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Estimating Incentive and Welfare Effects of Non-Stationary Unemployment Benefits

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The distribution of unemployment duration in our equilibrium matching model with spell-dependent unemployment benefits displays a time-varying exit rate. Building on Semi-Markov processes, we translate these exit rates into an expression for the aggregate unemployment rate. Structural estimation using a German micro-data set (SOEP) allows us to discuss the effects of a recent unemployment benefit reform (Hartz IV). The reform reduced unemployment by only 0.3%. Contrary to general beliefs, we find that both employed and unemployed workers gain (the latter from an intertemporal perspective). The reason is the rise in the net wage caused by more vacancies per unemployed worker.

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1 Introduction

Continental European unemployment is notorious for its persistence. France, Italy and Germany have had rising unemployment rates from the 1960s up to 2000 and even onward. There seems to be a consensus now that a combination of shocks and institutional arrangements lies at the origin of these high unemployment rates (Ljungqvist and Sargent, 1998, 2007a, b; Mortensen and Pissarides, 1999; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone explain the rise in unemployment: institutions have always been there but unemployment has not (at least not at this level) and shocks have hit many countries but not all countries have high unemployment rates. The step from this shock-institutions insight

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towards finding a solution to the European unemployment problem seems to be short: As shocks will not go, we need to address the institutions.[2]

A common suggestion to fight unemployment is to reduce long and generous unemployment benefits. This raises other questions, however: Should one reduce the length and level of unemployment benefits in order to reduce unemployment? One seems to be faced by a classic efficiency-equity trade-off. While reducing unemployment per se is beneficial, income of the unemployed and the insurance mechanism provided by unemployment benefits should not be neglected.

We examine the employment and welfare effects of a policy reform which reduces the length and level of unemployment benefits. We use Germany as an example of a continental European country for three reasons. First, the unemployment rate in Germany has been rising for many decades, just as e.g. in France or Italy. Second, the German unemployment benefit system has a two-tier structure which is typical of many OECD countries. Third, the so-called "Hartz IV reform" implemented in January 2005 comprises both the reduction of benefit levels and the cut of the duration of entitlement. Reforms of this type were undertaken in many other OECD countries as well (OECD, 2004).

Neglecting minor institutional details, the reform had two main effects. The maximum entitlement to unemployment insurance (UI) payments was (almost) uniformly reduced to 12 months (from a former maximum of 32 months). Unemployment assistance (UA) payments, formerly proportional to net earnings before the job loss, were replaced by a uniform benefit level. The effect of this new rule on UA payments on long-term unemployed workers was ambiguous. There are unemployed workers, mainly from low wage groups, whose benefit payments were lower before the reform than afterwards. Those were the "winners" of the reform (47% of long-term unemployed) - in a static sense. On the other hand, there were also long-term unemployed workers with relatively high wages before entering unemployment. These were affected negatively and their income has dropped (53% of long-term unemployed). Even though the fraction of "winners" and "losers" is roughly equal, aggregating gains and losses shows a loss of the average long-term unemployed worker of around 7% (Blos and Rudolph, 2005; OECD, 2007).

At first sight, the reform seems to have worked. The reported unemployment rate dropped from an annual average of 10.5% in 2004 (Bundesagentur für Arbeit, 2009) to 9.0% in 2007. On the other hand, growth rates in Germany were (for German standards) fairly high. While the German economy shrank in 2003, it has recovered since then and probably also created new jobs. The real GDP grew by 0.8% in 2005, by 3.0% in 2006 and by 2.5% in 2007 (Bundesagentur für Arbeit, 2009). The question therefore arises whether the drop in unemployment can be credited to the reform. It is also a priori unclear how strongly various groups were affected by the reform. Did utility of the (short- and long-term) unemployed or employed workers rise or fall? Did firms gain from the reform? What about social welfare?

We provide answers by using a model which combines various strands of the literature and adds some new and essential features. We employ an equilibrium matching framework and extend the standard textbook model for time-dependent unemployment benefits, endogenous

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[2] This conclusion is even stronger for papers which argue that changes in European unemployment can mainly be attributed to shifts in labour market institutions and to a lesser extent to the interaction of institutions with shocks (Nickell et al., 2005).

effort, risk-averse households, an exogenous "spell-effect" and Semi-Markov features. Each of these extensions is crucial. Unemployment benefits in our model need to depend on the length of the unemployment spell as this is a feature of basically all OECD unemployment benefit systems. Letting agents optimally choose their effort to find a job, we can analyze the incentive effects of (reforms of) the unemployment benefit system. Risk-averse households are required as we also want to evaluate insurance effects. The spell-effect captures the time-dependence of individual exit rates beyond the incentive effects induced by the two-tier system.[3] Finally, tools from the Semi-Markov literature are required as they allow us to deduce aggregate (un)employment from individual search behaviour. We can thereby compute macro efficiency effects resulting from micro incentives. Without these Semi-Markov tools, we would not be able to formulate an equilibrium model.

We solve this model numerically by looking at Bellman equations as differential equations. This gives us solutions which are as accurate as numerical precision and which do not require us to approximate the model in any way. Optimal behaviour implies an exit rate out of unemployment which is a function of the time spent in unemployment. We thereby obtain a sufficiently flexible endogenous distribution of unemployment duration which we employ for structural estimation by maximum likelihood.

The main theoretical contribution of our analysis is the explicit treatment of the Semi-Markov nature of optimal individual behaviour due to the presence of spell-dependent unemployment benefits: Optimal exit rates not only depend on whether the individual is unemployed (the current state of the worker) but also on how long an individual has been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from the applied mathematics literature, we obtain analytic expressions for individual employment probabilities contingent on current employment status and duration of unemployment - equations of the so-called Volterra type. They allow us to compute aggregate unemployment rates using a law of large numbers in our pure idiosyncratic risk economy. Given this link from optimal individual behaviour to aggregate outcomes, we can analyze the distribution and efficiency effects of changes in level and length of unemployment benefits.

The main empirical contribution is the careful structural modelling of exit rates out of unemployment. Falling unemployment benefits imply an increase of search effort and therefore also of *individual* exit rates over time. Empirical evidence shows, however, that *aggregate* exit rates tend to fall. We therefore combine individual incentive effects with an exogenous time-decreasing spell-effect and with unobserved heterogeneity. As is well known, the latter implies inter alia falling aggregate exit rates even though individual exit rates are rising. We find that along with significant upward pressure of the increasing search effort both exogenous spell-effect and unobserved heterogeneity have significant influence on the dynamics of the exit rates. Conditional on observed and unobserved characteristics, the exogenous spell-effect catches up with the effort effect of the benefit system in the first months of unemployment duration and overturns it in the later course of spell progress.

With a policy focus in mind, we emphasize and estimate the trade-off between insurance

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[3] The spell-effect $\eta(s)$ will be introduced and specified in detail below in (4), (21) and (25). Put simply, it is a function of the duration s of unemployment of an unemployed worker. The parameters of this function will be estimated.

and incentive effects of labour market policies. The degree of risk-aversion - crucial for understanding the insurance effect - is jointly estimated with exit rates and the spell-effect (and other model parameters). A comparative static analysis, using estimated parameters for the theoretical model, then allows us to derive precise predictions about the employment and distribution effects of changes in the length and level of unemployment benefits.

Providing a short preview of our results, we find that the reform did decrease the unemployment rate - which is the desirable effect - but only by 0.3%. Very much to our surprise, we also find that the reform increased wages and decreased profits. Usually, one would expect that a reduction of alternative income reduces wages. While this channel is present in our setup as well, the equilibrium effect of a rise in the number of vacancies per unemployed worker overcompensates the first effect - wages rise when benefits fall. The number of vacancies per unemployed worker rise as lower benefits induce the unemployed to search harder which in turn induces firms to open more vacancies per unemployed worker. As wages rise, profits fall.

This rise in wages is also the reason why the reform is social welfare increasing. The uninterested worker experiences a rise in expected utility. This is *not* due to a better insurance mechanism cause by the reform. In fact, the insurance mechanism is worsened when unemployment benefits are reduced. The rise in expected utility is entirely due to the equilibrium increase in the wage. The most surprising finding of our analysis is that even the long-term unemployed workers gain in an intertemporal perspective. Of course, this group loses instantaneously due to the drop of UA payments. But given the rise in the wage and the probability that they will find a job at some later point, they gain. Given our setup, the Hartz IV reform is Pareto-improving.

Our paper is related to various strands in the literature. From a theoretical perspective, we build on the search and matching framework of Mortensen (1982) and Pissarides (1985). Time-dependent unemployment benefits and endogenous effort have been originally analyzed by Mortensen (1977) in a one-sided job search model. Equilibrium search and matching models with time-dependent unemployment benefits include Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001).[4] In these models, exit rates are constant within each benefit regime. This does not fully capture continuously decreasing exit rates as observed in the data. There also exists a substantial literature that studies optimal insurance allowing for an arbitrary time path of unemployment benefit payments (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997, 2009; Shimer and Werning, 2007). Our focus is more of a positive nature trying to understand the welfare effects of existing systems which have a simpler benefit structure than the ones resulting from an optimization approach. We also allow for an unlimited number of transitions between employment and unemployment and take equilibrium effects of wages, vacancies and tax rates into account.[5]

From an empirical perspective, we estimate a structural parametric duration model with

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[4] Albrecht and Vroman (2005) and Coles and Masters (2007) also have time-dependent unemployment payments but they do not analyze the implications for individual effort. Albrecht and Vroman focus on the equilibrium wage dispersion and inefficient job rejection. Coles and Masters model aggregate uncertainty implying implicit transfers between firms and the stabilizing effect this has on the unemployment rate over the cycle.

[5] Acemoglu and Shimer (1999) and Moscarini (2005) use a general equilibrium model, but their setting is restricted to time-invariant benefits only.

a flexible distribution of unobserved heterogeneity. Time dependence of the hazard function due to time-dependent benefits is fully described by the equilibrium solution of our theoretical model. Structural econometric models with time-dependent benefits were originally estimated by van den Berg (1990) and Ferrall (1997).[6] Van den Berg and van der Klaauw (2010) extend the setting by introducing time dependence due to monitoring and sanctions. In contrast to our model, this literature deals with one-sided job search, which makes application of its estimates in an equilibrium analysis rather difficult. In addition to that, focus on the incentive effect is only partial (van den Berg and van der Klaauw, 2010) and insurance effect remains largely unaddressed. There also exists a broader empirical equilibrium search literature that deals with unemployment benefit heterogeneity (Bontemps et al., 1999), heterogeneity in workers abilities (Postel-Vinay and Robin, 2002) and heterogeneity in workers value of nonparticipation (Flinn, 2006). Unlike in our model, however, neither of these contributions takes time-dependent unemployment benefits or equilibrium wage, vacancy and tax effects into account.[7]

Semi-Markov methods are taken from the applied mathematical literature, see e.g. Kulkarni (1995) or Corradi et al. (2004). Economic papers which allowed for Semi-Markov features (e.g. Burdett et al., 1985, Aase, 1990, Magnac et al., 1995) focused on time-varying exit rates but did not exploit their full potential, i.e. they did not use Volterra equations which we need here for equilibrium considerations.

Finally, there is a very small academic literature which discusses the Hartz reforms. Heer (2006) provides a "tentative analysis" which does not explicitly look at the effects of a two-tier system. Fahr and Sunde (2009) focus on aspects of the Hartz reforms (Hartz I-III) which do not affect unemployment benefits. Franz et al. (2007) study the effects of Hartz IV in a CGE model focusing on the impact on various household types.

The structure of our paper is as follows. Section 2 presents the theoretical model, institutional setting, behaviour of supply and demand sides and the combination of both in economic welfare. Section 3 describes the equilibrium properties of the model. Section 4 illustrates the structural estimation and the underlying data. The simulation results and the evaluation of the institutional reform are presented in section 5. Section 6 concludes.

2 The model

We use a Mortensen-Pissarides type matching model and extend it for time-dependent unemployment benefits, endogenous effort, risk-averse households and an exogenous spell-effect. To solve it, we use Semi-Markov tools. The separation rate for jobs is constant and there is no search on the job. We focus on steady states in our analysis. Households are ex-ante identical but endogenously heterogenous in their unemployment duration.

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[6] Frijters and van der Klaauw (2006) develop a nonstationary search model with nonparticipation. Paserman (2008) provides a structural estimation taking hyperbolic discounting into account. See also Eckstein and van den Berg (2007) for literature review on nonstationary empirical search models.

[7] A general equilibrium model (of economic growth) is estimated by Lentz and Mortensen (2008).

2.1 Production, benefits and employment

The economy has a work force of exogenous constant size N. Employment is endogenous and given by L and the number of unemployed amounts to $N-L$. Firms produce under perfect competition on the goods market and each worker-firm match produces output A, which is constant. The match can be interrupted by exogenous causes which occur according to a time-homogenous Poisson process with a constant arrival rate λ.

Unemployed workers receive UI benefits b_{UI} and UA benefits b_{UA}. In basically all OECD countries, UI benefits are paid for a certain number of months, after which UA benefits are paid. We denote entitlement length to UI benefits by $\bar{s}$ and assume that it is identical for all individuals (as e.g. in Coles and Masters, 2006).[8] Hence, unemployment payments $b(s)$ are given by

$$b(s)=\begin{cases} b_{UI} & 0\leq s\leq \bar{s} \\ b_{UA} & \bar{s}<s \end{cases}. \tag{1}$$

We assume $b_{UI}>b_{UA}\geq 0$. Reflecting the institutional setup in most OECD countries and in Germany best, we consider b_{UA} and b_{UI} to be proportional to the net wage w earned at the moment the worker loses the job. With e.g. ξ_{UI} denoting the UI replacement rate, we obtain

$$b_{UI}=\xi_{UI}w. \tag{2}$$

This replacement rate will play a role in the wage setting equation and in the numerical implementation of the reform.

An unemployed worker finds a job according to a time-inhomogeneous Poisson process with arrival rate $\mu(.)$. This rate will also be called the job-finding rate, hazard rate or exit rate out of unemployment. We allow this rate to depend on effort $\phi(s)$ an individual exerts to find a job. Effort depends on the length s this individual has been spending in unemployment since her last job. If $s>\bar{s}$, the individual will be called long-term unemployed.

In addition to effort, the exit rate of an individual will also depend on aggregate labour market conditions and on something which we call a spell-effect. Labour market conditions are captured by labour market tightness θ, i.e. the ratio of the number of vacancies V divided by the number of unemployed,

$$\theta\equiv V/(N-L). \tag{3}$$

We assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies imply that any effort is in vain. The spell-effect captures all factors exogenous to the individual which affects her exit rate out of unemployment. This can include stigma (Vishvanath, 1989), ranking (Blanchard and Diamond, 1994) and gains or losses in individual search productivity. We denote this effect by $\eta(s)$. Assuming that a stigma becomes worse the longer s, we would expect $\eta(s)$ to fall in s. Summarizing, the exit rate will be of the form $\mu(\phi(s)\theta,\eta(s))$.[9]

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[8] Put differently, we do not let $\bar{s}$ be a function of past employment history. See the discussion after (13) for an extension.

[9] Given our focus on individual search behaviour, we start at the individual level and then derive a matching function (see the discussion following (22)) rather than the other way round. Both ways are of course equivalent.

There is a long discussion in the literature whether duration dependence taking the form of aggregate falling exit rates is due to a spell-effect (as modeled here by $\eta(s)$) or due to unobserved heterogeneity (Kiefer and Neumann, 1981, Flinn and Heckman, 1982, Heckman and Singer, 1984b and van den Berg and van Ours, 1996). We take unobserved heterogeneity into account in our empirical part and discuss its effects there.

The outcome of our time-varying exit rate will be an endogenous distribution of unemployment duration. Its density is given by (e.g. Ross, 1996, ch. 2)

$$f(s) = \mu(\phi(s)\theta, \eta(s))\, e^{-\int_0^s \mu(\phi(u)\theta, \eta(u))du}. \tag{4}$$

This density will be crucial later for various purposes including the estimation of model parameters. It is endogenous to the model, as the exit rate $\mu(\phi(s)\theta, \eta(s))$ is determined by the optimizing behaviour of workers and firms.

Unemployment benefit payments to short- and long-term unemployed are financed by a tax rate κ on gross wages. The labour tax κ implies that the net wage is $w = (1-\kappa)\, w^{gross}$. The number of short-term unemployed workers is $\int_0^{\bar{s}} f(s)\, ds\,(N-L)$ and $\int_{\bar{s}}^{\infty} f(s)\, ds\,(N-L)$ is the number of the long-term unemployed. The budget constraint of the government therefore reads

$$\left(b_{UI}\int_0^{\bar{s}} f(s)\, ds + b_{UA}\int_{\bar{s}}^{\infty} f(s)\, ds\right)(N-L) = \kappa\frac{w}{1-\kappa}L. \tag{5}$$

The government adjusts the wage tax κ such that this constraint holds at each point in time.

2.2 Optimal behaviour

Households are infinitely lived and do not save. They have a strictly positive time preference rate ρ. The present value of having a job is given by $V(w)$ and depends on the current endogenous wage w only. Employed workers enjoy instantaneous utility $u(w)$. The value $V(w)$ is constant in a steady state as the wage is constant, but differs across steady states. Whenever a worker loses her job, she enters the unemployment benefit system by obtaining insurance payments b_{UI} for the length of $\bar{s}$. Hence, the value of being unemployed when just having lost the job is given by $V(b_{UI}, 0)$ where 0 stands for a spell of length zero. This leads to a Bellman equation for the employed worker of

$$\rho V(w) = u(w) + \lambda\left[V(b_{UI}, 0) - V(w)\right]. \tag{6}$$

The Bellman equation for the unemployed worker reads

$$\rho V(b(s), s) = \max_{\phi(s)}\left\{u(b(s), \phi(s)) + \frac{dV(b(s), s)}{ds} + \mu(\phi(s)\theta, \eta(s))\left[V(w) - V(b(s), s)\right]\right\}. \tag{7}$$

The instantaneous utility flow of being unemployed, $\rho V(b(s), s)$, is given by three components. The first component shows the instantaneous utility resulting from consumption of $b(s)$ and effort $\phi(s)$. The second component is a deterministic change of $V(b(s), s)$ as the value of being unemployed changes over time. The third component is a stochastic change that occurs at the job-finding rate $\mu(\phi(s)\theta, \eta(s))$. When a job is found, an unemployed worker gains the difference between the value of being employed $V(w)$ and $V(b(s), s)$.

An optimal choice of effort $\phi(s)$ for (7) requires

$$u_{\phi(s)}\left(b(s),\phi(s)\right)+\mu_{\phi(s)}\left(\phi(s)\theta,\eta(s)\right)\left[V(w)-V(b(s),s)\right]=0, \tag{8}$$

where subscripts denote partial derivatives. It states that the utility loss resulting from increasing search effort must be equal to expected utility gain due to higher effort.

As unemployment benefits are discontinuous at $\bar{s}$, the question arises what happens to the value of being unemployed at this point. Value functions measure overall utility from optimal behaviour between now and the end of the planning horizon. As the value of being unemployed depends on unemployment benefits and unemployment duration only, it seems natural to assume that the value an instant before $\bar{s}$ and after $\bar{s}$ are identical. In fact, we assume that

$$V\left(b_{UI},\bar{s}\right)=V\left(b_{UA},\bar{s}\right). \tag{9}$$

The value of a job J to a firm is given by instantaneous profits $A-w/(1-\kappa)$, which is the difference between revenue A and the gross wage $w/(1-\kappa)$, reduced by the risk of being driven out of business

$$\rho J=A-w/(1-\kappa)-\lambda\left[J-J_0\right], \tag{10}$$

where $\rho>0$ stands for the interest rate (being identical to the discount rate of households) and where J_0 is the value of a vacancy.

Given that individual arrival rates are functions of the individual unemployment spell, the expected rate of exit out of unemployment is just the mean over individual arrival rates, given the endogenous distribution of the unemployment spell $f(s)$ from (4),

$$\bar{\mu}=\int_0^\infty \mu\left(\phi(s)\theta,\eta(s)\right)f(s)\,ds. \tag{11}$$

As a consequence, the vacancy filling rate is $\theta^{-1}\bar{\mu}$. The value of a vacant job is $\rho J_0=-\gamma+\theta^{-1}\bar{\mu}\left[J-J_0\right]$. With free entry, the value of holding a vacancy is $J_0=0$, leading to

$$J=\gamma\theta/\bar{\mu}. \tag{12}$$

Modelling wage setting for any country is a big challenge. Looking at Germany, almost two thirds of all wages and salaries are the outcome of negotiations between industry unions and employer federations.[10] Labour income not covered by central negotiations is determined either by individual bargaining, by wage posting or other. As we do not want to model heterogeneity in wage setting in this paper, we assume that all wages are the outcome of collective bargaining. The question then arises what the objective of unions and employer federations are. The main issue thereby is to what extend the interest of unemployed workers are taken into account. As almost all members of unions are employed, we assume here that wages are determined by insiders, i.e. those who currently have a job. Due to its analytical convenience, we also assume that wages are determined by Nash bargaining. We discuss alternatives in a moment.

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[10] This is in contrast to collective bargaining at the firm level as modelled e.g. by Cahuc and Lehmann (2000). Mortensen and Pissarides (1999) consider the case of a monopoly union which sets the share β of the surplus going to the worker. The firm responds by creating and destroying jobs.

In case of successful negotiations, the collective value of employed workers is $V\left(w\left(t\right)\right)L\left(t\right)$. If bargaining fails, workers receive unemployment benefits which - given institutional rules - depend on previous employment history and age. If we make entitlement length $\bar{s}$ from (1) a function of e.g. the employment history, we would obtain a distribution of $\bar{s}$. While this would not create major problems, we value the advantages of a theoretical model with a representative agent (estimation does take heterogeneity in $\bar{s}$ into account). We therefore give the same entitlement $\bar{s}$ to all individuals, independently of their employment history. Hence, if bargaining fails, the collective value of $L\left(t\right)$ workers is $V\left(b\left(0\right),0\right)L\left(t\right)$. The collective contribution of firms to the Nash product is simply $J\left(\frac{w(t)}{1-\kappa}\right)L\left(t\right)-J_0L\left(t\right)$. The generalized Nash product can therefore be written as $\left(V\left(w\left(t\right)\right)-V\left(b\left(0\right),0\right)\right)^{\beta}\left(J\left(\frac{w(t)}{1-\kappa}\right)-J_0\right)^{1-\beta}L\left(t\right)$.

Following the steps as in Pissarides (1985) for risk-neutral or Lehmann and van der Linden (2007) for risk-averse individuals, our setup for collective bargaining with a replacement rate (2) yields (see app. B.1.1)

$$\left(1-\beta\right)u\left(w\right)+\beta m_w\left(.\right)w=\left(1-\beta\right)u\left(b_{UI},\phi\left(0\right)\right)+\beta\left(1-\kappa\right)m_w\left(.\right)\left[A+\gamma\theta\frac{\mu\left(\phi\left(0\right),0\right)}{\bar{\mu}}\right], \tag{13}$$

where

$$m_w\left(w,b_{UI},\phi\left(0\right)\right)\equiv u_w\left(w\right)+\frac{\lambda}{\rho+\mu\left(\phi\left(0\right),0\right)}u_w\left(b_{UI},\phi\left(0\right)\right) \tag{14}$$

is generalized marginal utility from consumption. The first term $u_w\left(w\right)$ in (14) is marginal utility from consumption as an employed worker. The second term is the generalization due to the fact that b_{UI} is proportional to the previously earned wage: An increase in the bargained wage affects (the present value of expected) marginal utility from consumption if unemployed at a later point in time. If UI payments were not proportional to the previously earned wage, $m_w\left(.\right)$ would be given by $u_w\left(w\right)$. The left hand side of (13) corresponds to what in models with risk-neutrality is simply the wage rate w. On the right hand side, benefits for the unemployed (for risk-neutral households and no time-dependence of effort), are replaced by instantaneous utility from being unemployed, $u\left(b_{UI},\phi\left(0\right)\right)$. The contribution of the production side in squared brackets is translated into "utils" by multiplying with generalized marginal utility and takes tax effects into account.

As mentioned before, any real world economy exhibits a multitude of wage setting mechanisms. We are aware of the many alternatives to Nash bargaining and also to the structure of Nash bargaining used here. One alternative to its structure would consist in specifying an outside option where each individual worker would be entitled to UI payments according to past employment history. In the case of individual bargaining, an endogenous wage distribution would arise (see Albrecht and Vroman, 2005). With a distribution of employment history, there would be a distribution of outside options and therefore a distribution of wages. In our case of collective bargaining, however, we would still obtain a unique wage. With l denoting employment history, $\bar{s}\left(l\right)$ would denote the length of entitlement to UI payments.[11] One would then replace $V\left(b\left(0\right),0\right)$ by $\int_0^\infty V\left(b\left(0\right),0,\bar{s}\left(l\right)\right)g\left(l\right)dl$, where $g\left(l\right)$ is the distribution for employment duration. Clearly, $\int_0^\infty V\left(b\left(0\right),0,\bar{s}\left(l\right)\right)g\left(l\right)dl$ is a fixed quantity such that the wage would remain unique.

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[11] In an empirical implementation, age would be an additional argument for $\bar{s}\left(l\right)$.

An alternative to Nash bargaining itself consists in strategic bargaining. Strategic bargaining is the appropriate choice when payoffs change over time as Nash bargaining would correspond to myopic behaviour (Coles and Wright, 1998; Coles and Muthoo, 2003). Strategic bargaining was also used in the analysis of on-the-job search (Cahuc et al., 2006, Shimer, 2006) and in Hall and Milgrom (2008) who stress that employment fluctuations under Nash bargaining are too small. As our collective bargaining setup is the most appropriate assumption for Germany which implies that collective payoffs are stationary and as we do not focus on business cycle issues, we feel justified in using Nash bargaining here.

2.3 The social welfare function

In addition to the incentive effect of the reform, we would also like to understand the insurance effect. In a world without moral hazard, optimal unemployment insurance would require unemployment benefits to be equal to the net wage. With effort being a function of unemployment benefits, insurance considerations must take into account that effort decreases in unemployment benefits.

We can easily understand whether the insurance effect was taken into account in an appropriate way by computing expected utility of an individual being "behind the Rawlsian veil of ignorance". This is similar in spirit to social welfare functions employed by Hosios (1990) or Flinn (2006). One can alternatively look at this expected utility as average utility over all (employed and unemployed) workers. Expected utility EU is given by

$$EU \equiv \frac{L}{N} V(w) + \frac{N-L}{N} \left(\int_0^{\bar{s}} V(b_{UI}, s) f(s) \, ds + \int_{\bar{s}}^{\infty} V(b_{UA}, s) f(s) \, ds \right). \quad (15)$$

It adds the share L/N of employed workers times their welfare $V(w)$ to the share $(N-L)/N$ of unemployed workers times the average welfare of an unemployed. This average is obtained by integrating over all spells s, where $f(s)$ is the endogenous density (4), with exit rates $\mu(\phi(s)\theta, \eta(s))$ that follow from the steady state solution of the model, and the $V(b(s), s)$ are the values of being unemployed with a spell s and benefit payments $b(s)$ from (1).[12]

3 Equilibrium properties

3.1 Individual (un)employment probabilities

In models with constant job-finding and separation rates, the number (or measure) of employees can easily be derived by assuming a law of large numbers. Aggregate employment then follows $\dot{L} = \mu [N - L] - \lambda L$. With spell-dependent effort, individual arrival rates $\mu(.)$ are heterogeneous and the number of employees needs to be derived using techniques from the literature on Semi-Markov or renewal processes, e.g. Kulkarni (1995) or Corradi et al. (2004).

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[12] Our social welfare function does not take the value J of a firm into account. The resource cost γ for financing a vacancy are also neglected. We could transfer the balance $\pi L - \gamma M$ to households. As this balance rises in the reform in all of our quantitative analyses, our quantitative results below would not change if the balance was taken into account.

We need the number of employees in order to compute the unemployment rate and for computing income and expenditure in the government budget constraint. These Semi-Markov tools are therefore essential for any equilibrium model with time-dependent unemployment benefits.

The generalization of Semi-Markov processes compared to continuous time Markov chains consists in allowing the transition rate from one state to another to depend on the time an individual has spent in the current state. We apply this here and let the transition rate from unemployment to employment depend on the time s the individual has been unemployed. Hence, switching from a constant job-finding rate μ to a spell-dependent rate $\mu(s)$ implies switching from Markov to Semi-Markov processes. Processes are called "semi" as the history-dependence of the job finding rate $\mu(s)$ is not Markov. Processes are still called "Markov" as history no longer counts once an individual has found a job. This is also why these processes are related to renewal processes: whenever a transition to a new state occurs, the system starts from the scratch, it is "renewed" and history vanishes.

We start by looking at individual employment probabilities. Let $p_{ij}(\tau, s(t))$ describe the probability with which an individual, who is in state i (either e for employed or u for unemployed) today in t, will be in state $j \in \{e, u\}$ at some future point in time τ, given that her current spell is now $s(t)$. Starting with an individual that just lost her job, i.e. $s(t) = 0$, and taking into account that the separation rate λ remains constant, these expressions read (see app. A.5),

$$p_{uu}(\tau, 0) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v))\, p_{eu}(\tau - v)\, dv, \tag{16a}$$

$$p_{eu}(\tau) = \int_t^\tau e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0)\, dv. \tag{16b}$$

Expressions for complementary transitions are given by $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ and $p_{ee}(\tau) = 1 - p_{eu}(\tau)$, respectively.

These equations have a straightforward intuitive meaning. Consider first the case of τ being not very far in the future. Then all integrals (for $\tau = t$) are zero and the probability of being unemployed at τ is, if unemployed at t, one from (16a) and, if employed at t, zero from (16b). For a $\tau > t$, the part $e^{-\int_t^\tau \mu(s(y))dy}$ in (16a) gives the probability of remaining in unemployment for the entire period from t to τ. An individual unemployed today can also be unemployed in the future if she remains unemployed from t to v (the probability of which is $e^{-\int_t^v \mu(s(y))dy}$), finds the job in v (which requires multiplication with the exit rate $\mu(s(v))$) and then moves from employment to unemployment again over the remaining interval $\tau - v$ (for which the probability is $p_{eu}(\tau - v)$). Note that the probability $p_{eu}(\tau - v)$ allows for an arbitrary number of transitions in and out of employment between v and τ (see fig. 8 in app. A.5 for an illustration). As this path is possible for any v between t and τ, the densities for these paths are integrated. The sum of the probability of remaining unemployed all of the time and of finding a job at some v but being unemployed again at τ gives then the overall probability $p_{uu}(\tau, 0)$ of having no job in τ when having no job in t. The interpretation of (16b) is similar. The probability of remaining employed from t to v is simpler, $e^{-\lambda[v-t]}$, as the separation rate λ is constant. The individual then loses the job at v requiring the transition rate λ and then moves back and forth between unemployment and employment to eventually end up in unemployment at τ or earlier. The latter is captured by $p_{uu}(\tau - v, 0)$.

As we can see, these equations are interdependent: The equation for $p_{uu}(\tau, 0)$ depends on $p_{eu}(\tau - v)$ and the equation for $p_{eu}(\tau)$, in turn, depends on $p_{uu}(\tau - v, 0)$. Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first type (16b) and of the second type (16a). Integral equations can sometimes be transformed into differential equations, which will simplify their solution in practice. In our case, however, no transformation into differential equations is known.

After having computed the probability of being unemployed in τ when being unemployed in t for individuals that just became unemployed in t, i.e. who have a spell of length $s(t) = 0$, we will need an expression for $p_{uu}(\tau, s(t))$. This means, we will need the transition probabilities for individuals with an arbitrary spell $s(t)$ of unemployment. Luckily, given the results from (16a and b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \, p_{eu}(\tau - v) \, dv. \tag{17}$$

An unemployed with spell $s(t)$ in t has different exit rates $\mu(s(y))$ which, however, are known from our analysis of optimal behaviour at the individual level. Hence, only the integrals in (17) are different, the probabilities $p_{eu}(\tau - v)$ can be taken from the solution of (16a and b).

The notation $p_{uu}(\tau, s(t))$ in (17) and $p_{eu}(\tau)$ in (16b) nicely reflects the Semi-Markov nature of this setup: When employed in t, the probability $p_{eu}(\tau)$ of being unemployed in τ is *not* a function of the past and the only argument of $p_{eu}(\tau)$ is time τ. When unemployed in t, the probability $p_{uu}(\tau, s(t))$ of being unemployed in the future τ as well *is* a function of the past and this is captured by the argument $s(t)$.

3.2 Aggregate unemployment

We can now compute the expected number of unemployed for any cross-section distribution of spells $H(s(t))$,

$$E_t[N - L(\tau)] = [N - L(t)] \int_0^\infty p_{uu}(\tau, s(t)) \, dH(s(t)) \; + p_{eu}(\tau) L(t). \tag{18}$$

We start at the end of this equation, noting that there are $L(t)$ employed workers in t. The expected number of unemployed workers at some $\tau \geq t$ coming from currently employed workers is given by $p_{eu}(\tau) L(t)$. Again, one should keep in mind that the probability $p_{eu}(\tau)$ allows for an arbitrary number of switches between employment and unemployment between t and τ, i.e. it takes permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future by integrating over $p_{uu}(\tau, s(t))$ given the current distribution $H(s(t))$. Multiplying this by the number of unemployed today, $N - L(t)$, gives us the expected number of unemployed at τ out of the pool of unemployed in t. The sum of these two expected quantities gives the expected number of unemployed at some future point τ. Dividing by N gives the expected unemployment rate at τ.

When we focus on a steady state, we let τ approach infinity. In a steady state, the cross-section distribution $H(s(t))$ is identical to the distribution $F(s)$ whose density is given in (4). In order to obtain a simple expression for the aggregate unemployment rate, we exploit the pure idiosyncratic-risk structure where micro-uncertainty cancels out at the aggregate level. Hence, we assume that a law of large numbers holds and the population share of unemployed workers equals the average individual probability of being unemployed. This allows us to express (18) for a steady state as $(N-L)/N = [(N-L)/N] \int_0^\infty p_{uu}(s)\, dF(s) + p_{eu} L/N$. We have replaced $L(\tau) = L(t)$ by the steady state employment level L and the individual probabilities by the steady state expressions $p_{uu}(s)$ and p_{eu}. The probability p_{eu} is no longer a function of τ as this probability will not change in steady state, while there will always be a distribution of $p_{uu}(s)$, even in a steady state. Solving for the unemployment rates gives

$$\frac{N-L}{N} = \frac{p_{eu}}{p_{eu} + \left[1 - \int_0^\infty p_{uu}(s)\, dF(s)\right]} = \frac{p_{eu}}{p_{eu} + \int_0^\infty p_{ue}(s)\, dF(s)}, \tag{19}$$

where the second expression is more parsimonious.

If we assumed a constant job arrival rate here, we would get $p_{eu} = p_{uu} = \lambda/(\lambda+\mu)$ and $p_{ue} = \mu/(\lambda+\mu)$. Inserting this into our steady state results would yield the standard expression for the unemployment rate, $(N-L)/N = \lambda/(\lambda+\mu)$. In our generalized setup, the long-run unemployment rate is given by the ratio of individual probability p_{eu} to be unemployed when employed today divided by this same probability plus $\int_0^\infty p_{ue}(s)\, dF(s)$.

3.3 Functional forms

Estimation and a numerical solution require functional forms. We assume that the instantaneous utility function of an unemployed worker used e.g. in (7) is

$$u(b(s), \phi(s)) = \frac{b(s)^{1-\sigma} - 1}{1-\sigma} - \phi(s). \tag{20}$$

Effort is measured in utility terms. The utility function of an employed worker has the same structure only that consumption is given by w and there is no explicit effort. One could therefore look at ϕ as a measure of the difference between disutility from searching and disutility from work.

The spell-effect is captured by

$$\eta(s) = \eta_0 g(s), \tag{21}$$

where $g(s)$ is specified in the next section. The arrival rate of jobs $\mu(\phi(s)\theta, \eta(s))$ is assumed to obey

$$\mu(\phi(s)\theta, \eta(s)) = \eta(s)\left[\phi(s)\theta\right]^\alpha, \tag{22}$$

This specification can easily be made plausible when linking it to a matching function. The matching function represents the aggregate arrival rate and equals the sum over individual arrival rates, $m(N-L, V) = (N-L)\int \mu(s) f(s)\, ds$. Defining $\Omega \equiv \int \eta(s)\phi(s)^\alpha f(s)\, ds$ and using (3), we find $m(N-L, V) = \Omega [N-L]^{1-\alpha} V^\alpha$.[13] This shows that we succeed in identifying the elasticity α of vacancies as we assume that both effort and tightness have the same power α in (22).

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[13] Note that one could argue (see e.g. Cahuc and Lehman, 2000 or Fredriksson and Holmlund, 2001) that the individual arrival rate is a function of the ratio $V/(\Omega U)$ and not of the ratio $\theta = V/U$ as used here. The

4 Structural estimation

4.1 Exit rates out of unemployment

Before we estimate the model using data from the German Socio-Economic Panel (SOEP), we need to specify the functional forms for our spell-effect (21).[14] In order to do so, we consider the distributional aspects of our data on the observed unemployment duration. Fig. 1 shows the nonparametric estimates of the hazard functions from the entire sample of unemployment durations (solid line) and the subsample of individuals with the entitlement length equal to 12 months (dashed line).[15] The illustration focuses on the first 2.5 years of unemployment.

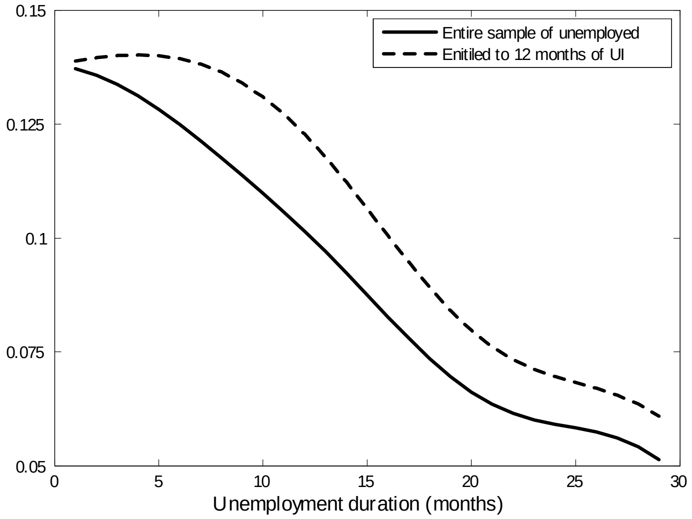

Figure 1 *Non-parametric hazard functions (entire sample and* $\bar{s} = 12$*)*

We see a clear downward time dependence of the exit risk. On the one hand, this may be the evidence of the true downward state dependence of an individual hazard rate (see e.g. van den Berg and van Ours, 1996, or Eckstein and Wolpin, 1995, for the evidence of this). On the other hand, this may be due to some neglected heterogeneity. Generally, individuals may differ with respect to certain observed and unobserved characteristics that condition the unemployment duration (see e.g. Heckman and Singer, 1984a,b). As far as the German benefit system is concerned, an additional source of unobserved heterogeneity

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former specification would assume a negative externality: If some other unemployed workers search harder, the arrival rate of an individual - ceteris paribus - decreases. Computing the aggregate matching function would then yield $m(U,V) = (\Omega U)^{1-\alpha} V^{\alpha}$. We do not believe that this will make a major quantitative difference and we therefore stick to our specification. For details, see app. B.1.2. We are grateful to Jean-Marc Robin for having raised this point.

[14] For more background on the SOEP, our sampling method and for descriptive statistics, see app. A.1.

[15] We use the Tanner and Wong (1983) smoothed nonparametric hazard function estimator with bias correction. Kernel function is Gaussian. Optimal bandwith is estimated by cross-validation discussed in Tanner and Wong (1984).

is the eligibility to UA benefits. Not all individuals receiving UI benefits may be eligible to UA benefits once the entitlement period expires. Eligibility to UA benefits is determined at the "means test", where an individual has to provide lengthy information about income sources of the household, number and age of dependents etc. If the means are sufficient, the person becomes ineligible to UA benefits, but might still claim social assistance, which eventually may or may not be provided. Unobservability for the econometrician results from the fact that, once exit out of unemployment occurs before the expiration of entitlement, an econometrician cannot know about the outcome of the means test. The individuals themselves, however, are very likely to know what the result of the test will be. If they expect to fail the test, they would search harder and therefore exit faster. This behaviour, if uncontrolled for, results in a decreasing nonparametric estimate of the hazard rate, just as it would be under the influence of some individual-specific unobserved component. Clearly, the true individual exit rate in this case may as well be constant, or increasing up to the expiration of entitlement and constant thereafter, as in Mortensen (1977), van den Berg (1991) and also in our theoretical model.[16] Finally, both true individual state dependence and unobserved heterogeneity may manifest themselves simultaneously (see e.g. van den Berg and van Ours, 1996, 1999, for evidence of this in U.S. and French data respectively).[17]

Summarizing, the individual exit rate derived from the theoretical model must be sufficiently flexible to capture both of these aspects. It needs to have a parametric form flexible enough to replicate the observed downward pattern even if no unobserved heterogeneity is present. At the same time, it needs to make provision for unobserved heterogeneity with respect to the outcome of the means test and unmeasured individual characteristics, so that it could match the observed downward pattern even if the true individual exit risks are increasing or constant. Our aim is to provide a fully structural econometric model that estimates the deep parameters of the theoretical model of sect. 2 adequately addressing both of these possibilities.

4.2 Econometric model

- Specification

Our data are sampled as a flow of entrants into unemployment and employment. The exit rate from our theoretical model is given by (22). The effort level $\phi(s)$ needs to be replaced by the optimal value implied by the first-order condition (8), i.e. $\phi(s)$ is a function of the duration s given the particular benefit environment (UI and UA payments together with the entitlement length $\bar{s}$), the spell-effect $\eta(s)$, the wage w and the labour market density θ. To stress this dependence but to keep notation short, we group these explanatory variables into a vector $\mathbf{z} \equiv \{b_{UI}, b_{UA}, \bar{s}, w, \theta\}$ and write $\phi = \phi(s; \mathbf{z})$. Please note that even though w and θ are endogenous to the theoretical model they are exogenous to the duration of unemployment which is our dependent variable. One can therefore either substitute them out using their

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[16] Gonzalez and Shi (2010) suggest that learning about one's own ability while searching increases the individual exit rate but might imply falling job-finding rates (probabilities in their discrete-time setup) for a given cohort.

[17] We define "not passing the means test" in our application as complete ineligibility both to UA benefits and social assistance.

theoretical solutions, which depend on the productivity A and vacancy costs γ among others, or one can use the data on w and θ directly. Using the data on w and θ directly simplifies an already complex numerical task of fitting the nonstationary model, making it faster by a factor of about 4. Moreover, it lifts the necessity of having employer-side data which are unavailable in SOEP as well as in any other typical panel household survey.

Clearly, there also exist other variables that may potentially influence both the search effort and the spell-effect, affecting thereby the exit rate out of unemployment (22). These variables may be either observable, such as gender, education level etc., or unobservable, like for instance individual ability of performing a job or looking for jobs. We group the observed explanatory variables in a vector $\mathbf{x}$ and assume that $\mathbf{x}$ enters the spell-effect and the separation rate with parameters $\boldsymbol{\zeta}_\eta$ and $\boldsymbol{\zeta}_\lambda$, respectively. Likewise, we represent the unobserved component by ν and assume that it influences the spell-effect and the separation rate via the parameters ξ_η and ξ_λ, respectively. Hence, the spell-effect (21) now reads $\eta(s;\mathbf{x},\nu) = \eta_0(\mathbf{x},\nu)\, g(s)$ and the separation rate reads $\lambda(\mathbf{x},\nu)$.[18]

Having introduced observed and unobserved characteristics, the exit rate from (22) can be written as

$$\mu_j(s;\mathbf{x},\mathbf{z},\nu) = \eta(s;\mathbf{x},\nu)\left[\phi(s;\mathbf{x},\mathbf{z},\nu)\,\theta\right]^\alpha, \qquad j = UI, UA, \tag{23}$$

where for ease of distinction j indicates either the UI or the UA regime.[19] In (23) also note that since the optimal search effort depends on both the spell-effect and the separation rate (see app. A.2), $\mathbf{x}$ and v will always implicitly enter the $\phi(s;\mathbf{z})$ via $\eta(s;\mathbf{x},\nu)$ and $\lambda(\mathbf{x},\nu)$.

We have four different types of labour market histories in our data set. The first group consists of individuals who enter unemployment with the right to claim UI benefits and exit unemployment before the expiration of entitlement period ($s \leq \bar{s}$). As argued in sect. 4.1, for these individuals we do not observe the outcome of the means test for eligibility to b_{UA}. We do assume, however, that individuals know about the outcome even before applying for b_{UA}. Let $\phi(s;\mathbf{z}|0)$ indicate the search effort given that $b_{UA} = 0$, which corresponds to the hypothetical failure at the test. Similarly, let $\phi(s;\mathbf{z}|b_{UA})$ stand for the hypothetical case in which the test will be passed (and so, $b_{UA} > 0$). Finally, let π denote the fraction of the individuals that pass the test. Then, given the observed and unobserved characteristics, the individual contribution to the likelihood in this group is

$$\begin{aligned}\ell(s;\mathbf{x},\mathbf{z},\nu) \;=\;& \pi\left[\mu_{UI}(s;\mathbf{x},\mathbf{z},\nu|b_{UA})\right]^{d_u} e^{-\int_0^s \mu_{UI}(u;\mathbf{x},\mathbf{z},\nu|b_{UA})du} \\ &+ (1-\pi)\left[\mu_{UI}(s;\mathbf{x},\mathbf{z},\nu|0)\right]^{d_u} e^{-\int_0^s \mu_{UI}(u;\mathbf{x},\mathbf{z},\nu|0)du},\end{aligned} \tag{24a}$$

where d_u is a dummy variable such that $d_u = 1$ if unemployment spell is uncensored.

The second group comprises individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to either UA or zero benefit

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[18] It would be desireable to integrate some aspects of the described ex-ante heterogeneity into the theoretical model right from the beginning. Our objective, however, is to focus on the implications of a two-tier benefit system under optimal effort with anticipated end date of the entitlement. We therefore leave theoretical treatment of complementary ex ante heterogeneity for future research. It is standard in the literature to model the basic feature of optimal behaviour one is interested in and capture the rest of heterogeneity in the econometric part (see e.g. van den Berg and Ridder, 1998, or Flinn, 2006).

[19] Appendix A.3 shows the differential equation that solves for $\mu_j(s)$ once optimal search effort is substituted out.

level and thereby reveal the outcome of the means test, and exit unemployment (or not) only after the expiration of entitlement ($s > \bar{s}$). For such individuals, the contribution to the likelihood is given by

$$\ell\left(s;\mathbf{x},\mathbf{z},\nu\right) = e^{-\int_0^{\bar{s}} \mu_{UI}(u;\mathbf{x},\mathbf{z},\nu)du} \pi^{d_t}\left(1-\pi\right)^{1-d_t}\left[\mu_{UA}\left(s;\mathbf{x},\mathbf{z},\nu\right)\right]^{d_u} e^{-\int_{\bar{s}}^{s} \mu_{UA}(u;\mathbf{x},\mathbf{z},\nu)du}, \tag{24b}$$

where d_t is a dummy variable such that $d_t = 1$ if we observe that an individual passes the means test.

The third group embarks all individuals who do not have the right to claim UI benefits and enter unemployment receiving lower UA benefits from the very beginning ($d_t = 1$) or not receiving benefits at all ($d_t = 0$). Their contribution to the likelihood is therefore

$$\ell\left(s;\mathbf{x},\mathbf{z},\nu\right) = \pi^{d_t}\left(1-\pi\right)^{1-d_t}\left[\mu_{UA}\left(s;\mathbf{x},\mathbf{z},\nu\right)\right]^{d_u} e^{-\int_0^{s} \mu_{UA}(u;\mathbf{x},\mathbf{z},\nu)du}. \tag{24c}$$

Our final group consists of entrants to employment. Conditional on observed and unobserved characteristics, the contribution is simply

$$\ell\left(l;\mathbf{x},\nu\right) = \left[\lambda\left(\mathbf{x},\nu\right)\right]^{d_j} e^{-\lambda(\mathbf{x},\nu)l}, \tag{24d}$$

where d_j is a dummy variable such that $d_j = 1$ if employment spell is uncensored and l is the duration of employment.

The parameterization of η_0 in the spell-effect is standard, namely $\eta_0\left(\mathbf{x},\nu\right) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta + \xi_\eta\nu\}$, where $\mathbf{x}$ contains an intercept. Similarly, the conditional separation rate is parameterized as $\lambda\left(\mathbf{x},\nu\right) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\lambda + \xi_\lambda\nu\}$. Since ν is unobservable, identification requires that either ξ_η or ξ_λ should be normalized. We set $\xi_\lambda = 1$. Finally, our parametric assumptions about the shape of $g\left(s\right)$ from the spell-effect (21) are

$$g\left(s\right) = S\left(s,\delta\right) + 1, \tag{25}$$

where $S\left(s,\delta\right)$ denotes a survivor function of the chi-square distribution with δ degrees of freedom (see app. B.3.2). Intercept in $\mathbf{x}$ plays a role of a scale parameter in the distribution of unemployment duration, adjusting $g\left(s\right)$ appropriately. Our choice of this particular functional form in (25) is motivated by the nonparametric estimates in fig. 1, where even in absence of unobserved heterogeneity the individual exit risk is well-matched by the reverse S-shaped curve. Sensitivity analyses with other parametric alternatives, e.g. the survivor function of the Weibull distribution, which has one parameter more, have shown no significant improvement in the model fit.

We are finally left with specifying the distribution of the unobserved component. As is common in the reduced-form literature starting with Heckman and Singer (1984b), as well as in the structural estimation literature (see e.g. van den Berg and Ridder, 1998 or Frijters and van der Klaauw, 2006), we assume that unobserved heterogeneity has a discrete distribution with K points of support, $\{\nu_k\}_{k=1}^{K}$. Let p_k denote the probability mass attached to the k-th point of support. With this assumption, the likelihood contribution of an unemployed individual in our sample finally reads

$$\ell\left(s;\mathbf{x},\mathbf{z}\right) = \Sigma_{k=1}^{K} p_k \ell\left(s;\mathbf{x},\mathbf{z},\nu_k\right), \tag{26a}$$

where $\ell(s;\mathbf{x},\mathbf{z},\nu_k)$ is any of the contributions in (24a)-(24c), depending on a particular type of the unemployed worker. Similarly, the likelihood contribution of the employed worker reads

$$\ell(l;\mathbf{x}) = \Sigma_{k=1}^{K} p_k \ell(l;\mathbf{x},\nu_k), \tag{26b}$$

where $\ell(l;\mathbf{x})$ is shown in (24d).

- Estimation procedure

Estimation of model parameters uses a part of the numerical solution method for the steady state. As described in app. A.2, for a given wage w and vacancy to unemployment ratio θ, the individual exit rate can be computed at any moment of the unemployment spell. Using the individual survey data implies that the wage w for each individual is known and the corresponding θ can be taken from macro data. Thus for any given parameter vector individual exit rates immediately follow. Details on the implementation can be found in app. A.3 and app. B.3.1.

Note that the model is estimated without explicitly specifying the wage setting mechanism. If we used linked employer-employee data, the model could be estimated by using the observable productivity data. This would also allow us to estimate the bargaining power parameter β as well as provide more information on the discrepancy between the observed wage and an endogenous wage solution implied by the model. For the rest of the parameters unrelated to wage setting mechanism, however, both approaches must be equivalent (assuming that wage setting in the second approach is correctly specified).

- Identification

The set of model parameters to estimate comprises α, σ, δ, π, $\boldsymbol{\zeta}_\lambda$, $\boldsymbol{\zeta}_\eta$, ξ_η and $\{\nu_k, p_k\}_{k=1}^{K}$. As just mentioned, bargaining power parameter β drops out as we use the household survey data only. Furthermore, we keep the rate of time preference ρ fixed.[20]

Identification of the model parameters comes from different sources. From (24d) and (26b) we can see that separation rate parameters $\boldsymbol{\zeta}_\lambda$ and the distribution of the unobserved component $\{\nu_k, p_k\}_{k=1}^{K}$ in $\lambda(\mathbf{x},\nu_k) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\lambda + \nu_k\}$, $\forall k$, are always identified from the data on the job duration l and observed individual characteristics $\mathbf{x}$. This holds as the distribution of job duration is simply a finite mixture of exponential distributions which is known to be identifiable. Since $\mathbf{x}$ contains an intercept, the first mass point in the distribution of unobserved component is normalized to zero (i.e. $\nu_1 = 0$). Finally, since $\{p_k\}_{k=1}^{K}$ is a probability mass function, we set $p_K = 1 - \Sigma_{k=1}^{K-1} p_k$.

Next we demonstrate identifiability of η_0, α and σ in a model where η_0 is a single parameter. To do so, we need to consider the endpoint condition that pins down the path of the optimal exit rate at infinity. This condition is shown in equation (A.9) in app. A.3 and corresponds to the exit rate in a completely stationary environment, where the two-step system and the exogenous spell-effect have no impact anymore. From equation (A.9) we see that σ is identified through the variation in the benefit level. For a fixed rate of time preference, conditioning on λ, the middle term in this equation further suggests that variation in the

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[20] We set ρ to 0.002, which corresponds to an annual interest rate of 2.4%.

value of employment, which itself stems from the variation in wages and benefits, will identify α and η_0 up to scaling, even if θ is a constant. From this follows that in the the solution for the exit rate μ_{UA} the unemployment duration, wage and benefit data will identify η_0, α and σ. As η_0 is identified from the endpoint condition and $\{\nu_k\}_{k=1}^{K}$ are identified from the data on the employment duration, the parameters $\boldsymbol{\zeta}_\eta$ in $\eta_0\left(\mathbf{x},\nu_k\right) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta + \xi_\eta\nu_k\}$, $\forall k$, are always identified from the observed individual characteristics $\mathbf{x}$ for a given ξ_η. Let us for the moment suggest that ξ_η is known and return to identification of this parameter later on.

Once the endpoint condition is identified, identification of the exogenous spell-effect comes from the optimal solution for the time path of the exit rate $\mu_j\left(s\right)$ itself. This path is shown in equation (A.8) in app. A.3. From the second term in this equation, we see that the spell-effect will always be identified as long as $\dot{\eta}\left(s\right)/\eta\left(s\right)$ remains a time-dependent function, which is true for our specification. Identification of the particular parameters in $g\left(s\right)$ will depend on its parametric form. With our specification in (25), δ is always identified being just a single parameter.

Finally, the fraction π of those who pass the means test is always identified at least by observability of the outcome of the means test in the two particular subsamples of our individuals. The first subsample contains all individuals who fail to exit unemployment before the entitlement period is over and transit to a lower-benefit regime. The second subsample comprises all those who have no entitlement to UI benefits from the very beginning. In both of these subsamples, the outcome of means test is observable as can be seen from (24b) and (24c), respectively.

Returning to the coefficient ξ_η that multiplies the unobserved component ν in $\eta_0\left(\mathbf{x},\nu\right)$, its identification in practice is less clear. This parameter should be identified from the unobserved component ν just by the data on unemployment duration that insure the appropriate scaling of ν in $\eta_0\left(\mathbf{x},\nu\right)$. However, all our attempts to fit the model keeping ξ_η as a free parameter were permanently plagued by very poor convergence and unreliable Hessian matrices evaluated at the maximizer. This has finally made us setting ξ_η to zero. Despite setting this parameter to zero the influence of unobserved component on the exit rate out of unemployment is still preserved, because this exit rate, as seen in (23), is a function of the optimal search effort and the optimal effort is determined by $\lambda\left(\mathbf{x},\nu\right)$. Thus, the exit rate retains the conditioning on ν and the mixture model (26a) is still valid for all three types of unemployed workers. This makes us believe that setting ξ_η to zero is not restrictive.

4.3 Estimation results

Table 1 below reports estimation results for three specifications. Specification I does not condition on the observed and unobserved individual characteristics, specification II conditions only on the observed characteristics, and specification III takes into account both observed and unobserved components. All specifications make provision for unobservability of the results of the means test.

Numerical complexity of the model makes us restrict attention on only a selected number of key characteristics, which are *sex* (=1 if male), *region* (=1 if an individual comes from East Germany), two education dummies: *medium-skilled* (=1 if middle vocational education) and *high-skilled* (=1 if higher vocational or higher education), and two *age* dummies that define the groups of workers up to 30 years old and 31 to 45 years old.

		(I)				**(II)**				**(III)**			
		Coeff.	SE	z-Stat.	p-Value	Coeff.	SE	z-Stat.	p-Value	Coeff.	SE	z-Stat.	p-Value
$\boldsymbol{\zeta}_\lambda$:	intercept	−4.4976	0.0563	−79.9007	0.0000	−4.4567	0.1805	−24.6865	0.0000	−5.6674	0.4370	−12.9688	0.0000
	sex					0.5403	0.1186	4.5552	0.0000	0.7117	0.1802	3.9494	0.0001
	region					0.8707	0.1140	7.6370	0.0000	1.1316	0.2054	5.5103	0.0000
	medium-skilled					−0.3855	0.1513	−2.5480	0.0108	−0.5102	0.2324	−2.1956	0.0281
	high-skilled					−0.7512	0.1889	−3.9773	0.0001	−0.8449	0.2594	−3.2566	0.0011
	age (up to 30)					−0.3134	0.1556	−2.0139	0.0440	−0.1687	0.2460	−0.6858	0.4928
	age (31 to 45)					−0.4102	0.1548	−2.6500	0.0080	−0.2583	0.2425	−1.0654	0.2867
$\boldsymbol{\zeta}_\eta$:	intercept	−3.5421	0.5414	−6.5427	0.0000	−3.8200	0.3455	−11.0553	0.0000	−4.0233	0.4737	−8.4936	0.0000
	sex					0.0650	0.1301	0.5000	0.6171	0.0897	0.1245	0.7206	0.4712
	region					−0.0201	0.1310	−0.1531	0.8783	0.0777	0.1395	0.5571	0.5774
	medium-skilled					0.2832	0.1627	1.7408	0.0817	0.2298	0.1737	1.3230	0.1858
	high-skilled					0.4758	0.2115	2.2499	0.0245	0.3937	0.2185	1.8016	0.0716
	age (up to 30)					0.7751	0.1726	4.4901	0.0000	0.7872	0.1711	4.6002	0.0000
	age (31 to 45)					0.6647	0.1581	4.2053	0.0000	0.6597	0.1616	4.0812	0.0000
α		0.4945	0.0783	6.3122	0.0000	0.4102	0.0762	5.3848	0.0000	0.4562	0.0750	6.0836	0.0000
σ		0.6538	0.1253	5.2196	0.0000	0.7748	0.1010	7.6678	0.0000	0.7022	0.1367	5.1378	0.0000
δ		11.8035	1.7599	6.7067	0.0000	14.5479	2.9778	4.8855	0.0000	14.8408	3.0935	4.7975	0.0000
π		0.2462	0.0297	8.2840	0.0000	0.2448	0.0294	8.3331	0.0000	0.2450	0.0295	8.3182	0.0000
ν										2.4717	0.1798	13.7493	0.0000
p										0.6404	0.0753	8.5083	0.0000
$\log \mathcal{L}$		−2920.36				−2853.94				−2819.37			

Table 1 *Estimation results*

As for the estimation results per se, our main finding is the significance of α. This means that changes in the optimal effort levels in response to any unemployment benefit reform, be it the reform of $b_{UI,UA}$ or of $\bar{s}$, will have a significant impact on the exit rate out of unemployment. As effort is a function of unemployment benefits via the first-order condition (8), this finding in particular contributes to the empirical reduced form literature that analyses the dependence between unemployment benefits and the probability of leaving unemployment. Evidence on this dependence are somewhat conflicting. Early work by Hujer and Schneider (1989) and Arulampalam and Stewart (1995) finds mostly no significant influence of benefits while later work by Carling et al. (2001) and Røed and Zhang (2003) state the opposite.[21] For German data, Fitzenberger and Wilke (2010) find significant effects of a reduction both of the level of benefits and of the entitlement length, the latter is visible only, however, for original entitlements above 12 months. Our entirely structural perspective provides an alternative view. While we do not rule out that for certain types of heterogeneous agents changing benefits may play no role, our result on the significance of α shows that in sufficiently aggregate terms there exists a positive significant relationship between the reemployment probability and a change in the level of unemployment benefit payments. Consequently, a change in the design of the unemployment benefit mechanism will induce a significant response on the macro level.

Our next important finding is on the role of unobserved heterogeneity. As for heterogeneity with respect to the outcome of the means test, tab. 1 shows that π is always different from zero and unity at 5% significance level. Together with the significance of α, this implies that the prospect of not passing the means test significantly increases search effort and exit probability. Moreover, looking at the results of specification III, we find that heterogeneity with respect to further unobserved characteristics also has an impact.[22] Our estimates show a two-point distribution of the unobserved component with the probability mass value p different from both zero and unity at 5% level. Along with the statistically significant estimate of the mass point, this shows a significant improvement of specification III over the specification II.

The importance of unmeasured heterogeneity is frequently highlighted in the literature on the structural estimation of search models. Much fewer attempts have been made to explicitly address the exogenous spell dependence within this framework. Since our model is nonstationary from the very beginning, accounting for such spell-dependence is particularly easy. Tab. 1 shows that the estimate of δ is significantly different from zero. However, absence of exogenous spell dependence is not nested in our model, because (25) does not include time-invariance as a special case. Therefore the appropriate test for exogenous spell-dependence is the Vuong (1989) likelihood ratio test for model selection where spell-effect in the competing model is absent, i.e. $\eta(s; \mathbf{x}, \nu) = \eta_0(\mathbf{x}, \nu)$.[23] Performing this test for the

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[21] To be fair, the analyses are undertaken for different countries. Moreover, later studies, notably Røed and Zhang (2003), provide more sophisticated treatment of unobserved heterogeneity in comparison to earlier ones.

[22] The estimated number of mass points in the distribution of the unobserved component is equal to two ($K = 2$). Adding the third mass point would always make us converge to any of the estimated two. With $\nu_1 = 0$ and $p_2 = 1 - p_1$, the estimates shown in tab. 1 are the estimates of ν_2 and p_1. As there is no ambiguity, subscripts are omitted.

[23] This is the test for strictly non-nested models as both models have no conditional distribution in common

richest specification, which is specification III, it turns out that the null hypothesis of the equivalence of these two models is clearly rejected in favour of the model with the spell-effect. This means that once key observed and unobserved components are accounted for, there still remains a significant degree of downward state dependence among long-term unemployed. Coexistence of unobserved heterogeneity and duration dependence aligns with the similar finding of van den Berg and van Ours (1996, 1999), who use an entirely different modelling approach.

Finally, our estimate of the utility parameter σ is also very comforting. First, the degree of risk aversion is high enough to reject the hypothesis of risk neutrality. Second, significance of σ provides the empirical evidence on the existence of a significant insurance effect of unemployment benefits.

Regarding the fit of the model, tab. 1 clearly shows an improvement from specification I to specification III, the latter being the best of all three. To illustrate the fit of our best specification for each of the four different subsamples (East Germany, West Germany, males and females), we predict the survivor function and plot it against the Kaplan-Meier nonparametric estimate of the survival probability in unemployment. These plots can be seen in app. A.4, fig. 7. We find that with the exception of the 3rd and 4th months of unemployment duration, where model predictions tend to lie outside the 95% confidence interval for the Kaplan-Meier estimate, our model provides quite an accurate fit to the data. This assures that all our simulations based on the estimated parameters will be well-grounded.

After having estimated all the parameters, we are left with determining labour productivity A and vacancy cost γ. The wage w and tightness θ were taken as exogenous in this first part of the estimation which was built on the household side of the model only. As the wage and tightness are endogenous in equilibrium, we now take the estimated parameters and compute parameters A and γ using the full equilibrium structure of our economy in the steady state (see app. B.2.2). We compute A and γ such that the equilibrium endogenous variables w and θ equalize with the average wage and labour market tightness from our data. See tab. 2 for results.

5 Evaluating the labour market reform

We now use the structurally estimated parameters in order to describe the steady state equilibrium of 2004 and to evaluate the reform effective as of January 2005.

5.1 The pre-reform steady state

Data is heterogeneous in many respects and we have vectors of $b_{UI,i}$, $b_{UA,i}$, $\bar{s}_i$ and the wage w_i. Building on the mean wage (used above e.g. to predict A and γ), UI payments for our representative agent are determined by ξ_{UI} as in (2). Quantitatively, the replacement rate

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(see Vuong, 1989, Definition 2). The appropriate test statistic is given in Vuong (1989), Theorem 5.1. We use the Schwarz correction factor to adjust for the difference in the number of parameters in the competing models (see Vuong, 1989, p.318). The calculated value of the adjusted test statistic is 57.13. Asymptotic distribution of the test statistic is standard normal.

is given by $\xi_{UI} = \omega\rho_{UI}$ where ρ_{UI} is the statutory replacement rate and ω is the share of individuals who are entitled to UI payments. As only an estimated share of $\pi = 24.4\%$ pass the means test (see tab. 1), UA payments for our representative agent are computed as the product of the statutory replacement rate ρ_{UA}, the previous wage and the share π, $b_{UA} = \pi\rho_{UA}w$. Average sample entitlement to UI payments is $\bar{s} = 12.22$ months, again for those entitled to UI payments. With these means for b_{UA} and $\bar{s}$, our representative agent receives the same amount of benefits at each point in time s as the mean in the data over a cohort of unemployed who all have an unemployment spell of s (see app. B.4.1). By representative we understand the average individual in our data set in 2004 (for descriptive statistics of individual characteristics in 2004 see tab. 3 in app. A.1).

All parameters used in this paper, apart from the ones presented in specification III of tab. 1, plus some selected endogenous variables are provided in tab. 2.[24] As in the estimation part, the time preference rate is chosen to fit an annual interest rate of 2.4%. The bargaining power β is set equal to .5. (Increasing the time preference rate to 3% or 4% or changing β to .3 or .7 in robustness checks had basically no effects.) We use specification III to from tab. 1 to predict the average separation rate λ and the parameter η_0 in (22) and (23) for the spell-effect. The corresponding separation rate λ, η_0 and the implied mean exit rate $\bar{\mu}$ can be seen in tab. 2.

exogenous parameters

ρ	β
.002	.5

predicted parameters

λ	η_0	A	γ
.010	.041	1453.6	11925.1

policy parameters

ρ_{UI}	ρ_{UA}	$\bar{s}$	ω
.6	.53	12.22	.56

equilibrium values

w	θ	κ	$\bar{\mu}$	$(N-L)/N$
1172.6€	.20	2.4%	0.11	8.7%

Table 2 *Parameters and selected equilibrium values*

Our equilibrium values fit perfectly by construction for the wage and labour market tightness. Our tax rate κ is sufficiently close to the actual social security contribution rate (this is the only purpose of taxes in our model). The average matching rate of 0.11 means that the probability to lose a job per month (as we use monthly data) is given by $e^{-\bar{\mu}*1}\bar{\mu} * 1 = 9.8\%$, which is reasonable as well. The only quantity which does not match aggregate statistics at first sight is the unemployment rate. While it is estimated at 8.7% in our model using micro-data, the official aggregate number is 10.5% (Bundesagentur für Arbeit, 2009). This is mainly caused by the different datasources used for our estimation and for official unemployment rates. We do not consider this to be a major problem as our main interest lies in understanding the *change* of the unemployment rate after the reform and not in its level and as the official unemployment rate lies within the 95% confidence interval of our estimated 8.7%.

For comparative statics below, we will take the exogenous parameters, the estimated and the predicted parameters and the replacement rate for short-term unemployed ρ_{UI} as given.

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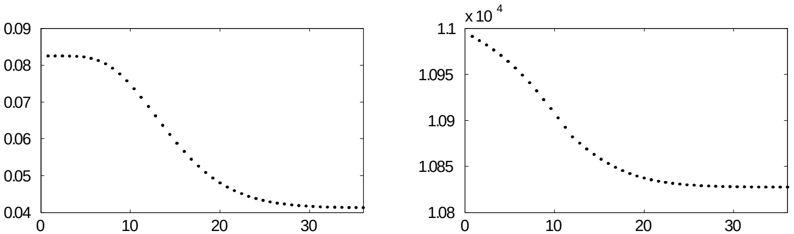
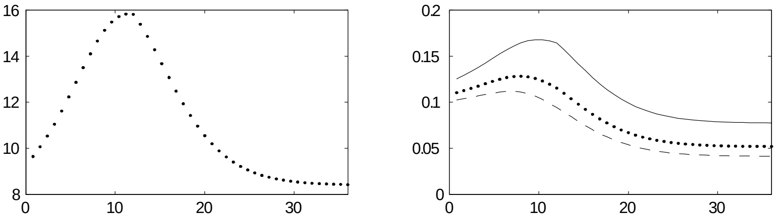
[24] See app. A.2 for a description of the numerical solution procedure.

We will then change long-term benefits b_{UA} and the entitlement period $\bar{s}$ to understand the effects on equilibrium values.



· estimated spell effect

· value of unemployment



· effort

· exit rates

Figure 2 *The spell effect $\eta(s)$, the value of being unemployed $V(b(s), s)$, effort $\phi(s)$ and the exit rate $\mu(b(s), s)$ ($\cdots$ b_{UA} as in model, $-$ $b_{UA} = 0$, $- -$ $b_{UA} = b_{UI}$) as a function of the spell s (in months)*

Although the economy is in the steady state, there are still dynamics on the micro level as illustrated in fig. 2. At any point in time, individuals find and lose jobs. The upper left panel shows that the estimated exogenous spell-effect from (21) with (25) falls over time. The value of being unemployed thereby unambiguously falls over time. This is shown by the upper right panel and needs to hold generally as (A.2) in the appendix shows. The intuition is simple: If there was *no* spell-effect ($\eta(s)$ is constant), a long-term unemployed would live in a stationary world and the value of being a long-term unemployed worker would be stationary as well. With a *negative* spell-effect, the job finding rate - taking optimally chosen effort into account - goes down and the value of being unemployed approaches a lower limit determined by the lower limit of $\eta(s)$. In *both* cases, the value of a short-term unemployed worker falls as the point in time where lower UA benefits are paid comes closer over time.

The optimal reaction of the unemployed worker is shown by the lower left panel. Effort increases during the first 12 months and then starts falling when entitlement to unemployment insurance ceases at $\bar{s} = 12.2$ months. Note that optimal effort could rise longer or fall earlier than at $\bar{s}$ as it is the outcome of the interplay of the spell-effect (lower $\eta(s)$ reduces optimal effort) and the incentive-effect, i.e. the potential gain from finding a job. As gains increase due to a falling value of being unemployed, this second effect tends to increase effort. This can be seen from the first-order condition in (8) or, more directly, from (A.1) in the appendix. The initial increase of effort clearly reflects the rising incentive to search harder the closer $\bar{s}$ approaches. For our estimation, after around 12 months, the increase in the gain of finding a job is no longer strong enough to compensate the "discouraging" spell-effect. Search effort falls and approaches a constant. The fact that unemployed workers finally "give up" is ultimately the effect of the exogenous negative spell-effect.

The lower right panel shows exit rates for $b_{UA} = 0$ (solid), average b_{UA} (dotted) and $b_{UA} = b_{UI}$ (dashed). Exit rates mirror effort dynamics conditional on the benefit level for $s > \bar{s}$. The case of $b_{UA} = b_{UI}$ does not provide any incentives to search harder over time. The exit probability is the lowest, being driven only by the spell-effect. The wider the gap between b_{UA} and b_{UI}, the less attractive is unemployment relative to employment. This implies higher upward pressure due to increased search effort and translates into a steeper increase of the exit rate out of unemployment. Exit rates are steepest and highest for $b_{UA} = 0$.

5.2 The effects of the reform

The reform was characterized by a reduction in UA benefits b_{UA} (which are given levels after the reform and no longer proportional to the previous wage) and entitlement length $\bar{s}$. Benefits decreased on average by 7%, average entitlement length dropped from 12.2 to 10.9 months, i.e. by 10.7%. We present here the effect of the reform, i.e. a joint decrease of both b_{UA} and $\bar{s}$. The replacement rate ξ_{UI} remains unchanged and UI payments are paid according to (2).

The horizontal axis of fig. 3 plots "Hartz-units". The "1" represents the situation before the reform. The "0" represents the situation after the reform and "-1" to "-4" shows the effects of a stronger reform, i.e. of reducing b_{UA} by another 7% and $\bar{s}$ by another 10.7%. The vertical axes plot changes relative to the pre-reform steady state which is normalized (for levels, see tab. 2) to 1. The pre-reform steady state is therefore always given by the point $(1, 1)$. The solid line shows the changes as predicted by our estimated model. The dashed lines above and below provide a 95% confidence interval for the statistic in the middle. This confidence interval is constructed using parametric bootstrap where draws are made from the multivariate normal distribution of the maximum likelihood estimates. For every new draw we recompute the equilibrium solution at each Hartz-unit. Due to the high numerical complexity of the equilibrium solution, confidence intervals at each Hartz-unit are based on 160 replications.[25]

Generally speaking, we find very weak effects of the reform. The qualitative effects of the reform are as intended by policy makers. Effort $\phi(0)$ when becoming unemployed rises the stronger the reform is, i.e. the further we are to the left of the upper left panel of fig. 3. Labour market tightness θ and the average job finding rate $\bar{\mu}$ from (11) rise as well. The increase in θ, i.e. the number of vacancies per unemployed worker, is crucial for our interpretation further below. Understanding this effect is simple, however: More effort by unemployed workers makes opening a vacancy more attractive. Hence, lower benefits induce a higher number of vacancies per unemployed worker. The quantitatively weak effect of the reform becomes visible when looking at the unemployment rate. It decreases from the pre-

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[25] To the best of our knowledge, we are the first to explicitly show confidence intervalls for our policy analysis - even though it turns out that not all reform-induced changes are significant.

reform steady state “1” to the reform level at “0” to (slightly more than) 97% only. Starting at an unemployment rate of around 10.5%, the effect of the reform would be to decrease the unemployment rate to 10.2%, i.e. by just 0.3 percentage points.

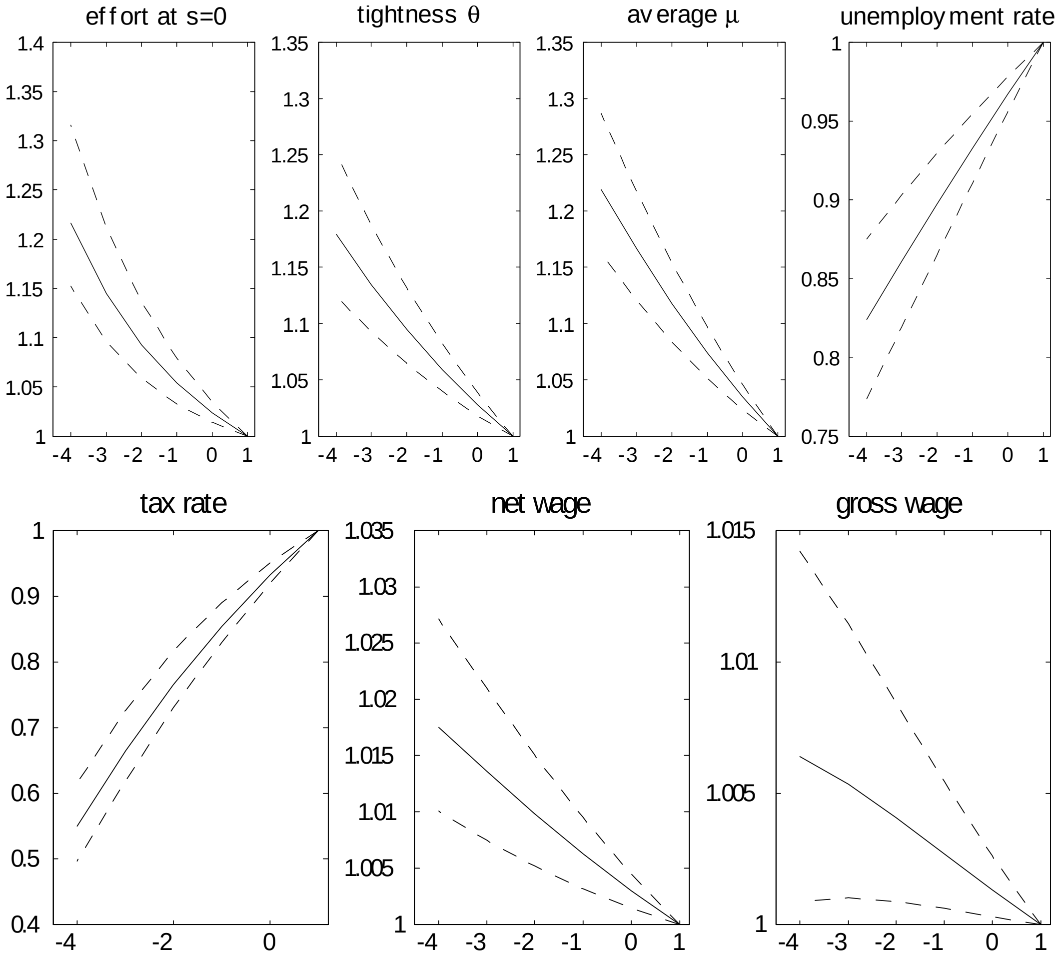

Figure 3 *Aggregate effects of UA payments b_{UA} and entitlement length $\bar{s}$*

The lower left panel shows that the tax rate falls. This has the following reasons: Lower benefit payments and a lower number of recipients reduce overall expenditure. This reduced expenditure is paid by more workers who earn higher gross wages.

One of the most surprising results is the increase in the net and gross wage. This increase is also the basis for the rise in social welfare which we will discuss later. The increase in the net wages becomes clear from the wage equation (13) when taking into account that the positive tightness effect dominates the negative effort effect. This is an interesting feature of this wage bargaining setup with endogenous effort and is in strong contrast to perfectly competitive setups, to bargaining setups with exogenous effort and to search setups where the reservation wage is a simple decreasing function of the outside option. Here, the outside option (utility from being an unemployed worker) decreases as well but this is overcompensated, given our estimates, by the positive effect of more vacancies per unemployed worker.

Confidence intervals for all the statistics in fig. 3 show that the changes induced by the reform are statistically significant, as both the upper and the lower bound lie strictly above (or strictly below) unity. Summarizing and ignoring distributional effects, the reform has the qualitatively desired effects but is quantitatively of hardly any importance: While the reduction of unemployment is statistically significant, economically, it is not.

When we look at welfare measures in fig. 4, we see the distributional nature of the reform. Employed workers gain from the reform. This might not appear surprising given that the net wage rises. Employed workers do also anticipate, however, the potential loss from the reform as they have a certain risk of becoming unemployed. If the value of being unemployed falls, the value of having a job could fall as well.

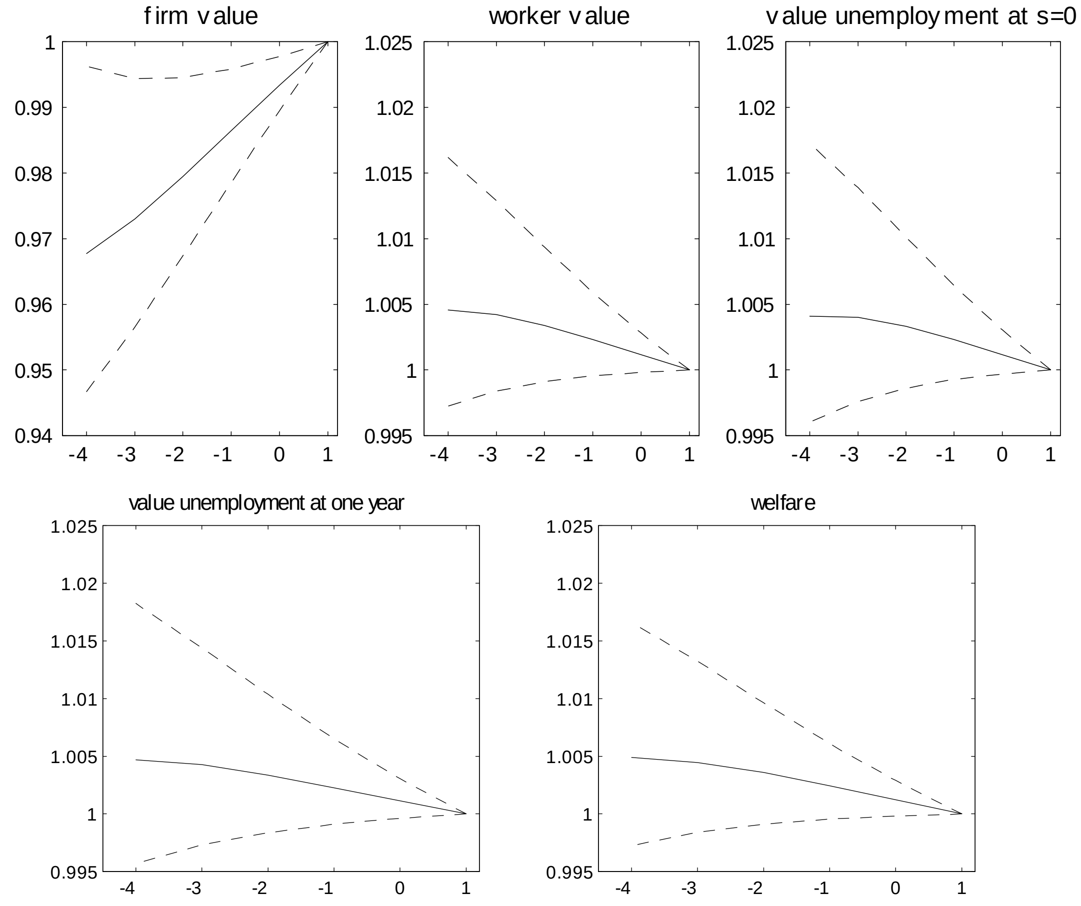

Figure 4 *Distributional effects of UA payments* b_{UA} *and entitlement length* $\bar{s}$

When we look at the value of being unemployed at $s = 0$, i.e. right after having lost the job, this value slightly *increases* as well due to the reform. This is entirely due to the anticipation effect of having a job. The short-term unemployed clearly loses as benefits will fall earlier and by more due to the reform. But a short-term unemployed worker also gains in an expected intertemporal sense as she will gain more due to the reform in case she finds a job. More formally, the value of unemployment depends negatively on search effort and positively on exit rate, which can be seen from equations (7) and (20). As both search effort and the exit rate go up, each effect could dominate. In our case, the positive exit-rate effect is the stronger one for the short-term unemployed. Our most stunning result is that even the long-term unemployed workers gain when we define long-term as those with an unemployment spell of 12 months. Looking at unemployed workers with a spell of two or three years, we also find that they still gain. Looking at the reason analytically (see app. B.4.2) confirms the intuition: The rise in the wage is so large that any temporary drop in consumption is overcompensated by expected future consumption.

Considering the 95% confidence intervals for each of these three statistics, though, we can see that despite the predicted wins and losses, all these wins and losses are not statistically significant. The confidence bounds lie above and below unity. What is significant, however, are the losses of the firms, i.e. a drop of J when benefits b_{UA} are reduced.

The loss of firms is also a surprising qualitative result of our analysis and is a direct consequence of the rise in the gross wage. Rewriting (10) slightly shows that the value of the firm J decreases in the gross wage, hence firms dislike the reform. Is this a strong disadvantage of our model given that employers generally were in favour of the reform in public discussions? We do not think so. The wage can rise or fall in our theoretical setup. Whether the worse outside option of employed workers reduces the wage more than the higher job-finding rate due to higher θ is theoretically an open question. Given our estimates, the "θ-effect" dominates. While firms urged the unemployed to search harder and the government to pay them lower benefits, they might not have seen the equilibrium effect that higher vacancies per unemployed worker finally makes workers stronger, wages rise and profits fall.

Given that both employed, short-term and long-term unemployed workers gain, it is then not surprising - despite the lack of statistical significance - that overall welfare as measured in (15) increases.

5.3 Understanding the effects of the reform

Generally speaking, we find very weak effects of the reform. This can be made plausible with a back-of-the-envelope calculation. As the unemployment rate is approx. 10% and only 1/3 becomes long-term unemployed, only 3.3% of the entire labour force are affected. Of these 3.3% only $\pi = 24.5\%$ pass the means test. In an intertemporal sense, income of the representative household is reduced only during $24.5\% * 3.3\% \approx 1\%$ of ones' lifetime. The duration of unemployment insurance payments is reduced by 10.7%, the level of the payments by 7%. Let this add up - to make this simple and high - to 18%. If 1% of lifetime income is reduced by 18%, overall lifetime income reduces by $1\% * 18\% \approx 0.2\%$. No surprise that quantitative effects are weak.

It is often argued that unemployment falls when there is economics growth. What is the contribution of growth to the reduction in unemployment in our setup? Real GDP in Germany grew by 0.8%, 3.0% and 2.5% in 2005, 2006 and 2007, respectively, i.e. by 6.4% over these three years. TFP grew by 4.0% (Ameco, 2010) in the same period and (annual averages of) unemployment in Germany fell from 10.5% to 9.0%. When the growth effect is taken into account by letting output A of a worker-firm match increase by 4.0% when evaluating the effects of the reform, the unemployment rate falls to 93.1% of its initial level (instead of 96.8% without TFP growth), as shown on the left-hand side of fig. 5. If we attribute the reduction to 96.8% to the reform and the remaining reduction (amounting to $96.8\% - 93.1\% = 3.7\%$) to economic growth, economic growth was slightly more successful in reducing unemployment than the reform.

The most surprising result of our analysis is that workers as a whole gain. This is surprising as one would expect that the insurance mechanism of unemployment benefit payments is reduced following the reduction in b_{UA}. Remember that in a world without moral hazard, unemployment benefits should equal the net wage. Given that a reduction in the length $\bar{s}$ and the level b_{UA} of unemployment benefits moves our economy further away from this setup, one should expect that the insurance mechanism is reduced. Why is it then that social welfare increases?

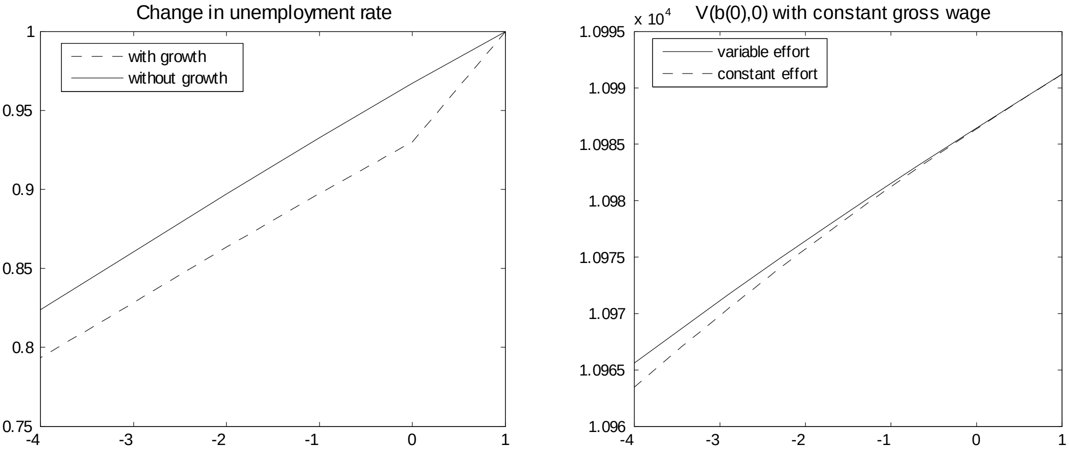

Figure 5 *The effect of economic growth on the change of the unemployment rate (left panel) and the pure insurance effect of the reform for short-term unemployed workers (right panel)*

To understand this, we divide the effects of the reform into two partial effects. The first partial effect is the insurance effect, i.e. the effect of the reform on social welfare when the gross wage is being held constant. What would happen if a reduction in b_{UA} implies a reduction in the tax κ and thereby an increase in the net wage w? The result of this thought-experiment is given in the right panel of fig. 5.

The horizontal axis plots the same Hartz-steps as in previous figures. We see (dashed line) that expected utility falls for a worker who just lost the job ($s = 0$) when effort is the same as in the pre-reform steady state (as in the lower left panel of fig. 2). The path of effort is therefore invariant to changes in benefit levels. This clearly reflects the fact that in a world with non-responding effort, unemployment benefits should equal the net wage. Any departure from this equality reduces expected utility. The same result holds when we allow effort to be a function of benefits, i.e. we allow incentives of the reform to play a role. The figure shows (solid line) that welfare decreases less (as incentives are now improved) but welfare still falls. The same qualitative figure holds for an employed worker and for unemployed workers for any s.

As we know that welfare rises in equilibrium, we can conclude that all beneficial effects of the reform come from the second partial effect, i.e. the increase in the wage due to the higher number of vacancies per unemployed worker. This stresses the importance of two points:

Academically speaking, an evaluation of labour market reforms should be undertaken in an equilibrium setup to provide a complete picture of the effects. More importantly, the reform *de*creased the insurance mechanism implied by unemployment benefits but - in the end - this was beneficial to the average worker.

One argument against our analysis would state that real hourly wages in Germany fell from 2004 onwards (see e.g. Brenke, 2009, fig. 2). While we admit this fact, it apparently must be caused by factors other than the reform which we analyse here. In order to check our statement, look at θ, the ratio of vacancies to the number of unemployed 3 years before and after the reform. According to fig. 3, it rises due to the reform.

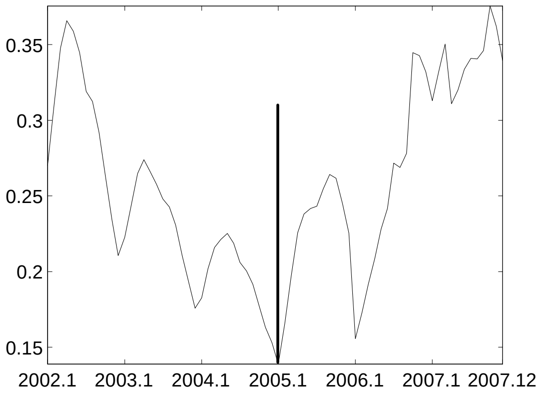

Figure 6 *Monthly vacancies per unemployed worker three years before and after the reform (Bundesagentur für Arbeit, 2010)*

As fig. 6 shows, vacancies per unemployed workers indeed increased after the reform, as predicted by our model.[26] Concerning wages, we conclude that there must have been some other factor independent of the reform which caused wages to decline. Hours worked - having increased by more than 4% between 2005 and 2007 - is one of the candidates.

6 Conclusion

Our project started by inquiring about the effects of the Hartz IV labour market reform on incentives and insurance mechanisms for the workforce. At the macro level, we investigate the effects on the unemployment rate and social welfare. We have developed an estimable search and matching model with endogenous effort under time-dependent unemployment benefits. The main extension compared to the existing search and matching literature is the endogenous distribution of unemployment duration that arises due to individual choice of search intensity in a nonstationary environment. A link between these micro-dynamics and macro quantities like the unemployment rate was developed using tools from the literature on Semi-Markov processes. The theoretical model provides the density of unemployment

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[26] We admit that the quantitative increase in the data is much larger than predicted by our model. We leave estimation of parameters using both micro and macro data for future research.

duration of an individual being a function of various model parameters. This density provides the basis for structural estimation via maximum likelihood. Equilibrium policy analyses were performed using the parameter estimates of the best fitting specification.

We find that the unemployment rate did decrease due to the reform. Unemployed workers have stronger incentives to search hard. The reduction of the unemployment rate was quantitatively very small, however. While statistically significant, the reduction amounts to 0.3% only. Concerning distributional effects, we find that employed workers and the short-term unemployed win. These results are not statistically significant, however. We also find - much to our own surprise and in contrast to the perception of the public - that even long-term unemployed workers win. Social welfare therefore increases! According to this setup, labour market reforms of this type are Pareto-improving.

Looking into the mechanisms in more detail, we find that the gain in social welfare is not due to an improvement in the insurance mechanism. Keeping the gross wage constant, the Hartz IV reform clearly reduced expected utility of a worker. As the reform did have equilibrium effects as well which made the wage *in*crease, expected utility of a worker *in*creased. Summarizing, while workers need to search harder, this higher search effort induces firms to open new vacancies. Firms open so many of them that the net and gross wage, overall utility for workers and social welfare increases.

These findings clearly give rise to new research questions of which one of the more pressing ones concerns the issue of ex-ante identical individuals. What if we had ex-ante heterogeneity in skill levels? One would expect that social welfare gains would not be as large. Long-term unemployed close to retirement age or with little educational background have low exit rates out of unemployment and they might lose from such a reform. Heterogeneity in separation rates could also be taken into account. With ex-ante heterogeneity, could a reform still be designed such that it is Pareto-improving? Further, how robust are the findings if individuals were allowed to self-insure through savings? Consumption drops would be smoother and welfare gains could be even larger than what was found here. If, however, low-income groups hardly hold any wealth, this would not be true for this group. Finally and maybe most importantly, the wage setting mechanism needs more empirical investigation. Given that our results are driven by the positive wage effect, one should inquire into the empirical plausibility of this channel. This can be done by using matched employer-employee data with reliable firm productivity measures and by adding macro channels which might also influence wage setting. Exploring this in more detail is left for future research.

A Appendix

A.1 Data

The data comes from the German Socio-Economic Panel (SOEP). The SOEP is a panel surveying households on an annual basis. The survey is coordinated by the Deutsche Institut für Wirtschaftsforschung (Berlin, see www.gsoep.de).

We draw a flow sample of entrants to employment and unemployment at each month of years 1997-98. The choice of the year of sampling is determined by the fact that no changes to either benefit levels or the entitlement length were made between the 1 January 1997 and 1 January 2005, when the Hartz IV reform was implemented. With December 2004 being the latest month of our observation period, we end up with a sample that describes a stationary entitlement-benefit environment and provides a fairly reliable information on long-term unemployment (only 9.12% of unemployment durations in our sample are right-censored). For each entrant we retrieve the duration of stay in the current state since the moment of entry. Following van den Berg and Ridder (1998, p.1194), we exclude individuals with transitions to states other than full-time employment and unemployment.

Units of measurement are months for the duration data and real EUR (based on 2004) for the wage and benefits data. Wage is the average monthly wage for the months of employment within a year prior to job loss, as these are the wage bases that conform with the observed benefit levels. Descriptive statistics can be found in tab. 3.

GSOEP does not contain information on the length of entitlement to UI benefits. Using statutory rules, however, allows computing the length of entitlement once we know the length of previous job durations and the age of an individual (see e.g. www.oecd.org/dataoecd/9/54/29730499.pdf). For this reason, for every person that enters unemployment we also have to retrieve her previous job history. In addition to that, previous job history provides us with the record of the latest wages earned.

The mean of the vacancy-unemployment ratio θ between 1997 and 1999 in Germany is 0.24 (data of the Institute for Employment Research, IAB, adjusted for underreporting). In the estimation we use a common measure for θ refraining from the variation across local labour markets. The reason is that making use of such variation will require introduction of market-specific fixed effects to separate the tightness effect from the rest of the local-market conditions. This will render the estimation computationally unfeasible.

	Unemployment			**Employment**			**Sample characteristics**	
	Mean	StD		Mean	StD		Mean	StD
Duration (s)	8.81	13.16	Duration (l),			Males	0.5380	0.4988
UI benefits (b_{UI})	727.46	294.94	censored	57.55	25.73	East Germans	0.4227	0.4942
Entitlement ($\bar{s}$)	12.18	5.48	Duration (l),			Medium-skill	0.5961	0.4909
Wage (w)	1166.26	538.07	all sample	40.68	30.13	High-skill	0.2090	0.4068
Share of entitled						Age up to 30	0.3843	0.4866
to UI (ω)	0.5657	0.4963	obs: total/cens.	694 / 392		Age 31 to 45	0.4264	0.4948
Share of $\bar{s} = 12$								
among entitled	0.4882	0.5010				obs: total		1067
Observed share of								
passing the test	0.2850	0.4525						
obs: total/cens.	373 / 34							

Table 3 *Descriptive statistics (months and EUR)*

Finally, in the predictions for the pre-reform steady state we use the average observed characteristics of the individuals sampled immediately before the reform, i.e. in 2004. For that we similarly draw a flow sample with entry between 01.2004-12.2004 and record personal characteristics. Descriptive statistics for these characteristics are shown in tab. 4.

	Sample characteristics	
	Mean	StD
Males	0.4957	0.5005
East Germans	0.3848	0.4870
Medium-skill	0.5109	0.5004
High-skill	0.2565	0.4372
Age up to 30	0.3413	0.4747
Age 31 to 45	0.4152	0.4933
obs: total		460

Table 4 *Descriptive statistics: Individual characteristics in 2004*

A.2 Steady state solution

We solve for the steady state of the model by separating the model into two "blocks".

Block 1 describes the behaviour of households. Given the functional forms for utility and the spell-effect in (20) and (22), the first-order condition for effort (8) reads

$$\phi(s) = \{\alpha\eta(s)\,\theta^{\alpha}\,[V(w) - V(b(s), s)]\}^{\frac{1}{1-\alpha}}\,. \tag{A.1}$$

It holds for both short- and long-term unemployed. Plugging this into the Bellman equation for the unemployed (7), using (22) and expressing it as a differential equation in s gives (see app. B.2.1)

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma} - 1}{1-\sigma} - \frac{1-\alpha}{\alpha}\,[\alpha\eta(s)\,\theta^{\alpha}]^{\frac{1}{1-\alpha}}\,[V(w) - V(b(s), s)]^{\frac{1}{1-\alpha}}\,, \tag{A.2}$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed is identical, we impose $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$ from (9) when solving this differential equation. Finally, since for an infinite unemployment spell, the spell-effect in (21) with (25) becomes a constant, $\lim_{s\to\infty} \eta(s) = \eta_0$ and all other quantities are stationary as well, we get the terminal condition for (A.2) by using $\lim_{s\to\infty} \dot{V}(b_{UA}, s) = 0$,

$$\rho V(b_{UA}) = \frac{b_{UA}^{1-\sigma} - 1}{1-\sigma} + \frac{1-\alpha}{\alpha}\,[\alpha\eta_0\theta^{\alpha}]^{\frac{1}{1-\alpha}}\,[V(w) - V(b_{UA})]^{\frac{1}{1-\alpha}}\,. \tag{A.3}$$

The Bellman equation for the employed worker (6) can be written with the explicit utility function as

$$V(w) = \frac{1}{\rho+\lambda}\left(\frac{w^{1-\sigma} - 1}{1-\sigma} + \lambda V(b_{UI}, 0)\right). \tag{A.4}$$

Now imagine we insert $V(w)$ from (A.4) into (A.2) and (A.3). Imagine further that we know all parameters and assume, for the time being, some values for w and θ. Then we can solve the differential equation (A.2) starting from some initial value $V(b_{UI}, 0)$ and see whether the solution for $s \rightarrow \infty$ is identical to $V(b_{UA})$ from (A.3). If it does not, we need to adjust our initial guess $V(b_{UI}, 0)$ until it does. Hence, with some exogenous w and θ, we have obtained the time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell-path of the value of being unemployed, $V(b(s), s)$, and the value of a job $V(w)$.

Given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ as a function of w and θ, block 2 allows to endogenize w and θ.

The Bellman equation for the firm and the free entry result, (10) and (12), gives us

$$\frac{A - \frac{w}{1-\kappa}}{\rho + \lambda} = \gamma \frac{\theta}{\bar{\mu}}. \tag{A.5}$$

The bargaining equation (13) is copied here for convenience,

$$(1-\beta) u(w) + \beta m_w(.) w = (1-\beta) u(b_{UI}, \phi(0)) + \beta (1-\kappa) m_w(.) \left[A + \gamma \theta \frac{\mu(\phi(0), 0)}{\bar{\mu}} \right], \tag{A.6}$$

where $m_w(.)$ is from (14) and $\phi(0)$ is the optimal search effort at the instant of entry into unemployment, which is given from (A.1). Equations (A.5) and (A.6) use the average exit rate $\bar{\mu}$ and the tax rate κ.

The average rate $\bar{\mu}$ is given by (11) which can easily be computed given that, after having solved block 1, the exit rates $\mu(.)$ are known from (22) and the density $f(s)$ can therefore be computed from (4). The tax rate κ makes the government budget constraint (5) hold and solves

$$b_{UI} U_{short} + b_{UA} U_{long} = \kappa \frac{w}{1-\kappa} L. \tag{A.7}$$

Given the density $f(s)$, one can compute the number of short-term and long-term unemployed on the right-hand side of this expression from $U_{short} = (N - L) \int_0^{\bar{s}} f(s) \, ds$ and $U_{long} = N - L - U_{short}$. The number of unemployed $N - L$ in turn follows from (19), using (16a,b) and (17), given again that exit rates are known from block 1.

Hence, we are basically left with (A.5) and (A.6) to determine the missing endogenous variables w and θ. After having solved block 1 with a guess of w and θ, we verify whether this guess fulfills (A.5) and (A.6). If not, the guess is adjusted until a solution is found.

A.3 Transition rates to employment

Transition rates to employment are fully described by the optimal search effort. Using first order condition for search effort (8) and the definition of the exit rate (22), we can therefore express value of unemployment as a function of the optimal exit rate. Inserting this value of unemployment into the Bellman equation for the unemployed (7) and expressing it as a differential equation in s, we obtain the time path of the optimal exit transition rate to employment as a result (see app. B.3.1 for a derivation). Omitting observed and unobserved characteristics for brevity, the differential equation that describes the exit rate out of unemployment both for the short-term and the long-term unemployed workers is

$$\begin{aligned}\dot{\mu}_j(s) &= \alpha\left[\mu_j(s)\right]^2 + \left(\frac{\dot{\eta}(s)}{\alpha\eta(s)} + \rho\right)\frac{\alpha}{1-\alpha}\mu_j(s) \\ &\quad - \frac{\alpha^2}{1-\alpha}\left[\eta(s)\,\theta^\alpha\right]^{\frac{1}{\alpha}}\left[\mu_j(s)\right]^{2-\frac{1}{\alpha}}\left[\rho V(w) - \frac{b_j^{1-\sigma}-1}{1-\sigma}\right], \qquad \text{(A.8)}\end{aligned}$$

where $j = UI, UA$ as in (23). App. B.3.1 further shows that the relevant endpoint condition for the second regime at $s \to \infty$ is

$$(1-\alpha)\,\mu_{UA} - \alpha\left[\eta_0\theta^\alpha\right]^{\frac{1}{\alpha}}\left[\mu_{UA}\right]^{1-\frac{1}{\alpha}}\left[\rho V(w) - \frac{b_{UA}^{1-\sigma}-1}{1-\sigma}\right] + \rho = 0 \qquad \text{(A.9)}$$

and that the condition for the first regime at $s = \bar{s}$ reads

$$\mu_{UI}(\bar{s}; b_{UI}) = \mu_{UA}(\bar{s}; b_{UA}). \qquad \text{(A.10)}$$

Once wages and market tightness are observed from the data, $V(w)$ can be obtained from the solution to block 1 in app. A.2. Consequently, under the assumption that observed w and θ are the direct results of the solution to block 2 in app. A.2, we can compute the equilibrium exit rates out of unemployment without requiring employer-side data.

A.4 Estimated and predicted survivor functions

The following figures show the predicted survivor functions (solid lines) for heterogeneous population groups joint with the Kaplan-Meier survivor probabilities (circles). Corresponding 95% confidence intervals are depicted by triangles. For a discussion, see towards the end of sect. 4.3.

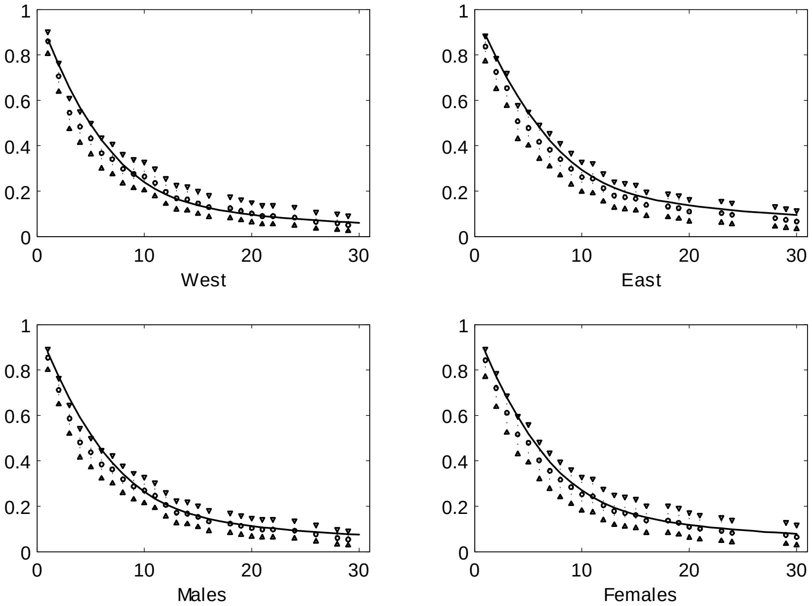

Figure 7 *Kaplan-Meier and predicted survivor functions*

A.5 A Semi-Markov process

This section provides a short introduction into Semi-Markov processes. Technically, it follows Kulkarni (1995) and Corradi et al. (2004). The original work is by Pyke (1961a,b). Due to their technical nature, these papers are less accessible and we hope that this appendix helps that these very useful methods become more widely used. To the best of our knowledge, this is the first application of these concepts in economics. For more details and the numerical implementation, see Schumm (2010, ch. 4). The first subsection describes the general approach to Semi-Markov processes while the second adapts it to our question.

A.5.1 The general approach

Let Y_n denote the state of a system after the nth transition. Let this state be i. Let the point in time of the nth transition be denoted by S_n. Define the probability that the system after the next transition is in j and that this transition takes place within a period of length x or shorter, conditional on the system being in i after the nth transition, as

$$Q_{ij}\left(x\right) \equiv P\left\{ Y_{n+1} = j, S_{n+1} - S_n \leq x \middle| Y_n = i \right\}.$$

The probability that any transition takes place is then given by summing up the probabilities for each j, $Q_i\left(x\right) = \Sigma_{j \neq i} Q_{ij}\left(x\right)$, not taking into account transitions from i to i.[27] The probability that the system will be in j in τ is given by

$$p_{ij}\left(\tau\right) = \left(1 - Q_i\left(\tau\right)\right)\delta_{ij} + \Sigma_{k \neq i} \int_0^{\tau} p_{kj}\left(\tau - x\right) dQ_{ik}\left(x\right). \tag{A.11}$$

The interpretation of this integral equation is as follows: the first part of the right hand side gives the probability that the system, being currently in state i, never leaves state i until τ. In this case $j = i$ and $\delta_{ij} = 1$, so $1 - Q_i\left(\tau\right)$ is the survival probability in state i. If $j \neq i$, $\delta_{ij} = 0$. The second part of the right hand side collects all cases in which the transition from i to j (which includes i) occurred via another state $k \neq i$. First, we take the probability that the process stayed in state i for a period of length x and passed to state k then (captured by $Q_{ik}\left(x\right)$). Then we need the probability that the process which is in state k after x will be in state j at τ (captured by $p_{kj}\left(\tau - x\right)$). As the transition from i to k can be anywhere between 0 and τ, we have to integrate over x in order to cover all possible transitions.

Equation (A.11) can slightly be rewritten, provided that $Q_{ik}\left(x\right)$ is once differentiable (which holds for our case), as

$$p_{ij}\left(\tau\right) = \left(1 - Q_i\left(\tau\right)\right)\delta_{ij} + \Sigma_{k \neq i} \int_0^{\tau} p_{kj}\left(\tau - x\right) \frac{dQ_{ik}\left(x\right)}{dx} dx. \tag{A.12}$$

The derivative $dQ_{ik}\left(x\right)/dx$ now gives the density of going from i to k after duration x. Multiplied by the probability $p_{kj}\left(\tau - x\right)$ of subsequently going from k to j (which may

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[27] We differ from the notation in the cited literature in that we explicitly write $j \neq i$ here or $k \neq i$ below. This is equivalent to setting the transition rate from i to i to zero. As our application does not have transitions from i to i (i.e. transition rates from i to i are zero), we find using $j \neq i$ explicitly more intuitive for our purpose. We are indebted to Ludwig Fahrmeir for various communications on Semi-Markov processes. For an excellent introduction in German, see Fahrmeir et al. (1981).

include many intermediate transitions to other states) gives the density of ending up in j after having gone to k after x. Integrating over all durations x gives the probability of starting in i and being in j at τ.

A.5.2 Our two-state system

We now need to adjust the notation such that it suits our purposes. We look at a worker who just moved in t (like today) into either employment e or unemployment u. Define $Q_{eu}(\tau)$ as the probability that a worker who just found a job in t "jumps" to u in a period of time shorter or equal to $\tau - t$. With a duration s dependent arrival rate $\lambda(s(v))$, this is then simply given by

$$Q_{eu}(\tau) = 1 - e^{-\int_t^\tau \lambda(s(v))dv}, \tag{A.13}$$

where $s(v) = v - t$ is the duration in her current state. In perfect analogy and using a spell-dependent arrival rate $\mu(s(v))$, we get $Q_{ue}(\tau) = 1 - e^{-\int_t^\tau \mu(s(v))dv}$. For the complementary events - remaining in a given state - the probabilities are simply $Q_{ee}(\tau) = 1 - Q_{eu}(\tau)$ and $Q_{uu}(\tau) = 1 - Q_{ue}(\tau)$. The probabilities that a transition takes place at all in this two state process are

$$Q_e(\tau) \equiv Q_{eu}(\tau), \; Q_u(\tau) \equiv Q_{ue}(\tau). \tag{A.14}$$

With two possible states, we have four transition probabilities for the future: an unemployed (employed) person can either be unemployed or employed at some future point in time τ. Two are redundant as the probability of e.g. an unemployed worker of being employed is complementary to the probability of being unemployed, $p_{ue}(\tau) = 1 - p_{uu}(\tau)$, and similarly $p_{ee}(\tau) = 1 - p_{eu}(\tau)$. Hence, we only focus on $p_{uu}(\tau)$ and $p_{eu}(\tau)$. These probabilities are, using the general equation (A.12),

$$p_{uu}(\tau) = 1 - Q_u(\tau) + \int_t^\tau p_{eu}(\tau - v)\frac{dQ_{ue}(v)}{dv}dv, \tag{A.15a}$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v)\frac{dQ_{eu}(v)}{dv}dv. \tag{A.15b}$$

These equations can be most easily be understood by looking at the following figure.

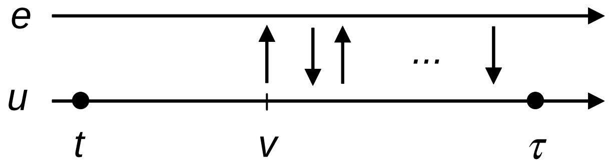

Figure 8 *Illustrating transition probabilities*

Let's consider $p_{uu}(\tau)$: An individual unemployed in t can be unemployed in τ by always remaining unemployed. This is the term $1 - Q_u(\tau)$. The individual can be unemployed in τ by remaining unemployed until v where she jumps into employment, the density for which is $dQ_{ue}(v)/dv$. After v, the probability of returning to unemployment in the remaining time span of $\tau - v$ is $p_{eu}(\tau - v)$. Note that this probability includes an arbitrary number of transitions larger than zero in this remaining period $\tau - v$. In contrast to integrating over x as in (A.12), we integrate over the point in time v here simply as this is more intuitive. The interpretation for $p_{eu}(\tau)$ is in perfect analogy.

As a last step, we need to determine the two derivatives $dQ_{ue}(v)/dv$ and $dQ_{eu}(v)/dv$. Given duration-dependent arrival rates, the derivatives of (A.13) are,

$$\frac{dQ_{ue}(v)}{dv} = e^{-\int_t^v \mu(s(y))dy} \frac{d}{dv} \int_t^v \mu(s(y))\, dy = e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \tag{A.16a}$$

$$\frac{dQ_{eu}(v)}{dv} = e^{-\int_t^v \lambda(s(y))dy} \frac{d}{dv} \int_t^v \lambda(s(y))\, dy = e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)). \tag{A.16b}$$

Given (A.14) and the derivatives, the equations (A.15) become

$$p_{uu}(\tau) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau p_{eu}(\tau - v)\, e^{-\int_t^v \mu(s(y))dy} \mu(s(v))\, dv,$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v)\, e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v))\, dv.$$

The final adjustment we need to make is to replace $\lambda(s(v))$ by λ as the separation rate is assumed to be constant. This then gives equations (16) in the main text.

B Appendix

All references to appendices starting with B are available upon request.

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