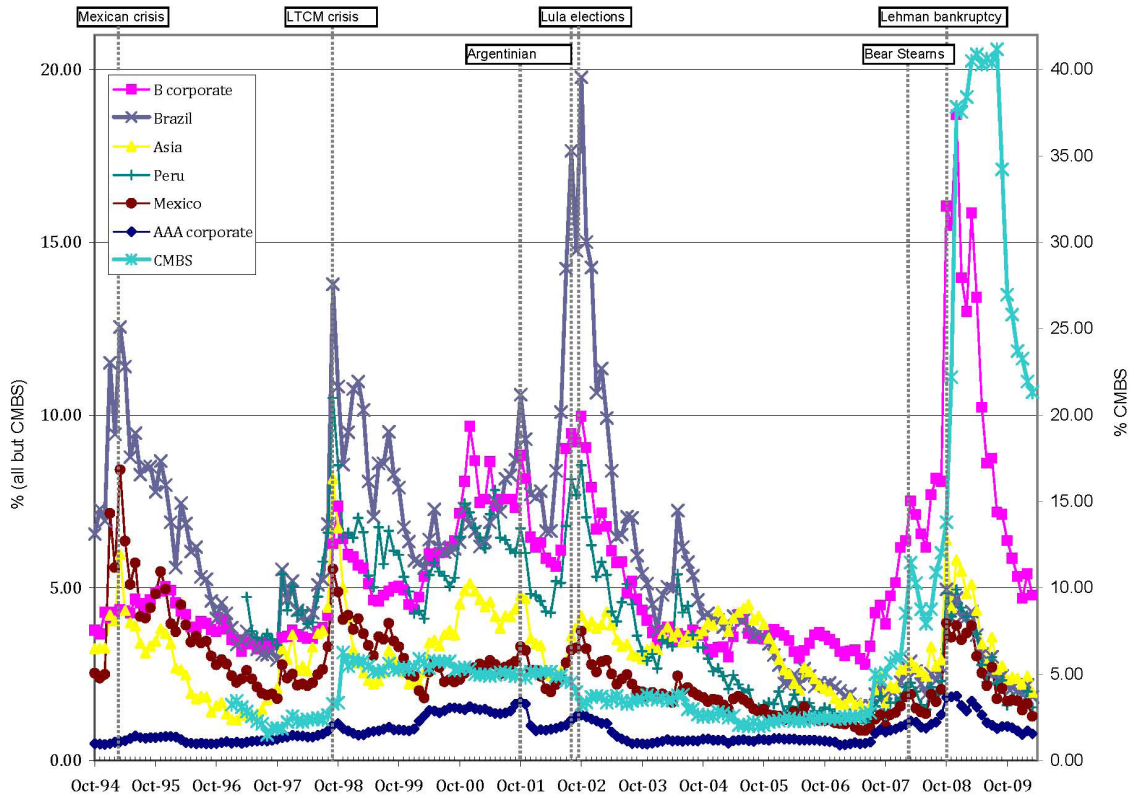
Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Bulgaria, Mexico, Peru, the yield spread of AAA corporate bonds and an index of the yield spread of B-graded corporate bonds and BB-graded comercial mortgage-backed assets (with scale on the right side of the figure) between October 1994 and April 2010. Source: Datastream, St. Louis Fed.

career concerns of the uninformed fund managers. Every period, each investor has a manager working for him. At the end of the period, the investor updates his belief based on his manager's performance, and decides whether to retain him or to hire a new one. The investors' firing decision distorts the investment decision of uninformed managers who would like to be perceived as informed.

One of our main results is that managers' career concerns impose a premium on the return of risky bonds, which may be positive or negative depending on the default probability. Uninformed fund managers try to time the market in order to behave as if they were informed and knew in advance if there was default or not. Default hurts the reputation of uninformed managers who invest in the risky bond, and no default hurts the reputation of uninformed managers who invest in the riskless asset. Thus, when the default risk is high, the premium for investing in the risky bond is positive to compensate for the risk of being fired. When instead the default risk is low, the risky bond will trade at a negative premium. This

reputational premium amplifies the bond price volatility, in comparison to an economy with no career concerns.

We also explore a more general version of the model, where we allow for persistent default risk. In this case, career concerns have additional effects on asset price volatility. In particular, the reputational premium now varies not only with the default probability but also with the measure of employed managers who are informed, which is not stationary anymore. The more employed managers are informed, the higher is the informational content of future prices, which increase the expected utility of employed uninformed managers. This makes reputation more valuable and increases the distorting effect of career concerns. This implies that asset price movements cannot be explained purely by fundamentals. When the measure of informed managers is infinitesimal, we show that a higher degree of persistence dampens the reputational discount when the default risk is low and magnifies the reputational premium when the default risk is high.

It is well known that the premium on risky assets is time-varying: the difference between the expected return on risky asset and riskless return is lower in good times than in bad times. The more common explanations appeal to time-varying marginal utility of consumption due to habit formation, time-varying probability of disasters or slow-moving component in consumption risk.[1] In our model the premium changes, because of the time-varying risk for a manager of being fired. In contrast to the consumption-based explanations, our mechanism implies that in good times (low risk) some managers are willing to take risky bets for returns that do not compensate them enough for the risk.

Our mechanism is consistent with several empirical observations in different markets. For example, Coval, Jurek and Stafford (2008) argue that between 2004 and September of 2007 collateralized debt obligations (CDOs) provided too little compensation for risk compared to portfolios of securities of the same pay-off structure. As CDOs are traded only by institutional investors, our mechanism is a good candidate to explain this phenomenon.

On a similar line, Brunnermeier and Nagel (2004) argue that hedge funds were investing heavily in technological stocks during the dotcom bubble, although they seemed aware of the mispricing. Our model suggests that hedge funds were willing to buy technological stocks at highly inflated prices because of their fear of losing reputation and hence funds if they missed the high returns generated by the bubble. This is consistent with the additional fact, reported in Brunnermeier and Nagel (2004), that the largest hedge fund, Tiger Fund, which refused to invest in technology stocks, experienced severe fund outflows in 1999 compared to its main competitor who did invest in technology stocks, Quantum Fund.

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[1]See Cochrane (2006) for a detailed review.

Finally, Duffie et al. (2003) document that the implied short spread of Russian bonds was spectacularly low during the first 10 months of 1997. Moreover, their estimation shows that in one short interval in 1997, bond prices were so high that the implied risk-neutral default-adjusted short spread was negative. Although this implication relies heavily on their specific term structure model, it is worthwhile to point out that this is inconsistent with most established explanations of time-varying premium, but consistent with our model.

Literature review. To our knowledge, this is the first paper to show that fund managers' career concerns amplify the reaction of asset prices to shocks, generating a counter-cyclical premium.

Our paper is related to herding models, such as Scharfstein and Stein (1990), Zwiebel (1995), and Ottaviani and Sorensen (2006). These papers are close to our work because agents with career concerns distort their decisions to convince their clients that they are informed. However, our mechanism does not feature herding behavior. In these papers, each agent herds on others' decisions because going against the average action is a bad signal about his ability. In our model, at the equilibrium prices, fund managers make their investment decisions regardless of other managers' decision. That is, there are no strategic complementarities. Moreover, this literature typically concentrates on partial equilibrium models while our focus is on the interaction of career concerns and asset prices. An exception is Rajan (1994), which is very close to our spirit. Rajan (1994) shows that herding may motivate bank executives to overextend credit in good times, hence amplifying the effect of real shocks. However, in bad times banks provide the right amount of credit and there is no amplification effect. In contrast, in our paper career concerns always generate an amplification effect, given that when the default risk is high (bad times) managers are worried of being fired and underinvest in the risky bonds.

There is also a growing literature which analyzes the effect of delegated portfolio management on asset prices, such as Allen and Gorton (1993), Shleifer and Vishny (1997), Vayanos (2003), He and Krishnamurthy (2008), and Cuoco and Kaniel (2010). However, these papers take managers' distorted incentives as given. In particular, the two papers mainly related to ours are Dasgupta and Prat (2008) and Dasgupta, Prat and Verardo (2008) who introduce reputational concerns into a Glosten-Milgrom type of sequential trading model.[2] They show that reputational concerns can lead to excessive trading, slow revelation of information and (if the market maker has market power) biased prices. They are the first to use the term reputational premium and to point out that reputation may lead managers to choose bets

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[2]More recently, Vayanos and Woolley (2008) show that learning about managerial ability can explain momentum and reversal, while Malliaris and Yan (2010) connect reputational concerns of hedge fund managers to the skewness of their returns and slow moving capital.

with negative net present value. However, both these papers take the reputational concerns as exogenously given and the mechanism behind the reputational premium is based on herding behavior. Also, they do not explore the effect of reputation on asset price volatility, which is the focus of our paper. Dasgupta and Prat (2006) is the first paper to microfound reputational concerns of fund managers who are afraid to be fired. However, the mechanism is again based on herding and the paper does not explore asset price volatility.

Our paper is also related to a large literature on the propagation and amplification of fundamental shocks due to the interaction between asset values and collateralized lending. Seminal papers in this area are Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Gromb and Vayanos (2002)[3]. The main difference with our mechanism is that these papers have typically an asymmetric distortion, given that collateral constraints build into the model an external finance premium, usually generating underinvestment. In our model, instead, we microfound the financial distortion and we generate a premium that can be either positive or negative.

The rest of the paper is organized as follows. In Section 2, we introduce an example to illustrate the main mechanism of our model. In Section 3, we describe the model, and define and characterize an equilibrium. In Section 4, we analyze an extended version of the model, where the supply of risky bonds and the default decision are endogenous. In Section 5, we analyze an economy where productivity is persistent and we propose some numerical exercises. Finally, Section 6 concludes. The Appendix includes all the proofs which are not in the text.

2   An Example

In this section, we introduce a simple example to show the main mechanism of the model, that is, how prices may be distorted by career concerns of fund managers.

Assume that a large group of risk-neutral fund managers have to decide whether to invest a unit of capital in a risky asset or in a riskless asset. The risky asset has price p and pays 1 if the good state realizes and 0 if the bad state realizes. The probability of the bad state is equal to q. The riskless asset pays the safe return $R < 1/p$. The riskless asset is in infinite supply, while the supply of the risky bond is fixed and smaller than the total capital invested by the managers. Assume that managers get a fraction γ of the investment returns and obtain a constant reward W if and only if their investment is successful, that is, if

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[3]See also Aghion, Banerjee and Piketty (1999), Rampini (2004), Krishnamurthy (2003), Danielsson, Shin and Zigrand (2004), Morris and Shin (2004), Bernardo and Welch (2004), Gai, Kondor and Vause (2005), and Guerrieri and Lorenzoni (2009).

they invest in the risky bond if and only if there is no default. This reward scheme may be interpreted as the reduced form of a model where unsuccessful managers are fired and W represents the value of being retained.

It is straightforward that the bond market clears if and only if managers are indifferent between investing in the risky bond and in the riskless asset. Hence, the equilibrium price of the risky bond has to satisfy the following indifference condition:

$$(1 - q)(\gamma/p + W) = \gamma R + qW. \tag{1}$$

The left-hand side of equation (1) represents the expected payoff of a manager who invests in the risky bond. With probability $1 - q$ there is no default and the manager gets a return γ/p and the reward W. If instead there is default, the manager gets zero revenues and no reward. Similarly, the right-hand side of equation (1) represents the expected payoff of a manager who invests in the riskless asset. He gets always a return γR, but he obtains the reward only if there is default.[4]

In order to characterize the price distortion generated by W, we define the *premium* Π as the difference between the expected return and the risk free rate

$$\Pi \equiv \frac{1 - q}{p} - R.$$

The indifference condition (1) immediately implies that $\Pi = 0$ when there is no reward scheme, that is, $W = 0$. In this case, fund managers care only about the expected returns of the bond and the premium is zero. When instead $W > 0$, the premium can be negative or positive. In particular, if $q > 1/2$, the payoff of the risky bond is skewed to the left as the probability of default is larger than the probability of no default. In this case, investing in the riskless asset has an advantage over the risky bond as this ensures the reward payment with larger probability. If the expected return of the two assets were equal, all managers would prefer the riskless one, because of this advantage. Thus, in equilibrium there must be a positive premium on the risky bond to induce managers to hold it. Similarly, if $q < 1/2$ the payoff of the risky bond is skewed to the right. In this case, the risky bond has an advantage and the premium is negative.

This simple example is suggestive, but it clearly calls for microfoundations behind the reward scheme. The story we have in mind is a story of career concerns, which needs both a dynamic environment and some form of heterogeneous information. In the rest of the paper, we build a dynamic general equilibrium model of delegated portfolio management with in-

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[4]The equilibrium price is consistent with the assumption that $1/p > R$ if $R > W(1 - 2q)/q$.

formed and uninformed fund managers, where the default risk is time-varying. Investors need fund managers to manage their capital. Based on their investment performance, investors learn about their type and make their firing decision. This model generates an incentive scheme similar to this example, where W is an equilibrium object equal to the discounted expected utility of an uninformed manager who retains his job. In particular, uninformed managers' career concerns generate a reputational premium analogous to the one described above. We show that the presence of such a reputational premium magnifies the volatility of asset prices.

3   Model

Consider an infinite-horizon economy, set in discrete time, populated by three groups of agents: investors, fund managers and bond sellers.

There is a measure Γ of infinitely lived, risk neutral investors with discount rate β. At the beginning of each period, they receive a unit endowment of consumption goods. This endowment can be invested in two ways: in a riskless asset which pays a fixed gross rate of return $R > 1$ at the end of the period or in risky bonds that have a random payoff 0 or 1 at the end of the period.

The risky bonds are traded at the beginning of period t at the price p_t. Their end-of-period payoff depends on the aggregate shock $\chi_t \in \{0, 1\}$. If $\chi_t = 1$ there is "default" and they pay zero, if $\chi_t = 1$ there is no default and they pay one unit of consumption. The probability of default is given by q_t, which is an iid random variable drawn at the beginning of period t from the cumulative distribution function $F(q)$ with support $[\underline{q}, \overline{q}]$.

Risky bonds are supplied by bond sellers, whose behavior is mechanical. At the beginning of each period, they need to borrow b_t, that is, there is an inelastic nominal supply b_t of bonds. The bond supply b_t is a random variable drawn independently each period from a uniform distribution on $[\underline{b}, \overline{b}]$.[5] The supply shock b_t is not observed by investors and fund managers. Its role is to ensure that the bond market price is not always fully revealing.

Notice that all the assets pay back at the end of the period so that the investment takes place within the period and there is no technology to transfer capital across periods. This drastically simplifies the investors' behavior given that there is no saving decision: at the beginning of each period they fully invest their unit endowment and at the end of the period they fully consume the returns of their investment.

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[5]It is possible to derive the behavior of bond sellers from first principles, by assuming that there are overlapping generations of two-period lived bond sellers who invest in risky projects and default on their debt as in Guerrieri and Kondor (2009). For simplicity of exposition, here we omit this treatment.

Investors cannot invest their endowment on their own, they need to employ a fund manager. Fund managers are also infinitely lived, risk neutral and have discount factor β. Each investor can employ only one fund manager and a fund manager can work only for a single investor. For simplicity, we fix the contract between investors and fund managers and assume that fund managers keep a share γ of the returns and leave the rest to the investors. We also assume that fund managers must fully consume their share of returns in each period.

There are two types of fund managers: informed (I) and uninformed (U). There is a mass M^I of informed managers and a large continuum of uninformed managers. The manager's type is his own private information. Informed managers have an informational advantage: they observe the realization of χ_t at the beginning of period t, while all other agents only observe χ_t at the end of the period.[6] Uninformed managers only observe the probability of default q_t at the beginning to the period. After the investment payoffs are realized, the investor decides whether to retain his manager or to fire him and hire a new one. We will specify in a moment how the labor market for managers works.

To complete the description of the environment we need to specify the timing of the agents' actions and the functioning of the bond market and of the labor market.

Timing. Each period is divided in two stages. In the first stage, each employed manager chooses how to invest the unit of capital he manages and the bond market clears. In the second stage, investors observe the return of their manager's investment and decide whether to fire him. After firing decisions have been made, the investors without a manager are randomly matched to unemployed managers on the labor market.

The bond market. In the first stage of period t, each manager submits a demand schedule for risky bonds to an auctioneer. For simplicity, we restrict managers to three choices: invest zero, invest 1 unit of consumption goods, or declare indifference between 0 or 1, an option we denote by $\{0,1\}$. Therefore, a demand schedule is a map $d : \mathcal{R}_+ \to \{0, 1, \{0, 1\}\}$ which for any price $p \geq 0$ gives the manager's demand $d(p)$. The auctioneer collects all the demand schedules, selects the equilibrium price and assigns the bonds to the managers. In particular, if p is the equilibrium price, managers with demand $d(p) = 1$ receive $1/p$ bonds, managers with $d(p) = 0$ receive no bonds, and managers with demand $d(p) = \{0, 1\}$ are selected randomly to receive 0 or $1/p$ bonds so as to clear the market.

The labor market. In the second stage of period t, an investor observes whether his manager invested in the riskless or in the risky asset in the previous stage and observes the

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[6]The extreme assumption that informed managers have perfect information is not crucial for our argument. However, as we will see, it greatly simplifies the analysis of the investors' beliefs, making the model more tractable.

realized value of χ_t. For technical reasons, it is useful to assume that, at the same time, the investor receives an additional exogenous signal. This signal is denoted by $\sigma_{i,t}$, for manager i at time t, and can take two values, 0 and 1. If the manager is informed the signal is always $\sigma_{i,t} = 0$. If the manager is uninformed, the signal is $\sigma_{i,t} = 0$ with probability ω and $\sigma_{i,t} = 1$ with probability $1-\omega$. Therefore, with probability $1-\omega$ the type of the uninformed manager is perfectly revealed. We will discuss the role of this exogenous signal when we analyze the equilibrium. Given all the information available, the investor updates his beliefs about the manager's type and chooses whether to retain him or fire him and hire a new one.

To ensure that the pool of unemployed managers always contains informed managers, we assume that in the second stage of period t an investor-manager match is exogenously terminated with probability $1 - \delta$. At the end of period t, all investors who do not have a manager—either because they fired him or because of exogenous termination—search for one. At the same time, unemployed managers choose either to pay a cost $\kappa \leq \gamma R$ and look for a job or to stay inactive.[7] Then matching takes place. The matching technology is Leontief: given S searching investors and N unemployed managers looking for a job, the number of matches created is $\min\{S, N\}$. Therefore, the probability of being matched is $\min\{S, N\}/S$ for the investors and $\min\{S, N\}/N$ for the managers. Our assumptions ensure that, in equilibrium, investors are always on the short side of the market, that is, $S < N$, so that investors are always matched with probability 1.

Given that there is a continuum of agents, an investor will never meet the same manager twice. Moreover, we assume that an investor can only observe the trading history of the manager he employs.[8] Therefore, from the point of view of the investors, all newly employed managers are observationally equivalent and the probability that a newly employed manager is informed is equal to the fraction of informed managers in the unemployment pool.

In specifying preferences, contracts and the market structure, we have made a number of simplifying assumptions. The role of these assumptions is to allow us to focus the analysis on two key decision variables: the fund managers' decision whether to invest in the riskless or in the risky asset and the investors' decision to retain or fire their managers at the end of each period. Investors acquire information on whether their fund manager is informed or uninformed by observing their investment decisions and their realized returns. They fire the fund manager whenever their belief about the quality of the manager is lower than the average quality of a newly hired manager. This firing decision is the source of career concerns

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[7]The assumption $\kappa \leq \gamma R$ is sufficient to ensure that it is profitable for an informed manager to search for a job.

[8]This assumption is necessary because some jobs are exogenously terminated with probability δ. In Guerrieri and Kondor (2009) we assume that managers die instead of being exogenously separated and hence we can allow managers' histories to be public information.

for the fund managers.

4 Equilibrium

We now define and construct a stationary symmetric equilibrium. In general, given the signalling nature of the game, there are multiple equilibria, as informed managers can take different actions to signal their type. Here we focus on equilibria where informed managers signal their type by making the "right" investment decisions, i.e., by making investment decisions that maximize expected returns conditional on their information. Moreover, we study equilibria where prices are not fully revealing so that informed managers make higher expected returns than uninformed managers and investors strictly prefer hiring informed managers.

4.1 Equilibrium Definition

In a stationary equilibrium, the measures of employed managers, informed and uninformed, are constant and equal, respectively, to Γ^I and Γ^U. As we will show below, investors are on the short side of the job market, so they are always matched and $\Gamma^I + \Gamma^U = \Gamma$. All informed managers submit the same demand schedule contingent on the realization of the default shock χ_t. We denote this demand schedule by $d^I(p; \chi_t)$. Similarly, all uninformed managers submit the same demand schedule contingent on the default probability q_t, which we denote by $d^U(p; q_t)$.

The auctioneer picks the price and the bond allocation consistent with the demand schedules submitted by the agents and with market clearing. At each time t, there are three aggregate shocks: the default probability q_t, the default shock χ_t, and the supply shock b_t. The equilibrium price depends on the aggregate shocks according to the function $p_t = P(q_t, \chi_t, b_t)$. The equilibrium bond allocation also depends on the aggregate shocks (q_t, χ_t, b_t), and is described by a function $X(d; q_t, \chi_t, b_t)$ which gives the equilibrium probability of investing in risky bonds for a manager demanding $d \in \{0, 1, \{0, 1\}\}$. In order to be consistent with the demand schedules, the function X must satisfy $X(0; q_t, \chi_t, b_t) = 0$ and $X(1; q_t, \chi_t, b_t) = 1$, while $X(\{0, 1\}; q_t, \chi_t, b_t)$ can take any value in $[0, 1]$.

Let $x_t^I \equiv X(d^I(p_t; \chi_t); q_t, \chi_t, b_t)$ and $x_t^U \equiv X(d^U(p_t; q_t); q_t, \chi_t, b_t)$ denote the equilibrium probabilities of investing in risky bonds respectively for informed and uninformed managers. By the law of large numbers, x_t^I and x_t^U are also equal to the fraction of informed and uninformed managers investing in risky bonds. Therefore, market clearing on the bond

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market requires

$$\Gamma^I x_t^I + \Gamma^U x_t^U \le b_t,$$

with equality when the equilibrium price p_t is strictly positive.

Let $\theta_{i,t} \in \{0,1\}$ denote the realized investment in the risky asset for manager i. Since managers can choose to demand $\{0,1\}$—i.e., they can tell the auctioneer that they are indifferent between 0 and 1—$\theta_{i,t}$ is, in general, a random variable and the probability of $\theta_{i,t} = 1$ is x_t^I for the informed agent and x_t^U for the uninformed agent.

At the beginning of each period t, there is a distribution of existing investor-manager matches. The investor matched with manager i believes that with probability $\eta_{i,t}$ the manager is informed. Then, after observing the realized investment $\theta_{i,t}$, the exogenous signal $\sigma_{i,t}$, the default shock χ_t, and the price p_t, the investor updates his belief to

$$\eta_{i,t+1} = H\left(\eta_{i,t}, \theta_{i,t}, \sigma_{i,t}, \chi_t, p_t\right).$$

Since $\eta_{i,t}$ appears in this expression, beliefs depend in general on the whole past history of the investor-manager match. Therefore, the distribution of beliefs is the only equilibrium object which does not depend only on (q_t, χ_t, b_t). However, thanks to our simplifying assumptions, we can focus on equilibria where the equilibrium firing decision only depends on the current values of $\theta_{i,t}, \sigma_{i,t}, \chi_t, p_t$ and where we do not need to keep track of belief dynamics explicitly. In particular, we describe the firing strategy using the function $\phi_{i,t} = \Phi\left(\theta_{i,t}, \sigma_{i,t}; \chi_t, p_t\right)$, where $\phi_{i,t} = 1$ corresponds to firing and $\phi_{i,t} = 0$ to retention. Let us denote by ξ_t^s the equilibrium probability that a manager of type s is fired, with $s = I, U$, that is, $\xi_t^s \equiv E\left[\Phi\left(\theta_{i,t}, \sigma_{i,t}; \chi_t, p_t\right) | x_t^s, s\right]$, where the expectation is taken with respect to $\theta_{i,t}$ and $\sigma_{i,t}$. Again, by the law of large numbers, ξ_t^I and ξ_t^U are also equal to the fraction of informed and uninformed that are fired at time t.

In a stationary equilibrium, the values of Γ^I and Γ^U have to be consistent with equilibrium job market flows. Let μ denote the ratio of searching investors to searching managers, and, in particular, let N_t^I and N_t^U denote the measures of informed and uninformed managers looking for a job. At the end of each period, a fraction ξ_t^s of employed managers of type s is fired and a fraction $1 - \delta$ of the remaining managers is exogenously separated. Since μN_t^s managers of type s are matched at the end of each period, the following condition ensures the stationarity of Γ^s:

$$[\xi_t^s + (1 - \xi_t^s)(1 - \delta)]\Gamma^s = \mu N_t^s. \tag{2}$$

Given that ξ_t^s is a function of the shocks (q_t, χ_t, b_t), equation (2) shows that N_t^s will also be, in general, functions of the same shocks. This gives us our last equilibrium object, the

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fraction of informed managers in the unemployment pool, which corresponds to the ratio $\varepsilon_t = N_t^I / \left(N_t^I + N_t^U \right)$, and is then also a function of (q_t, χ_t, b_t).

We can now write down the optimization problems of investors and managers. From now on we drop the time subscripts from all the stationary objects.

At the end of time t, an investor with posterior belief η' that his manager is informed, chooses to fire him only if the value of hiring a random new manager, informed with probability ε, is higher than the value of keeping him. Given that the probability for an investor of hiring a new manager is always equal to 1, his firing decision ϕ solves

$$J \left(\eta', \varepsilon \right) = \max_{\phi} (1 - \phi) V \left(\eta' \right) + \phi V \left(\varepsilon \right), \tag{3}$$

where $V \left(\eta \right)$ denotes the value of being matched with a manager informed with probability η, that is,

$$V \left(\eta \right) = E \left[(1 - \gamma) \left(\theta \left(1 - \chi \right) / p + (1 - \theta) R \right) + \beta J \left(H \left(\eta, \theta, \sigma, \chi, p \right), \varepsilon \right) | \eta \right].$$

Notice that the prior belief η affects the right-hand-side of the above expression in two ways: directly through the Bayes rule H and through the distribution of θ. A manager informed with probability η invests in the risky bond, $\theta = 1$, with probability $\eta x^I + (1 - \eta) x^U$. If the manager invests in the risky bond, the investor gets $(1 - \gamma) / p$ only if there is no default, while if the manager invests in the riskless asset, the investor gets $(1 - \gamma) R$ for sure. Moreover, the investor updates his belief about the manager type after observing his investment θ, his signal σ, the default state χ, and the price p, according to Bayes rule $\eta' = H \left(\eta, \theta, \sigma, \chi, p \right)$. Given the posterior belief, he makes his firing decision to maximize problem (3) and this determines his continuation utility.

Let us now turn to the managers' behavior. The uninformed managers' behavior is characterized by the Bellman equation[9]

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\begin{aligned}
W \quad = \quad E[\max_{d \in \{0,1,\{0,1\}\}} & E[X \left(d; a \right) \left(\gamma \left(1 - \chi \right) / p + [1 - \Phi \left(1, \sigma; \chi, p \right)] \delta \beta W \right) + \\
& (1 - X \left(d; a \right)) \left(\gamma R + [1 - \Phi \left(0, \sigma; \chi, p \right)] \delta \beta W \right) | p, q]],
\end{aligned}
\tag{4}
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where W denotes the expected utility of an employed uninformed manager at the beginning of a period, before the realization of q. The maximization problem on the right-hand side can be interpreted as follows. Given his choice of d, the manager receives risky bonds

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[9]For consistency of notation, we adopt the convention that when $p = 0$ and $\chi = 1$ the rate of return $(1 - \chi) / p$ is zero. Notice also that the managers' demand at off-the-equilibrium-path prices is not pinned down by individual optimality.

with probability $X(d; a)$. Then, if there is no default he receives the current return γ/p, while if there is default his current return is zero. If he is not fired—with probability $1 - \Phi(1, \sigma; \chi, p)$—and the match is not exogenously terminated—with probability δ—he keeps his job and receives the continuation utility βW. If he loses his job, he gets zero continuation utility, given the free entry condition (5). With probability $1 - X(d; a)$ the manager receives the riskless bond. He then always receives the safe current return γR. His continuation utility is computed as above, except that his firing probability is now $\Phi(0, \sigma; \chi, p)$. The Bellman equation that characterizes the informed managers' behavior is the same, except that the expectation can be conditioned on p and χ.

Finally, since we assumed that there is a large continuum of uninformed managers and the cost of searching is κ, the following free entry condition must hold:

$$\mu W - \kappa = 0. \tag{5}$$

Given that the informed managers have a richer information set than the uninformed ones, they can always mimic them and their expected utility of being employed is higher than W.[10] This implies that unemployed informed managers get positive expected utility when searching for a job, so that they all actively search and

$$N_t^I = M^I - (1 - \xi_t^I)\delta\Gamma^I \text{ for all } t. \tag{6}$$

The timeline below summarizes the timing in a stationary environment.
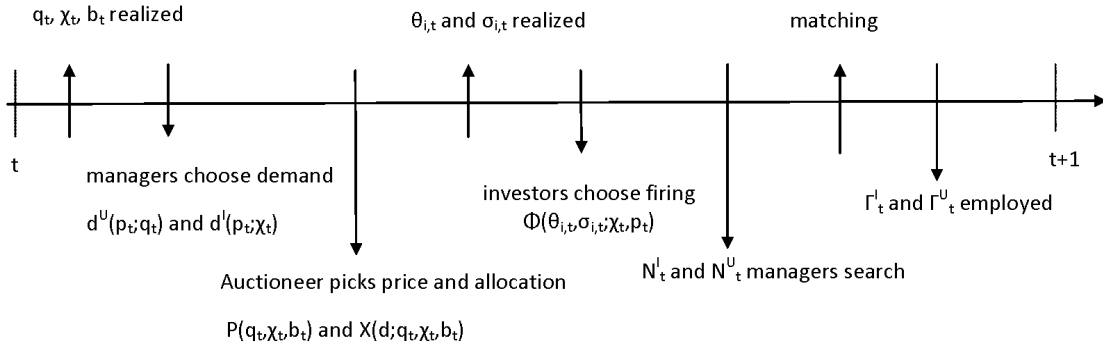
We are now ready to define an equilibrium.

Definition 1 *A stationary symmetric equilibrium is given by demand schedules* $d^I(p; \chi)$ *and* $d^U(p; q)$, *a price function* $P(q, \chi, b)$, *a bond allocation* $X(d; q, \chi, b)$, *a firing strategy*

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[10]The value of being employed informed managers satisfies the same Bellman equation (4) for W, except that the expectation is conditional on p and χ, where recall that $q = E[\chi]$.

$\Phi(\theta, \sigma; \chi, p)$, *a law of motion for the investor beliefs* H, *a measure of employed informed managers* Γ^I *and a matching probability for unemployed managers* μ, *such that:*

1. *the fund managers' demand schedules are optimal, taking as given the equilibrium price and the investors' firing strategy;*

2. *the investors' firing strategy is optimal given investors' beliefs, the equilibrium price and the managers' demand schedules;*

3. *the bond allocation is consistent with the managers' demand schedules;*

4. *the bond market clears;*

5. *investors' beliefs* H *are consistent with Bayes' law on the equilibrium path;*

6. Γ^I *and* μ *are consistent with stationary labor market flows and with managers' free entry.*

4.2 Characterization

We now show that, under appropriate assumptions, a stationary equilibrium exists. In particular, we assume that

$$\bar{b} < \Gamma - M^I, \ \underline{b} > M^I, \text{ and } \bar{b} - \underline{b} > M^I, \tag{A1}$$

$$\omega < \frac{1}{1+\delta}, \tag{A2}$$

$$\underline{q} > \left(1 + \frac{1}{\delta \omega \beta}\right)^{-1} \tag{A3}$$

We will discuss the role of these assumptions as we describe the equilibrium.

Given these assumptions, we can construct an equilibrium in which equilibrium prices are sometimes fully revealing and sometimes completely uninformative, depending on the realization of the variable $z(\chi, b) \equiv b - (1 - \chi) \Gamma^I$. The variable z can be interpreted as the supply of risky bonds net of the potential demand of informed managers, who are willing to demand risky bonds when $\chi = 0$.

Proposition 1 *Under assumptions A1-A3, there exists a stationary symmetric equilibrium in which the price function is*

$$P(q, \chi, b) = \begin{cases} \frac{1}{R} & \text{if } z(\chi, b) \in [\underline{b} - \Gamma^I, \underline{b}) \\ \tilde{P}(q) & \text{if } z(\chi, b) \in [\underline{b}, \bar{b} - \Gamma^I] \\ 0 & \text{if } z(\chi, b) \in (\bar{b} - \Gamma^I, \bar{b}] \end{cases}, \tag{7}$$

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for some function $\tilde{P}(q)$ which takes values in $(0, 1/R)$, the demand schedules are

$$d^I(p; \chi) = \begin{cases} \{0, 1\} & \text{if } p = 1/R \\ 1 - \chi & \text{otherwise} \end{cases}$$

and

$$d^U(p; q) = \begin{cases} 0 & \text{if } p = 0 \\ \{0, 1\} & \text{otherwise} \end{cases},$$

the bond allocation is

$$X(d; q, \chi, b) = \begin{cases} d & \text{if } d \in \{0, 1\} \\ \frac{b}{\Gamma} & \text{if } d = \{0, 1\} \text{ and } z(\chi, b) \in [\underline{b} - \Gamma^I, \underline{b}) \\ \frac{z(\chi, b)}{\Gamma^U} & \text{if } d = \{0, 1\}, \ z(\chi, b) \in [\underline{b}, \bar{b} - \Gamma^I] \\ 0 & \text{if } d = \{0, 1\} \text{ and } z(\chi, b) \in (\bar{b} - \Gamma^I, \bar{b}] \end{cases},$$

and the firing rule is

$$\Phi(\theta, \sigma; \chi, p) = \begin{cases} 0 & \text{if } \sigma = 0 \text{ and either } p = 1/R \text{ or } \theta = 1 - \chi \\ 1 & \text{otherwise} \end{cases}.$$

The equilibrium features three possible regimes of information revelation. First, if the bond price is equal to zero default is revealed, no manager invests in risky bonds and hence no manager is fired, except if $\sigma = 1$. Second, if the bond price is equal to $1/R$ no default is revealed and the two assets have the same safe return. In this case, all managers are indifferent between the two assets and are never fired, except if $\sigma = 1$. Third, if the price is equal to $\tilde{P}(q)$ no information is revealed. In this case, the informed managers demand the risky bond if and only if there is no default, while the uninformed managers are the marginal traders and are indifferent between the risky bond and the risk-free asset. The auctioneer allocates the risky bonds to the informed managers if they demand any. The residual bonds are allocated randomly to a fraction of uninformed managers to clear the market. Informed managers are never fired while uninformed managers are fired whenever their investment reveals that they are not informed.

Let us now describe more in detail how equilibrium prices reveal information. If $p = 1/R$, then $z \in [\underline{b} - \Gamma^I, \underline{b})$. In this case, uninformed managers learn that $\chi = 1$ because z can be smaller than $\underline{b}$ only if a positive mass of informed managers is demanding risky bonds, which only happens if $\chi = 1$. If $p = 0$, then $z \in (\bar{b} - \Gamma^I, \bar{b}]$. In this case, uninformed managers learn that $\chi = 0$ because z can be greater than $\bar{b} - \Gamma^I$ only if no informed managers are demanding risky bonds, which only happens if $\chi = 0$. Finally, when $p = \tilde{P}(q)$, then $z \in [\underline{b}, \bar{b} - \Gamma^I]$ and

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the uninformed managers' updated beliefs are:

$$\Pr(\chi = 1 | p = \tilde{P}(q)) = \frac{\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - \Gamma^I])}{\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - \Gamma^I]) + \Pr(\chi = 0, z \in [\underline{b}, \overline{b} - \Gamma^I])}. \tag{8}$$

Since b is independent of χ and uniformly distributed on $[\underline{b}, \overline{b}]$, we have

$$\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - \Gamma^I]) = q \Pr(b \in [\underline{b}, \overline{b} - \Gamma^I]) = q \frac{\overline{b} - \underline{b} - \Gamma^I}{\overline{b} - \underline{b}}$$

and

$$\Pr(\chi = 0, z \in [\underline{b}, \overline{b} - \Gamma^I]) = (1 - q) \Pr(b \in [\underline{b} + \Gamma^I, \overline{b}]) = (1 - q) \frac{\overline{b} - \underline{b} - \Gamma^I}{\overline{b} - \underline{b}},$$

where Assumption A1 guarantees that $\overline{b} - \underline{b} > \Gamma^I$, so that these are strictly positive probabilities. Substituting in (8) it follows that $\Pr(\chi = 1 | p = \tilde{P}(q)) = q$ and the price $\tilde{P}(q)$ is completely uninformative.

Next, let us verify that the managers' demand schedules are optimal. For informed managers, it is easy to see that their demand is optimal both because it maximizes their current expected returns and because it maximizes their continuation utility, by ensuring that they are never fired.

Turning to uninformed managers, their behavior is characterized by the Bellman equation (4). When prices are fully revealing, it is easy to check that the uninformed managers' strategy is optimal, as it perfectly mimics the informed managers' behavior. Let us then focus on the case of non-revealing prices, when $p = \tilde{P}(q)$. In this case, substituting the investors' firing rule, the maximization problem in (4) becomes

$$\max_{d \in \{0,1,\{0,1\}\}} E \left[X(d; q, \chi, b)(1 - \chi) \left(\gamma \frac{1}{p} + \omega \delta \beta W \right) + (1 - X(d; q, \chi, b))(\gamma R + \chi \omega \delta \beta W) \mid p, q \right].$$

We need to check that when $p = \tilde{P}(q)$ it is optimal for the uninformed manager to demand $d = \{0, 1\}$. In the appendix, we show that the allocation probability $X(\{0, 1\}; q, \chi, b)$ is independent of χ, conditional on q and $p = \tilde{P}(q)$. This implies that $d = \{0, 1\}$ is optimal for uninformed managers if and only if

$$(1 - q) \left(\gamma \frac{1}{\tilde{P}(q)} + \omega \delta \beta W \right) = \gamma R + q \omega \delta \beta W. \tag{9}$$

This condition is analogous to condition (1) in the example of Section 2. The left-hand side represents the expected payoff of investing in risky bonds and the right-hand side represents the expected payoff of investing in the riskless asset. Notice that when investing in risky

bonds the manager is fired when there is no default—with probability $(1 - q)$—while when investing in riskless bonds he is fired when default occurs—with probability q. Moreover, in both cases he is fired if the exogenous signal σ reveals that he is uninformed—with probability ω. Rearranging condition (9), we obtain an explicit expression for the equilibrium price under no information revelation, for given W:

$$\tilde{P}(q) = \frac{\gamma(1-q)}{\gamma R - (1-2q)\,\delta\omega\beta W} \text{ for all } q \in [\underline{q}, \overline{q}]. \tag{10}$$

Assumption A3 is sufficient to ensure that $\tilde{P}(q) \in (0, 1/R)$ to avoid extreme equilibria where managers are willing to accept returns smaller than R in every state of the world just to improve their reputation.

To complete the characterization of equilibrium prices, it remains to solve for W, the expected utility of employed uninformed managers. Using the Bellman equation (4), after some algebra, we obtain

$$W = \gamma R + [\alpha + (1 - \alpha)\,E(q)]\,\delta\omega\beta W, \tag{11}$$

where $\alpha \equiv \Gamma^I/(\overline{b} - \underline{b})$ is the probability that the price is fully revealing. To interpret this expression, notice that an uninformed manager is indifferent between following his equilibrium strategy and always buying the riskless asset.[11] Under this strategy, the uninformed manager always receives current returns equal to γR. Then, if prices are fully revealing he is only fired for exogenous reasons, while if prices are non-revealing he is also fired when default occurs, which happens with expected probability $E(q)$. Also, notice that W is increasing in the measure of employed informed managers Γ^I. This is because the more employed managers are informed, the higher is the probability that prices are fully revealing and hence that uninformed manager are not fired.[12] Interestingly, expression (10) shows that W, and hence Γ^I, has a non-monotonic effect on prices: $\tilde{P}(q)$ is increasing in Γ^I if and only if $q < 1/2$.

It is easy to check that the bond allocation $X(d; q, \chi, b)$ is consistent with the managers' demand and that the bond market always clears. In particular, when $z(\chi, b) \in (\overline{b} - \Gamma^I, \overline{b}]$, default is fully revealed and the price is 0. In this case, both supply and demand of bonds are zero, as no manager demands the risky bond. When $z \in [\underline{b} - \Gamma^I, \underline{b})$, no default is revealed

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[11] Always buying the riskless asset is optimal when prices are fully revealing, since either risky bonds always default or they are equivalent to riskless bonds. It is also optimal when prices are non-revealing, because then, by construction, the price makes uninformed managers indifferent between the two assets.

[12] Although in our model the amount of information in prices switches between the extremes, the externality would survive in more general frameworks with regimes of partial revelation. In fact, the same externality is the basis of the classic Grossman-Stiglitz paradox: more informed agents increase the information content of prices, which improves the relative profitability of uninformed agents' trades.

and all managers are indifferent between risky bonds and the riskless asset. In this case, risky bonds are randomly allocated to all managers, informed and uninformed, and the probability of investing in risky bonds is equal to b/Γ. When $z(\chi, b) \in [\underline{b}, \overline{b} - \Gamma^I]$, informed managers invest in the bond if and only if there is default and only uninformed managers are indifferent. Hence, to clear the market, the probability of investing in the bond for a manager who is indifferent must be equal to $(b - \Gamma^I)/\Gamma^U$ if $\chi = 0$ and to b/Γ^U if $\chi = 1$, or, more compactly, to $z(\chi, b)/\Gamma^U$. Assumption A1 ensures that in all these cases we have $X(d; q, \chi, b) \in (0, 1)$, the details are in the appendix.

Next, we need to show that the investors' firing rule $\Phi(\theta, \sigma, \chi, p)$ is optimal. As problem (3) shows, each period investors' current payoffs are given by a share $1 - \gamma$ of the return on their current investment. The expected return made by informed and uninformed managers is the same when $p = 1/R$ or $p = 0$, and there is full revelation. However, when $p = \tilde{P}(q)$, the expected return of an informed manager is higher. Therefore, $V(\eta)$ is increasing in η and investors prefer to have informed managers investing their capital. Given the updated belief η' and the fraction of informed managers in the unemployment pool ε, problem (3) implies that an investor will fire his manager if and only if $\eta' < \varepsilon$. Therefore, to check that the firing rule is optimal we need to show that, for any belief η that can arise in equilibrium, the updated belief η' is greater than ε whenever $\sigma = 0$ and either $p = 1/R$ or $\theta = 1 - \chi$ and is smaller than ε otherwise. The second part of this statement is easy to check, because $\sigma = 1$ or $\theta \neq 1 - \chi$ and $p < 1/R$ can only happen when the manager is uninformed. Therefore, in this case $\eta = 0$ which is always smaller than $\varepsilon > 0$.[13] That is, when the manager is exogenously revealed to be uninformed or when he makes a mistake at a non-revealing price, he is immediately identified and fired. The first part of the statement above is harder to check because η' depends on the history of the match and ε depends on the current shocks. In the appendix, we show that Assumption A2 is sufficient to ensure that in this case $\eta' > \varepsilon$ always holds.

Finally, in the appendix we derive the equilibrium values of μ, Γ^I, and Γ^U consistent with stationary labor market flows and with managers' free entry. We also show that $\mu \in (0, 1)$ so that investors are always on the short side of the market.

4.3 Amplification

We now compare the behavior of our model with a benchmark model with no career concerns. This allows us to derive our main result: managers' career concerns magnify the price volatility of risky bonds.

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[13]See the appendix for the proof that $\varepsilon > 0$.

As a benchmark model with no career concerns, consider our model with $M^I = 0$. In this case, all managers are uninformed, so investors are indifferent between keeping the manager working for them and hiring a new one. Then, there exists an equilibrium where managers are never fired and maximize their expected returns in each period. We call this equilibrium the *benchmark equilibrium*. The bond price in the benchmark equilibrium is determined by the standard no-arbitrage condition

$$P^B(q) = \frac{1-q}{R} \text{ for all } q \in [\underline{q}, \overline{q}]. \tag{12}$$

Similarly to section 2, when there is no information revelation, let $\Pi(q)$ be the difference between the expected repayment and the risk free rate R, that is,

$$\Pi(q) \equiv \frac{1-q}{p_t} - R. \tag{13}$$

We call $\Pi(q)$ the *reputational premium* because it characterizes the price distortion generated by the career concerns of the uninformed managers. Condition (12) immediately implies that the reputational premium in the benchmark equilibrium with no career concerns, $\Pi^B(q)$, is equal to zero for all $q \in [\underline{q}, \overline{q}]$.

When instead there is a positive measure of informed managers, $M^I > 0$, the reputational premium can be negative or positive. In particular, when $p_t = 1/R$ and there is full revelation of no default the premium is equal to zero, while when $p_t = 0$ and there is full revelation of default there is no bond trade and the premium is not well defined. When instead there is no revelation and $p_t = \tilde{P}(q)$, it is easy to check that $\Pi(q)$ is negative if and only if $q < 1/2$. Assumption A3 ensures that there exists an equilibrium with $\underline{q} < 1/2 < \overline{q}$, so that the equilibrium premium switches sign depending on the realization of q. When the default probability is particularly low, investing in the risky bond is a relatively safe bet because there is a higher chance to mimic the investment of the informed managers. Hence, uninformed managers have a high probability of not being fired and this compensates them for a negative premium (discount) on the bond. When instead the default probability is high, uninformed managers investing in the risky bond have a large probability of being fired and hence they demand a positive premium. In short, the equilibrium price reflects this preference for large probability events. It follows that in equilibrium, the reputational premium varies with q, magnifying the volatility of prices.

Figure 2 represents graphically the price schedule $\tilde{P}(q)$ defined in (10) and the price schedule for the benchmark equilibrium $P^B(q)$ defined in (12). The intersection of these two functions at the realized default probability q, give the prices in our equilibrium and
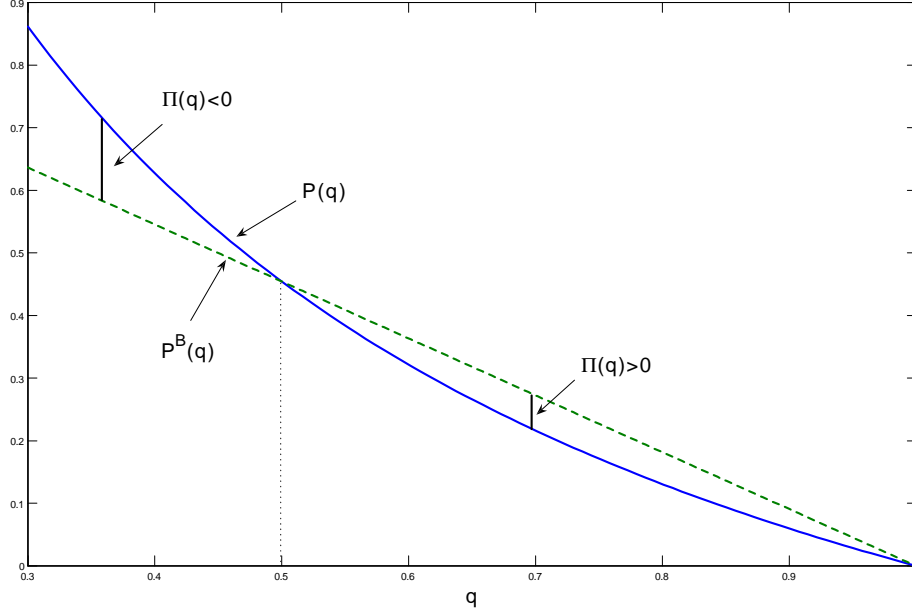
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Figure 2: The figure plots the price schedule in the model with career concerns $P(q)$ together with the price schedule for the benchmark equilibrium $P^B(q)$. The parameters used are: $M^I = .5$, $\Gamma = 2$, $\beta = .99$, $\delta = .85$, $\omega = .5$, $\kappa = .1$, $\bar{b} - \underline{b} = .71$, $\underline{q} = .3$, $\bar{q} = 1$, $E(q) = .6$.

in the benchmark one respectively. The figure shows that both the pricing schedules are monotonically decreasing in q and they intersect at $q = 1/2$, so that $\Pi(q) > 0$ if and only if $q > 1/2$. Moreover, $\tilde{P}(q)$ is steeper than $P^B(q)$ at $q = 1/2$ and for q not too close to 1. This immediately implies that the price of the risky bond reacts more to a change in q in our model in comparison to the benchmark as long as q is not too high. However, we can prove a more general result. Next proposition states our main amplification result: the volatility of log prices (and hence of log spreads) is always higher in our model with career concerns relative to the benchmark equilibrium.

Proposition 2 *In equilibrium, the reputational premium* $\Pi(q)$ *is positive whenever* $q < 1/2$, *and negative otherwise. Moreover, equilibrium prices are more volatile than in the benchmark equilibrium, that is,* $Var(\log P(q, \chi, b)) \geq Var(\log P^B(q))$.

Proposition 2 shows that managers' career concerns amplify the price reaction of risky bonds to changes in their default risk. In particular, when the default risk suddenly increases, the economy can switch from regimes with low bond spreads (high p) to regimes with high bond spreads (low p). The first type of regimes are frequently described as regimes of *abundant liquidity* or with *traders reaching for yield*. To describe phenomena where the

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economy switches to the second type of regime, common terms are *flight-to-quality, flight-to-liquidity, disappeared liquidity*, or *drop in risk appetite*. In our model, phenomena of this type can arise even if fund managers are risk-neutral and their aggregate funds are constant. In good times, when the default probability of credit instruments is low, it is very attractive for uninformed managers to invest in these instruments, because they are likely to gain high returns and hence improve their reputation. If suddenly the default probability increases, investing in the risky asset becomes less appealing because their reputation starts deteriorating. Hence, prices increase not only because of the higher probability of default, but also because of an additional premium coming from career concerns.

It is well established in the literature that the premium on risky assets is time-varying and, in particular, that it in good times is lower than in bad times. However, standard explanations are based on time-varying marginal utility of consumption, on time-varying probability of disasters or on slow-moving component in consumption risk. A common denominator of these different mechanisms is that the premium is always positive. Our model generates a time-varying component of the premium on risky assets that can be negative. In good times some managers are willing to take risky bets without the sufficient compensation in returns. This unique implication of our model seems consistent with a number of empirical observations that we have described in the introduction (Duffie et al., 2003, Brunnermeier and Nagel, 2004, and Coval, Jurek and Stafford, 2008).

The amplification effect is going to be stronger the more serious career concerns are, that is, the more managers care about their reputation. In particular, this is the case when the entry cost κ is higher. Given that in equilibrium there is free-entry of uninformed managers, the higher is the entry cost, the smaller is the measure of uninformed managers who look for a job. This increases the hiring probability for all managers and hence increases the measure of informed managers who are employed. As we have discussed above, employed informed managers generate a positive externality for the uninformed managers. The more employed managers are informed the higher is the probability that prices will reveal information about the default state, hence increasing the expected utility of employed managers who are uninformed. This makes their reputation more valuable and amplifes the distortion generated by career concerns. This suggests that we should expect higher price volatility in markets of more complex financial instruments where setting up a fund requires more resources. The next proposition summarizes this result.

Proposition 3 *The amplification effect is stronger the higher is the entry cost κ.*

The effect on amplification of the discount factor, β, and of the probability of an exogenous revealing signal, $1 - \omega$, is ambiguous. On the one hand, for given Γ^I, when β

is higher, managers care more about their future and hence about their reputation. Also, career concerns are stronger when there is a lower chance that managers lose their job for exogenous reasons and hence when ω is higher. However, on the other hand, an increase in either one of these parameters also increases the expected utility of being employed. More uninformed managers will search for a job, reducing the hiring probability and the measure of informed employed managers. This makes price less informative and hence decreases the expected value of being employed and the returns to reputation. The two effects go in opposite directions and make the overall effect in general ambiguous.

4.4 Limit Equilibrium

It is interesting to explore our model in the limit case with $M^I \to 0$. This limit is very tractable and insightful at the same time. We show that as $M^I \to 0$, the sequence of stationary equilibria constructed so far converges to a *limit equilibrium* where the bond price never reveals any information, and is constant over time. Intuitively, this can be the case because as the fraction of informed managers is infinitesimal, the uninformed managers demand essentially all the bonds supplied and hence don't learn any information from the equilibrium price.

Proposition 4 [14] *Under assumption A1-A3, when $M^I \to 0$, there exists a limit equilibrium where the price $P(q) \in (0, 1/R)$ is determined by the indifference condition of the uninformed managers, the demand schedule for informed managers is $d^I(p; \chi) = 1 - \chi$ for all p and χ, the demand schedule for the uninformed managers is $d^U(p; q) = \{0, 1\}$ for all p and q, the bond allocation is*

$$X(d; q, \chi, b) = \begin{cases} d & \text{if } d \in \{0, 1\} \\ \frac{b}{\Gamma} & \text{if } d = \{0, 1\} \end{cases},$$

and the investors' strategy is

$$\Phi(\theta, \sigma; \chi) = \begin{cases} 0 & \text{if } \theta = 1 - \chi \\ 1 & \text{if } \theta \neq 1 - \chi \text{ or } \sigma = 1 \end{cases}.$$

In a limit equilibrium, prices never reveal any information and uninformed managers are always the marginal traders. Informed managers demand the risky bond if and only if there is no default, while uninformed managers cannot follow the same strategy. The equilibrium price will make them indifferent between demanding the risky bond and the riskless asset, so that bonds can be allocated to clear the market. At the end of the period, investors

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[14]The proof of this proposition is an obvious generalization of the proof of Proposition 1 and hence omitted.

fire managers who failed to mimic the informed managers' strategy and hence revealed to be uninformed. Clearly, informed managers are never fired. Once again, assumption A1 guarantees that $b/\Gamma \in (0, 1)$ for any b, so that there are always some uninformed managers investing in the risky bond and some investing in the risk-free asset.

For a given default probability q, the equilibrium price $P(q)$ is determined by the same indifference condition (10), where the expected continuation utility of an employed uninformed manager W satisfies condition (11) with $\Gamma^I \to 0$, that is,

$$W = \gamma R + E(q) \delta \omega \beta W. \tag{14}$$

Combining (10) and (14) gives the equilibrium price level for a given default probability, that is,

$$P(q) = \frac{1 - q}{R} \left[\frac{1 - \delta \omega \beta E(q)}{1 - \delta \omega \beta (E(q) + 1 - 2q)} \right] \text{ for all } q \in [\underline{q}, \overline{q}].$$

The equilibrium price immediately shows that also in the limit equilibrium the reputational premium $\Pi(q)$ varies with q and can be positive or negative depending on q being below or above $1/2$. This shows that the reputational premium does not disappear when the informational asymmetry becomes infinitesimal, that is, as $M^I \to 0$. It is interesting to notice that there is a discontinuity at $M^I = 0$, given that the benchmark equilibrium does not survive as long as $M^I > 0$, even if $M^I \to 0$.

5 Persistent Default Risk

In this section, we generalize the model to allow for persistent default probability q_t. In particular, we allow q_t to be distributed according to a first-order Markov process with cumulative density function $G(q_t|q_{t-1})$ with the same support $[\underline{q}, \overline{q}]$. The environment is a natural generalization of the one with iid shocks.

Using a numerical example, we illustrate that when q_t is persistent, there are two effects on the reputational premium. First, there is the direct effect that a higher default probability today increases the chance of a higher default probability in the future. This effect leads to larger premium when the default risk is high, and lower discount when the default risk is low. Second, there is an indirect effect due to the fact that the measure of employed informed managers is now varying over time. When there are more employed informed managers the probability that prices reveal information is higher and then the expected utility of uninformed managers is higher. This increases the reputational effect of career concerns and implies that the price of the risky bond might change even if the fundamentals, that is, the

perceived default risk, do not.

5.1 Equilibrium with persistent shocks

First, we define an equilibrium for the model where the default probability q follows a first-order Markov process. In this case, a stationary equilibrium does not exist and we focus on Markovian equilibria where q and Γ^I become state variables.[15]

Definition 2 *A Markovian symmetric equilibrium is given by demand schedules $d^I(p; \chi)$ and $d^U(p; q)$, a price function $P(q, \chi, b, \Gamma^I)$, a bond allocation $X\left(d; q, \chi, b, \Gamma^I\right)$, a firing strategy $\Phi(\theta, \sigma; \chi, p)$, a law of motion for the investor beliefs H, a law of motion for the measure of employed informed managers $G\left(q, \Gamma_I\right)$, and a matching probability for unemployed managers $\mu\left(q, \Gamma^I\right)$, such that:*

1. *the fund managers' demand schedules are optimal, taking as given the equilibrium price and the investors' firing strategy;*

2. *the investors' firing strategy is optimal given investors' beliefs, the equilibrium price and the managers' demand schedules;*

3. *the bond allocation is consistent with the managers' demand schedules;*

4. *the bond market clears;*

5. *investors' beliefs H are consistent with Bayes' law on the equilibrium path;*

6. *$G\left(q, \Gamma_I\right)$ and $\mu\left(q, \Gamma^I\right)$ are consistent with stationary labor market flows and with managers' free entry.*

We now characterize a Markovian equilibrium with similar features to the equilibrium in the baseline model. In particular, prices can be fully revealing or completely uninformative, depending on the realization of $z(\chi, b, \Gamma^I) = b - (1 - \chi)\Gamma^I$. As before, the variable z represents the supply of bonds net of the potential demand of informed managers and now depends on the state Γ^I as well. In such a Markovian equilibrium, the price function is

$$P(q, \chi, b, \Gamma^I) = \begin{cases} \frac{1}{R} & \text{if } z(\chi, b, \Gamma^I) \in [\underline{b} - \Gamma^I, \underline{b}) \\ \tilde{P}\left(q, \Gamma^I\right) & \text{if } z(\chi, b, \Gamma^I) \in [\underline{b}, \bar{b} - \Gamma^I] \\ 0 & \text{if } z(\chi, b, \Gamma^I) \in (\bar{b} - \Gamma^I, \bar{b}] \end{cases}, \tag{15}$$

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[15] Note, that we still omit the time subscript for Γ^I, even if it is not constant anymore.

for some function $\tilde{P}\left(q, \Gamma^I\right)$ which takes values in $(0, 1/R)$; the demand schedules are

$$d^I(p; \chi) = \begin{cases} \{0, 1\} & \text{if } p = 1/R \\ 1 - \chi & \text{otherwise} \end{cases} \quad \text{and} \quad d^U(p; q) = \begin{cases} 0 & \text{if } p = 0 \\ \{0, 1\} & \text{otherwise} \end{cases} \quad ; \quad (16)$$

the bond allocation is

$$X(d; q, \chi, b, \Gamma^I) = \begin{cases} d & \text{if } d \in \{0, 1\} \\ \frac{b}{\Gamma} & \text{if } d = \{0, 1\} \text{ and } z(\chi, b, \Gamma^I) \in [\underline{b} - \Gamma^I, \underline{b}) \\ \frac{z(\chi, b, \Gamma^I)}{\Gamma - \Gamma^I} & \text{if } d = \{0, 1\}, \ z(\chi, b, \Gamma^I) \in [\underline{b}, \overline{b} - \Gamma^I] \\ 0 & \text{if } d = \{0, 1\} \text{ and } z(\chi, b, \Gamma^I) \in (\overline{b} - \Gamma^I, \overline{b}] \end{cases} \quad ; \quad (17)$$

and the firing rule is

$$\Phi\left(\theta, \sigma; \chi, p\right) = \begin{cases} 0 & \text{if } \sigma = 0 \text{ and either } p = 1/R \text{ or } \theta = 1 - \chi \\ 1 & \text{otherwise} \end{cases} \quad . \quad (18)$$

This equilibrium is very similar to the one constructed in Section 4.2 for the model with iid q, with the exception that the price, the bond allocation, and the labor market flows now depend on the states q and Γ^I. There are three revelation regimes: default is revealed, non default is revealed and no information is revealed. Informed managers always maximize the expected returns conditional on their information and hence demand the bond if and only if there is default whenever $p < 1/R$ and are indifferent when $p = 1/R$. Uninformed managers mimic the informed whenever the default state is revealed, while when there is no revelation, they are indifferent between risky bonds and riskless assets. The auctioneer picks the price and the bond allocation which are consistent with the managers' demand and ensure market clearing. Finally, an investor fires his manager whenever either his investment or the exogenous signal reveal that he is uninformed.

Let us now discuss the main differences with the iid case. First, let us look at the labor market. As mentioned above, the flows are now not stationary, so that Γ^I becomes one state variable. As in the baseline model, given that there is a large continuum of uninformed managers the hiring probability $\mu\left(q, \Gamma^I\right)$ must be consistent with the following free entry condition:

$$\mu\left(q, \Gamma^I\right) W\left(q, \Gamma^I\right) = \kappa, \quad (19)$$

where $W\left(q, \Gamma^I\right)$ denotes the expected value of being employed for an uninformed manager at the end of the period which now depends on the states q and Γ^I. Given that the expected value of being employed is always higher for an informed manager than for an uninformed

manager, all unemployed informed managers at time t search for a job. In equilibrium, no informed manager is fired. Hence, the measure of employed informed managers at the beginning of time $t + 1$ must be equal to the measure of informed managers employed at time t whose job was not exogenously terminated, plus the measure of unemployed informed managers who found a job at the end of time t, that is, the law of motion for Γ^I must solve

$$\Gamma^{I\prime} = G\left(q, \Gamma^I\right) \equiv \delta\Gamma^I + \mu\left(q, \Gamma^I\right)\left(M^I - \delta\Gamma^I\right). \tag{20}$$

One of the key equilibrium strategies is the bond demand of the uninformed agents. Their optimization problem can be described with a Bellman equation similar to (4) with the difference that W now depends on the two states q and Γ^I. As in the baseline model, the allocation probability $X(d; q, \chi, b, \Gamma^I)$ is independent of χ so that the indifference condition for the uninformed managers is given by the analog of equation (9). Hence, when there is no information revelation, the equilibrium price can be written as

$$\tilde{P}\left(q, \Gamma^I\right) = \frac{\gamma\left(1 - q\right)}{\gamma R - \left(1 - 2q\right)\delta\omega\beta W\left(q, \Gamma^I\right)}, \tag{21}$$

where

$$W\left(q, \Gamma^I\right) = \gamma R + \left[\alpha\left(\Gamma^I\right) + \left(1 - \alpha\left(\Gamma^I\right)\right)E[q'|q]\right]\delta\omega\beta E[W\left(q', G\left(q, \Gamma_I\right)\right)|q], \tag{22}$$

where $\alpha\left(\Gamma^I\right) = \Gamma^I/(\bar{b} - \underline{b})$ is the probability that the price is fully revealing. It is interesting to highlight the direct effect of q and Γ^I on the expected utility W. On the one hand, the higher is the default probability q, the higher is the expected default probability tomorrow and hence the lower is the chance to be fired if investing in the riskless asset. Given that the price makes uninformed managers indifferent between investing in the two assets, this implies that the higher q, the higher is their expected utility. On the other hand, as more employed managers are informed, that is, the higher is Γ^I, the higher is the probability that prices are fully revealing. When there is full revelation, uninformed managers are better off because they are never fired and their expected utility is higher.

Finally, for the proposed equilibrium to exist, it must be that uninformed managers who mimic the informed ones are never fired in equilibrium. In the iid baseline model, assumption A2 was a sufficient condition to ensure that this was the case. With persistent q, we need to tighten such an assumption as follows:

$$\omega < \left(1 - \frac{M^I}{\Gamma}\right)\left[\frac{1}{(1 - \delta)(1 - \delta\omega\beta)} + \delta\right]^{-1}. \tag{A2'}$$

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Proposition 5 *If there exist three functions $W\left(q, \Gamma^I\right)$, $\mu\left(q, \Gamma^I\right)$, and $G\left(q, \Gamma_I\right)$ that satisfy equations (20)-(22) and assumptions A1, A2' and A3 hold, there is a Markovian equilibrium, where prices satisfy (15), managers' demand schedules (16), the bond allocation (17), and the firing rule (18).*

Using numerical methods, we find that the equilibrium exists for a wide range of parameters.[16] Also, in all simulations we find similar qualitative properties. In the next section we describe a representative numerical example.

Before moving to simulations, it is insightful to mention the limit case with $M^I \to 0$, that is, where the informational asymmetry becomes infinitesimal. This case is more tractable because the measure of informed managers who are employed Γ^I also converges to 0, and hence the only state variable is q. In particular, condition (19) reduces to $\mu\left(q\right) = \kappa/W\left(q\right)$ where

$$W\left(q\right) = \gamma R + E[q'|q]\delta\omega\beta E[W\left(q'\right)|q].\tag{23}$$

It is straightforward to see that the right-hand-side of the previous equation is a contraction and hence that there exists an equilibrium function $W\left(q\right)$. This implies that in the limit case, an equilibrium always exists if assumptions A1, A2', and A3 are satisfied.

5.2 Amplification

In this section, we are interested in comparing the baseline model with iid default risk with the model where the default risk is persistent. In particular, we want to explore the changes in the reputational premium and hence in the amplification effect. For simplicity, we consider the two-state case where $q_t \in \left\{q^L, q^H\right\}$ with $q^L < 1/2 < q^H$. We assume that in the iid case $\Pr\left(q_{t+1} = q^H | q_t = q^\sigma\right) = \pi^*$ for $\sigma = L, H$, while in the persistent case $\Pr\left(q_{t+1} = q^H | q_t = q^\sigma\right) = \pi^\sigma$ for $\sigma = H, L$, with $\pi^L < \pi^* < \pi^H$. To help the comparison, we denote the equilibrium variables of the iid case with an asterisk.

As in the iid case, it is interesting to see how the reputational premium varies when there is no information revelation. When the default probability q is persistent, the reputational premium depends not only on q, but also on the measure of employed informed managers Γ^I. In particular, under no revelation,

$$\Pi\left(q, \Gamma^I\right) \equiv \frac{1-q}{P\left(q, \Gamma^I\right)} - R = -\frac{\left(1 - 2q\right)\delta\omega\beta W\left(q, \Gamma^I\right)}{\gamma}.\tag{24}$$

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[16]Unfortunately, the assumptions of standard fixed-point theorems (e.g. Schauder, Banach) do not hold in our case and the existence of $W\left(q, \Gamma^I\right)$, $\mu\left(q, \Gamma^I\right)$, and $G\left(q, \Gamma_I\right)$ satisfing equations (20)-(22) is not guaranteed for all parameters.

where we have substituted for $P(q, \Gamma^I)$ using (21) and where $W(q, \Gamma^I)$ is defined by expression (22). As in the iid case, the absolute value of the premium is increasing in the expected utility of employed uninformed managers $W(q, \Gamma^I)$, which represents the reward to good reputation. However, in the iid case W was constant over time, while now it varies with both q and Γ^I, as we have discussed above.

5.2.1 Limit Case

Let us start by considering the limit case with $M^I \to 0$. This case is particularly tractable because the measure of informed employed managers also converges to 0 and the only state variable is q. In this case, we can explicitly compute the expected utility of employed informed managers contingent on the state, that is,

$$W(q^\sigma) = \gamma R + [\pi^\sigma (q^H - q^L) + q^L] \delta \omega \beta [\pi^\sigma (W(q^H) - W(q^L)) + W(q^L)]. \qquad (25)$$

In order to explore how the degree of persistency of the default risk q affects amplification, suppose, without loss of generality, that $\pi^H \equiv \pi (1 - \lambda) + \lambda$ and $\pi^L \equiv \pi (1 - \lambda)$, with $\pi \in [0, 1]$ and $\lambda \in [0, 1]$. In the two extreme cases, when $\lambda = 1$ the shock becomes permanent and when $\lambda = 0$ we are back to the iid benchmark. We are then interested in exploring how the reputational premium varies with λ, that is, as the persistence degree of the stochastic process for q increases.

From condition (25), given that $q^H > q^L$ we can easily show that $W(q^H) > W(q^L)$. It follows that $W(q^\sigma)$ is increasing in π^σ. Moreover, from the definition of π^σ we obtain

$$\frac{d\pi^H}{d\lambda} = 1 - \pi \text{ and } \frac{d\pi^L}{d\lambda} = -\pi.$$

This implies that as λ increases, or q becomes more persistent, $W(q^H)$ increases while $W(q^L)$ decreases. This means that when $q = q^H$ and the reputational premium is positive, the premium is amplified by persistency. On the contrary, when $q = q^L$ and there is a reputational discount, such a discount is dampened by persistency.

5.2.2 A numerical example

The limit case is tractable at the expense of losing the effect of the measure of informed employed managers on the reputational premium. To explore this feature of the model, we now turn to a numerical example with $M^I > 0$.[17]

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[17]We make sure that the parameters that we pick for the numerical example satisfy assumptions A1, A3, and condition (34), which is a weaker version of assumption A2' derived in the Appendix.
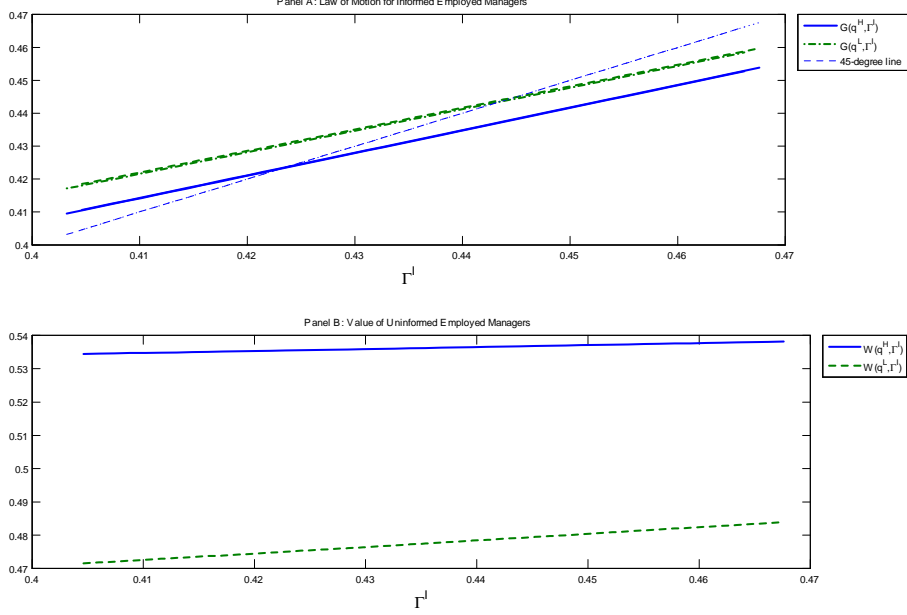
Figure 3: Panel A plots the law of motion for informed employed managers in the high and law state, $G\left(q^H, \Gamma^I\right)$ and $G\left(q^L, \Gamma^I\right)$ together with the 45-degree line. Panel B plots the value function for employed uninformed managers in the high and low state, $W\left(q^H, \Gamma^I\right)$ and $W\left(q^L, \Gamma^I\right)$. The parameters used are: $M^I = .5$, $\bar{\Gamma} = 2$, $\beta = .99$, $\delta = .85$, $\omega = .5$, $\kappa = .1$, $\bar{b} - \underline{b} = .71$, $\underline{q} = .3$, $\bar{q} = .9$, $\pi^L = .1$, and $\pi^H = .9$.

First of all, we want to understand the dynamics of Γ^I, which are endogenously determined according to condition (20). Panel A of Figure 3 shows $G\left(q^L, \Gamma^I\right)$ and $G\left(q^H, \Gamma^I\right)$ as functions of Γ^I, together with the 45-degree line. The figure shows that both functions are increasing in Γ^I: the more informed managers are employed today, the more are going to be employed in the future if the default state does not change. However, $G\left(q^H, \Gamma^I\right) < G\left(q^L, \Gamma^I\right)$ for all Γ^I, meaning that when the default risk is high, everything else equal, more informed managers are going to be employed. Both $G\left(q^H, \Gamma^I\right)$ and $G\left(q^L, \Gamma^I\right)$ cross the 45-degree line only once and their slope is smaller than 1 at each intersection. Let $\overline{\Gamma} \equiv G\left(q^H, \overline{\Gamma}\right)$ and $\underline{\Gamma} \equiv G\left(q^L, \underline{\Gamma}\right)$. Then $\left[\underline{\Gamma}, \overline{\Gamma}\right]$ denotes the ergodic set for Γ^I, that is, if $\Gamma^I_0 \in \left[\underline{\Gamma}, \overline{\Gamma}\right]$, Γ^I_t will remain in $\left[\underline{\Gamma}, \overline{\Gamma}\right]$ for all $t > 0$. Moreover, Γ^I will gradually increase whenever $q = q^L$ and gradually decrease whenever $q = q^H$. This is illustrated in Panel A of Figure 4, which plots the simulated path for Γ^I for a specific realization of sequence of shocks $\{q_t\}_{t \in [1,50]}$. The same figure shows the constant value Γ^{I*} for the baseline case where q is iid, where $\underline{\Gamma} < \Gamma^{I*} < \overline{\Gamma}$. So Γ^I fluctuates around its iid counterpart.

Once we understood the dynamics of Γ^I, we can turn to the dynamics of the expected

utility of employed uninformed managers $W\left(q, \Gamma^I\right)$, which are key to determine the reputational premium according to equation (24). Panel B in Figure 3 represents $W\left(q^L, \Gamma^I\right)$ and $W\left(q^H, \Gamma^I\right)$ as functions of Γ^I and shows that $W\left(q^L, \Gamma^I\right) < W\left(q^H, \Gamma^I\right)$ for all Γ^I. This is because the higher is the default probability q, the higher is the expected default probability tomorrow and hence the lower is the chance to be fired if investing in the riskless asset. Given that the price makes uninformed managers indifferent between investing in the two assets, this implies that the higher q, the higher is their expected utility. This implies that W jumps whenever the default state changes. Moreover, the figure shows that both $W\left(q^L, \Gamma^I\right)$ and $W\left(q^H, \Gamma^I\right)$ are increasing functions in Γ^I. For a given default risk q, as more employed managers are informed (higher Γ^I), the higher is the probability that prices are fully revealing. When there is full revelation, uninformed managers are better off because they are never fired and their expected utility is higher. Together with the dynamics of Γ^I, this implies that $W\left(q, \Gamma^I\right)$ keeps decreasing whenever the default risk remains q^H and keeps increasing whenever the default risk remains q^L.
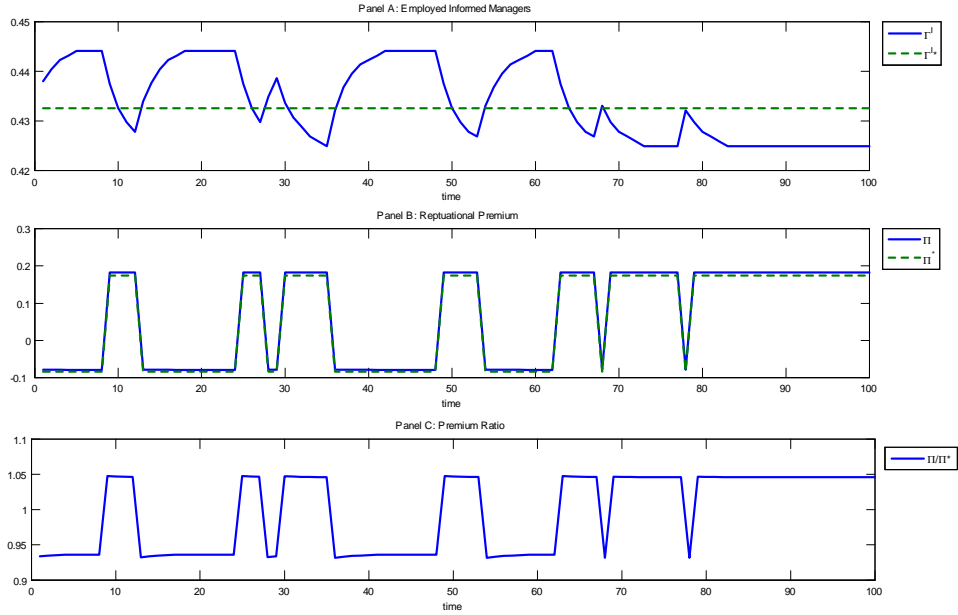

Figure 4: Panel A compare the evolution of Γ^I and Γ^{I*} over time for a specific sequence of shocks. Panel B compares the evolution of Π and Π^* for the same simulation. Panel C plots the ratio of Π/Π^*. The parameters used are: $M^I = .5$, $\Gamma = 2$, $\beta = .99$, $\delta = .85$, $\omega = .5$, $\kappa = .1$, $\bar{b} - \underline{b} = .71$, $\underline{q} = .3$, $\overline{q} = .9$, $\pi^L = .1$, and $\pi^H = .9$.

Finally, we can turn to the dynamics of the reputational premium $\Pi\left(q, \Gamma^I\right)$. Given equation (24), the fact that $W\left(q, \Gamma^I\right)$ is increasing in Γ^I immediately translates in $\Pi\left(q, \Gamma^I\right)$

being increasing Γ^I. Moreover, there are two effects of q on the premium. On the one hand, there is a direct positive effect of q on $\Pi\left(q, \Gamma^I\right)$, that is, the higher is the default risk the higher is the firing probability and hence the reputational premium. On the other hand, there is the positive effect of q on $W\left(q, \Gamma^I\right)$, that increases the premium in the high default risk state and decreases the absolute size of the discount in the low default risk state.

Panel B in Figure 4 compares the simulated pattern of the premium for a realized sequence of default risk for the baseline model with iid q and the persistent model. Panel C in the same figure plots the ratio Π/Π^* so that the comparison is more evident. First, it is interesting to notice that the persistence of q magnifies the reputational premium when positive $(q = q^H)$ and dampens it when negative $(q = q^L)$. This comes from the direct effect of q on $W\left(q, \Gamma^I\right)$ and from the fact that Γ^I increases (decreases) whenever $q = q^L$ (q^H), driving $W\left(q, \Gamma^I\right)$ and hence the absolute value of the premium up (down). Second, it is striking that the premium varies over time even for a sequence of realizations where the default risk does not change. For example, if the default risk stays equal to q^L for a sequence of periods, Γ^I keeps increasing and so does the expected utility $W\left(q^L, \Gamma^I\right)$. This implies that the reputational discount increases in absolute value, even though the default probability does not change. In contrast, if the economy experiences a sequence of high realizations of default risk q^H, Γ^I decreases, hence reducing $W\left(q^L, \Gamma^I\right)$ and dampening the reputational premium.

To sum up, when the default risk is persistent, there is an additional source of volatility in asset price dynamics, driven by the labor market. In this example, the bond price can vary even when the fundamentals of the risky bond do not change. In particular, the measure of employed informed agents changes the future informational content of prices, which changes career prospects of uninformed managers, which, in turn, affect current prices. Moreover, we show that the degree of persistency magnifies the amplification effect in states with high default probability and dampens it when the default risk is low.

6   Conclusion

In this paper, we have proposed a general equilibrium model of delegated portfolio management with time-varying default risk, where career concerns distort asset prices. In particular, risky bonds trade with a reputational premium, which may be positive or negative depending on the default risk. For example, when the default probability is high, the return on the risky bond has to be high to compensate the uninformed managers for the high risk of being fired. As the default risk changes over time, the reputational premium amplifies the volatility of the risky bond price.

For future research, it would be interesting to introduce alternative risky assets in the

portfolio choice of the managers. In this case, our mechanism would generate contagion. Imagine that there are two risky bonds and a riskless asset. The reputational cost of investing in the risk-less asset depends on the default probability of both the risky bonds. If none of them defaults, the manager who invests in the risk-less asset loses his reputation. Thus, if the probability of default of any of the risky bonds decreases, the risk-less asset will be less attractive, and the prices of both bonds will have to increase in order to make uninformed managers indifferent between different investment opportunities.

Finally, it would be interesting to develop the supply side of the model in the context of sovereign debt.[18] A large literature on business cycle characteristics of emerging markets [19] highlights that emerging market bond spreads are very volatile. In particular, the magnitude of volatility of interest rates is hard to reconcile with models where bond prices are determined by the standard no-arbitrage condition. Our model provides an appealing framework to think about this excess volatility.

Appendix

Proof of Proposition 1

Here we complete the proof that, under assumptions A1-A3, there exists a stationary equilibrium with the features described in Proposition 1. The main part of the proof is in the text, but we need three additional steps: first, we show that the allocation probability $X(\{0,1\}; q, \chi, b)$ is independent of χ, conditional on q and $p = \tilde{P}(q)$; second, we derive the equilibrium values for Γ^I, Γ^U, and μ that are consistent with stationary labor market flows and managers' free entry; third, we complete the proof that the investors' firing strategy is optimal, by showing that assumption A2 is sufficient to ensure that the belief that an employed manager is informed if he did not reveal to be uninformed is always higher than the probability that a newly hired manager is informed.

Step 1. First, we want to show that the allocation probability $X(\{0,1\}; q, \chi, b)$ is independent of χ, conditional on q and $p = \tilde{P}(q)$, that is,

$$E\left[X(\{0,1\}; q, \chi, b) | q, \chi = 1, p = \tilde{P}(q)\right] = E\left[X(\{0,1\}; q, \chi, b) | q, \chi = 0, p = \tilde{P}(q)\right].$$

From the equilibrium price schedule (7), we know that $p = \tilde{P}(q)$ if $z \in [\underline{b}, \overline{b} - \Gamma^I]$. Assumption

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[18]Guerrieri and Kondor (2009) proposes a first attempt in this direction.

[19]See Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2006), and Arellano (2008).

A1 ensures that this happens with positive probability. Recall that $z = b - (1 - \chi) \Gamma^I$. Hence, when $p = \tilde{P}(q)$ and $\chi = 1$ it must be that $b \in [\underline{b}, \overline{b} - \Gamma^I]$, while when $p = \tilde{P}(q)$ and $\chi = 0$ it must be that $b \in [\underline{b} + \Gamma^I, \overline{b}]$. One can then derive

$$E\left[X\left(\{0,1\}; q, \chi, b\right) | q, \chi = 1, p = \tilde{P}(q)\right] = \int_{\underline{b}}^{\overline{b} - \Gamma^I} \frac{b}{\Gamma^U} dF(b) = \frac{1}{2\Gamma^U}[(\overline{b} - \Gamma^I)^2 - \underline{b}^2],$$

$$E\left[X\left(\{0,1\}; q, \chi, b\right) | q, \chi = 0, p = \tilde{P}(q)\right] = \int_{\underline{b} + \Gamma^I}^{\overline{b}} \frac{b - \Gamma^I}{\Gamma^U} dF(b) = \frac{1}{2\Gamma^U}[(\overline{b} - \Gamma^I)^2 - \underline{b}^2].$$

It follows that these two expressions are the same, completing the proof.

Step 2. The firing probabilities consistent with the equilibrium firing strategy according to $\xi_t^s = E\left[\Phi\left(\theta_{i,t}, \sigma_{i,t}, \chi_t, p_t\right) | x_t^s\right]$ can be reduced to $\xi_t^I = 0$ for all q_t, χ_t, and b_t and

$$\xi_t^U = \begin{cases} 1 - \omega \left[x_t^U (1 - \chi_t) + \left(1 - x_t^U\right) \chi_t\right] & \text{if } z_t \in [\underline{b}, \overline{b} - \Gamma^I] \\ 1 - \omega & \text{if } z_t \notin [\underline{b}, \overline{b} - \Gamma^I] \end{cases}. \tag{26}$$

Using $\xi_t^I = 0$, condition (2) with $s = I$, and (6), we obtain

$$\Gamma^I = \frac{\mu M^I}{1 - \delta (1 - \mu)}. \tag{27}$$

Also, equation (5) can be rewritten as

$$W = \frac{\kappa}{\mu}. \tag{28}$$

From condition (11), we obtain

$$W = \frac{\gamma R}{1 - \delta \omega \beta \left[\frac{\Gamma^I}{\overline{b} - \underline{b}} + \left(1 - \frac{\Gamma^I}{\overline{b} - \underline{b}}\right) E(q)\right]} \tag{29}$$

and by combining (27)-(29) we obtain an equation in μ only $g(\mu) = 0$, where

$$g(\mu) = \frac{\kappa}{\mu} - \gamma R \left\{1 - \delta \omega \beta \left[E(q) + \frac{M^I (1 - E(q))}{(\overline{b} - \underline{b}) \left(\delta + \frac{1 - \delta}{\mu}\right)}\right]\right\}^{-1}. \tag{30}$$

Notice that $\lim_{\mu \to 0} g(\mu) = \infty$, $\lim_{\mu \to 1} g(\mu) < 0$ thanks to the assumption that $\kappa < \gamma R$, and $g'(\mu) < 0$ by inspection. It immediately follows that there exists a unique $\mu \in (0, 1)$ such that $g(\mu) = 0$. Given μ, one can use equation (27) to solve for a unique $\Gamma^I < M^I$, and hence

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a unique $\Gamma^U = \Gamma - \Gamma^I$, and equation (28) to solve for a unique W.

Step 3. By definition ε_t satisfies

$$\varepsilon_t = \frac{N_t^I}{N_t^I + N_t^U} > 0, \tag{31}$$

that is, the probability that a newly hired manager is informed is equal to the ratio of unemployed informed managers relative to all the unemployed managers. When manager i realizes $\theta_{i,t} = 1 - \chi_t$ and/or $p_t \in \{0, 1/R\}$, the investor's belief is updated according to $\eta_{i,t+1} = \eta_{i,t}/[\eta_{i,t} + (1 - \xi_t^U)(1 - \eta_{i,t})]$, where ξ_t^U defined in equation (26) denotes the proportion of uninformed managers who are fired. Next, we show that assumption A2 is sufficient to make sure that in equilibrium $\eta_{i,t+1} \geq \varepsilon_t$ for any ξ_t^U and $\eta_{i,t+1} > 0$.

First, consider an investor who has just hired manager i at the end of $t - 1$ and hence, by definition, has prior belief $\eta_{i,t} = \varepsilon_{t-1}$. In this case, if $\theta_{i,t} = 1 - \chi_t$ and/or $p_t \in \{0, 1/R\}$, then $\eta_{i,t+1} = \varepsilon_{t-1}/[\varepsilon_{t-1} + (1 - \xi_t^U)(1 - \varepsilon_{t-1})]$. Next, we want to show that $\eta_{i,t+1} \geq \varepsilon_t$. This condition can be rewritten as

$$\frac{1 - \varepsilon_t}{\varepsilon_t} \geq \left(\frac{1 - \varepsilon_{t-1}}{\varepsilon_{t-1}}\right)(1 - \xi_t^U). \tag{32}$$

Using expression (31) for ε_t with $N_t^I = M^I - \delta\Gamma^I$ from condition (6), we have that $(1 - \varepsilon_t)/\varepsilon_t = N_t^U/(M^I - \delta\Gamma^I)$, and, hence, condition (32) can be rewritten as $N_t^U/N_{t-1}^U \geq 1 - \xi_t^U$, where $N_t^U = (1 - \delta(1 - \xi_t^U))\Gamma^U/\mu$. Hence, in order for (32) to be satisfied it must be that $1 - \delta(1 - \xi_t^U) > (1 - \delta(1 - \xi_{t-1}^U))(1 - \xi_t^U)$, which is ensured by assumption A2, given that $\xi_t^U \in [1 - \omega, 1]$ for all t.

Let us now consider managers who were working for an investor for longer than 1 period. First, notice that the investors' beliefs about any manager who is still working at time t but was hired at time $t' < t$ must be higher than the initial belief $\varepsilon_{t'-1}$, given that if he was not fired he never made any mistake, that is, $\eta_{i,t} \geq \varepsilon_{t'-1}$. Hence, the posterior belief about a manager who was hired at time t' and did not make a mistake at time t is

$$\eta_{i,t+1} = \frac{\eta_{i,t}}{\eta_{i,t} + (1 - \xi_t^U)(1 - \eta_{i,t})} \geq \frac{\varepsilon_{t'-1}}{\varepsilon_{t'-1} + (1 - \xi_{t'}^U)(1 - \varepsilon_{t'-1})}.$$

It follows that a sufficient condition for this manager not being fired is $(1 - \varepsilon_t)/\varepsilon_t \geq (1 - \xi_{t'}^U)(1 - \varepsilon_{t'})/\varepsilon_{t'}$, which, by the same argument, is satisfied when assumption A2 holds, completing the proof.

Proof of Proposition 2

Taking logs of expression (7), one obtains

$$
\log P(q,\chi,b) = \begin{cases} -\log R & \text{if } z(\chi,b) \in [\underline{b} - \Gamma^I, \underline{b}) \\ \log \tilde{P}(q) & \text{if } z(\chi,b) \in [\underline{b}, \bar{b} - \Gamma^I] \\ \log 0 & \text{if } z(\chi,b) \in (\bar{b} - \Gamma^I, \bar{b}] \end{cases},
$$

where combining (10) and expression (11) to substitute for W, we obtain

$$
\tilde{P}(q) = \frac{1-q}{R} \left[\frac{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta}{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta - (1-2q) \delta\omega\beta} \right],
$$

where $\alpha = \Gamma^I / (\bar{b} - \underline{b})$. Recall that Assumption A3 ensures that $\tilde{P}(q) \in (0, 1/R)$ for any $q \in [\underline{q}, \bar{q}]$. Taking logs of this expression and of equation (12) we can define

$$
\begin{aligned}
h(q) &\equiv \log P^B(q) = \log(1-q) - \log R, \\
g(q) &\equiv \log \tilde{P}(q) = \log(1-q) - \log R - \log \left[\frac{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta}{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta - (1-2q) \delta\omega\beta} \right].
\end{aligned}
$$

Next, differentiate the last two expressions with respect to q and obtain $h'(q) = -1/(1-q)$ and $g'(q) = -1/(1-q) - 2\beta\delta\omega/[1 - \beta\delta\omega(1 - 2q + \alpha + (1-\alpha) E(q))]$. Assumptions A1 and A3 guarantee that $|g'(q)| > |h'(q)|$. Define $\bar{h} \equiv \int h(q) dF(q)$, $\bar{g} \equiv \int g(q) dF(q)$, and q_0 such that $g(q_0) = \int g(q) dF(q)$. Then

$$
Var(g(q)) = \int (g(q) - \bar{g})^2 dF(q) = \int (g(q) - g(q_0))^2 dF(q) > \int (h(q) - h(q_0))^2 dF(q),
$$

where the last inequality follows from $|g'(q)| > |h'(q)|$ and the monotonicity of both h and g. Moreover, from a standard property of the second moment, we can write

$$
\int (h(q) - h(q_0))^2 dF(q) = \int (h(q) - \bar{h})^2 dF(q) + \int (\bar{h} - h(q_0))^2 dF(q).
$$

Combining the last two expressions we then obtain

$$
Var(g(q)) > \int (h(q) - \bar{h})^2 dF(q) + \int (\bar{h} - h(q_0))^2 dF(q) \geq Var(h(q)).
$$

This implies that $Var(\log P(q,\chi,b)) > Var(\log P^B(q))$ whenever $z(\chi,b) \in [\underline{b}, \bar{b} - \Gamma^I]$. For any other $z(\chi,b) \notin [\underline{b}, \bar{b} - \Gamma^I]$, $\log P(q,\chi,b) = \log P^B(q)$, completing the proof that $Var(\log P(q,\chi,b)) \geq Var(\log P^B(q))$.

Proof of Proposition 3

From the proof of Proposition 2, it is straightforward that the amplification effect is going to be stronger, the higher is the absolute value of $d \log \tilde{P}(q)/dq$, or $|g'(q)|$ in the notation of the proof. Such an object is larger, the larger is $y(q; \kappa)$, where

$$y(q; \kappa) \equiv 2\beta\delta\omega \left[1 - \beta\delta\omega \left(1 - 2q + \frac{\Gamma^I(\kappa)}{(\bar{b} - \underline{b})}(1 - E(q)) + E(q) \right) \right]^{-1},$$

where with some slight abuse of notation I define by $\Gamma^I(\kappa)$ the equilibrium measure of informed employed managers as a function of the parameter κ. Recall that Assumption A3 is sufficient to ensure that $y(q; x) > 0$ for all $q \in [\underline{q}, \bar{q}]$. We can then differentiate this expression and obtain that for any given $q \in [\underline{q}, \bar{q}]$ we have $dy(q; \kappa)/d\kappa = (dy/d\Gamma^I)(d\Gamma^I/d\kappa)$, where

$$\frac{dy}{d\Gamma^I} = 2(\beta\delta\omega)^2 \left(\frac{1 - E(q)}{\bar{b} - \underline{b}} \right) \left[1 - \beta\delta\omega \left(1 - 2q + \frac{\Gamma^I(\kappa)}{(\bar{b} - \underline{b})}(1 - E(q)) + E(q) \right) \right]^{-2} > 0.$$

Next, differentiating condition (27) we obtain

$$\frac{d\Gamma^I}{d\mu} = \frac{(1 - \delta)^2 M^I}{[1 - \delta(1 - \mu)]^2} > 0.$$

Moreover, we can find $\mu'(\kappa)$ applying the implicit function theorem to (30), where, with some slight abuse of notation, we can write

$$g(\mu; \kappa) \equiv \frac{\kappa}{\mu} - \gamma R \left\{ 1 - \delta\omega\beta \left[E(q) + \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)} \right] \right\}^{-1} = 0.$$

We can then derive

$$g_\kappa = \frac{1}{\mu},$$

$$g_\mu = -\frac{1}{\mu^2} \left[\kappa + \gamma R\omega\beta \frac{1-\delta}{\delta} \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)^2} \left\{ 1 - \delta\omega\beta \left[E(q) + \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)} \right] \right\}^{-2} \right],$$

which gives $\mu'(\kappa) = -g_\kappa/g_\mu > 0$, so that $d\Gamma^I/d\kappa > 0$. This implies $dy(q; \kappa)/d\kappa > 0$, completing the proof.

Proof of Proposition 5

Most of the arguments used in the existence proof for q iid case go through for q persistent. Given that we assumed that there exist three functions $W\left(q,\Gamma^I\right)$, $\mu\left(q,\Gamma^I\right)$, and $G\left(q,\Gamma_I\right)$ satisfying equations (19), (20), and (22), we only need to prove that the investors' firing strategy is optimal. That is, we show that assumption A2' is sufficient to ensure that the belief that an employed manager is informed if he did not reveal to be uninformed is always higher than the probability that a newly hired manager is informed.

More precisely, we now show that under A2', the posterior probability that manager i is informed if $\theta_{i,t} = 1 - \chi_t$ is larger than the probability that an unemployed manager at time t is informed, ε_t. The proof follows closely the one for the iid case. First, consider an investor who has just hired manager i so that his prior belief $\eta_{i,t} = \varepsilon_t$. In this case, if $\theta_{i,t} = 1 - \chi_t$, then $\eta_{i,t+1} = \varepsilon_{t-1} / \left[\varepsilon_{t-1} + \left(1 - \xi_t^U\right)\left(1 - \varepsilon_{t-1}\right)\right]$. Next, we want to show that $\eta_{i,t+1} \geq \varepsilon_t$. This condition can be rewritten as (32) and, substituting for ε_t using expression (31), we obtain

$$1 - \xi_t^U \leq \frac{N_t^U / \left(M^I - \delta\Gamma_t^I\right)}{N_{t-1}^U / \left(M^I - \delta\Gamma_{t-1}^I\right)}, \tag{33}$$

where $\Gamma - \Gamma_{t+1}^I = \delta\left(1 - \xi_t\right)\left(\Gamma - \Gamma_t^I\right) + \mu_t N_t^U$. Hence, we can rewrite condition (33) as follows:

$$1 - \xi_t^U \leq \frac{\Gamma - \Gamma_{t+1}^I - \delta\left(1 - \xi_t\right)\left(\Gamma - \Gamma_t^I\right)}{\Gamma - \Gamma_t^I - \delta\left(1 - \xi_{t-1}\right)\left(\Gamma - \Gamma_{t-1}^I\right)} \frac{\mu_{t-1}\left(M^I - \delta\Gamma_{t-1}^I\right)}{\mu_t\left(M^I - \delta\Gamma_t^I\right)}.$$

Given that $\xi_t^U \in [1 - \omega, 1]$ and $\mu_t = \kappa / W\left(q_t, \Gamma_t^I\right)$, a sufficient condition is then

$$\omega \leq \left[\frac{\Gamma - \bar{\Gamma}}{\Gamma - \underline{\Gamma}} - \delta\omega\right] \frac{M^I - \delta\bar{\Gamma}}{M^I - \delta\underline{\Gamma}} \frac{\underline{W}}{\overline{W}}, \tag{34}$$

where $[\underline{\Gamma}, \bar{\Gamma}]$ is the ergodic set for Γ^I, $\underline{W} \equiv W\left(q^L, \underline{\Gamma}\right)$ and $\overline{W} \equiv W\left(q^H, \bar{\Gamma}\right)$.[20] From expression (23), it is straightforward that $W\left(q_t, \Gamma_t^I\right) \in [\gamma R, \gamma R / \left(1 - \delta\omega\beta\right)]$ and hence a stricter condition is

$$\omega \leq \left[\frac{\Gamma - M^I}{\Gamma} - \delta\omega\right]\left(1 - \delta\right)\left(1 - \delta\omega\beta\right),$$

which ensures that A2' is sufficient for condition (33) to be satisfied. A similar argument to the iid case applies when managers have been employed for more than 1 period, completing the proof.

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[20]We use the weaker condition (34) instead of A2' when we pick parameters for our numerical example.

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